Title: Majorana bound states in magnetic skyrmions

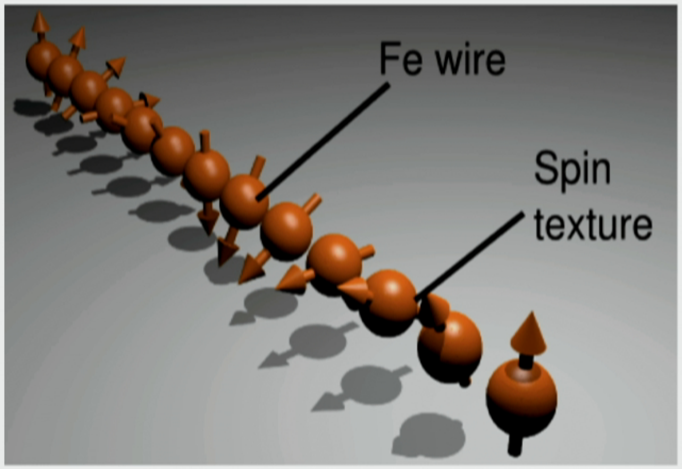
Date: Jul 21, 2016 03:30 PM

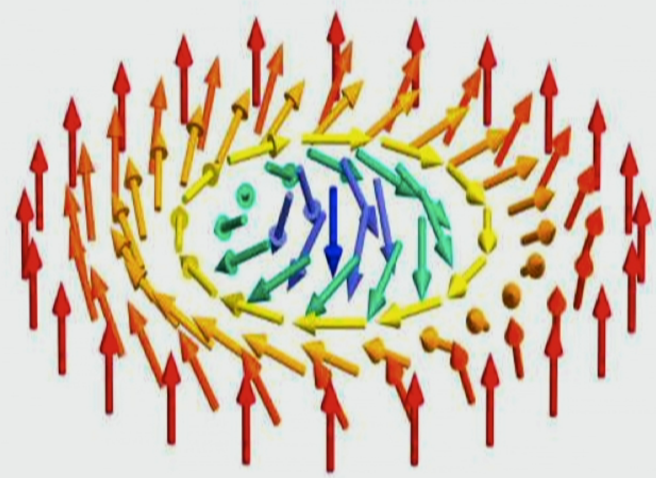
URL: http://pirsa.org/16070068

Abstract: <p>Magnetic skyrmions are highly mobile nanoscale topological spin textures. We show, both analytically and numerically, that a magnetic skyrmion of an even azimuthal winding number placed in proximity to an s-wave superconductor hosts a zero-energy Majorana bound state in its core, when the exchange coupling between the itinerant electrons and the skyrmion is strong. This Majorana bound state is stabilized by the presence of a spin-orbit interaction. We propose the use of a superconducting tri-junction to realize non-Abelian statistics of such Majorana bound states.</p>

Majorana Bound States in Magnetic Skyrmions

arXiv:1602.00968





Guang Yang,
Center for Emergent Matter Science, RIKEN

Perimeter Institute, July 21, 2016

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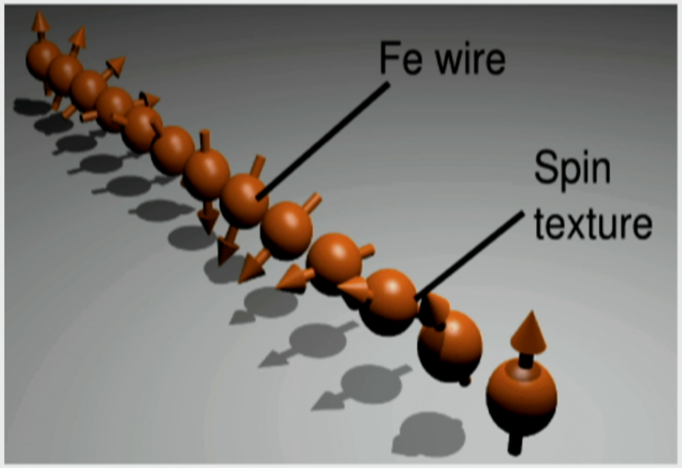

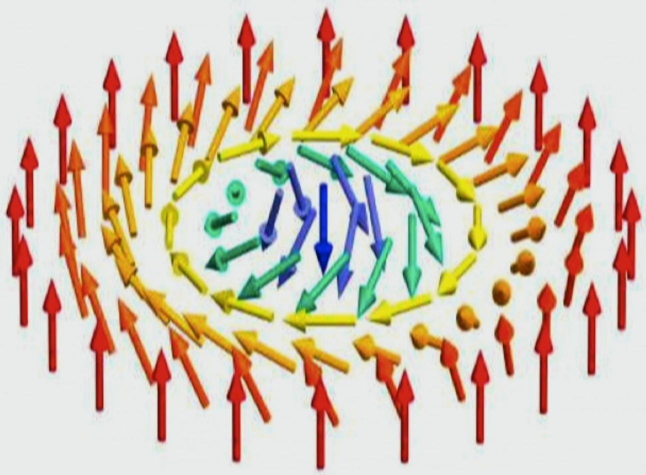

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Collaborators

Peter Stano
(RIKEN)

Jelena Klinovaja
(Basel)

Daniel Loss
(RIKEN & Basel)

Outline

- Introduction: Majorana fermions in solid state systems
- Introduction: magnetic skyrmions
- Majorana bound states in magnetic skyrmions
- Braiding Majorana bound states in magnetic skyrmions

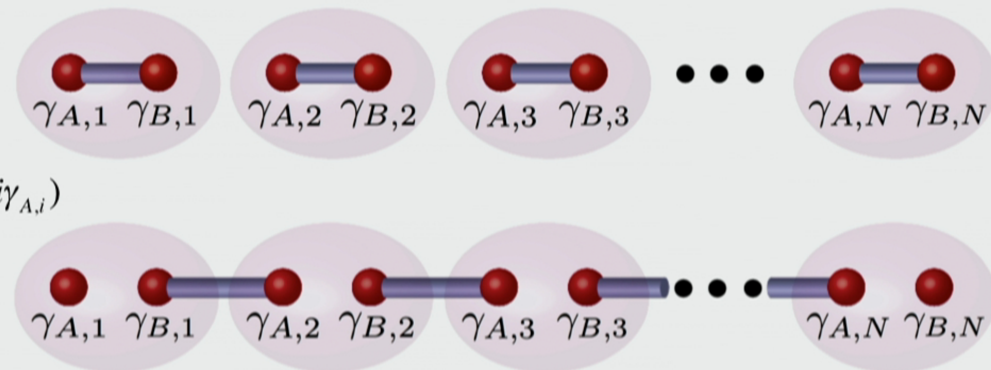
What are Majorana fermions?

- Proposed for neutrinos (Majorana, 1937)
  - self-adjoint: $\psi = \psi^{+}$
  - governed by a real Dirac equation: $\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$
  - chargeless: $\partial_{\mu} \xrightarrow{\times} D_{\mu} = \partial_{\mu} + ieA_{\mu}$
- In condensed matter $\psi \sim c^{+} + c$
  - emergent quasiparticles in a "spinless" superconductor
  - equal weight on particle and hole parts

1D toy model: The Kitaev chain

$$H - \mu N = -\mu \sum_i c_i^+ c_i - \frac{1}{2} \sum_i (t c_i^+ c_{i+1} + \underbrace{\Delta c_i c_{i+1}}_{p\text{-wave pairing}} + \text{H.c.})$$

Kitaev 2001

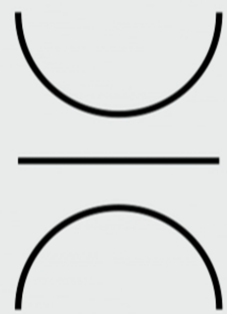


$|\mu| > t$

$c_i = \frac{1}{2}(\gamma_{B,i} + i\gamma_{A,i})$

$|\mu| < t$

Fig. from Alicea, RPP, 2012



Majorana bound states: $\gamma_{A,1}$, $\gamma_{B,N}$

ground state degeneracy: $|0\rangle$, $|1\rangle = f^+|0\rangle$

$$f = \gamma_{A,1} + i\gamma_{B,N}$$

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p-wave pairing

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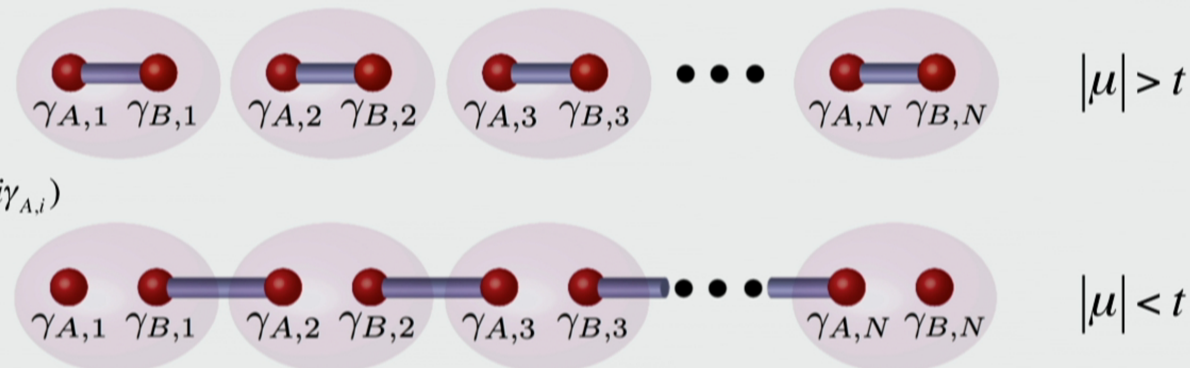

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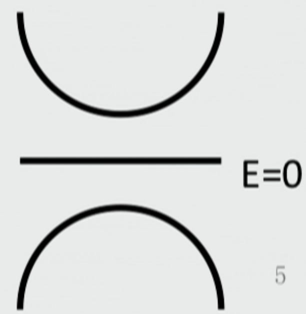

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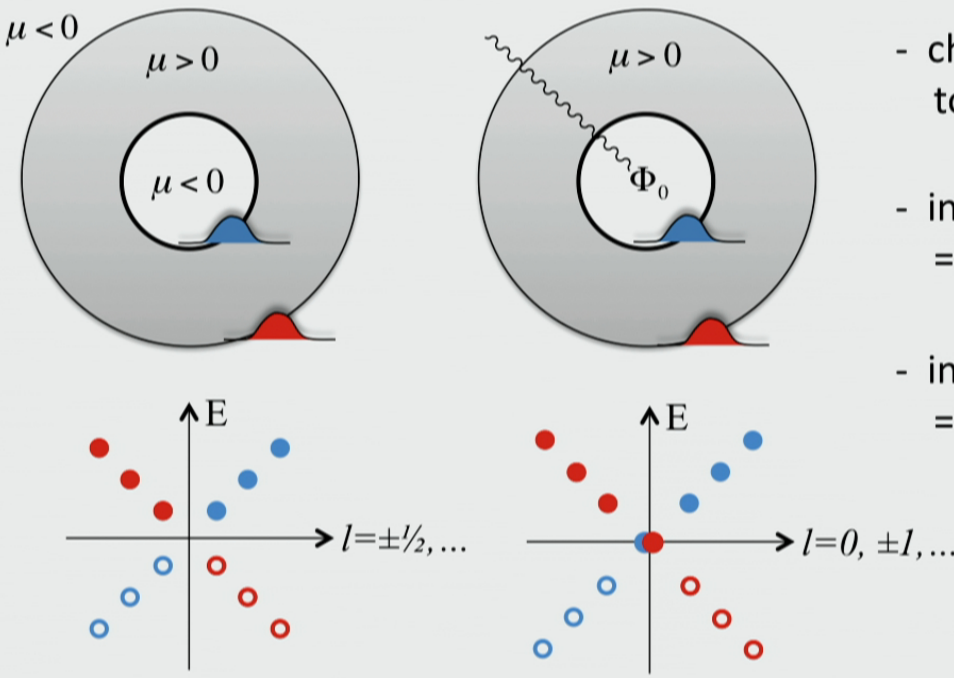
2D toy model: p+ip superconductor

$$H = \int d^2r \left\{ \psi^{+} \left(-\frac{\nabla^2}{2m} - \mu \right) \psi + \left(\frac{\Delta}{2} \psi \underbrace{(\partial_x + i\partial_y)}_{p_x + ip_y \text{ pairing}} \psi + \text{H.c.} \right) \right\}$$

Read & Green 2000



- chiral Majorana edge states at topological/trivial interfaces
- insert flux quanta $n\Phi_0$ (***n* is odd**) => Majorana zero modes
- infinitesimal interfaces (vortices) => Majorana bound states

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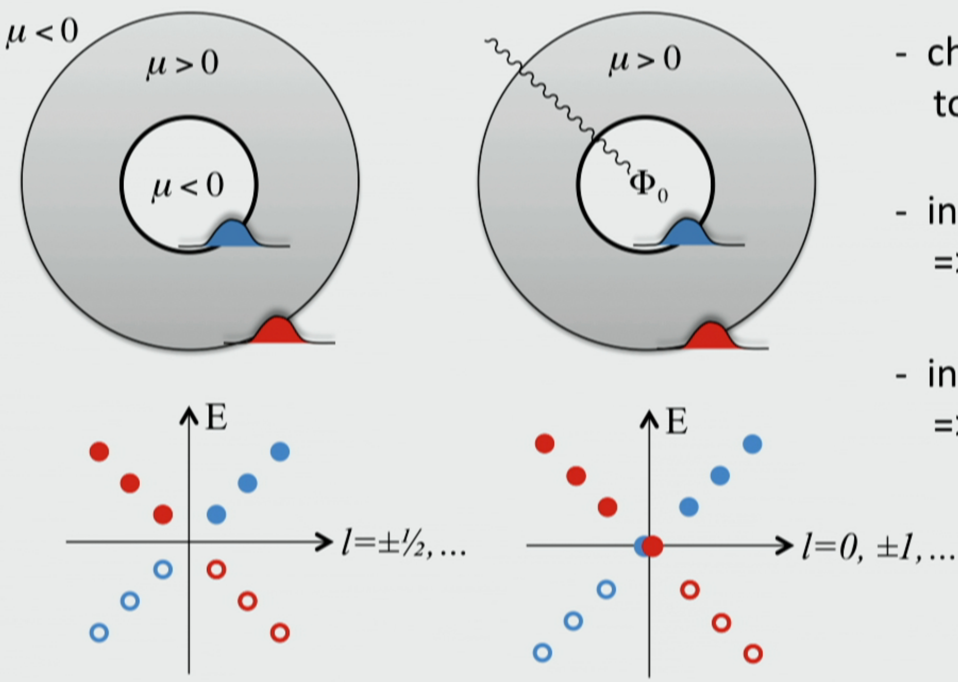

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Topological quantum computing

- Ground state degeneracy
  - N Majoranas: dim= $2^{N/2}$
  - N=4: $|0\rangle, c_1^+|0\rangle, c_2^+|0\rangle, c_1^+c_2^+|0\rangle$

$$c_1 = \frac{1}{2}(\gamma_1 + i\gamma_2),\ \ c_2 = \frac{1}{2}(\gamma_3 + i\gamma_4)$$

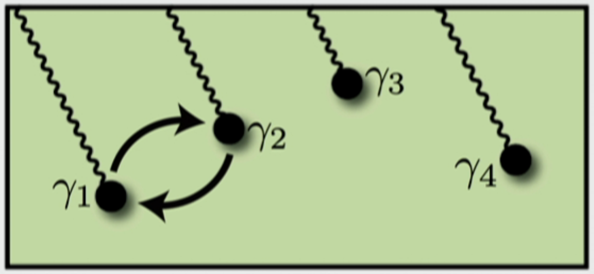

Fig. from Alicea, RPP, 2012

- Non-Abelian exchange rules (Ivanov 2001)

$$\gamma_1 \rightarrow \gamma_2,\ \ \gamma_2 \rightarrow -\gamma_1$$

- Quantum computing with anyons (Kitaev 1997)

Immune to disorder and local perturbations!

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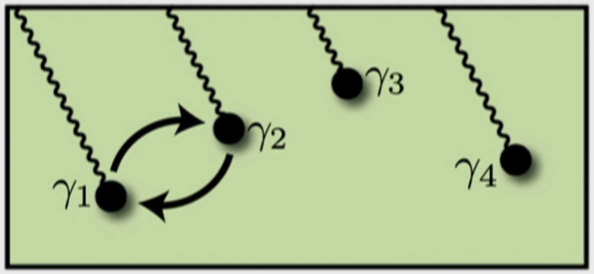

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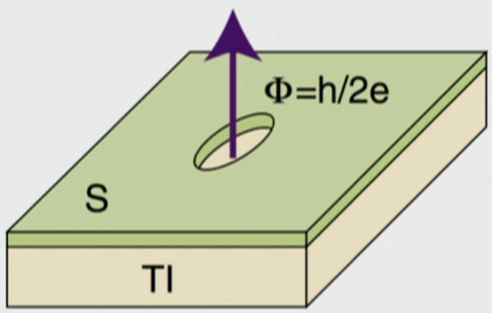
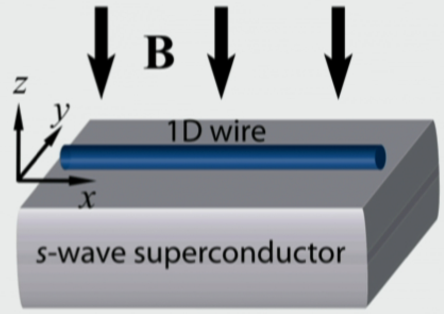
Practical realization in solid state systems

- Intrinsic *p+ip* superconductivity: Sr_2RuO_4
- The Moore-Read quantum Hall state (Moore & Read, 2000)
- Synthetic topological superconductivity
  - topological insulator/s-wave S.C. (Fu & Kane, 2008)
  - fermion superfluid in cold atoms (Sato et al., 2009)
  - magnetic insulator (field)/SOC semiconductor/s-wave S.C. (Sau et al., 2010; Lutchyn et al., 2010; Oreg et al., 2010; …)



Fu & Kane, PRL, 2008



Lutchyn et al., PRL 2010

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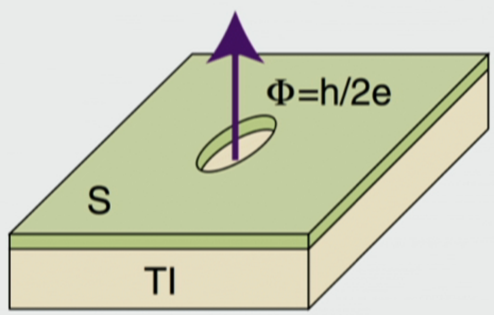

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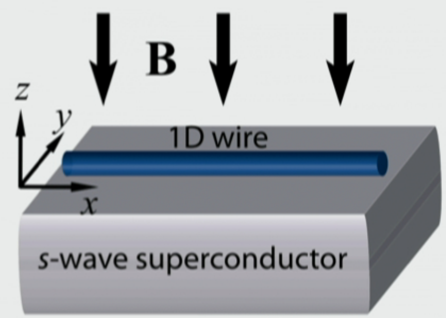

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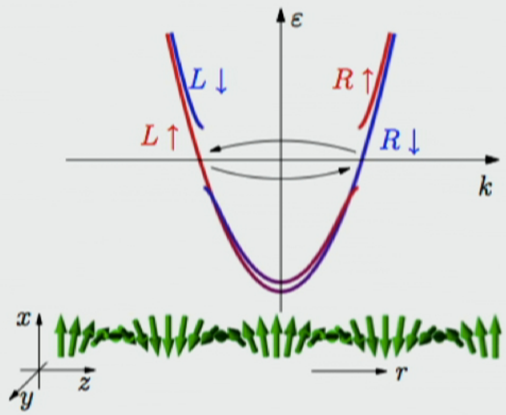
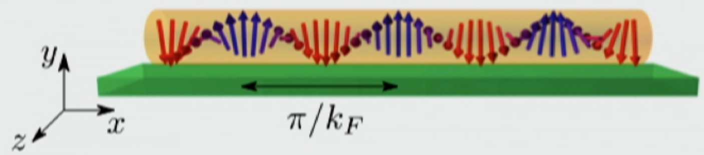
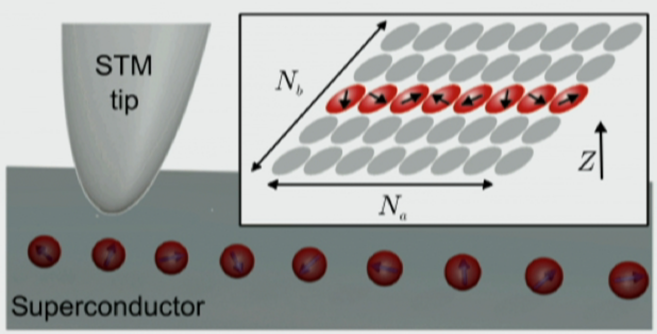
Majorana fermions from helical magnetic order



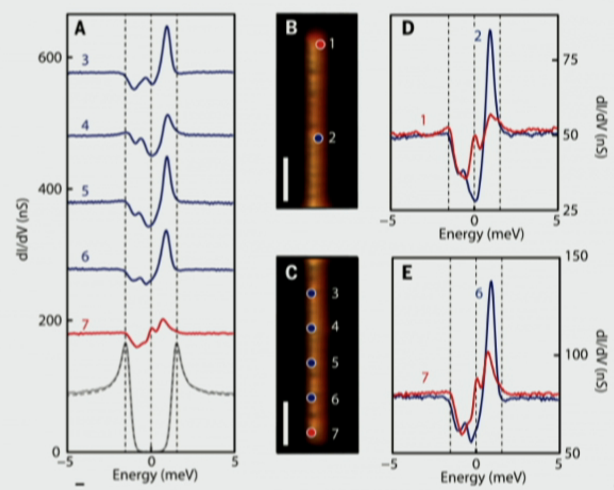
Braunecker et al., PRB, 2010



Klinovaja et al., PRL, 2013
Braunecker & Simon, PRL, 2013
Vazifeh & Franz, PRL, 2013



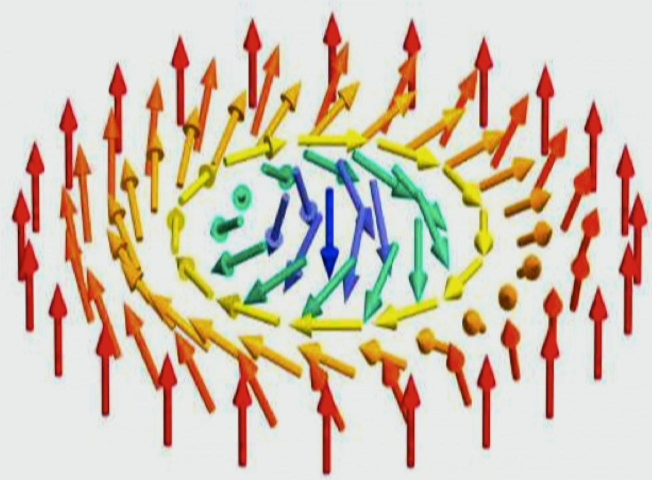
Nadj-Perge et al., PRB, 2013



Nadj-Perge et al., Science, 2014

Can we generalize this idea to two dimensions?

Non-coplanar magnetic order: Magnetic Skyrmions



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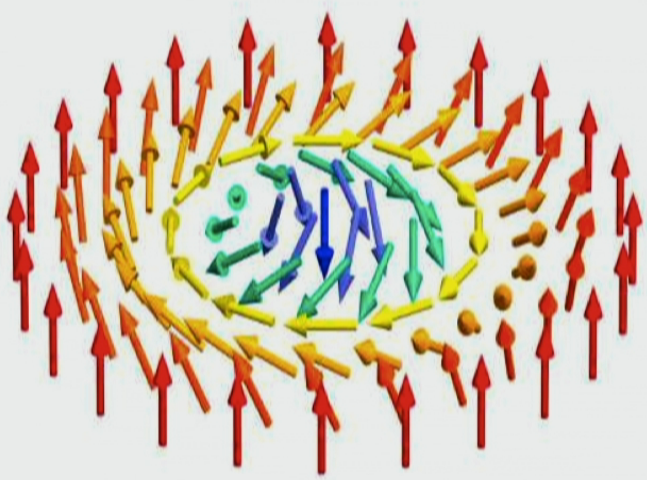

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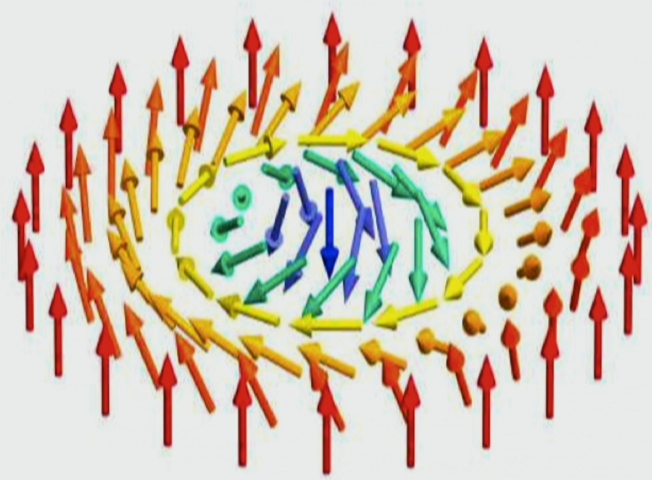
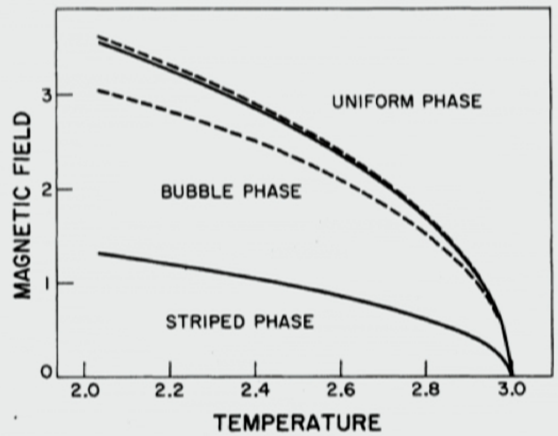


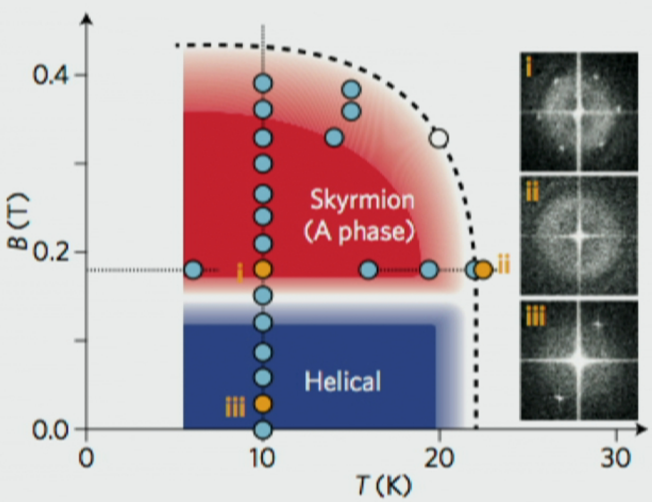
Magnetic skyrmions

1. magnetic dipolar interaction
   & perpendicular easy-axis anisotropy

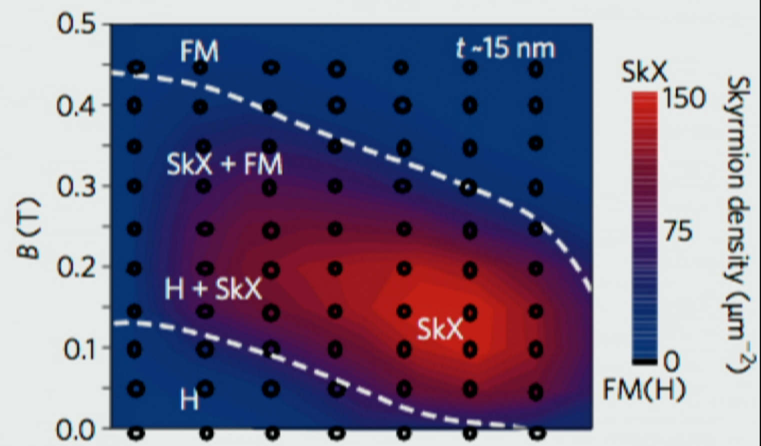
2. Dzyaloshinskii-Moriya interaction



Garel & Doniach, PRB 26, 325 (1982)



Tonomura et al., Nano Lett. 12, 1673 (2012)



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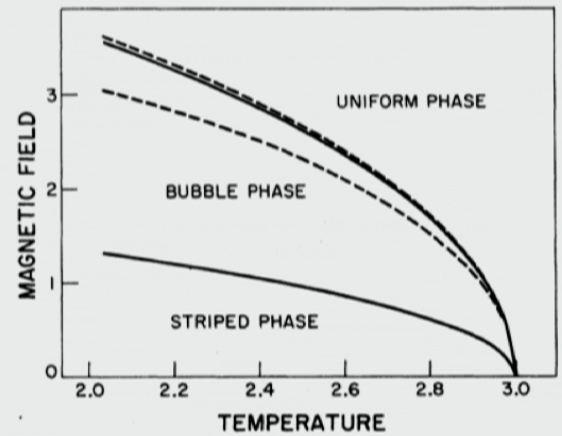

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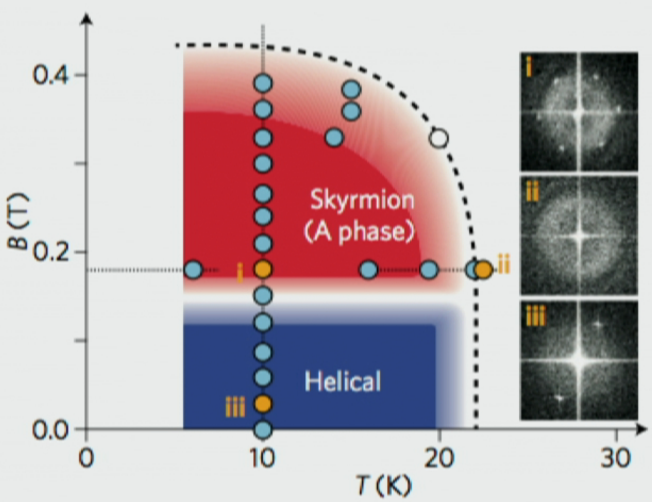

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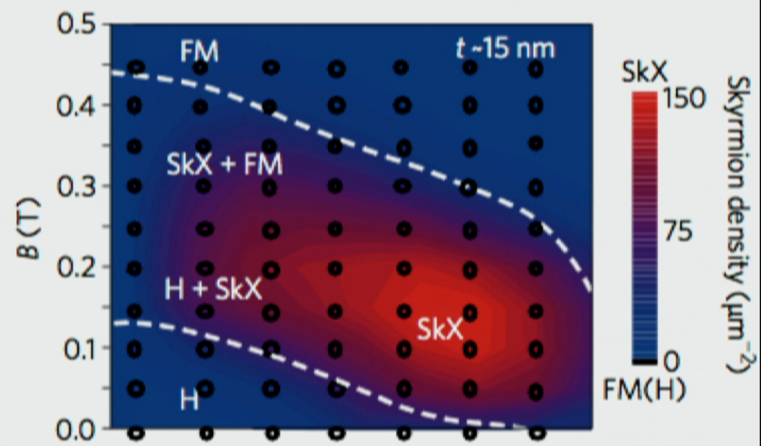

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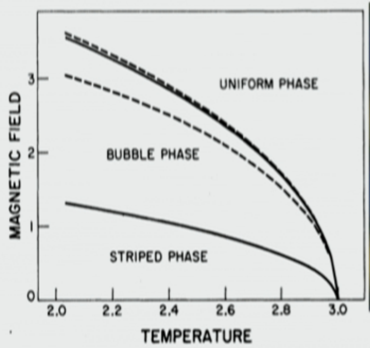
Magnetic skyrmions

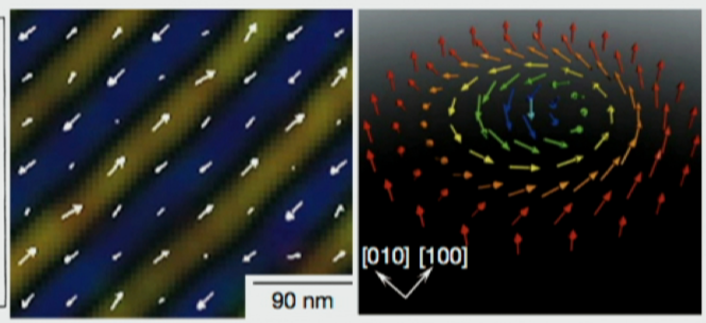
A topological spin texture

$$\hat{N}(\vec{r}) = (\sin f(r)\cos n\theta, \sin f(r)\sin n\theta, \cos f(r))$$

$$f(r=0)=0; \quad f(r\to\infty)=\pi$$

topological charge: $n = \dfrac{1}{8\pi}\int d^2\vec{r}\,\varepsilon^{\alpha\beta}\hat{N}\cdot(\partial_\alpha\hat{N}\times\partial_\beta\hat{N})$





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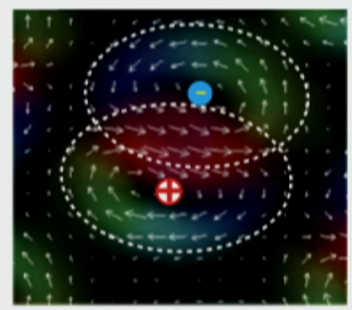

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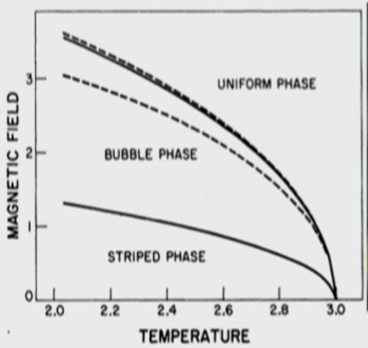

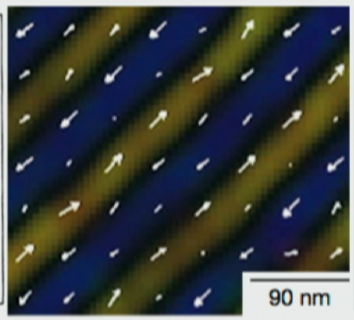

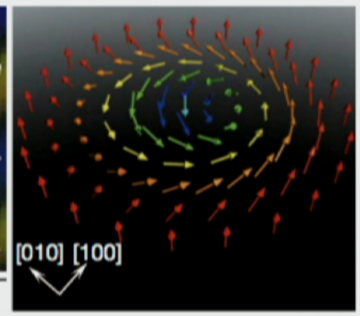

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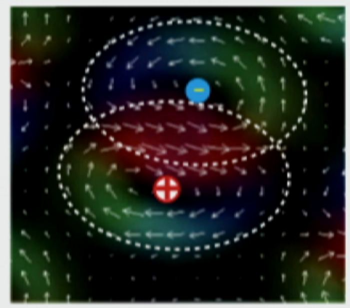

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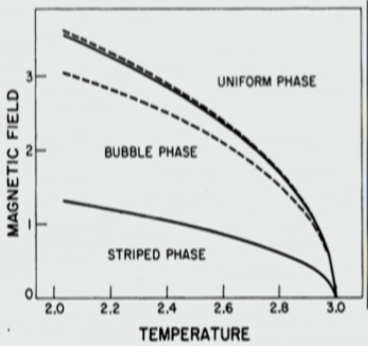

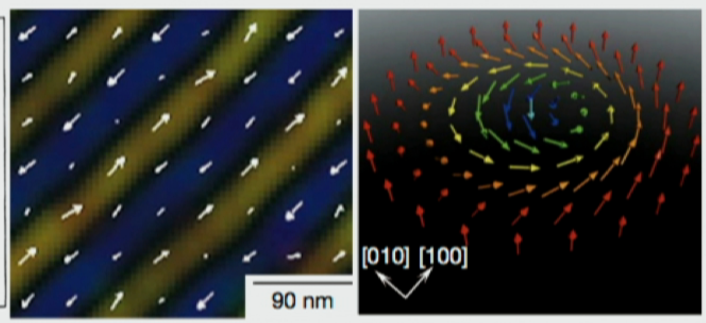

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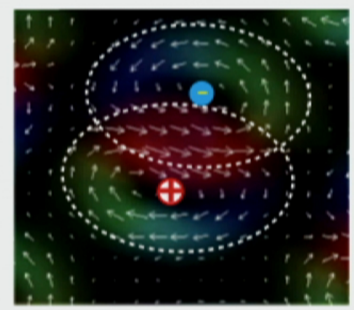
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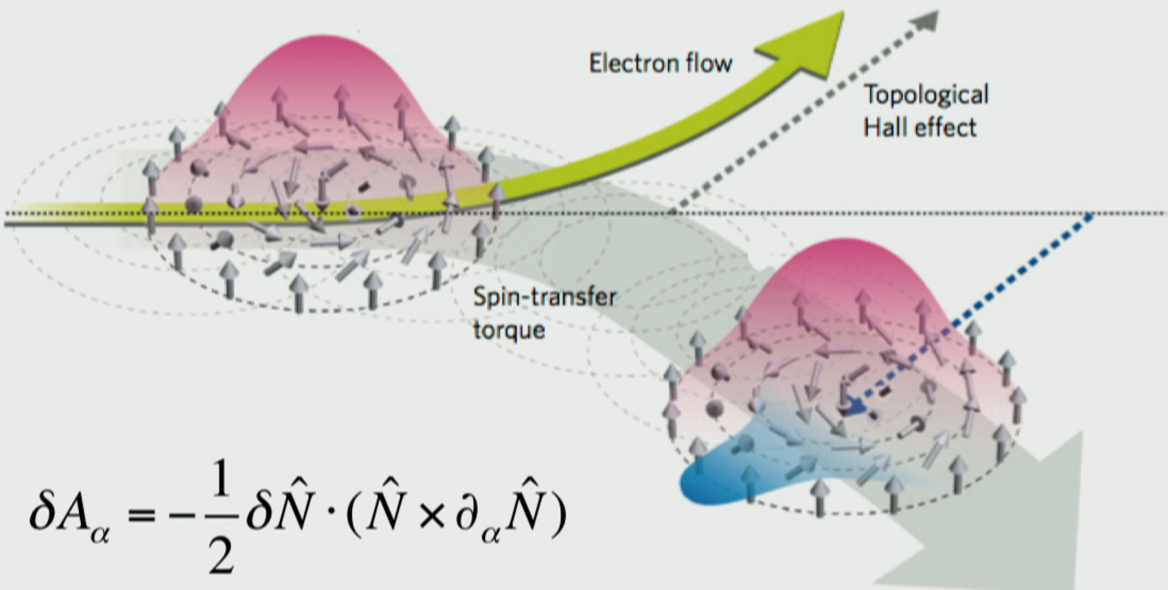
Driving magnetic skyrmions

Spin transfer torque



$$\delta A_\alpha = -\frac{1}{2}\delta\hat{N}\cdot(\hat{N}\times\partial_\alpha\hat{N})$$

$$L \sim j_\alpha A^\alpha$$

$$[\partial_t + \vec{j}\cdot\vec{\nabla}]\hat{N}(\vec{r},t) = 0$$

Nagaosa & Tokura,
Nat. Nanotechnol. 8, 899 (2013)

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Driving magnetic skyrmions

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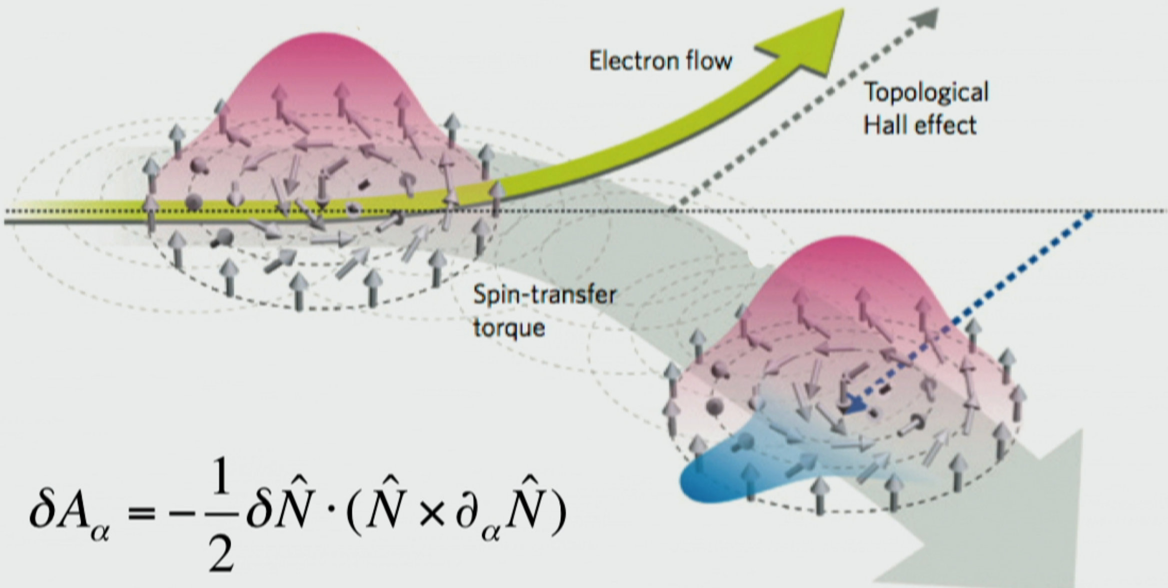

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Nagaosa & Tokura,
Nat. Nanotechnol. 8, 899 (2013)

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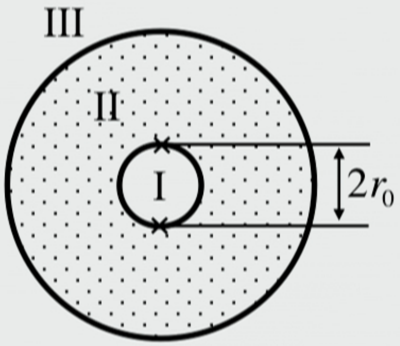
Model

- Skyrmion spin texture

$$\hat{N}(\vec{r}) = \left(\sin f(r)\cos n\theta, \sin f(r)\sin n\theta, \cos f(r)\right)$$

$$f(r) = \begin{cases} 0; & r < r_0 \quad \text{(Region I)} \\ \frac{\pi}{R}(r - r_0); & r_0 < r < r_0 + pR \quad \text{(Region II)} \\ \pi; & r > r_0 + pR \quad \text{(Region III)} \end{cases}$$



- Hamiltonian

$$H = H_0 + H_S$$

exchange coupling

$$H_0 = \int d^2\vec{r} c^+(\vec{r})(-\frac{\nabla^2}{2m} - \mu + \alpha\hat{N}\cdot\vec{\sigma})c(\vec{r})$$

$$H_S = \int d^2\vec{r}(\Delta c_\uparrow^+(\vec{r})c_\downarrow^+(\vec{r}) + \text{H.c.}) \qquad \Delta = \Delta_0 e^{i\varphi}$$

Pre-analysis of spectrum

- BdG equation

$$H^{BdG}\Psi(\vec{r}) = E\Psi(\vec{r}) \qquad\qquad \Psi = [u_\uparrow, u_\downarrow, v_\downarrow, -v_\uparrow]^T$$

- A good quantum number: angular momentum l

$$L = -i\partial_\theta + (n/2)\sigma_z$$

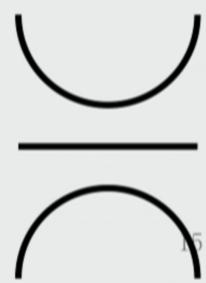
$$\boxed{\Psi_l(\vec{r}) = e^{i(\varphi/2)\tau_z} e^{i\theta(l-n\sigma_z/2)}\Psi_l(r)}$$

even n: integer l
odd n: half-integer l

single-valued as $\theta \to \theta + 2\pi$

- Under particle-hole transformation C

$$l \Rightarrow -l$$



E=0
l=0

A non-degenerate zero mode must have l=0 (for **even *n***)

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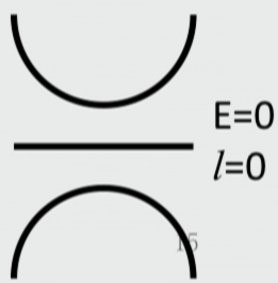
$$\boxed{\Psi_l(\vec{r}) = e^{i(\varphi/2)\tau_z} e^{i\theta(l - n\sigma_z/2)}\Psi_l(r)}$$

single-valued as $\theta \to \theta + 2\pi$

even n: integer l
odd n: half-integer l

- Under particle-hole transformation C

$$l \Rightarrow -l$$



A non-degenerate zero mode must have l=0 (for **even *n***)

Pre-analysis of spectrum

- BdG equation

$$H^{BdG}\Psi(\vec{r}) = E\Psi(\vec{r}) \qquad \Psi = [u_\uparrow, u_\downarrow, v_\downarrow, -v_\uparrow]^T$$

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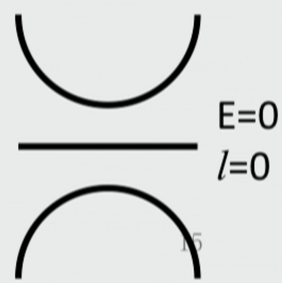

A non-degenerate zero mode must have l=0 (for **even *n***)

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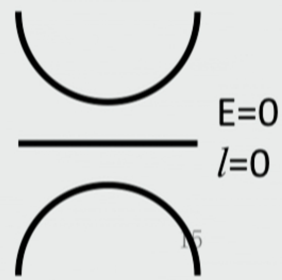


A non-degenerate zero mode must have l=0 (for **even *n***)

Majorana zero modes

- Defining conditions

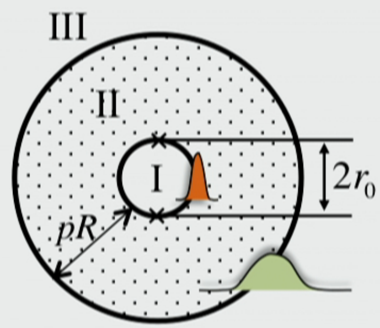
$$H^{BdG}\Psi_0(\vec{r})=0 \qquad C\Psi_0(\vec{r})\propto\Psi_0(\vec{r})$$

- BdG equation

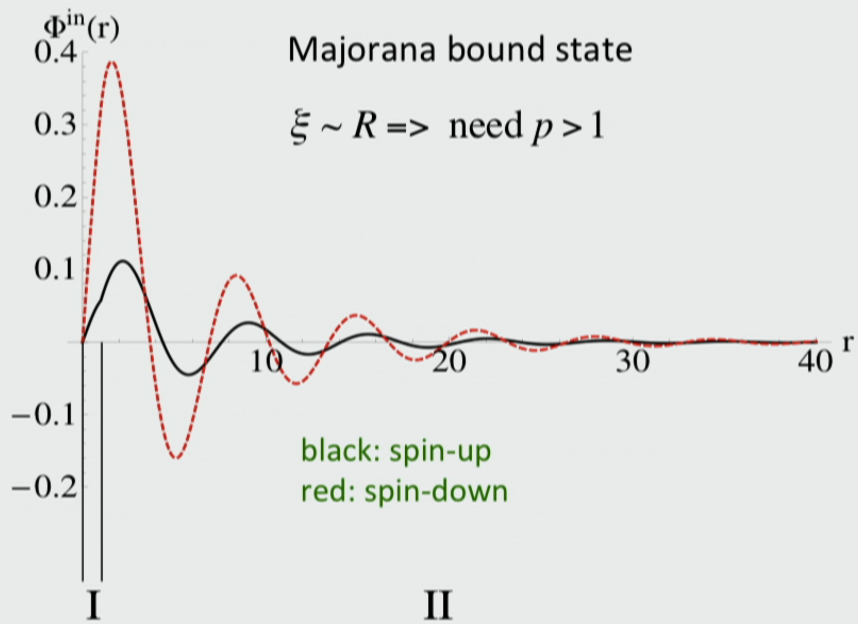
$$\begin{pmatrix} -\frac{1}{2m}(\partial_r^2+\frac{1}{r}\partial_r-\frac{1}{r^2})-\mu+\alpha\cos f & \alpha\sin f+\eta\Delta_0 \\ \alpha\sin f-\eta\Delta_0 & -\frac{1}{2m}(\partial_r^2+\frac{1}{r}\partial_r-\frac{1}{r^2})-\mu-\alpha\cos f \end{pmatrix}\begin{pmatrix} u_\uparrow(r) \\ u_\downarrow(r) \end{pmatrix}=0$$

$$\Psi_0(r)=[u_\uparrow(r),u_\downarrow(r),v_\downarrow(r),-v_\uparrow(r)]^T \qquad v_{\uparrow,\downarrow}(r)=\eta u_{\uparrow,\downarrow}(r) \qquad \eta=\pm1$$

Majorana wave functions



- Inner Majorana: bound state $(\alpha^2 > \tilde{\mu}^2 + \Delta_0^2)$
- Outer Majorana: delocalized state

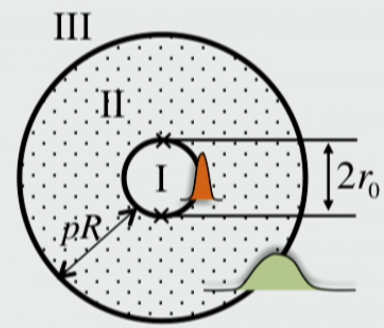


Majorana bound state

$\xi \sim R$ => need $p > 1$

black: spin-up
red: spin-down

Majorana wave functions



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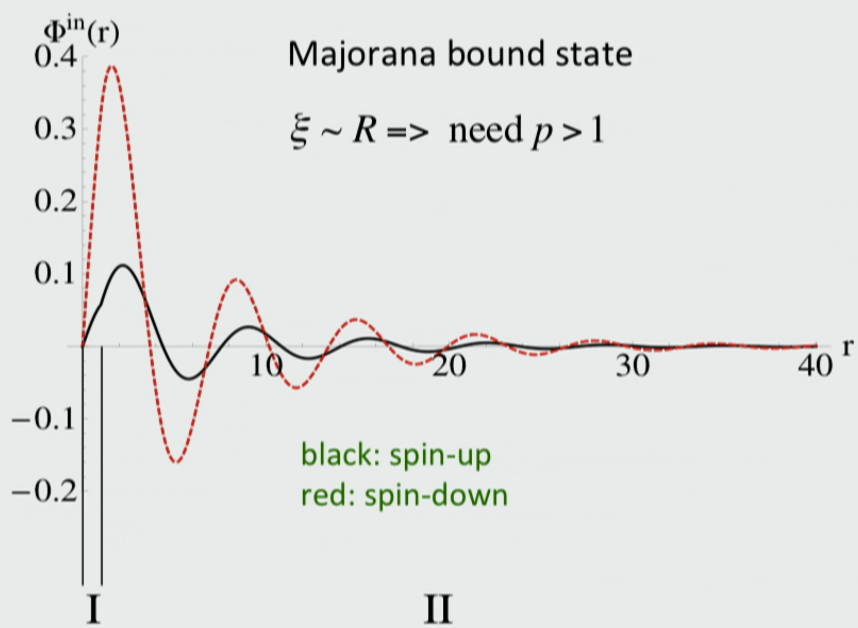
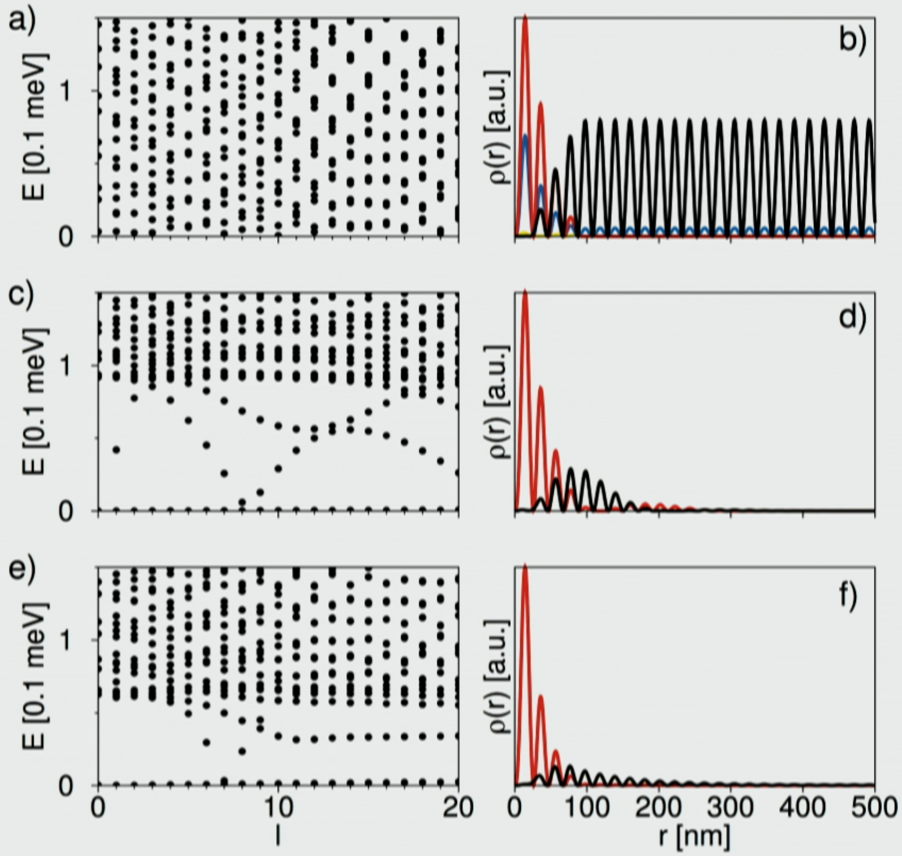


Quasiparticle spectrum



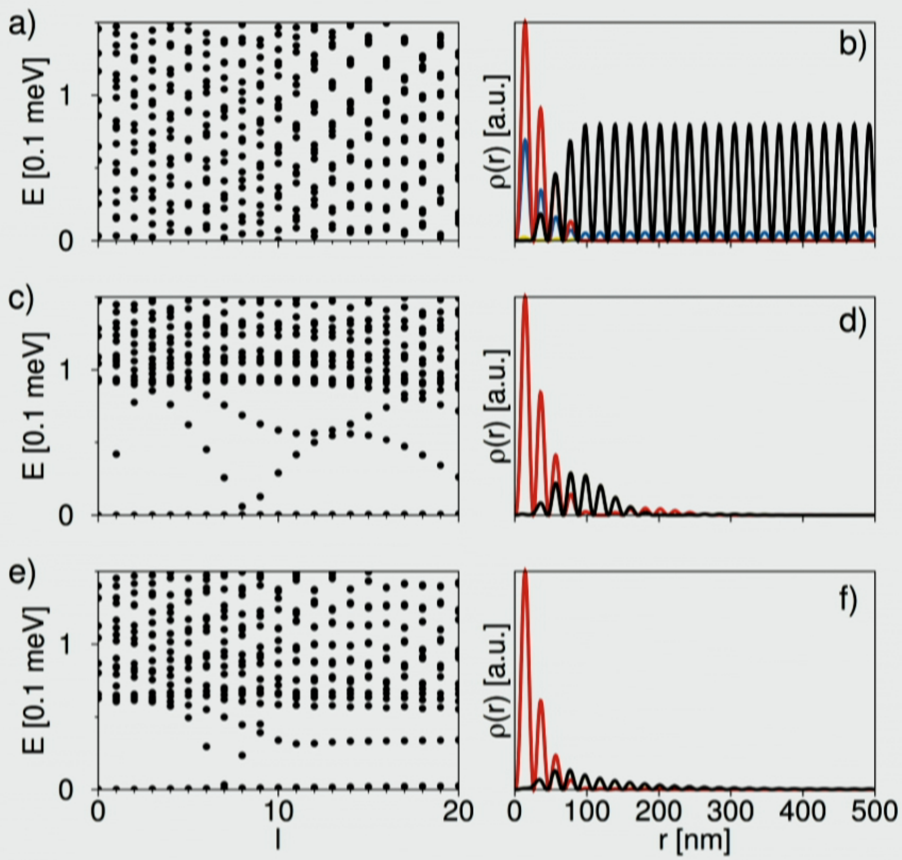
(a-b): p=1 skyrmion

(c-d): p=10 skyrmion

(e-f): p=1 skyrmion with spin-orbit interaction

black: spin-up; red: spin-down

Quasiparticle spectrum



(a-b): p=1 skyrmion

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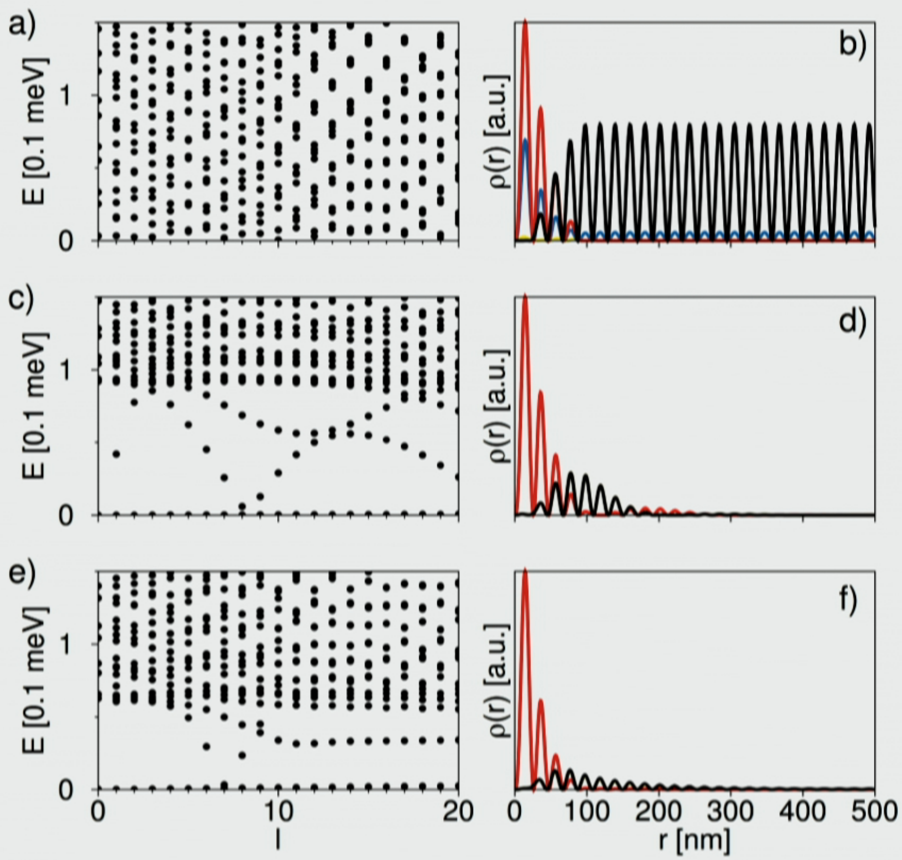
Quasiparticle spectrum



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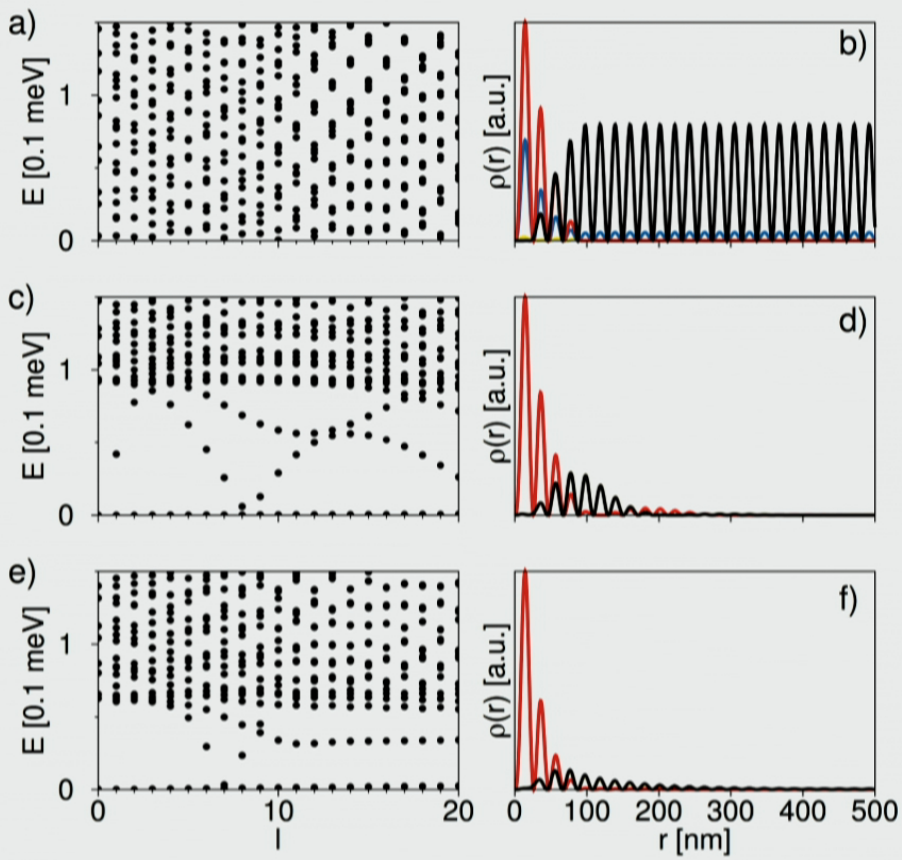

(a-b): p=1 skyrmion

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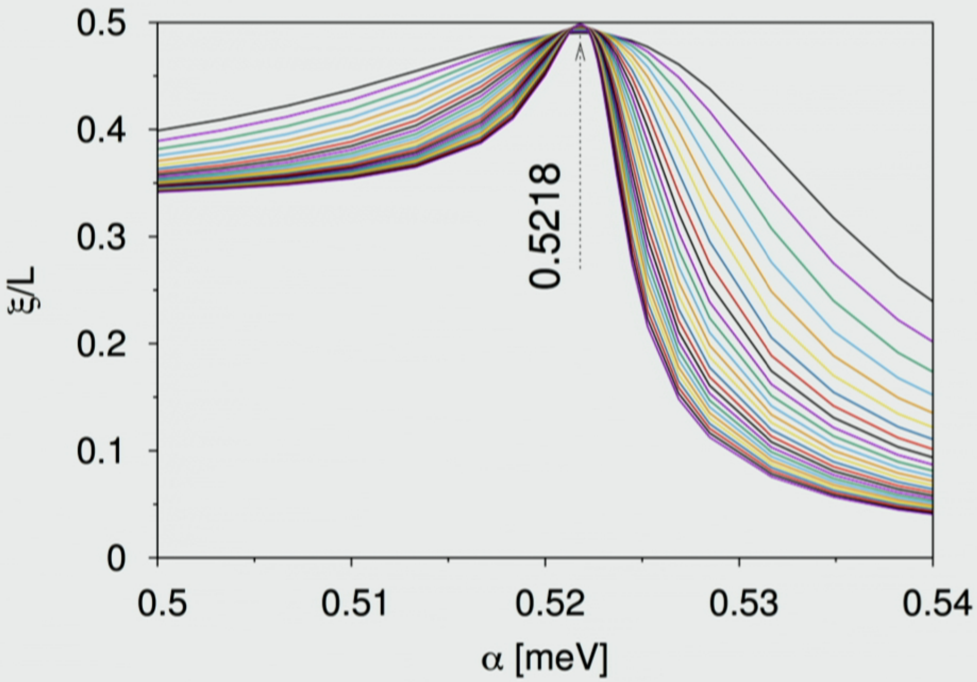
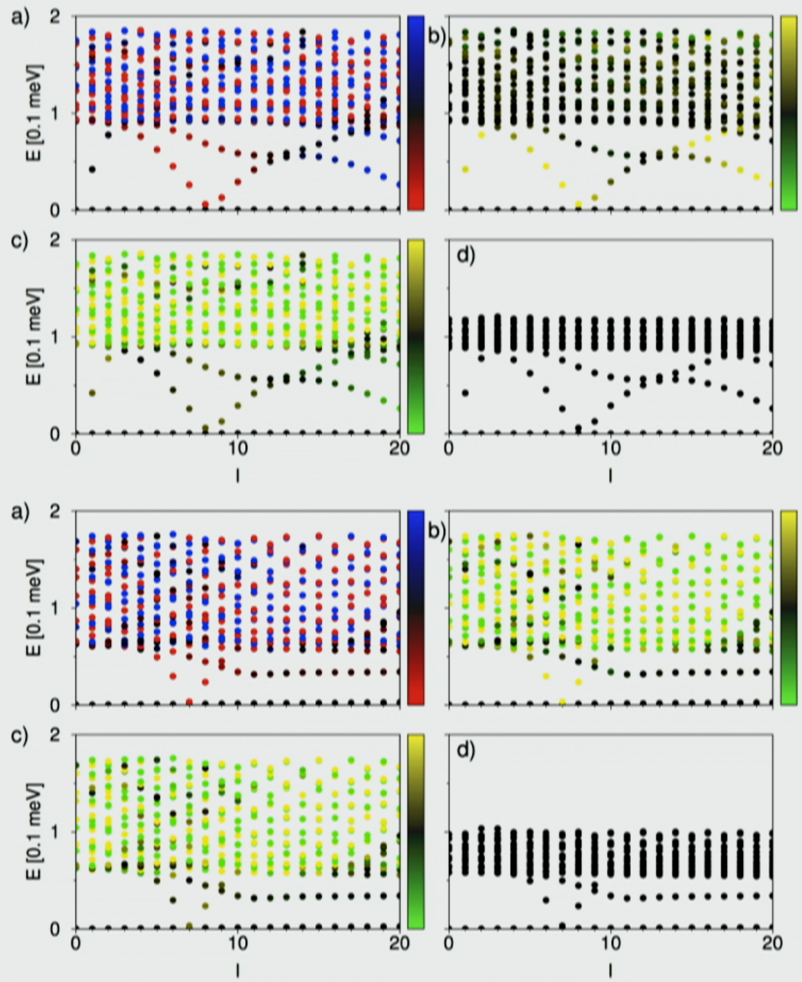
Topological phase transition



$\Delta_0 = 0.5\text{meV}$ $\mu = 0$ $R = 25\text{nm}$ $m = m_e$

phase transition point: $\alpha^2 = \tilde{\mu}^2 + \Delta_0^2$ ➡ $\alpha = 0.5220\text{meV}$ 21

Subgap states

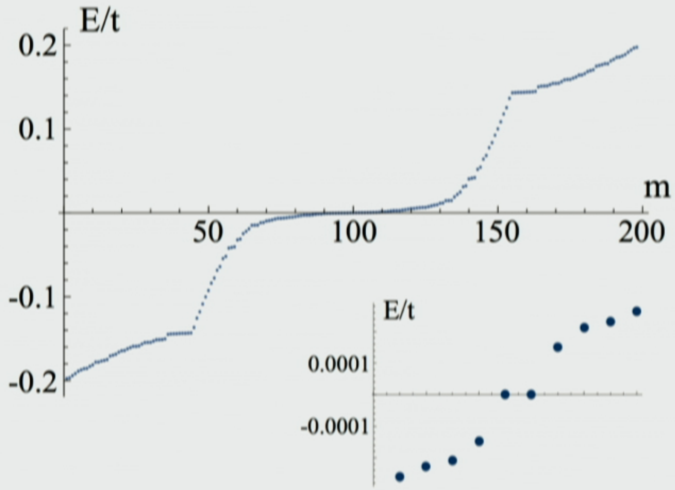


a) charge polarization
b) spin polarization along z
c) spin polarization along N
d) doubled system size

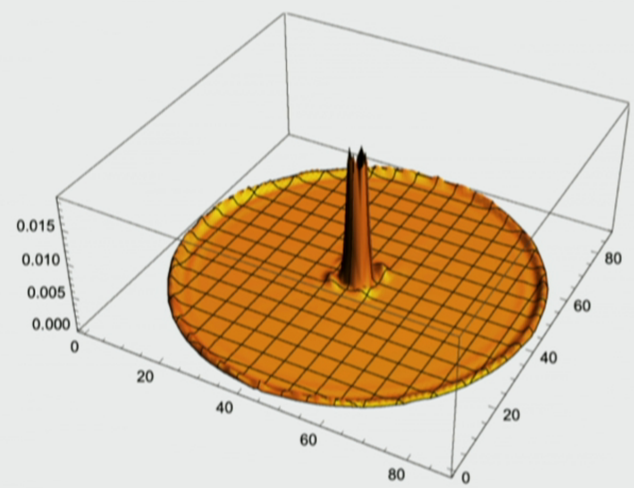
color bar: <-0.1, 0.1>

2D tight-binding calculation

$$H^{BdG} = (-\frac{\nabla^2}{2m} - \mu)\tau_z + \alpha \hat{N} \cdot \vec{\sigma} + \Delta\tau_+ + \Delta^*\tau_-$$



quasiparticle spectrum



Majorana bound state

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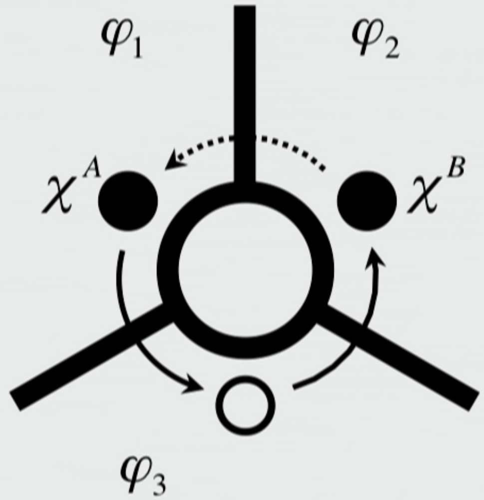
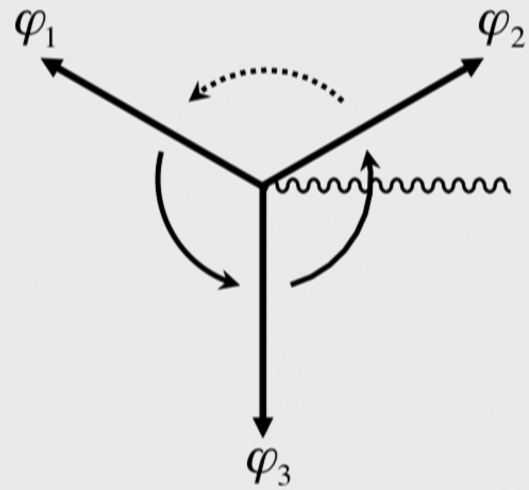
Non-Abelian statistics



● Initial/final positions

○ Intermediate position



$$\Psi_0 \propto e^{i(\varphi/2)\tau_z}$$

$$\Delta = \Delta_0 e^{i\varphi}$$

"Y"-shaped phase differences:

$$\chi_A \to -s\chi_B \quad \chi_B \to s\chi_A \quad (s = \pm 1)$$

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Summary

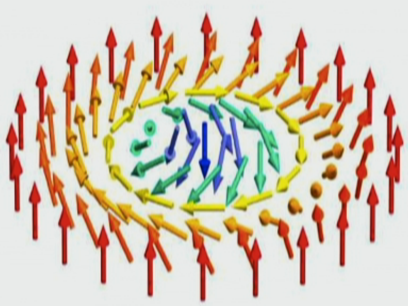
- A Majorana bound state in the core of a magnetic biskyrmion

  when $\alpha^2 > \tilde{\mu}^2 + \Delta_0^2$

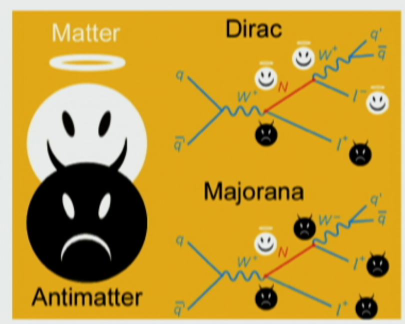
- An extended Majorana fermion outside the biskyrmion

- Well-protected Majorana bound state when $p >> 1$

- Braiding Majorana bound states with superconducting tri-junctions



Thank you !



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