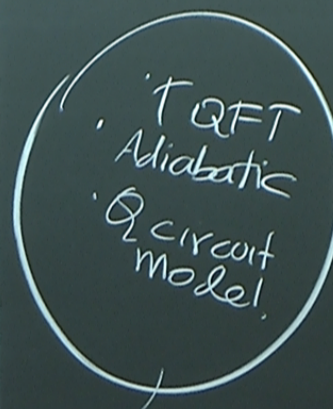
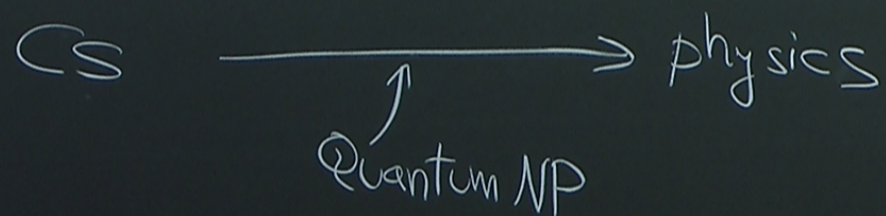
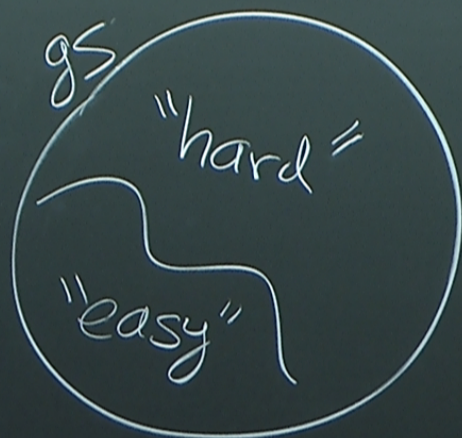


Title: Quantum NP and the Complexity of Ground States

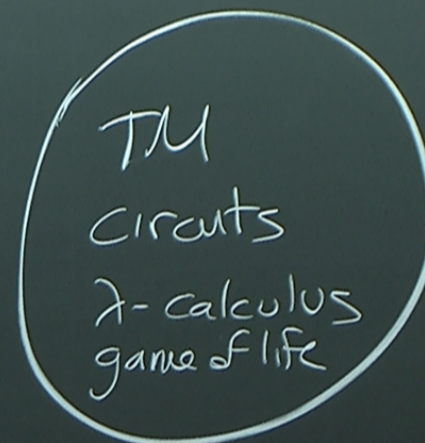
Date: Jul 28, 2016 09:00 AM

URL: <http://pirsa.org/16070051>

Abstract:



Quantum.



classical

$$1) H = - \sum_{i \sim j} J_{ij} S_i S_j, \leq a.$$

J_{ij} input, output: Is there $\vec{S}_i$ s.t

$$E(S_1 \cdot S_n) \leq a.$$

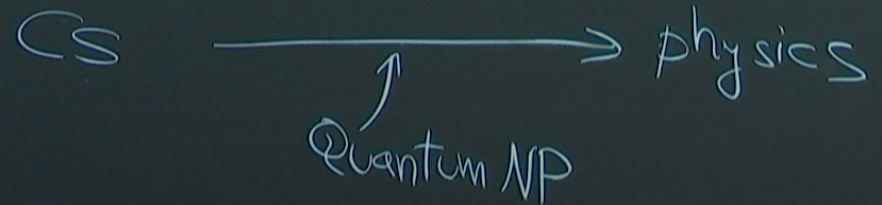
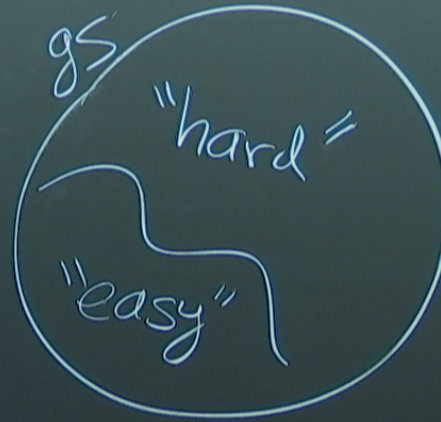
2) factoring, N, b is the p|N
 $p \leq b.$

Quantum NP & complexity of GS.

P, BQP, NP, hardness, reduction
QMA-complete



- how long would it take for a system to relax to its GS?
- how complex is the ent in its GS?

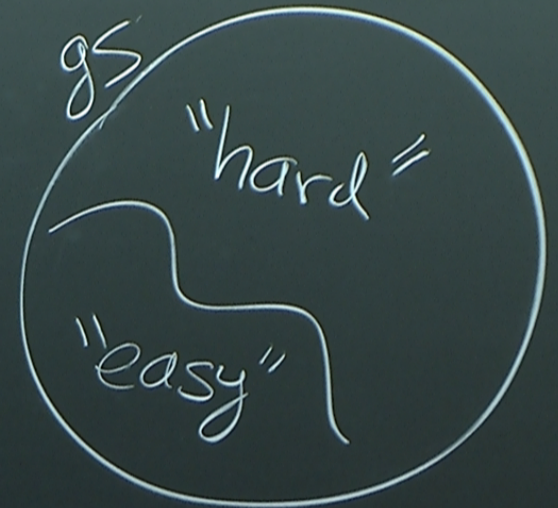


P: $L \in P$ if $\exists$

classical Algorithm which
runs in polynomial time
s.t. $\forall n, \forall x$ of n bits

$$x \in L \Rightarrow A(x) = 1$$

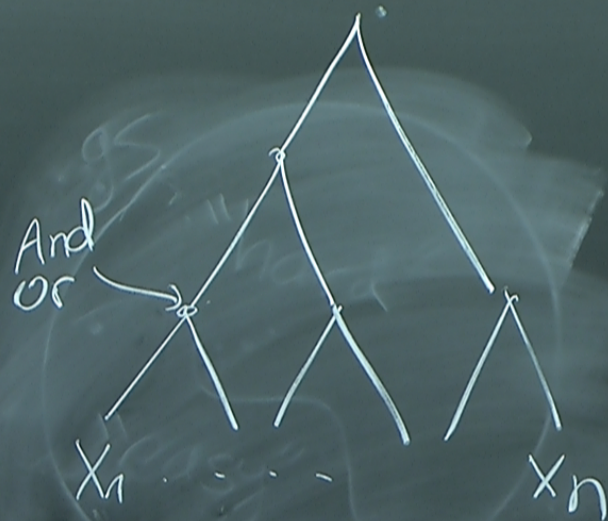
$$x \notin L \Rightarrow A(x) = 0$$



CS

Quantum

$\exists$
 hm which
 ial time
 bits
 max to



$$\mathcal{L} \subseteq \bigcup_{n=1}^{\infty} \{0, 1\}^n$$

TQFT
 Adiabatic
 Q circuit
 model

Quantum.
 BQP

← Bounded prob polynomial

BPP - Algorithm (described by
a sequence of circuits C_n
with access to random bits

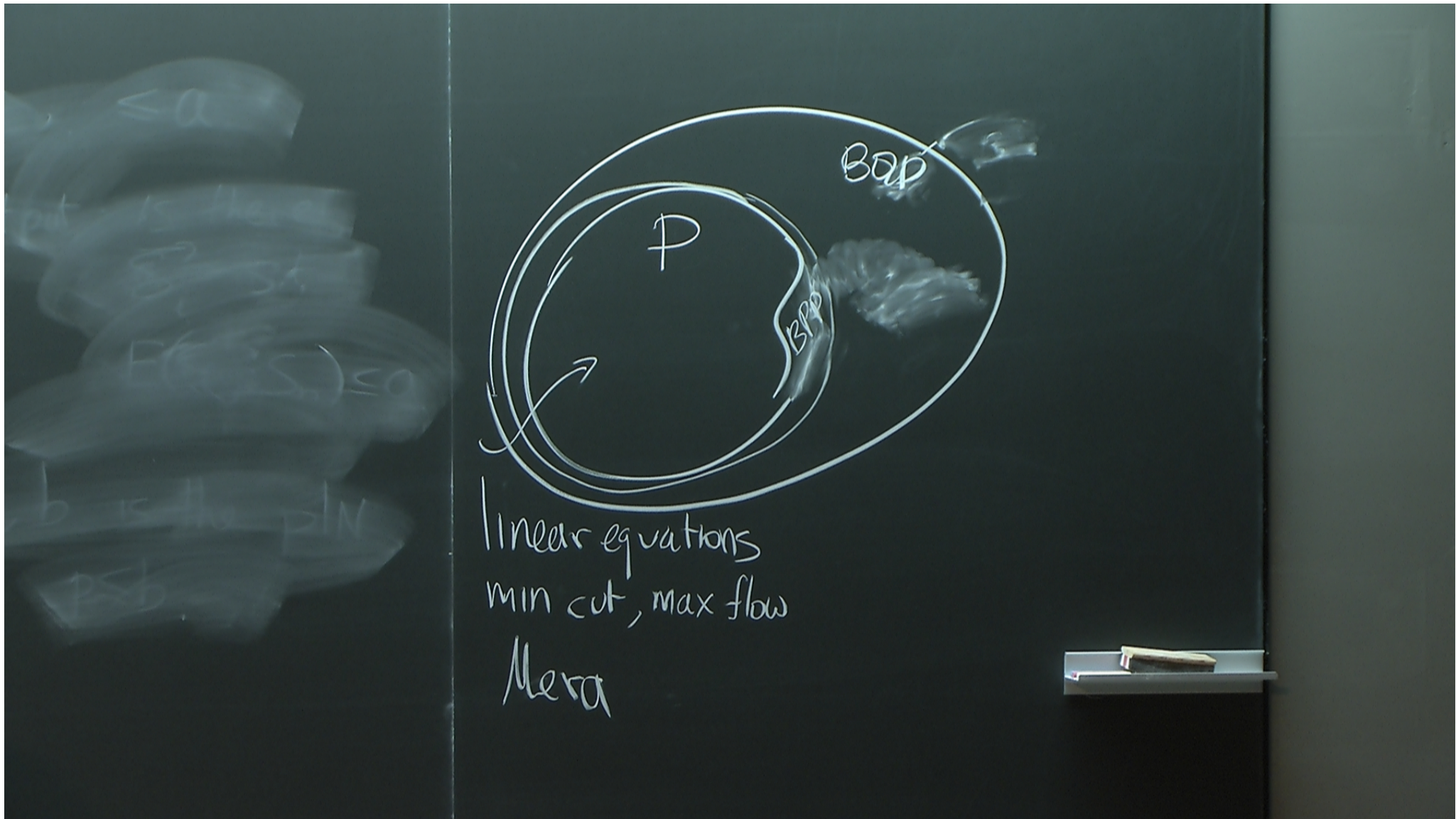
$$x \in L \Rightarrow \Pr_r(A(x, r) = 1) \geq \frac{2}{3}$$

$$x \notin L \Rightarrow \Pr_r(A(x, r) = 0) \geq \frac{2}{3}$$

$$1) H = - \sum_{i,j} \dots$$

J_{ij} input

2) factoring



polynomial

(described by
bits C_n)

random bits

$$A(x, r) = 1) \geq \frac{2}{3}$$

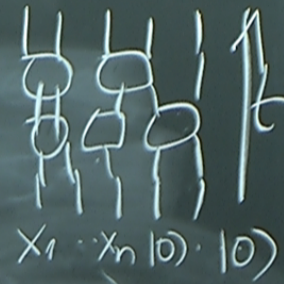
$$A(x, r) = 0) \geq \frac{2}{3}$$

BQP $L \in \text{BQP}$ if $\exists$ poly time

Q algorithm $\{Q_n\}_{n=1}^{\infty}$

$$x \in L \Rightarrow \Pr(Q_n(x) = 1) \geq \frac{2}{3}$$

$$x \notin L \Rightarrow \Pr(Q_n(x) = 0) \geq \frac{2}{3}$$



$$\langle 0^k x | Q_n^\dagger \Pi_1 Q_n | x 0 \rangle$$

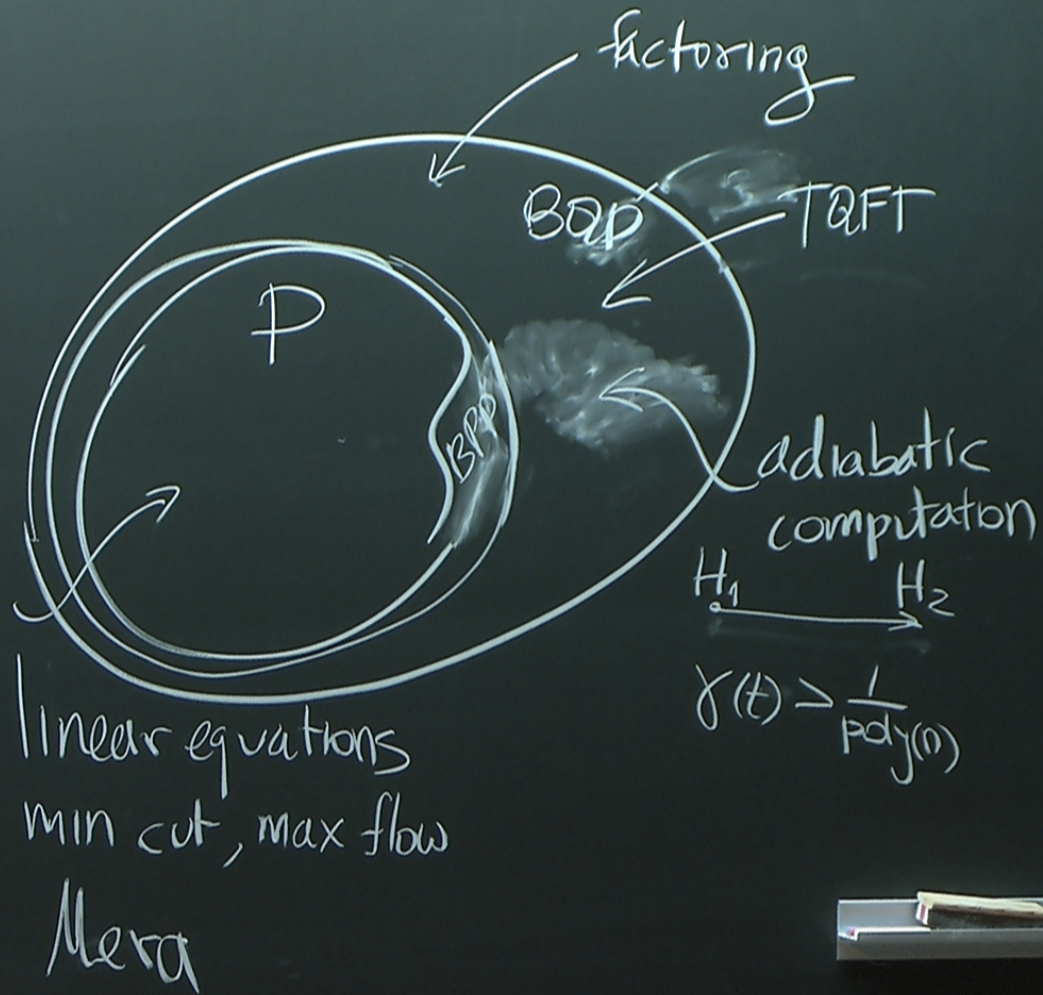
$$m \text{ trials} \Rightarrow \frac{1}{2^m}$$

linear eq
min cut,
Mera

$\exists$ poly time

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$x_1 \cdots x_n | 0 \cdots 0 \rangle$$



Suppose
I have
a way
to solve
 L_2

"hardness"

L is "hard" for $\mathcal{C}$ if
any other problem $L' \in \mathcal{C}$
 L' can be reduced to L

"complete"

a problem L is complete for
a class $\mathcal{C}$ if it is in $\mathcal{C}$
and hard for the class.

BPP, NP,
QNP

BPP

a sequence
with a

$x \in L =$

$x \notin L =$

BQP, NP,
QNP

$L' \in C$

to L

complete for

QECTT - every physically
realizable comp model can
be simulated efficiently by
a Q computer.

BQP $\subseteq$

Q algo

$x \in L \Rightarrow$

$x \notin L \Rightarrow$

$(\text{ok} \times | \Phi_n^T \Pi_n$

$m \text{ trn}$

NP $L \in NP$ if $\exists p(y) A$
s.t. $\forall x$

$$x \in L \Rightarrow \exists y \quad A(x, y) = 1$$

$$x \notin L \Rightarrow \forall y \quad A(x, y) = 0$$

$$H = -\sum_{i,j} J_{ij} S_i S_j, a$$

Garey-Johnson

K-SAT

K-SAT

$$F = (z_1 \vee z_2 \vee \bar{z}_3) \wedge (z_4 \vee \bar{z}_5 \vee z_6) \dots$$

$\underbrace{\hspace{10em}}_{K=3}$

Cook-Levin Thm

QMA (QNP)

$L \in \text{QNP}$ if $\exists \{Q_n\}_{n=1}^{\infty}$
of poly size $s +$

$$x \in L \Rightarrow \exists |y\rangle \Pr(Q_n | x \rangle | y \rangle = 1) \geq \frac{2}{3}$$

Quantum!!!

$$x \notin L \Rightarrow \forall |y\rangle, \Pr(Q_n | x \rangle | y \rangle = 0) \leq \frac{1}{3}$$

BQP $L \in$

Q algorithm

$$x \in L \Rightarrow \Pr$$

$$x \notin L \Rightarrow \Pr$$

$$\langle 0 | K | x \rangle \langle 0 | Q_n^\dagger \Pi_1 Q_n$$

m trials

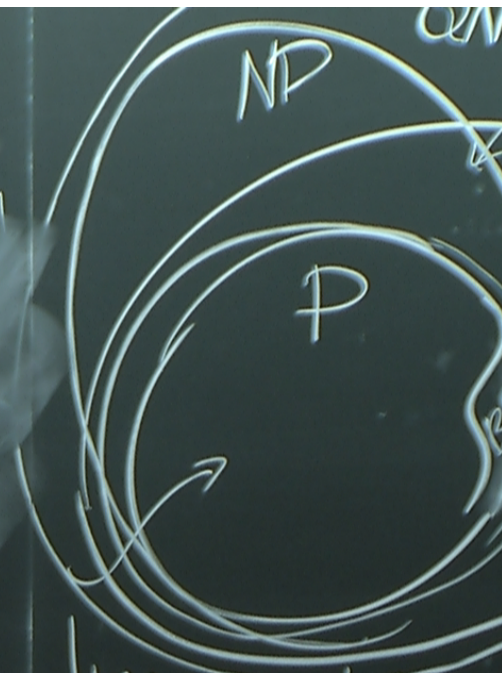
k-Local Hamiltonian Problem

$$H = \sum_{j=1}^m H_j, \text{ on } n \text{ qubits}$$

H_j is k -local, b, a , so

$$b - a \geq \frac{1}{\text{poly}(n)}$$

g.v of $H \leq a$ or $\geq b$?



linear equations
min cut, max flow
Mera

$$BQP \neq P$$

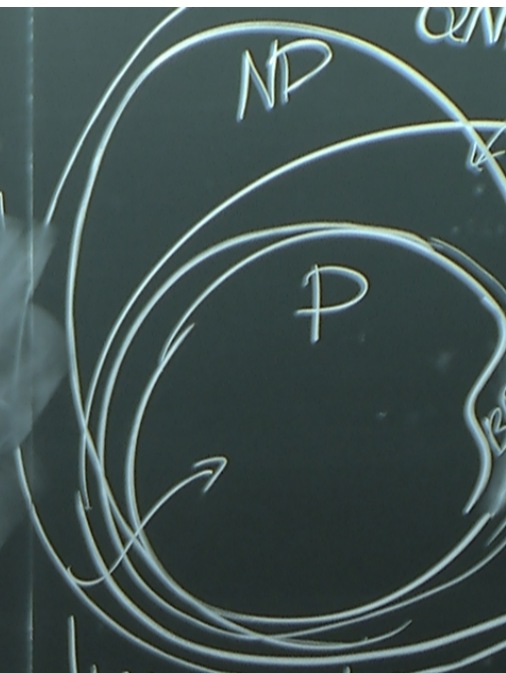
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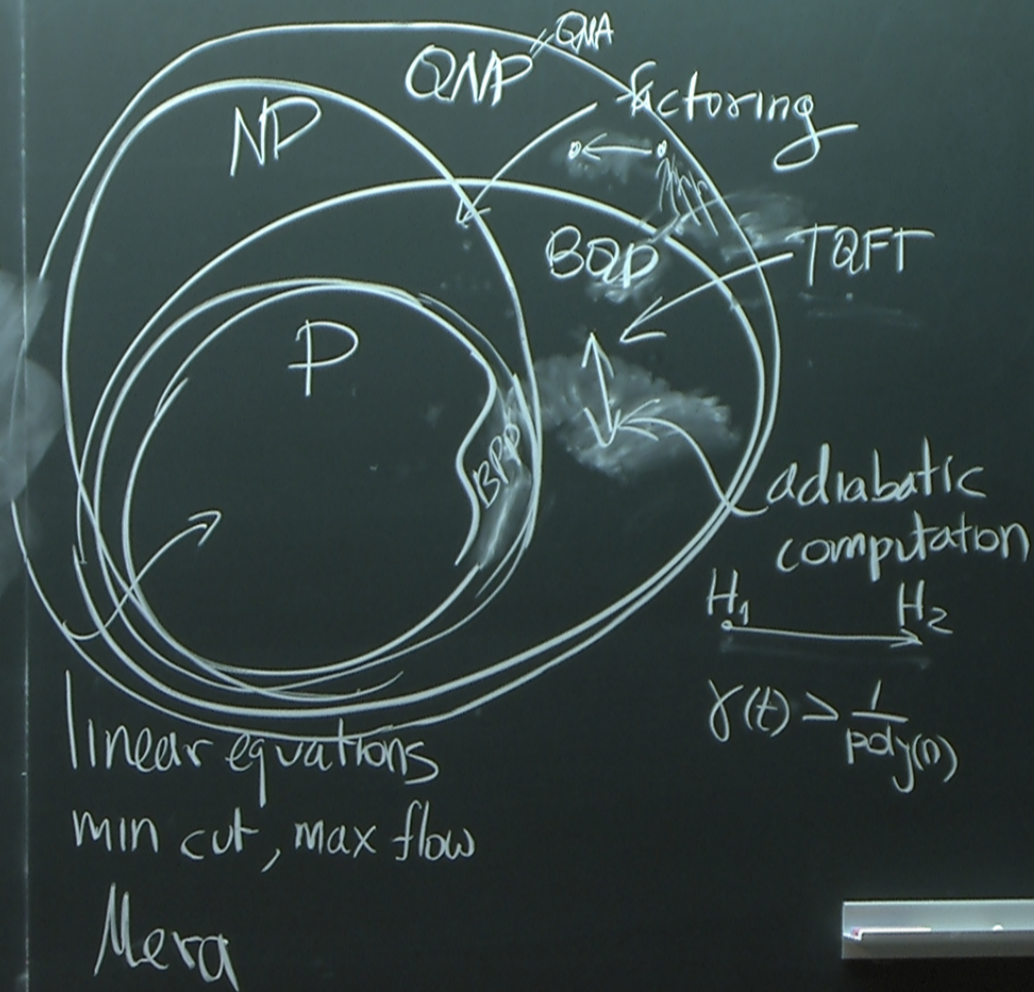
Problem

qubits

1, b, a, so

PN

or $\geq b$?



lim: TI LH are "QMA" complete

Hubbard Model

Feizenberg Model

P-complete

stency of density matrices

in: Garey-Johnson

K-SAT

$$F = (z_1 \vee z_2 \vee \bar{z}_3) \wedge (z_4 \vee \bar{z}_5 \vee z_6) \dots$$

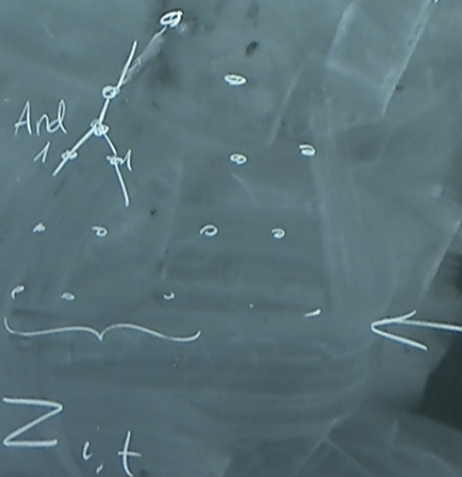
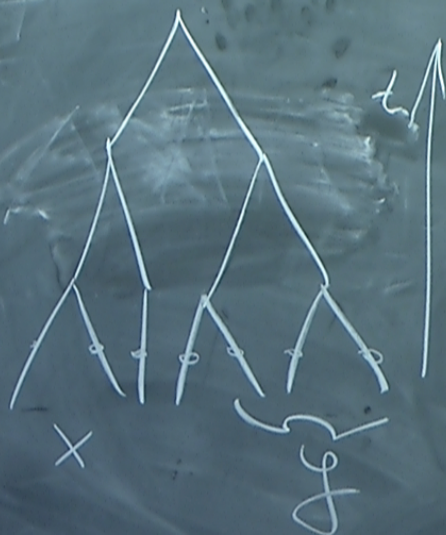
$K=3$

Cook-Levin Thm $\rightarrow$ Quantum

$$H_1 = |001\rangle \langle 001|_{123}$$

$$H = \sum H_i$$

reduced $LENP \rightarrow 3-SAT$



K-SAT

$$F = (z_1 \vee z_2 \vee \bar{z}_3) \wedge (z_4 \vee \bar{z}_5)$$

$K=3$

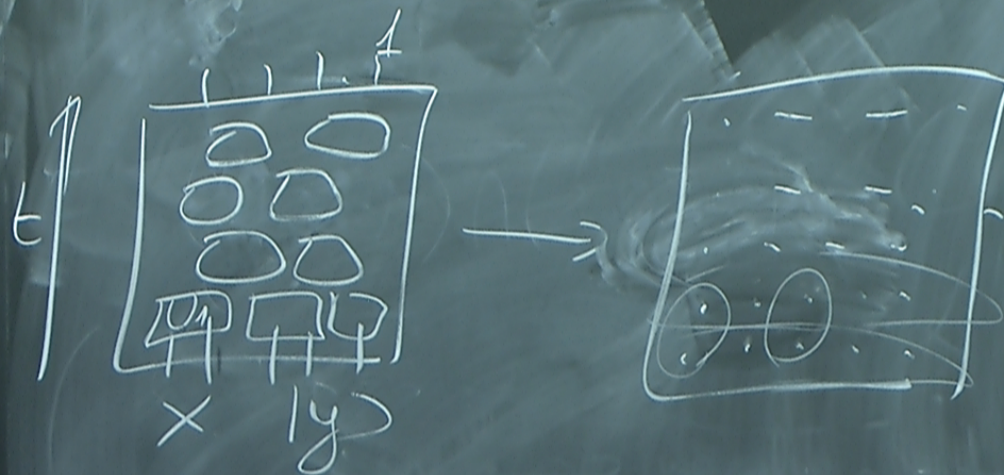
Cook-Levin Thm

$$H_1 = \langle 1001 \rangle \langle 001 \rangle$$

$$H = \sum H_i$$

3-SAT

$L \in QMA \rightarrow LH$



$$|\gamma_0\rangle \rightarrow U_1 |\gamma_0\rangle \rightarrow U_2 U_1 |\gamma_0\rangle \rightarrow$$

$$|\alpha\rangle, |\beta\rangle$$

$$\frac{|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle}{\sqrt{2}}$$

QMA

$L \in QMA$
of poly

$x \in L$

$x \notin L$

3-SAT

QMA $\rightarrow$ LH

QMA (Q)

$L \in QNP$

of p...

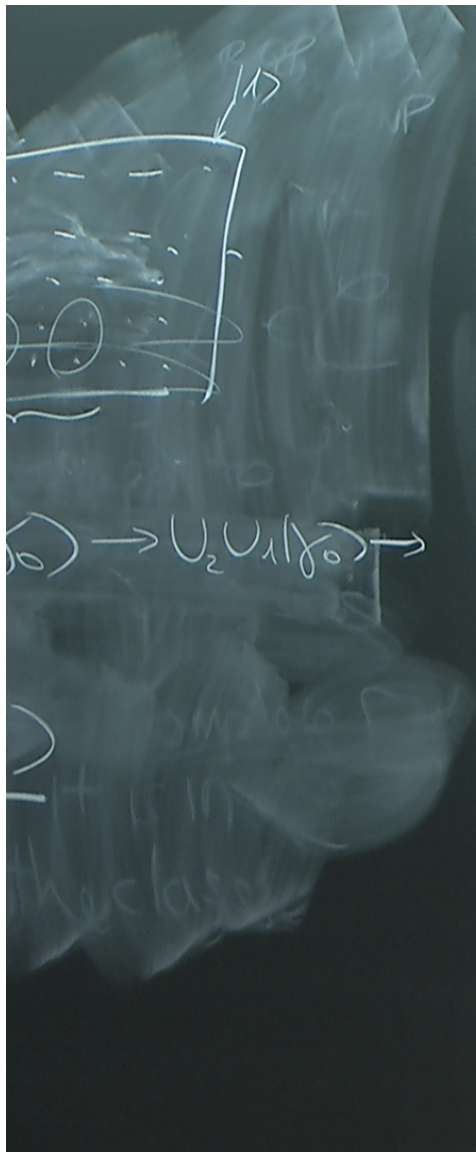
$x \in$

$|\gamma_0\rangle \rightarrow U_1 |\gamma_0\rangle \rightarrow U_2 U_1 |\gamma_0\rangle \rightarrow$

$|\alpha\rangle, |\beta\rangle$

$|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle$

$\frac{|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle}{\sqrt{2}}$



$$\mathcal{H}_{\text{prop}} = \sum \mathcal{H}_{\text{prop}, t}$$

$$\mathcal{H}_{\text{init}} \quad \Pi_{\text{input qubits being } 1} \otimes |t=0\rangle \langle t=0|$$

$$\mathcal{H}_{\text{output}} \quad \Pi_{\text{out is } 1} \otimes |t=T\rangle \langle t=T|$$

history state:

$$|\eta\rangle = \sum |\chi(t)\rangle \otimes |t\rangle$$

$$\begin{aligned} \mathcal{H}_{\text{prop}, t} = & I \otimes |t\rangle \langle t-1| + I \otimes |t-1\rangle \langle t-1| \\ & - U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \end{aligned}$$

k-Local Hamiltonian

$$\mathcal{H} = \sum_{j=1}^m \mathcal{H}_j, \quad 0$$

$\mathcal{H}_j$ is k -

$$b-a \geq p$$

g.v of $\mathcal{H}$