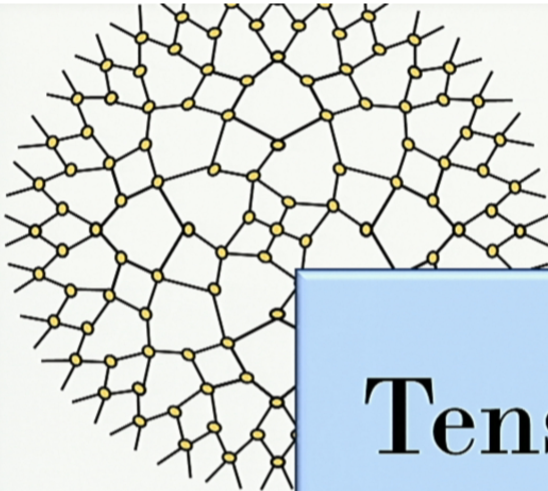
Title: Tensor Networks

Date: Jul 27, 2016 09:00 AM

URL: http://pirsa.org/16070046

Abstract:

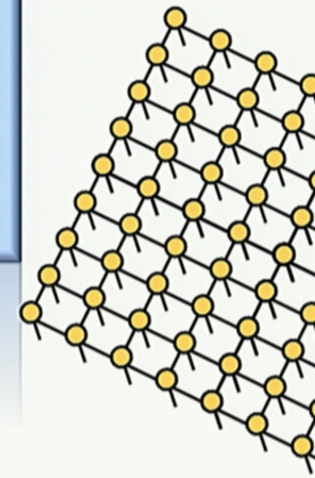


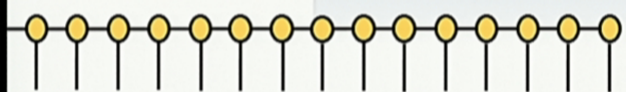
It from Qubit

Summer School

July 27th, 2016

Tensor Networks





Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

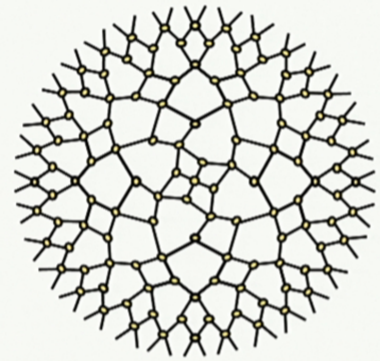
compute canada | calcul canada

TENSOR NETWORKS INITIATIVE

SIMONS FOUNDATION

NSERC

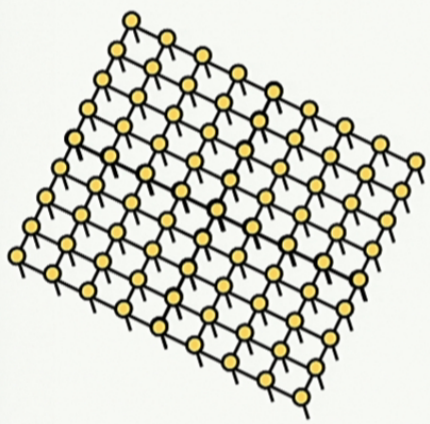
Outline:



Generalities
- Area law and tensor networks
- Definition
- Useful tensor networks

Examples
- D=1 MPS vs MERA
- D>1 PEPS, branching MERA

MERA
- quantum circuit
- RG transformation
- AdS/CFT



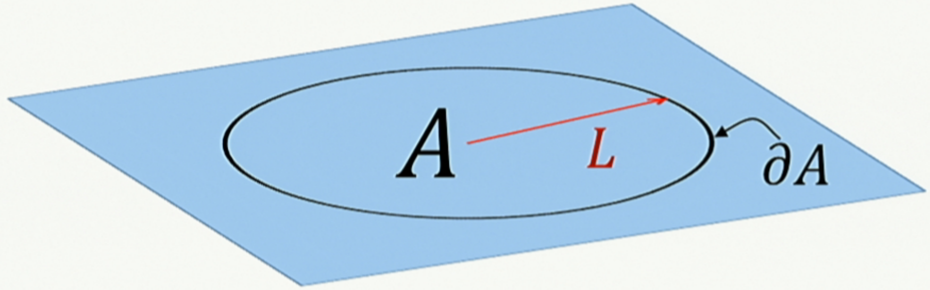
Area law

$\rho_A = Tr_B(|\Psi\rangle\langle\Psi|)$

$S = -Tr(\rho_A \log \rho_A)$

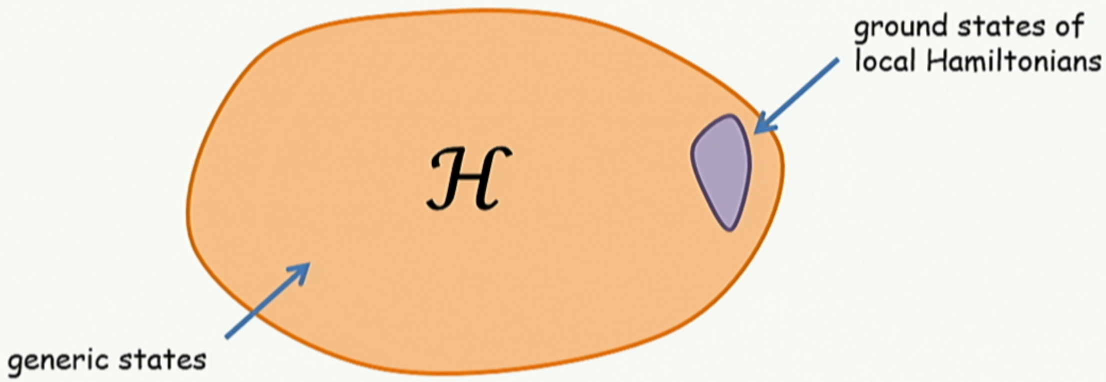
ground states of
local Hamiltonians

$$S \sim |\partial A| \sim L^{D-1}$$



generic states
(lattice model)

$$S \sim |A| \sim L^{D}$$

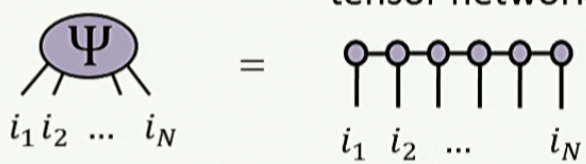


Many-body wave-function of N spins

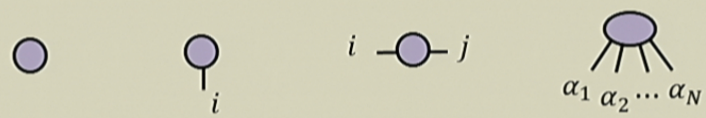
$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters

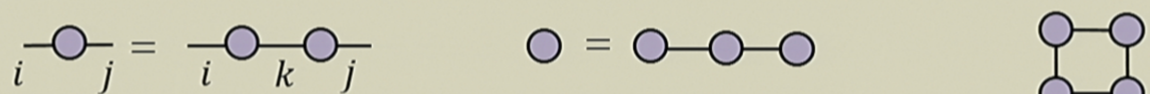
tensor network



graphical notation

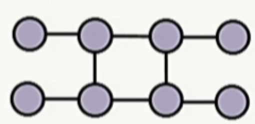


a $\quad$ b_{i} $\quad$ c_{ij} $\quad$ $d_{\alpha_1 \alpha_2 \cdots \alpha_N}$



$T_{ij} = \sum_k R_{ik} S_{kj}$ $\quad$ $a = \vec{y}^\dagger \cdot M \cdot \vec{x}$ $\quad$ $tr(ABCD)$

why bother?

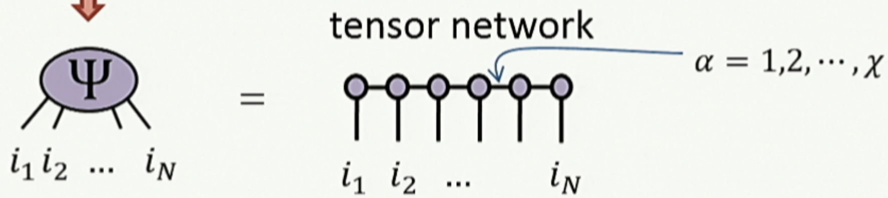


$$\sum_{ijklmnop} A_{ijk} B_{jlm} C_{nko} D_{kmr} x_i y_l z_n v_r$$

Many-body wave-function of N spins

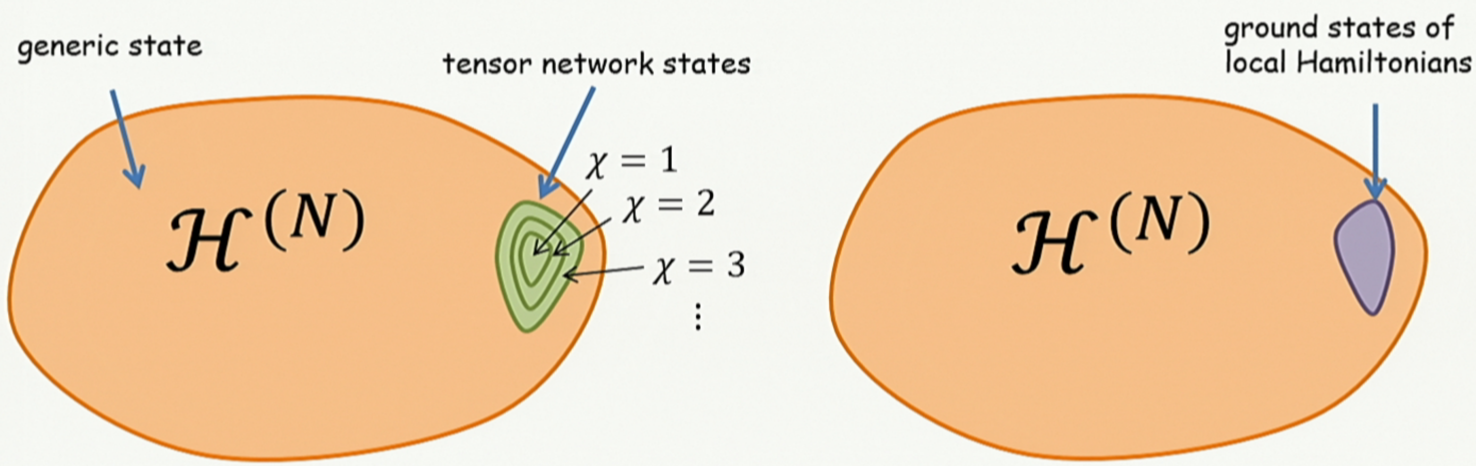
$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



2^N parameters → $O(N2\chi^2)$ parameters

inefficient → **efficient**

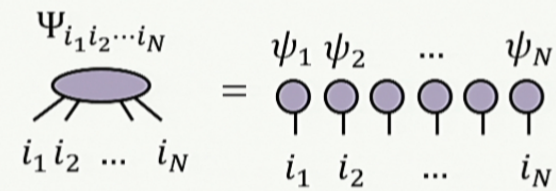


Why bond indices?

Product (unentangled) state

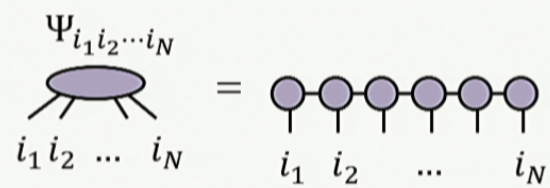
$$|\Psi\rangle = \ \psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$$

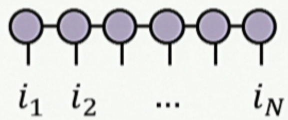
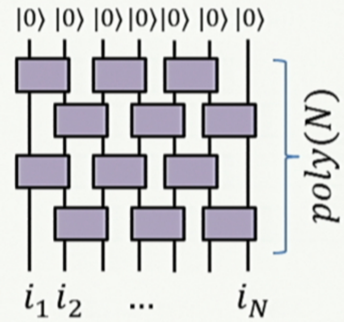
$$= \left(\sum_{i_1} (\psi_1)_{i_1} |i_1\rangle\right) \otimes \left(\sum_{i_2} (\psi_2)_{i_2} |i_2\rangle\right) \otimes \cdots \otimes \left(\sum_{i_N} (\psi_N)_{i_N} |i_N\rangle\right)$$



Entangled state

$$|\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$$

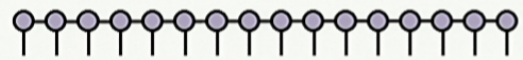


Computational efficiency	matrix product state (MPS) 	quantum circuit 
A: Efficient representation $\lvert\Psi\rangle \quad \leftrightarrow \quad poly(N)$ coefficients	✓	✓
B: Efficient manipulation $\langle\Psi\lvert\Psi\rangle$ $\langle\Psi\lvert\sigma_i^x\rvert\Psi\rangle$ $\vdots$ computational time and memory $poly(N)$	✓	✗

Matrix product state (MPS)

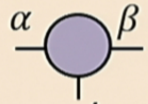
$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

$\Psi_{i_1 i_2 \cdots i_N}$



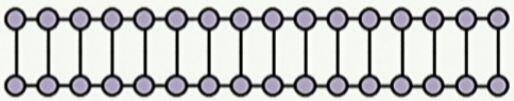
A: Efficient representation ?

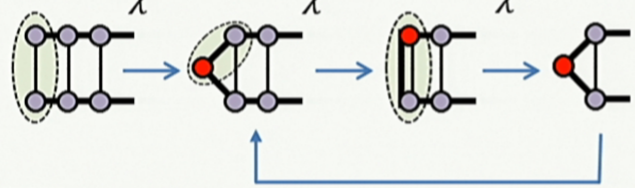
$O(N\chi^2)$ parameters



B: Efficient computation?

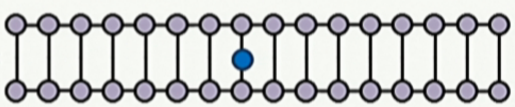
$\langle\Psi|\Psi\rangle =$
norm

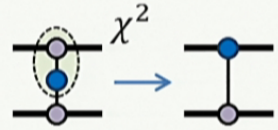




$O(N\chi^3)$!!!
computational time

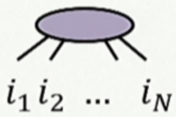
$\langle\Psi|\sigma_i^z|\Psi\rangle =$
local expectation value



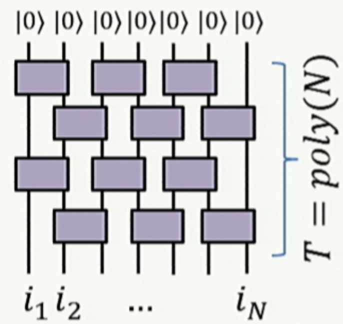


Quantum circuit

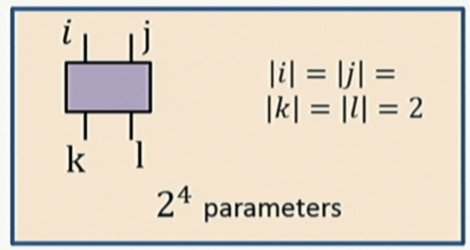
$\Psi_{i_1 i_2 \cdots i_N}$



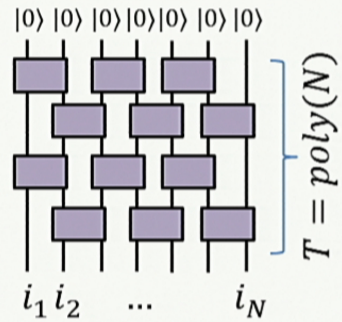
2^N

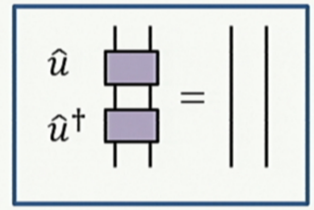


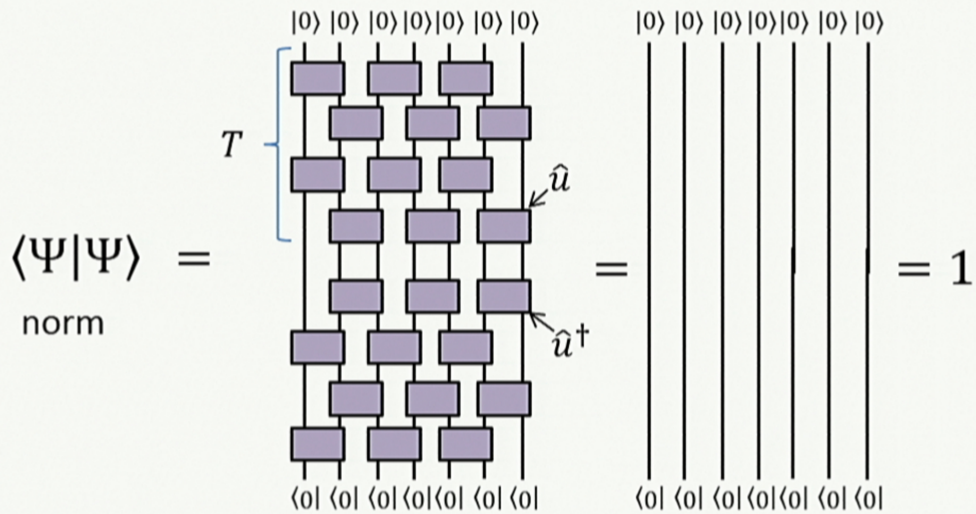
A: Efficient representation ?



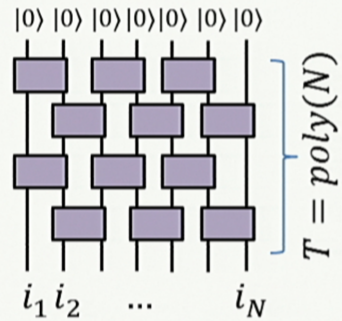
B: Efficient manipulation ?

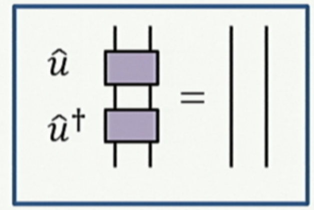






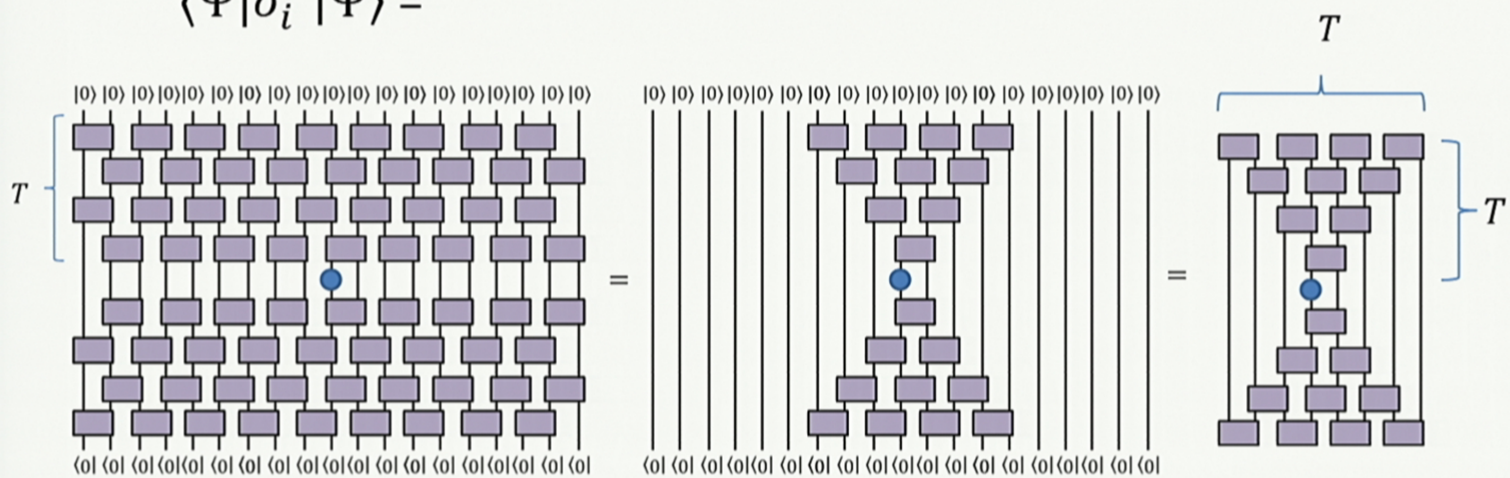
B: Efficient manipulation ?





local expectation value

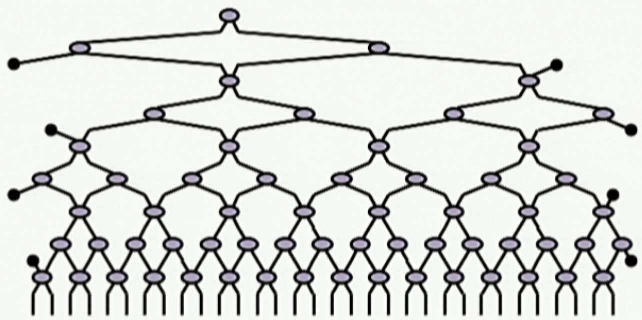
$$\langle \Psi | \sigma_i^x | \Psi \rangle =$$



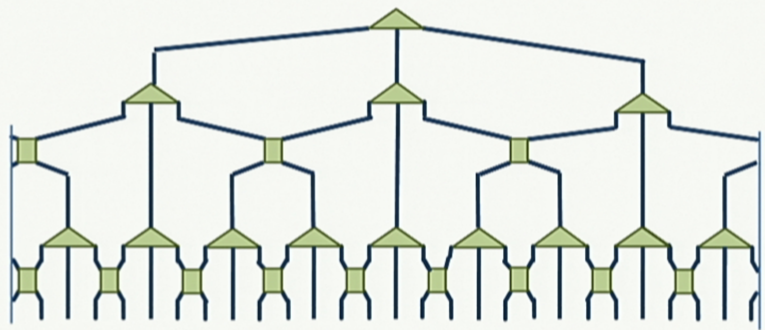
computational time and memory

$$\sim \exp(T) = \exp(N^q)$$

MERA



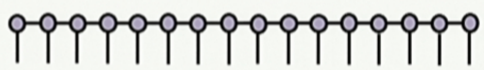
also MERA !



A: Efficient representation?

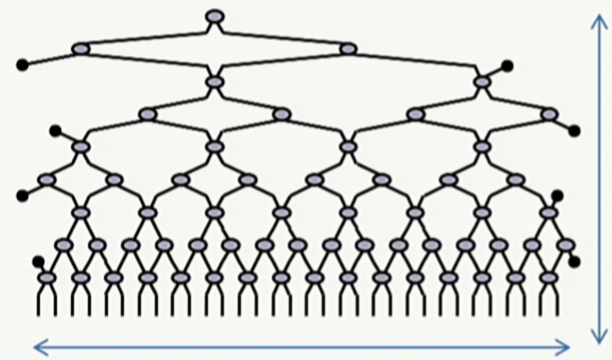
$$N + \frac{N}{2} + \frac{N}{4} + \ldots = N\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) \leq 2N$$

Matrix product state
(MPS)



N sites $\Rightarrow$ N tensors
$\Rightarrow O(N)$ parameters

Multi-scale entanglement
renormalization ansatz
(MERA)



N sites $\Rightarrow N\log(N)$ tensors ?

$2N$ tensors $\Rightarrow O(N)$ parameters

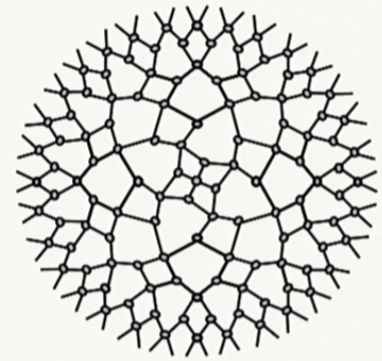
B: Efficient manipulation?

$\langle\Psi|\Psi\rangle$ norm

$\langle\Psi|\sigma_i^x|\Psi\rangle$ local expectation values

Focus lecture by
Markus Hauru
MERA: A tensor network for scale invariant systems
Wed 5pm Bob room

Outline:



Generalities
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- Definition
- Useful tensor networks

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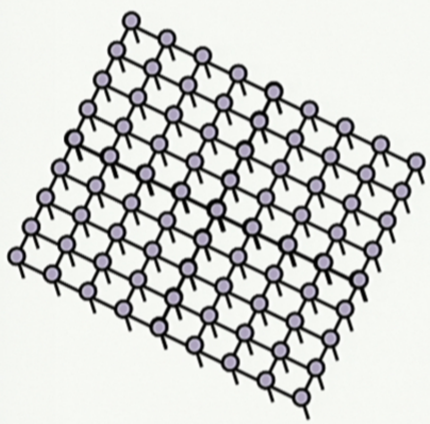
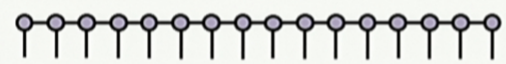
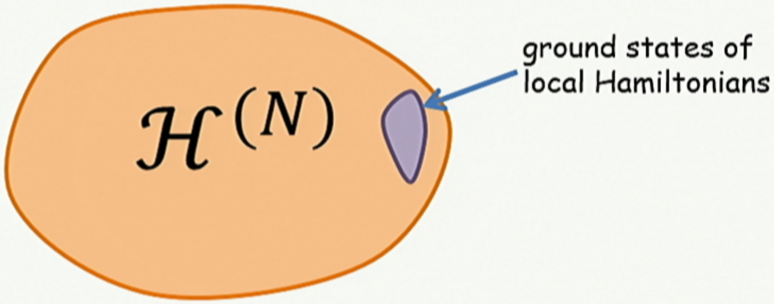
- D=1 MPS vs MERA
- D>1 PEPS, branching MERA

MERA
- quantum circuit
- RG transformation
- AdS/CFT







Area law

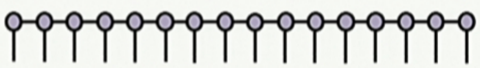
$$S \sim |\partial A| \sim L^{D-1}$$

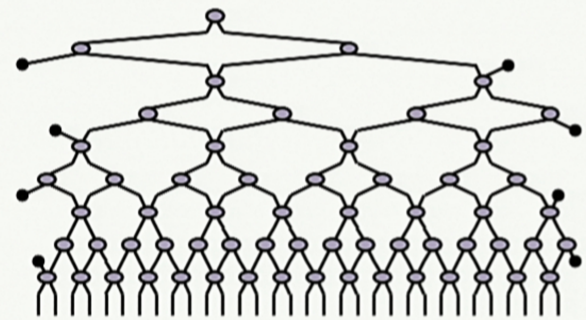
D=1 spatial dimensions

Area law: $S \sim |\partial A| \sim L^{0}$ gapped Hamiltonians

exponential decay of correlations $c(L) \sim e^{-L/\xi}$

Structural properties

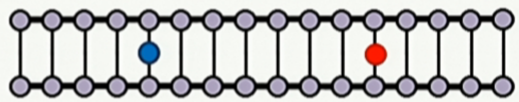
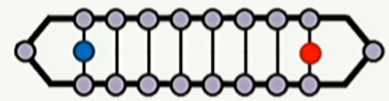


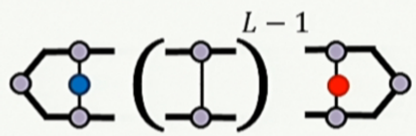


- Decay of correlations
- Scaling of entanglement

$\langle\Psi|\hat{o}(0)\,\hat{o}(L)|\Psi\rangle$ MPS

Problem session

$=$  $=$ 

$=$ 

$\approx\ a\lambda^L = a e^{-L/\xi}$

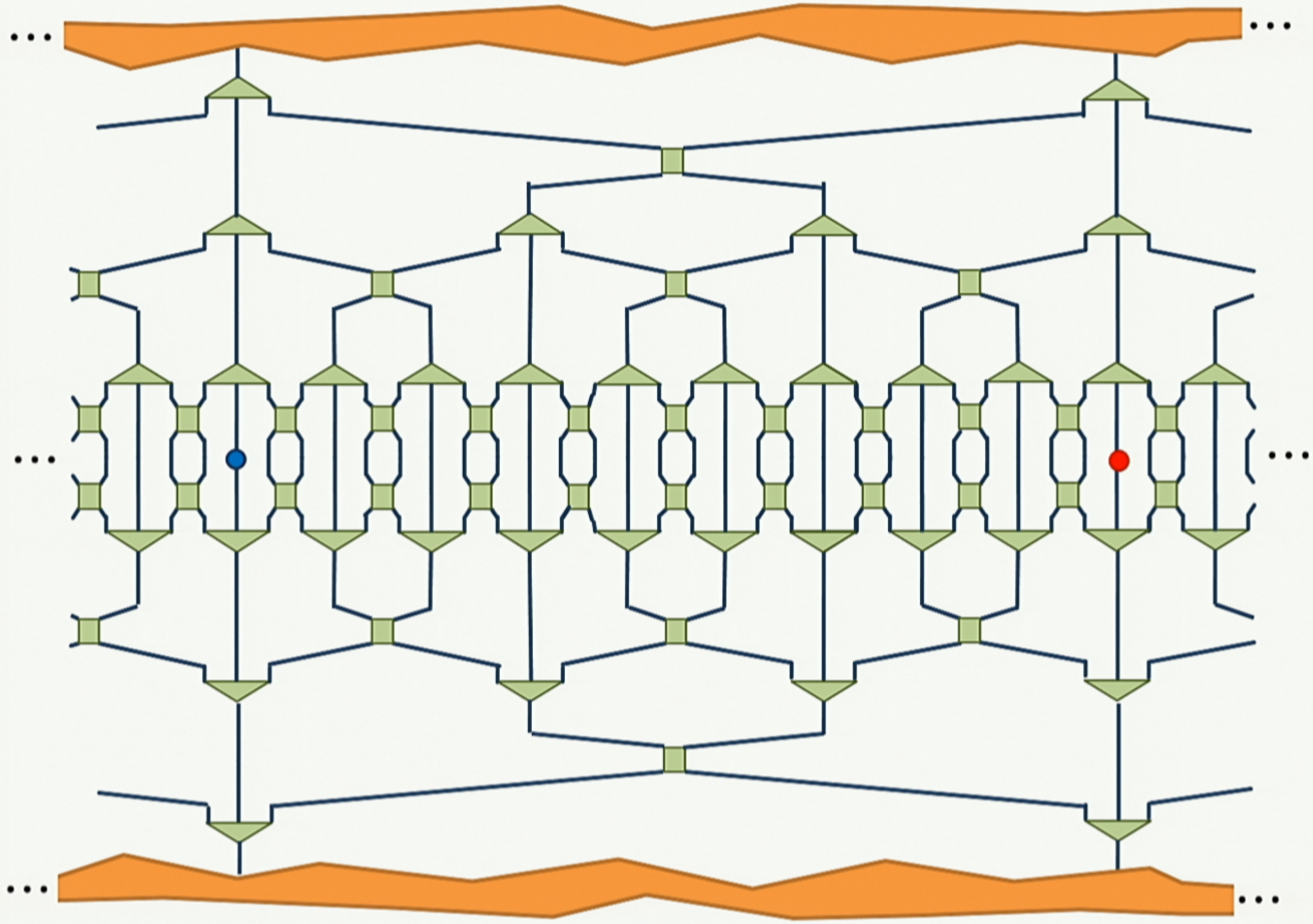
$$\xi \equiv -\frac{1}{\log \lambda}$$

$\Rightarrow$ exponential decay of correlations

MERA

$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle\ =$

Focus lecture by
Markus Hauru

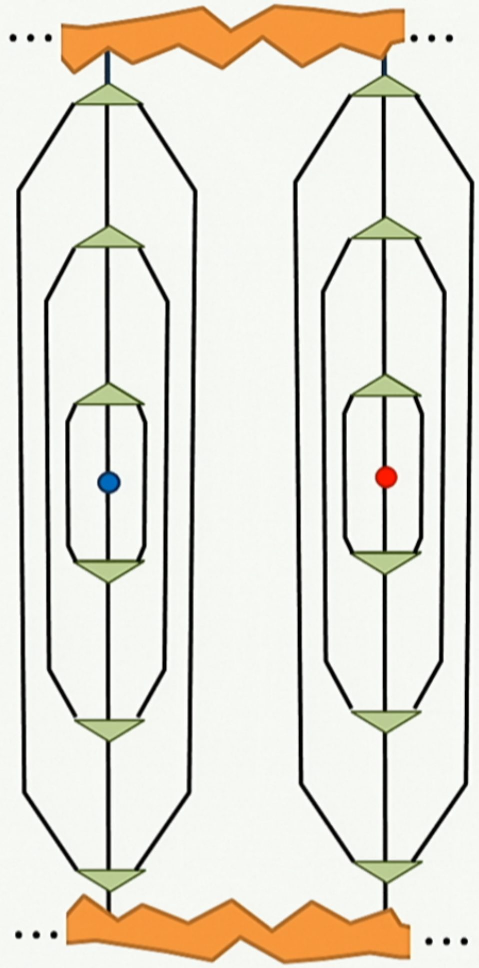


$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

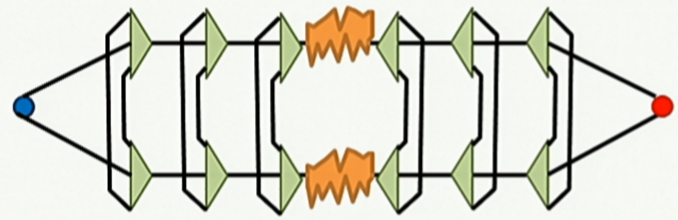
MERA

Focus lecture by
Markus Hauru

$=$



$=$



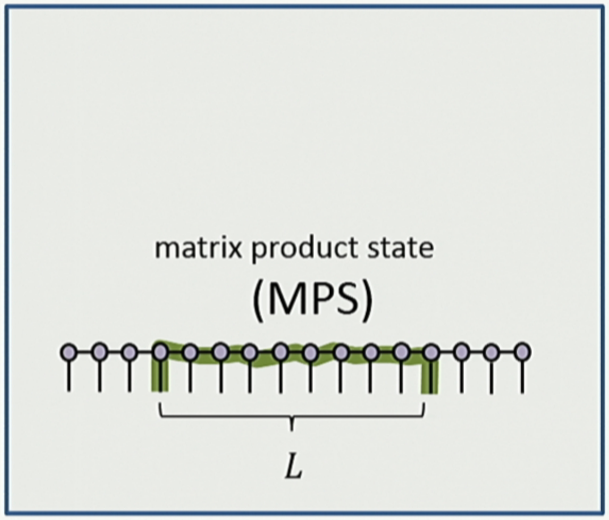
$\approx (\lambda)^{\log_3(L)}(\lambda)^{\log_3(L)}$

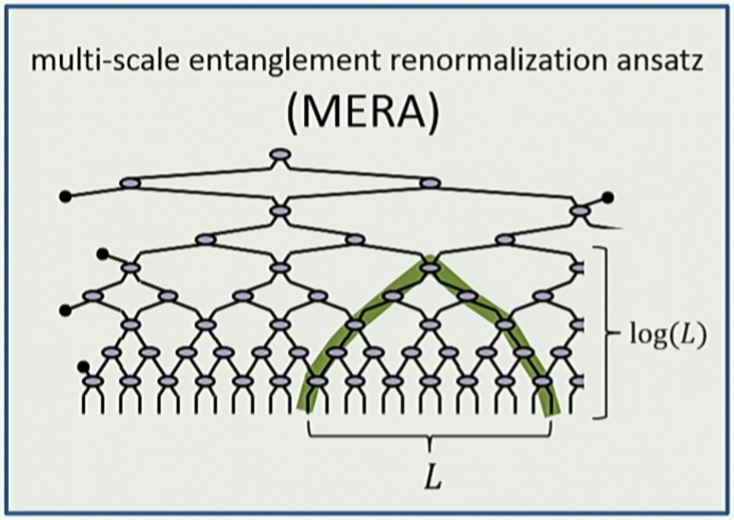
$= \lambda^{2\log_3(L)} \quad = L^{2\log_3(\lambda)} = L^{-p}$

$x^{\log_3(y)} = y^{\log_3(x)} \qquad p \equiv -2\log_3(\lambda)$

$\Rightarrow$ polynomial decay of correlations

Correlations: geometric interpretation





structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$$

exponential

structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx \frac{1}{L^{2\Delta}}$$

power-law

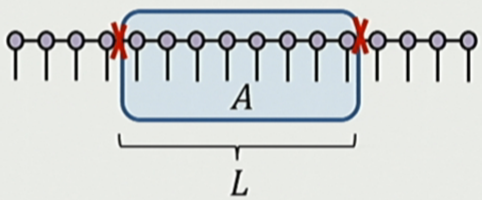
Entanglement entropy

Problem session

Focus lecture by Markus Hauru

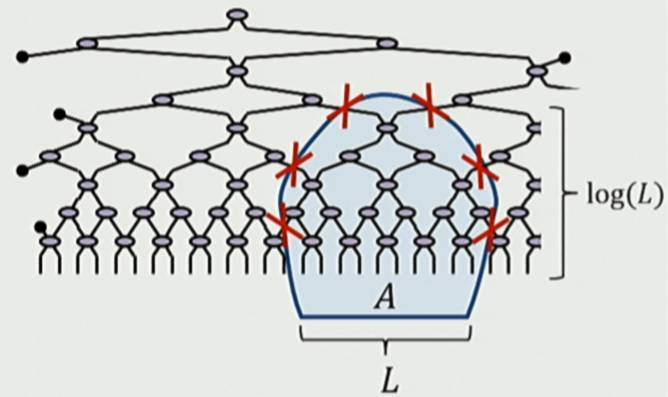
matrix product state

(MPS)



multi-scale entanglement renormalization ansatz

(MERA)



connectivity:

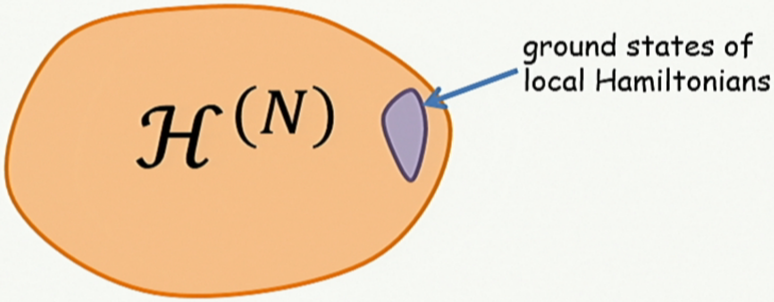
$S(A) \leq const$

area law!

connectivity:

$S(A) \leq \log L$

logarithmic correction!



Area law

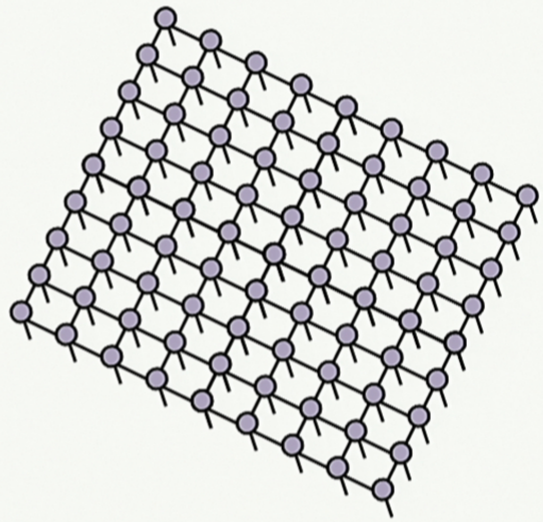
$$S \sim |\partial A| \sim L^{D-1}$$

space dimension \ energy spectrum	gapped $\Delta > 0$	gapless $\Delta = 0$: **small** surface of zero modes	gapless $\Delta = 0$: **large** [(D-1)- dimensional] surface of zero modes
D=1 (L)	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 (L, L)	$S_L \approx L$	$S_L \approx L$	$S_L \approx L \log(L)$
D=3 (L, L, L)	$S_L \approx L^2$	$S_L \approx L^2$	$S_L \approx L^2 \log(L)$

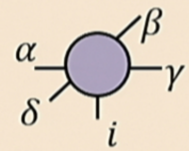
area law $S_L \approx L^{D-1}$ (gapped and small-surface gapless columns)

$S_L \approx L^{D-1} \log(L)$ logarithmic correction

Projected entangled pair states (PEPS)



A: Efficient representation?



$|\alpha| = |\beta| =$
$|\gamma| = |\delta| = \chi$
$|i| = 2$

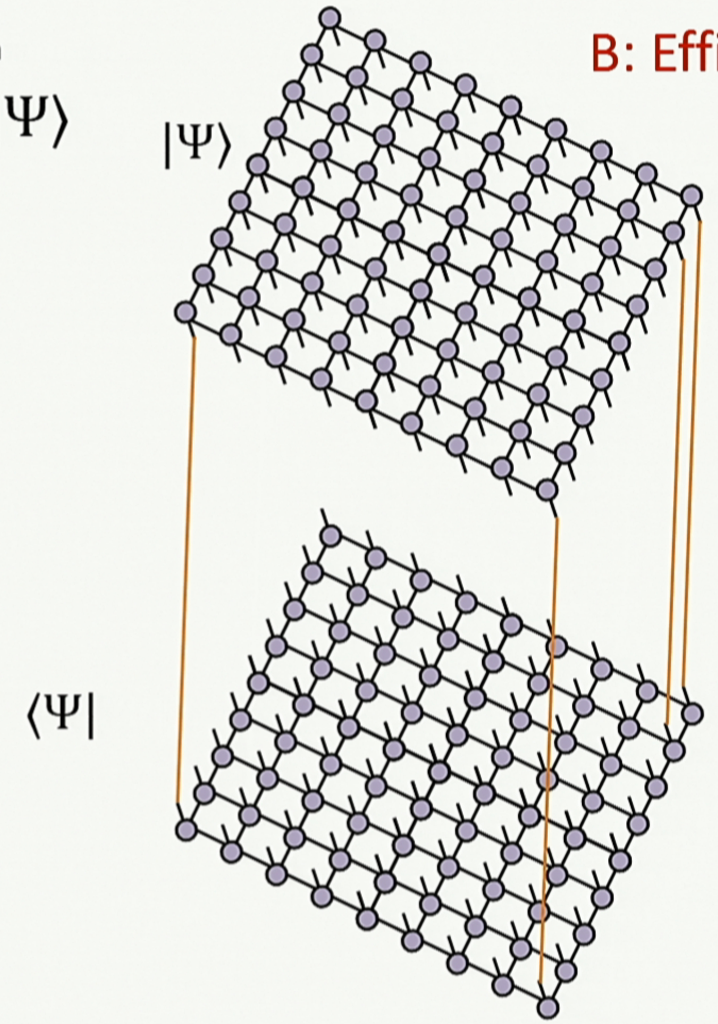
$2\chi^4$ parameters

✓ $O(N\chi^4)$ parameters

Projected entangled pair states (PEPS)

Norm

$\langle\Psi|\Psi\rangle$



B: Efficient manipulation?

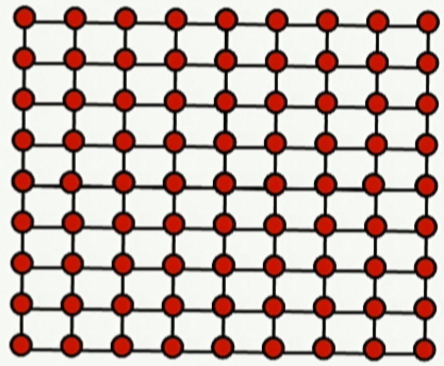
Projected entangled pair states (PEPS)

Norm?

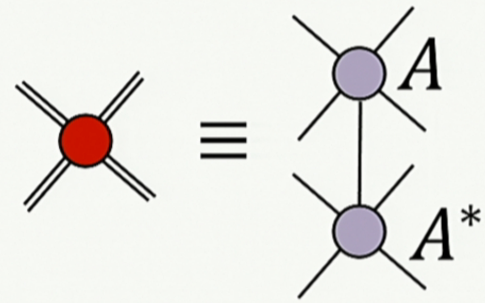
B: Efficient manipulation?

$\langle \Psi | \Psi \rangle$

N_y



N_x



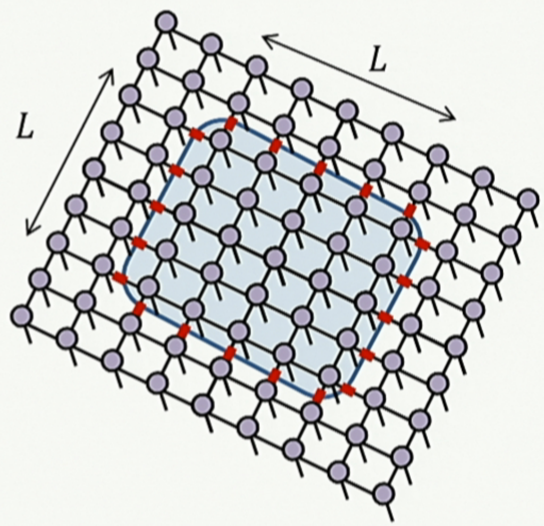
Cost exact contraction $N_x \exp(N_y)$ (if $N_y < N_x$)

Cost approximate contraction $O(N_x N_y)$

Projected entangled pair states (PEPS)

Structural properties

Entanglement entropy:



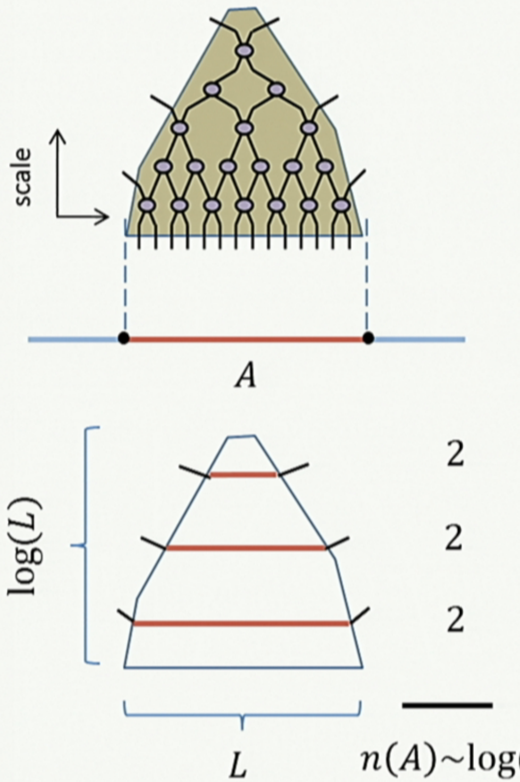
area law

$S_L \leq 4L \log(\chi) \quad (= L^{D-1})$

Entanglement entropy in MERA

D=1 dimensions

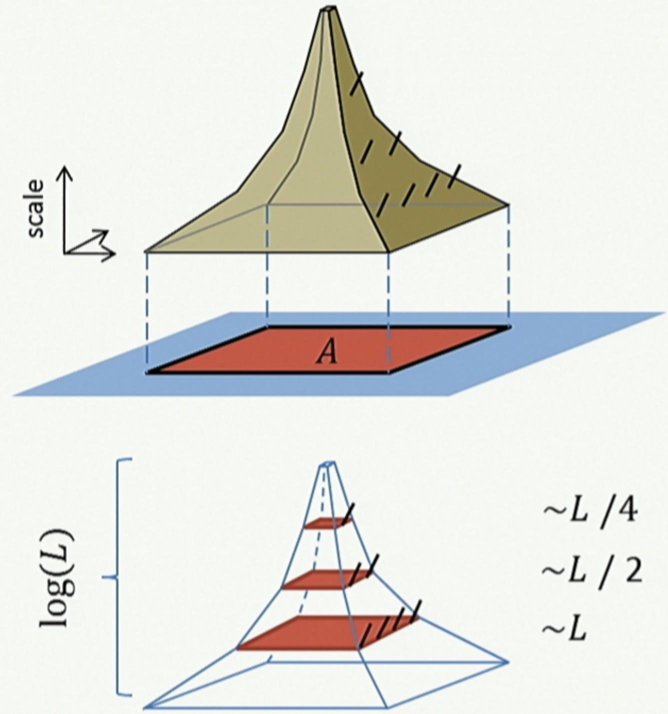
$S \sim \log(L)$ logarithmic correction



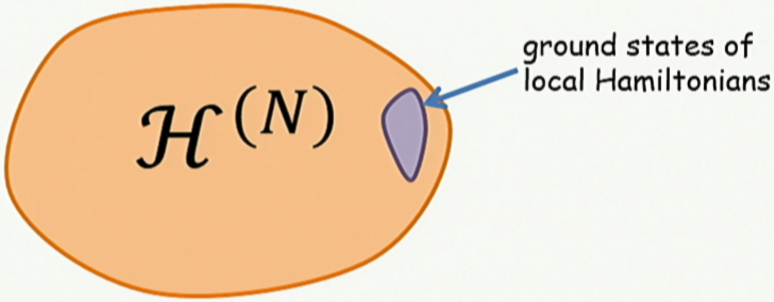
$n(A) \sim \log(L)$

D=2 dimensions

$S \sim L$ area law



$$n(A) \sim L(1 + \frac{1}{2} + \frac{1}{4} + \cdots)$$
$$\leq 2L$$



Area law

$$S \sim |\partial A| \sim L^{D-1}$$

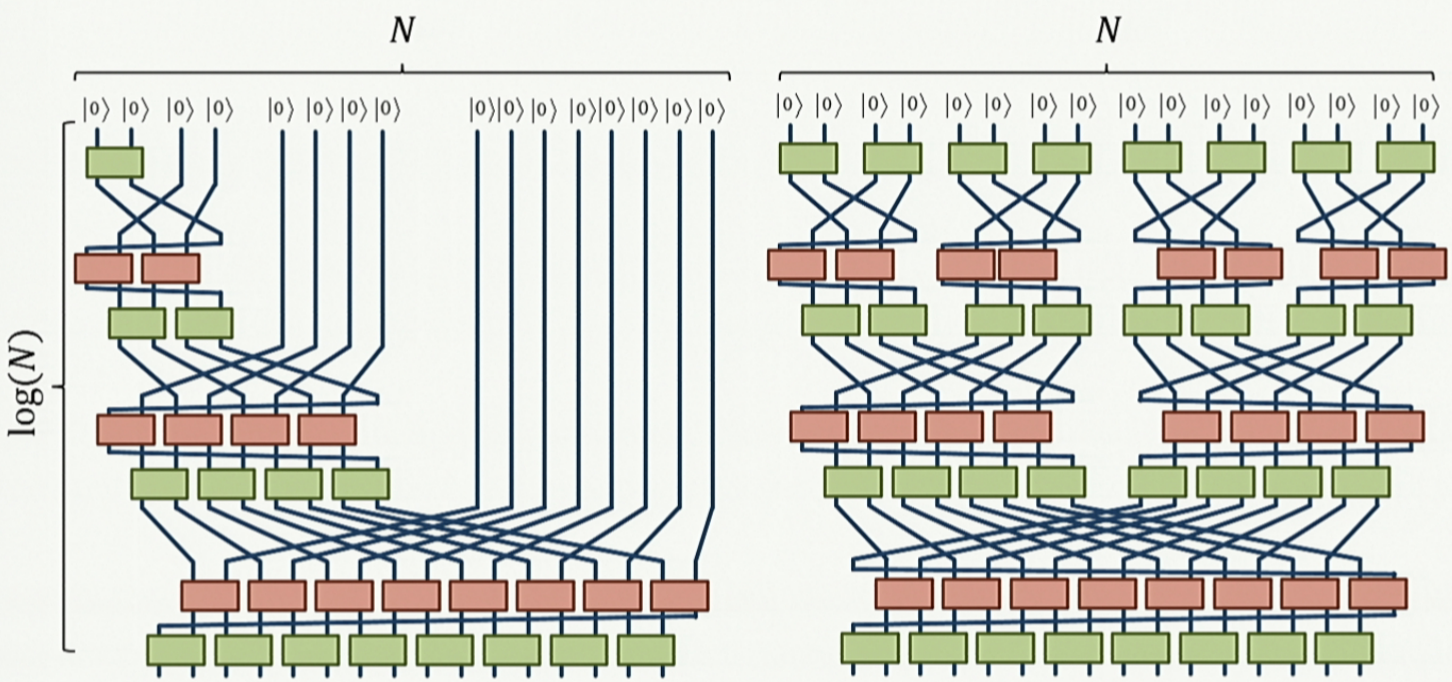
energy spectrum / space dimension	gapped $\Delta > 0$	gapless $\Delta = 0$: **small** surface of zero modes	gapless $\Delta = 0$: **large** [(D-1)- dimensional] surface of zero modes
D=1 (L)	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 (L, L)	$S_L \approx L$ PEPS	$S_L \approx L$ MERA	$S_L \approx L \log(L)$
D=3 (L, L, L)	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ MERA	$S_L \approx L^2 \log(L)$

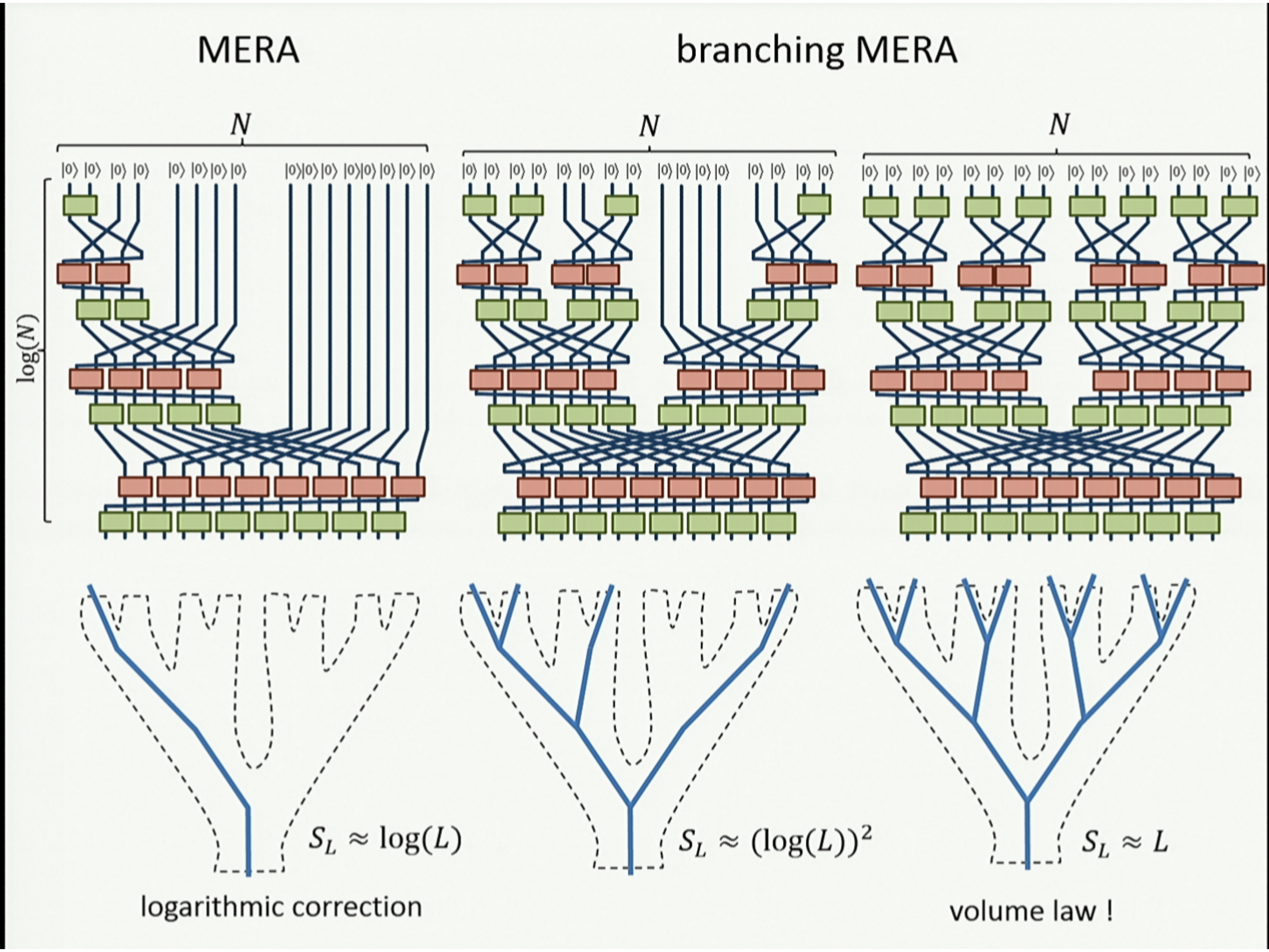
area law $S_L \approx L^{D-1}$ (gapped and small surface of zero modes)

$S_L \approx L^{D-1} \log(L)$ logarithmic correction

MERA

branching MERA

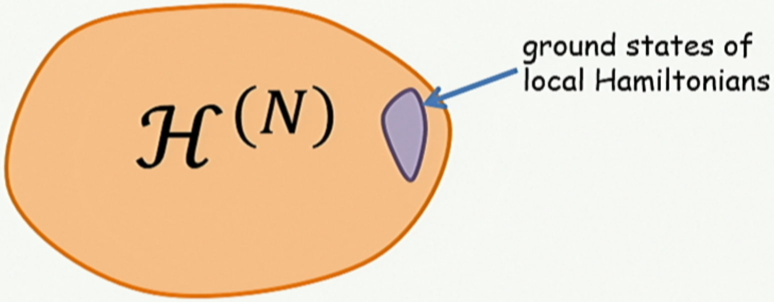



MERA
branching MERA
N
N
N
log(N)
|0⟩
$S_L \approx \log(L)$
$S_L \approx (\log(L))^2$
$S_L \approx L$
logarithmic correction
volume law !

branching MERA

D=1 spatial dimensions	$S_L \approx \log(L)$	…		$S_L \approx L$

D>1 spatial dimensions	$S_L \approx L^{D-1}$	…	$S_L \approx L^{D-1}\log(L)$	…	$S_L \approx L^D$



Area law

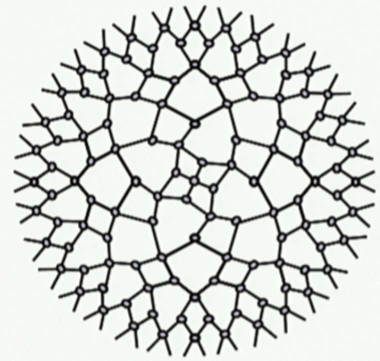
$$S \sim |\partial A| \sim L^{D-1}$$

space dimension \ energy spectrum	gapped $\Delta > 0$	gapless $\Delta = 0$ — **small** surface of zero modes	gapless $\Delta = 0$ — **large** [(D-1)- dimensional] surface of zero modes
D=1 (L)	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 (L, L)	$S_L \approx L$ PEPS	$S_L \approx L$ MERA	$S_L \approx L\log(L)$ branching MERA
D=3 (L, L, L)	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ MERA	$S_L \approx L^2\log(L)$ branching MERA

area law $S_L \approx L^{D-1}$ (gapped and small-surface gapless columns)

$S_L \approx L^{D-1}\log(L)$ logarithmic correction

Outline:



Generalities Area law and tensor networks

Definition

Useful tensor networks

Examples D=1 MPS vs MERA

D>1 PEPS, branching MERA

MERA quantum circuit

RG transformation

AdS/CFT