Title: Quantum Simulations

Date: Jul 25, 2016 02:30 PM

URL: http://pirsa.org/16070073

Abstract:

Simulating Thermal states

Ham $H = \sum_k E_k |\psi_k\rangle\langle\psi_k|$

Temp T

task: i) prepare $\rho_T = \dfrac{e^{-H/T}}{Z}$

ii) $\mathrm{tr}(A\rho_T)$ obs. $\underline{A}$

Simulating Thermal states

Ham $H = \sum_k E_k |\psi_k\rangle\langle\psi_k|$

Temp T

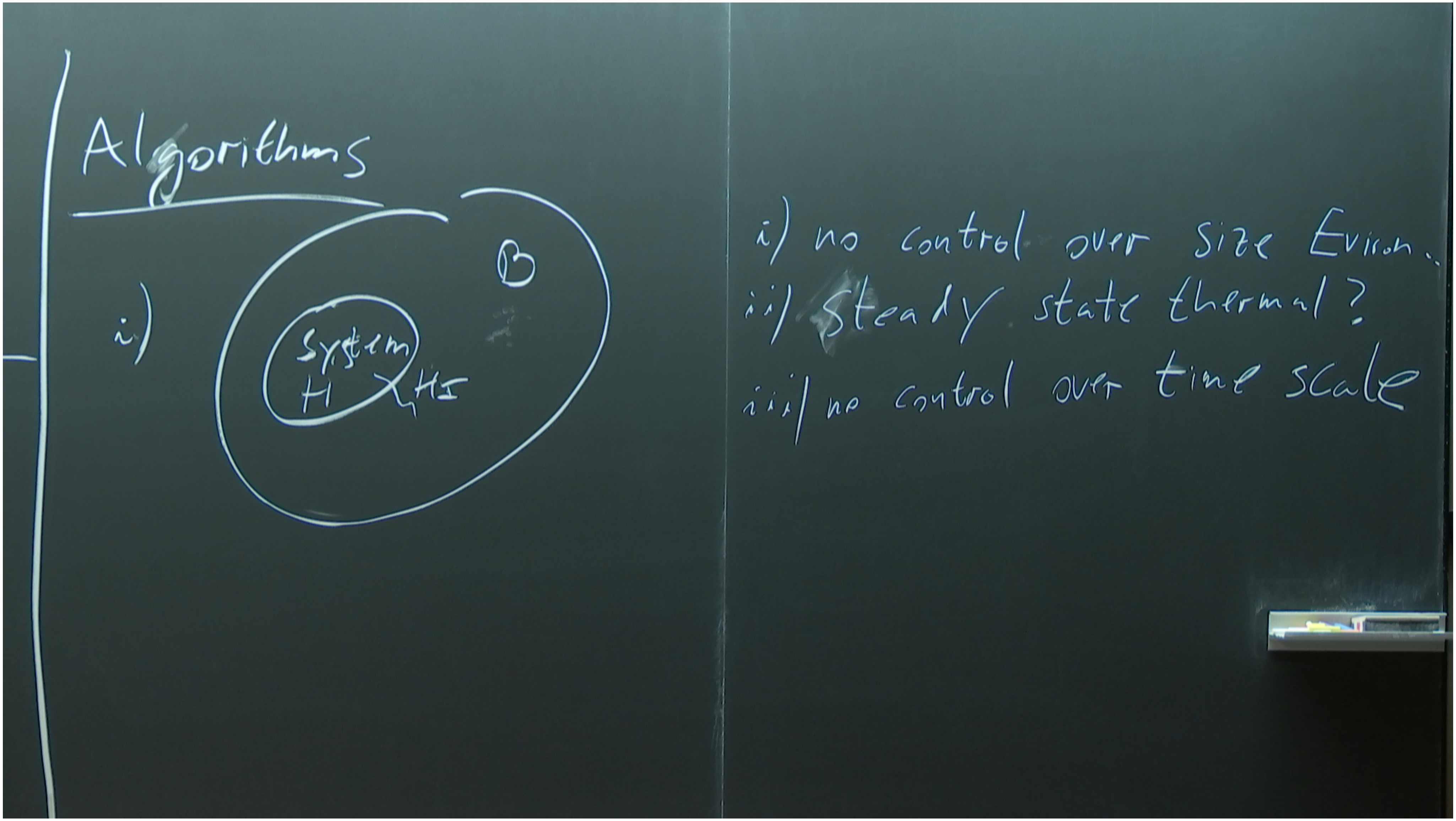
task: i) prepare $\rho_T = \frac{e^{-H/T}}{Z}$

ii) $\mathrm{tr}(A\rho_T)$ obs. $\underline{A}$

No

No-go: No general efficient alg for (poly(n, 1/error)) Gibbs state preparation @ Const T

(unless NP $\subseteq$ BQP; spin glasses)


Algorithms
i)
System
H
H_I
B
i) no control over size Eviron..
ii) Steady state thermal?
iii) no control over time scale

Monte Carlo Methods

(e.g. Metropilis)

Classical: $H = J\sum_{\langle i,j\rangle} Z_i Z_j - h\sum_i Z_i$

$\underbrace{|0,1,0,\ldots,1\rangle}_{n}$

Markov chain on configurations

steady distribution

$p(x) = \frac{e^{-E_x/T}}{Z}$

tions
tribution
x/T

$$\Pr\big(\overbrace{(x_1 \ldots x_i \ldots x_n)}^{x} \to \overbrace{(x_1 \ldots \bar{x}_i \ldots x_n)}^{x'}\big)$$

$$= \frac{1}{n} \max\left(1, e^{-(E_{x'} - E_x)/T} / Z\right)$$

How quickly it converges?

T_c finite correlation

0 $\exp(c\sqrt{n})$ T

Fast convergence
$(O(n\log(n)))$

Algo

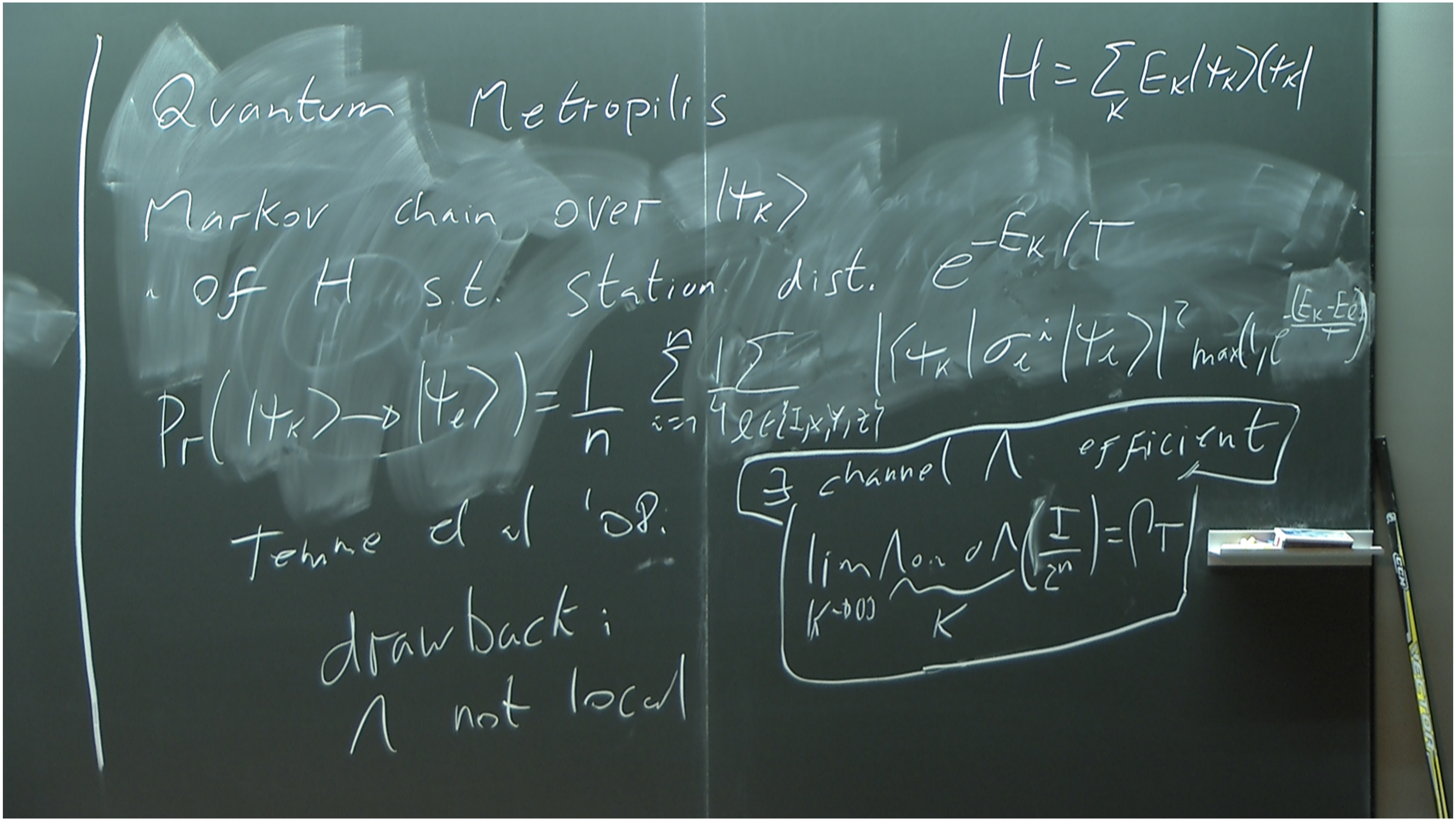
i)

Quantum Metropilis

$$H = \sum_k E_k |\psi_k\rangle\langle\psi_k|$$

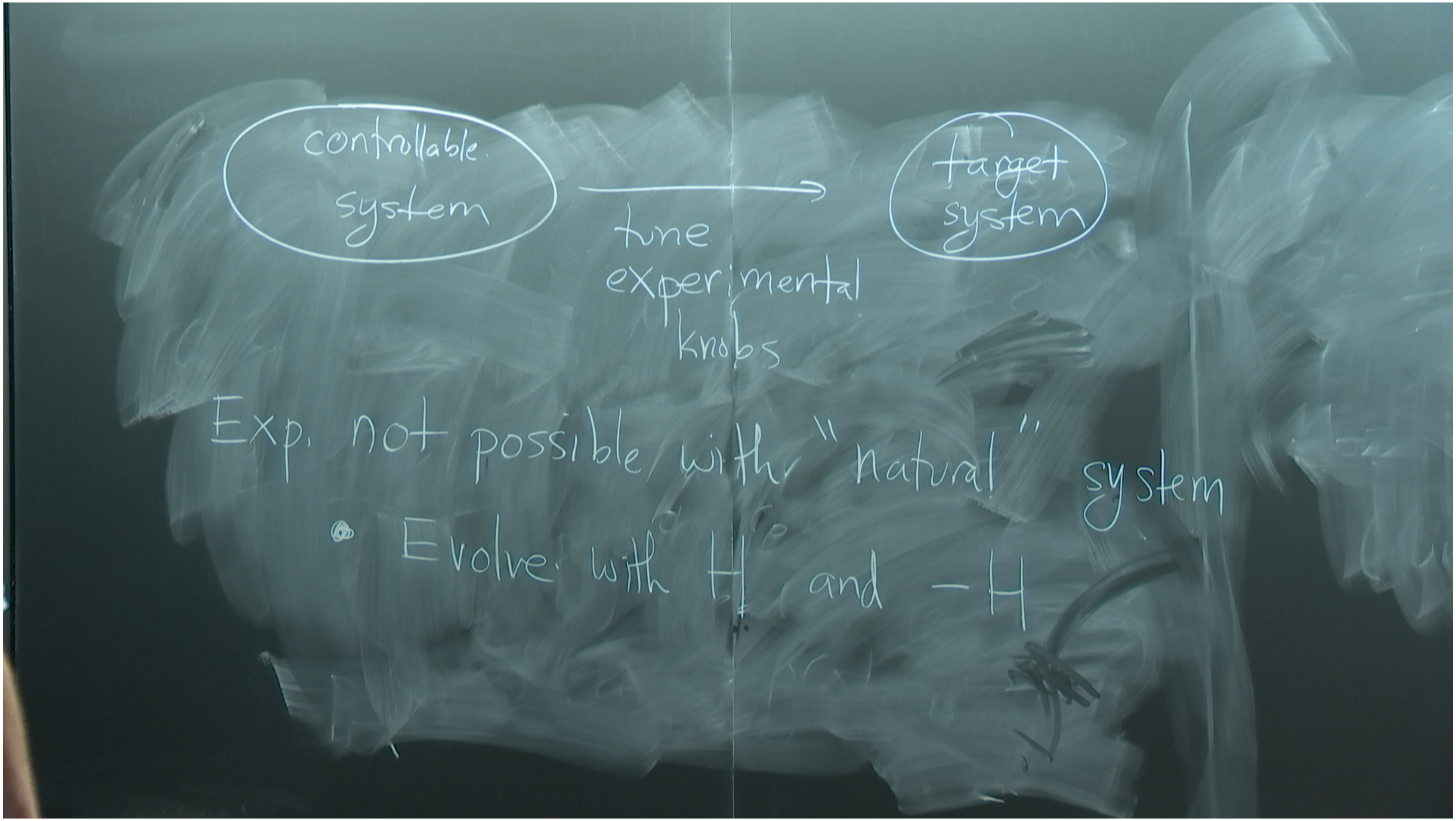
Markov chain over $|\psi_k\rangle$ of H s.t. station. dist. $e^{-E_k/T}$

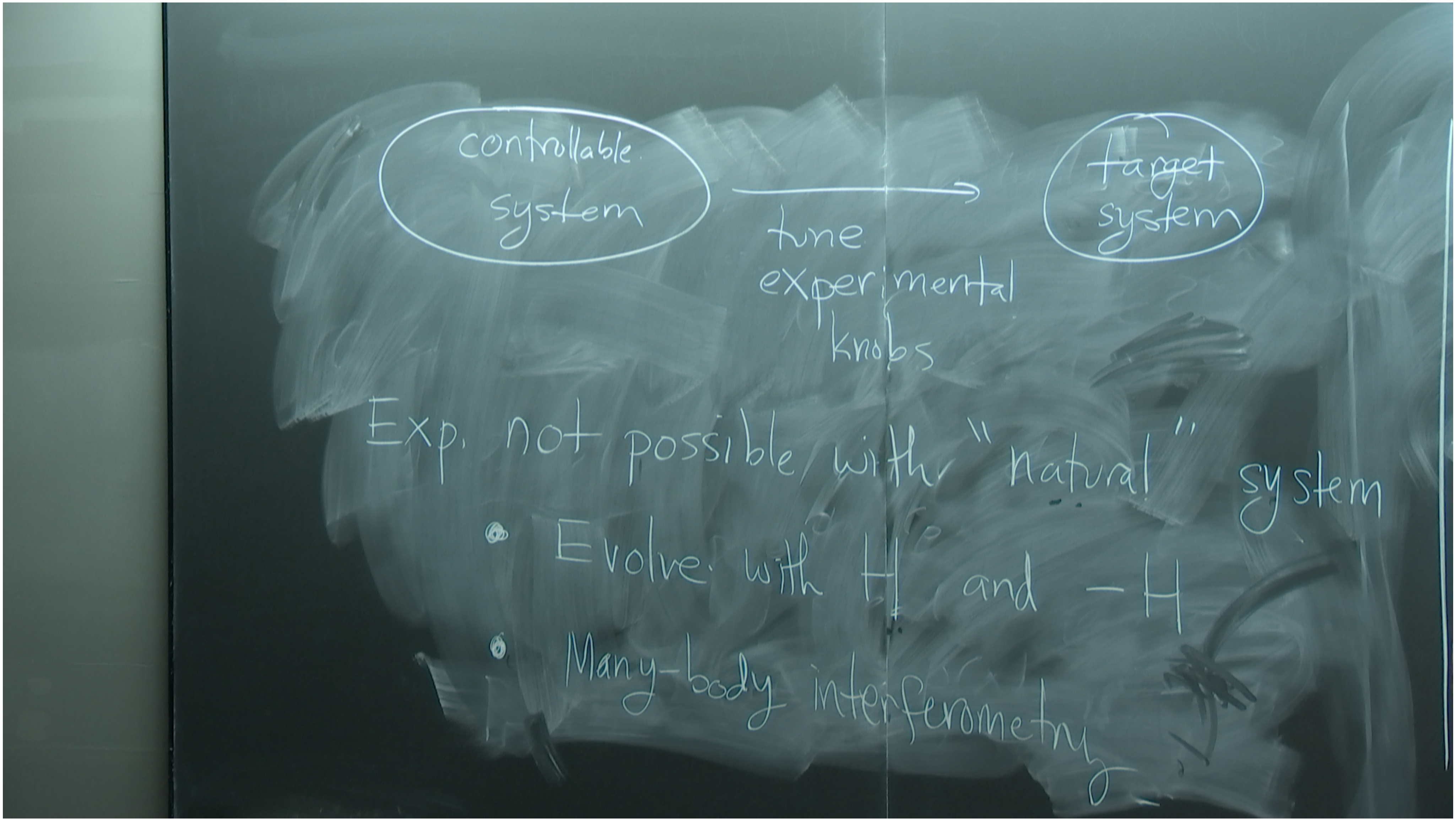
$$\Pr(|\psi_k\rangle \to |\psi_\ell\rangle) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{4}\sum_{\ell\in\{I,x,y,z\}} |\langle\psi_k|\sigma_\ell^i|\psi_\ell\rangle|^2 \max\left(1, e^{-\frac{(E_k-E_\ell)}{T}}\right)$$

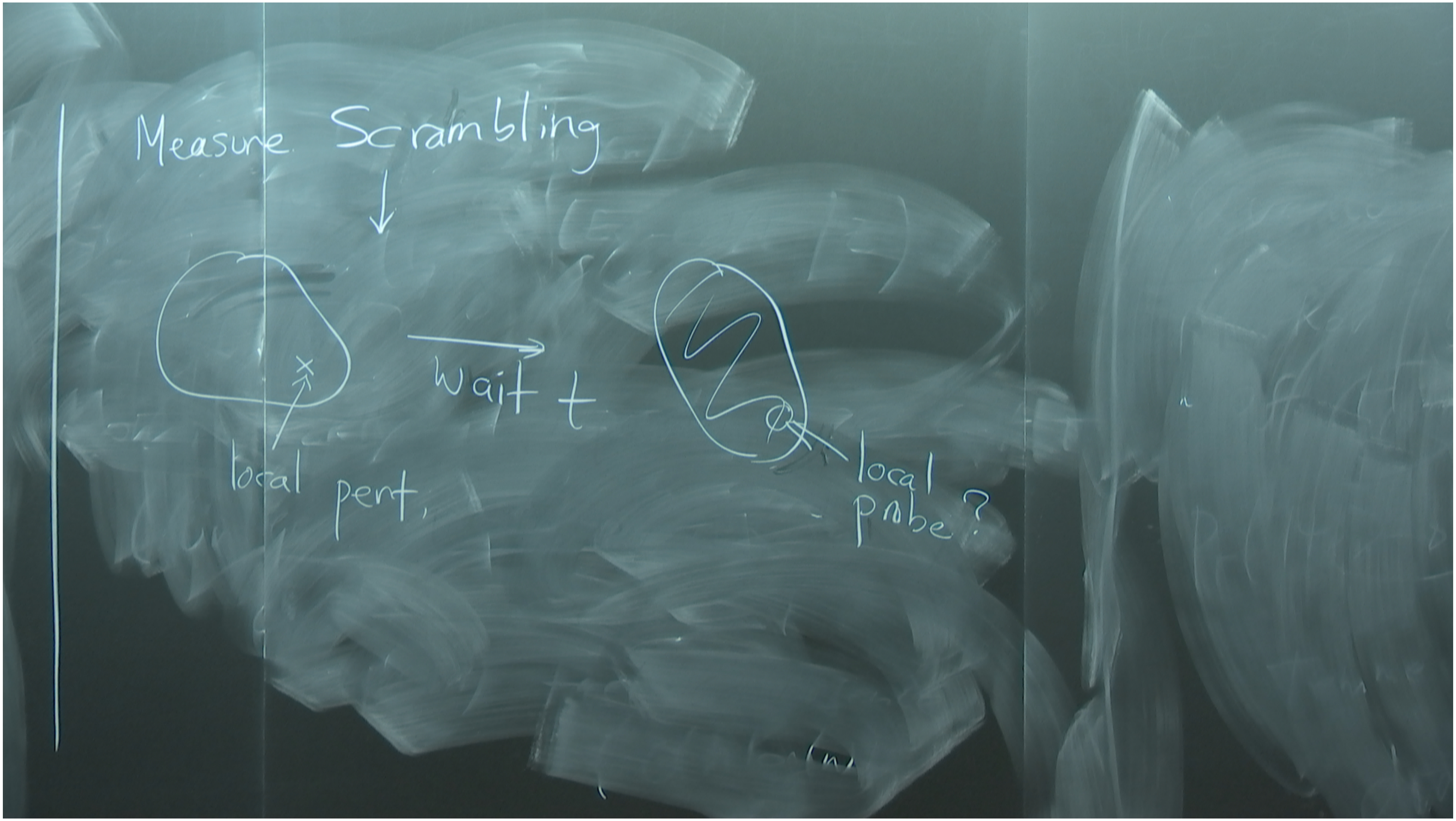

Quantum Metropilis
$H=\sum_K E_K|\psi_K\rangle\langle\psi_K|$
Markov chain over $|\psi_K\rangle$ of H s.t. station. dist. $e^{-E_K/T}$
$\Pr(|\psi_K\rangle \to |\psi_\ell\rangle) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{4} \sum_{\ell\in\{I,x,y,z\}} |\langle\psi_K|\sigma_\ell^i|\psi_\ell\rangle|^2 \max(1, e^{-\frac{(E_K-E_\ell)}{T}})$
Temme et al '08:
drawback: Λ not local
$\exists$ channel Λ efficient
$\lim_{K\to\infty} \underbrace{\Lambda\circ\cdots\circ\Lambda}_{K}\left(\frac{I}{2^n}\right) = \rho_T$

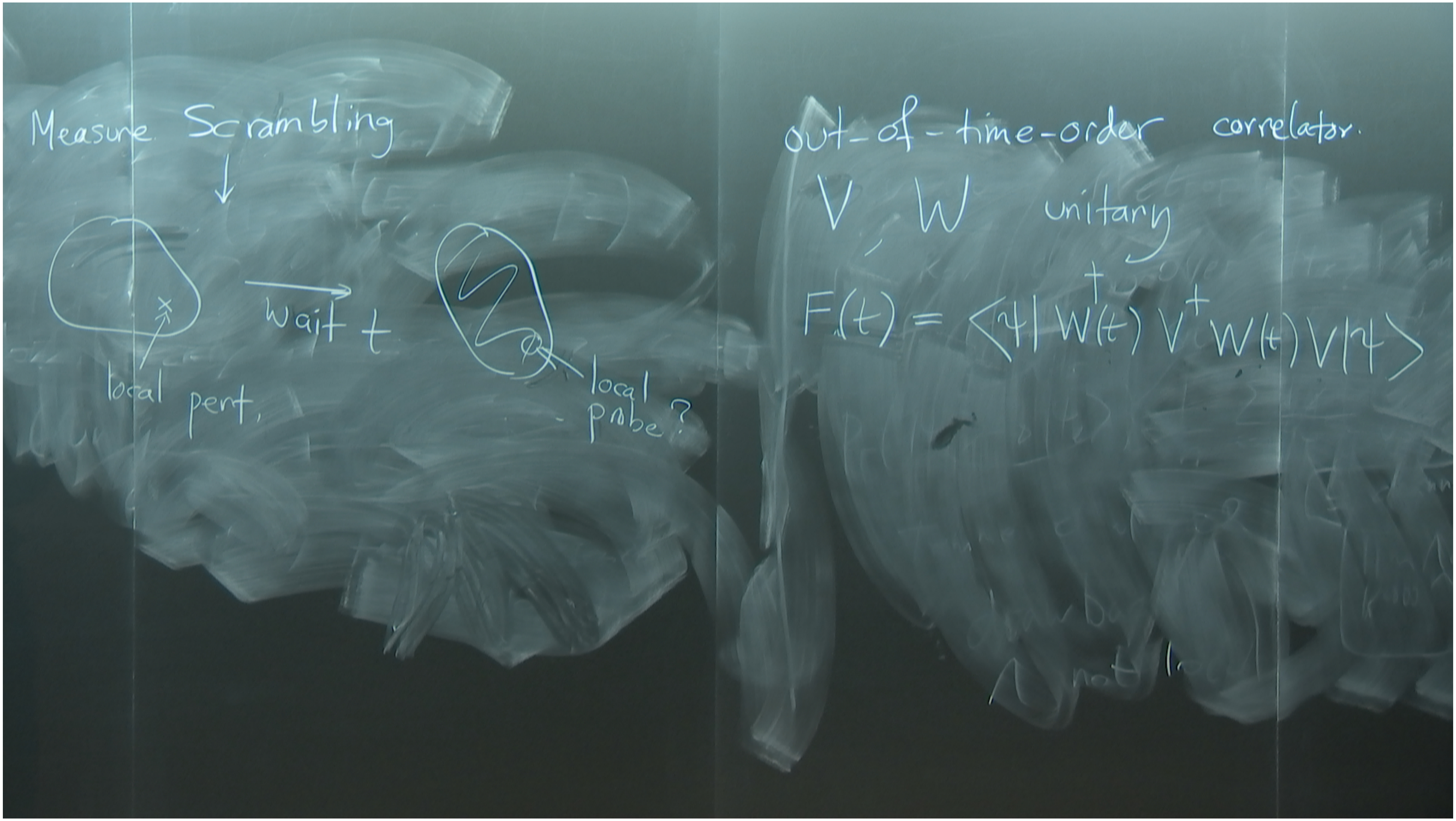
Q1. "local quantum metropolis" alg?

Q2. understand convergence?

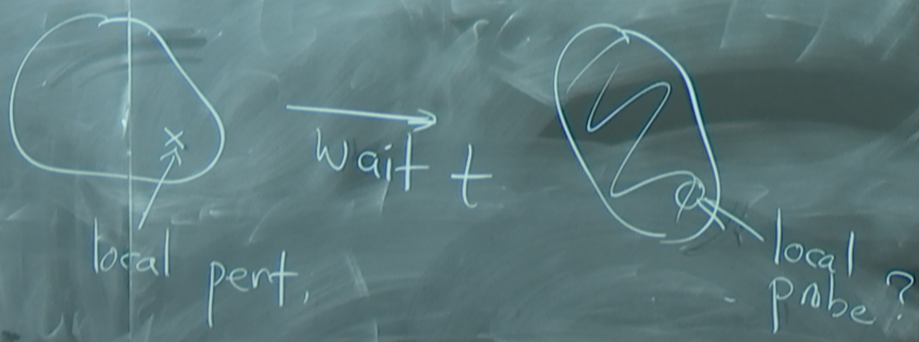

controllable system
tune experimental knobs
target system
Exp. not possible with "natural" system
• Evolve with H and −H


controllable system
target system
tune experimental knobs
Exp. not possible with "natural" system
Evolve with H and −H
Many-body interferometry


Measure Scrambling
wait t
local pert.
local
probe ?


Measure Scrambling
wait t
local pert.
local probe?
out-of-time-order correlator.
V, W unitary
$F(t) = \langle \psi | W^\dagger(t) V^\dagger W(t) V | \psi \rangle$

Measure Scrambling



out-of-time-order correlator.

V, W unitary

$$F(t) = \langle \psi | W^\dagger(t) V^\dagger W(t) V | \psi \rangle$$

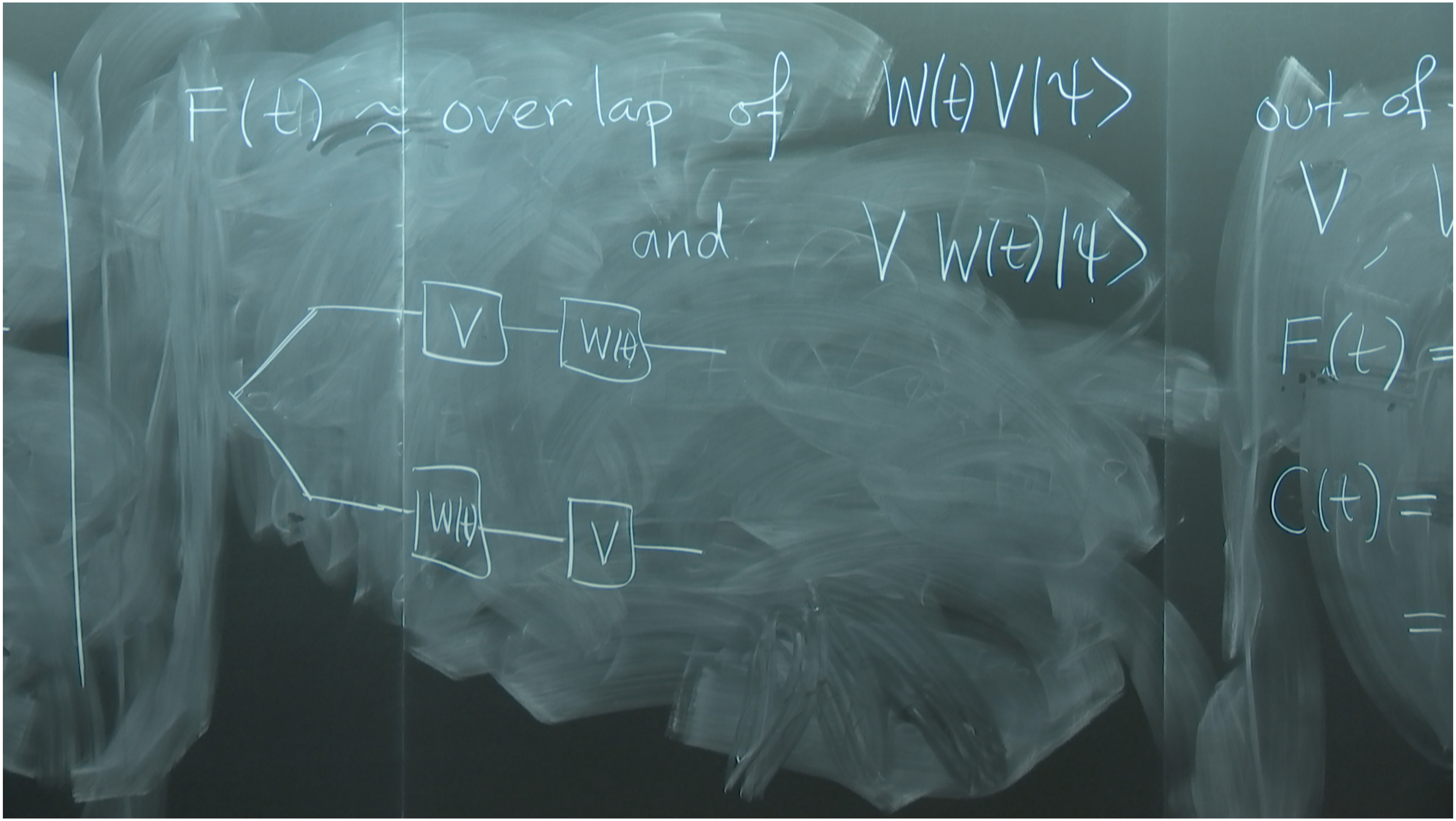
$$C(t) = \langle \psi | [W(t), V]^\dagger [W(t), V] | \psi \rangle$$
$$= 2 - 2\,\mathrm{Re}[F]$$

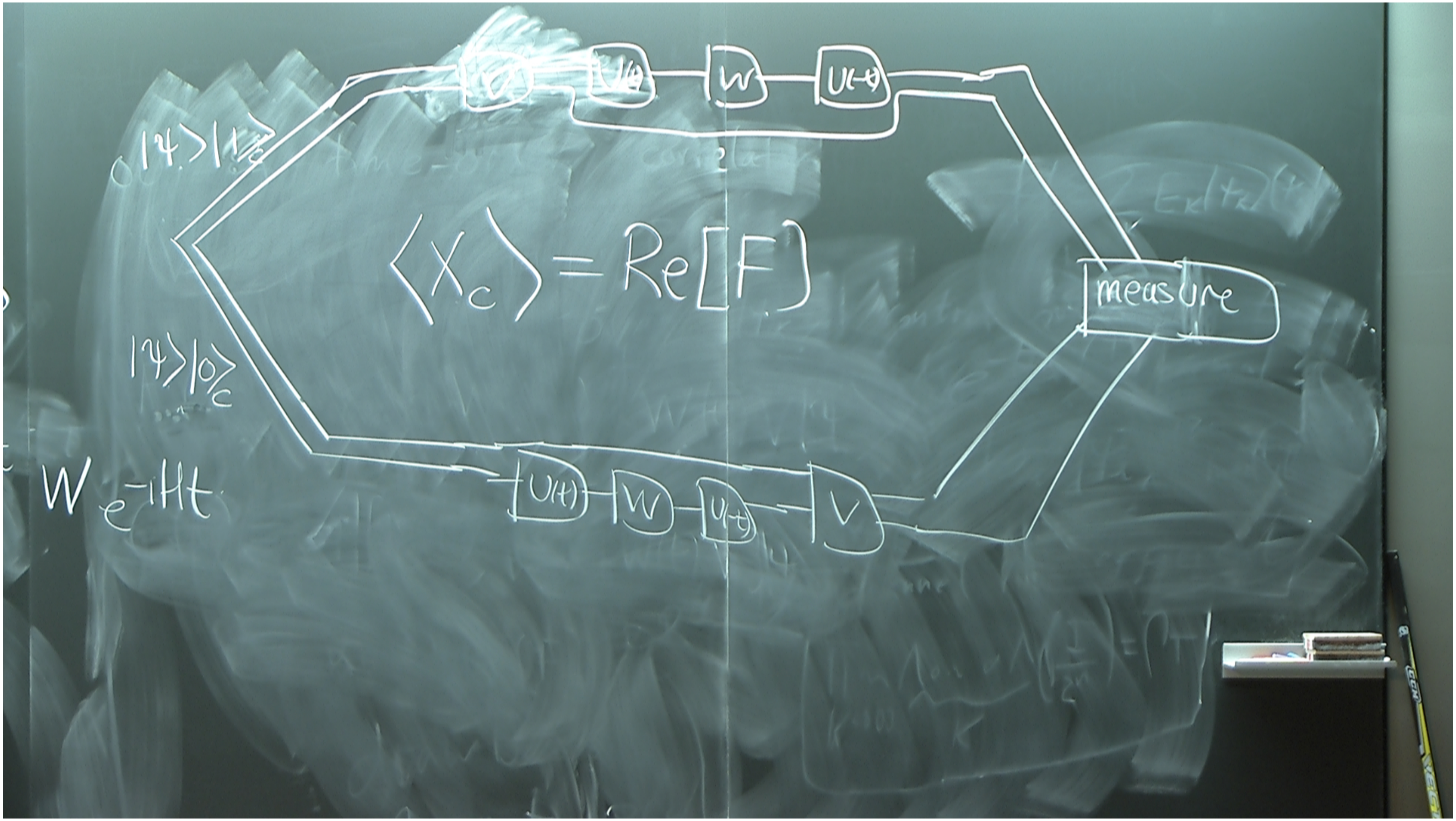
Motivations

- fingerprint for dual BH

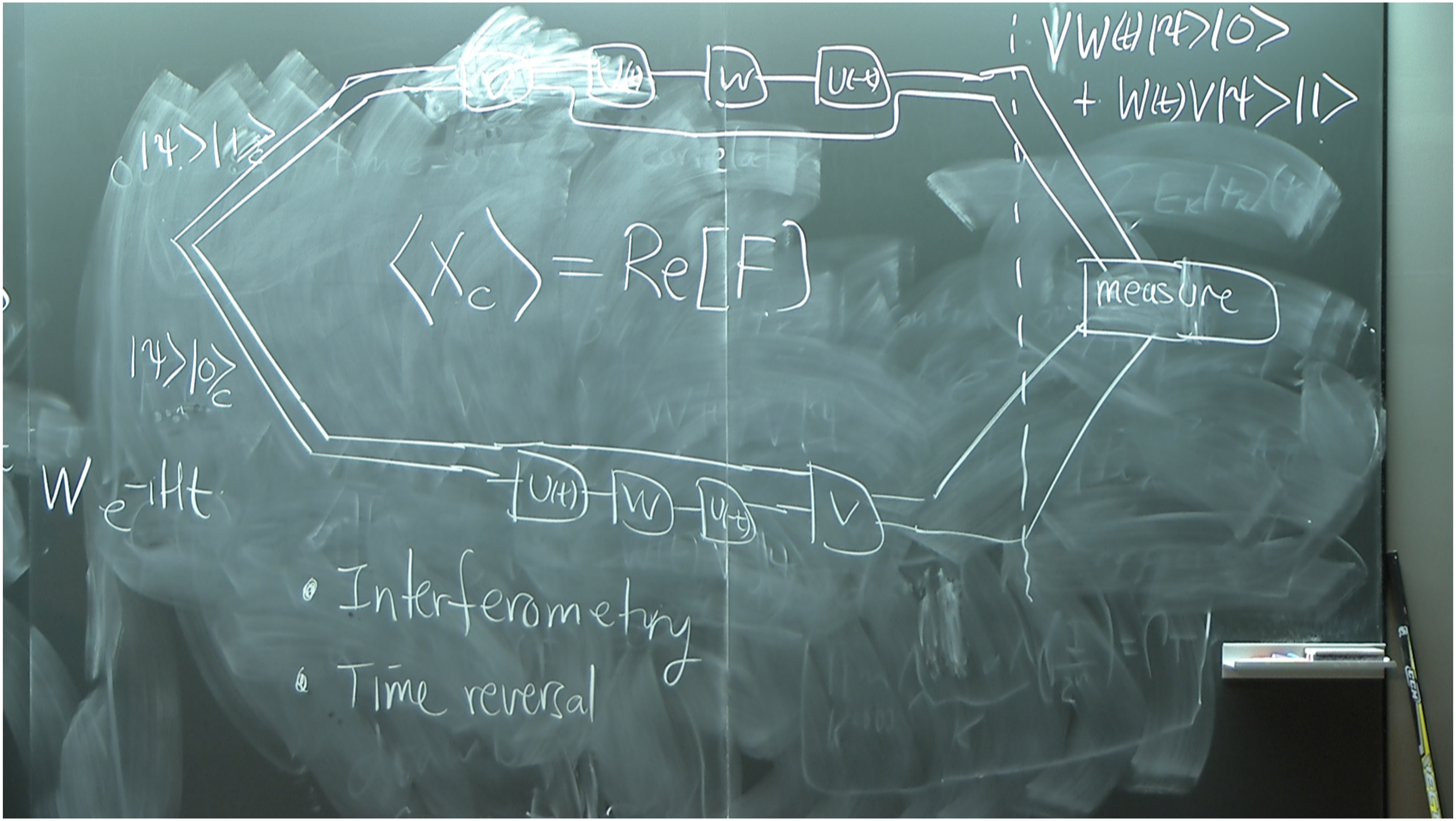
$$F \sim 1 - \frac{1}{S} e^{\lambda t} \qquad \lambda = 2\pi T$$

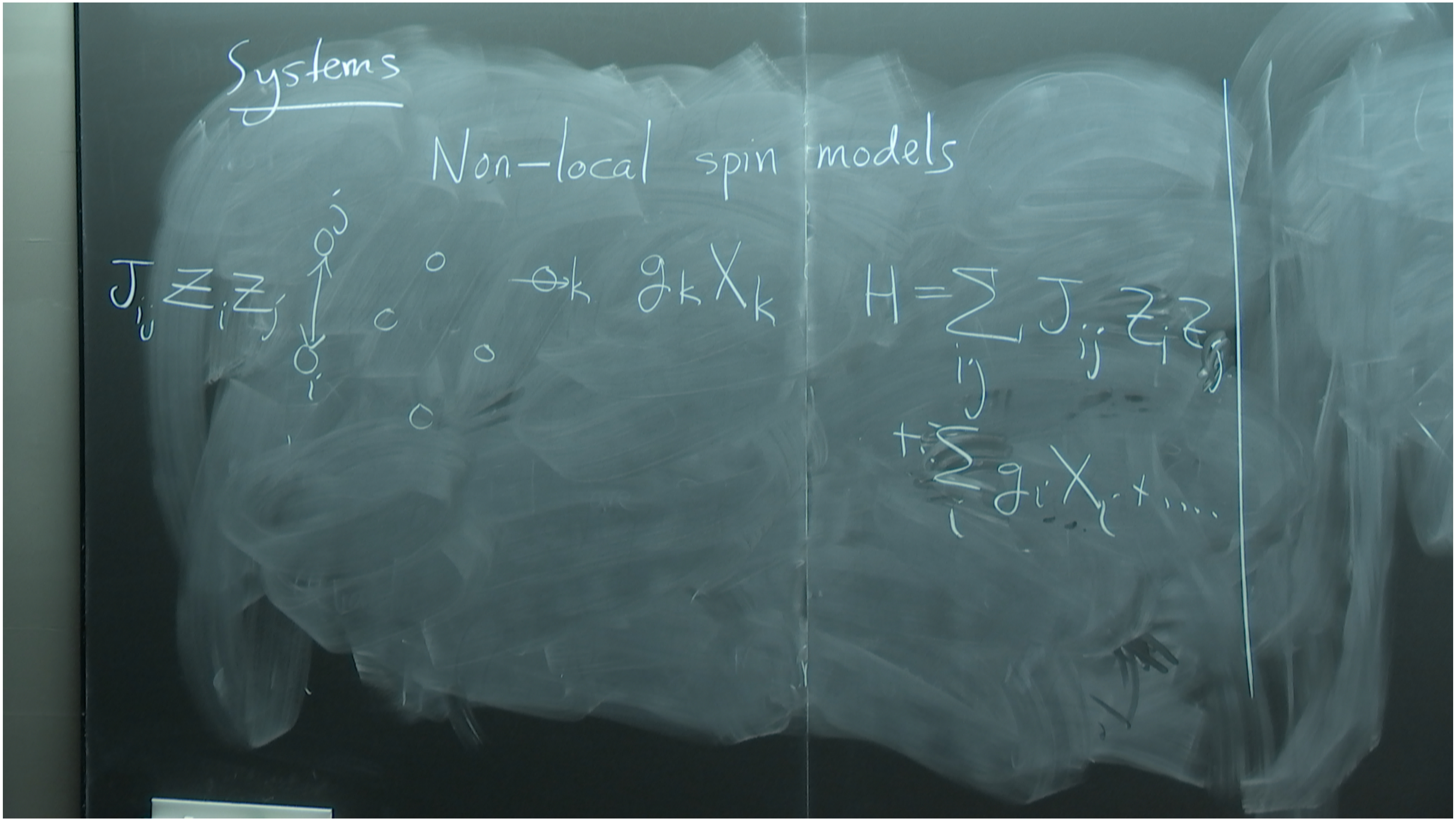
- how special are BHs?
- Quantum Lyapunov?

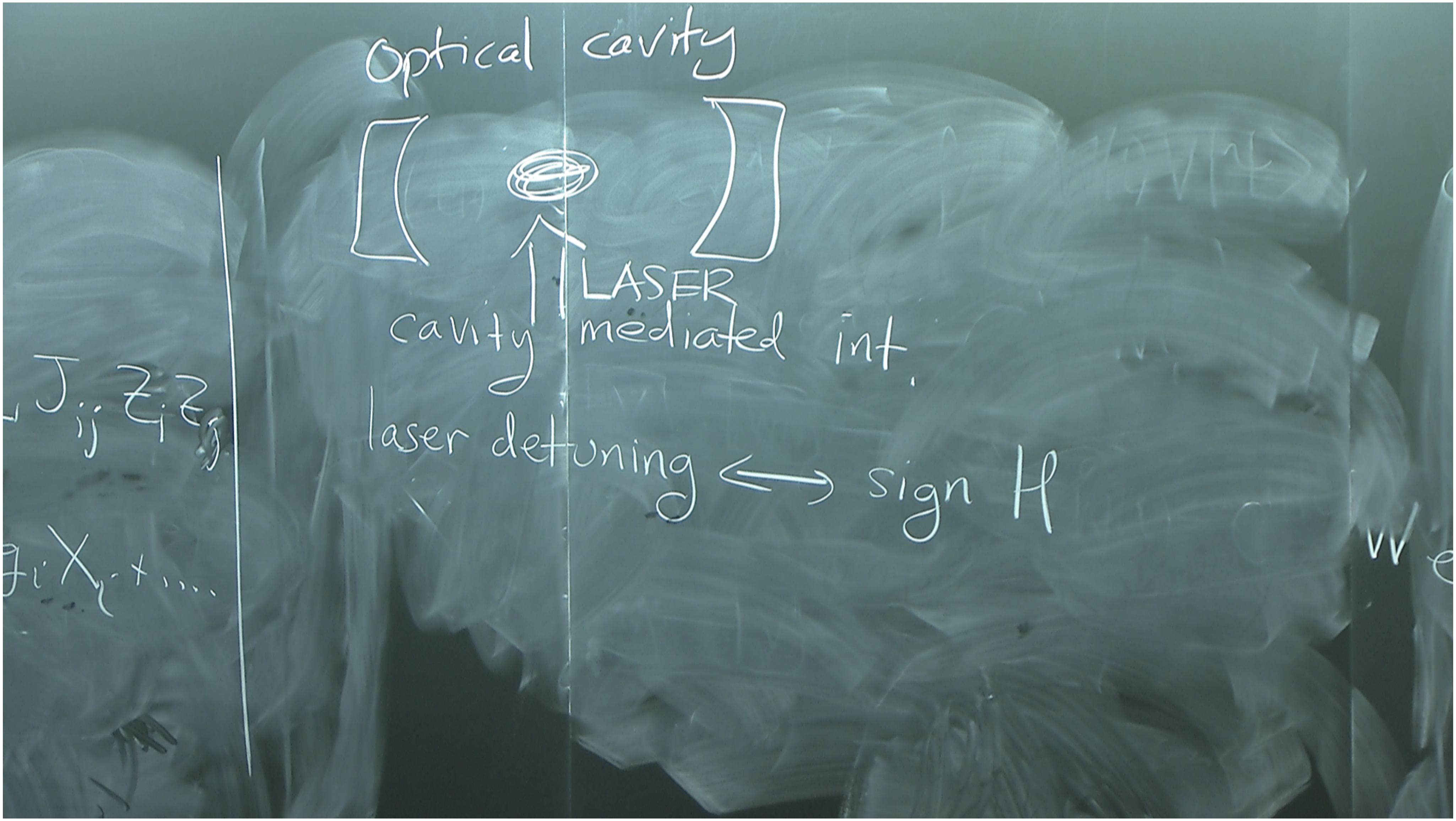

F(t) ≈ overlap of W(t) V|ψ⟩
and V W(t)|ψ⟩
V
W(t)
W(t)
V
out-of-
V
F(t) =
C(t) =
=



$|\psi\rangle|1\rangle_c$
$|\psi\rangle|0\rangle_c$
$W e^{-iHt}$
$\langle X_c \rangle = \mathrm{Re}[F]$
U(t)
W
U(-t)
V
measure


$V W(t)|\psi\rangle|0\rangle$
$+ W(t)V|\psi\rangle|1\rangle$
$|\psi\rangle|1\rangle_c$
$|\psi\rangle|0\rangle_c$
$\langle X_c \rangle = \mathrm{Re}[F]$
measure
U(t)
W
U(-t)
V
$W \quad e^{-iHt}$
• Interferometry
• Time reversal

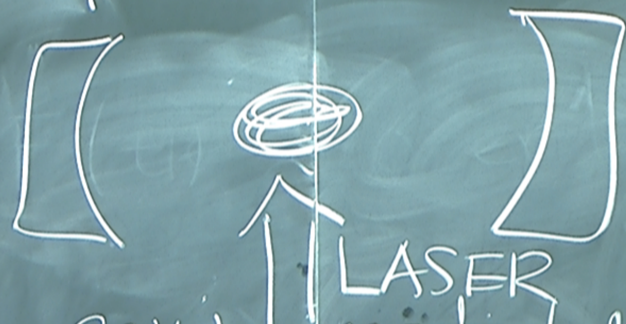

Systems
Non-local spin models
$J_{ij} Z_i Z_j$
$g_k X_k$
$H = \sum_{ij} J_{ij} Z_i Z_j + \sum_i g_i X_i + \ldots$


Optical cavity
LASER
cavity mediated int.
laser detuning ⟷ sign H

$J_{ij} Z_i Z_j$

$g_i X_i + \dots$

1. Optical cavity



cavity mediated int.

laser detuning $\longleftrightarrow$ sign H

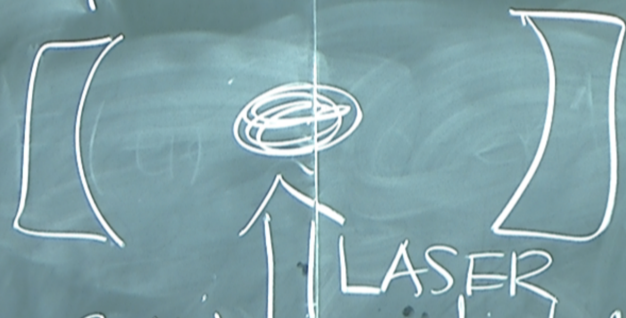
2. Rydberg

W e

$\ldots J_{ij} Z_i Z_j$

$g_i X_i + \ldots$

1. Optical cavity



cavity mediated int.

laser detuning $\longleftrightarrow$ sign H

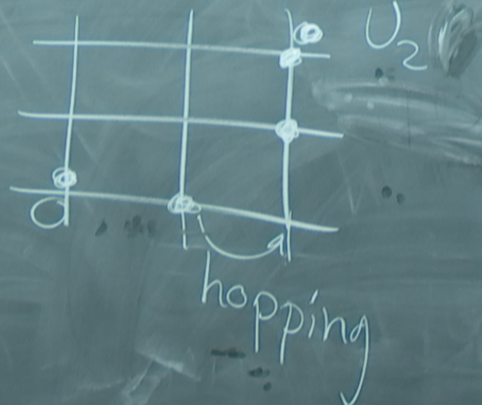
2. Rydberg

3. SYK Danishita et al.

W e

Local models

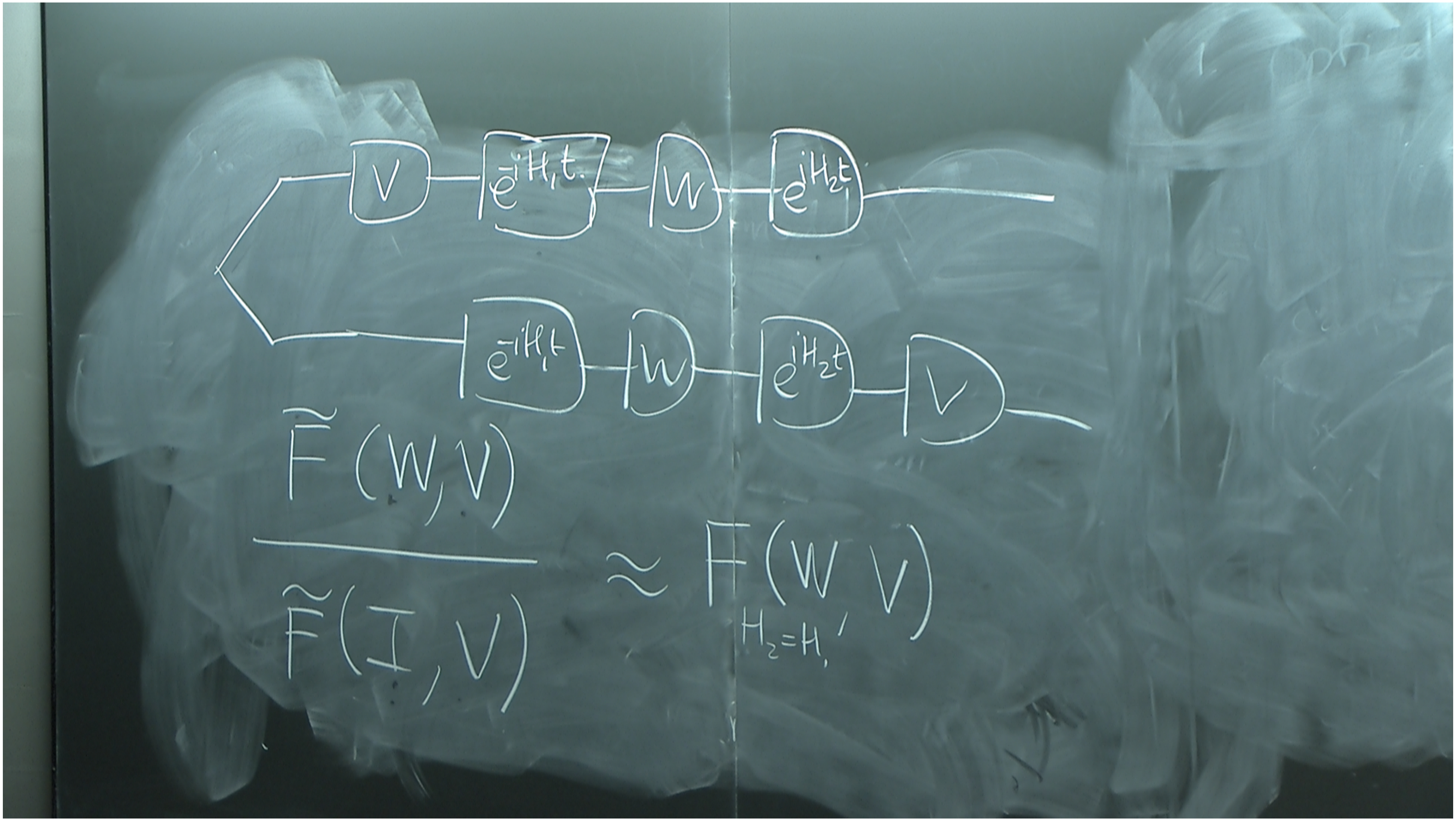
1. optical lattice

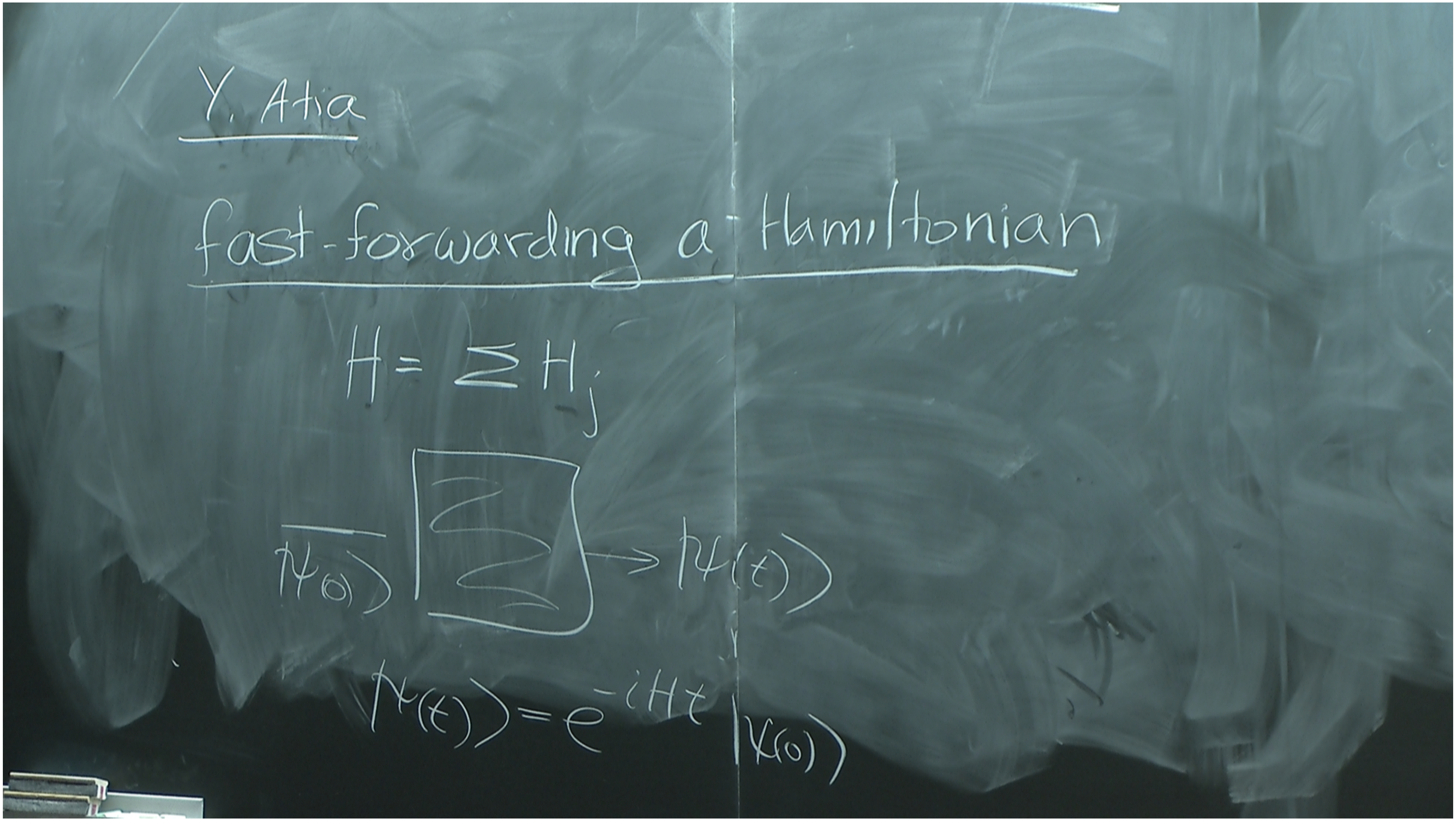


Feshbach resonance

+

lattice modulation.


V
e^{-iH_1 t}
W
e^{iH_2 t}
e^{-iH_1 t}
W
e^{iH_2 t}
V
F̃(W,V) / F̃(I,V) ≈ F(W,V)
H_2=H_1


Y. Atia
fast-forwarding a Hamiltonian
$H = \sum H_j$
$|\psi(0)\rangle \rightarrow |\psi(t)\rangle$
$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Y. Atia

fast-forwarding a Hamiltonian

$$H = \sum_j H_j$$

Dan Roberts

Lenny Susskind

$$|\psi(0)\rangle \; \boxed{} \longrightarrow |\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$$

Y. Atia

fast-forwarding a Hamiltonian

$$H = \sum_j H_j$$

$$|\psi(0)\rangle \longrightarrow |\psi(t)\rangle$$

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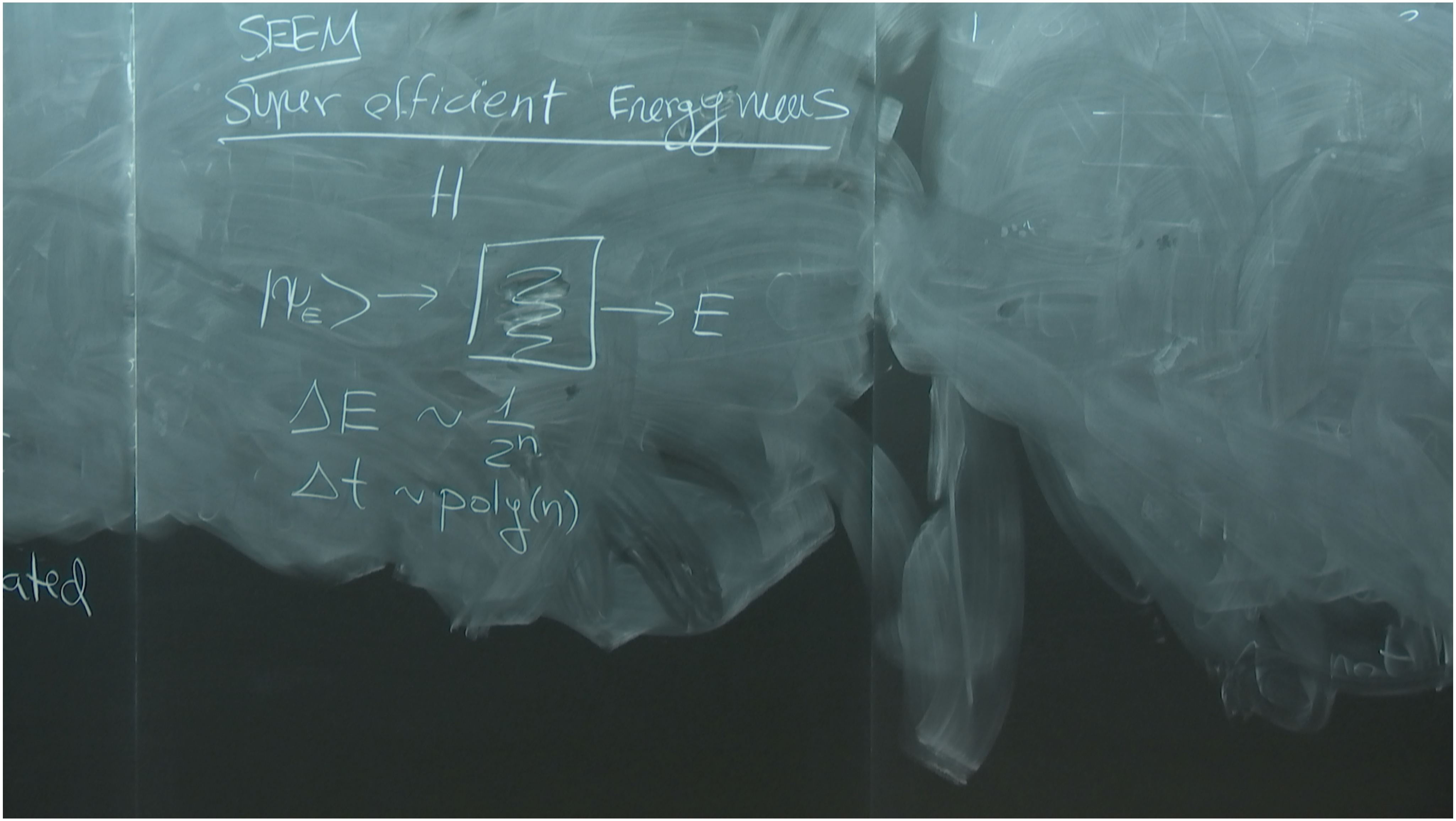
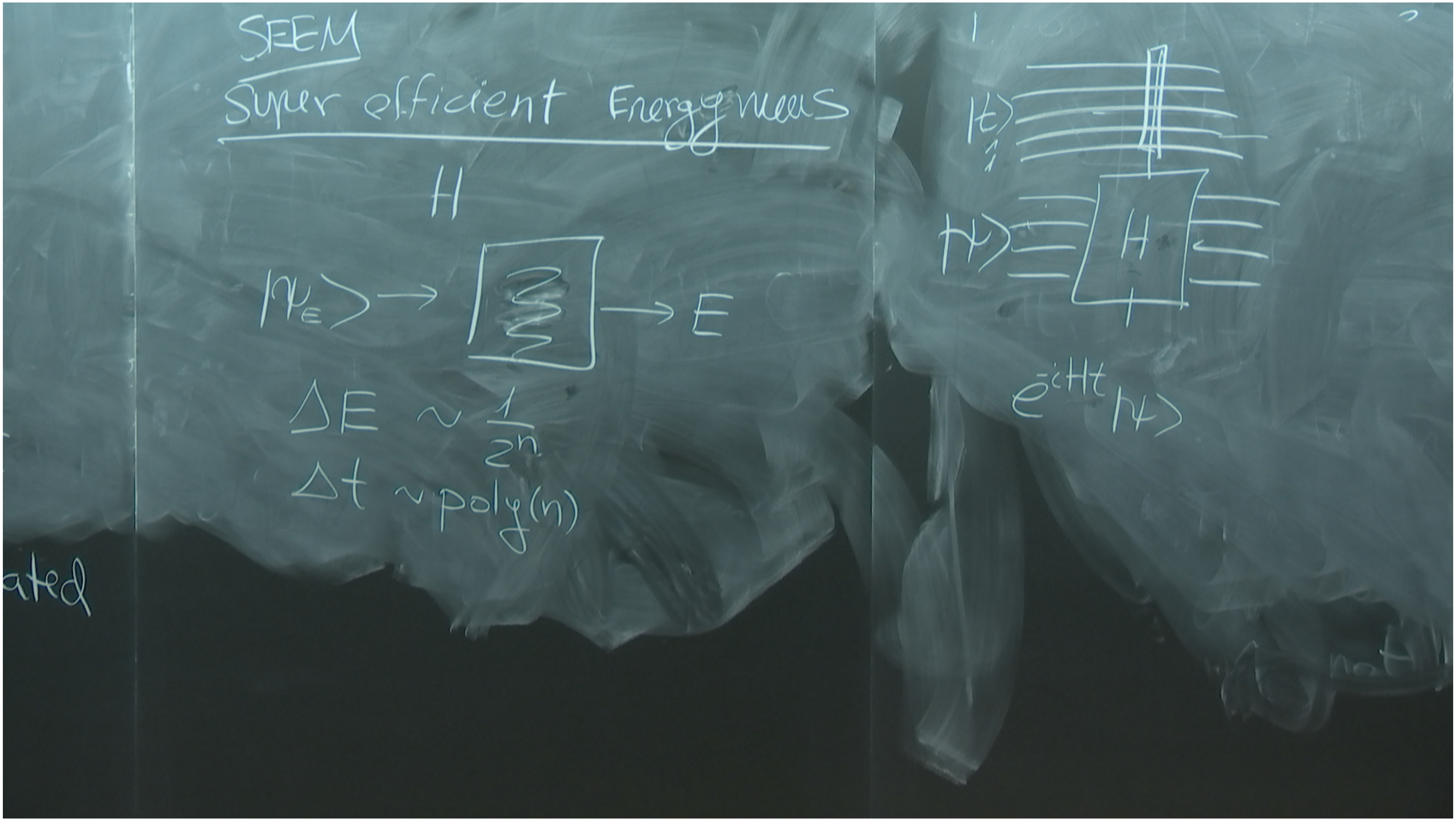
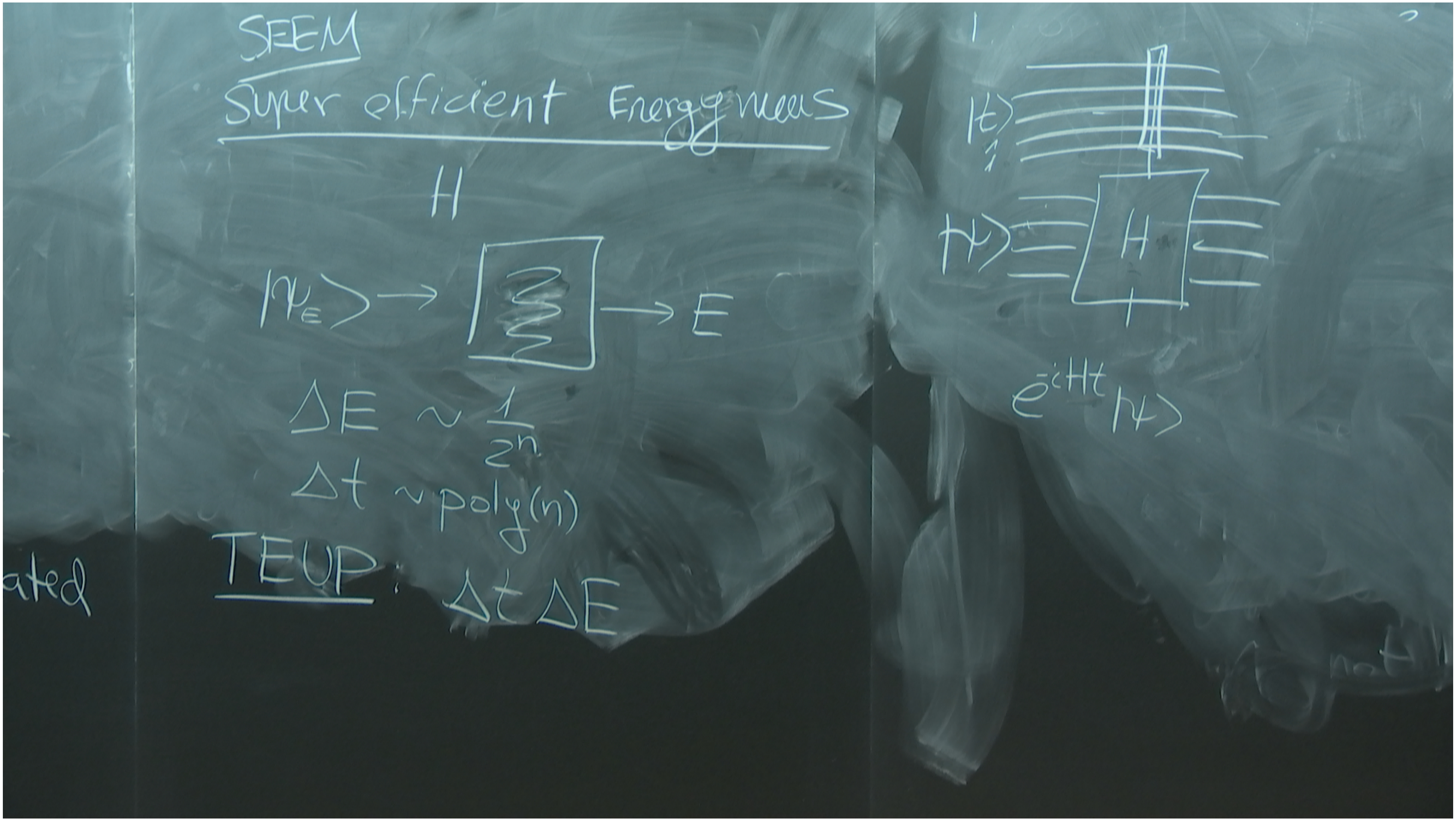
Dan Roberts
Lenny Susskind

H can be FF if
e^{-iHt} can be simulated
in time $\log(t)$


SEEM
Super efficient Energy meas
H
|ψE⟩ → → E
ΔE ~ 1/2^n
Δt ~ poly(n)
ated


SEEM
Super efficient Energy meas
H
|ψ_E> → E
ΔE ~ 1/2^n
Δt ~ poly(n)
ated
|t>
|ψ>
H
e^{-iHt}|ψ>


SEEM
Super efficient Energy meas
H
|ψE> → → E
ΔE ~ 1/2n
Δt ~ poly(n)
TEUP: ΔtΔE
ated
|t>
|ψ>
H
e-iHt |ψ>

Aharonov Bohm '61

TEUP violated

Y. Aharonov Massar Popescu '02

hold

<u>CTEUP</u>

ΔE (comp complexity of meas) $\geq \Omega(1)$

Aharonov Bohm '61

TEUP violated

Y. Aharonov Massar Popescu '02

hold

<u>CTEUP</u>

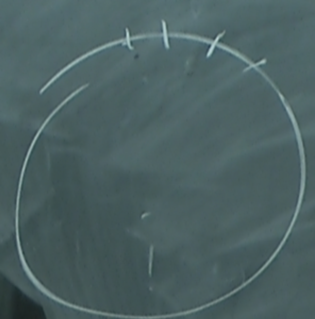
ΔE (comp complexity of meas) $\geq \Omega(1)$

$$H = \sum_{i=1}^{n} \sigma_x^i$$

$N \qquad n = \lceil \log N \rceil$

$y \qquad \gcd(N, y) = 1$

$y, y^2, y^3, y^4 \cdots \cdots y^r = 1 \bmod N$



$U|a\rangle = |a \cdot y \bmod N\rangle$

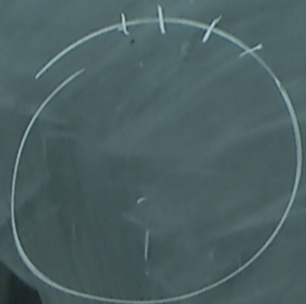
$$r\left\{ \begin{pmatrix} \begin{matrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 1 & & & 0 \end{matrix} & \\ & (\quad) \end{pmatrix} \right.$$

$e^{i 2\pi k / r}$

$N \quad n = \lceil \log N \rceil$

$y \quad \gcd(N, y) = 1$

$y, y^2, y^3, y^4 \cdots y^r = 1 \bmod N$



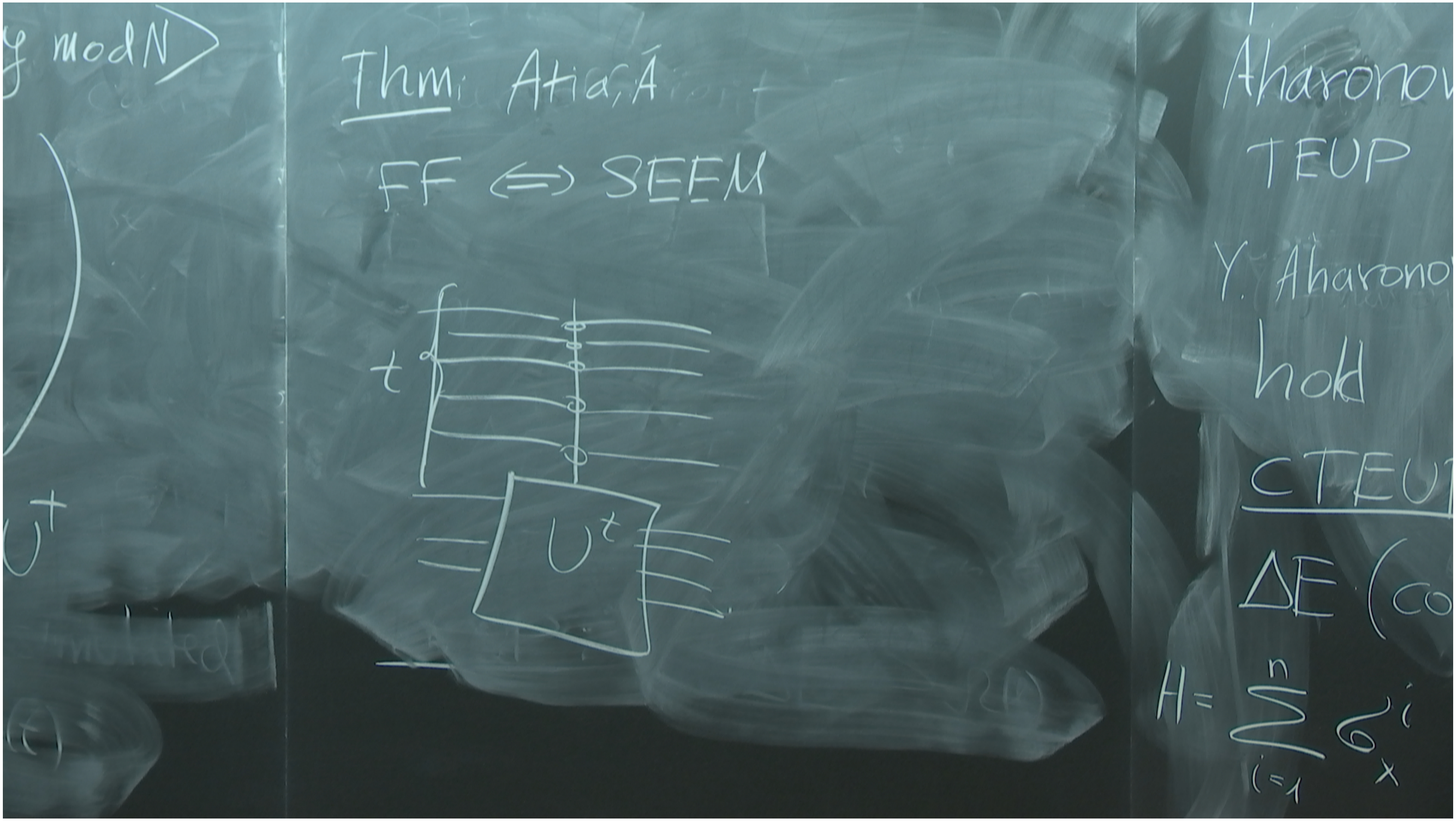
$U|a\rangle = |a \cdot y \bmod N\rangle$

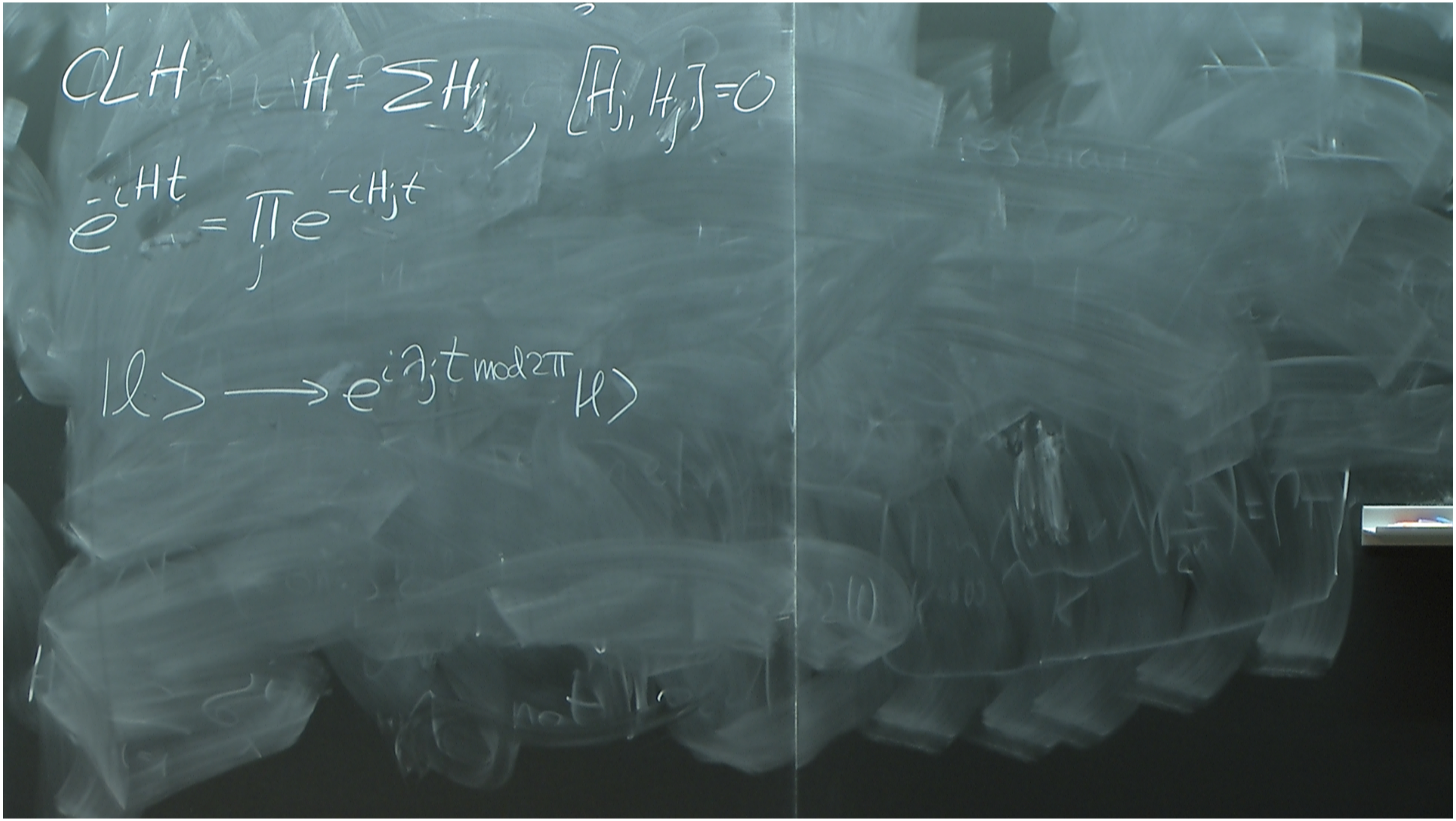
$$r\left\{\begin{pmatrix} \begin{matrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 1 & & & 0 \end{matrix} & \\ & \begin{pmatrix} & \end{pmatrix} \\ & & \ddots \end{pmatrix}\right.$$

$e^{i\frac{2\pi k}{r}}$

$e^{iH} \quad \Big| \quad H = U + U^\dagger$

$2\cos\frac{2\pi k}{r}$


y mod N⟩
Thm: Atia, Á
FF ⟺ SEEM
t
U^t
Aharonov
TEUP
Y. Aharonov
hold
CTEU
ΔE (co
H = Σ_{i=1}^{n} σ_x^i


CLH $H=\sum H_j$, $[H_j, H_{j'}]=0$
$e^{-iHt} = \prod_j e^{-iH_j t}$
$|\ell\rangle \longrightarrow e^{i\lambda_j t \bmod 2\pi}|\ell\rangle$

1100

CLH $\quad H=\sum_j H_j \,,\quad [H_j, H_{j'}]=0$

$$e^{-iHt} = \prod_j e^{-iH_j t}$$

$$|l\rangle \longrightarrow e^{i\lambda_j t \bmod 2\pi}|l\rangle$$

Anderson localization