

Title: A Primer on Bulk Reconstruction: Part 2

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URL: <http://pirsa.org/16070043>

Abstract:

# A Primer on Bulk Reconstruction. Part 2

1. HKLL ( $\phi = ?$ )
2. Einstein Eqns ( $G_{\mu\nu} = T_{\mu\nu}$ )

Extrapolate

$$O(x) = \lim_{z \rightarrow 0} z^{-\Delta} \phi(x, z)$$



$$(\nabla^2 - m^2) \phi = 0$$

$$\phi(x, z) = \int dx' K(x, z; x') O(x')$$

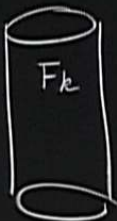
on. Part 2

$$z^{-\Delta} \phi(x, z)$$

$$U(x')$$

mode sums

annihilation  
↓

$$\phi(x, z) = \int dk a_k F_k(x, z) + c.c.$$




on Part 2

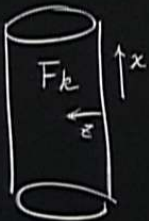
$$z^{-\Delta} \phi(x, z)$$

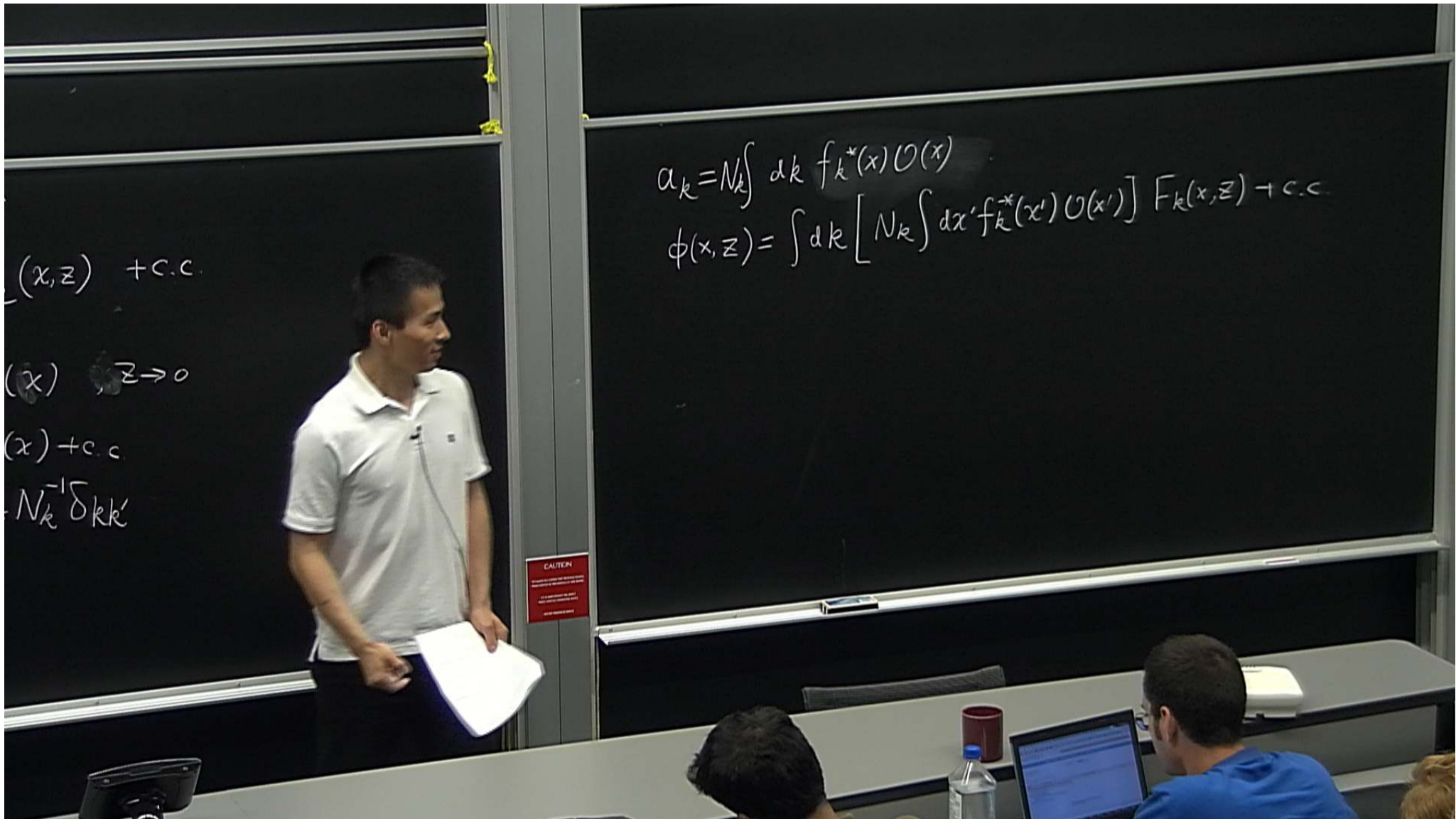
$$\psi(x')$$



mode sums

annihilation  
↓


$$\phi(x, z) = \int dk a_k F_k(x, z) + c.c.$$
$$F_k(x, z) \sim z^\Delta f_k(x) \quad z \rightarrow 0$$
$$\psi(x) = \int dk a_k f_k(x) + c.c.$$
$$\int dx f_k^*(x) f_{k'}(x) = N_k^{-1} \delta_{kk'}$$



$(x, z) + c.c.$   
 $(x) \quad z \rightarrow 0$   
 $(x) + c.c.$   
 $N_k^{-1} \delta_{kk}$

$$a_k = N_k \int dk f_k^*(x) \psi(x)$$
$$\phi(x, z) = \int dk \left[ N_k \int dx' f_k^*(x') \psi(x') \right] F_k(x, z) + c.c.$$

CAUTION

$(x, z) + c.c.$

$(x) \quad z \rightarrow 0$

$(x) + c.c.$

$N_k^{-1} \delta_{kk'}$

$$a_k = N_k \int dx f_k^*(x) \psi(x)$$
$$\phi(x, z) = \int dk \left[ N_k \int dx' f_k^*(x') \psi(x') \right] F_k(x, z) + c.c.$$
$$= \int dx' \left[ \int dk N_k^{-1} F_k(x, z) + c.c. \right] \psi(x')$$

CAUTION

$$\begin{aligned}
 a_k &= N_k \int dx f_k^*(x) \psi(x) \\
 \phi(x, z) &= \int dk \left[ N_k \int dx' f_k^*(x') \psi(x') \right] F_k(x, z) + c.c. \\
 &= \int dx' \underbrace{\left[ \int dk N_k f_k^*(x') F_k(x, z) + c.c. \right]}_{K(x, z; x')} \psi(x')
 \end{aligned}$$

c.c.

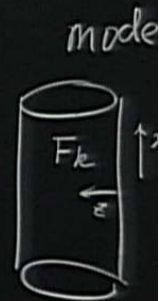
→ 0

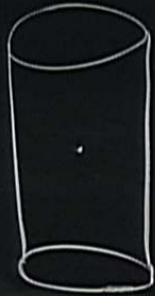
CAUTION



$$ds^2 = \frac{R^2}{\cos^2 \rho} (-dt^2 + d\rho^2 + \sin^2 \rho d\Omega_{d-1}^2)$$

$0 \leq \rho < \frac{\pi}{2}$   
↑ center      ↑ AdS





$$ds^2 = \frac{R^2}{\cos^2 \rho} (-dt^2 + d\rho^2 + \sin^2 \rho d\Omega_{d-1}^2)$$

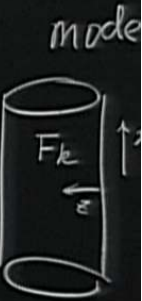
$$0 \leq \rho < \frac{\pi}{2}$$

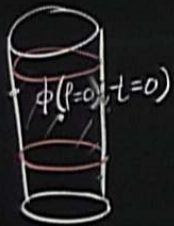
$\uparrow$  center                       $\uparrow$   $\partial \text{AdS}$

$$\phi(\chi, z) = \sum_{n=0}^{\infty} \sum_{lm} a_{nlm} e^{-i(zn + (l+z)t)}$$

$$\times (\sin \rho)^l (\cos \rho)^{\Delta} P_n^{\left(\Delta - \frac{d}{2}, l + \frac{d}{2} - 1\right)}(-\cos^2 \rho) Y_{lm}(\Omega) + \text{c.c.}$$

$z \sim \cos \rho$

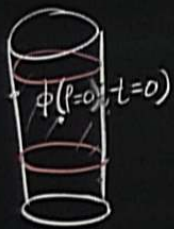




$$K(p=0, t=0; t') = \frac{\Gamma(\Delta - \frac{d}{2} + 1) \Gamma(1 - \frac{d}{2})}{\pi \text{Vol}(S^{d-1}) \Gamma(\Delta - d + 1)} \underbrace{(2 \cos t')^{\Delta - d}}_{\cos t' \geq 0} \theta(\text{spacelike})$$

Even-dim AdS

$$a_k = N_k \int \phi(x, z) =$$



$$K(\rho=0, t=0; t') = \frac{\Gamma(\Delta - \frac{d}{2} + 1) \Gamma(1 - \frac{d}{2})}{\pi \text{Vol}(S^{d-1}) \Gamma(\Delta - d + 1)} (2 \cos t')^{\Delta - d} \theta(\text{spacelike})$$

Even-dim AdS

$\cos t' \geq 0$

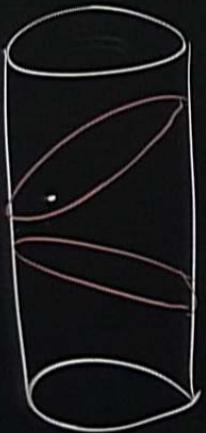
$$0 = \int dx \sum_{k>0} c_k e^{i(\Delta - k)t} \quad \psi(x)$$

$$K \rightarrow K + \sum_{k>0} c_k e^{i(\Delta - k)t}$$

$$a_k = N_k \int$$
$$\phi(x, z) =$$

$d$ -dim AdS

$\Theta$  (spacelike)



AdS-Rindler

$D[A] = \text{domain of dep of } A$

timelike/null

$= \{ P \mid \text{All causal curves passing } P \text{ must intersect } A \}$



CAUTION

$d$ -dim AdS

$\theta$  (spacelike)



AdS-Rindler

$D[A] =$  <sup>fully</sup> domain of dep of A timelike/null  
 $= \{ P \mid \text{All causal curves passing } P \text{ must intersect } A \}$

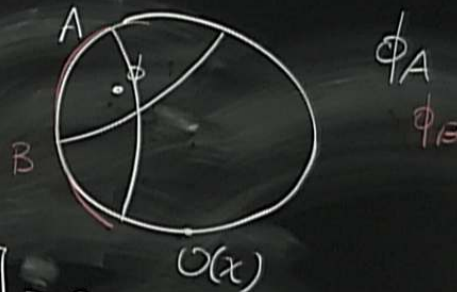


$K(x, z; x')$ ,  $\text{supp}(K) = D[A]$   
 $\forall (x, z) \in W_R[A]$

CAUTION

# A Primer on Bulk Reconstruction. Part 2

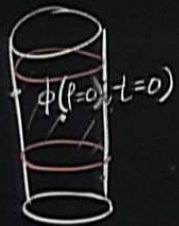
1. HKLL ( $\phi = ?$ )
2. Einstein Eqns ( $G_{\mu\nu} = T_{\mu\nu}$ )



① Commutator:  $[\phi, \mathcal{O}(x)] = 0$

②  $\phi \sim \phi_{AB} \sim \phi_{BC} \sim \phi_{AC}$





$$K(p=0, t=0, t') = \frac{\Gamma(\Delta - \frac{d}{2} + 1) \Gamma(1 - \frac{d}{2})}{\pi \text{Vol}(S^{d-1}) \Gamma(\Delta - d + 1)} (2 \cos t')^{\Delta - d} \theta(\text{spacelike})$$

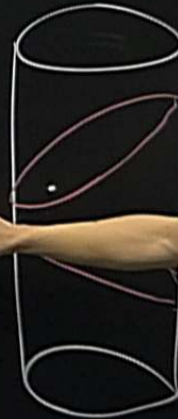
Even-dim AdS

$$0 = \int_{k>0} dk \sum_{k>0} c_k e^{i(\Delta - k)t} \quad \psi(x)$$

$$\cos t' \geq 0$$

$$\phi = \int K \psi$$

$$K \rightarrow K + \sum_{k>0} c_k e^{i(\Delta - k)t}$$



CAUTION

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$$\rho \rightarrow \rho + \delta\rho$$

$$K_\rho \equiv -\log \rho$$

$$\delta S = \delta \langle K_\rho \rangle$$

① Cominator:

②

$$S = \frac{\text{Area}(\text{min})}{4G_N}$$

$$K_\rho = \int (\dots T_{tt})$$

$\downarrow$   
 $g_{tt}$

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$$g_{tt}$$

Commutator:

