Title: Toy Holography

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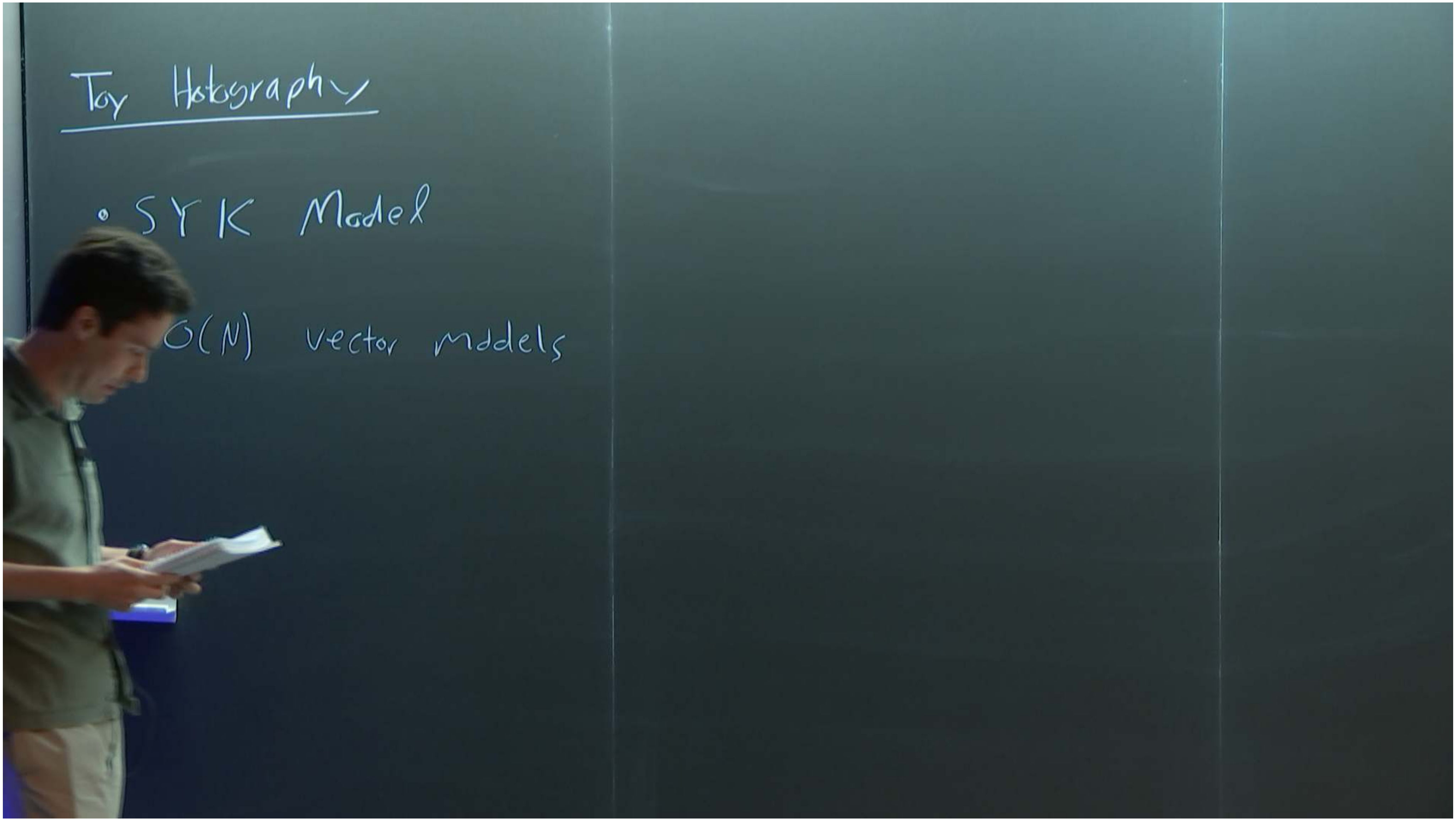
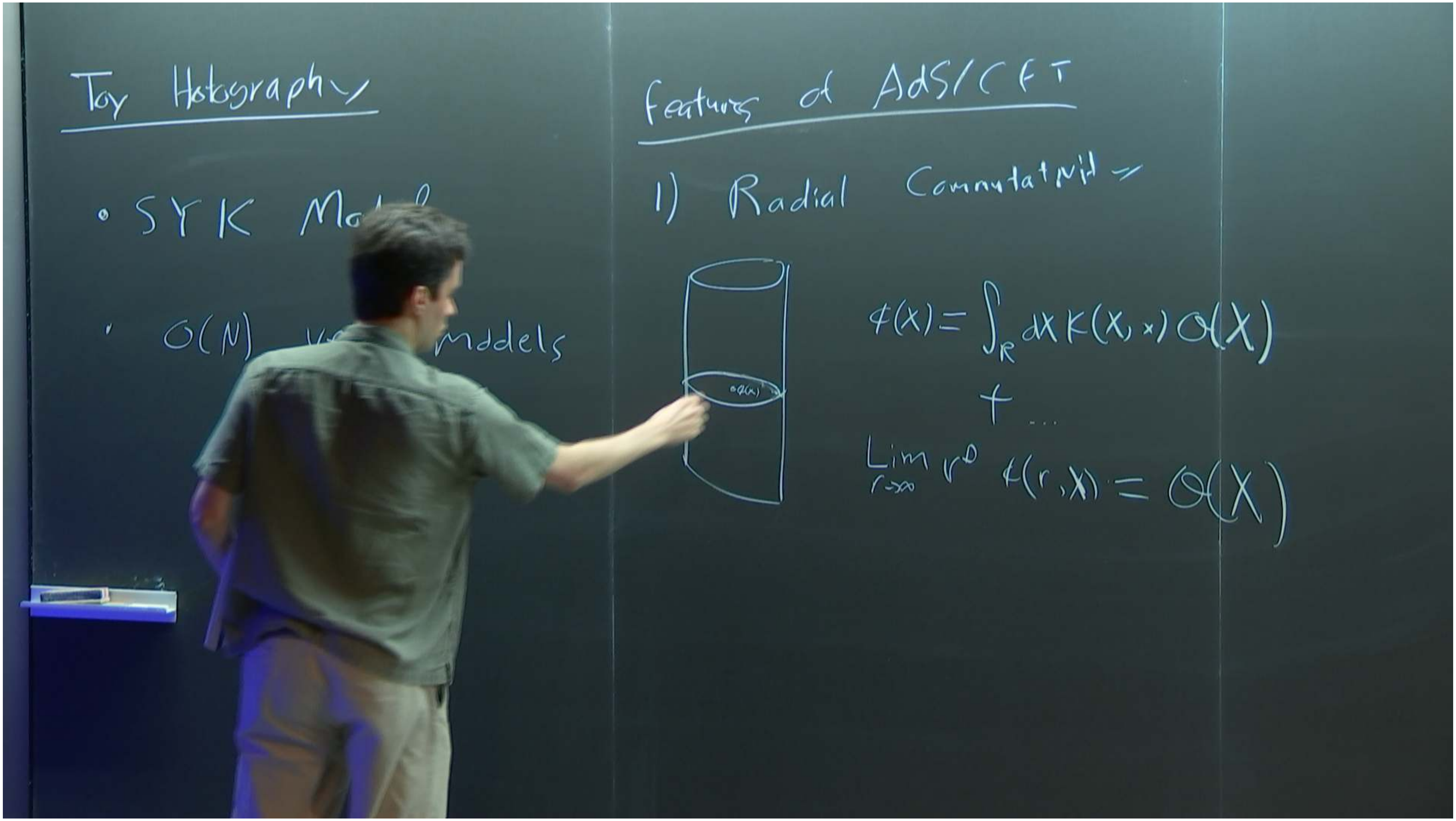
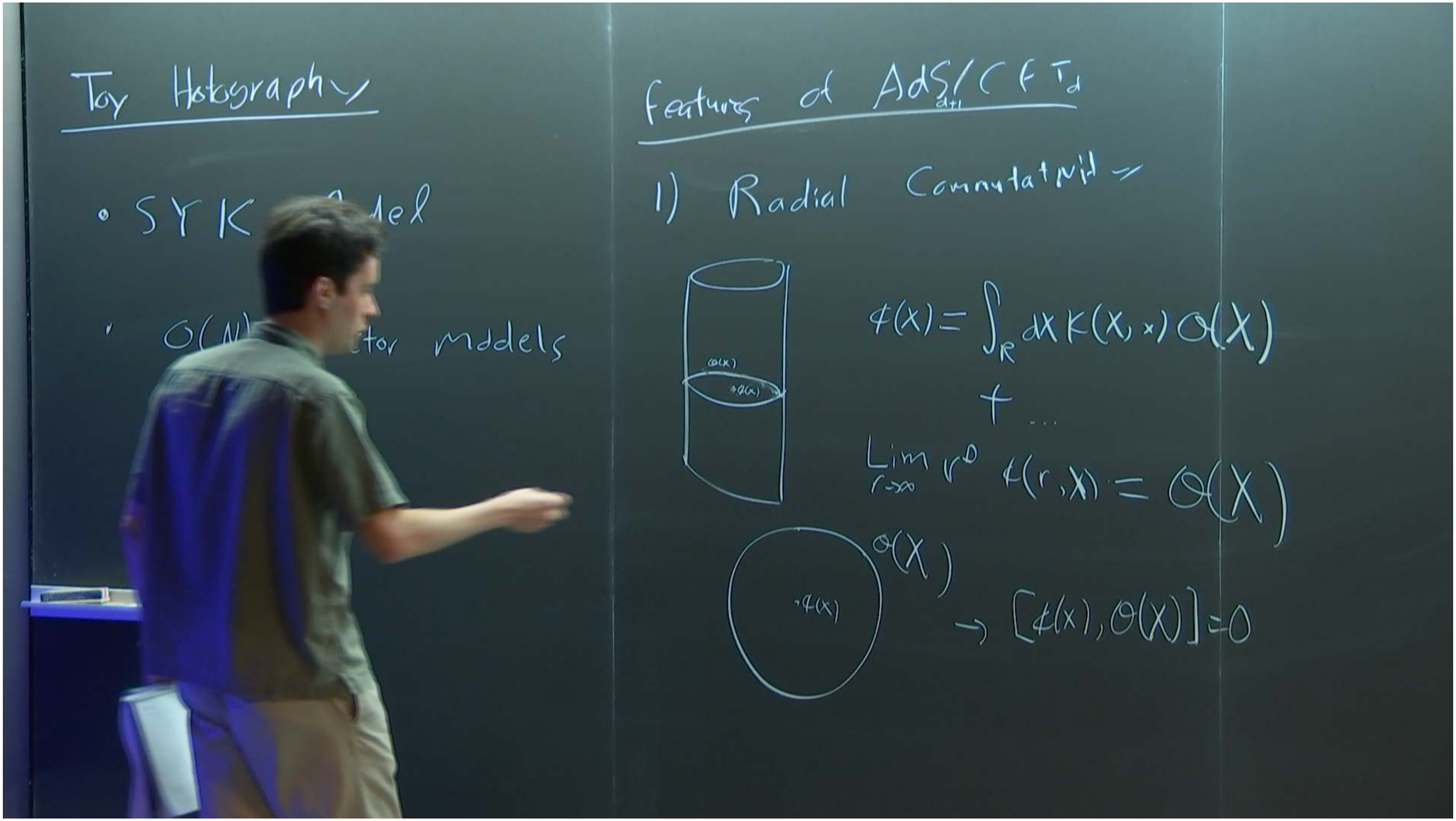
URL: http://pirsa.org/16070057

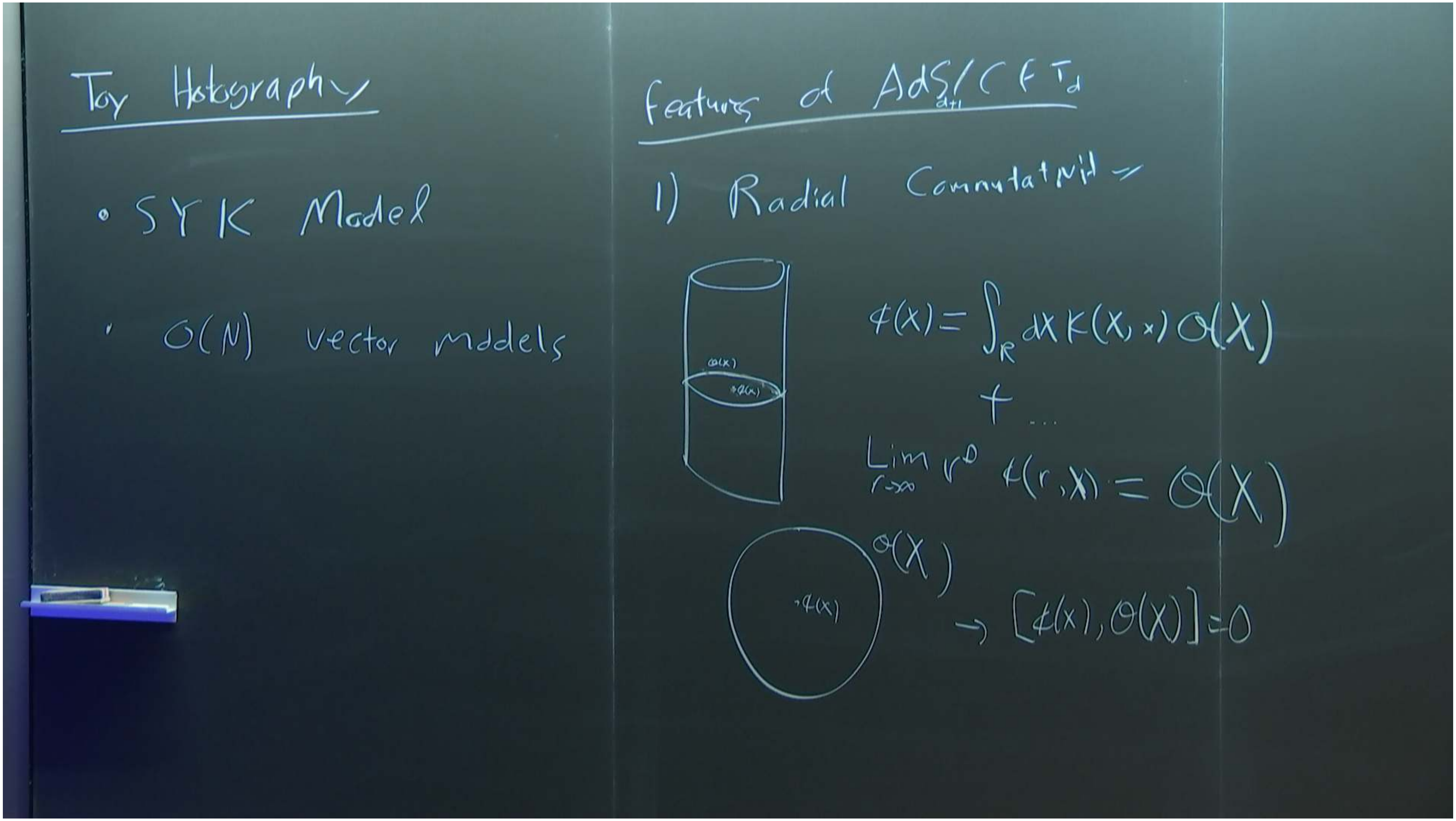
Abstract:

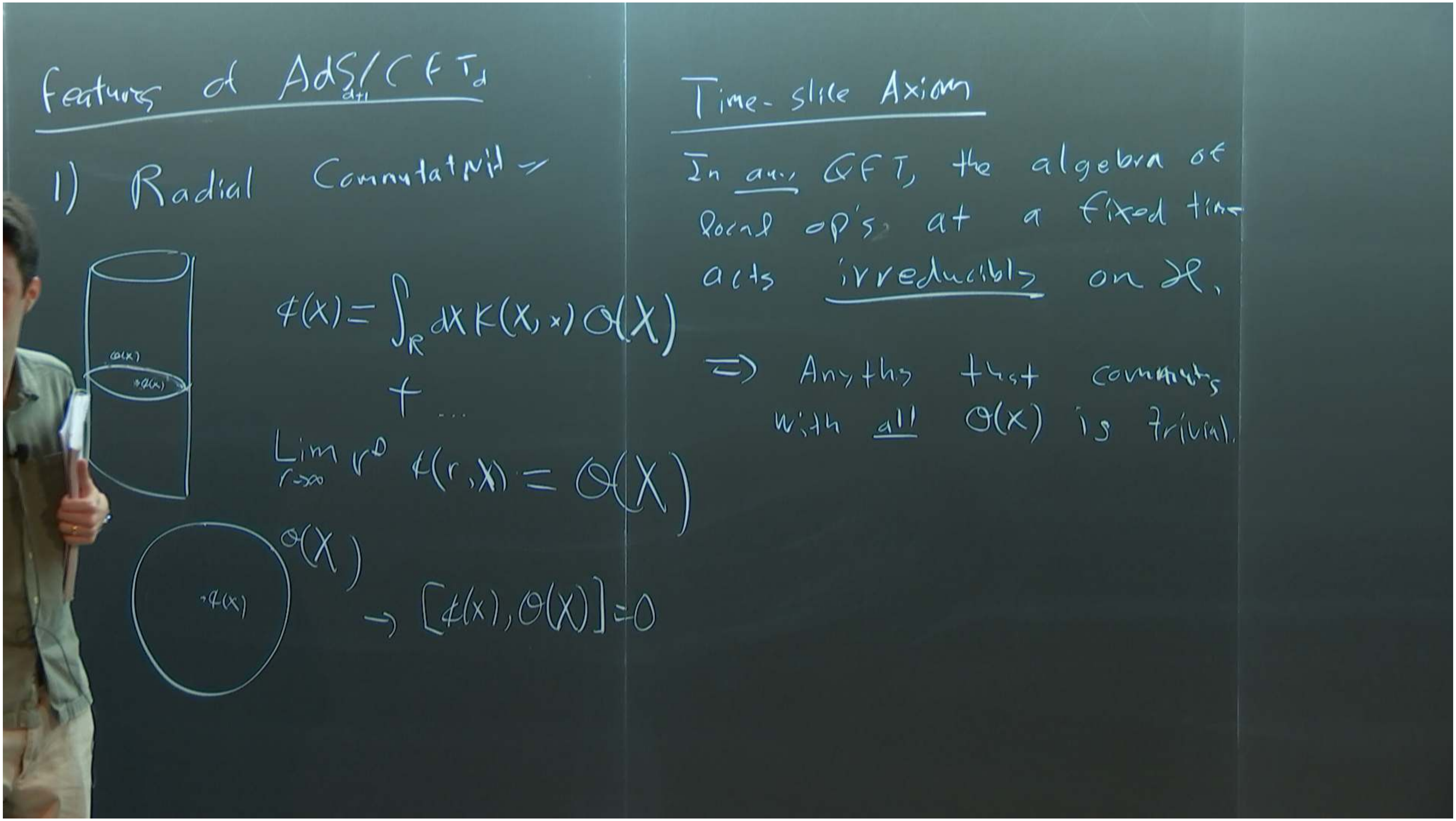

Toy Holography
• SYK Model
O(N) vector models

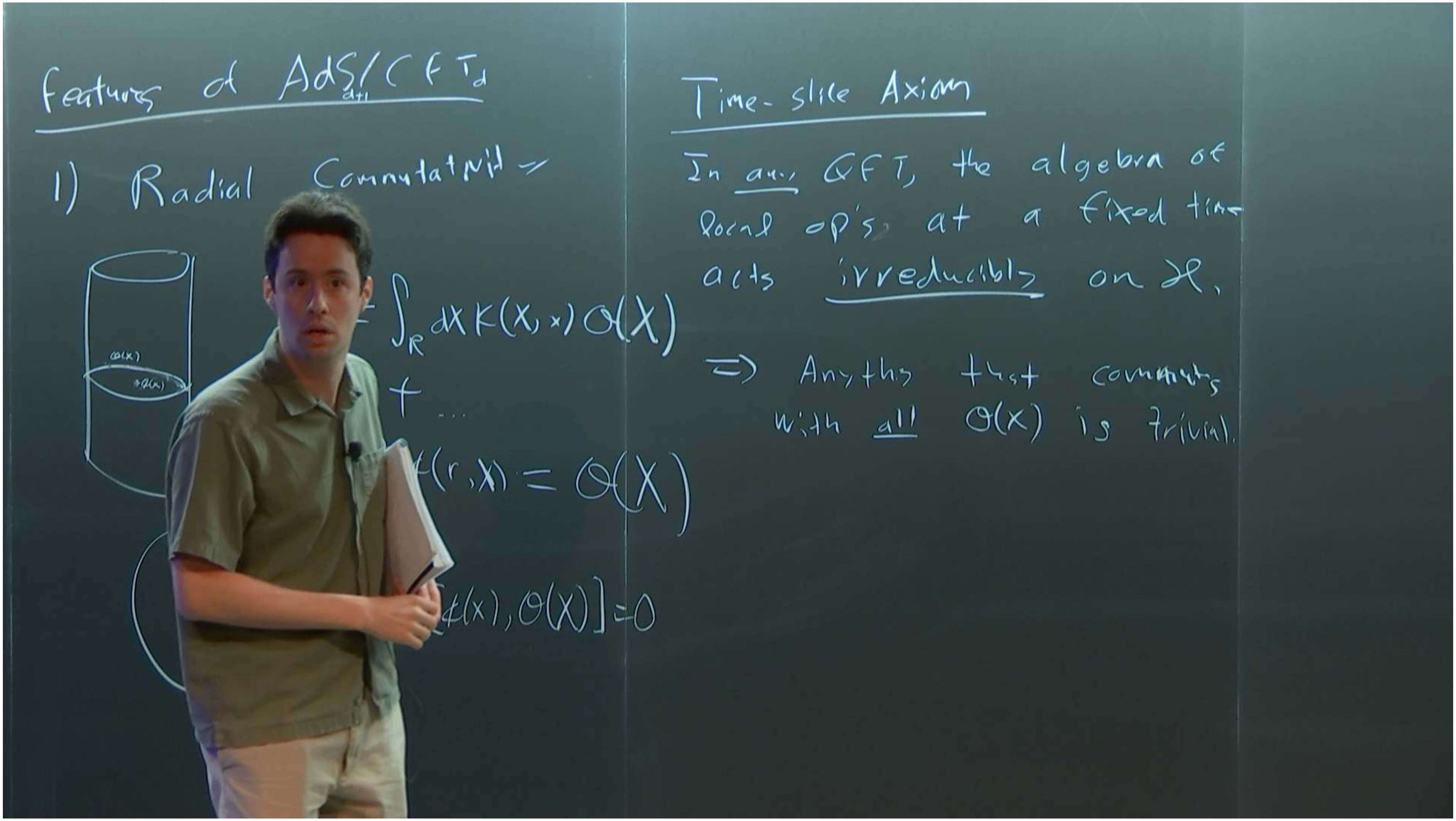

Toy Holography
• SYK Mo
• O(N)
models
Features of AdS/CFT
1) Radial Commutativity
φ(X) = ∫_R dx K(X, x) O(x)
+ ...
Lim_{r→∞} r^Δ φ(r, X) = O(X)



Toy Holography
• SYK
• O(N)
models
Features of AdS/CFT
1) Radial Commutativity
$\phi(X) = \int_R dx\, K(X,x)\, O(X)$
$+ \ldots$
$\lim_{r\to\infty} r^{\Delta} \phi(r,X) = O(X)$
$O(X)$
$\phi(X)$
$\to [\phi(X), O(X)] = 0$


Toy Holography
• SYK Model
• O(N) vector models
Features of AdS$_{d+1}$/CFT$_d$
1) Radial Commutativity
O(x)
φ(x)
φ(x) = ∫$_R$ dX K(X, x) O(X) + ...
Lim$_{r\to\infty}$ r$^{\Delta}$ φ(r, X) = O(X)
O(X)
φ(x)
→ [φ(x), O(X)] = 0


Features of AdS/CFT
1) Radial Commutativity
Time-slice Axiom
In any QFT, the algebra of local op's at a fixed time acts irreducibly on H.
⇒ Anything that commutes with all O(x) is trivial.


Features of AdS/CFT
1) Radial
Time-slice Axiom
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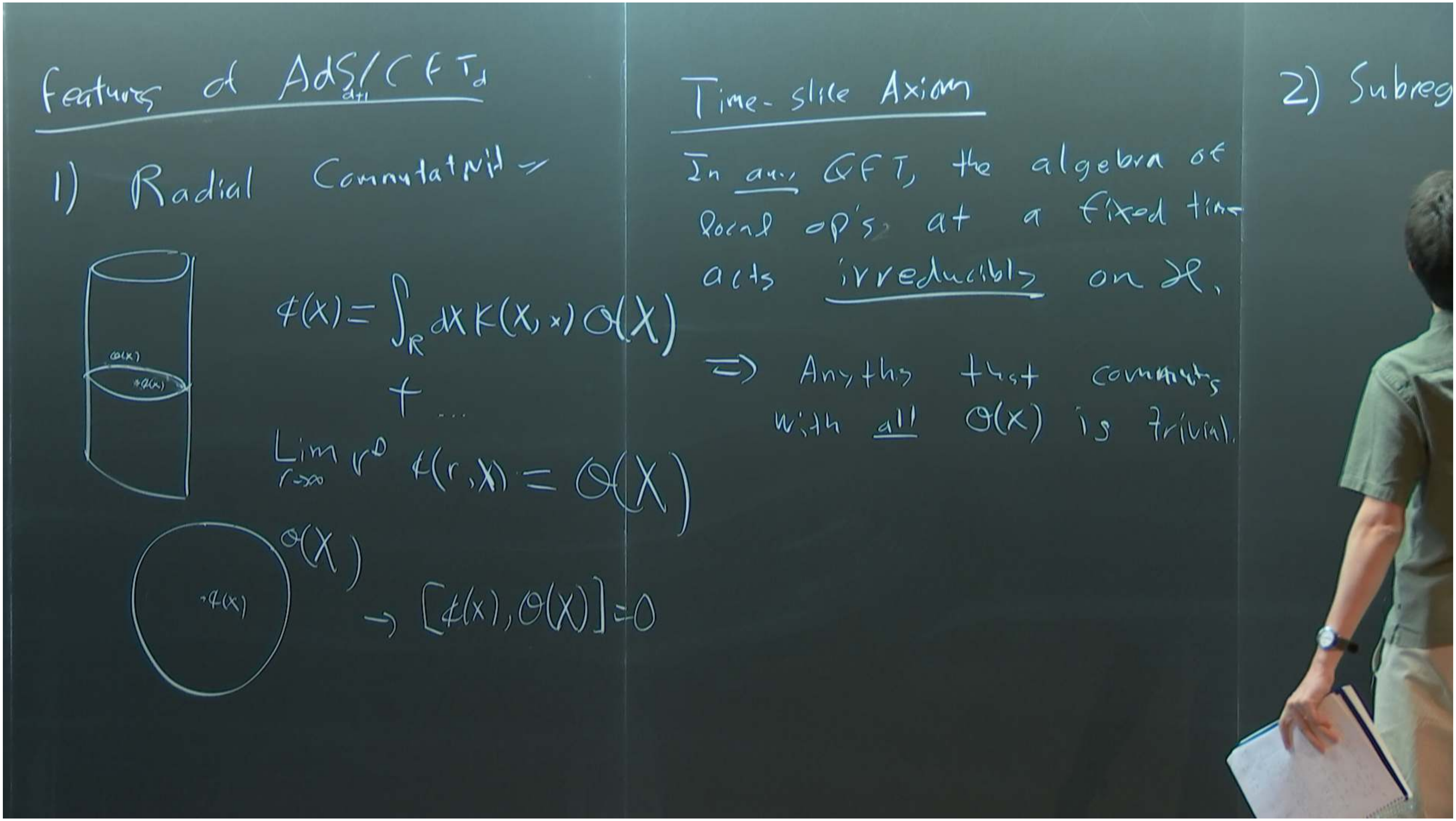
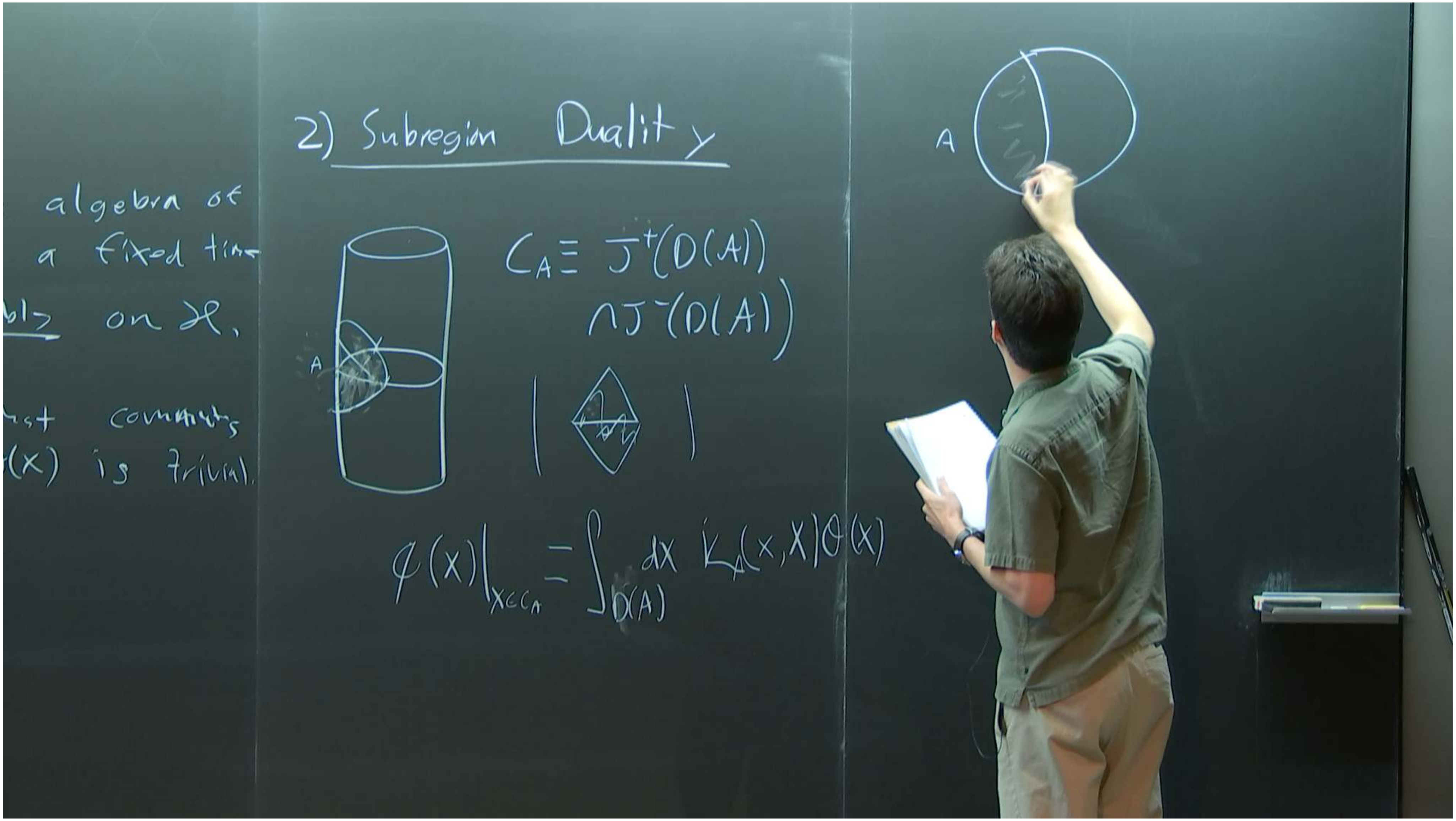
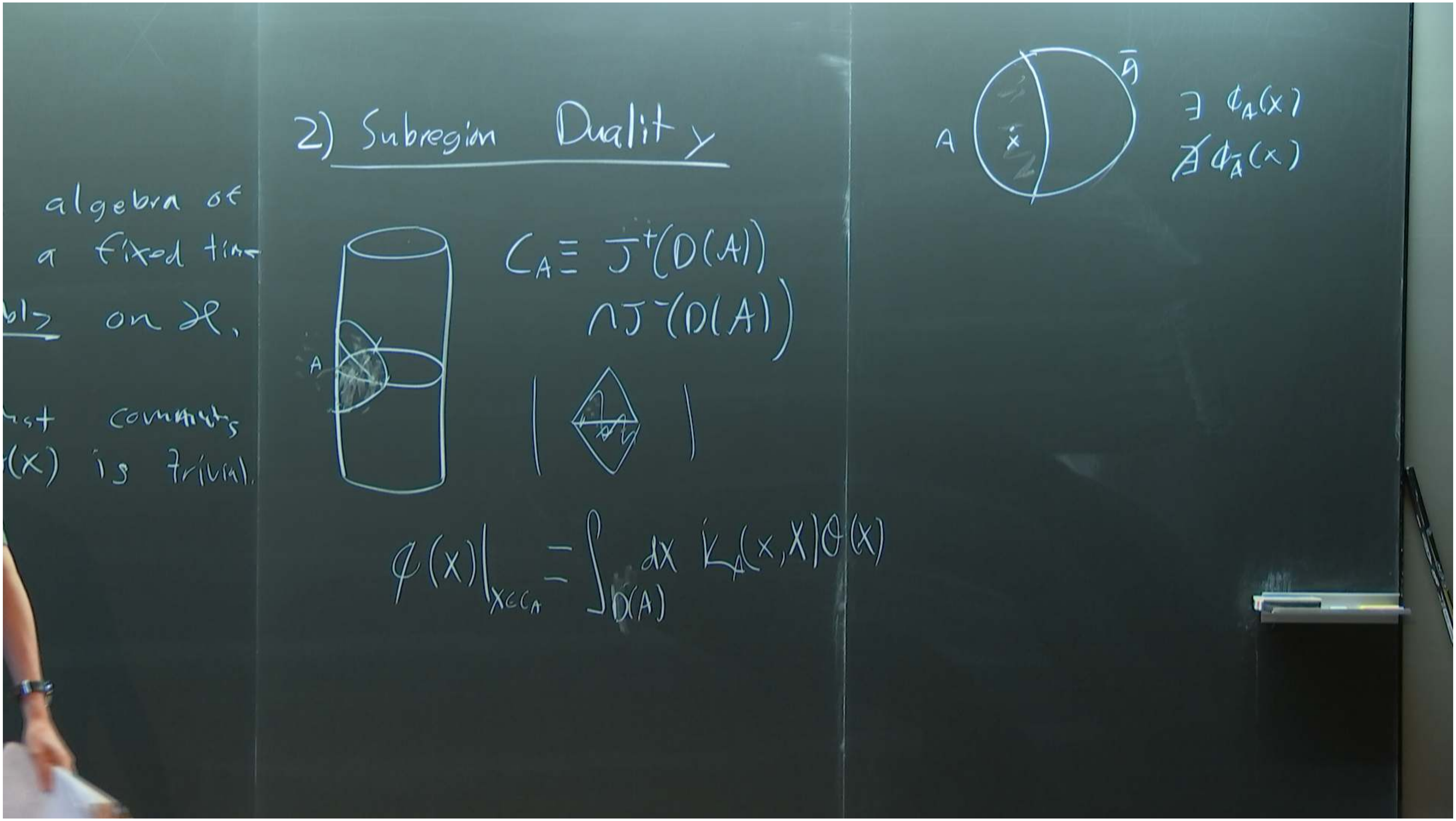
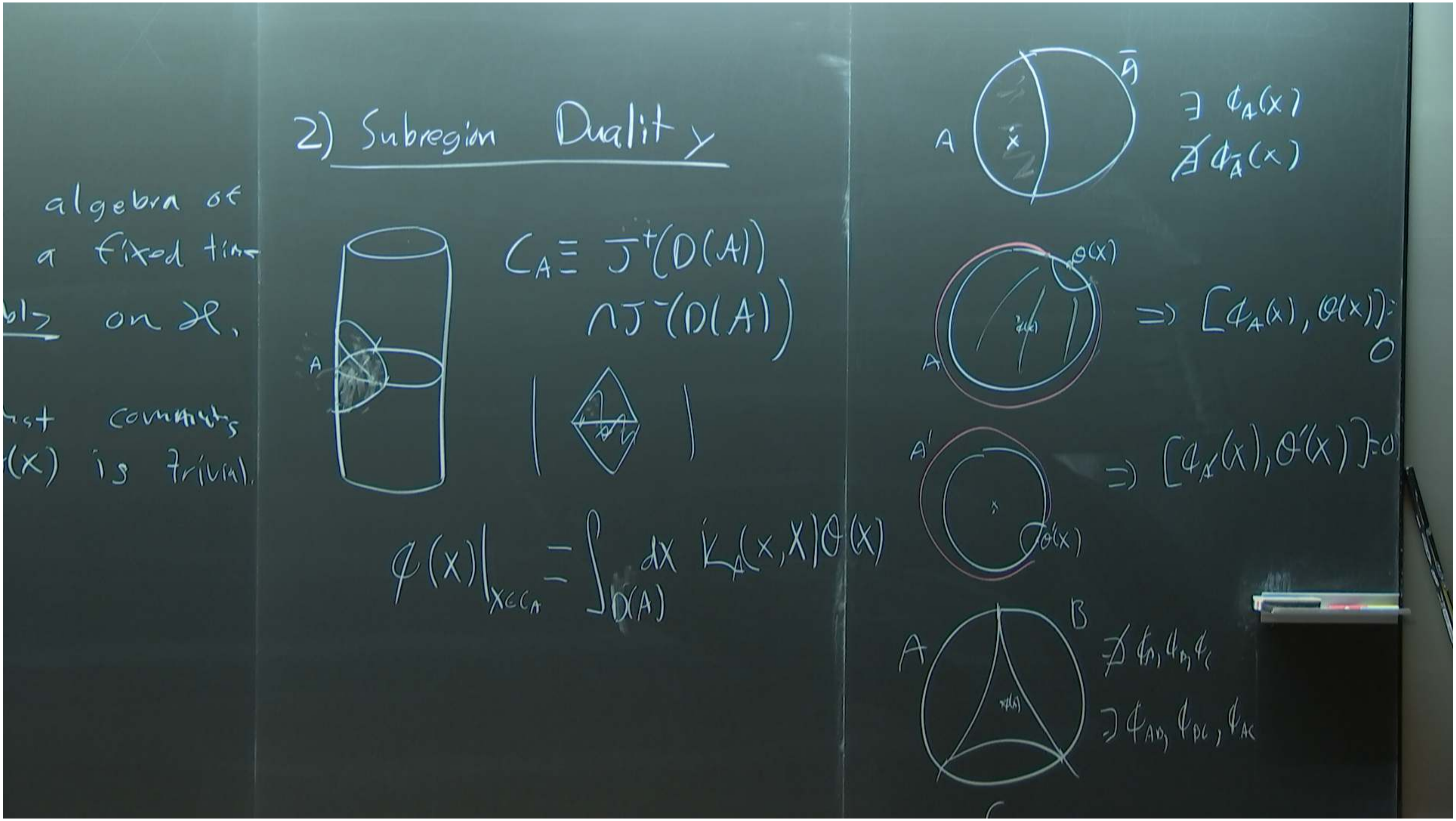
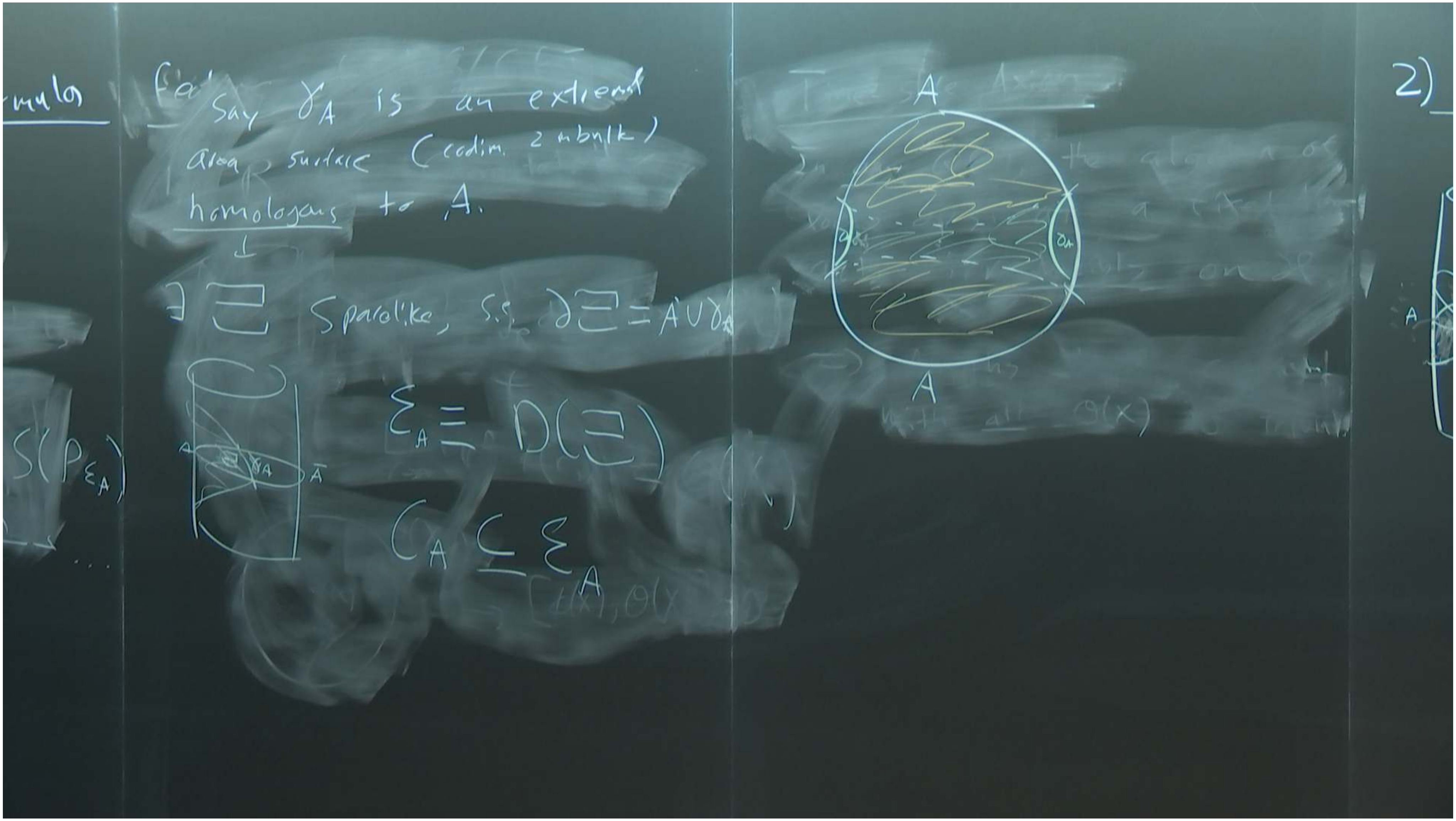

Features of AdS$_{d+1}$/CFT$_d$
1) Radial Commutativity
$\phi(x) = \int_R dX\, K(X, x)\, \mathcal{O}(X)$
+ ...
$\lim_{r\to\infty} r^{\Delta} \phi(r, X) = \mathcal{O}(X)$
$\mathcal{O}(X)$
$\phi(x)$
$\rightarrow [\phi(x), \mathcal{O}(X)] = 0$
Time-slice Axiom
In any QFT, the algebra of local op's at a fixed time acts irreducibly on $\mathcal{H}$.
$\Rightarrow$ Anything that commutes with all $\mathcal{O}(X)$ is trivial.
2) Subreg

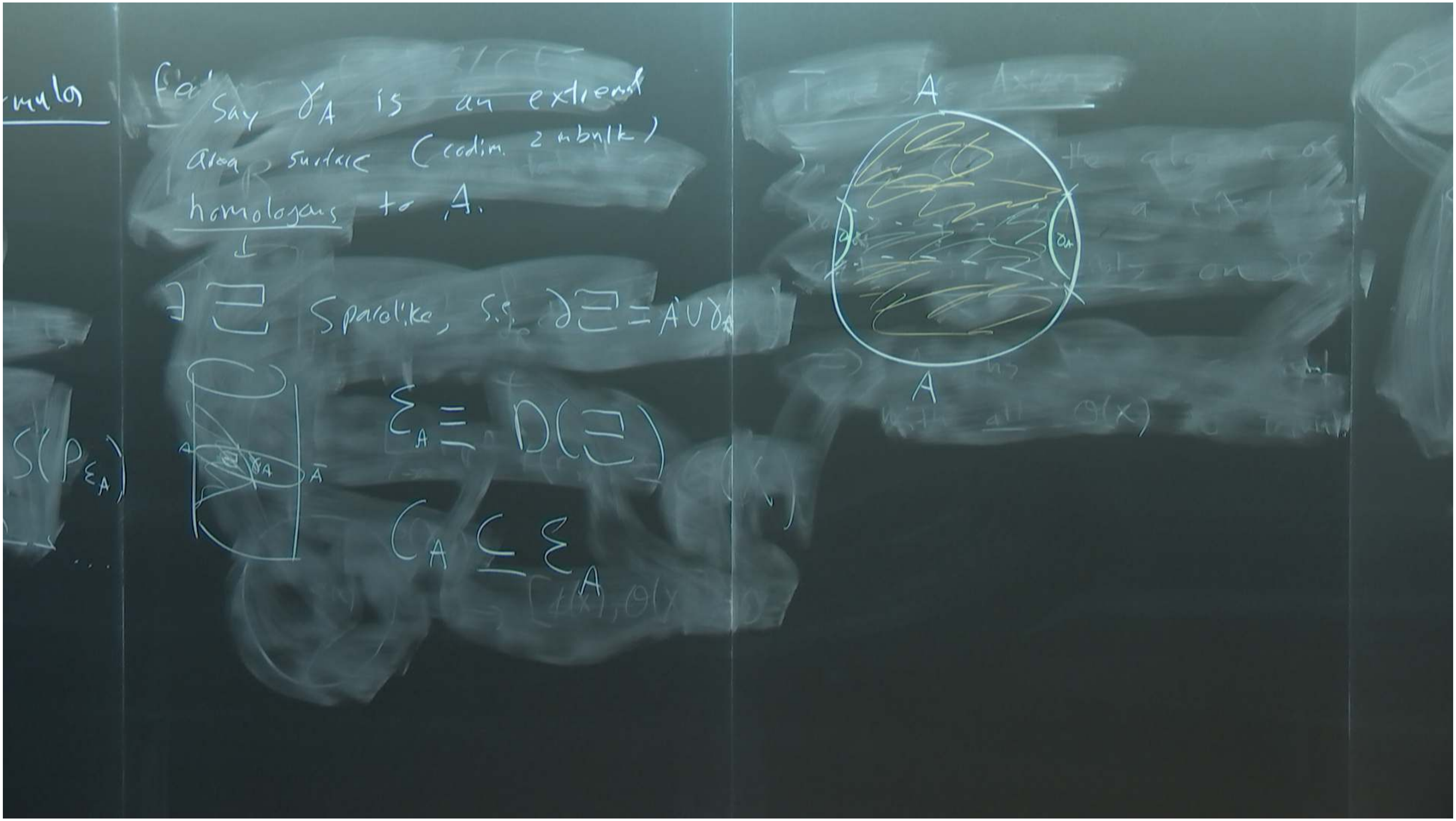

2) Subregion Duality
algebra of
a fixed time
on $\mathcal{H}$.
commutes
is trivial.
A
$C_A \equiv J^+(D(A)) \cap J^-(D(A))$
$\phi(x)|_{x \in C_A} = \int_{D(A)} dX \, K_A(x,X) \, \mathcal{O}(X)$
A



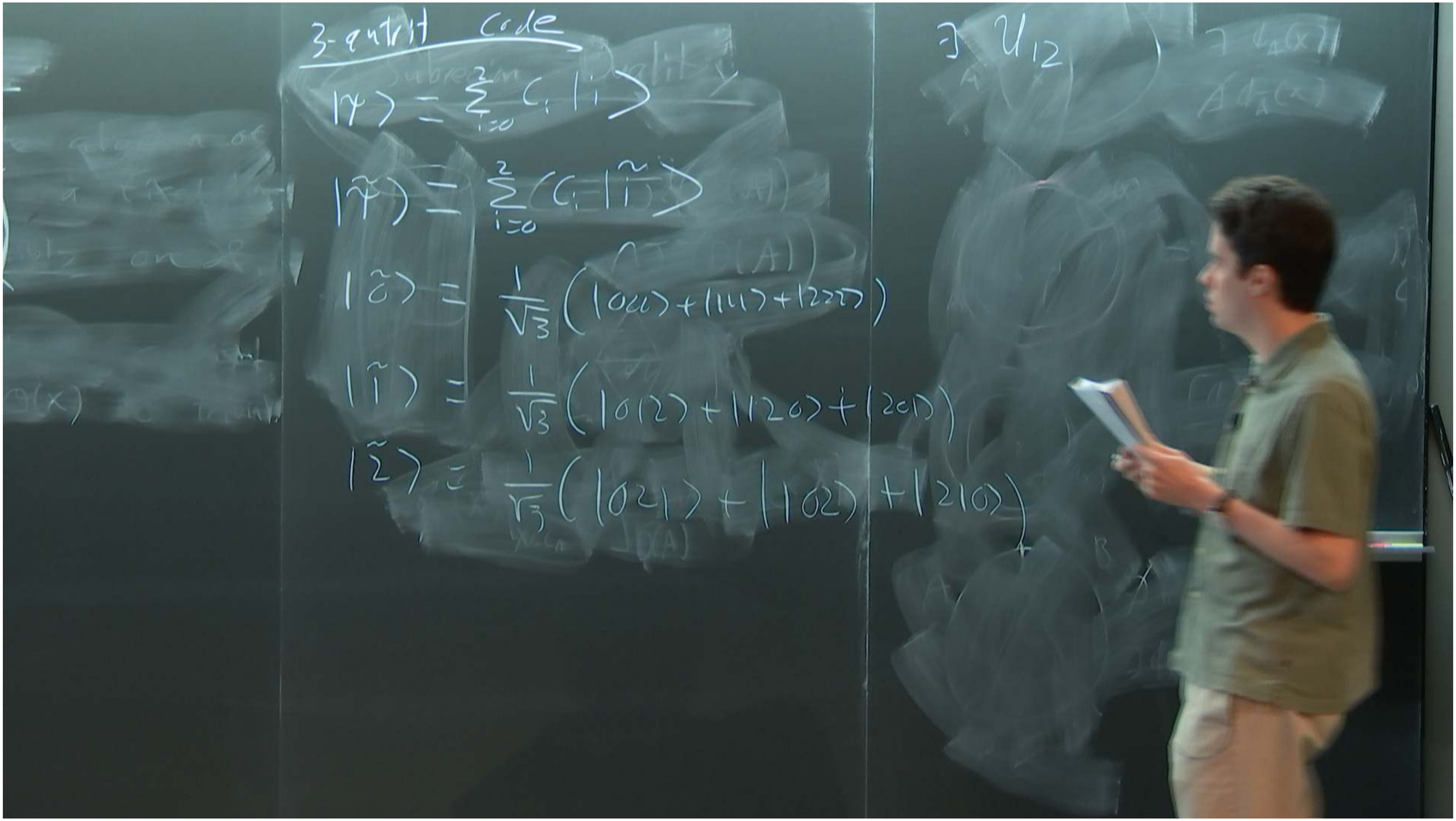
algebra of
a fixed time
on $\mathcal{H}$.
commutes
(x) is trivial.
2) Subregion Duality
$C_A \equiv J^+(D(A)) \cap J^-(D(A))$
A
$\phi(x)|_{x \in C_A} = \int_{D(A)} dX \, K_A(x,X) \, O(X)$
A
x
$\bar{A}$
$\exists \, \phi_A(x)$
$\nexists \, \phi_{\bar{A}}(x)$

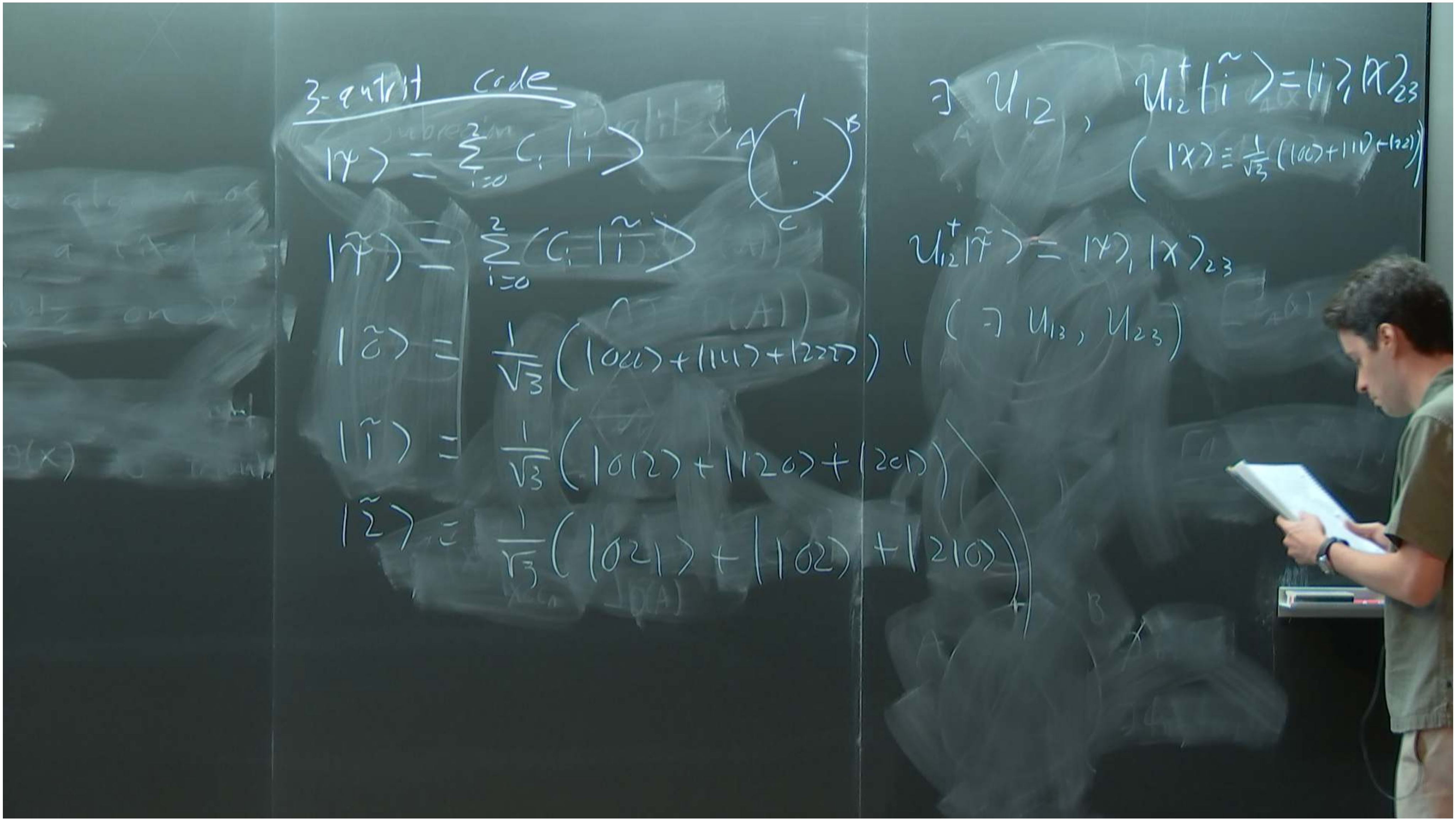

2) Subregion Duality
algebra of
a fixed time
on ℋ.
commutes
(x) is trivial.
C_A ≡ J⁺(D(A)) ∩ J⁻(D(A))
A
φ(x)|_{x∈C_A} = ∫_{D(A)} dX K_A(x,X) O(X)
∃ φ_A(x)
∄ φ_Ā(x)
O(x)
⇒ [φ_A(x), O(x)] = 0
A'
O'(x)
⇒ [φ_A(x), O'(x)] = 0
B
C
∄ φ_A, φ_B, φ_C
∃ φ_AB, φ_BC, φ_AC


rmula
Say γA is an extremal
area surface (codim. 2 in bulk)
homologous to A.
∃ Ξ spacelike, s.t. ∂Ξ = A ∪ γA
ℰA ≡ D(Ξ)
CA ⊆ ℰA
S(ρℰA)
A
2)



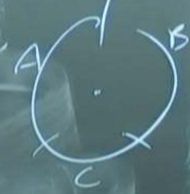
Say γ_A is an extremal
area surface (codim. 2 in bulk)
homologous to A.
∃ Ξ spacelike, s.t. ∂Ξ = A ∪ γ_A
Σ_A = D(Ξ)
C_A ⊆ Σ_A
S(ρ_{Σ_A})
A
A


3-qutrit code
|ψ⟩ = Σ_{i=0}^{2} c_i |i⟩
|ψ̃⟩ = Σ_{i=0}^{2} c_i |ĩ⟩
|0̃⟩ = 1/√3 (|000⟩ + |111⟩ + |222⟩)
|1̃⟩ = 1/√3 (|012⟩ + |120⟩ + |201⟩)
|2̃⟩ = 1/√3 (|021⟩ + |102⟩ + |210⟩)
∃ U_12


3-qutrit code
|ψ> = Σ_{i=0}^{2} c_i |i>
|ψ̃> = Σ_{i=0}^{2} c_i |ĩ>
|0̃> = 1/√3 (|000> + |111> + |222>)
|1̃> = 1/√3 (|012> + |120> + |201>)
|2̃> = 1/√3 (|021> + |102> + |210>)
A
B
C
∃ U_12 , U_12^† |ĩ> = |i>_1 |χ>_23
(|χ> = 1/√3 (|00> + |11> + |22>))
U_12^† |ψ̃> = |ψ>_1 |χ>_23
(∃ U_13 , U_23)

3-qutrit code

$$|\psi\rangle = \sum_{i=0}^{2} c_i |i\rangle$$



$$|\tilde\psi\rangle = \sum_{i=0}^{2} c_i |\tilde i\rangle$$

$$|\tilde 0\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right),$$

$$|\tilde 1\rangle = \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right)$$

$$|\tilde 2\rangle = \frac{1}{\sqrt{3}}\left(|021\rangle + |102\rangle + |210\rangle\right)$$

$\exists\ U_{12}$, $U_{12}^\dagger |\tilde i\rangle = |i\rangle_1 |\chi\rangle_{23}$

$\left(|\chi\rangle \equiv \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)\right)$

$U_{12}^\dagger |\tilde\psi\rangle = |\psi\rangle_1 |\chi\rangle_{23}$

$(\exists\ U_{13},\ U_{23})$

Say $\tilde O |\tilde i\rangle = \sum_j (O)_{ji} |\tilde j\rangle$

$$O_{12} \equiv U_{12} O_1 U_{12}^\dagger$$

$\Rightarrow \exists\ O_{13},\ O_{23}$

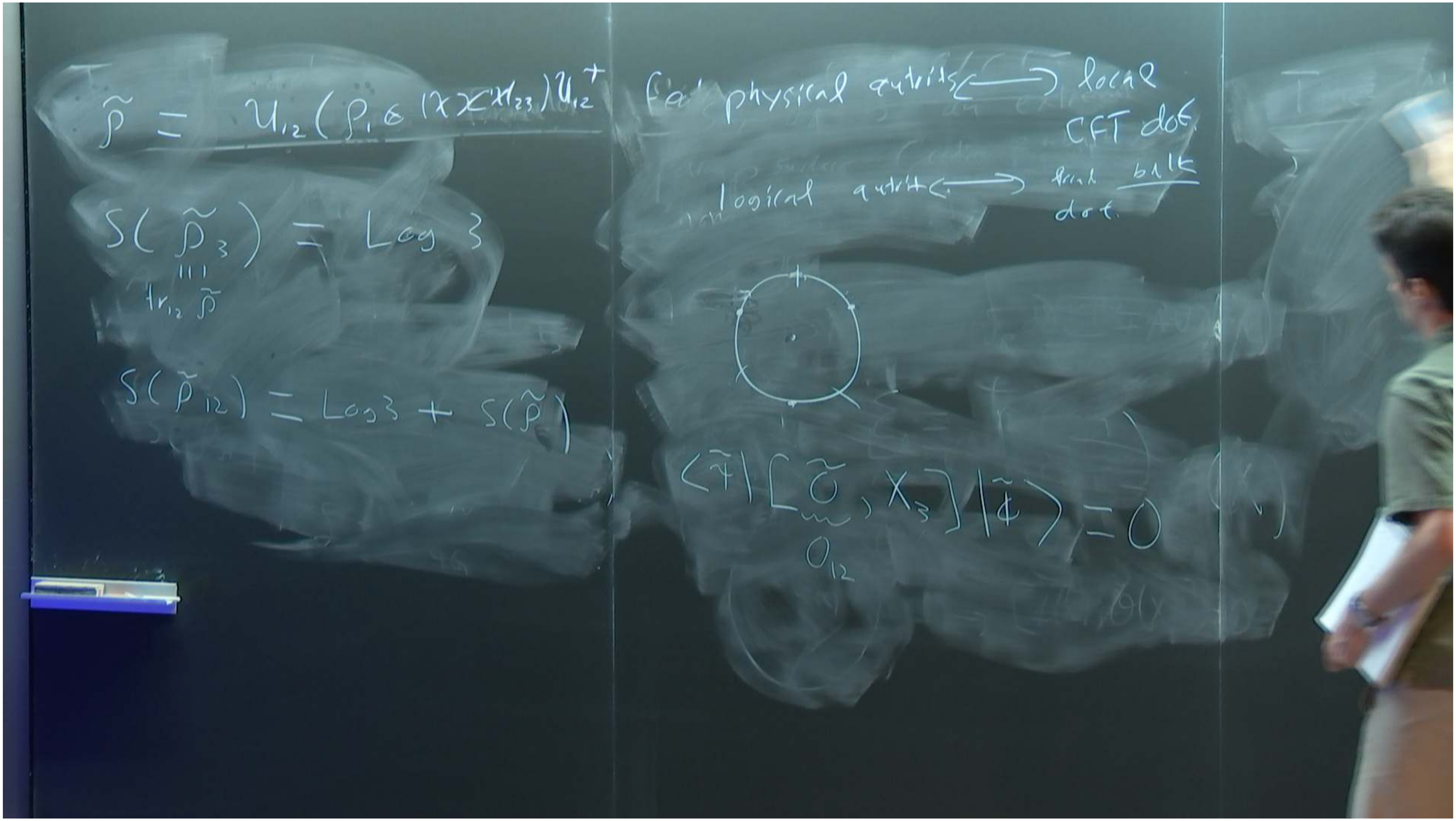
$$\tilde{\rho} = U_{12}(\rho_1 \otimes |\chi\rangle\langle\chi|_{23})U_{12}^\dagger$$

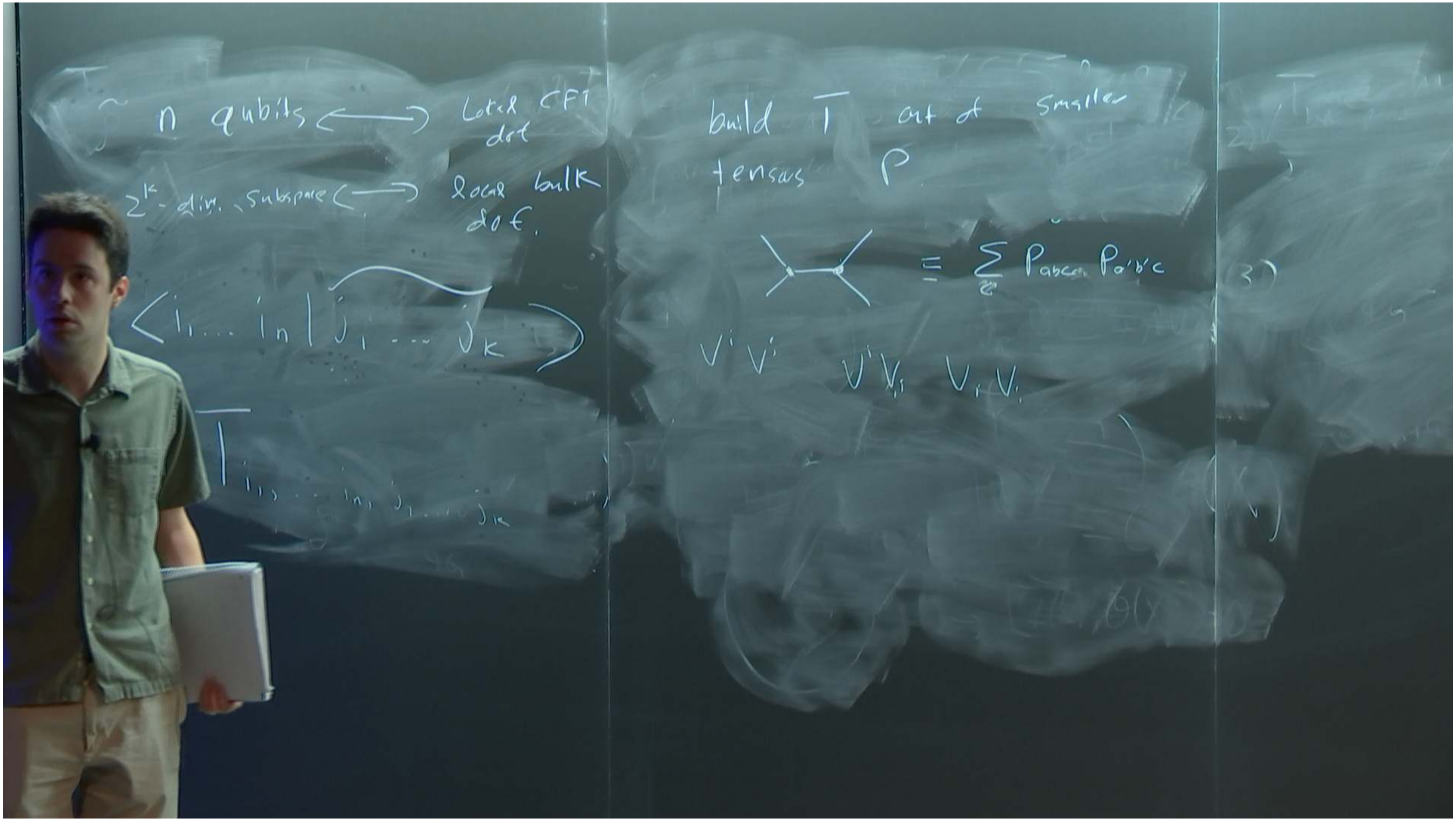
physical qutrits ⟷ local CFT dof

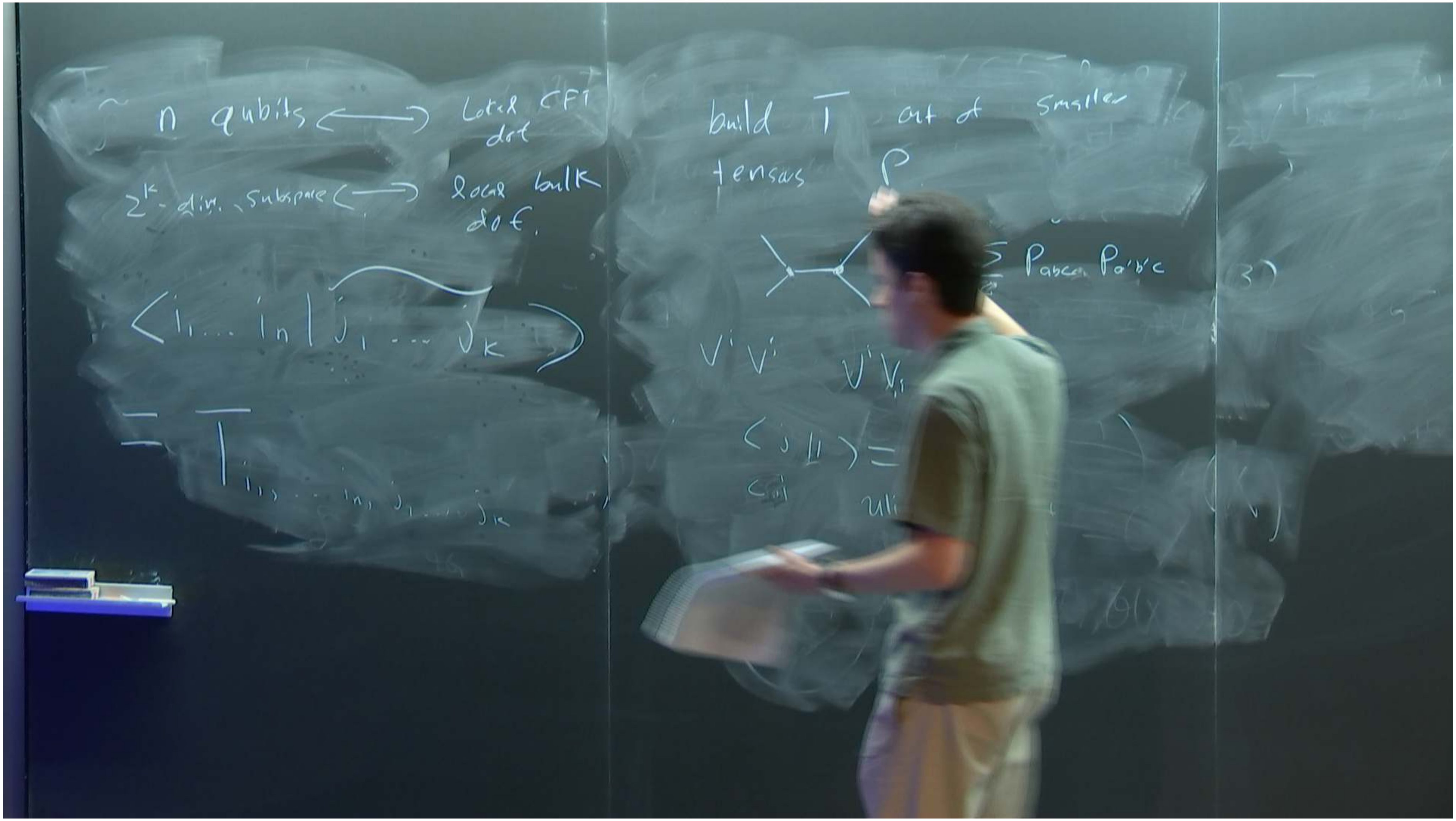
$$S(\tilde{\rho}_3) = \log 3$$

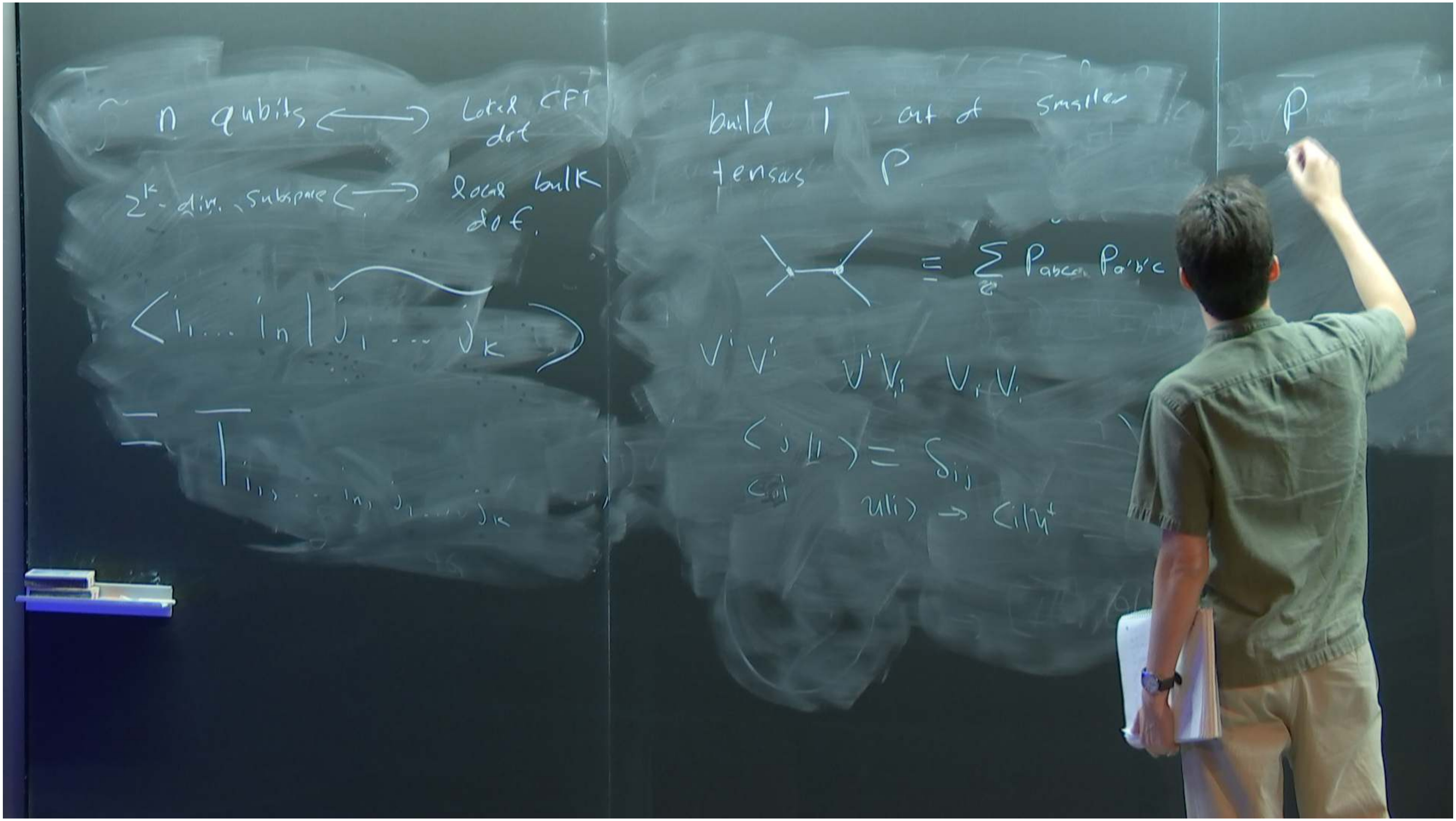
$$\tilde{\rho}_3 = \mathrm{tr}_{12}\,\tilde{\rho}$$

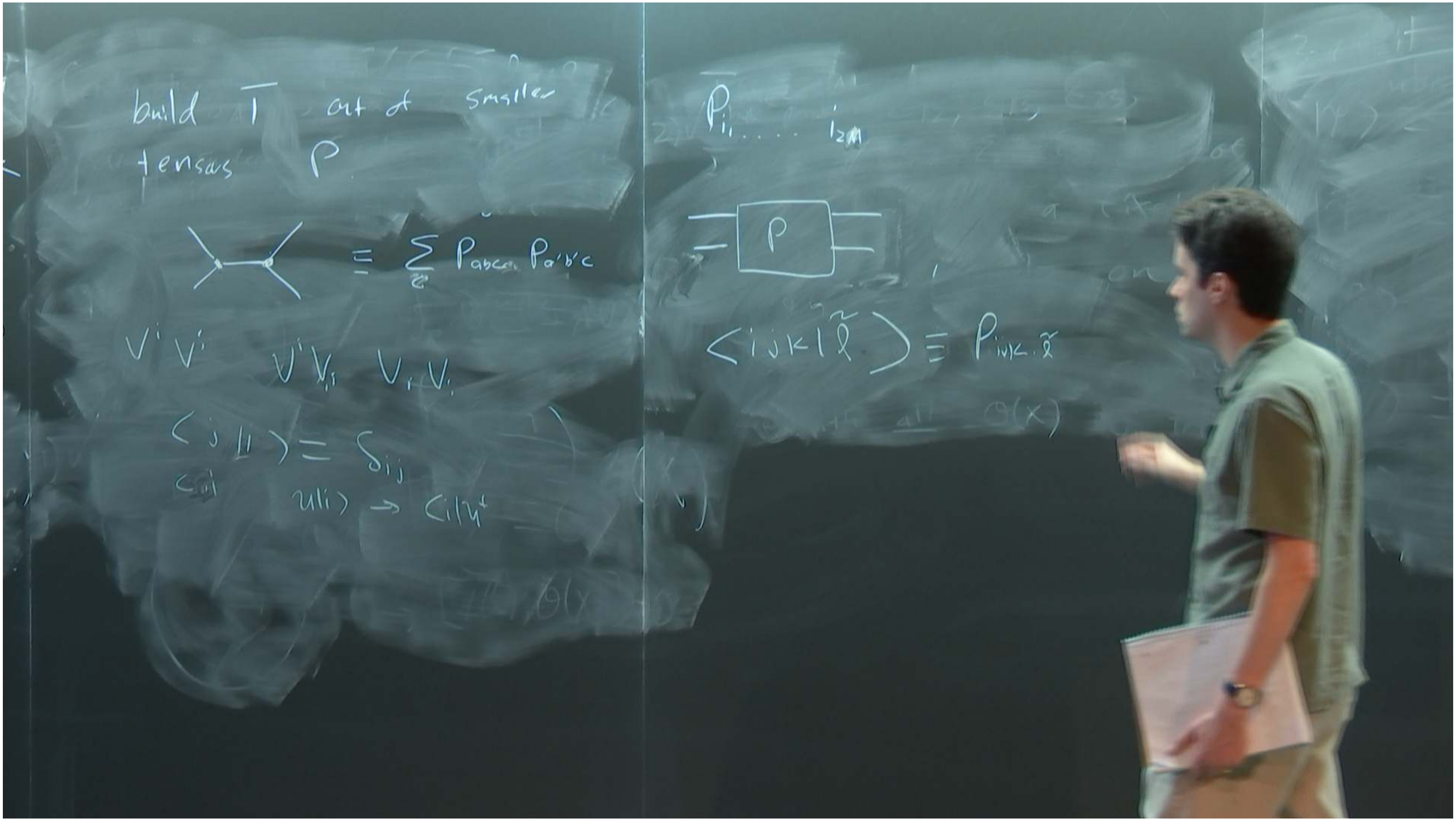
$$S(\tilde{\rho}_{12}) = \log 3 + S(\tilde{\rho})$$

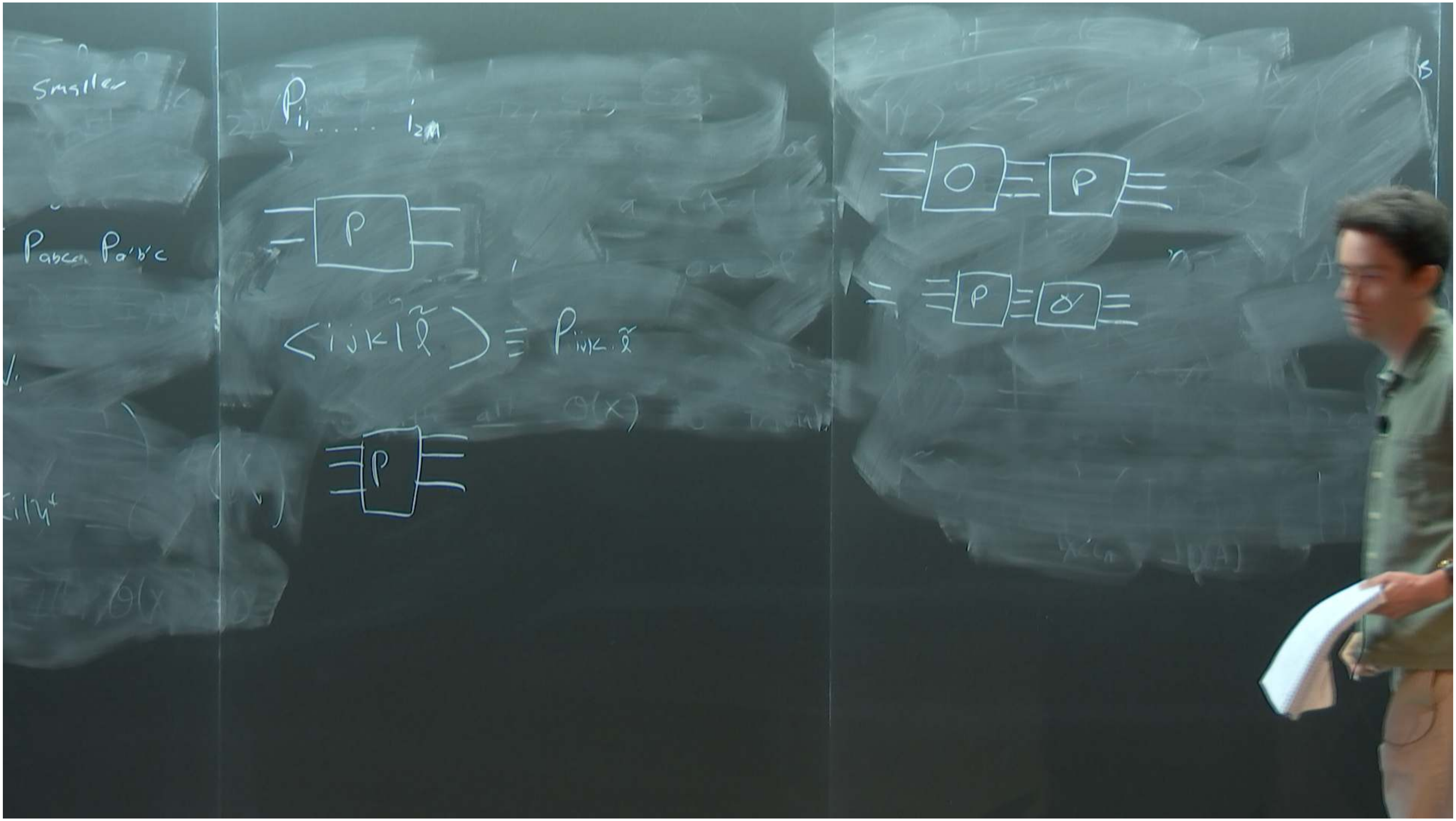

physical qutrits ⟷ local CFT dof
logical qutrit ⟷ bulk dof
S(ρ̃ 3) = Log 3
S(ρ̃ 12) = Log 3 + S(ρ̃)

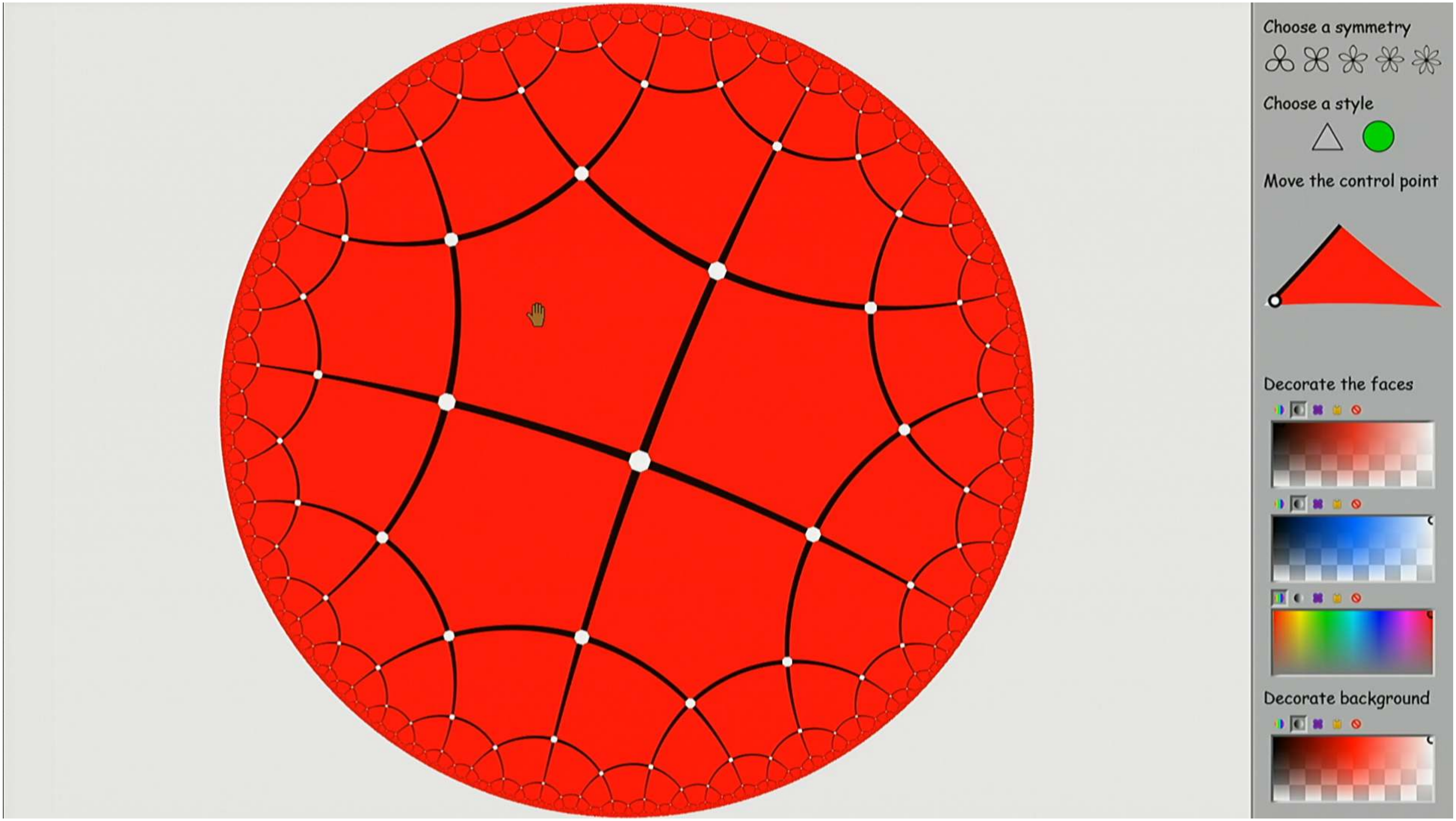

n qubits ⟷ local CFT dof
2^k-dim. subspace ⟷ local bulk dof.
build T out of smaller tensors P
= Σ P_{abce} $P_{a'b'c}$

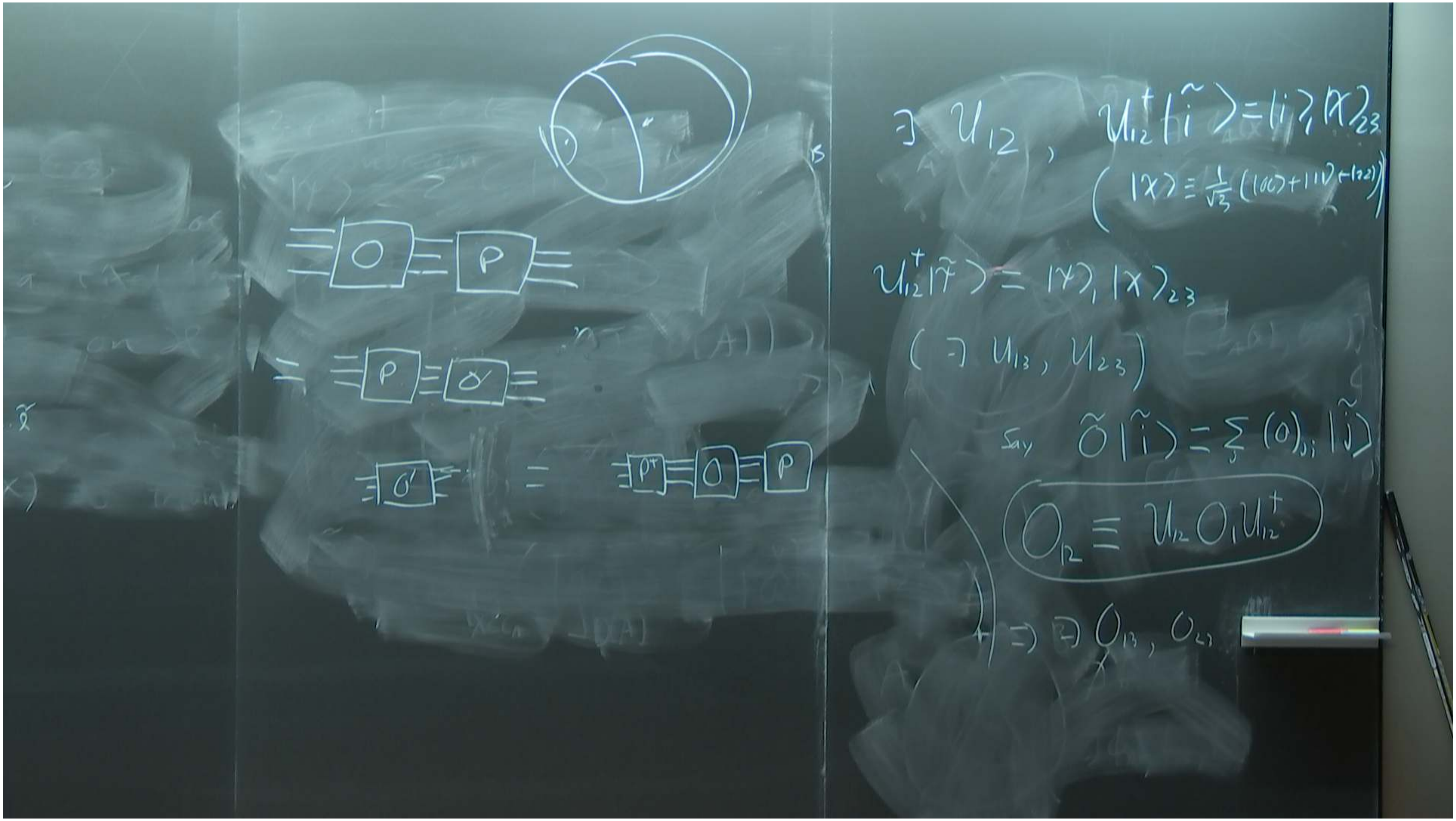

n qubits ⟷ local CFT dof
2^k-dim. subspace ⟷ local bulk dof.
build T out of smaller tensors


n qubits
local CFT dof
local bulk dof.
build T out of smaller
tensors

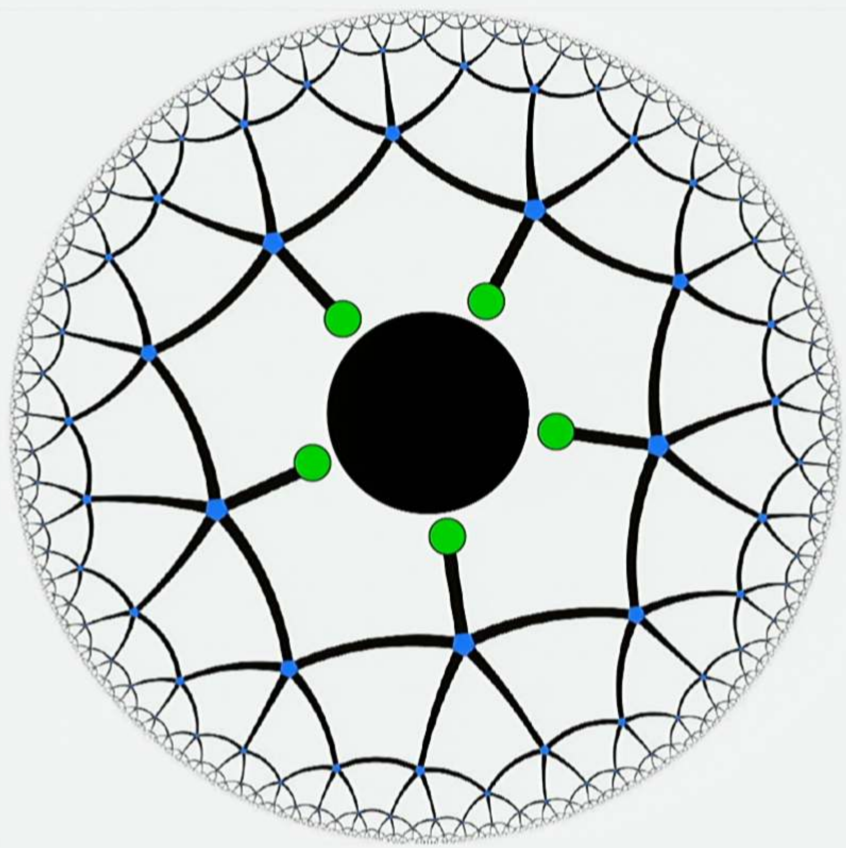

build T out of smaller
tensors P
P

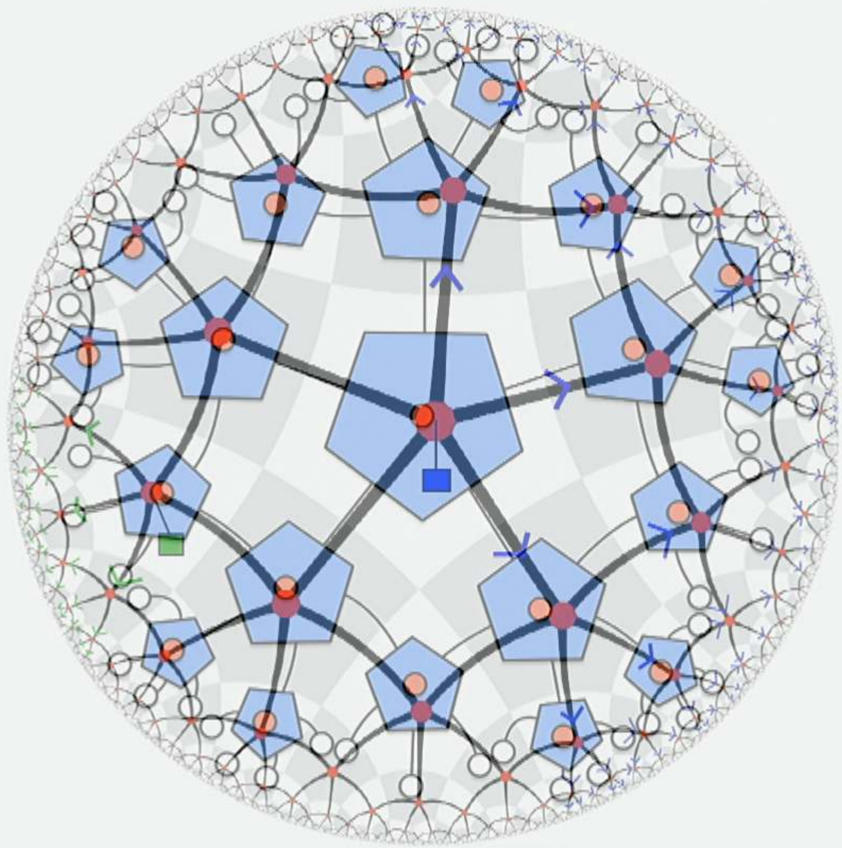

smaller
Pabca Pa'b'c
P
O
P
P
O

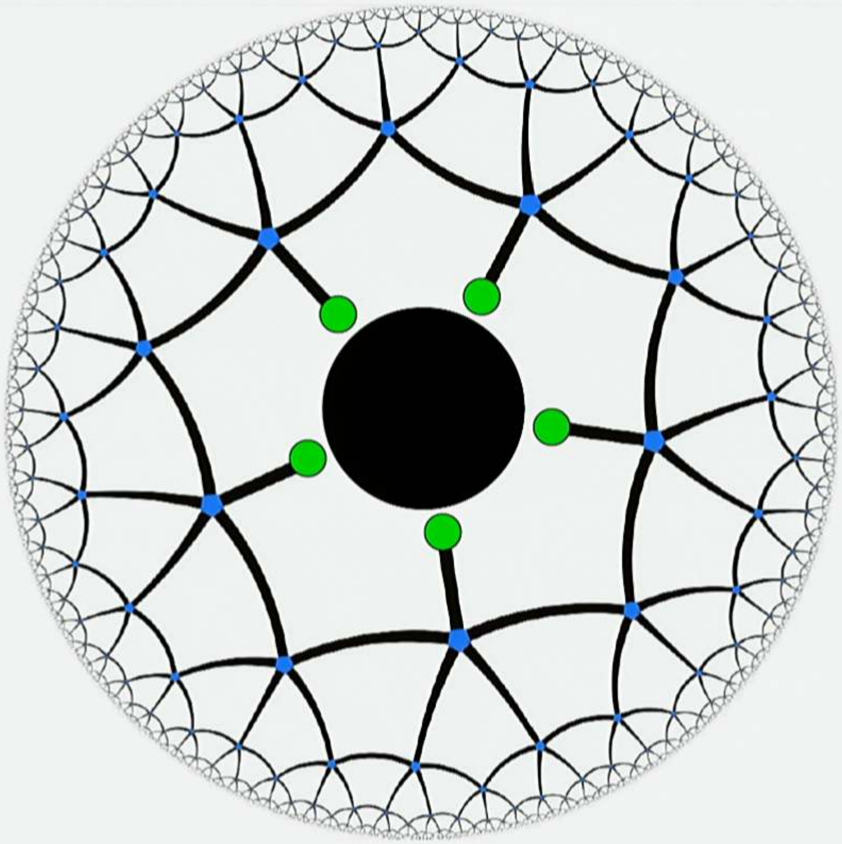

Choose a symmetry
Choose a style
Move the control point
Decorate the faces
Decorate background



O
P
P
O'
O'
P†
O
P
∃ U12 , U12† |ĩ⟩ = |i⟩1 |χ⟩23
(|χ⟩ ≡ 1/√3 (|00⟩ + |11⟩ + |22⟩))
U12† |χ̃⟩ = |χ⟩1 |χ⟩23
(∃ U13 , U23)
Say Õ |ĩ⟩ = Σj (O)ji |j̃⟩
O12 ≡ U12 O1 U12†
⇒ ∃ O13 , O23







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