Title: Boundary States and Entanglement Spectrum from Strong Subadditivity

Date: Jul 21, 2016 05:00 PM

URL: http://pirsa.org/16070061

Abstract: In this talk I will consider quantum states satisfying an area law for entanglement (e.g. as found in quantum field theory or in condensed matter systems at sufficiently low temperature). I will show that both the boundary state and the entanglement spectrum admit a local description whenever there is no topological order. The proof is based on strong subadditivity of the von Neumann entropy. For topological systems, in turn, I'll show that the topological entanglement entropy quantifies exactly how many extra bits are needed in order to have a local description for the boundary state. This latter result is based on a recent strengthening of strong subadditivity.

Based on joint work with Kohtaro Kato (University of Washington)

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Quantum Physics of Information

Coordinators: Fernando Brandao, Veronika Hubeny, Stephen Jordan, Renato Renner

Scientific Advisors: Daniel Harlow, Patrick Hayden, John Preskill

Quantum information science began about 30 years ago as the union of two of the biggest scientific developments of the last century, quantum mechanics and computer science. The original motivation was to understand the new possibilities offered by quantum mechanics to information processing and computation. In recent years, it has emerged that the field also offers a new perspective for the study of physics, from condensed matter and thermodynamics to quantum gravity.

Recent breakthroughs in the application of quantum information in other areas of physics have served to increase the interest of researchers in high energy, gravity, and condensed matter, to work at the intersection of these fields. This program has the objective of cultivating these growing interdisciplinary discussions. It is the hope that the program will be a turning point in the exciting, rapidly developing dialogue between quantum information and physics as a whole.

Some of the topics covered in the program are quantum computational complexity in physics and the simulation of physical systems, advances in concepts and methods of quantum information of relevance to many body physics and thermodynamics, as well as the role of quantum information in quantum gravity, field theory, and the foundations of quantum theory.

DATES

Sep 18, 2017 - Dec 15, 2017

INFORMATION

Apply

Application deadline is:
Oct 2, 2016.

Deadline: October 2

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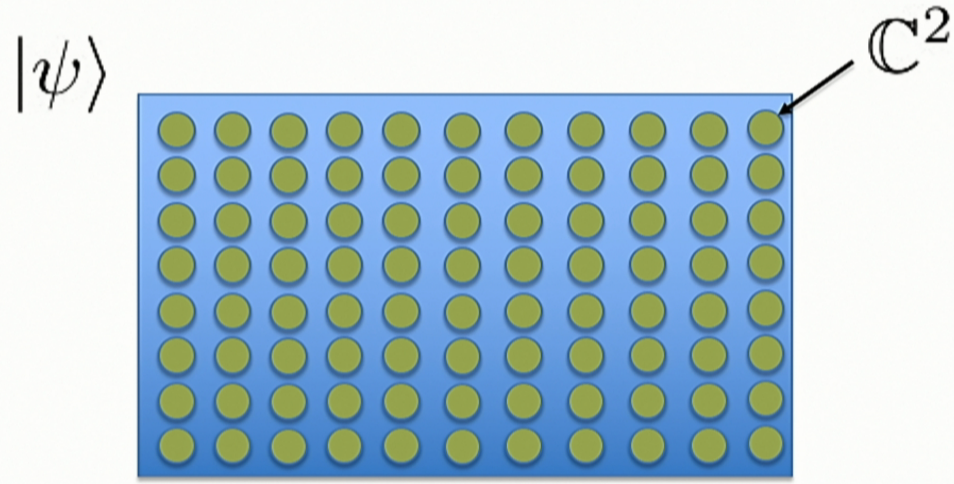
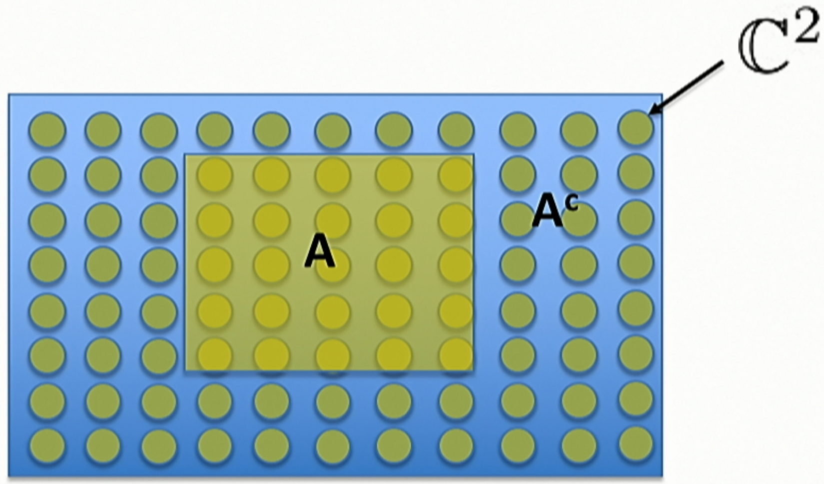
Entanglement in Many-Body Quantum States

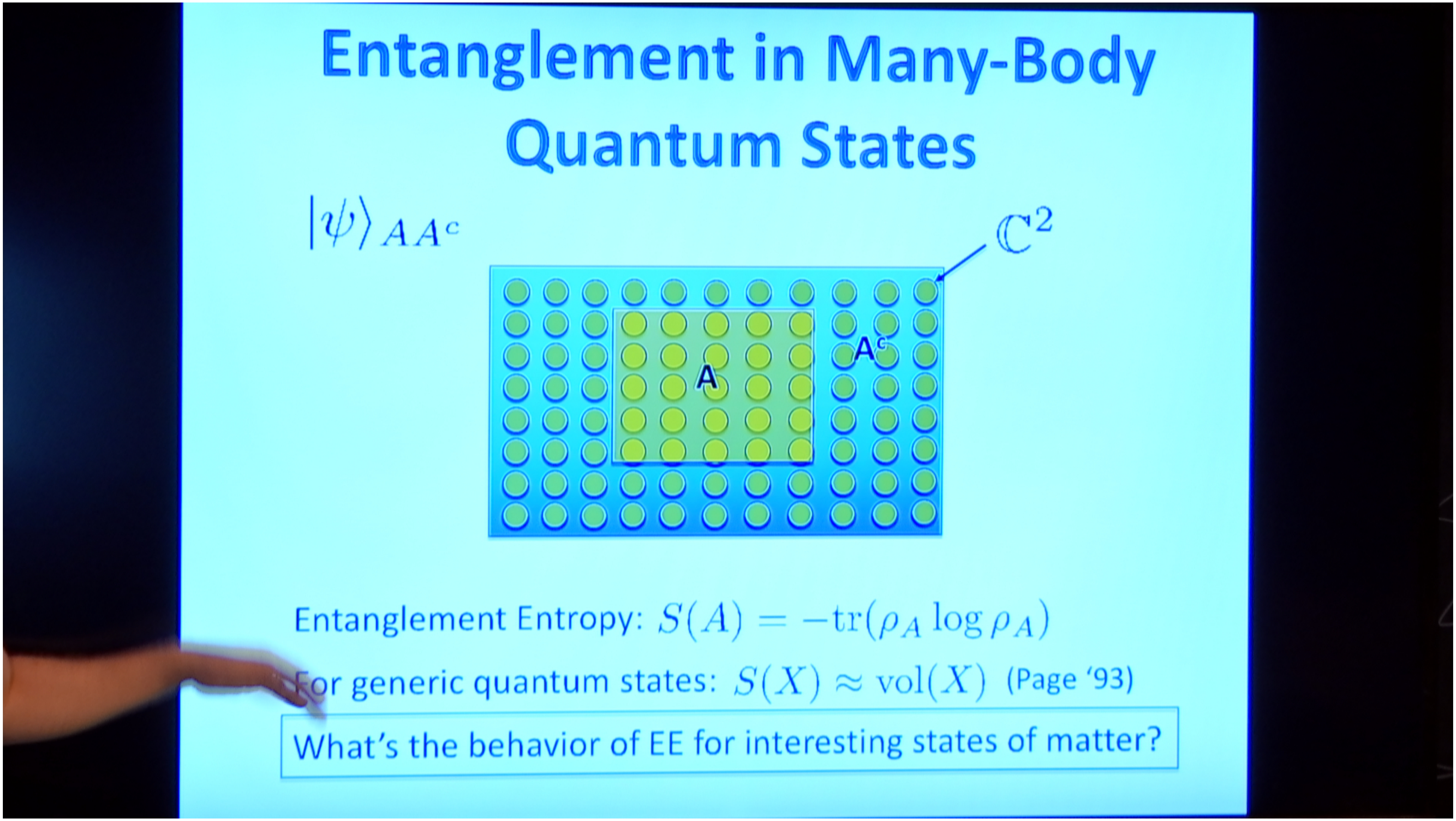


Entanglement in Many-Body Quantum States

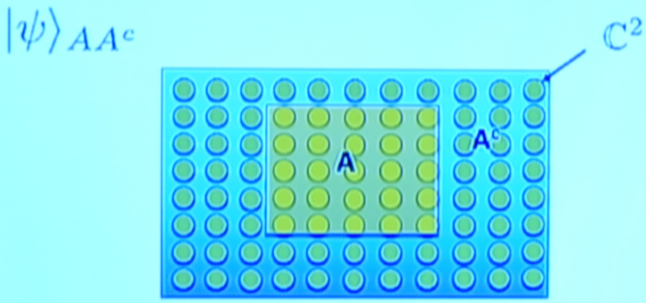
$|\psi\rangle_{AA^c}$



Entanglement Entropy: $S(A) = -\mathrm{tr}(\rho_A \log \rho_A)$


Entanglement in Many-Body Quantum States
$|\psi\rangle_{AA^c}$
$\mathbb{C}^2$
A
A^c
Entanglement Entropy: $S(A) = -\mathrm{tr}(\rho_A \log \rho_A)$
For generic quantum states: $S(X) \approx \mathrm{vol}(X)$ (Page '93)
What's the behavior of EE for interesting states of matter?

Area Law

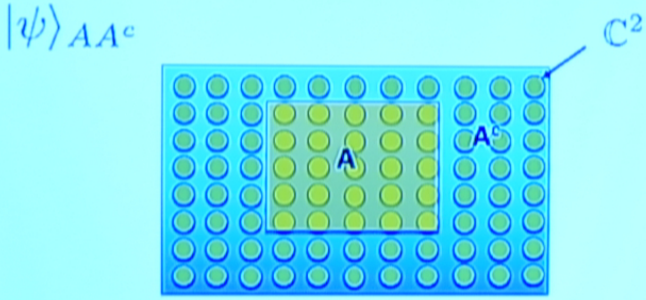


Entanglement is "localized", concentrated around the boundary

For every region *X*: $S(X) = \alpha|\partial X| - \gamma + \ldots$

e.g. gapped models, 2+1 CFT (from RT formula)

Area Law



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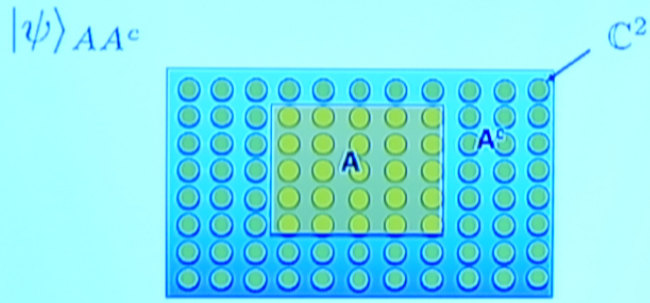
For every region *X*: $S(X) = \alpha|\partial X| - \gamma + \ldots$

γ: Topological EE
(signature topological order)

$\gamma = \log \mathcal{D}, \quad \mathcal{D} = \sqrt{\sum_a d_a^2}$ *D*: Quantum dimension

Area Law



Entanglement is "localized", concentrated around the boundary

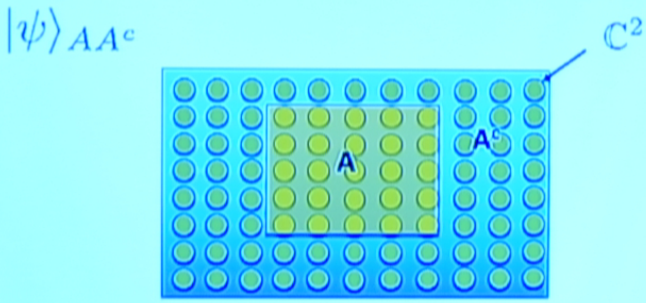
For every region *X*: $S(X) = \alpha|\partial X| - \gamma + \ldots$

- Topological EE quantifies "non-local entanglement"

(Kitaev '12) $\gamma = 0$: state is adiabatically connected to trivial phase
(Kim '13) $\log(N) \leq 2\gamma$ N := number topologically protected states

⋮

Area Law

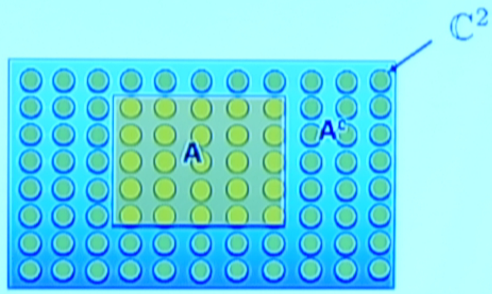


Entanglement is "localized", concentrated around the boundary

What are the consequences of an area law?
What's the influence of TEE on the boundary?

Area Law

$|\psi\rangle_{AA^c}$



Entanglement is “localized”, concentrated around the boundary

What are the consequences of an area law?
What’s the influence of TEE on the boundary? This talk:

Area Law $\rightarrow$ TEE determines locality of
i) Boundary State
ii) Entanglement Spectrum

by strong subaddivitity and
stronger subaddivitity

Quantum Information 1.01: Fidelity

... it's a measure of distinguishability between two quantum states.

Given two quantum states their fidelity is given by

$$F(\rho,\sigma) := \mathrm{tr}((\rho^{1/2}\sigma\rho^{1/2})^{1/2})$$

It tells how distinguishable they are by any quantum Measurement

Ex 1: F=1: same state

Ex 2: F=0 : perfectly distinguishable states

Quantum Information 1.01: Relative Entropy

... it's another measure of distinguishability between two quantum states.

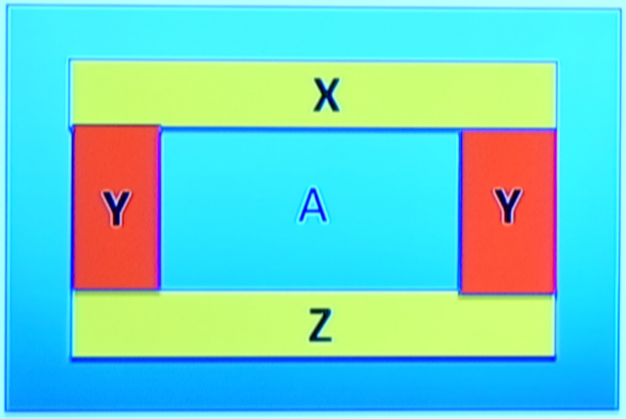
Def: $S(\rho\|\sigma) := \mathrm{tr}(\rho(\log(\rho) - \log(\sigma)))$

Gives optimal exponent for distinguishing the two states

Pinsker's inequality: $S(\rho\|\sigma) \geq -\frac{1}{2}\log F(\rho,\sigma)$

$$S(\rho\|\sigma) \approx 0 \implies \rho \approx \sigma$$

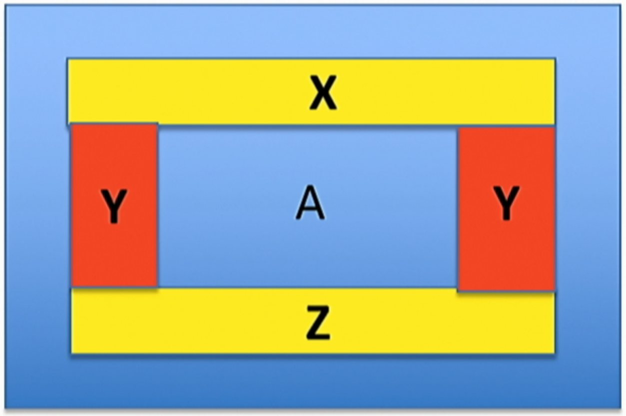
Topological EE and Locality of Boundary States



ρ_{XYZ}: reduced state on XYZ

XYZ Boundary of A

Topological EE and Locality of Boundary States



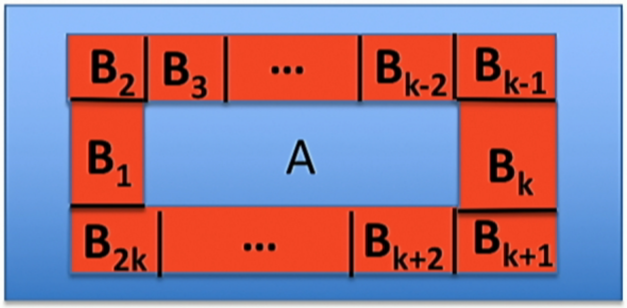
ρ_{XYZ}: reduced state on XYZ

XYZ Boundary of A

Result 1. If $S(X) = \alpha|\partial X| - \gamma + \ldots$:

$$\gamma \quad \approx \quad \min_{H_{XY}, H_{YZ}} S(\rho_{XYZ} \| \exp(H_{XY} + H_{YZ})/\mathrm{tr}(\ldots))$$

Topological EE and Locality of Boundary States



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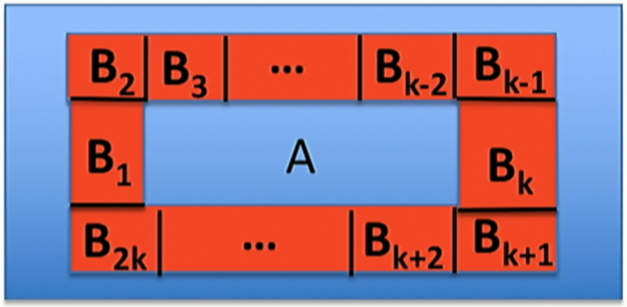
$$\gamma \approx \min_{H_{XY}, H_{YZ}} S(\rho_{XYZ} \| \exp(H_{XY} + H_{YZ})/\mathrm{tr}(...))$$

$$\approx \min_{H_{B_1 B_2}, \ldots, H_{B_{2k-1} B_{2k}}} S(\rho_{B_1 \ldots B_{2k}} \| \exp(H_{B_1 B_2} + \ldots + H_{B_{2k-1} B_{2k}})/\mathrm{tr}(...))$$

Topological EE and Locality of Boundary States



ρ_{XYZ}: reduced state on XYZ

XYZ Boundary of A

Obs 1: $\gamma = 0$

$$\Longrightarrow \quad \rho_B \approx \exp(H_{B_1 B_2} + \ldots H_{B_{2k-1} B_{2k}}/\mathrm{tr}(\ldots))$$

Obs 2: Thermal states has same on-site symmetries as original state

Obs 3: Thermal state is max entropy state consistent with local constraints

TEE gives number of non-local bits

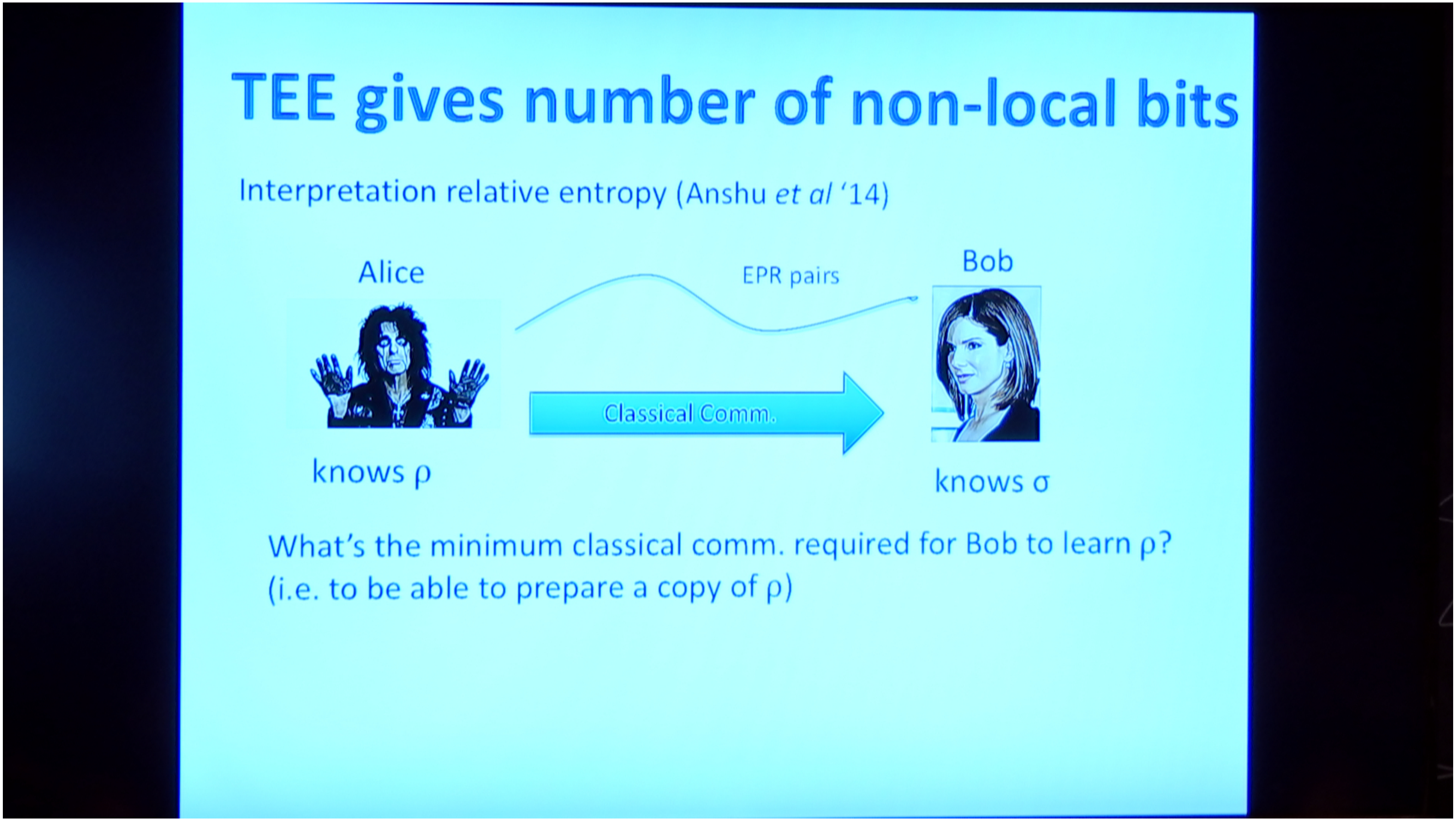
Interpretation relative entropy (Anshu *et al* '14)

Alice

knows ρ

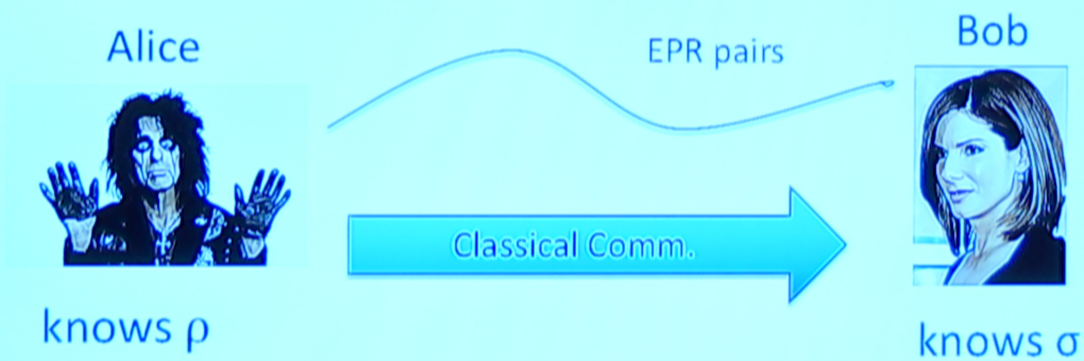
Bob

knows σ


TEE gives number of non-local bits
Interpretation relative entropy (Anshu et al '14)
Alice
EPR pairs
Bob
Classical Comm.
knows ρ
knows σ
What's the minimum classical comm. required for Bob to learn ρ?
(i.e. to be able to prepare a copy of ρ)

TEE gives number of non-local bits

Interpretation relative entropy (Anshu *et al* '14)



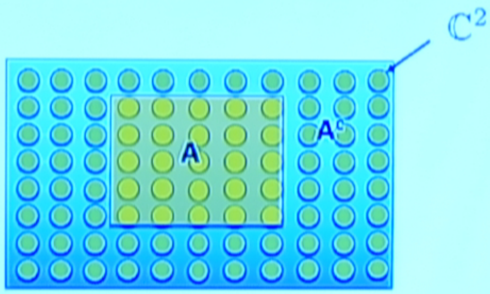
$\approx S(\rho\|\sigma)$ necessary and sufficient for Bob to prepare a copy of ρ

$$\boxed{\gamma \approx \min_{\sigma \in \text{Local Gibbs State}} S(\rho_{B_1 \ldots B_{2k}} \| \sigma) \text{ gives number of non-local bits of } \rho}$$

obs: Consistent with γ = log(quantum dimension)

Entanglement Spectrum

$|\psi\rangle_{AA^c}$



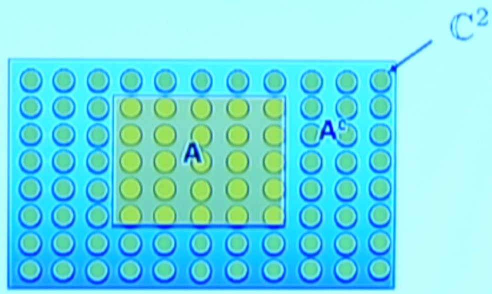
$\lambda(\rho_A)$: eigenvalues of ρ_A
Entanglement Spectrum

Area law statement about $-\sum_i \lambda_i \log \lambda_i$

What can we say about the whole spectrum?

Entanglement Spectrum

$|\psi\rangle_{AA^c}$



$\lambda(\rho_A)$: eigenvalues of ρ_A
Entanglement Spectrum

Area law statement about $-\sum_i \lambda_i \log \lambda_i$

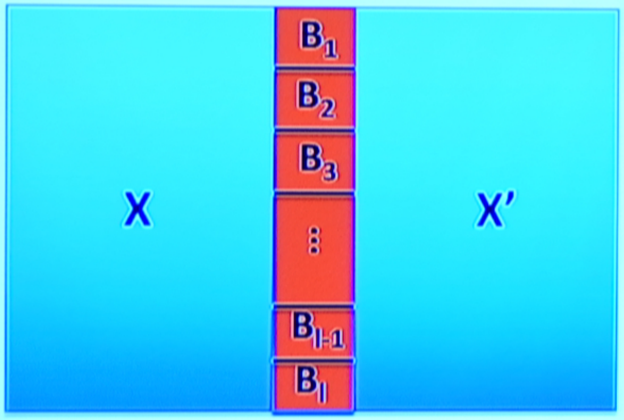
What can we say about the whole spectrum?

(Haldane, Li '08, Cirac, Poiblanc, Schuch, Verstraete '11, ...)

γ=0: matches spectrum thermal state local model

$\gamma\neq$0: matches spectrum thermal state local model after projecting into topological superselection sector

Entanglement Spectrum



We assume translation invariance s.t. $\rho_X = \rho_{X'}$

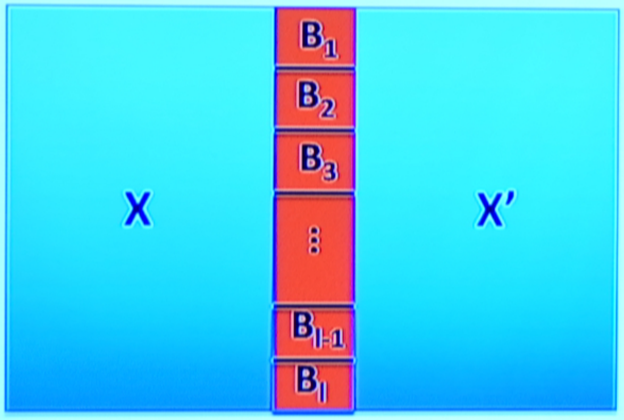
Result 2: If $S(X) = \alpha|\partial X| - \gamma + \ldots$:

$$\gamma = 0 \implies \lambda(\rho_X)^{\otimes 2} \approx \lambda(e^{\sum_k H_{B_k, B_{k+1}}})$$

$$\gamma \neq 0 \implies \lambda(\rho_X)^{\otimes 2} \approx \lambda(\sigma),$$

$$\mathrm{tr}_{B_1}(\sigma) = e^{\sum_{k>1} H_{B_k, B_{k+1}}}$$

Result 2 from 1



From area law assumption:
(more later)

$$\rho_{XX'} \approx \rho_X \otimes \rho_{X'}$$

$$\lambda(\rho_{XX'}) = \lambda(\rho_B) \Longrightarrow \lambda(\rho_X) \otimes \lambda(\rho_{X'}) \approx \lambda(\rho_B)$$

Uhlmann's theorem There is an isometry U : B -> $B_X B_{X'}$ s.t.

$$U|\psi\rangle_{XBX'} \approx |\phi\rangle_{XB_X} \otimes |\phi'\rangle_{XB_{X'}} \quad \rho_X = \mathrm{tr}_{B_X}(|\phi\rangle\langle\phi|_{XB_X})$$

U maps degrees of freedom of X and X' into B

Why does it hold?

We want to show:

$$\boxed{\begin{aligned} \gamma \;&\approx \min_{H_{XY},H_{YZ}} S(\rho_{XYZ} \| \exp(H_{XY} + H_{YZ})/\mathrm{tr}(...)) \\ &\approx \min_{H_{B_1B_2},\ldots,H_{B_{2k-1}B_{2k}}} S(\rho_{B_1\ldots B_{2k}} \| \exp(H_{B_1B_2} + \ldots + H_{B_{2k-1}B_{2k}})/\mathrm{tr}(...)) \end{aligned}}$$

$\chi = 0$: follow from *strong subadditivity* (SSA) (Lieb, Ruskai '73)

$$S(AB) + S(BC) \leq S(ABC) + S(B)$$

$\chi \neq 0$: fol[illegible]s from a *strengthening* of SSA (Fawzi and Renner '14)

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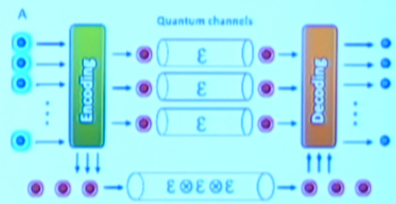
Applications of SSA

Used to prove optimal rates for nearly every quantum information protocol.

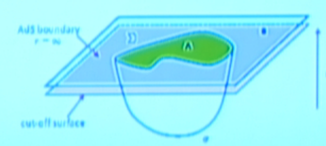
- Channel capacities (classical, quantum, private)
- Distillable Entanglement
- ....



(Casini, Huerta, Myers ...) SSA + Lorentz Invariance:

- Entropic proof of the *c*-theorem (irreversibility of renormalization flow)
- Proof of Bekenstein's and Bousso's bound

(Ryu-Takayanagi, Headrick, ...) Test for holographic proposals of entropy



Many others...

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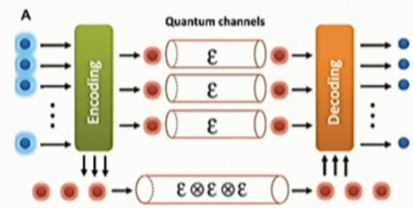

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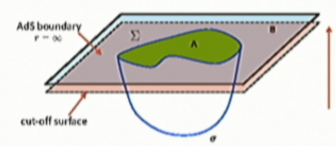

Many others...

Conditional Mutual Information

Given ρ_{ABC},

$$\begin{aligned} I(A:C|B) &:= S(AB) + S(BC) - S(ABC) - S(B) \\ &= S(\rho_{ABC} \| \exp(\log(\rho_{AB}) + \log(\rho_{BC}) - \log(\rho_B))) \end{aligned}$$

Strong subadditivity: $I(A:C|B) \geq 0$

Conditional Mutual Information

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Strong subadditivity: $I(A:C|B) \geq 0$

Stronger subadditivity (Fawzi-Renner '14):

$$I(A:C|B) \geq \frac{1}{2} \min_{\Lambda: B \to BC} -\log(F(\rho_{ABC}, \Lambda(\rho_{AB})))$$

$$I(A:C|B) \approx 0 \implies I_A \otimes \Lambda^{B \to BC}(\rho_{BC}) \approx \rho_{ABC}$$

quantum channel

Conditional Mutual Information

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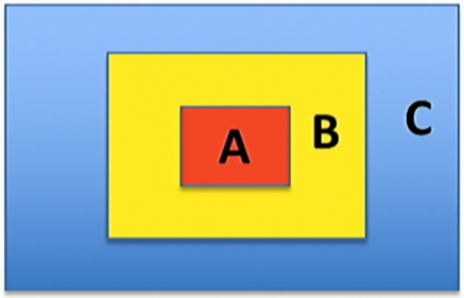
Stronger subadditivity (Fawzi-Renner '14):

$$I(A:C|B) \geq \frac{1}{2} \min_{\Lambda: B \to BC} -\log(F(\rho_{ABC}, \Lambda(\rho_{AB})))$$

Can reconstruct the state ABC from reduction on AB by acting on B only

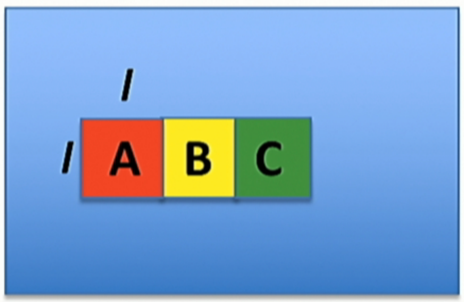
$$A \longrightarrow B \quad \Longrightarrow \quad A \longrightarrow B \longrightarrow C$$

Consequence of Area Law: State Reconstruction



For every ABC with trivial topology:

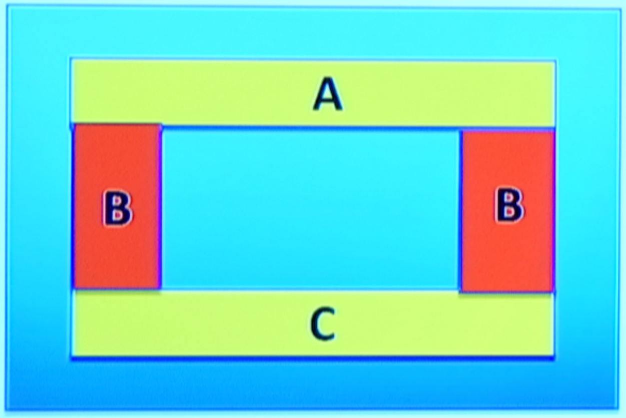
$$I(A:C|B) \approx 0$$



$$\begin{aligned} & I(A:C|B) \\ = \quad & S(AB) + S(BC) - S(ABC) - S(B) \\ = \quad & \alpha(|\partial(AB)| + |\partial(BC)||\partial(ABC)| - |\partial(B)|) + \dots \\ = \quad & \alpha(6l + 6l - 8l - 4l) + \dots \end{aligned}$$

TEE as Conditional Mutual Info

(Kitaev, Preskill '05, Levin, Wen '05)



$$\gamma = I(A:C|B) + \ldots$$

$$\begin{aligned} & I(A:C|B) \\ = \; & S(AB) + S(BC) - S(ABC) - S(B) \\ = \; & \alpha(\partial(AB) + |\partial(BC)| - |\partial(ABC)| - |\partial(B)|) - \gamma - \gamma + \gamma + 2\gamma + \ldots \\ = \; & \gamma + \ldots \end{aligned}$$

Non zero TEE gives an obstruction to reconstruct ρ_{ABC} from ρ_{AB} by acting on B

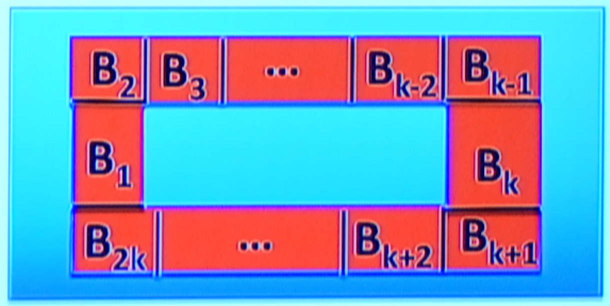
Why does it work?

We want to show:

$$\gamma \approx \min_{H_{XY},H_{YZ}} S(\rho_{XYZ} \| \exp(H_{XY}+H_{YZ})/\mathrm{tr}(...))$$

$$\approx \min_{H_{B_1B_2},\dots,H_{B_{2k-1}B_{2k}}} S(\rho_{B_1\dots B_{2k}} \| \exp(H_{B_1B_2}+\dots+H_{B_{2k-1}B_{2k}})/\mathrm{tr}(...))$$

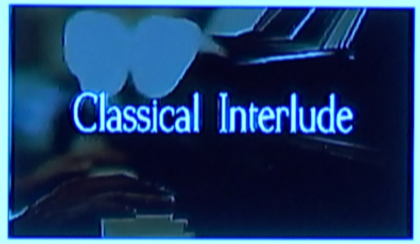
Let's start with the case ɣ=0.



Need to show $\rho_{B_1\dots B_{2k}}$ is close to thermal assuming all conditional mutual information are small, i.e. **approximately independence**

$$I(B_1\dots B_{j-1} : B_{j+1}\dots B_{2k-1} | B_j B_{2k}) \approx 0$$

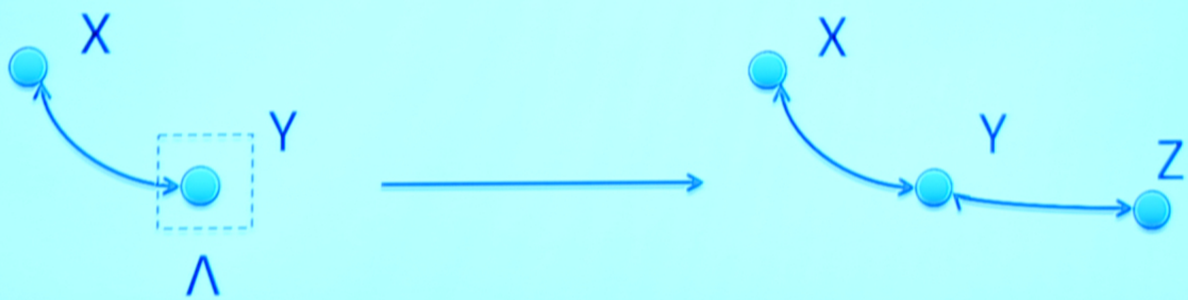
Markov Chain



X, Y, Z with distribution p(x, y, z)

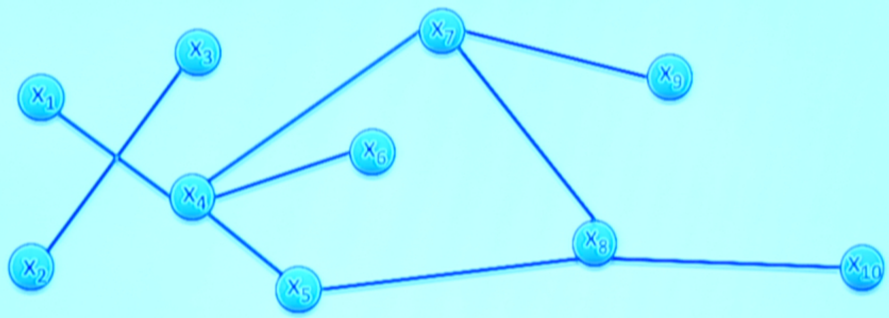
i) *X-Y-Z* Markov if *X* and *Z* are independent conditioned on *Y*

ii) *X-Y-Z* Markov if there is a channel $\Lambda : Y \rightarrow YZ$ s.t. $\Lambda(p_{XY}) = p_{XYZ}$

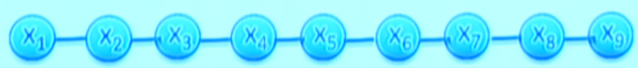


iii) $I(X:Y|Z)_p \;=\; \mathbb{E}_{z \sim p(z)} I(X:Y)_{p(x,y|z=z')}$

Markov Networks

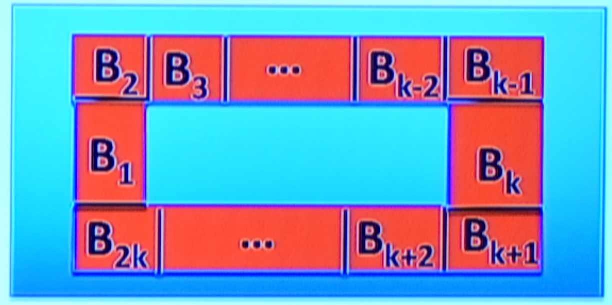


We say $X_1, \ldots, X_n$ on a graph G form a Markov Network if X_i *is* indendent of all other X's conditioned on its neighbors

Ex: Markov chains 

Going Back

Need to show $\rho_{B_1 \ldots B_{2k}}$
is close to thermal assuming all
conditional mutual information
are small (approximately independence)



$$I(B_1 \ldots B_{j-1} : B_{j+1} \ldots B_{2k-1} | B_j B_{2k}) \approx 0$$

We want a **quantum** and **approximate** version of **Hammersley-Clifford**,
but only for 1D chains

Quantum Markov Chain

Classical: X, Y, Z with distribution p(x, y, z)

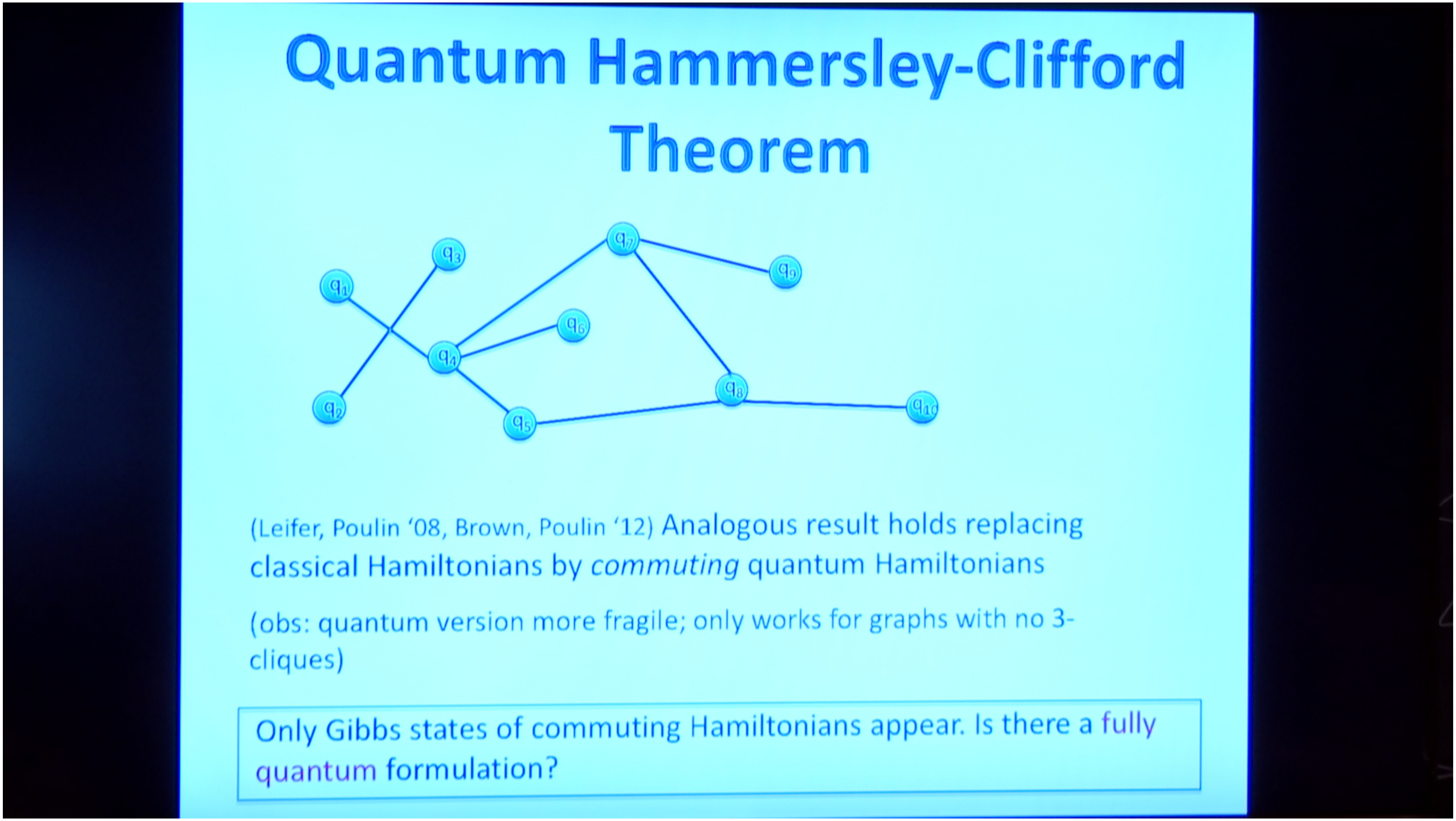
i) *X-Y-Z* Markov if *X* and *Z* are independent conditioned on *Y*

ii) *X-Y-Z* Markov if there is a channel $\Lambda : Y \to YZ$ s.t. $\Lambda(p_{XY}) = p_{XYZ}$

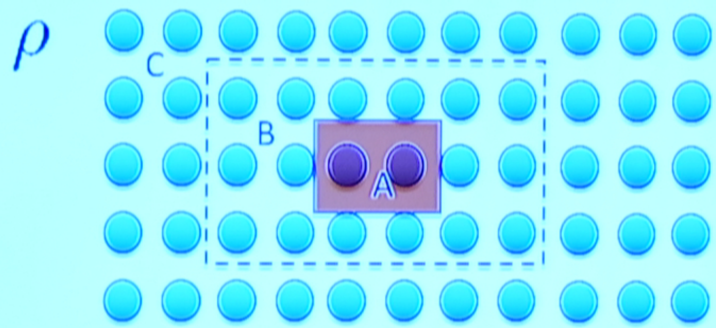
Quantum: (Hayden, Jozsa, Petz, Winter '03)

i) ρ_{ABC} Markov quantum state if A and C are "independent conditioned" on B, i.e. $H_B \simeq \bigoplus_k H_{B_{L,k}} \otimes H_{B_{R,k}}$ and

$$\rho_{ABC} = \bigoplus_k p_k \rho_{AB_{L,k}} \otimes \rho_{B_{R,k}C}$$


Quantum Hammersley-Clifford Theorem
q1
q3
q7
q9
q6
q4
q2
q8
q10
q5
(Leifer, Poulin '08, Brown, Poulin '12) Analogous result holds replacing classical Hamiltonians by *commuting* quantum Hamiltonians
(obs: quantum version more fragile; only works for graphs with no 3-cliques)
Only Gibbs states of commuting Hamiltonians appear. Is there a fully quantum formulation?

Q. Approximate Markov States



ρ quantum approximate Markov if for every A, B, C

$$I(A:C|B) \to 0 \text{ when } \mathrm{dist}(A,C) \to \infty$$

Conjecture

Quantum Approximate Markov $\Leftrightarrow$ Gibbs state local Hamiltonian

$$\rho = e^{\sum_k H_k}$$

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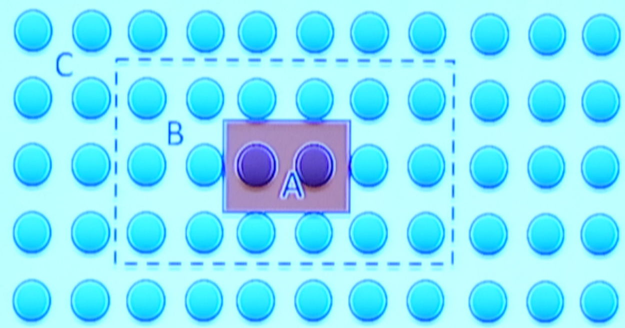

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Approximate Quantum Markov Chains are Thermal

thm

1. Let H be a local Hamiltonian on n qubits. Then

$$I(A:C|B)_{\rho_T} \leq e^{-c'\sqrt{|B|}+e^{c/T}}$$

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2. Let $\rho_{1\ldots n}$ be a state on n qubits s.t. for every split ABC with $|B| > m$, $I(A:C|B) \leq \varepsilon$. Then

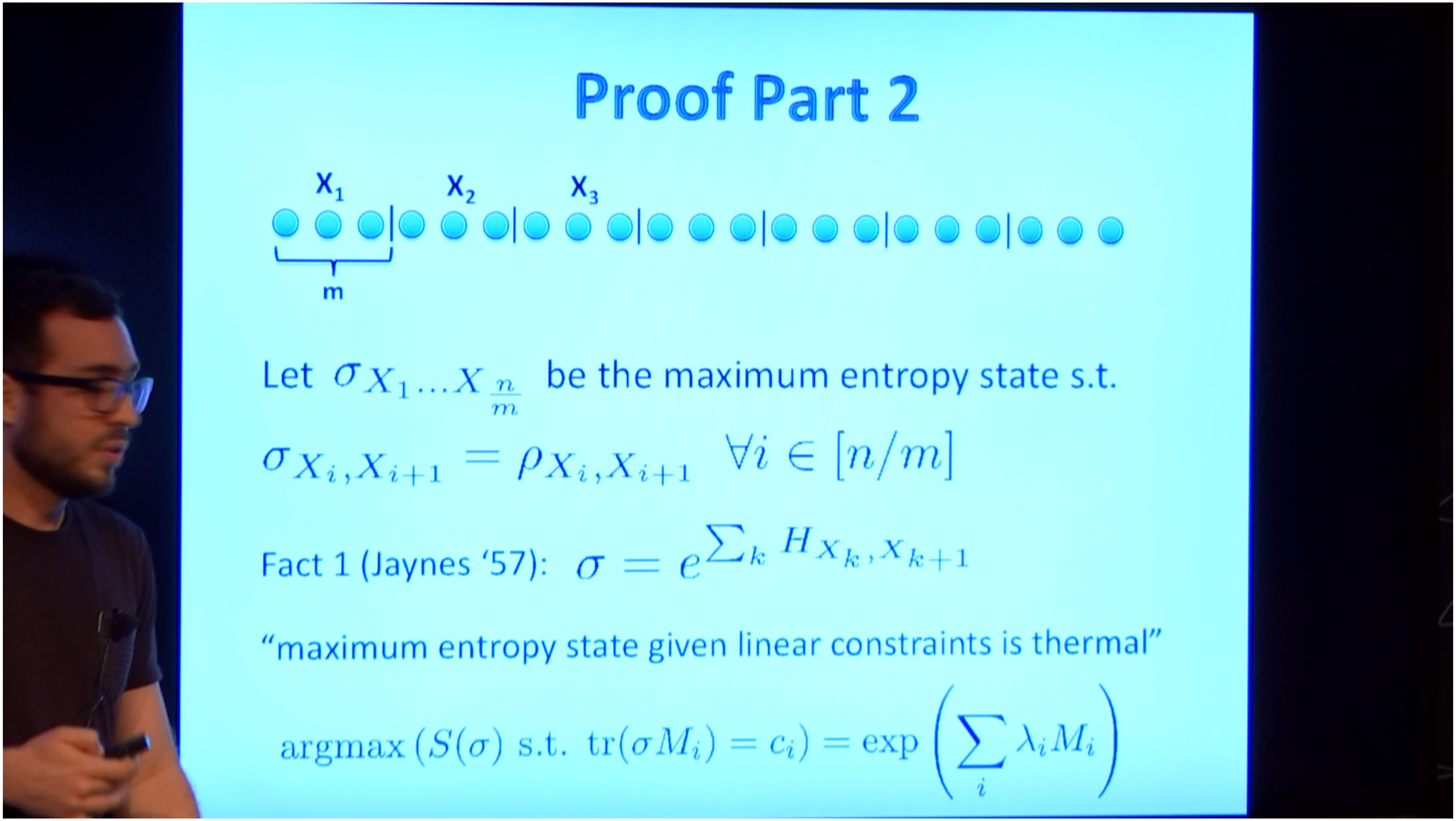
$$\min_{H \in \mathcal{H}_{2m}} S\left(\rho || e^{H}\right) \leq \varepsilon \frac{n}{m}$$

$$\mathcal{H}_{2m} := \{H : H = \sum_k H_{k,k+1},\ \forall k \ \mathrm{supp}(H_{k,k+1}) \leq 2m\}$$

Proof Part 2

Let $\sigma_{X_1 \ldots X_{\frac{n}{m}}}$ be the maximum entropy state s.t.

$$\sigma_{X_i, X_{i+1}} = \rho_{X_i, X_{i+1}} \ \forall i \in [n/m]$$


Proof Part 2
X_1 X_2 X_3
m
Let $\sigma_{X_1 \ldots X_{\frac{n}{m}}}$ be the maximum entropy state s.t.
$\sigma_{X_i, X_{i+1}} = \rho_{X_i, X_{i+1}} \; \forall i \in [n/m]$
Fact 1 (Jaynes '57): $\sigma = e^{\sum_k H_{X_k, X_{k+1}}}$
"maximum entropy state given linear constraints is thermal"
$\mathrm{argmax}\,(S(\sigma) \text{ s.t. } \mathrm{tr}(\sigma M_i) = c_i) = \exp\left(\sum_i \lambda_i M_i\right)$

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Fact 2
$$\begin{aligned} \min_{H \in \mathcal{H}_{2m}} S(\rho \| e^H / Z) &\leq -S(\rho) - \mathrm{tr}(\rho \log \sigma) \\ &= S(\sigma) - S(\rho) \end{aligned}$$

Let's show it's small

Proof Part 2

$$
\begin{aligned}
& S(X_1 \ldots X_{n/m})_\sigma \\
\leq\ & S(X_1X_2)_\sigma - S(X_2)_\sigma + S(X_2 \ldots X_{n/m})_\sigma \\
\leq\ & S(X_1X_2)_\sigma - S(X_2)_\sigma + S(X_2X_3)_\sigma - S(X_3)_\sigma + S(X_3 \ldots X_{n/m})_\sigma \\
\leq\ & \sum_i S(X_iX_{i+1})_\sigma - S(X_{i+1})_\sigma \\
=\ & \sum_i S(X_iX_{i+1})_\rho - S(X_{i+1})_\rho
\end{aligned}
$$

Since $\sigma_{X_i,X_{i+1}} = \rho_{X_i,X_{i+1}} \quad \forall i \in [n/m]$

Proof Part 1

Recap: Let *H* be a local Hamiltonian on *n* qubits. Then

$$I(A:C|B)_{\rho_T} \leq e^{-c'\sqrt{|B|}+e^{c/T}}$$

We show there is a recovery channel from B to BC reconstructing the state on ABC from its reduction on AB.

More technical. Uses **Quantum Belief Propagation** equations of Hastings.

Summary

- Locality of EE (area law) implies locality of boundary states and entanglement spectrum
- Quantum Approximate Markov Chains are Thermal

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- Quantum Approximate Markov Chains are Thermal

Open Questions:

- Applications to high energy/holography?
- Are two copies of entanglement spectrum needed?
- Is the conjecture about approximate Markov chains true?
- Thermal state has same symmetries as original state. Mapping from 2D (zero temperature) to 1D (thermal). Is it useful for classification of (symmetry-protected) phases?