Title: Quantum Information and Symmetry

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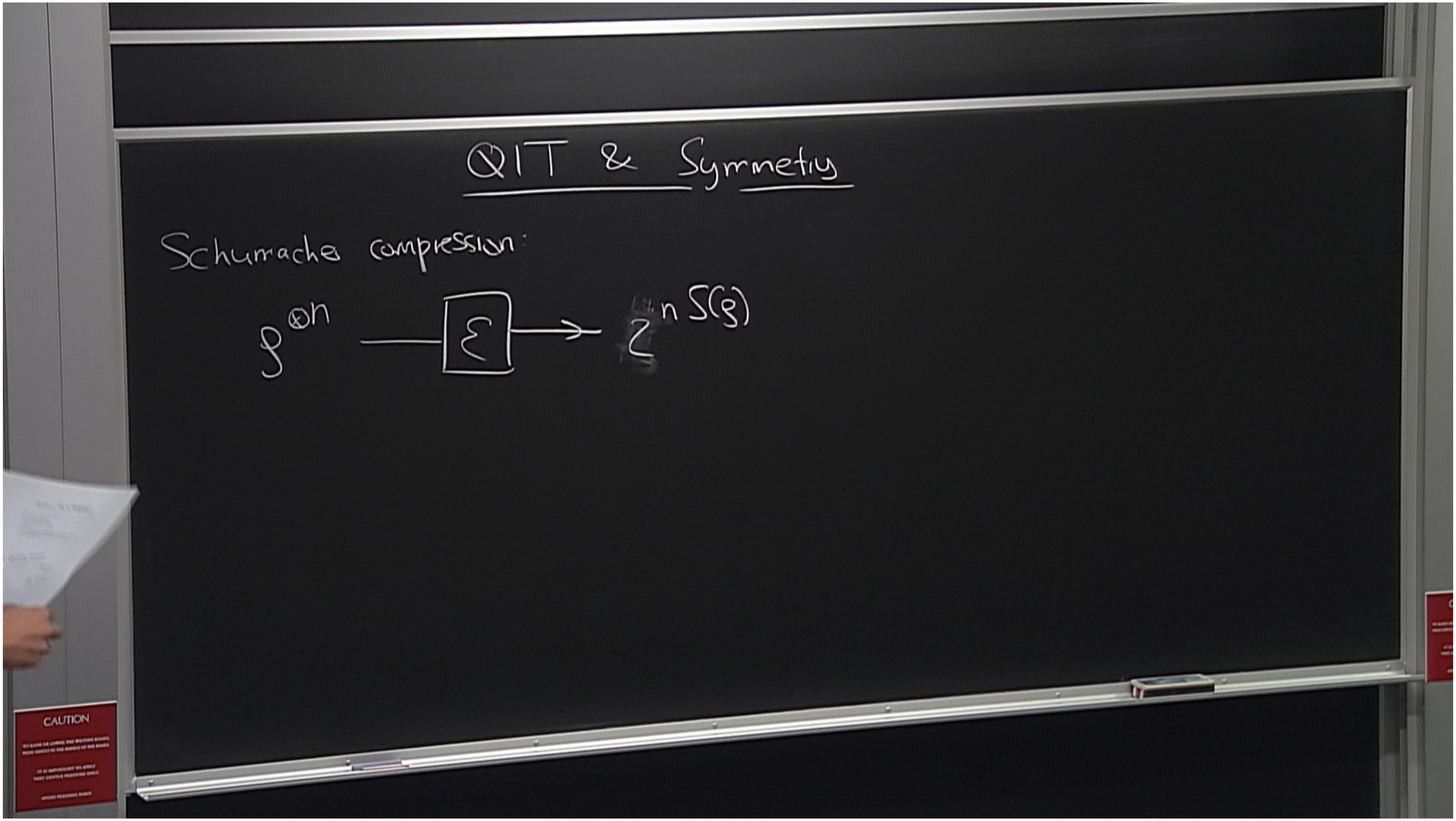
URL: http://pirsa.org/16070044

Abstract:


QIT & Symmetry
Schumacher compression:
$\rho^{\otimes n} \longrightarrow \boxed{\mathcal{E}} \longrightarrow 2^{n S(\rho)}$

$$\underbrace{\rho^{\otimes n}}_{\text{permut-symm}} \longrightarrow \boxed{\mathcal{E}} \longrightarrow 2^{n S(\rho)}$$

qubits

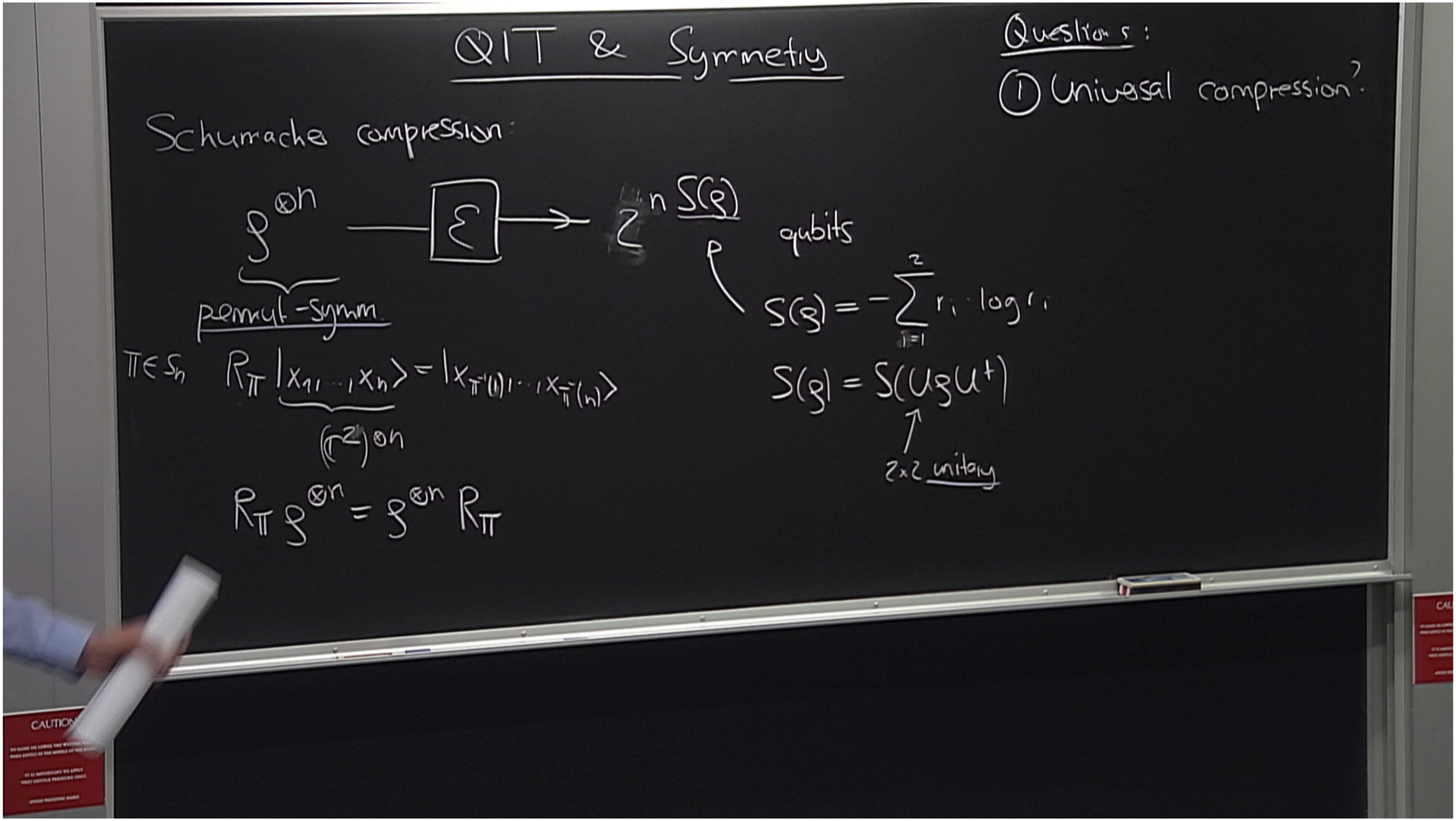
$$S(\rho) = -\sum_{i=1}^{2} r_i \cdot \log r_i$$

$$S(\rho) = S(U \rho U^{+})$$

2×2 unitary

$$\pi \in S_n \quad R_\pi \underbrace{|x_1, \ldots, x_n\rangle}_{(\mathbb{C}^2)^{\otimes n}} = |x_{\pi^{-1}(1)}, \ldots, x_{\pi^{-1}(n)}\rangle$$

$$R_\pi \rho^{\otimes n} = \rho^{\otimes n} R_\pi$$


QIT & Symmetry
Questions:
① Universal compression?
Schumacher compression:
$\rho^{\otimes n} \longrightarrow \boxed{\mathcal{E}} \longrightarrow 2^{n S(\rho)}$
qubits
permut.-symm.
$S(\rho) = -\sum_{i=1}^{2} r_i \log r_i$
$\pi \in S_n$ $R_\pi |x_1, \ldots, x_n\rangle = |x_{\pi^{-1}(1)}, \ldots, x_{\pi^{-1}(n)}\rangle$
$(\mathbb{C}^2)^{\otimes n}$
$S(\rho) = S(U \rho U^\dagger)$
2x2 unitary
$R_\pi \rho^{\otimes n} = \rho^{\otimes n} R_\pi$

Two copies: $\rho^{\otimes 2}$

$$F = \sum_{i,j} |ij\rangle\langle ji|$$

$$\rho,\ \rho \longrightarrow \boxed{F} \longrightarrow \pm 1$$

$$\vdots$$

$$\varphi,\ \varphi \longrightarrow \boxed{F} \longrightarrow \pm 1$$

$$\mathrm{tr}\, \rho^{\otimes 2} F = \mathrm{tr}\, \rho^2 = r_1^2 + r_2^2 = \left(\tfrac{1}{2}+r\right)^2 + \left(\tfrac{1}{2}-r\right)^2$$

$$= \sum_{i,j} \langle i|\rho|j\rangle\langle j|\rho|i\rangle$$

$\downarrow$

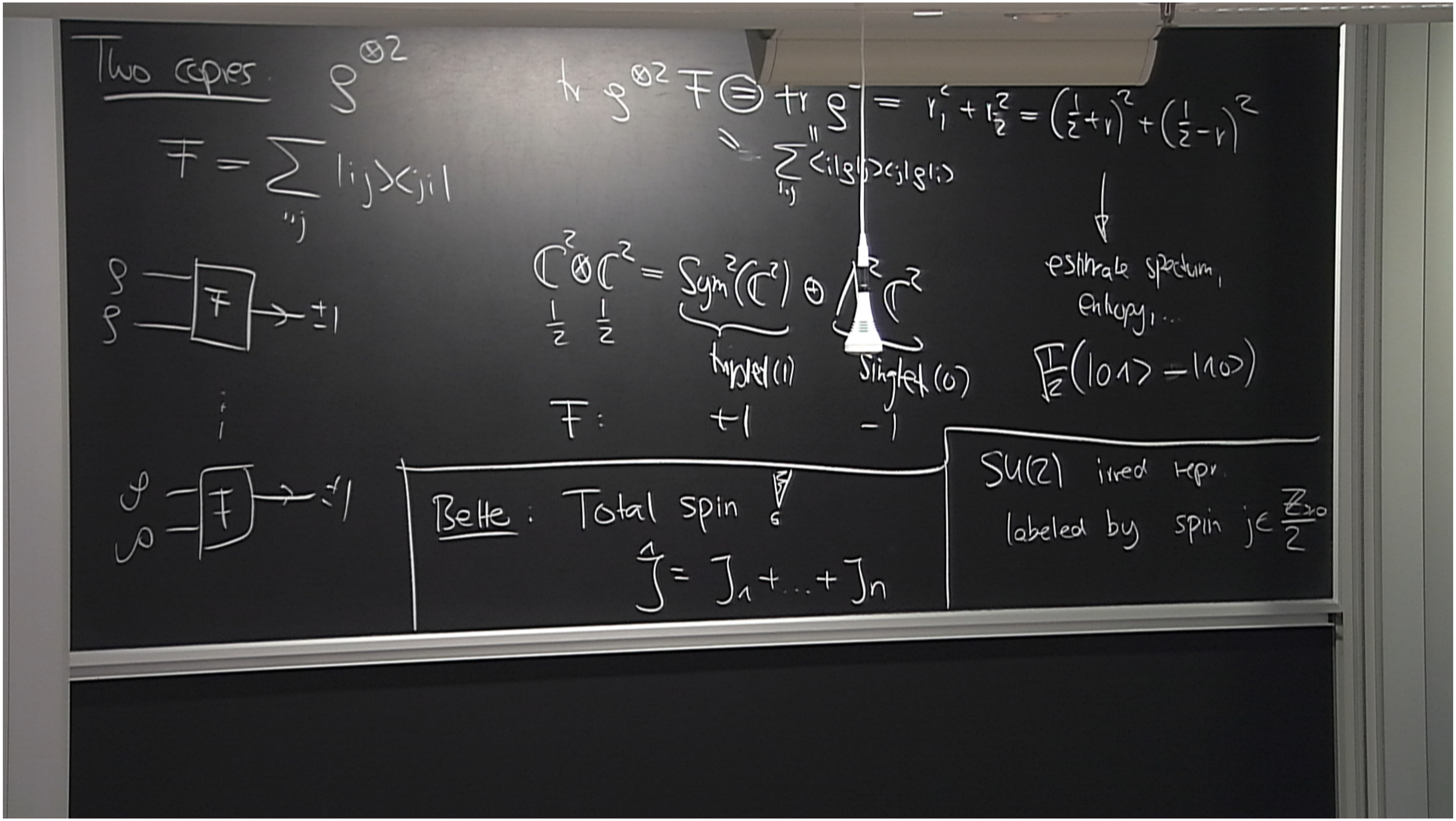
estimate spectrum, entropy, ...

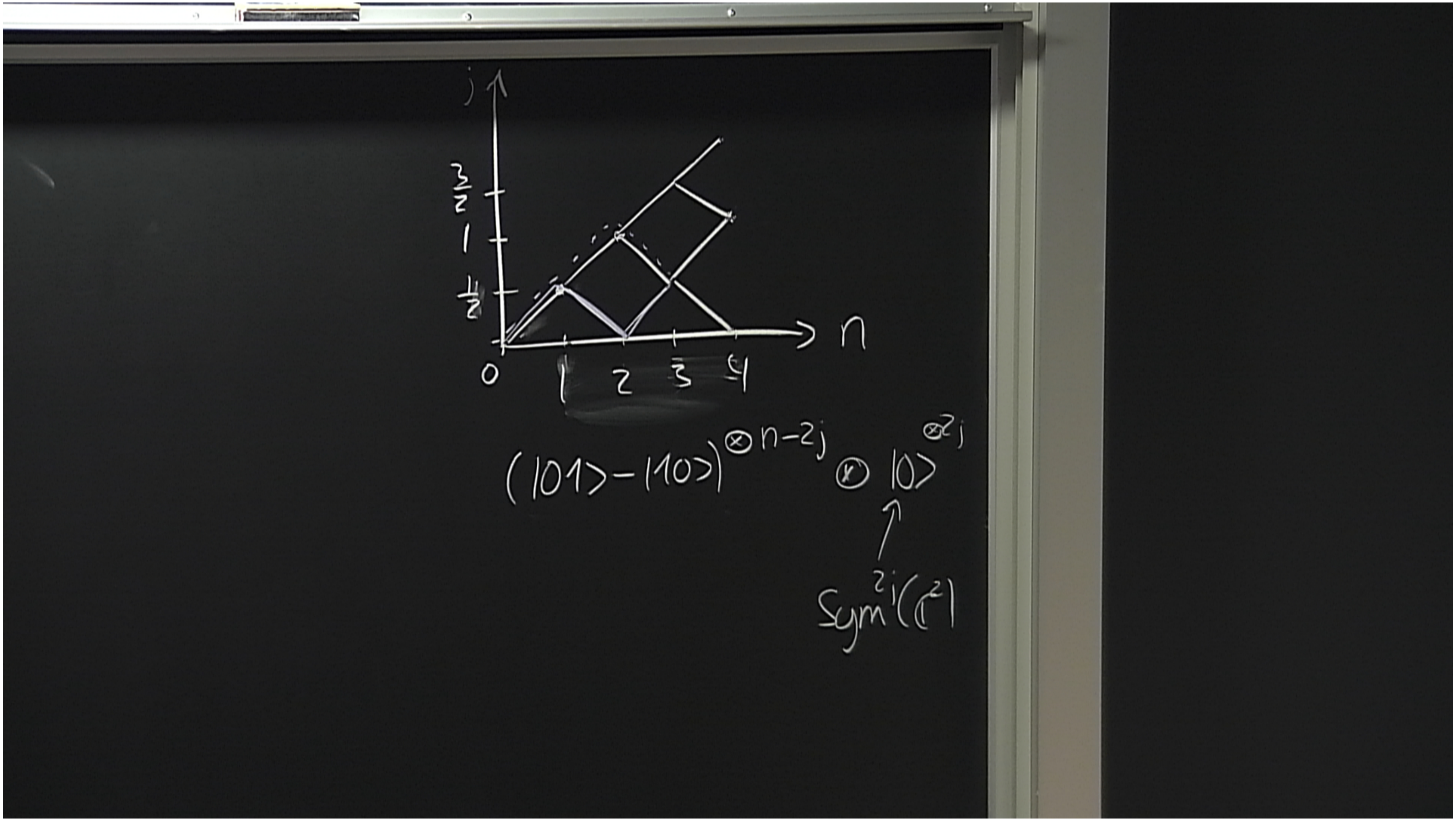
$$\underset{\frac{1}{2}}{\mathbb{C}^2} \otimes \underset{\frac{1}{2}}{\mathbb{C}^2} = \underbrace{\mathrm{Sym}^2(\mathbb{C}^2)}_{\text{triplet } (1)} \oplus \underbrace{\Lambda^2 \mathbb{C}^2}_{\text{singlet } (0)}$$

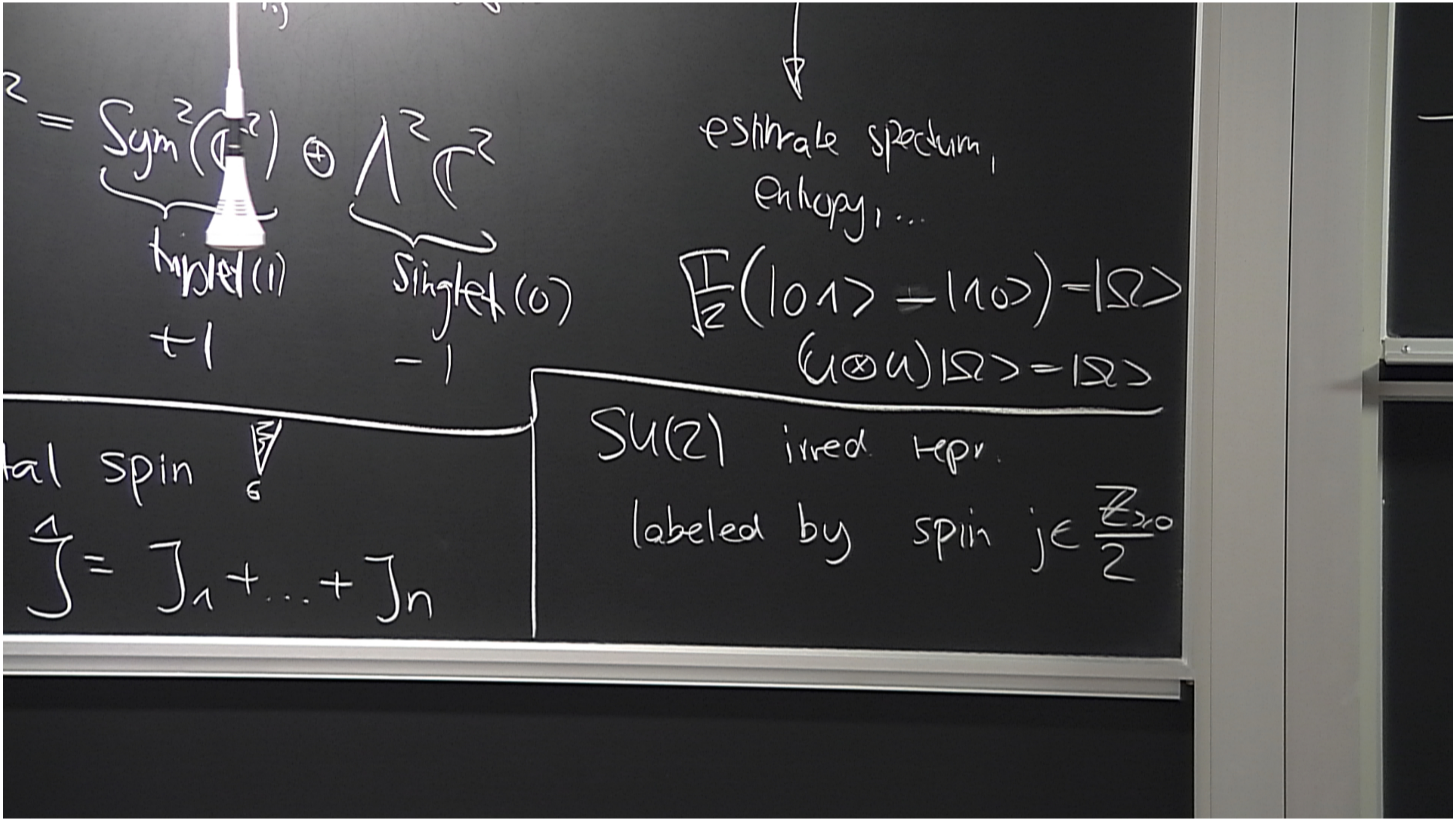
F: $\quad +1 \quad\quad -1$

Better: Total spin!

$$\hat{J} = J_1 + \ldots + J_n$$


Two copies: $\rho^{\otimes 2}$
$\mathbb{F} = \sum_{i,j} |ij\rangle\langle ji|$
$\operatorname{tr} \rho^{\otimes 2} \mathbb{F} = \operatorname{tr} \rho^2 = r_1^2 + r_2^2 = (\frac{1}{2}+r)^2 + (\frac{1}{2}-r)^2$
$= \sum_{i,j} \langle i|\rho|j\rangle\langle j|\rho|i\rangle$
estimate spectrum, entropy, ...
ρ, ρ, $\mathbb{F}$, ± 1
$\mathbb{C}^2 \otimes \mathbb{C}^2 = \mathrm{Sym}^2(\mathbb{C}^2) \oplus \Lambda^2 \mathbb{C}^2$
$\frac{1}{2}$ $\frac{1}{2}$
triplet (1)
singlet (0)
$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
$\mathbb{F}$: +1 -1
Better: Total spin
$\hat{J} = J_1 + \dots + J_n$
SU(2) irred repr. labeled by spin $j \in \frac{\mathbb{Z}_{\geq 0}}{2}$


j
n
$\frac{3}{2}$
1
$\frac{1}{2}$
0
1 2 3 4
$(|01\rangle - |10\rangle)^{\otimes n-2j} \otimes |0\rangle^{\otimes 2j}$
$\mathrm{Sym}^{2j}(\mathbb{C}^2)$


= Sym²(ℂ²) ⊕ Λ²ℂ²
triplet (1)
+1
singlet (0)
−1
estimate spectrum,
entropy, ...
1/√2 (|01> − |10>) = |Ω>
(U⊗U)|Ω> = |Ω>
tal spin
J = J1 + ... + Jn
SU(2) irred repr.
labeled by spin j ∈ Z≥0 / 2

How does it work?

spin-j repr. matrix of U

$$\mathrm{tr}\, \cancel{U^{\otimes n}}\, g^{\otimes n} P_j = m_j^n \cdot \mathrm{tr}\, R_j(\cancel{U}\, g)$$

$SU(2) \hookrightarrow SL(2)$

$GL(2)$

$$(\lambda U)^{\otimes n} = \lambda^n \cdot U^{\otimes n}$$

$j \otimes \frac{1}{2} = \left[j+\frac{1}{2}\right] \oplus \underbrace{\left[j-\frac{1}{2}\right]}_{\text{unless neg.}}$

$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

$$(\mathbb{C}^2)^{\otimes n} = \underbrace{\tfrac{1}{2} \otimes \tfrac{1}{2} \otimes \cdots \otimes \tfrac{1}{2}}_{n} = (1 \oplus 0) \otimes \underbrace{\tfrac{1}{2} \otimes \cdots \otimes \tfrac{1}{2}}_{n-2} = \left(\tfrac{3}{2} \oplus \tfrac{1}{2} \oplus \tfrac{1}{2}\right) \otimes \underbrace{\tfrac{1}{2} \otimes \cdots \otimes \tfrac{1}{2}}_{n-3}$$

$$= \bigoplus_{j=0}^{n/2} \boxed{m_j^n \cdot j}$$

total spin $= j$ sector

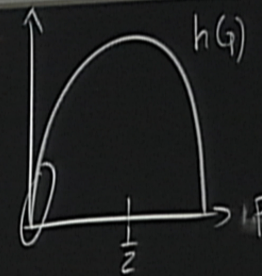
multiplicity

P_j = projector onto

irred SU(2)-repr

irred repr of S_n

$$\boxed{\begin{aligned} (\mathbb{C}^2)^{\otimes n} &\cong \bigoplus_j j \otimes \mathbb{C}^{m_j^n} \\ U^{\otimes n} &\cong \bigoplus_j R_j(U) \otimes I_{m_j^n} \end{aligned}}$$

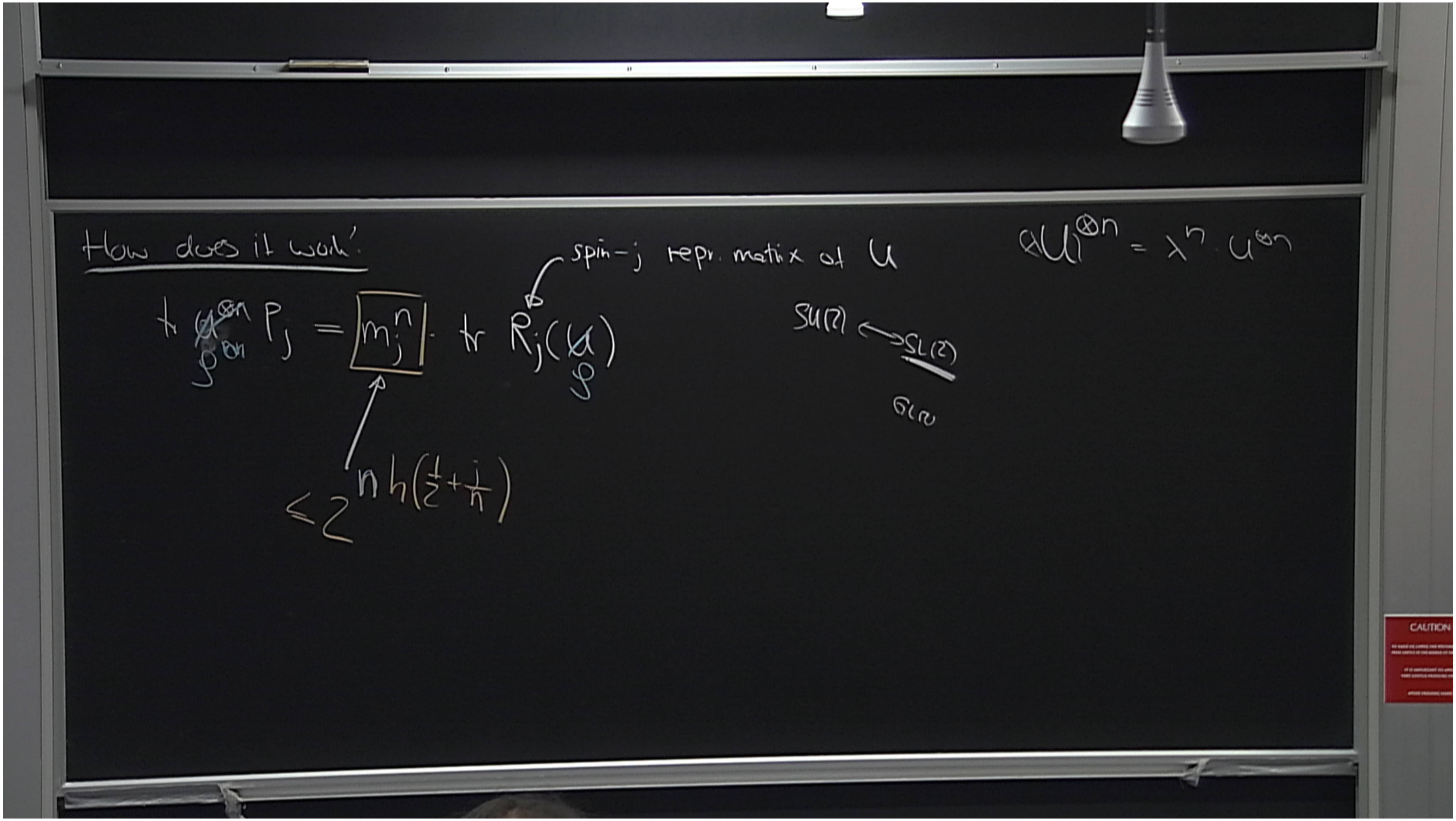


Result: (Keyl & Werner)

$$\left\{ \begin{array}{c} \text{probability of} \\ \text{outcome } j \end{array} \right\} = \mathrm{tr}\, \rho^{\otimes n} P_j \lesssim 2^{-n\left|\frac{j}{n} - r\right|}$$

$\leq d\{$

Y. diagram $\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$


How does it work?
spin-j repr. matrix of U
$\mathrm{tr}\, \cancel{U^{\otimes n}}\, \rho^{\otimes n} P_j = \boxed{m_j^n} \cdot \mathrm{tr}\, R_j(\cancel{U}\, \rho)$
$\leq 2^{n h\left(\frac{1}{2} + \frac{j}{n}\right)}$
$SU(2) \longleftrightarrow SL(2)$
$GL(2)$
$(\lambda U)^{\otimes n} = \lambda^n \cdot U^{\otimes n}$