Title: Quantum Gravity and Quantum Chaos

Date: Jul 29, 2016 09:00 AM

URL: http://pirsa.org/16070056

Abstract:

SS, D. Stanford, "Black holes and the butterfly effect"

" "Stringy effects in scrambling"

Maldacena + " "A bound on chaos"

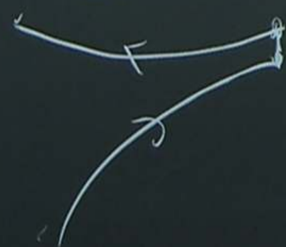
Roberts, SS, Stanford, "Localized shocks."

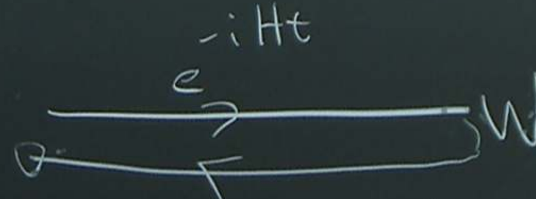
"Black holes and the
butterfly effect"
"Stringy effects in
scrambling"
"A bound on chaos"
"Localized shocks."

$$Q.C. \longleftrightarrow Q.G.$$

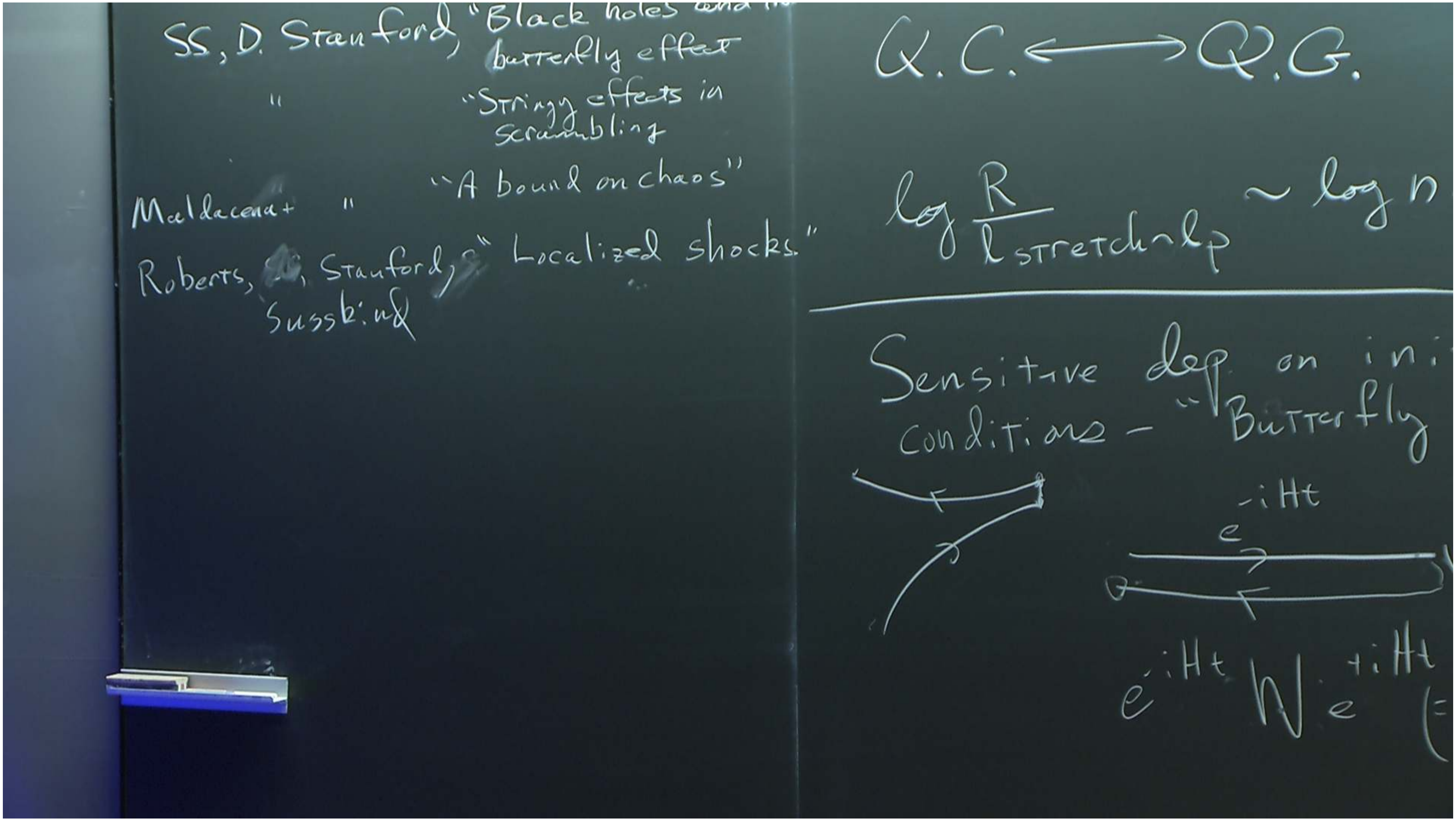
$$\log \frac{R}{l_{\text{stretch}} \sim l_p} \sim \log n \sim \log S$$

Sensitive dep. on initial conditions — "Butterfly effects"





$$e^{-iHt}\, W\, e^{+iHt} \; (= W(t))$$


SS, D. Stanford, "Black holes and the butterfly effect"
" "Stringy effects in scrambling"
Maldacena + " "A bound on chaos"
Roberts, , Stanford, " Localized shocks."
Susskind
Q.C. ⟷ Q.G.
log R / l stretch ~ l p ~ log n
Sensitive dep. on ini
conditions - "Butterfly
e^{-iHt}
e^{iHt} W e^{-iHt}

$\log S$

$\ldots l$

"effects"

$W(t))$

$$H = \sum_i a_i^\dagger a_{i+1} + c.c. + a_i^\dagger a_{i+1} a_{i+2}^\dagger a_{i+2}$$

$$a_j = W$$

$$W(t) = a_j + [H, a_j] + [H,[H,a_j]] + \ldots$$

$a\,a_j\,a$ $\qquad$ $a\,a_j\,a\,a\,a\,a$

φ "grows"

$+ a_i^{\dagger} a_{i+1} a_{i+2}^{\dagger} a_{i+2}$

$[H,[H,a_j]] + \cdots$

$a\, a\, a\, a\, a\, a$

Non-local, 2-local, k-local

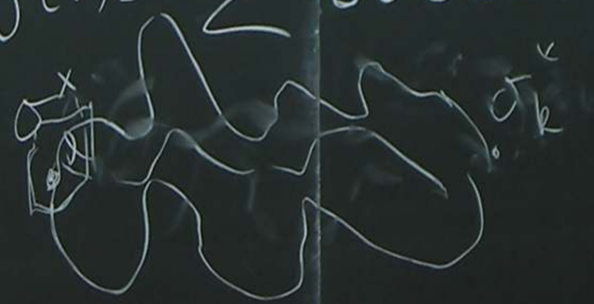
k finite

$$\sum_{i,j=1}^{N} \sigma_i^{\alpha} \sigma_j^{\beta} J_{ij}^{\alpha\beta}$$

$$W(t) = \sigma^x + \sigma\sigma + \sigma\sigma\sigma$$

Lieb-Robinson

$$W(t) = \sum \sigma\sigma\sigma\sigma\sigma$$



$$\| [\sigma_\ell, \sigma_k(t)] \|$$

ocks"

$$C(t) = \left\langle -[\sigma_\ell, \sigma_k(t)]^2 \right\rangle_\beta$$

$$\boxed{\sim \frac{1}{N} e^{\lambda_L t}} \qquad \sim 1 \sim \frac{1}{N} e^{\lambda_L t_*}$$

$$t_* \sim \frac{1}{\lambda_L} \log N$$

Lyapunov

$$|\delta q(\tau)| \sim e^{\lambda_L t}\,|\delta q(0)|$$

$$\frac{\partial q(t)}{\partial q(0)} = \{q(t), p(0)\}_{PB} \longrightarrow \frac{1}{i\hbar}\left[\hat{q}(t), \hat{p}(0)\right]$$

$$C(t) = \hbar^2 e^{2\lambda_L t}, \qquad \hbar \sim \frac{1}{e^S}$$

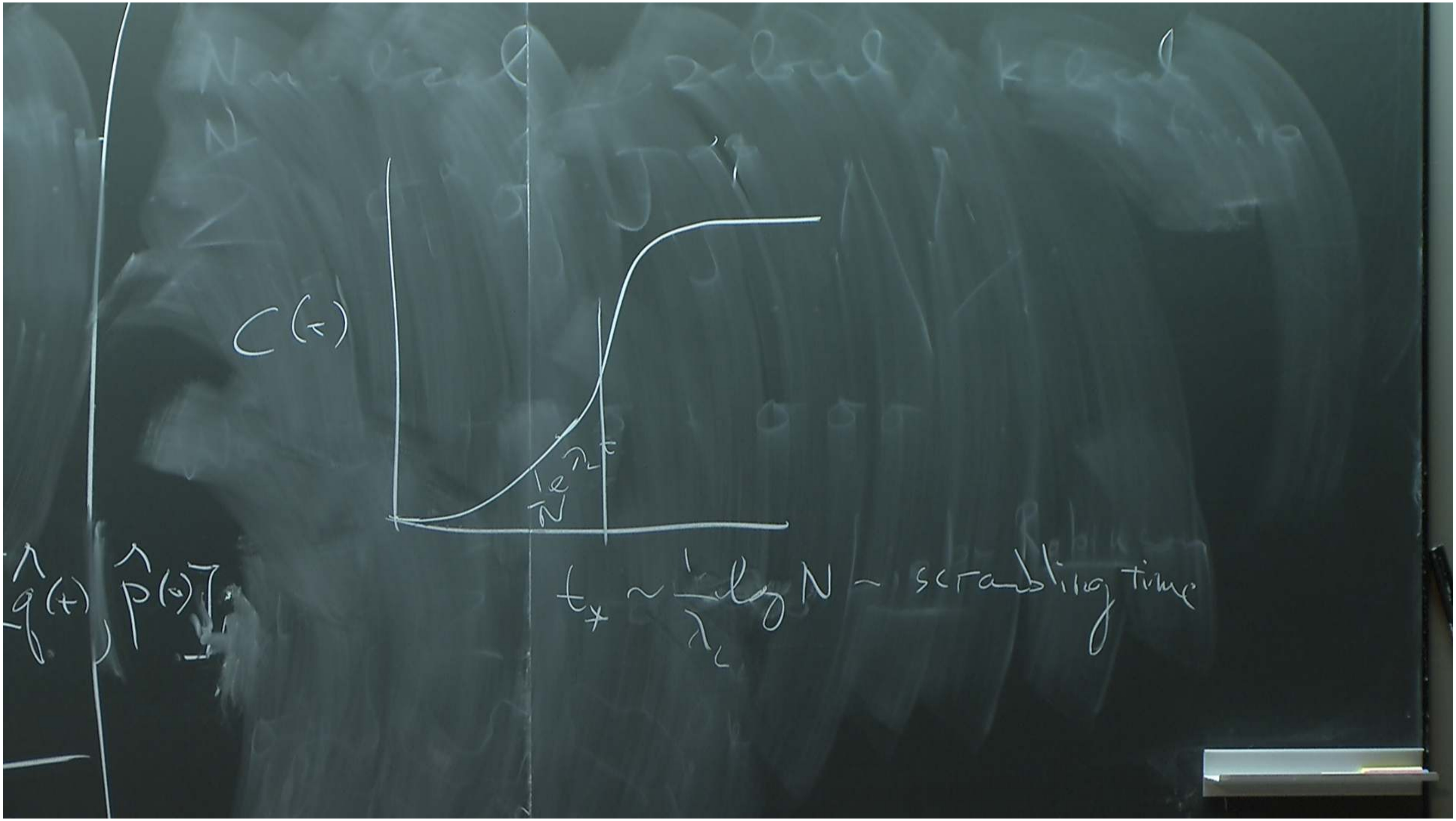
$$\log\left(\frac{L^D}{\hbar^{D/2}}\right) \sim \frac{D}{2}\log\frac{1}{\hbar}$$

β

t_*

$\log N$

$\sum_{i,j=1}^{N}$


C(t)
1/N e^{λ_L t}
t_* ~ 1/λ_L log N ~ scrambling time
[q̂(t), p̂(t)]

ia

s''

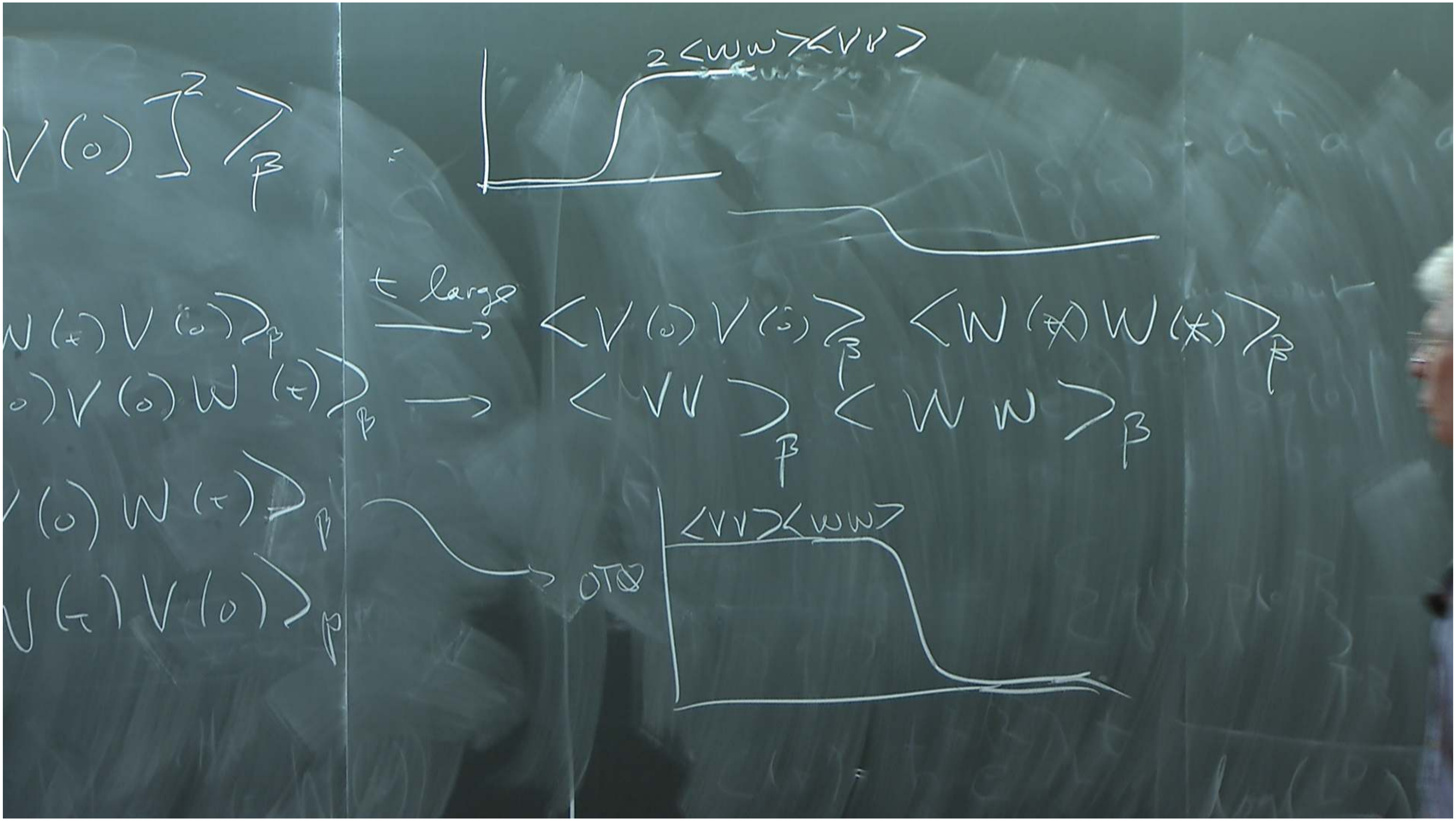
shocks.''

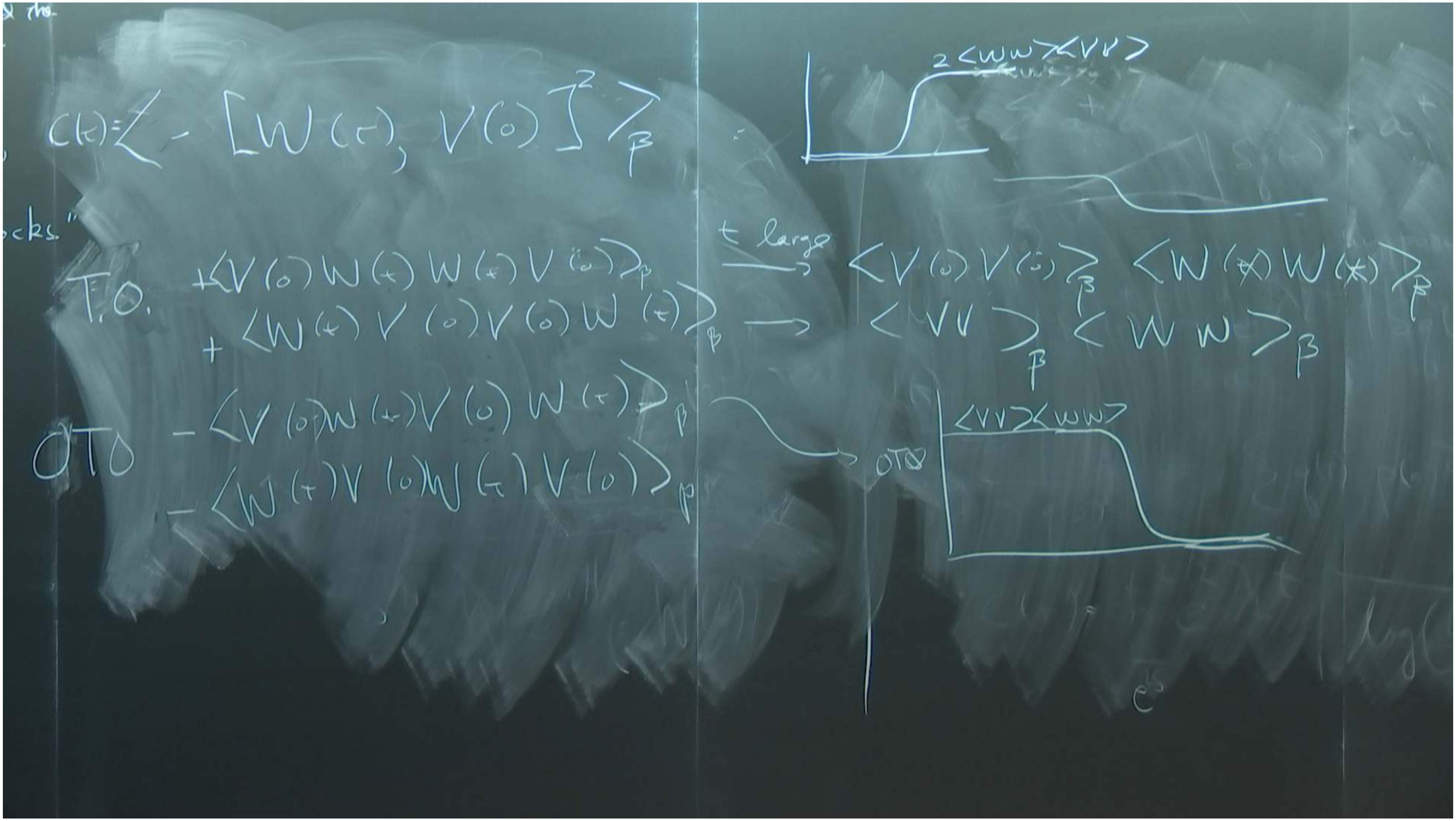
$$C(t) = \langle -[W(t), V(0)]^2 \rangle_\beta$$

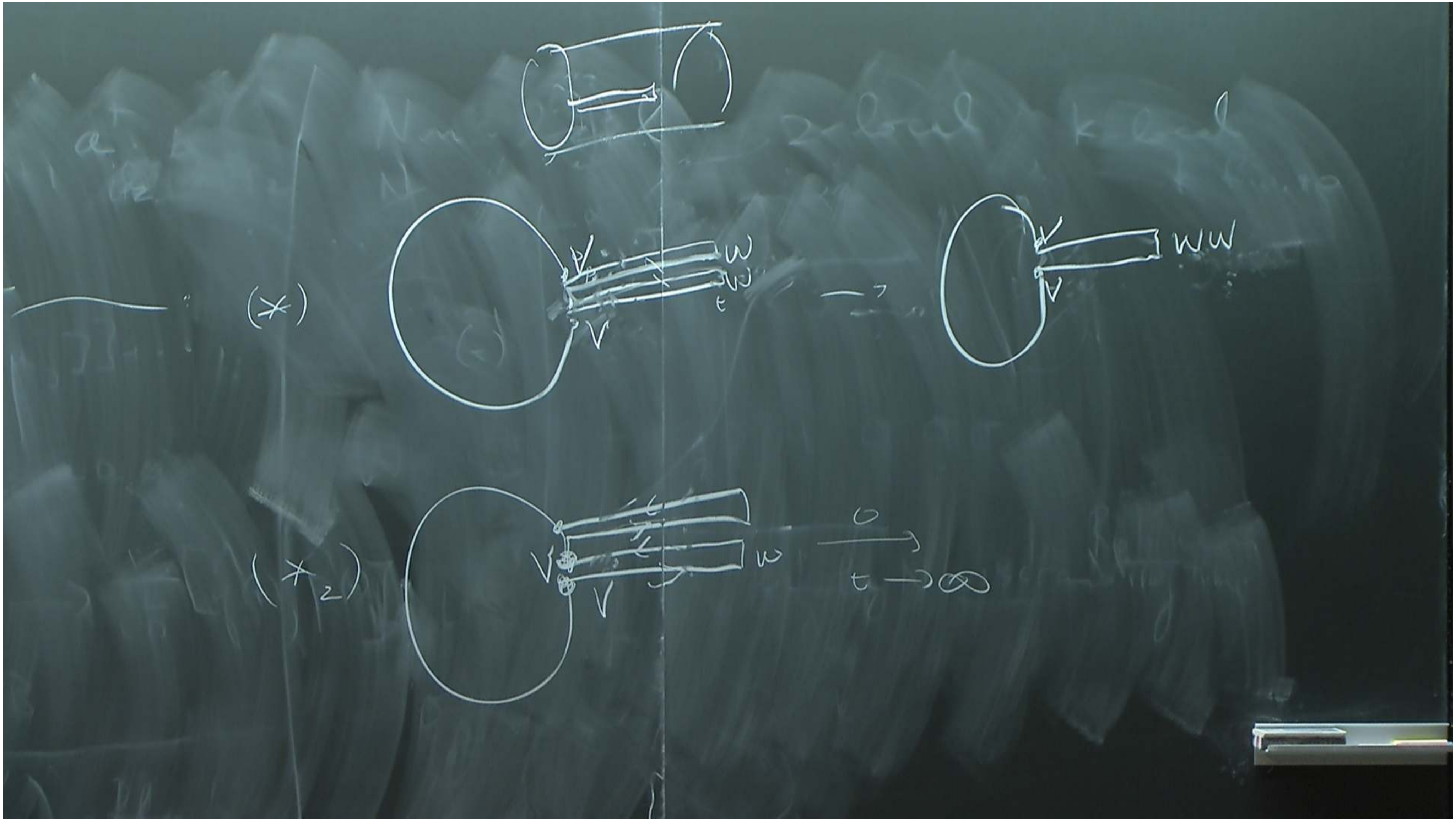
$$= \langle V(0)W(t)W(t)V(0)\rangle + \langle W(t)V(0)V(0)W(t)\rangle - \langle V(0)W(t)V(0)W(t)\rangle - \langle W(t)V(0)W(t)V(0)\rangle$$

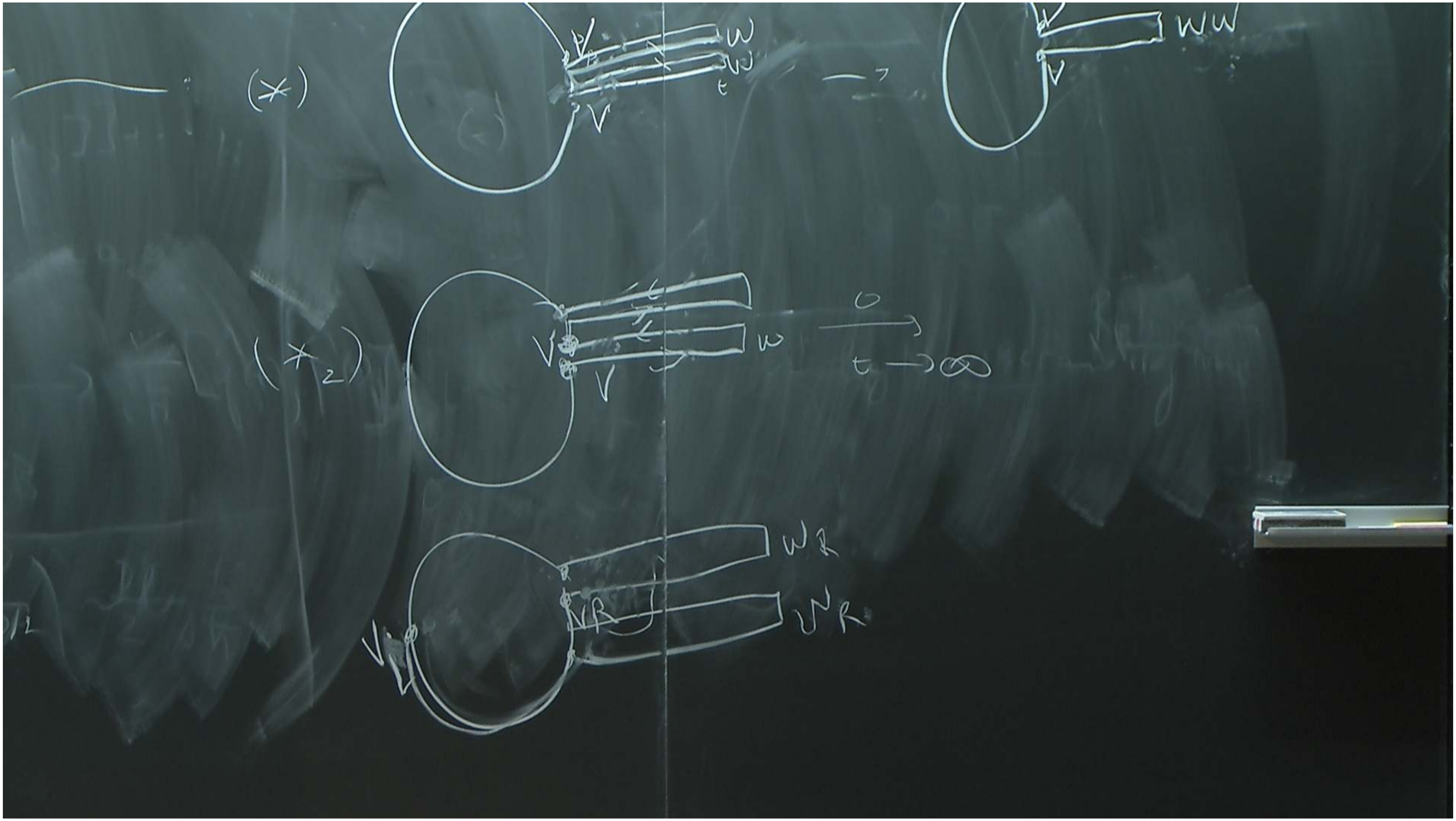
g(0

g(0


2<WW><VV>
V(0)]² >β
t large
W(t)V(0)>β → <V(0)V(0)>β <W(x)W(x)>β
0)V(0)W(t)>β → <VV>β <WW>β
V(0)W(t)>β
W(t)V(0)>β
OTO
<VV><WW>


$C(t) = \langle - [W(t), V(0)]^2 \rangle_\beta$
T.O.
$+ \langle V(0) W(t) W(t) V(0) \rangle_\beta$
$+ \langle W(t) V(0) V(0) W(t) \rangle_\beta$
OTO
$- \langle V(0) W(t) V(0) W(t) \rangle_\beta$
$- \langle W(t) V(0) W(t) V(0) \rangle_\beta$
t large
$\langle V(0) V(0) \rangle_\beta \langle W(t) W(t) \rangle_\beta$
$\langle VV \rangle_\beta \langle WW \rangle_\beta$
$2\langle WW \rangle \langle VV \rangle$
OTO
$\langle VV \rangle \langle WW \rangle$


(*)
V
W
W
t
V
WW
V
(*2)
V
V
w
0
t → ∞


(*)
W
W
t
V
WW
V
(*₂)
V
V
W
0
t → ∞
WR
VR
V
WR

effect
...ts in
chaos''
...l shocks.''

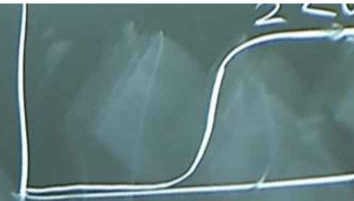
$$D(t) = \langle V(t_1)\, W(t_2)\, V(t_3)\, W(t_4) \rangle_\beta$$

early late early late

$$= \langle \Psi | \Psi' \rangle$$

$$|\Psi'\rangle = V(t_3)\, W(t_4)\, |TFD\rangle$$

$$|\Psi\rangle = W^\dagger(t_2)\, V^\dagger(t_1)\, |TFD\rangle$$

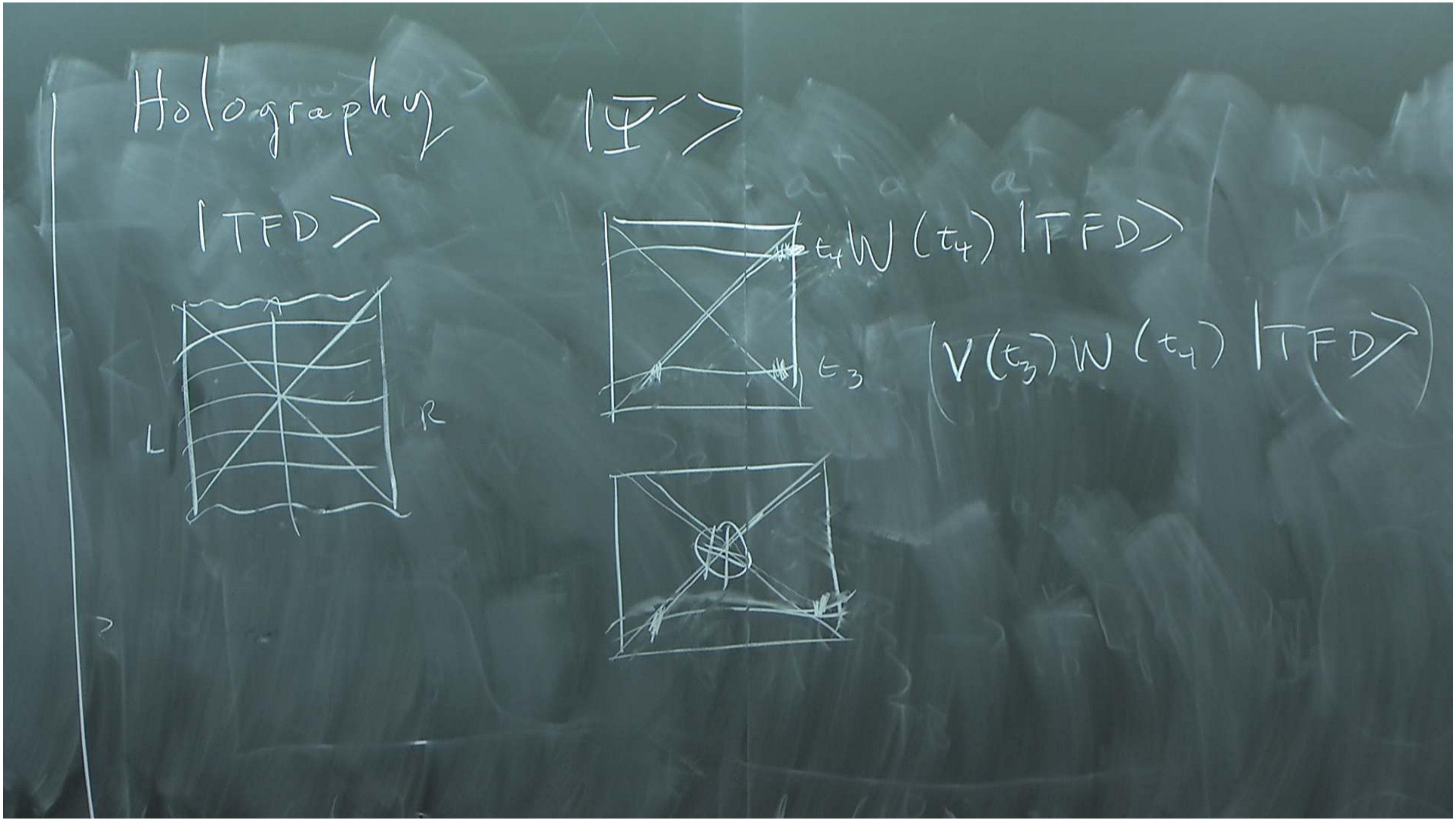


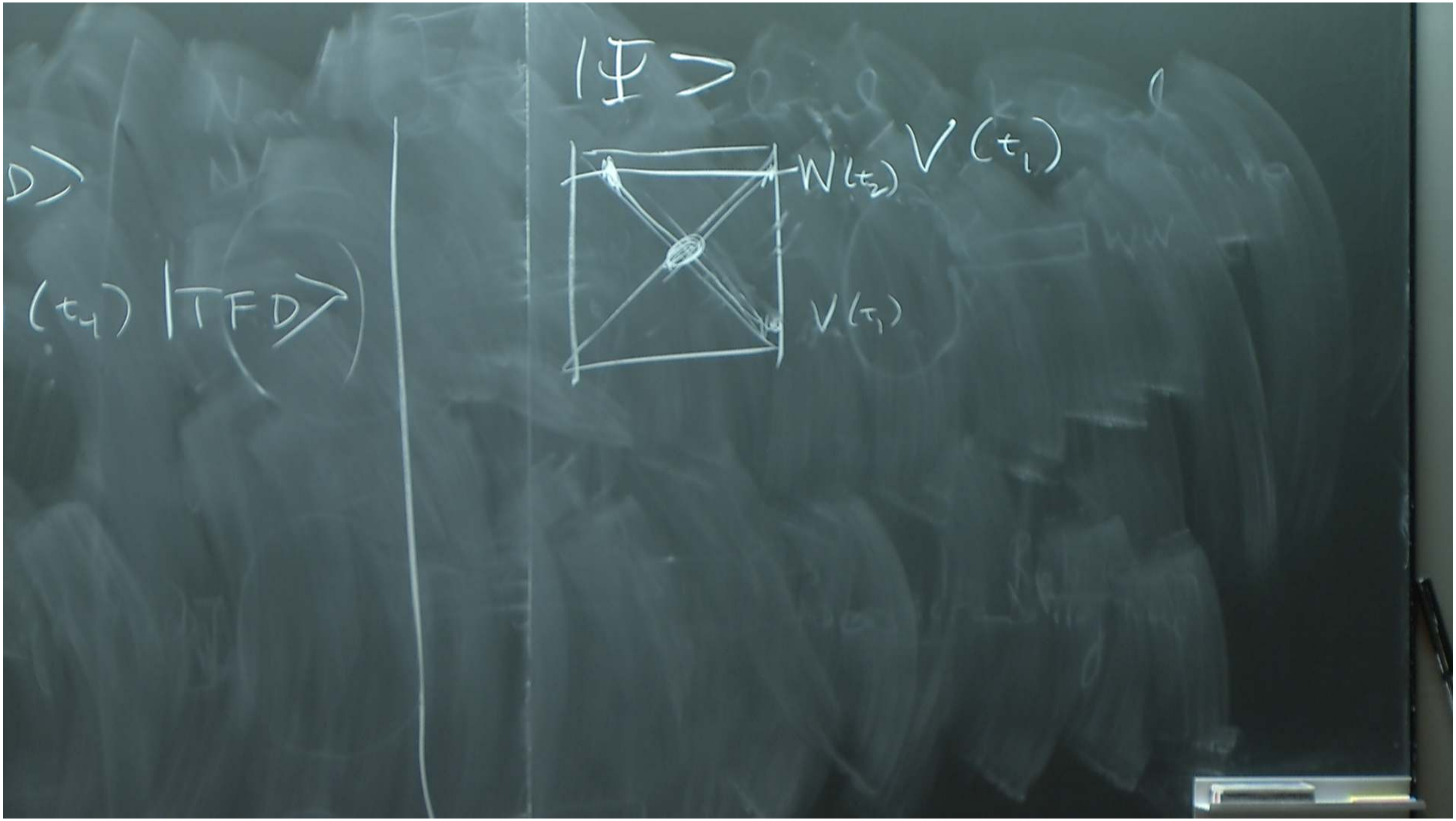
$\langle V(0)\, V(0$ ...

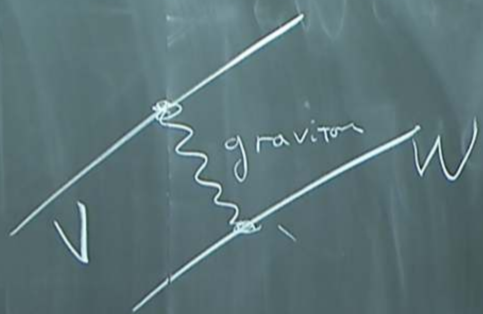
$\langle VV \rangle_\beta$

$\langle VV \rangle \langle$ ...

OTO

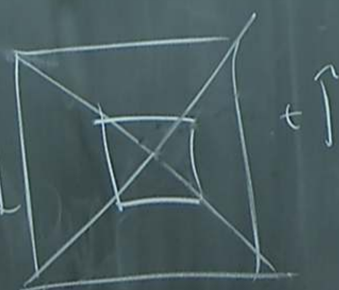

Holography
|Ψ'>
|TFD>
L
R
t4 W (t4) |TFD>
t3
V(t3) W (t4) |TFD>


|Ψ>
W(t₂) V(t₁)
V(t₁)
D>
(t₄) |TFD>



$\delta \sim G_N\, s \left(\text{transverse, AdS..}\right)$, $\quad s = E_{cm}^2$

$V(0) \sim E_V \sim T$



$E_W \sim e^{\frac{2\pi t}{\beta}}\, T$

$E_{cm} \sim E_V E_W \sim T^2 e^{2\pi t/\beta}$

$$D(t) = \langle V(t_1)\, W(t_2)\, V(t_3)\, W(t_4) \rangle_\beta$$

early late early late

$$\delta \sim G_N s \sim \frac{T\, l_{AdS}^2}{N^2} e^{2\pi t/\beta}$$

$$c_0 - c_1 \frac{T^2}{N^2} e^{2\pi t/\beta} + \cdots$$

Holography

$$\lambda_L = \frac{2\pi}{\beta} = 2\pi T$$

Einstein Grav.

$$D(t) = \langle V(t_1)\, W(t_2)\, V(t_3)\, W(t_4) \rangle_\beta$$

early late early late

$$\delta \sim G_N s \sim \frac{T \ell_{AdS}^2}{N^2} e^{2\pi t/\beta}$$

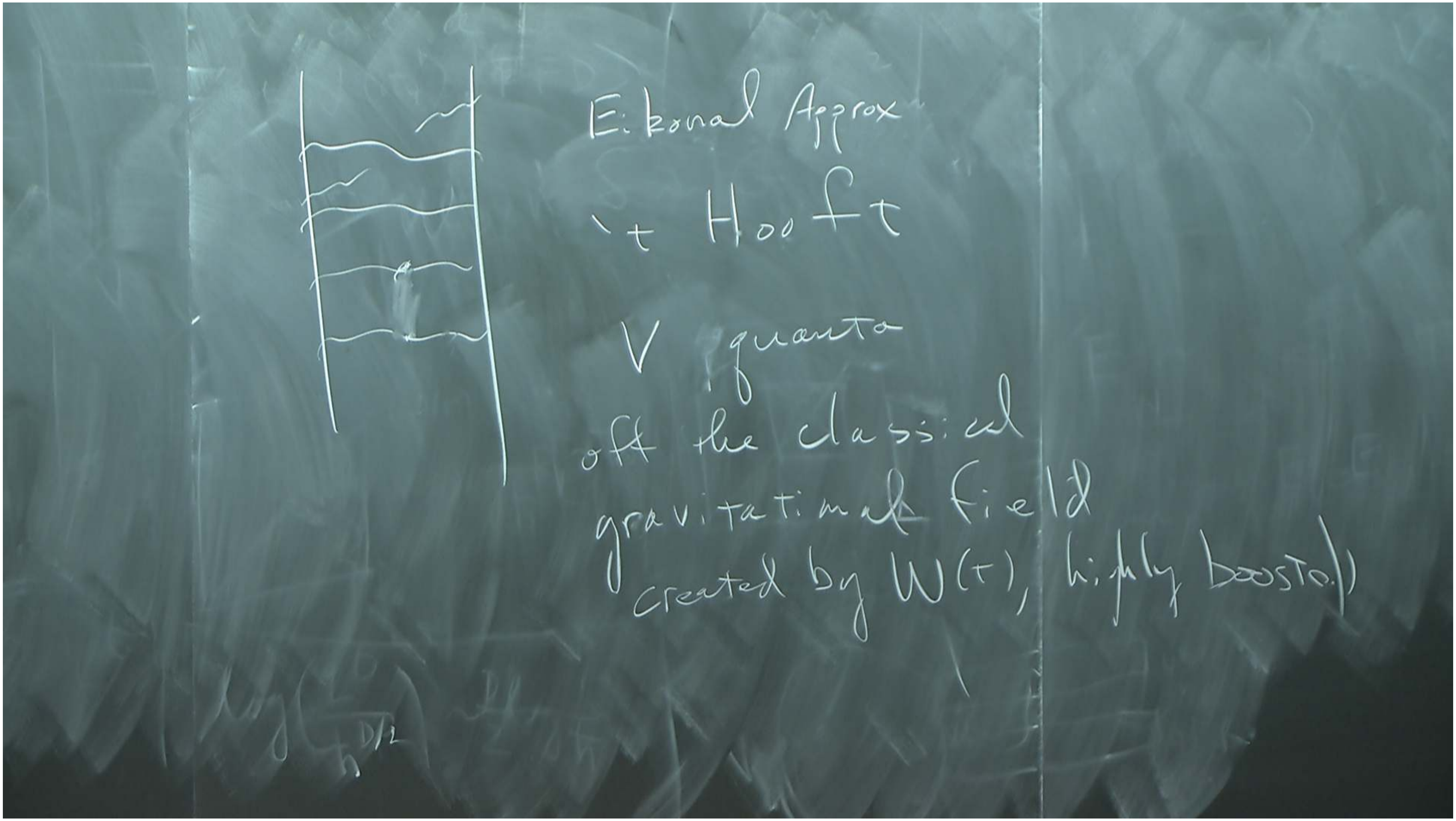
$$c_0 - c_1 \frac{T^2}{N^2} e^{2\pi t/\beta} + \ldots$$

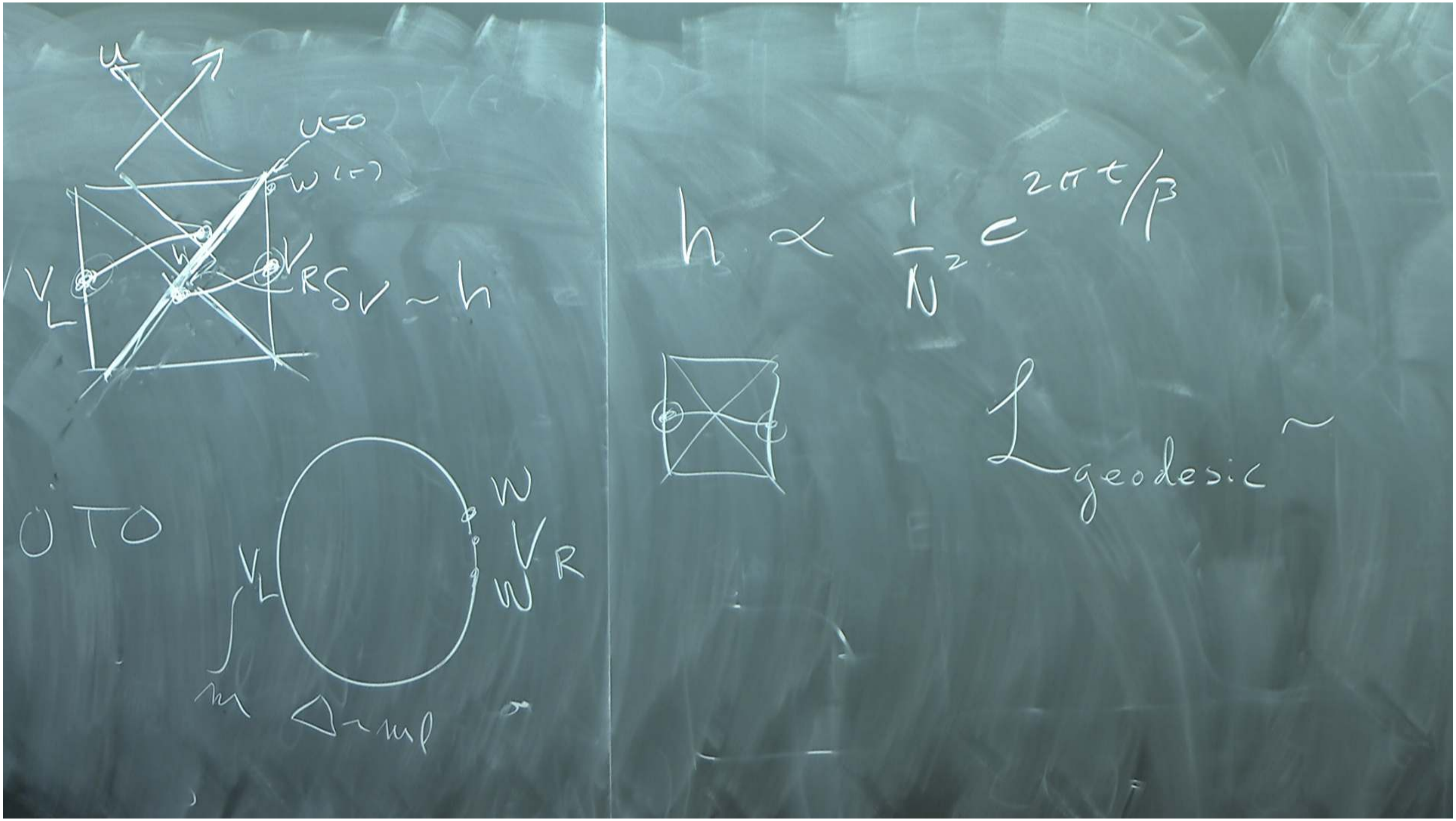
$$t_* \sim \frac{\beta}{2\pi} \log N^2 = \frac{\beta}{2\pi} \log S$$

Holography

$$\lambda_L = \frac{2\pi}{\beta} = 2\pi T$$

Einstein Grav.


Eikonal Approx
't Hooft
V quanta
off the classical
gravitational field
created by W(t), highly boosted)


u
u=0
W(-t)
V_L
V_R
δv ~ h
OTO
W
V_R
W
V_L
m
Δ ~ ml
h ∝ 1/N² e^{2πt/β}
L_geodesic ~

$$\frac{1}{N^2}\, e^{2\pi t/\beta}$$

$$\langle V_L V_R \rangle_W$$

$$\mathcal{L}^{(\text{reg})}_{\text{geodesic}} \sim 2\, \ell_{AdS} \log\left(1 + \frac{h}{z}\right)$$

$$\langle V_L V_R \rangle_W \sim e^{-m \mathcal{L}_{\text{geodesic}}}$$

$$\frac{h}{2}\Big)$$

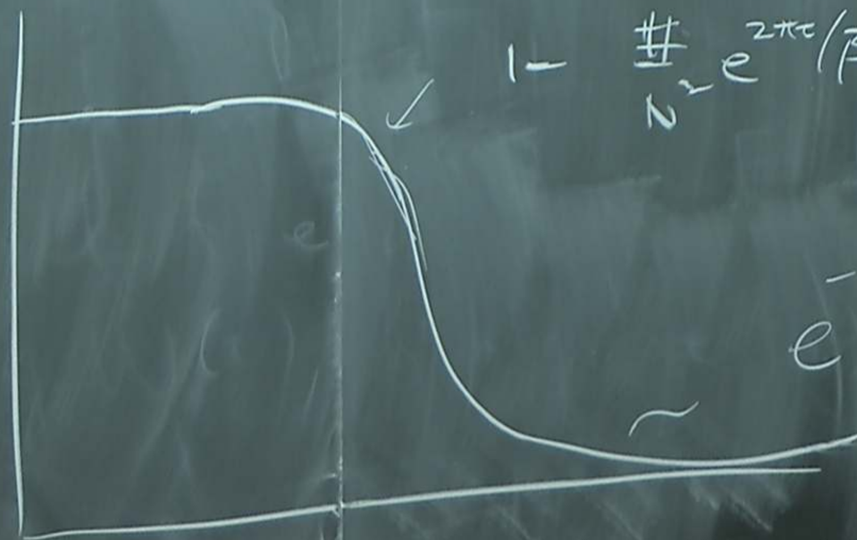
$$\sim \left(\frac{1}{1 + \frac{1}{N^2} e^{2\pi t/\beta}} \right)^{2 m l \Delta}$$

$\frac{h}{2})$

$$\langle V_L V_R \rangle_W \sim e^{-m \mathcal{L}_{geodesic}}$$

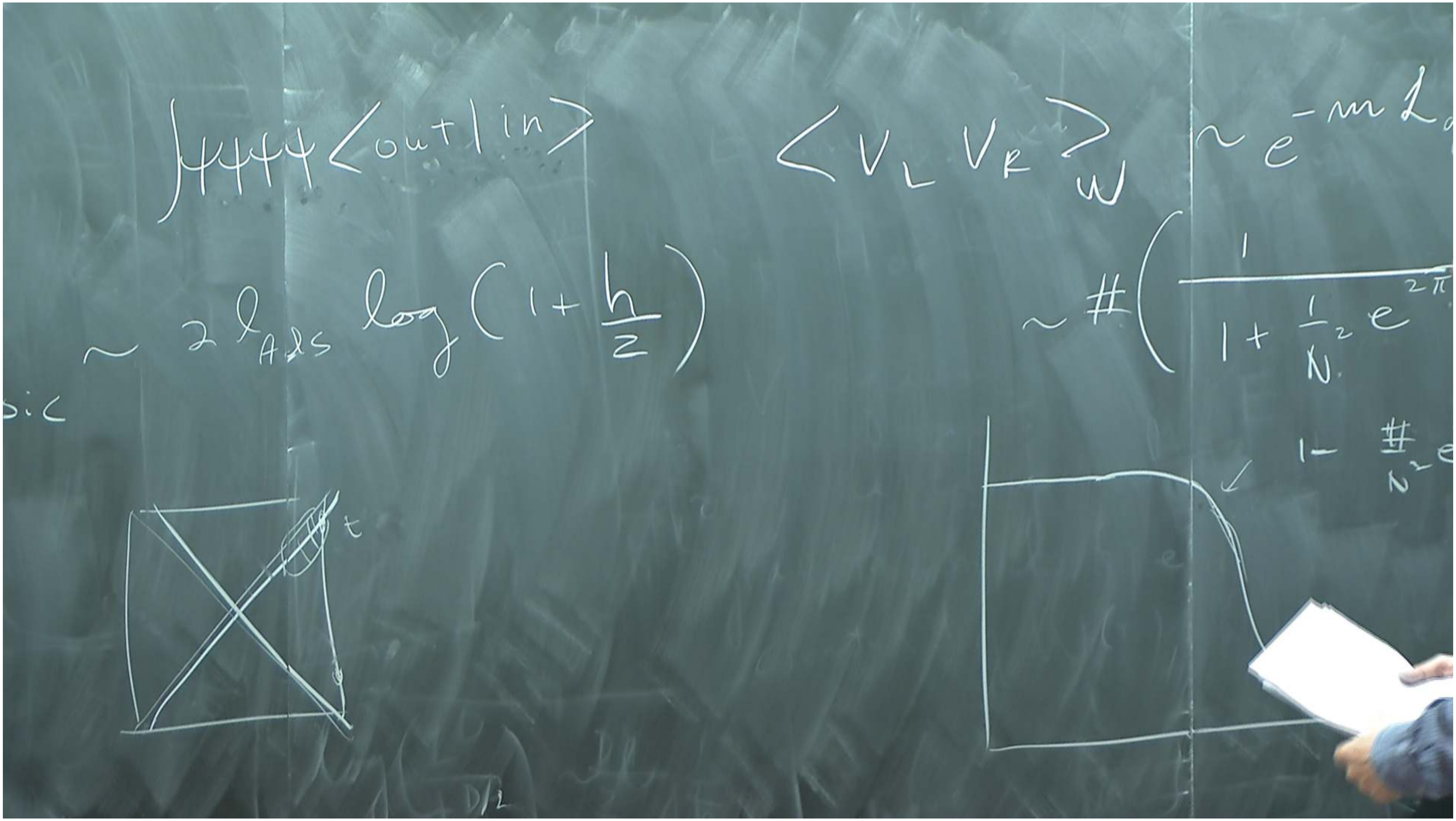
Δ_ℓ

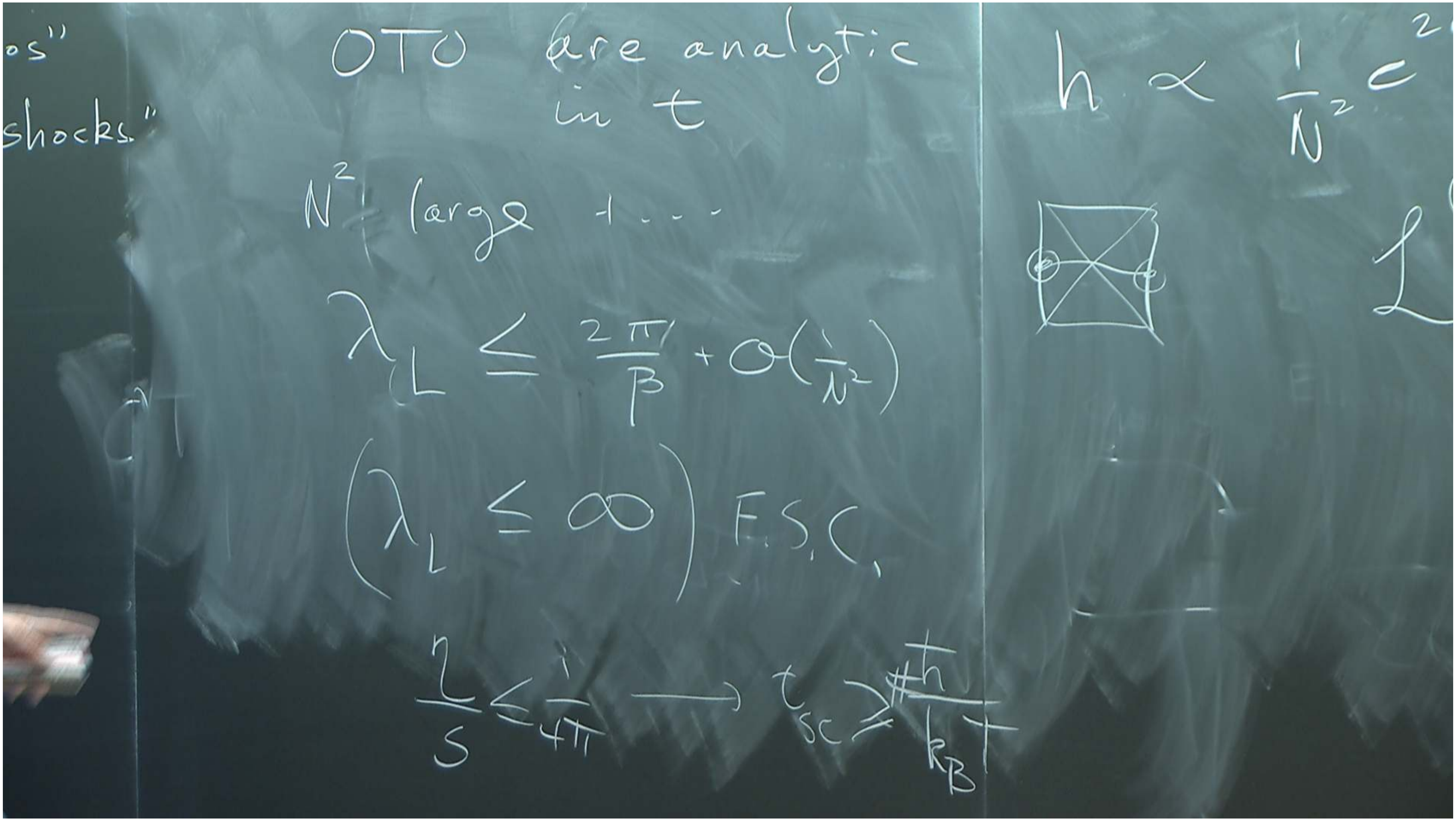
$$\sim \# \left(\frac{1}{1+\frac{1}{N^2} e^{2\pi t/\beta}} \right)^{2m\ell}$$



$1 - \frac{\#}{N^2} e^{2\pi t/\beta}$

$\sim e^{-(2\pi t/\beta \cdot 2\Delta_i)}$


$\int \psi\psi\psi\psi \langle out | in \rangle$
$\sim 2 \ell_{AdS} \log\left(1 + \frac{h}{z}\right)$
$\langle V_L V_R \rangle_W \sim e^{-m L}$
$\sim \# \left(\frac{1}{1 + \frac{1}{N^2} e^{2\pi}} \right.$
$1 - \frac{\#}{N^2} e$
t


"os"
"shocks."
OTO are analytic in t
N² large + ...
λ_L ≤ 2π/β + O(1/N²)
(λ_L ≤ ∞) F.S.C.
η/s ≤ 1/4π ⟶ t_sc ≥ ħ/k_B T
h ∝ 1/N² e^2