Title: Focus Lecture

Date: Jul 27, 2016 05:00 PM

URL: http://pirsa.org/16070049

Abstract:

Outline and references

Recap	Tensor networks? MERA?
Why MERA?	MERA as a quantum circuit ER: an RG transformation Efficient computation
Structural properties	Efficient computation Structure of two-point correlators Structure of entanglement

References:

0512165, 0912.1651, 0610099: Basics by Vidal

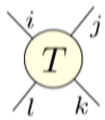
0707.1454: Algorithms by Evenbly & Vidal

1106.1082: Tensor networks and geometry by Evenbly & Vidal

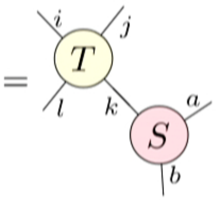
0810.0580: Scale invariant MERA by Pfeifer, Evenbly & Vidal

What are tensor networks again?

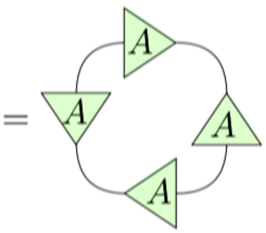
$T_{ijkl} =$



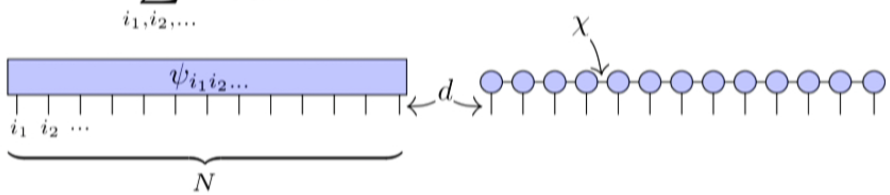
$\sum_k T_{ijkl} S_{kab} =$

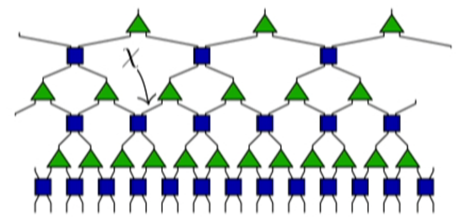


$\mathrm{Tr}\left[A A^{\mathrm{T}} A A^{\mathrm{T}} \right] =$



$|\psi\rangle = \sum_{i_1, i_2, \ldots} \psi_{i_1 i_2 \ldots} |i_1 i_2 \ldots\rangle$





d^N elements

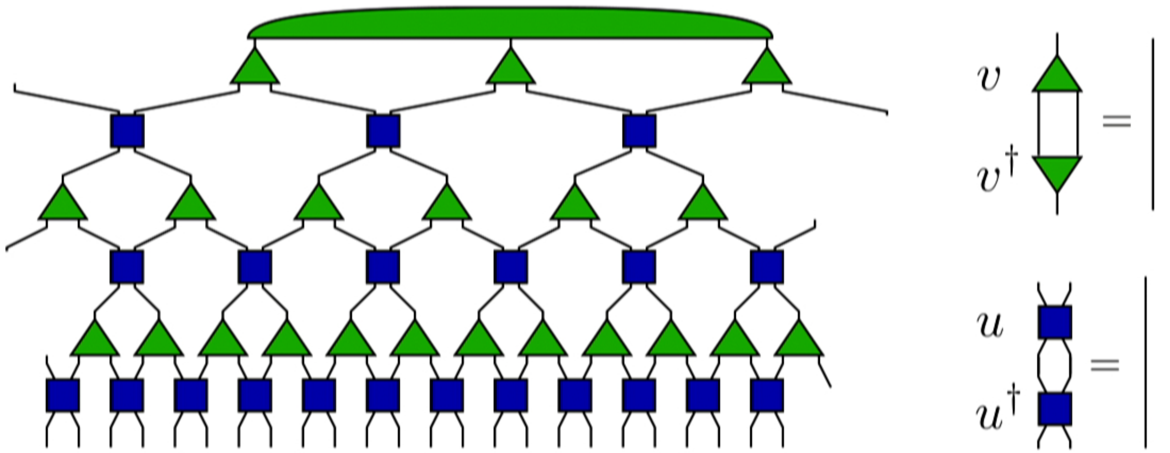
$O(N\chi^p)$ elements

Efficient computation

Structure of the state manifest

What is MERA again?

(Multiscale Entanglement Renormalization Ansatz)



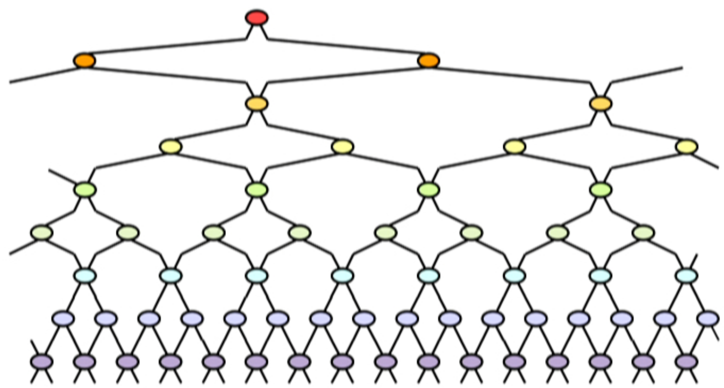
Storage: $O(\chi^4\ N)$

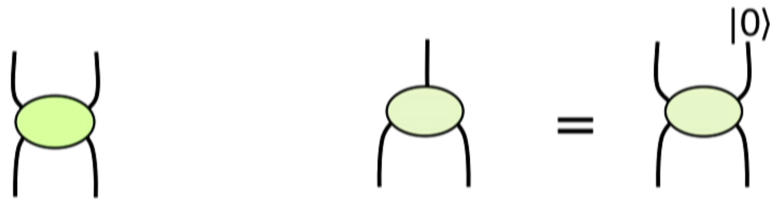
Computation: $O(\chi^{8\text{-}9} \log N)$

A good ansatz for scale invariant states with

- polynomial decay of correlators
- logarithmic scaling of entanglement entropy

MERA as a quantum circuit





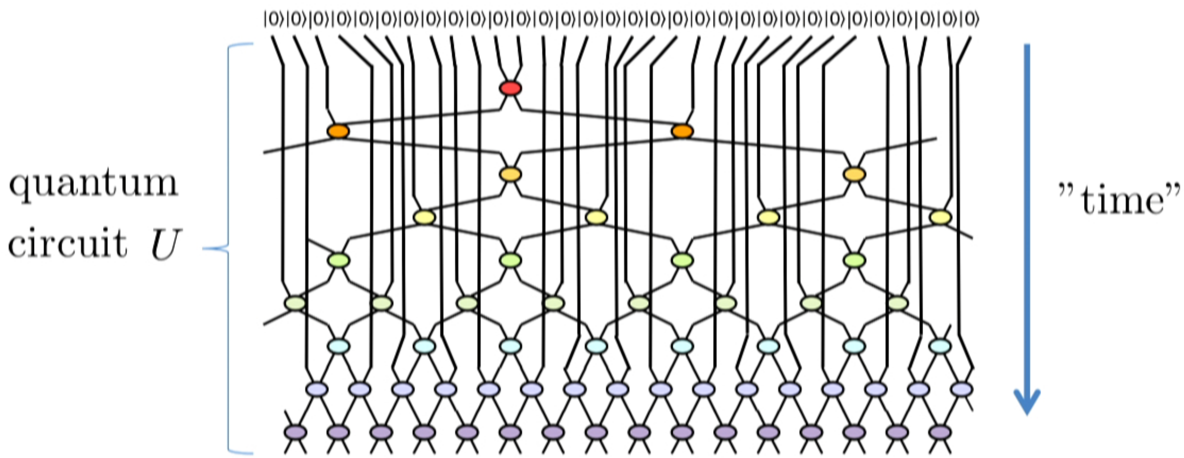
Unitary
("disentangler")

Isometry
("coarse-grainer")

Also a unitary

Figure from Guifre Vidal

MERA as a quantum circuit



Ground state ansatz: $|\psi\rangle = U|0\rangle^{\otimes N}$

Entanglement introduced by gates at different times (i.e. length scales)

Figure from Guifre Vidal

Entanglement renormalization:
A real space RG transformation

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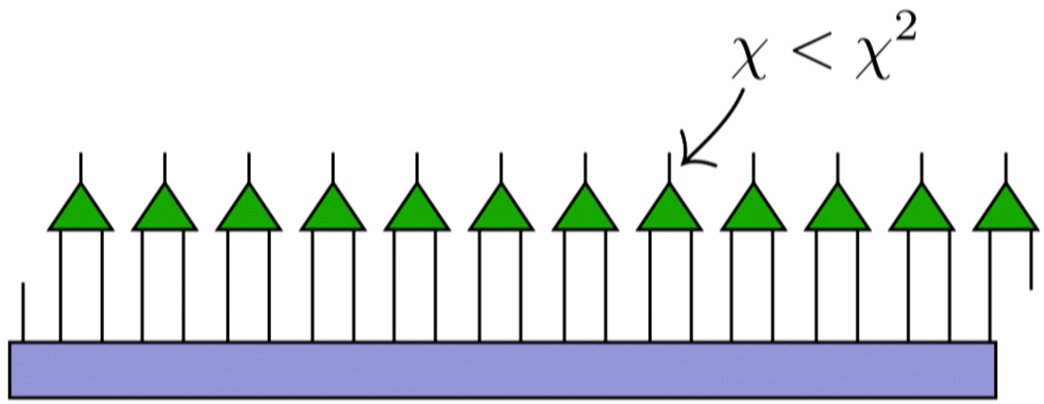
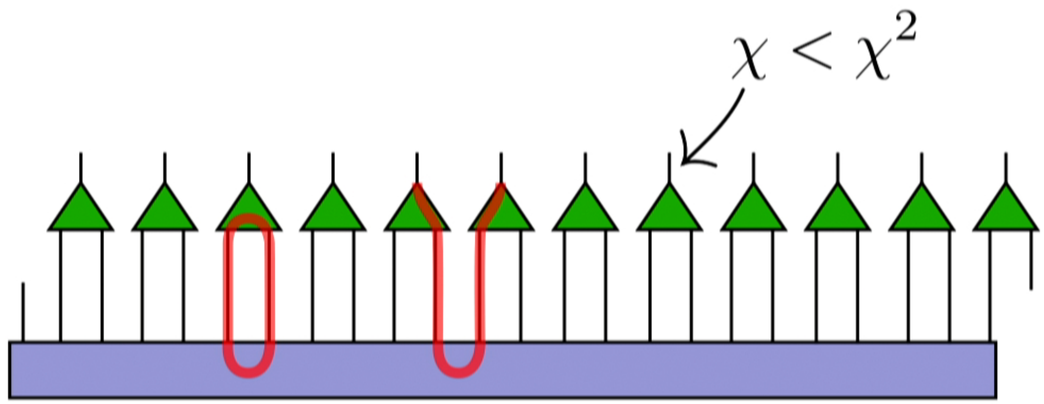
Entanglement renormalization:
A real space RG transformation



Kadanoff's splin blocking, 1966
White's density matrix renormalization group, 1992

Entanglement renormalization:
A real space RG transformation



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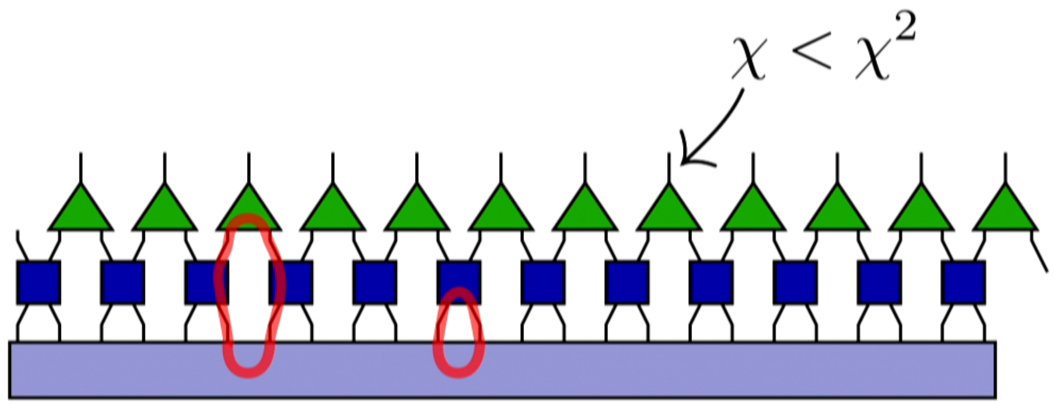
Entanglement renormalization:
A real space RG transformation



Vidal's entanglement renormalization, 2005

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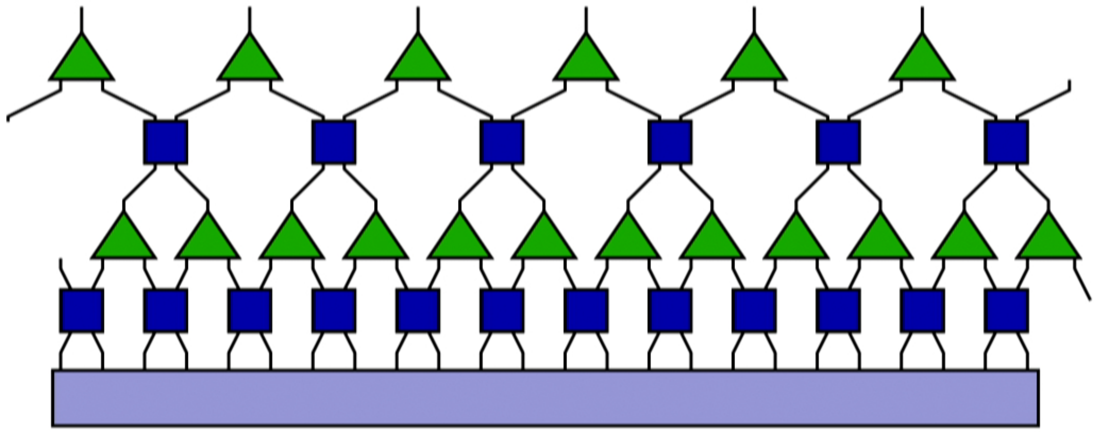
Entanglement renormalization:
A real space RG transformation



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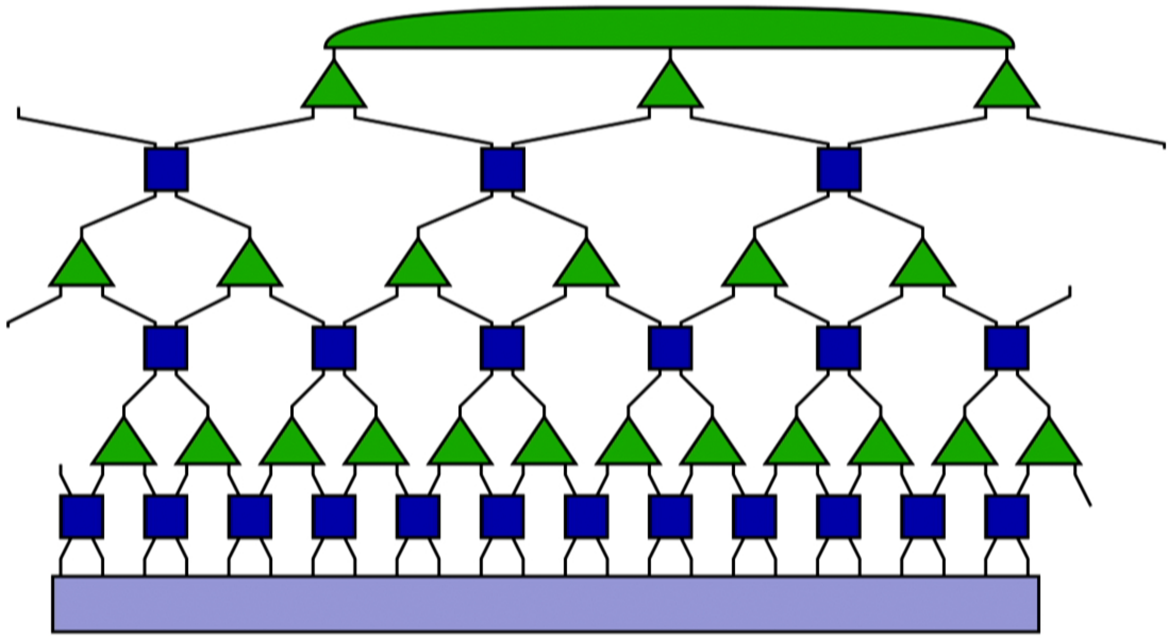
Entanglement renormalization:
A real space RG transformation



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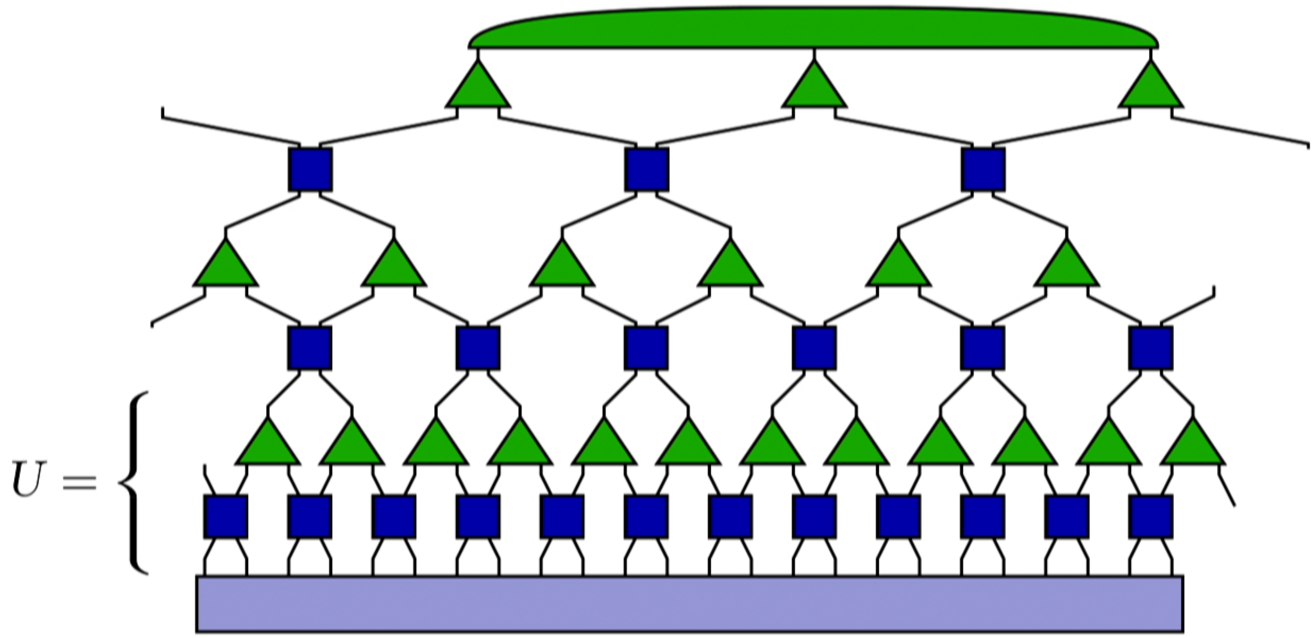
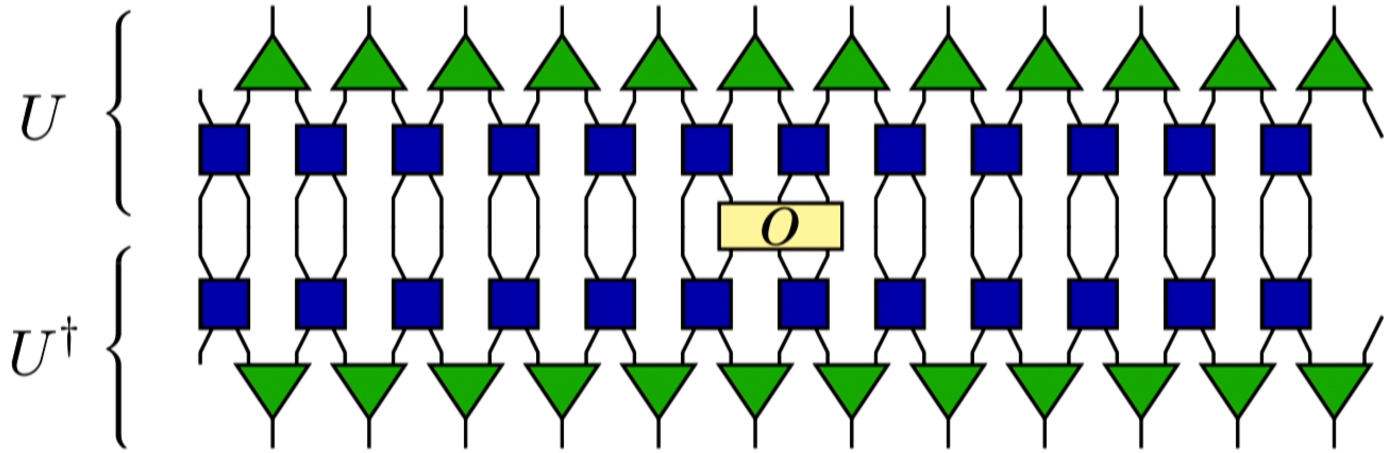
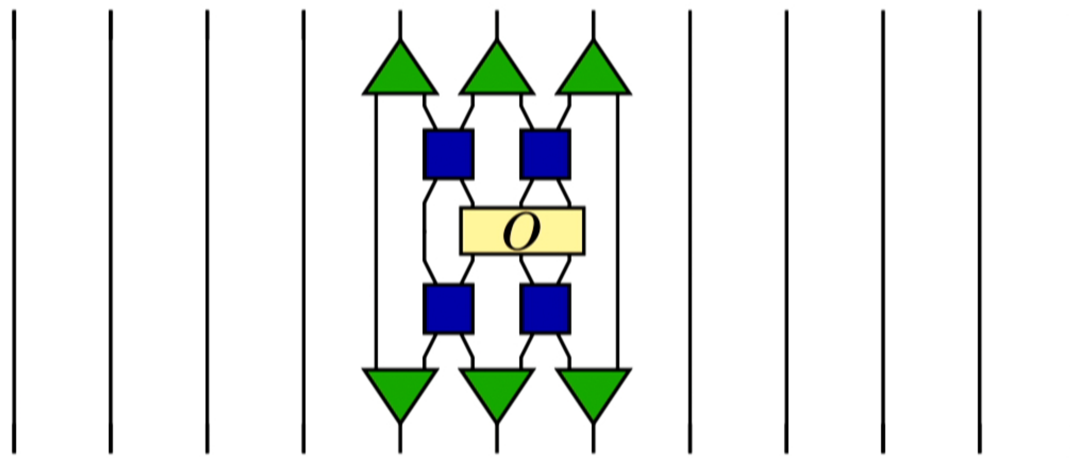
Entanglement renormalization:
A real space RG transformation



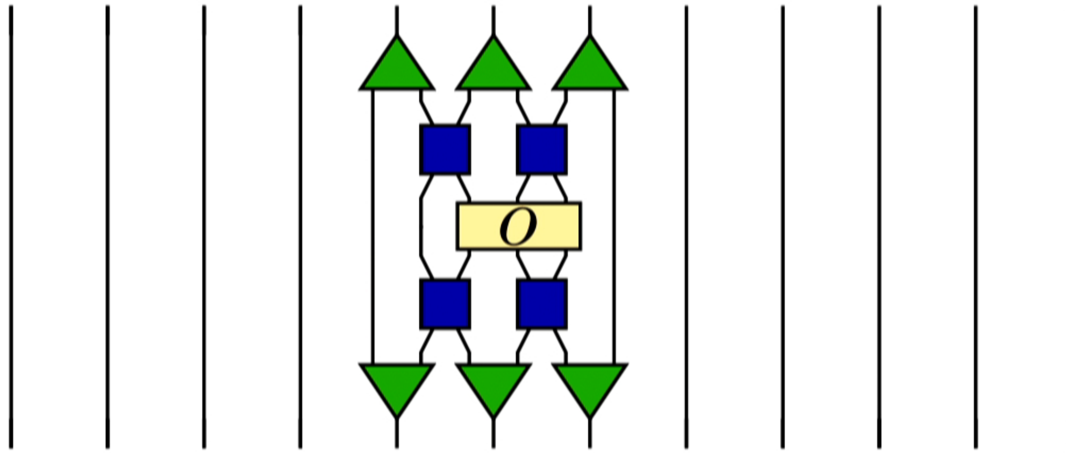
Entanglement renormalization on an operator



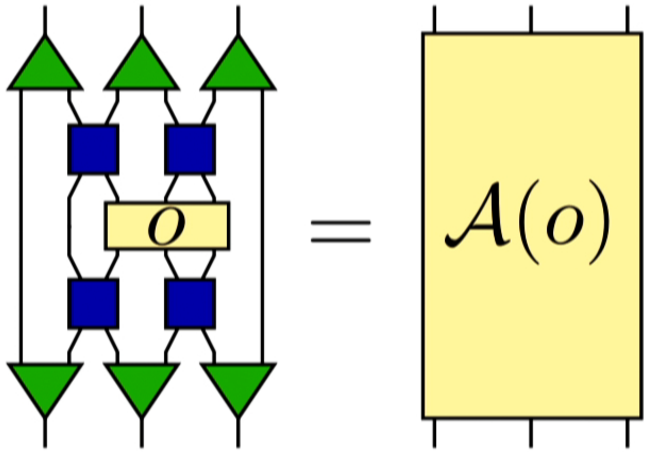
Entanglement renormalization on an operator



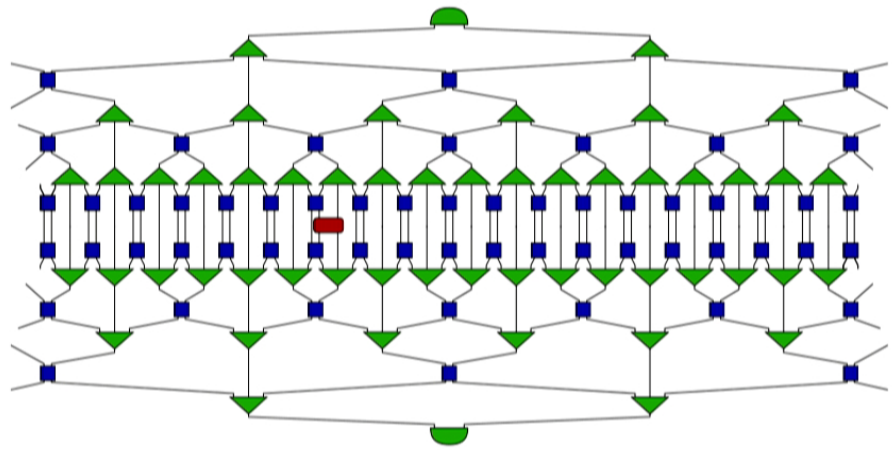
Entanglement renormalization on an operator



Entanglement renormalization on an operator



Efficient computation of expectation values

$$\langle \mathrm{MERA} |\, o \,| \mathrm{MERA} \rangle =$$ 

Efficient computation of expectation values

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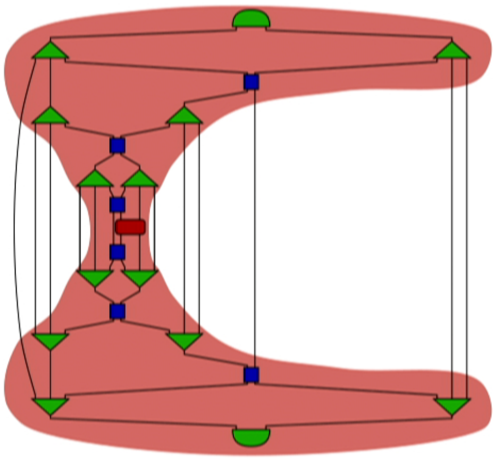
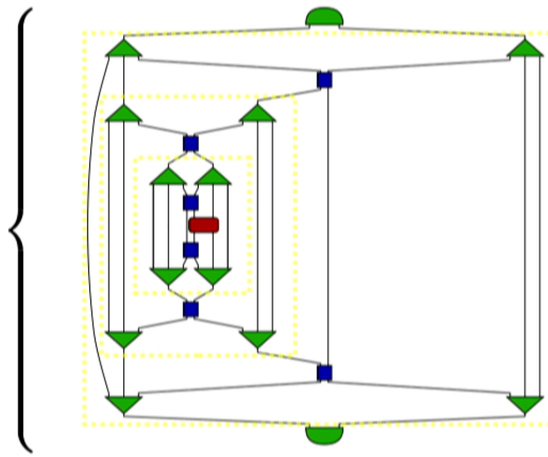


Efficient computation of expectation values

log N layers

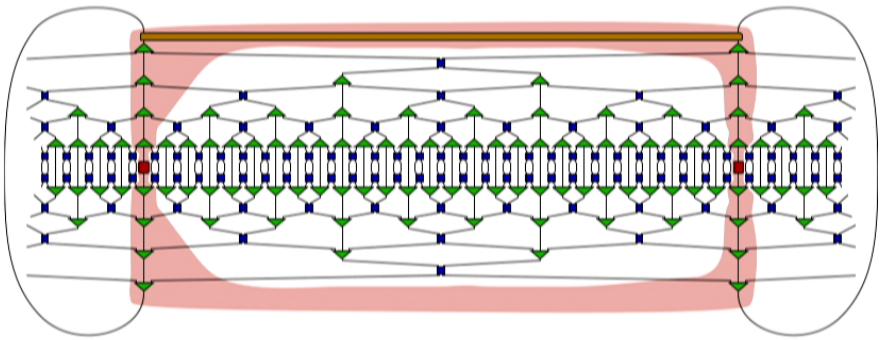


In summary: One only needs to consider tensors in the causal cone. Then the computation reduces to log N applications of the ascending superoperator.

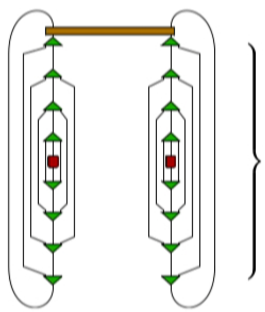
Two-point correlators

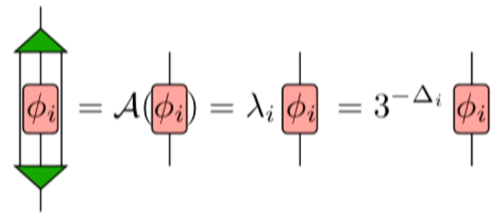
$$\langle o_0 o_L \rangle$$

$$||$$



$$||$$

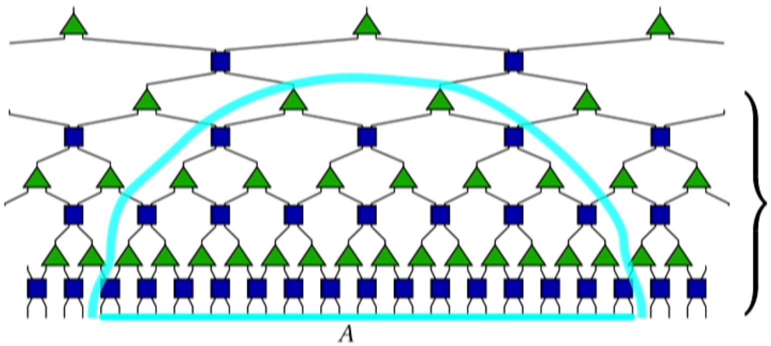




$$
\begin{aligned}
\langle \phi_i \phi_j \rangle &= \mathrm{Tr}\left[\left(\mathcal{A}^{\log_3 L}(\phi_i) \otimes \mathcal{A}^{\log_3 L}(\phi_j)\right)\rho\right] \\
&= \left(3^{-\Delta_i - \Delta_j}\right)^{\log_3 L} \mathrm{Tr}\left[\left(\phi_i \otimes \phi_j\right)\rho\right] \\
&= L^{-\Delta_i - \Delta_j}\, \mathrm{Tr}\left[\left(\phi_i \otimes \phi_j\right)\rho\right] \\
&= \frac{C_{ij}}{L^{\Delta_i + \Delta_j}}
\end{aligned}
$$

Block entropy for MERA

In all tensor network states, an upper bound for entanglement entropy of a region is given by a minimal cut that separates that region.



$\log |A|$ layers, two cuts per layer

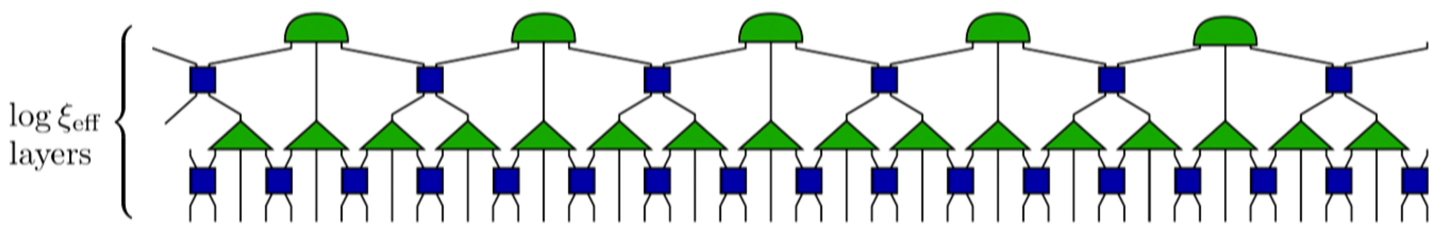
$\Rightarrow S(A) \leq 2 \log |A| \cdot \log_2 \chi$

Typically, this bound is saturated.

Hayden et al., 1601.01694

Finite range MERA

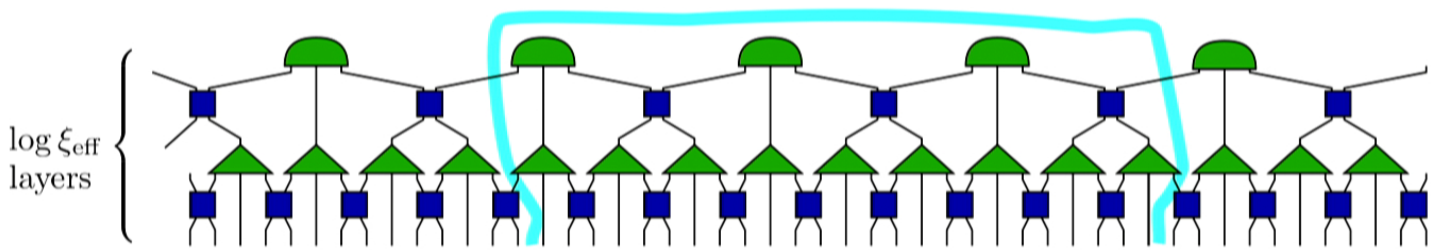
A cap/maximum number of layers corresponds to a finite correlation length.



Finite range MERA

A cap/maximum number of layers corresponds to a finite correlation length.

$S \sim$ minimal cut

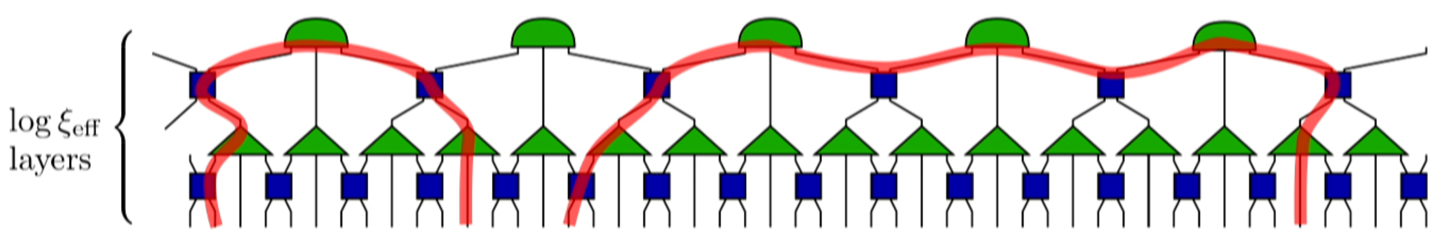


Finite range MERA

A cap/maximum number of layers corresponds to a finite correlation length.

$S \sim$ minimal cut

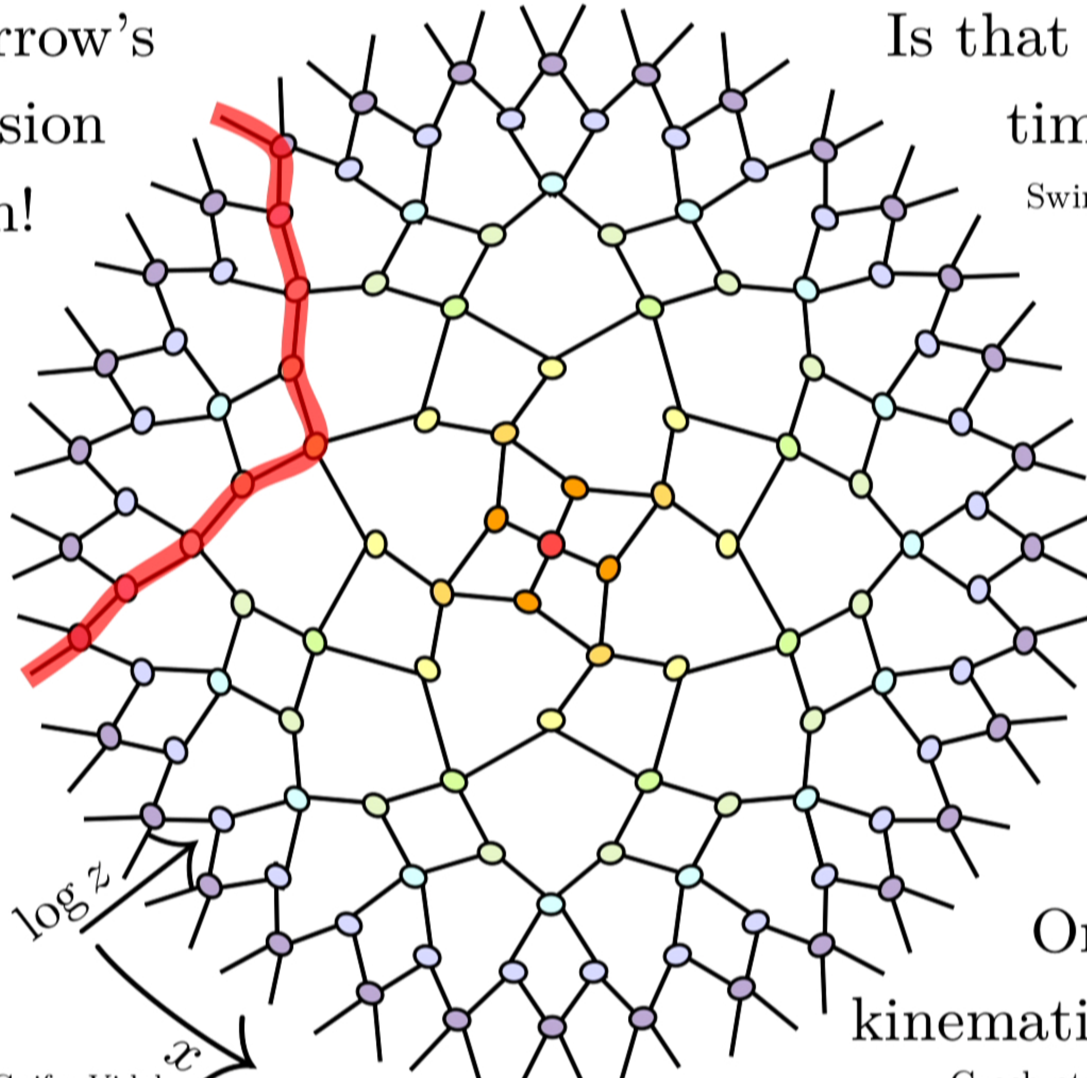
2-point correlator $\sim e^{-(\text{length of "geodesic"})}$



Tomorrow's discussion session!

Is that an AdS time slice?

Swingle, 0905.1317



Or is it its kinematic space?

Czech et al. 1512.01548

Figure from Guifre Vidal

Other stuff I don't have time for

- Thermal MERA
- 2D
- Continuous MERA (Guifre Vidal's talk yesterday)
- Branching MERA (beyond logarithmic corrections to area law)

Summary

MERA is a class of tensor network states that can be thought of as

- quantum circuits that take a product state to hierarchically entangled state.
- results of a real space RG procedure that removes all local correlations.

It

- is a good ansatz for ground states of gapless local Hamiltonians:
  - $S(L) \sim \log L$
  - $\langle o(0)o(L)\rangle \sim L^{-p}$
- has tantalizing connections to the holographic principle.