Title: EPlus Keynote: Stephen Green

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Abstract:

General Relativity and Gravitational Waves

Stephen R. Green

EinsteinPlus Workshop
Perimeter Institute
July 8, 2016

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

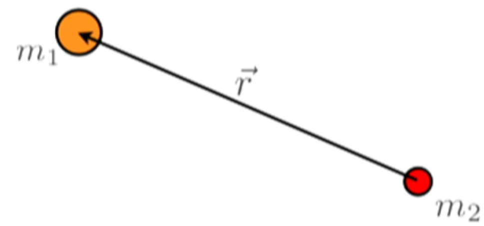
Outline

- Review of Newtonian gravity
- General relativity vs Newtonian gravity
- Black holes and gravitational waves
- Detection of gravitational waves
- My research: Black hole instabilities

Newtonian gravity review

- Force that a body exerts on another in Newtonian gravity

$$\vec{F} = \frac{G_N m_1 m_2}{r^2}\hat{r}$$



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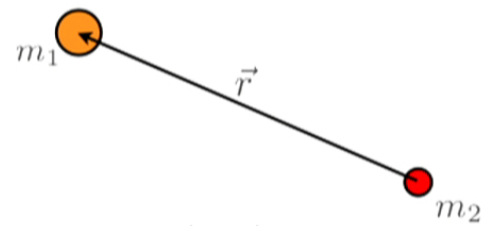
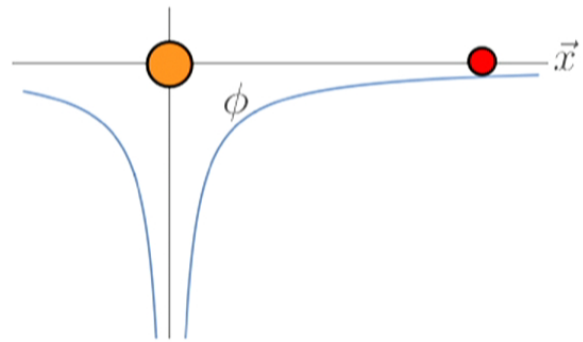


- Useful to think in terms of Newtonian gravitational potential $\phi(t, \vec{x})$

  In this case, the potential of object 1 is $\phi = -\frac{G_{\rm N} m_1}{r}$

  Then the force on object 2 is $\vec{F} = -m_2 \vec{\nabla}\phi\Big|_{\vec{x}_2}$



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- Left hand side is now Lorentz invariant
- But this equation is inconsistent with observations, and there are theoretical reasons why it doesn't work.

General relativity

- In general relativity, we replace the Newtonian potential with spacetime geometry.

Poisson equation $\nabla^2\phi = 4\pi G_{\rm N}\rho$

Einstein equation $G_{ab} = 8\pi G_{\rm N} T_{ab}$

Curvature of spacetime geometry

Matter stress-energy-momentum

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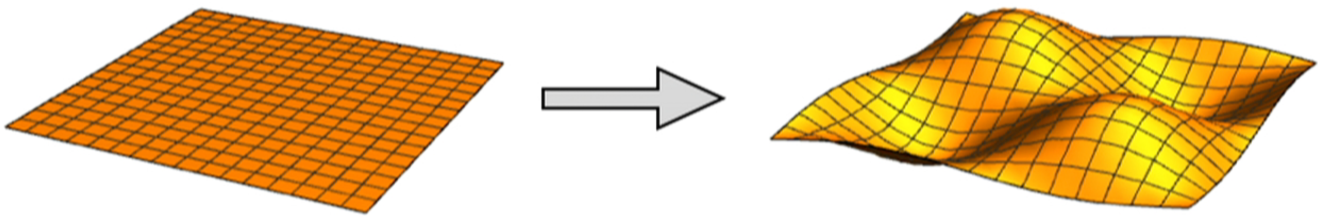
- New idea: Gravity as spacetime geometry

  E.g., in 2 dimensions



- Difficult to visualize, but *in 4 dimensions, space and time are (together) curved, rather than flat.* Useful to think in terms of 2 dimensional curved surfaces.

  Spacetime curvature is determined by the Einstein equation. A solution to the Einstein equation is a spacetime metric, or geometry.

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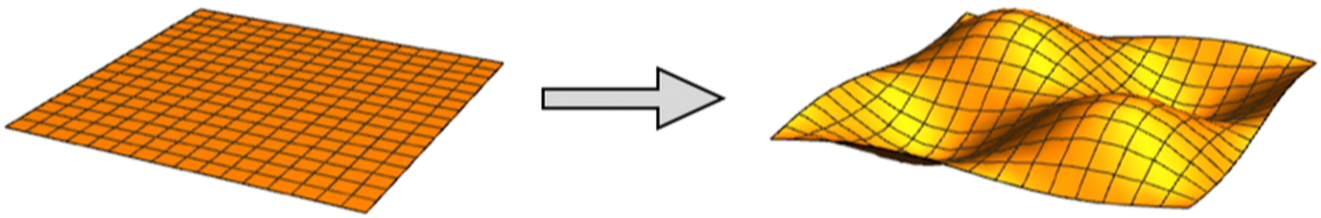

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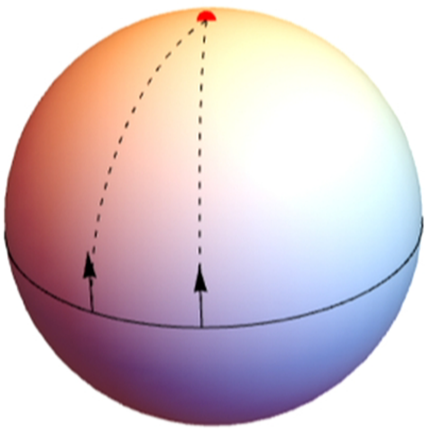
- But how can spacetime geometry be the same as gravity?

- Objects in spacetime follow *geodesics.*

  In flat space, initially parallel trajectories remain parallel.

Not true in curved space:



In curved spacetime, worldlines of objects can intersect because of the spacetime curvature. This is how geometry gives rise to gravity.

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  - Nonlinear

    Superposition does not hold: sum of two solutions is not a solution.

    Physically, gravitational field carries energy itself, and any type of energy gravitates itself. Thus, *gravity gravitates*. (E.g., black hole)

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    - Not all 10 equations are independent. Linked by 4 Bianchi identities $\nabla^b G_{ab} = 0$

      Thus only 6 independent equations. Likewise, we have freedom to choose our 4 spacetime coordinates.

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  - Equation is hyperbolic and elliptic, whereas Poisson equation is elliptic. Thus we have propagating wave solutions.

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    2 of the 6 independent equations are hyperbolic -> 2 polarizations of gravitational waves.

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    2 of the 6 independent equations are hyperbolic -> 2 polarizations of gravitational waves.

- However, all of this means that solutions to the Einstein equation can be much more interesting. Indeed, there are even interesting solutions to the vacuum equation, with no matter included:

$$G_{ab} = 0$$

Black holes

- Solution to Einstein equation with

$$G_{ab} = 0$$

Black holes

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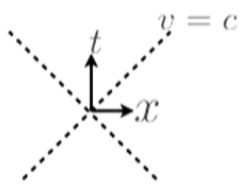
- Spacetime diagram useful representation



Each point of diagram represents a sphere

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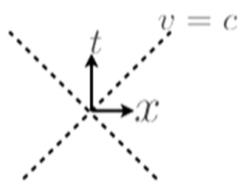

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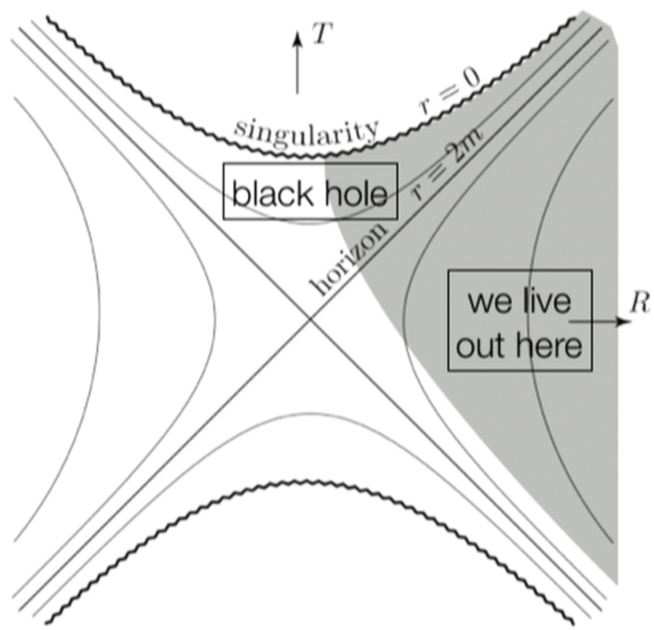

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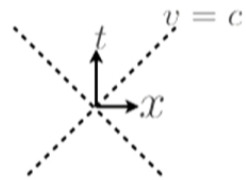

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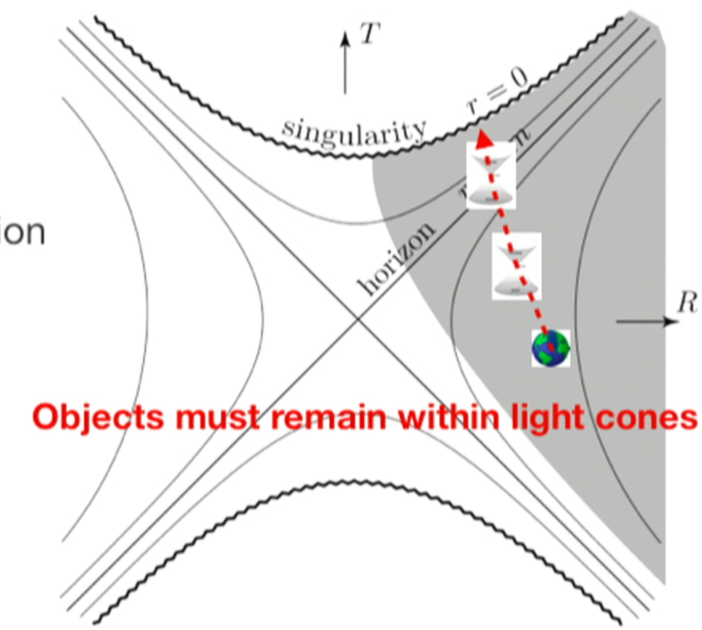

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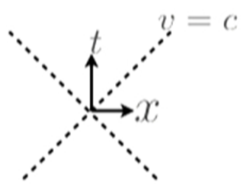
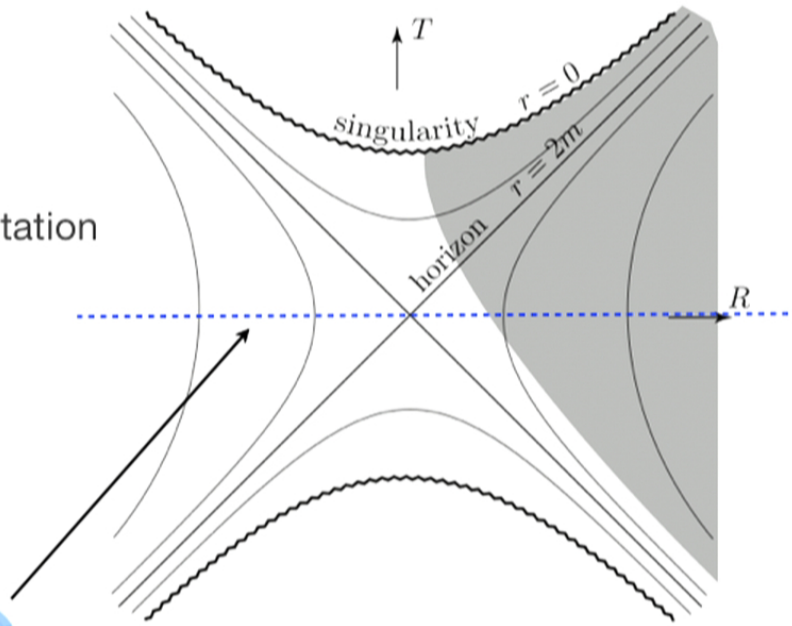
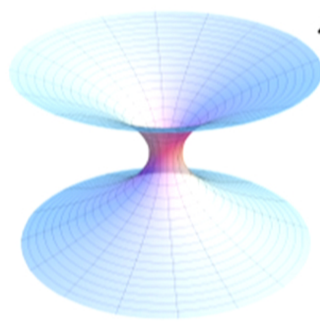


Each point of diagram represents a sphere



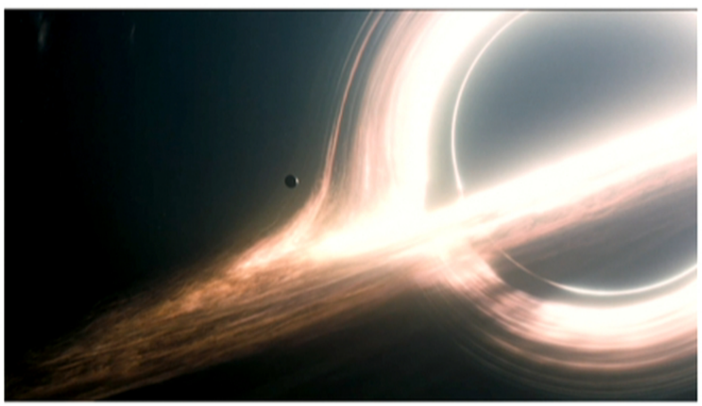


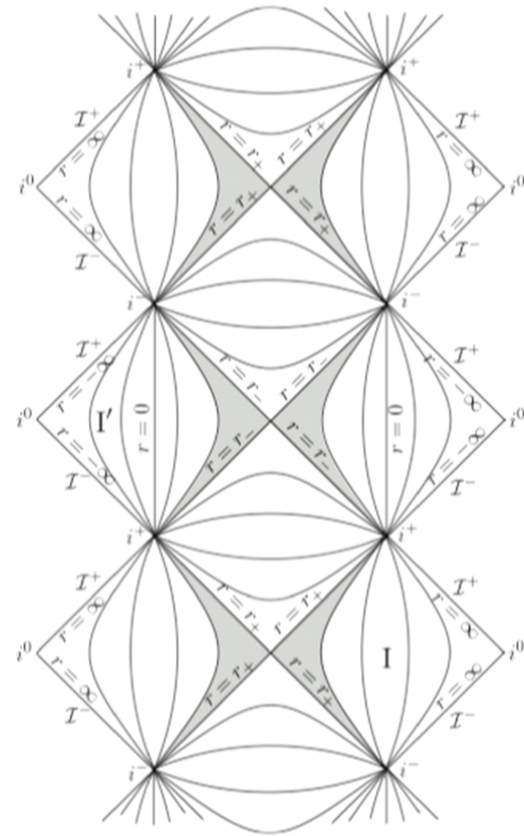
embedding diagram of throat

Black holes

- Black holes can have much more complicated structure.

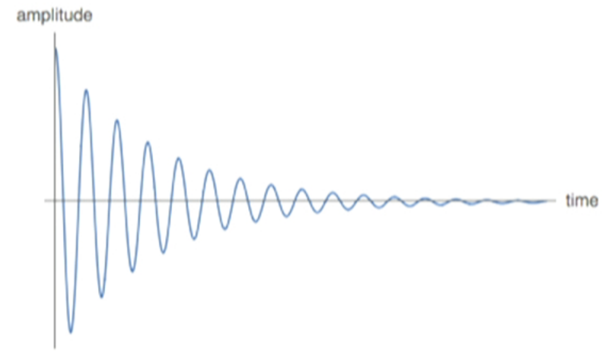
  For a rotating black hole,





Black hole quasinormal modes

- When perturbed, a black hole rings like a bell



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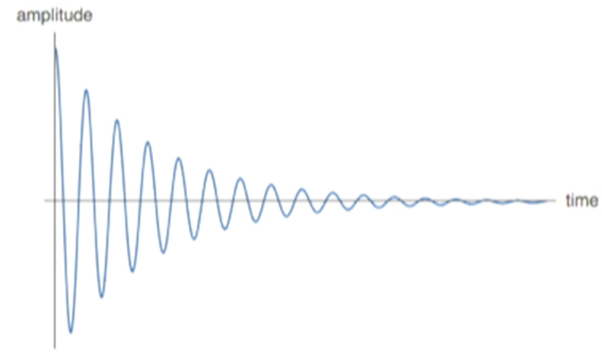
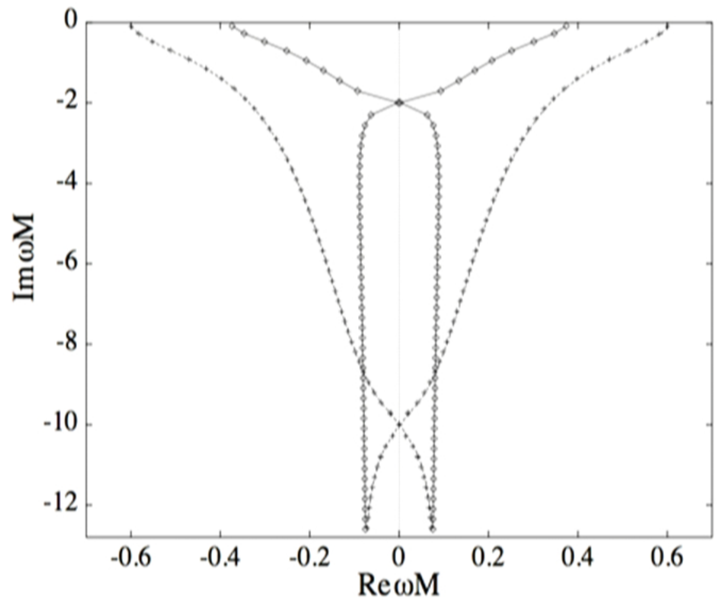
- When perturbed, a black hole rings like a bell
- Characteristic complex frequencies depend on the particular black hole perturbed.





perturbation $\sim e^{-i\omega t}$

Figure: Kokkatas and Schmidt, 1999

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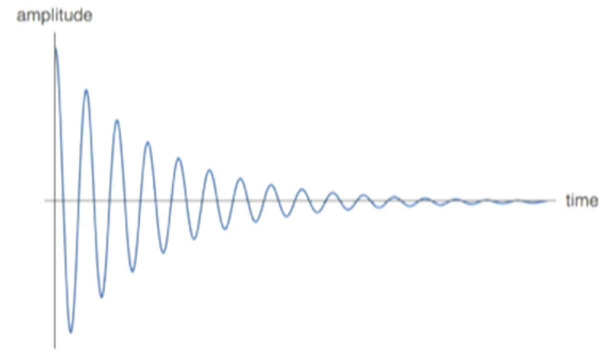
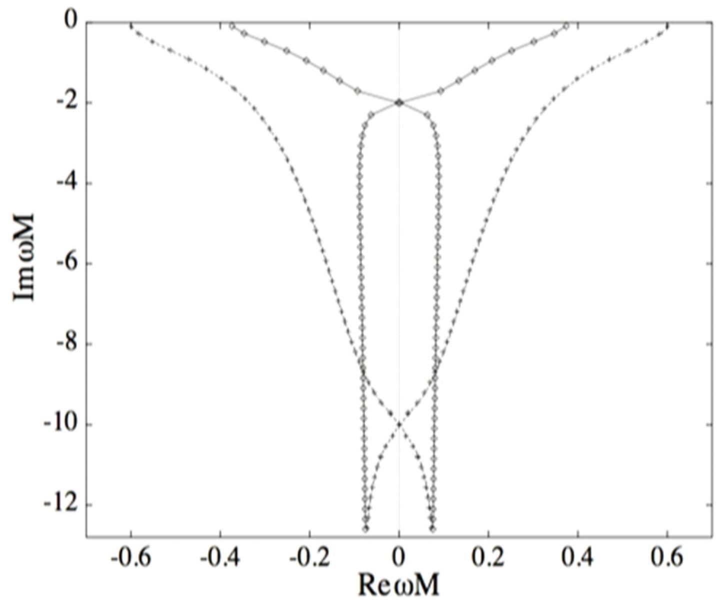




$$\text{perturbation} \sim e^{-i\omega t}$$

Figure: Kokkatas and Schmidt, 1999

Gravitational waves

- Vacuum solutions, like the black hole: $G_{ab} = 0$
- Produced by violent events, such as merger of two black holes, or neutron stars.
- Propagate long distances at the speed of light, as ripples in spacetime.
- Two polarizations: +, x

Summary: Newtonian gravity vs general relativity

- General relativity is *much richer* than Newtonian gravity

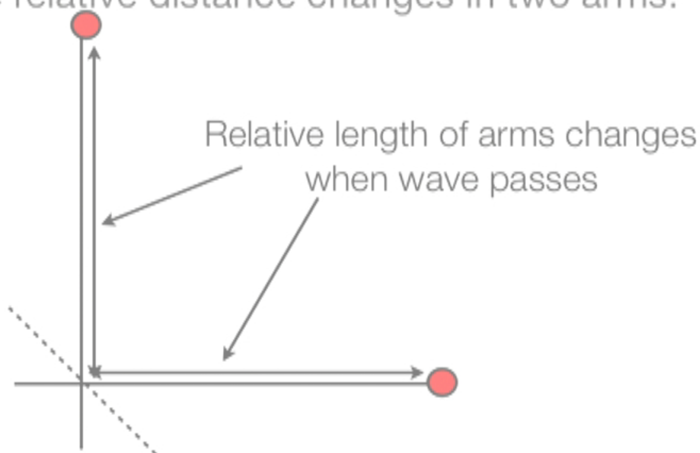
	Newtonian gravity	general relativity
field	potential	spacetime geometry
field equation	$\nabla^2\phi = 4\pi G_{\mathrm{N}}\rho$	$G_{ab} = 8\pi G_{\mathrm{N}} T_{ab}$
components	1	10
	linear	nonlinear
	elliptic	hyperbolic/elliptic
propagating degrees of freedom	0	2

More dynamical solutions

- More complicated solutions cannot be obtained analytically. They require numerical simulations on supercomputers
- For example, the merger of two black holes. See video.

Detection of gravitational waves

- When a gravitational wave passes, it distorts space, changing the amount of space between objects
- For example, the merger of two black holes. See video.
- To detect the wave passage, one measures this change. Use an interferometer to measure relative distance changes in two arms.



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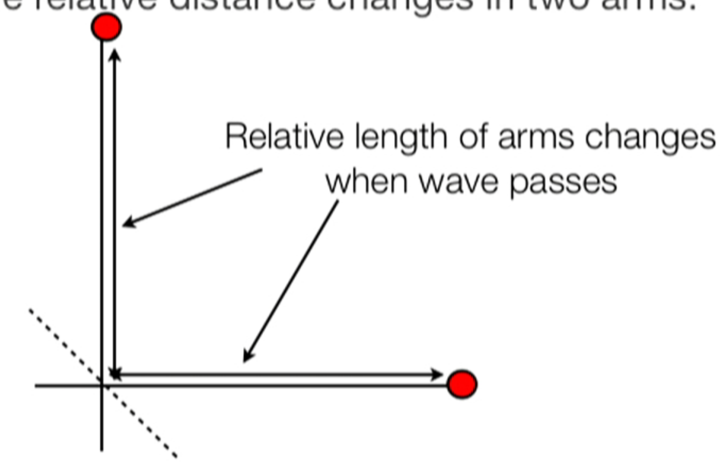
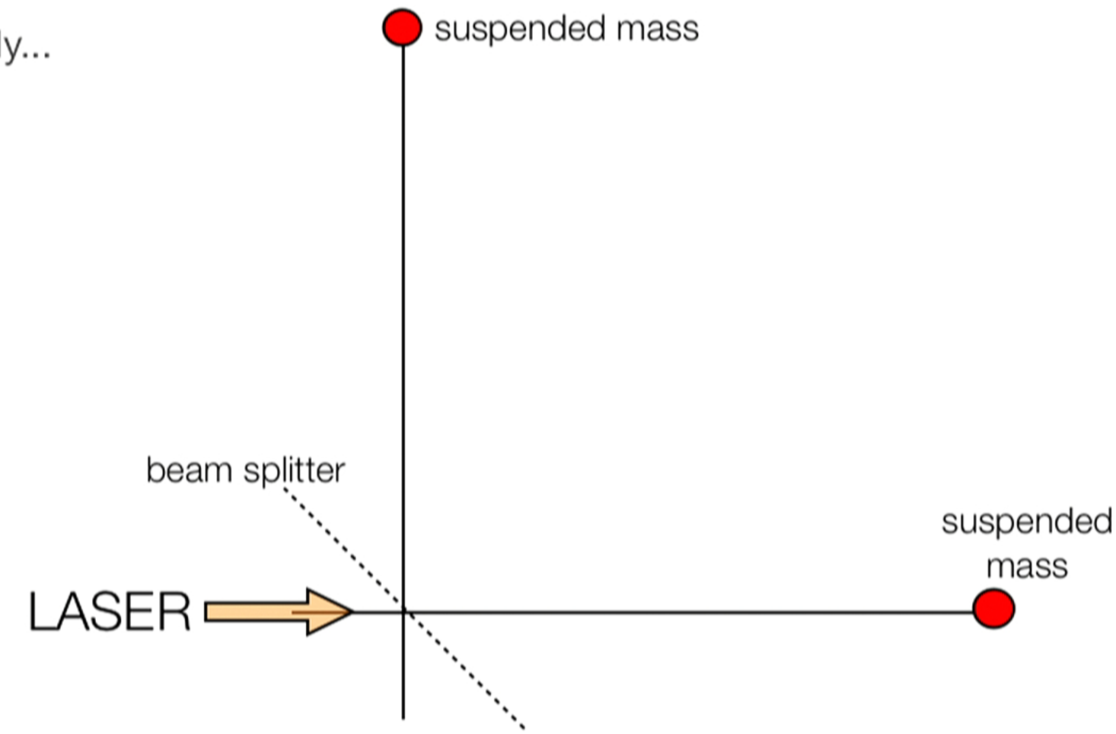


Detection of gravitational waves

- Schematically...



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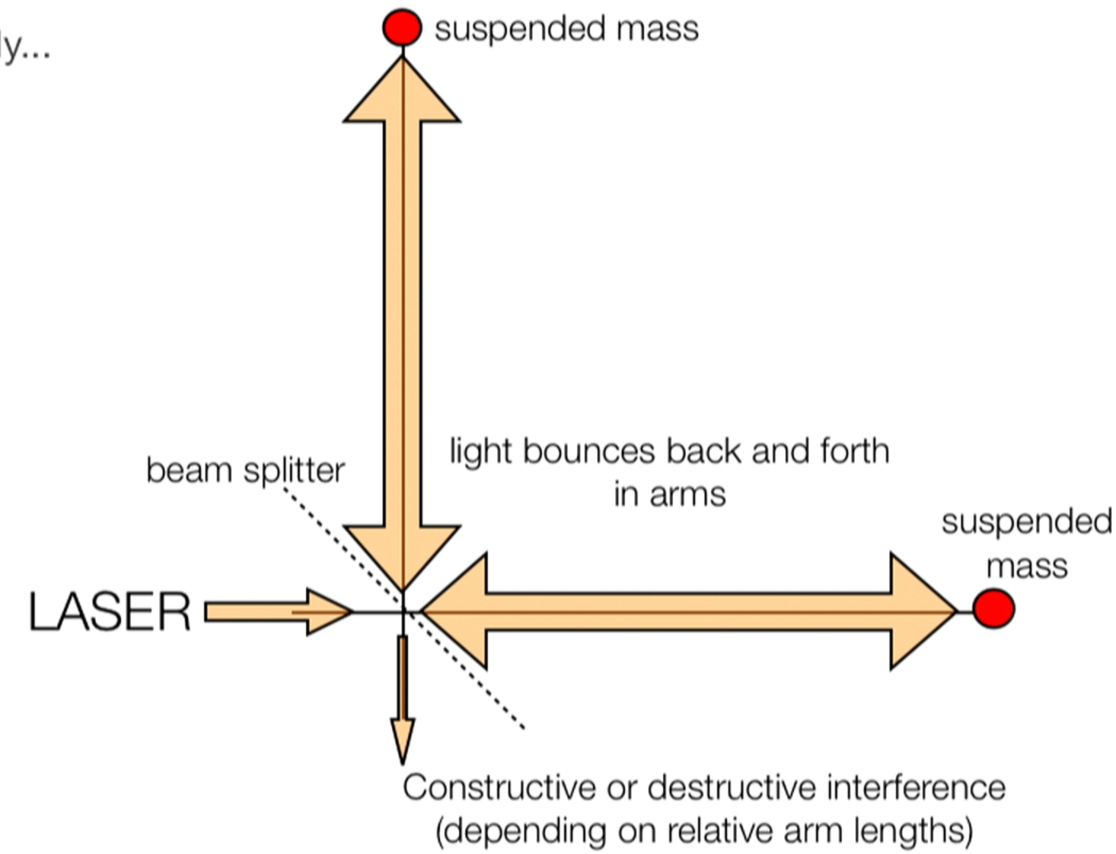
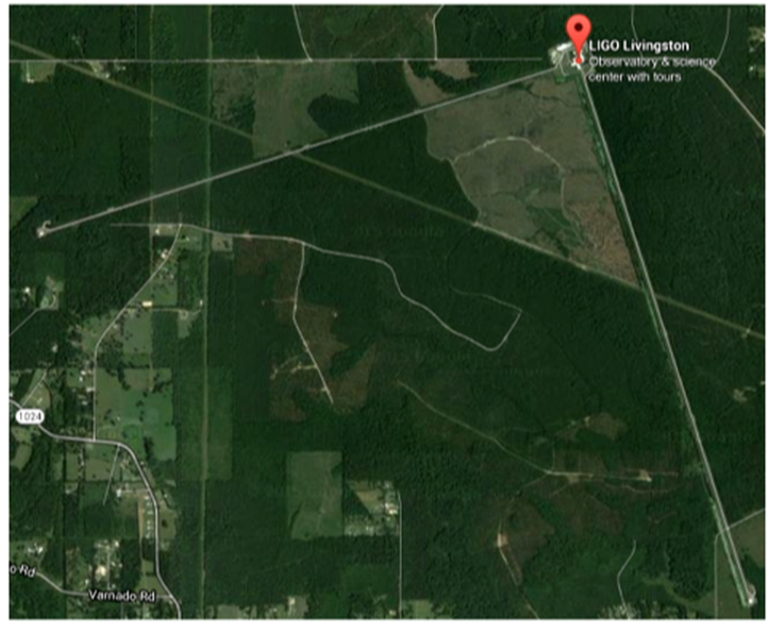


Laser Interferometer Gravitational-Wave Observatory (LIGO)

- Two detectors

Hanford, WA



Livingston, LA

Detection of gravitational waves

- Gravitational wave detector network

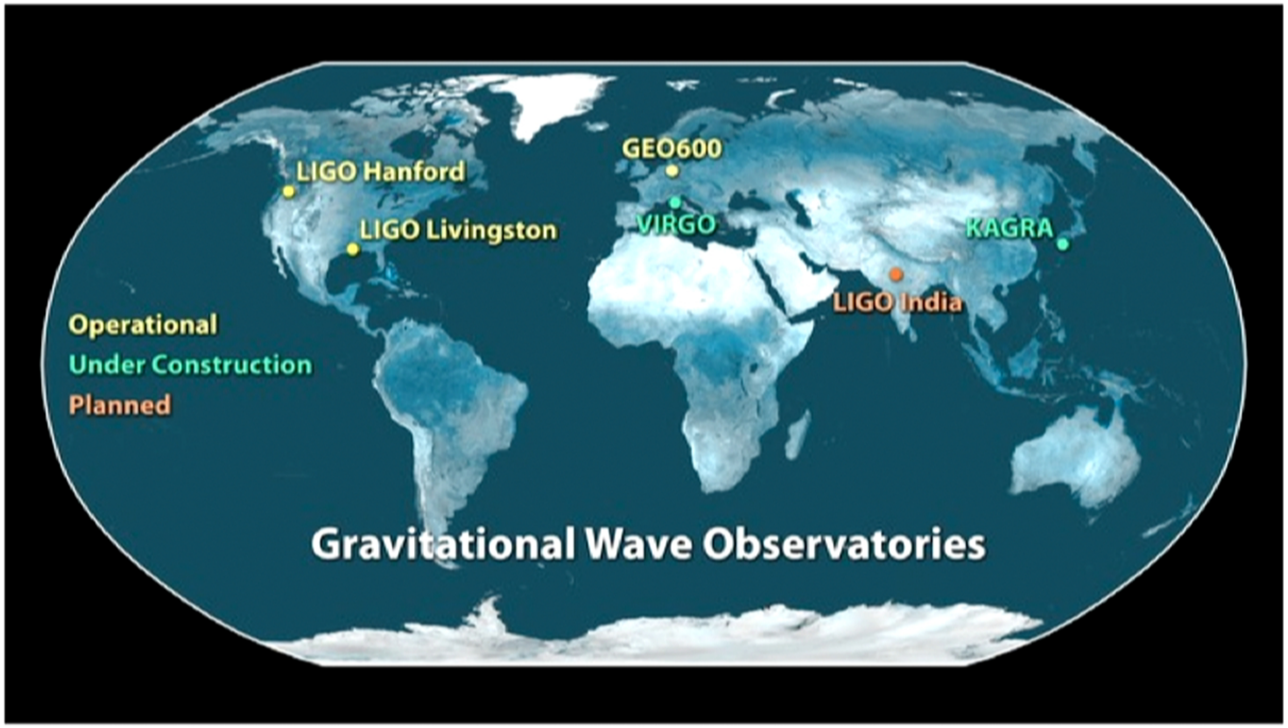
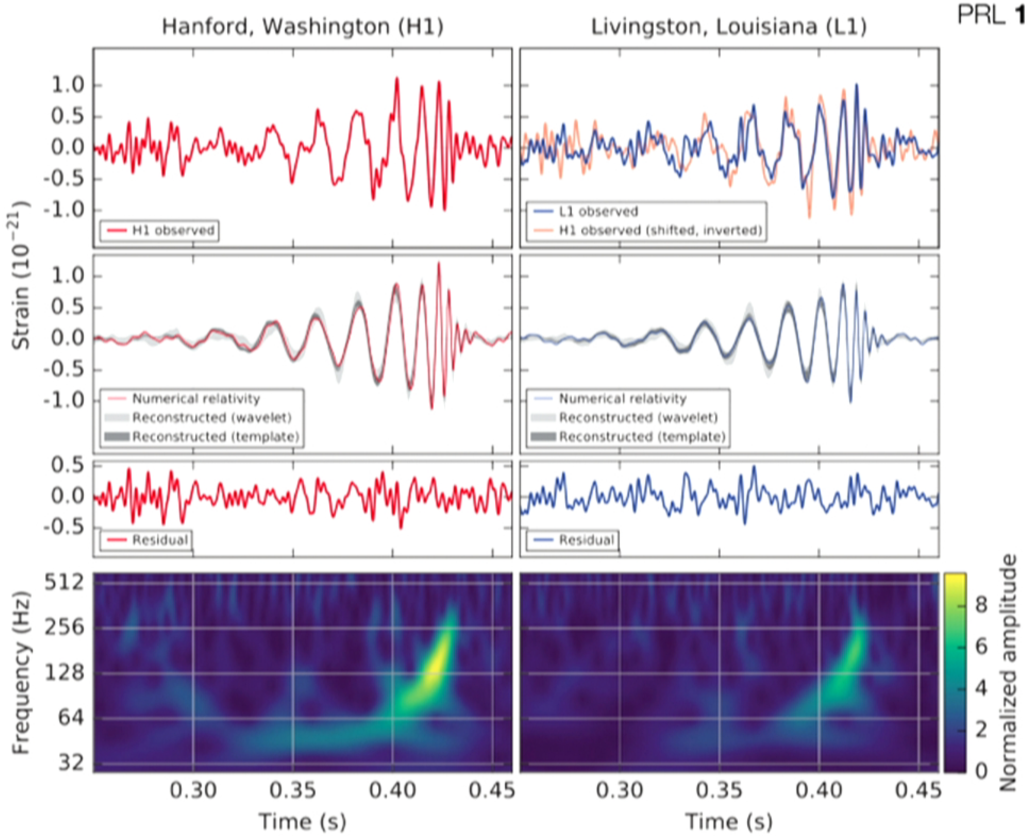


Image: Caltech/MIT/LIGO Lab

Detection GW150914

PRL **116**, 061102 (2016)



Detection GW150914

- Given the waveform, identify process that produced the waves by comparing to numerical simulations.

  Inspiral, merger, ringdown of system seen in gravitational wave signal.

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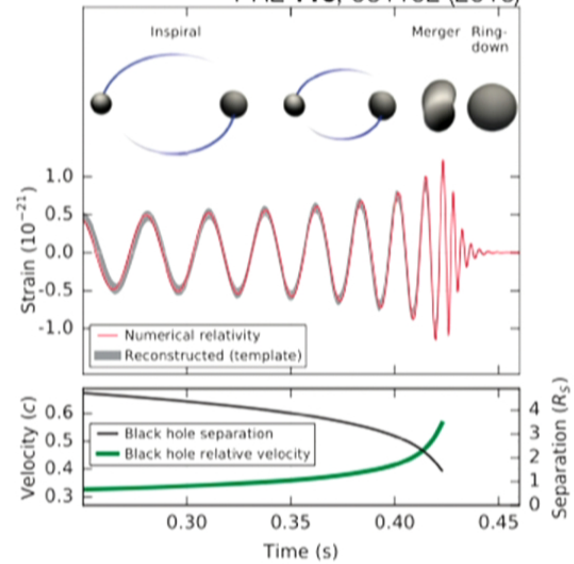

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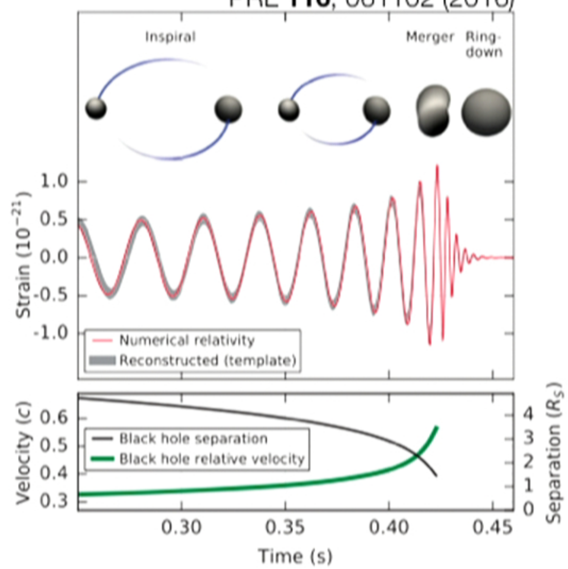

- This event was a merger of 2 black holes, of 36 and 29 solar masses.

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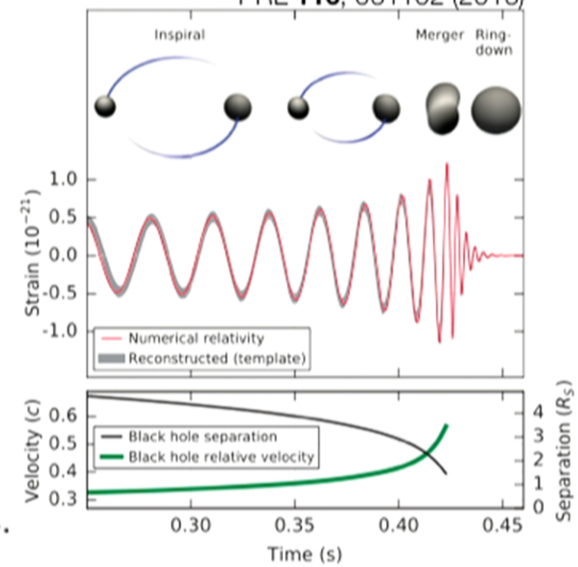

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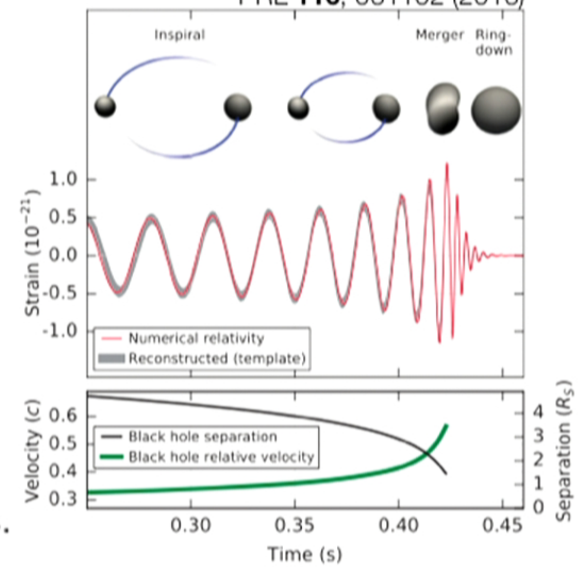

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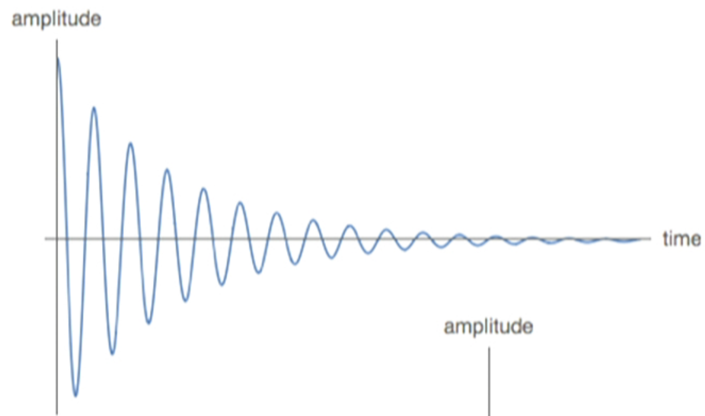
- Final black hole 62 solar masses, so 3 solar masses of energy released in gravitational waves.
- Waves propagated for 1.3 billion years before reaching Earth.

Summary

- General relativity is described by the Einstein equation. Solutions correspond to a spacetime geometry.
- Einstein equation is much more complicated than the Poisson equation for Newtonian gravity, but the resulting solutions are much more interesting. Solutions include black holes, gravitational waves, merging black holes, expanding universes (cosmology), etc.
- Detection of gravitational waves last year from binary black hole merger confirms predictions of general relativity in the strong field regime.

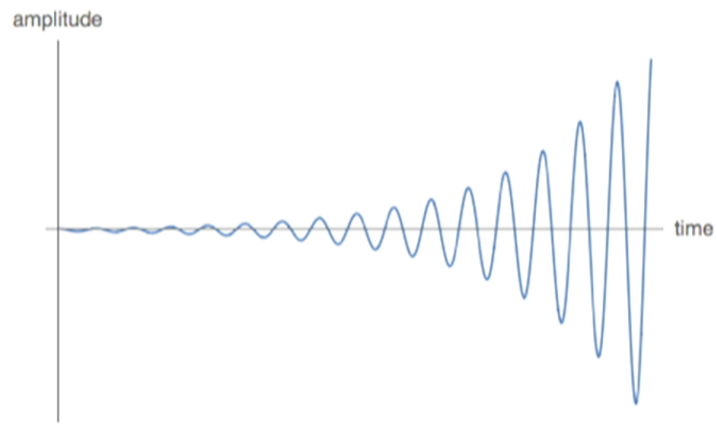
Example of my work: Black hole instabilities

- Recall LIGO observation: After merger, black hole rings down to a stationary black hole.



- But are there circumstances under which a perturbation can grow?

"Black hole instability"



Extraction of mass from black hole

- In addition to having a mass, black holes can carry angular momentum and electric charge.

- The area of the event horizon of a charged, rotating black hole is

$$\text{Area} = 4\pi \left(M + \sqrt{M^2 - a^2 - e^2} \right)^2$$

mass

angular momentum

charge

- **Theorem:** **Area of a black hole can never decrease.**

  - *Hence, a physical process is not forbidden from extracting mass from a black hole, as long as sufficient charge and angular momentum are also extracted, such that area increases.*

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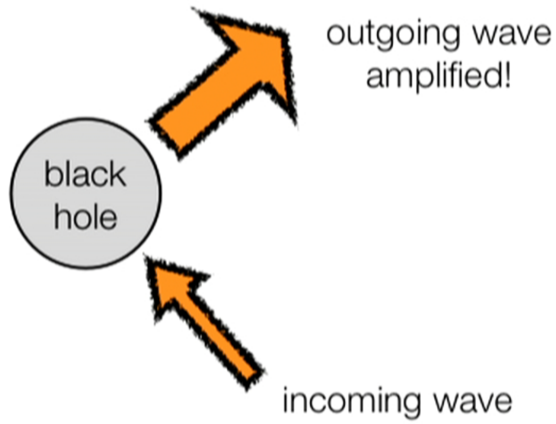
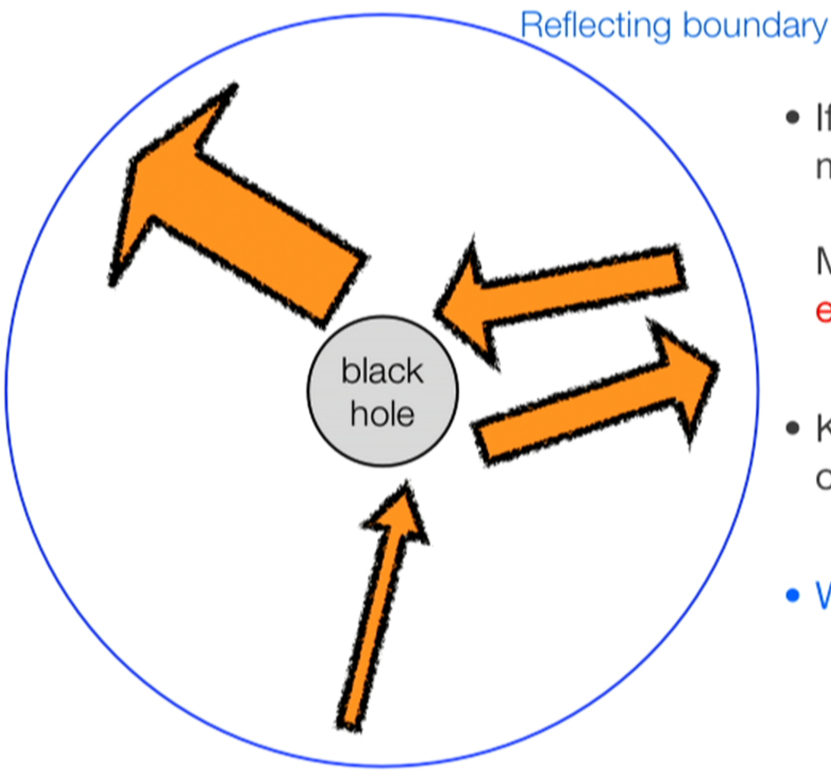
Extraction of mass from black hole

- So if matter with a sufficiently large charge/mass or angular momentum/mass ratio falls into the black hole, the area theorem says that it cannot be absorbed by the black hole. This would cause a decrease in area.

  Instead, more matter comes out!



Superradiant instability



- If black hole surrounded by a mirror, an instability develops.

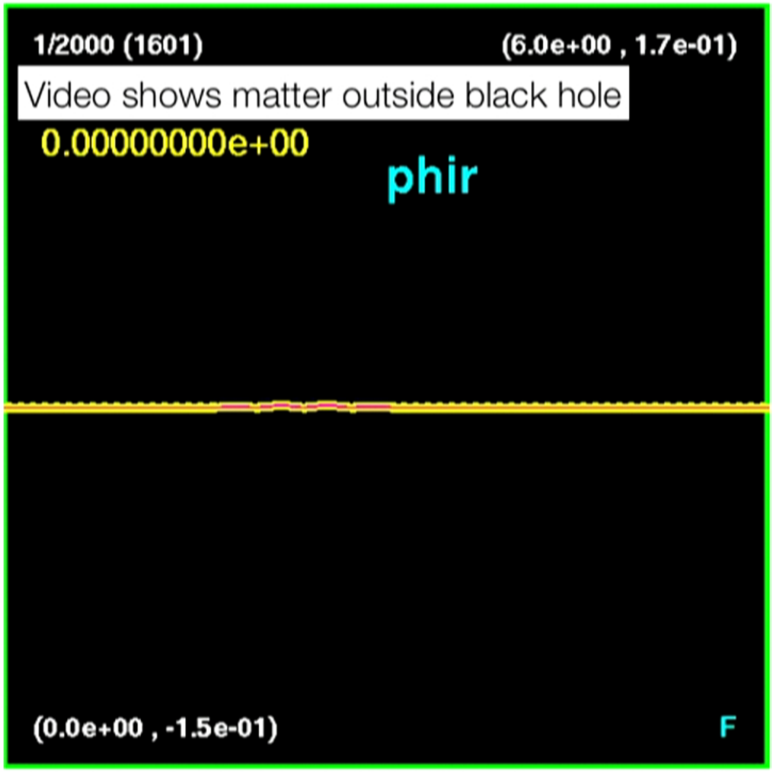
  Matter outside grows exponentially in amplitude.

- Known as the black hole bomb, or superradiant instability.

- What is the final state?

Superradiant instability

- We performed numerical simulations to study this process, solving the Einstein and matter equations.
- Charged case.
- Throughout process, black hole area increases while wave grows in amplitude
- Eventually, most of the charge is extracted, process settles down.



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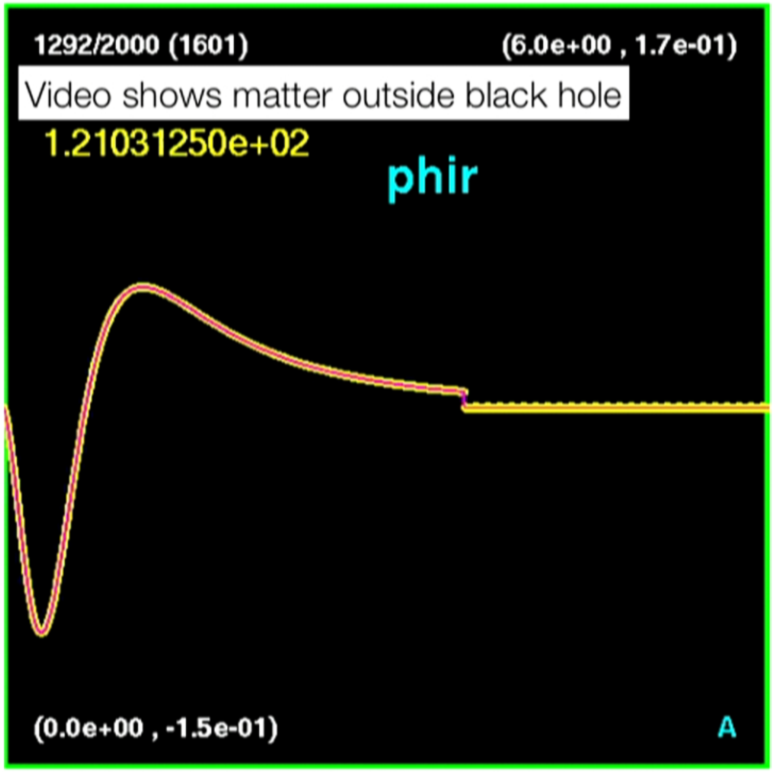

reflecting mirror

black hole horizon

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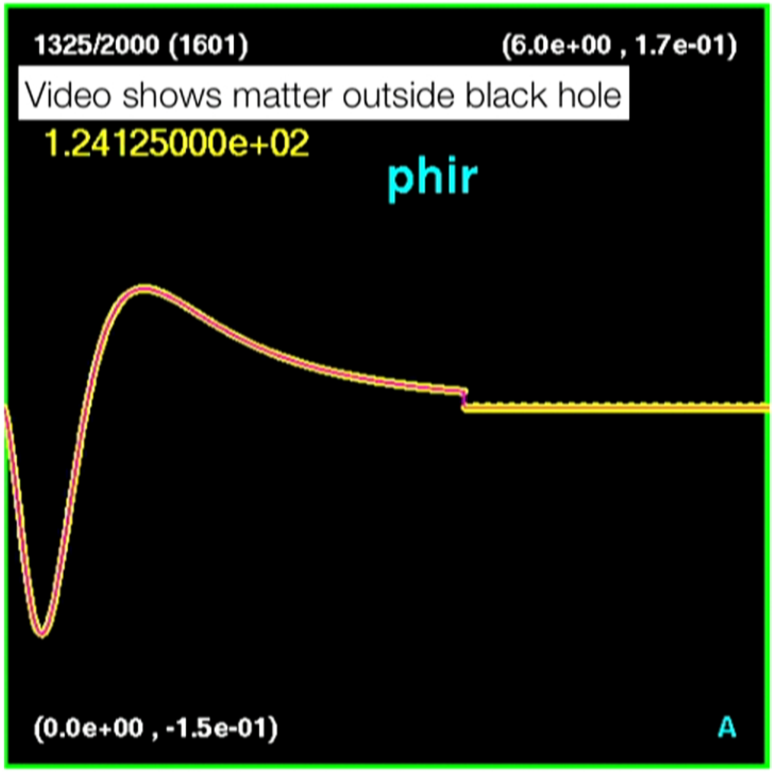
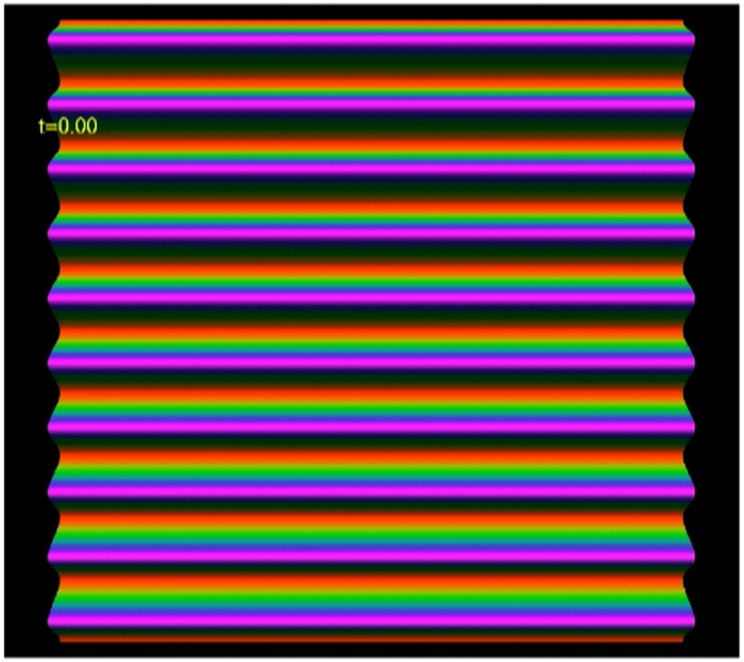


reflecting mirror

black hole horizon

Another example: Black hole turbulence

- For a special type of black hole, gravitational waves in the vicinity display fluid-like behavior.



- Video shows the behavior of the perturbation.
- Turbulent cascades of energy, formation of vortices.
- Will it be realized astrophysically?

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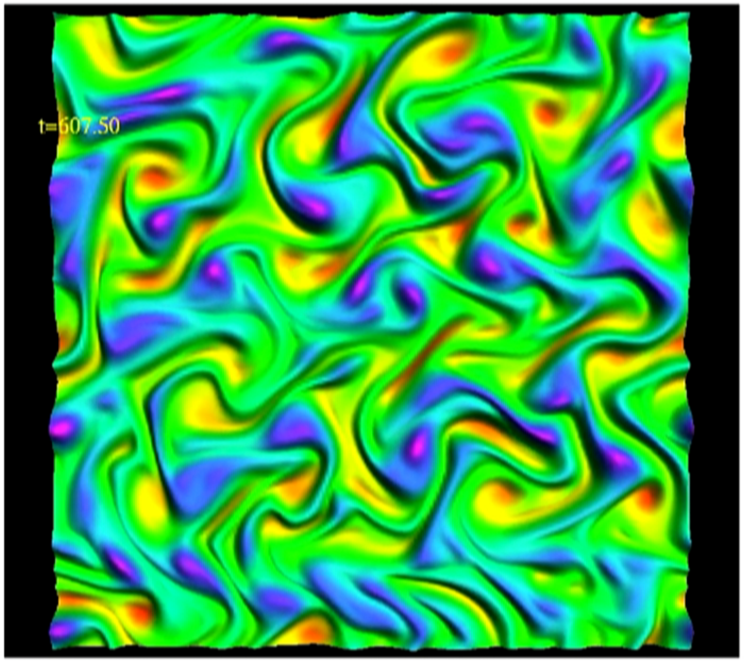
- Turbulent cascades of energy, formation of vortices.
- Will it be realized astrophysically?



Conclusions

- General relativity predicts black holes, gravitational waves, and allows for complicated dynamics.
- With LIGO, we have begun to probe the strong field (black hole) regime of general relativity. We will soon observe mergers involving neutron stars as well. In the next few years we will have far more data to confirm general relativity predictions, and to do astrophysics.
- We continue to discover interesting properties of the Einstein equation, and we hope to probe them experimentally in the future.