

Title: Quantum Information and Symmetry

Date: Jul 26, 2016 05:00 PM

URL: <http://pirsa.org/16070044>

Abstract:

QIT & Symmetry

Schumacher compression:

$$\rho^{\otimes n} \longrightarrow \boxed{\mathcal{E}} \longrightarrow 2^{nS(\rho)}$$

CAUTION

DO NOT TOUCH THE SURFACE OF THE BOARD.
IF IT IS NECESSARY TO APPLY
PRESSURE, USE THE ERASER.
AVOID PRESSURE ON THE BOARD.

$$\underbrace{g^{\otimes n}}_{\text{permut-symm}} \xrightarrow{\boxed{\mathcal{E}}} 2^n S(g)$$

qubits

$$S(g) = -\sum_{i=1}^2 r_i \log r_i$$

$$S(g) = S(UgU^\dagger)$$

$\uparrow$
2x2 unitary

$$\pi \in S_n \quad R_\pi \underbrace{|x_1, \dots, x_n\rangle}_{(r^Z)^{\otimes n}} = |x_{\pi^{-1}(1)}, \dots, x_{\pi^{-1}(n)}\rangle$$

$$R_\pi g^{\otimes n} = g^{\otimes n} R_\pi$$

CAUTION

THE BOARD HAS A WARNING LABEL. PLEASE HANDLE IT WITH CARE. IT IS NOT TO BE USED FOR ANY OTHER PURPOSE. IT IS NOT TO BE USED FOR ANY OTHER PURPOSE. IT IS NOT TO BE USED FOR ANY OTHER PURPOSE.

QIT & Symmetry

Questions:

① Universal compression?

Schumacher compression:

$$\underbrace{\mathcal{H}^{\otimes n}}_{\text{permut.-symm}} \xrightarrow{\mathcal{E}} 2^{n S(\mathcal{E})} \text{ qubits}$$

$$S(\mathcal{E}) = -\sum_{i=1}^2 r_i \log r_i$$

$$S(\mathcal{E}) = S(U \mathcal{E} U^\dagger)$$

$\uparrow$
 2×2 unitary

$$\pi \in S_n \quad R_\pi \underbrace{|x_1, \dots, x_n\rangle}_{(r^2)^{\otimes n}} = |x_{\pi^{-1}(1)}, \dots, x_{\pi^{-1}(n)}\rangle$$

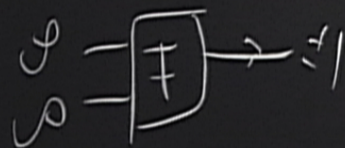
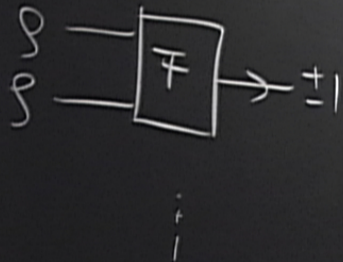
$$R_\pi \mathcal{H}^{\otimes n} = \mathcal{H}^{\otimes n} R_\pi$$

CAUTION

DO NOT TOUCH THE BOARD
IF IT IS NECESSARY TO
PLEASE ASK THE LECTURER
FOR PERMISSION

Two copies $S^{\otimes 2}$

$$F = \sum_{i,j} |ij\rangle\langle jil|$$



$$\begin{aligned} \text{tr } S^{\otimes 2} F &\equiv \text{tr } S = r_1^2 + r_2^2 = \left(\frac{1}{2} + r\right)^2 + \left(\frac{1}{2} - r\right)^2 \\ &= \sum_{i,j} \langle i|S|i\rangle\langle j|S|j\rangle \end{aligned}$$

↓
estimate spectrum,
entropy, ...

$$\begin{aligned} \mathbb{C}^2 \otimes \mathbb{C}^2 &= \underbrace{\text{Sym}^2(\mathbb{C}^2)}_{\text{triplet (1)}} \oplus \underbrace{\wedge^2 \mathbb{C}^2}_{\text{singlet (0)}} \\ F: \quad &+1 \quad \quad -1 \end{aligned}$$

Bell: Total spin $\vec{S}$

$$\vec{J} = J_1 + \dots + J_n$$

Two copies $\mathfrak{g}^{\otimes 2}$

$$F = \sum_{ij} |ij\rangle\langle jil$$

$$\begin{array}{c} \mathfrak{g} \\ \mathfrak{g} \end{array} \xrightarrow{F} \pm 1$$

$$\begin{array}{c} \mathfrak{g} \\ \mathfrak{g} \end{array} \xrightarrow{F} \pm 1$$

$$\text{tr } \mathfrak{g}^{\otimes 2} F \equiv \text{tr } \mathfrak{g} = r_1^2 + r_2^2 = \left(\frac{1}{2} + r\right)^2 + \left(\frac{1}{2} - r\right)^2$$

$$= \sum_{ij} \langle i | \mathfrak{g} | j \rangle \langle j | \mathfrak{g} | i \rangle$$

$$\mathbb{C}^2 \otimes \mathbb{C}^2 = \underbrace{\text{Sym}^2(\mathbb{C}^2)}_{\text{Triplet (1)}} \oplus \underbrace{\wedge^2 \mathbb{C}^2}_{\text{Singlet (0)}}$$

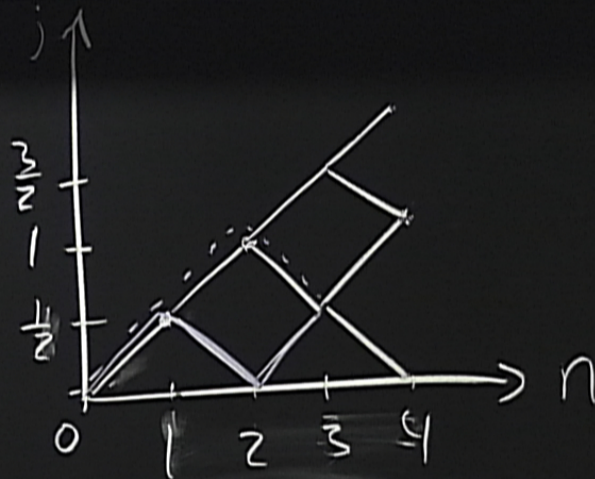
$F: \quad \quad \quad +1 \quad \quad \quad -1$

↓
estimate spectrum,
entropy, ...

$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Bethe: Total spin $\vec{J}$
 $\vec{J} = \vec{J}_1 + \dots + \vec{J}_n$

$SU(2)$ irred repr.
labeled by spin $j \in \frac{\mathbb{Z}_{\geq 0}}{2}$



$$(|01\rangle - |10\rangle)^{\otimes n-2j} \otimes |0\rangle^{\otimes 2j}$$

$\uparrow$
 $\text{Sym}^{2j}(\mathbb{C}^2)$

$$\mathbb{C}^2 = \underbrace{\text{Sym}^2(\mathbb{C}^2)}_{\text{triplet (1)} \atop +1} \oplus \underbrace{\wedge^2 \mathbb{C}^2}_{\text{singlet (0)} \atop -1}$$

estimate spectrum,
entropy, ...

$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = |\Omega\rangle$$

$$(u \otimes u)|\Omega\rangle = -|\Omega\rangle$$

total spin $\frac{3}{2}$

$$\vec{J} = \vec{J}_1 + \dots + \vec{J}_n$$

$SU(2)$ irred repr.

labeled by spin $j \in \frac{\mathbb{Z}_{\geq 0}}{2}$

How does it work?

$$\text{tr} \rho_j = m_j^n \cdot \text{tr} \rho_j(U)$$

spin- j repr. matrix of U

$$\text{SU}(2) \longleftrightarrow \text{SL}(2)$$

$\text{GL}(2)$

$$\rho(U)^{\otimes n} = \lambda^n \cdot \rho(U)^{\otimes n}$$

$$j \otimes \frac{1}{2} = \left[j + \frac{1}{2} \right] \oplus \underbrace{\left[j - \frac{1}{2} \right]}_{\text{unless neg.}}$$

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

$$(\mathbb{C}^2)^{\otimes n} = \underbrace{\frac{1}{2} \otimes \frac{1}{2} \otimes \dots \otimes \frac{1}{2}}_n = (1 \oplus 0) \otimes \underbrace{\frac{1}{2} \otimes \dots \otimes \frac{1}{2}}_{n-2} = \left(\frac{3}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} \right) \otimes \underbrace{\frac{1}{2} \otimes \dots \otimes \frac{1}{2}}_{n-3}$$

$$= \bigoplus_{j=0}^{n/2} \boxed{m_j^n \cdot j} \quad \text{total spin} = j \text{ sector}$$

multiplicity

$P_j = \text{projector onto}$

probability of outcome j

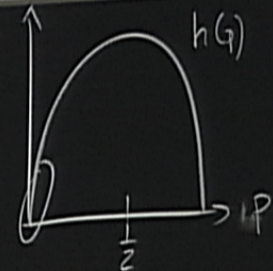
$$= \text{tr } S^{\otimes n} P_j \approx 2^{-n \left| \frac{j}{n} - \frac{1}{2} \right|}$$

$\leq d \left\{ \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \right\}$ & diagram $\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$

$$(\mathbb{C}^2)^{\otimes n} \cong \bigoplus_{j=0}^n \left(j \otimes \underbrace{\mathbb{C}^{m_j^n}}_{\text{irred } \text{SU}(2)\text{-repr.}} \right)$$

irred repr. of $\text{SU}(n)$

$$U^{\otimes n} \cong \bigoplus_j R_j(U) \otimes I_{m_j^n}$$



How does it work?

$$\text{tr}_{\mathcal{H}^{\otimes n}} P_j = \boxed{m_j^n} \cdot \text{tr} R_j(U)$$

$$\leq 2^{n h(\frac{1}{2} + \frac{j}{n})}$$

spin- j repr. matrix of U

$$SU(2) \longleftrightarrow \underline{SL(2)} \\ \text{flow}$$

$$\lambda U^{\otimes n} = \lambda^n \cdot U^{\otimes n}$$