

Title: Entanglement Entropy in nonAbelian Gauge Theories

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Abstract:

Basis of gauge-invariant states in LGT?

Spin network basis

simplest state / vacuum

(Donnelly) notion of entanglement entropy

(Donnelly & Freedman) notion of subsystem (gluing)

(Casini et al) "electric centre"

* inconvenient for coarse graining

Fusion basis: multi-scale

simplest state (TQFT vacuum)

another notion of entanglement entropy

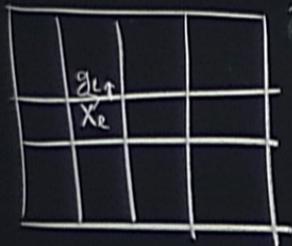
another notion of subsystem (gluing)

generalization of "magnetic centre"
(Need an "electric part")

Fusion basis in Non Ab. Lattice gauge theory

LGT basics

(holonomy) (d flux)



$$u_s \xrightarrow{g_e, X_e} u_t$$

$$g_e \in G, X_e = X_e^i \tau_i \in \text{Lie Alg.}$$

gauge trafo $u_t^{-1} g_e u_s, u_t^{-1} X_e u_s$

$$\{g_e, X_e^i\} = \tau^i g_e$$

$$\{X_e^i, X_e^j\} = f^{ijk} X_e^k$$

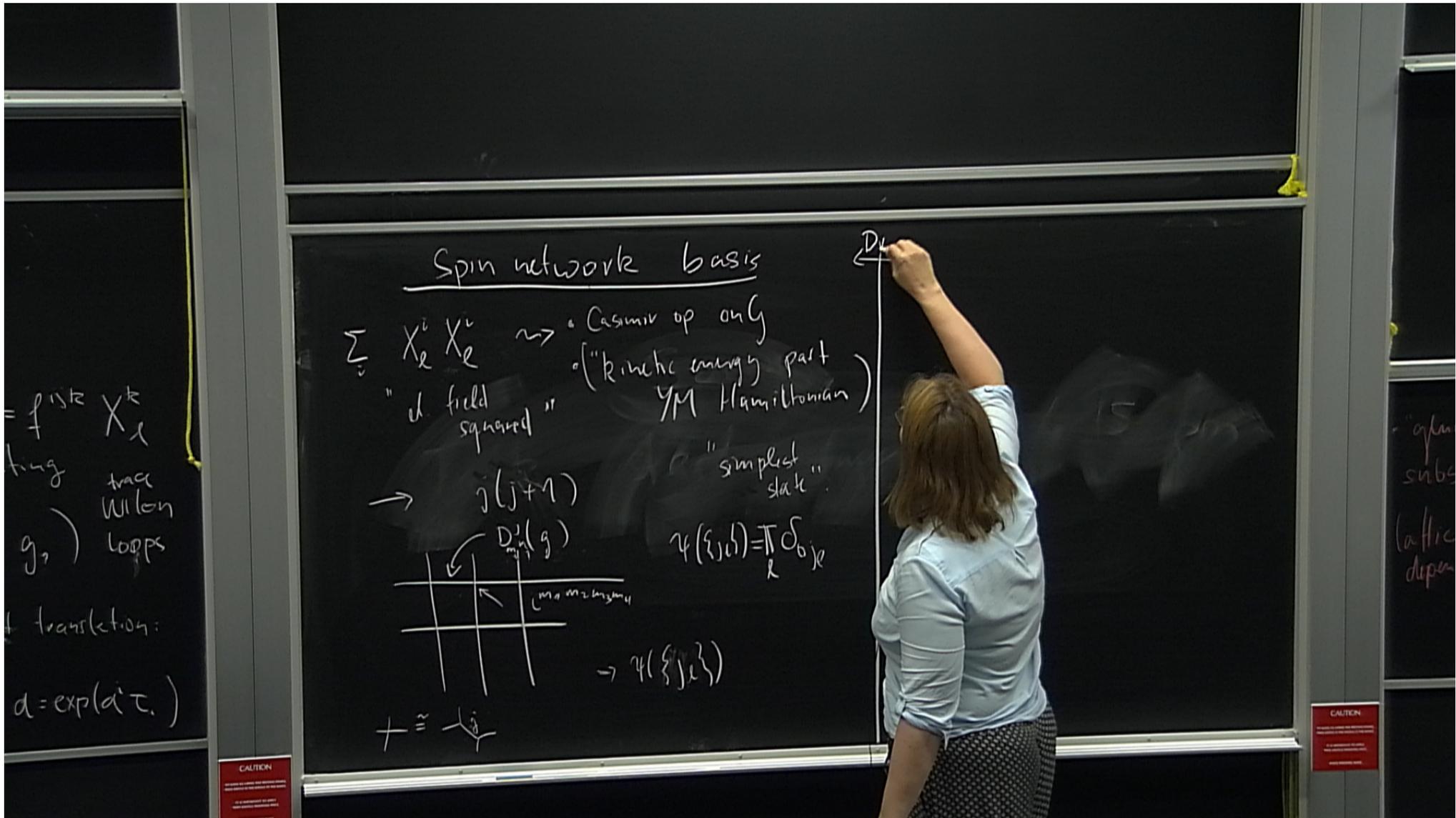
non-commuting

holonomy polarization: $\mathcal{Z}(\{g_e\})$

$\rightarrow \hat{f}(g_e)$ act by mult.

$\rightarrow X_e^i$ act as (li-group) derivatives $\rightsquigarrow \exp(a, \hat{X}^i)$ as (convention) left translation:

For finite group $\Rightarrow \exp(a, \hat{X}^i) f(g) = f(a^{-1} g) \quad a = \exp(a^i \tau_i)$

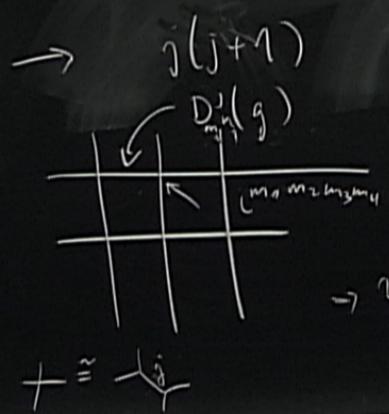


Spin network basis

$\sum_e X_e^i X_e^j \rightsquigarrow$ Casimir op on g
 "field squared" ("kinetic energy part YM Hamiltonian")

"simplest state"

$$\psi(\{j_i\}) = \prod_e \delta_{j_e}^e$$



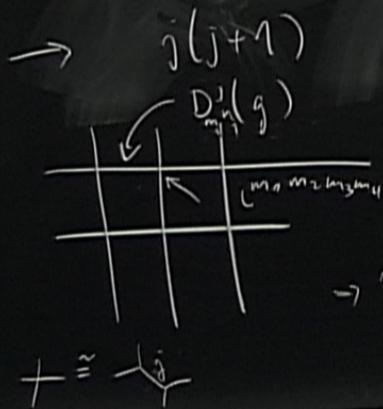
$\rightarrow \psi(\{j_e\})$

first X_e^k
 trace Wilson
 g_i loops
 transition:
 $d = \exp(a^i \tau_i)$

open
 sub
 lattice
 depen

Spin network basis

$\sum_e X_e^i X_e^i \rightsquigarrow$ Casimir op on G
 "field squared"
 ("kinetic energy part of YM Hamiltonian")

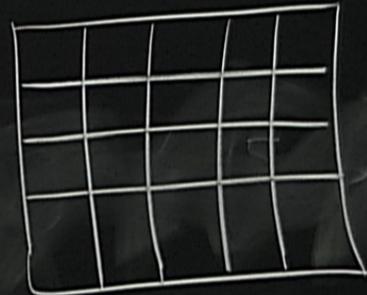


$$\psi(\{j_i\}) = \prod_{\ell} \delta_{\ell}^{j_i}$$

$\rightarrow \psi(\{j_i\})$

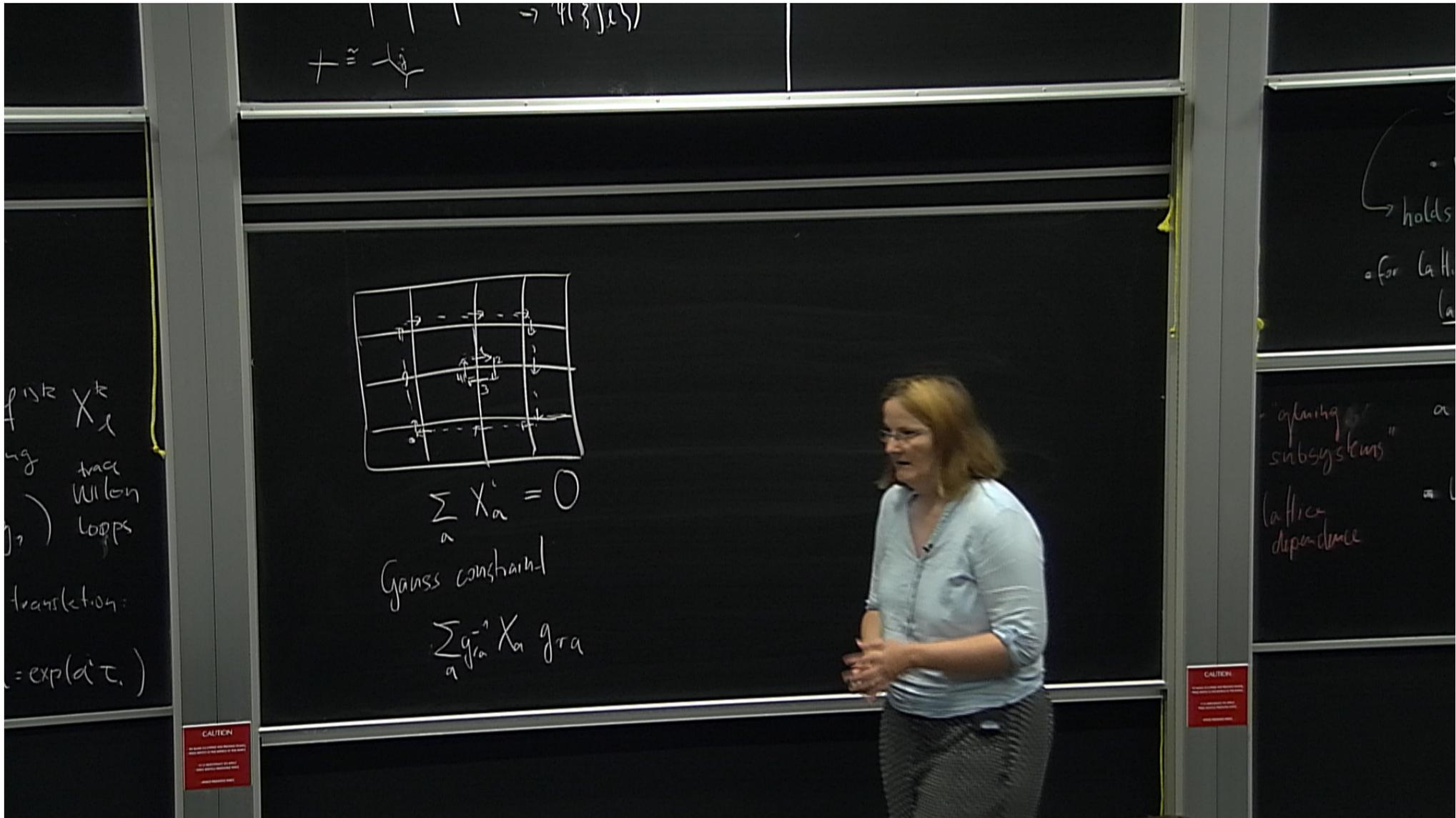
Dual basis

Fusion basis

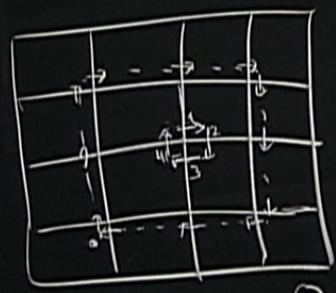


"hierarchical set of tr(Wilson loop)"





$$+ \approx - \rightarrow (1, 3, 2, 5)$$



$$\sum_a X_a = 0$$

Gauss constraint

$$\sum_a g_a^{-1} X_a g_a$$

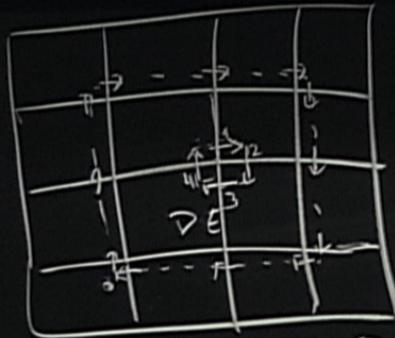
first trace
Wilson
loops
translation:
 $= \exp(a \cdot \tau)$

holds
for lattice
lattice
dependence
subsystems

CAUTION
DO NOT TOUCH THE BOARD
OR THE CHALK

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OR THE CHALK

$$+ \approx \text{diagram}$$



$$\sum_a X_a = 0$$

Gauss constraint

$$\sum_a g_a^{-1} X_a g_a = \text{Wilson loop}$$

$$D_{\text{HM}}^R(\exp a_i X_{\text{loop}}) \quad d\alpha = X(R)$$

$$[\alpha, G] = 0$$

↑
stabilizer of holonomy

"Ribbon operator" : Wilson loop
"Flux loop"

→ hierarchical choice of closed ribbon op

⇒ Fusion basis: $(C, R) = S$

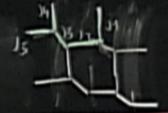
(non-Abelian) Super sectors
 • SNW basis state /

SNW basis

diagonal operators

$$\sum_e X_e^c X_e^c$$

For each link (and expanded links)



* local

Fusion basis

hierarchical set of closed ribbon op → trace Wilson loop
 → (projection of) Flux loop (related to 't Hooft op's)



* non-local / multi-scale

Simplest state / vacuum

$$X_e \equiv 0 \quad (\text{strong coupling})$$

→ all labels trivial

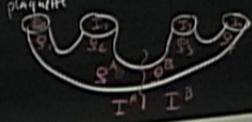
"BF-vacuum". $\text{curl} = 0$, gauge invat: Torsion = 0
 all labels trivial. (no electric & magn charge)

extension of Hilbert space

violates Gauss constraint for each bdry link



violates one flatness and one Gauss constraint (allows for one add. defect)



at the ends of open ribbons

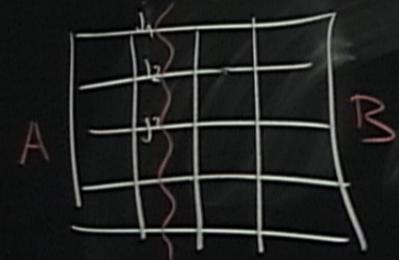
$$= X(R)$$

loop
loop "

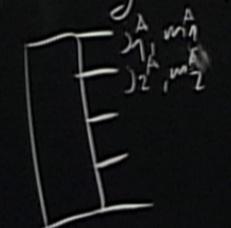
$$R = S$$

\Rightarrow FADIAN GINS (L.N.)

\mathcal{H} (gauge inv. states)



"extending H.S." $\frac{B}{j_1}, m_1^B$



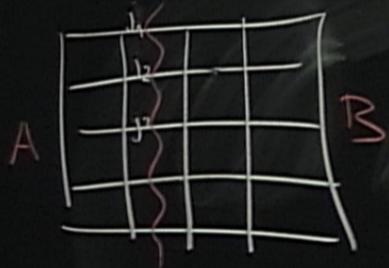
X^k
 ℓ
trace
Wilson
loops
 t_0
 (τ)

CAUTION
BEWARE OF HOT SURFACES
DO NOT TOUCH THE BOARD
WHEN IT IS HOT

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⇒ FUSION RULES (L.N. 3)

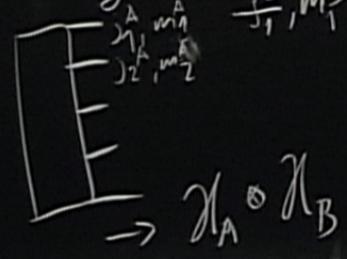
\mathcal{H} (gauge inv. states)



$$\mathcal{H} \ni \Psi \hookrightarrow \mathcal{H}_A \otimes \mathcal{H}_B \rightarrow \text{trace out } B$$

$$S = H(\{s_j\}_{\text{bdry}}) + \langle \sum_{l \text{ bdry}} (\ln \text{dim})_{|e} \rangle + S_A(\{s_j\}_{\mathcal{H}_A})$$

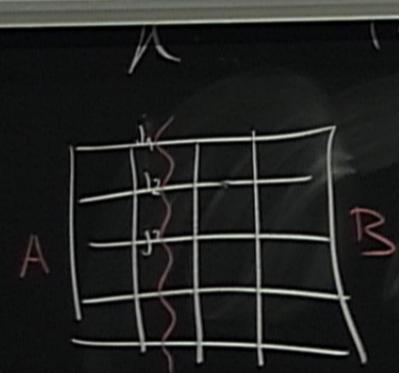
"extending H/S"



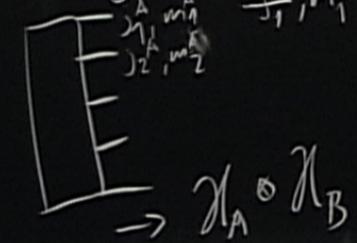
χ^k
trace
Wilson
loops
t.o.s.
 $\chi(\tau)$

CAUTION
DO NOT TOUCH THE BOARD
OR THE CONTENTS OF THE BOARD

$$\sum_a g_a^{-1} \chi_a g_a = \chi_{\text{loop}} \rightarrow \text{hierarchical choice of closed ribbon op} \Rightarrow \text{Fusion basis: } (C, R) = S$$



"extending H/S" $\frac{B}{1}, m_1^B$



$$\chi \ni \psi \hookrightarrow \chi_A \otimes \chi_B \rightarrow \text{trace out } B$$

$$S = H(\{S_a\}) + \langle \dots \ln \det(\rho_B) \dots \rangle + S_A(\{S_a\})$$

$$D = \begin{pmatrix} D & & & \\ & D & & \\ & & D & \\ & & & D \end{pmatrix}$$

"electric center"

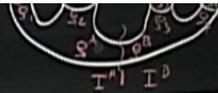
CAUTION

CAUTION

$R = S$
 all out B
 $B \rangle$
 "electric center"

SNW basis
 $\sum_e X_e^i X_e^j$ $\xrightarrow{l: \text{boundary}}$ $\left\{ \begin{array}{l} j \text{ labels} \\ \text{super selection} \\ \text{sectors} \end{array} \right.$
 centre in operator algebra (non-Abelian?)
 entanglement entropy for
 $S = \sum_{\text{boundary links}} \ln \dim(j_e)$
 • SNW basis state / Wilson lines applied to vacuum
 • vacuum ($X=0$): $S=0$
 \rightarrow holds in ($X=0$ a.c.) continuum limit
 • for lattice BF vacuum: lattice dependent, divergent in continuum limit

Fusion basis
 closed ribbon operator along boundary \rightarrow $\left\{ \begin{array}{l} S \text{ labels} \\ \text{super selection} \\ \text{sectors} \end{array} \right.$
 (+ representation indices)
 • Fusion basis state
 $S = \ln \dim(\mathcal{B}_0)$
 $\left\{ \begin{array}{l} \text{generated from BF vac=} \\ \text{by applying } N \text{ oper} \\ \text{ribbons} \end{array} \right.$
 $S = \sum_{a=1}^N \ln \dim(\rho_a)$
 • BF vacuum $S=0$
 \rightarrow holds in ($\text{Cur} = \text{BF}$) continuum limit



• vacuum $(X=0)$: $S=0$
 holds in $(X=0)$ a.e. continuum limit
 • for lattice BF vacuum, lattice dependent, divergent in continuum limit

• BF vacuum $S=0$
 $S = \sum_{a=1}^4 \ln |a| (S_a)$
 holds in $(Curv=0)$ a.e. continuum limit
 = (BF) TQFT

"opening subsystems"
 lattice dependence

assumes $X=0$ away from links
 = lattice generated from Wilson lines/loops

assumes $Curv=0, Tors=0$ near boundary

only depends on position of defect excitations, generated at the ends of open ribbons