

Title: AdS/CFT correspondence

Date: Jul 26, 2016 11:00 AM

URL: <http://pirsa.org/16070042>

Abstract:

$$\mathcal{S}_\beta = e^{-\beta H}$$

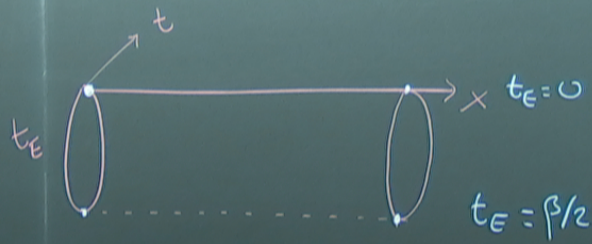
$$|TFD\rangle_\beta = \mathcal{H}_L \otimes \mathcal{H}_R$$

$$\text{Tr}_{R,L}(|TFD\rangle_\beta \langle TFD|) = \mathcal{S}_\beta^{L,R}$$

$$|TFD\rangle_\beta = \sum_i e^{-\beta E_i/2} |E_i^L E_i^R\rangle$$

$$\mathcal{Z}(\beta) = \text{Tr}(\mathcal{S}_\beta)$$

$$= \langle TFD | TFD \rangle_\beta$$



$$S_\beta = e^{-\beta H}$$

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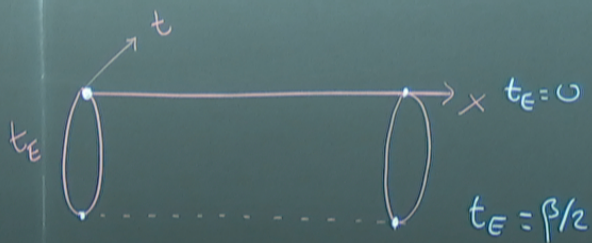
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$$Z(\beta) = \text{Tr}(S_\beta)$$

$$= \langle TFD | TFD \rangle_\beta$$

$$\log Z(\beta) = -\beta F(\beta)$$

$$S(T) = -\text{Tr}(S_\beta \ln S_\beta)$$



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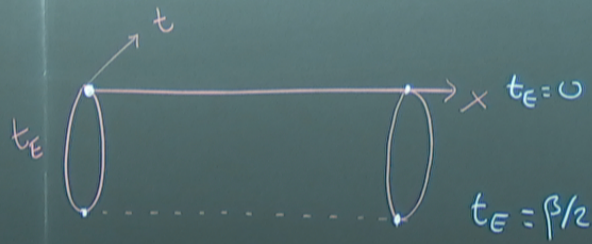
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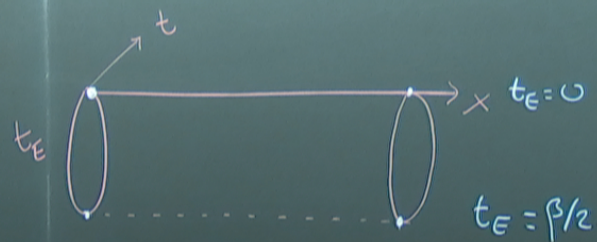
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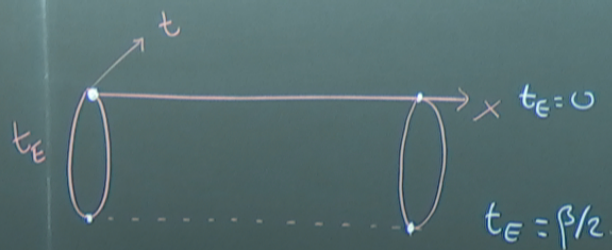
Since $Z(\beta)$ only cares about energy spectrum, truncate bulk dynamics to dynamics of pure gravity

$$Z(\beta) = Z_{\text{string}}(\text{Bdy}) S'_{t_E \sim t_E + \beta}$$

$$= \int [Dg] e^{-S_{\text{string}}}$$

$$\approx e^{-I_{\text{grav}}}$$

$$S_{\text{grav}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R$$



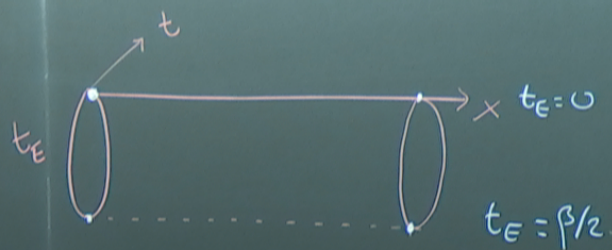
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$$Z_{\text{string}}(\text{Body: } S'_{t_E \sim t_E + \beta})$$

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$$S_{\text{grav}} = \frac{1}{16\pi G_N} \int \sqrt{-g} (R - 2\Lambda)$$

$$\Lambda = -\frac{d(d-1)}{2l_{\text{AdS}}^2}$$



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$$\simeq e^{-I_{\text{grav}}} \quad c_{\text{eff}} \rightarrow \infty$$

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$$c_{\text{eff}} = \frac{l_{\text{AdS}}^{d-1}}{16\pi G_N}$$

i) CFT_d on $\mathbb{R}^{d-1,1}$ @ β
equivalent to ECFT_d on $\mathbb{R}^{d-1} \times S'_\beta$

$$S_{\text{grav}} = \frac{1}{16\pi G_N} \int F_g (R - 2\Lambda)$$

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 $ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[f(z) dt^2 + \frac{dz^2}{g(z)} \right]$

Einstein's eqns $R_{AB} + d g_{AB} = 0$

$C_{\text{eff}} \rightarrow \infty$

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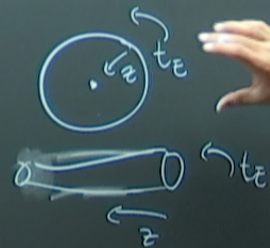
Einstein's eqns $R_{AB} + d g_{AB} = 0$

2 possible topologies:

(t_E, z) plane

- $\mathbb{B}_2$ disc

- $S^1_\beta \times \mathbb{R}$



Solns:

$S^1_\beta \times \mathbb{R}$ $f(z) = g(z) = 1$

$\mathbb{B}_2$ $f(z) = g(z) = 1 - \frac{z^d}{z_+^d}$

EAdS_{d+1}
 $t_E \sim t_E + \beta$

$$S_{\text{grav}} = \frac{1}{16\pi G_N} \int Fg(R - 2\Lambda)$$

$$C_{\text{eff}} = \frac{l_{\text{AdS}}^{d-1}}{16\pi G_N}$$

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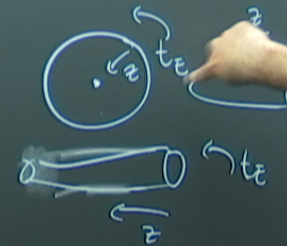
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smoothness: $\beta = \frac{4\pi l_{\text{AdS}}^2}{d z_+}$

EAdS_{d+1}
 $t_E \sim t_E + \beta$

$-2\Lambda)$

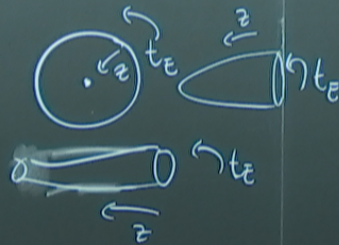
$$\Lambda = -\frac{d(d-1)}{2l_{\text{AdS}}^2}$$

2 possible topologies:

(t_E, z) plane

- B_2 disc

- $S^1_P \times \mathbb{R}$



To understand which soln dominates the saddle-pt we compute on-shell action

$$S_{\text{grav}} = S_{\text{bulk}} + S_{\text{bdy}}$$

$$\frac{1}{8\pi G} \int \sqrt{-g} K + \text{ct. terms}$$

variational principle

converge

@ β

on $\mathbb{R}^{d-1} \times S^1_P$

$$E + \frac{dz^2}{g(z)} + d\vec{x}_{d-1}^2$$

Solns:

$S^1_P \times \mathbb{R}$

$$f(z) = g(z) = 1$$

$$E \text{AdS}_{d+1}$$

$$t_E \sim t_E + \beta$$

B_2

$$f(z) = g(z) = 1 - \frac{z^d}{z_+^d}$$

smoothness:

$$\beta = \frac{4\pi l_{\text{AdS}}^2}{dz_+}$$

$$\rightarrow S(T) = T^{d-1} V_x$$

$-2\Lambda)$

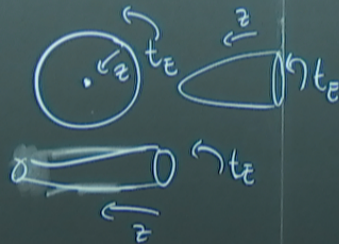
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$$E + \frac{dz^2}{g(z)} + d\vec{x}_{d-1}^2$$

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$$t_E \sim t_E + \beta$$

$\mathbb{B}_2$

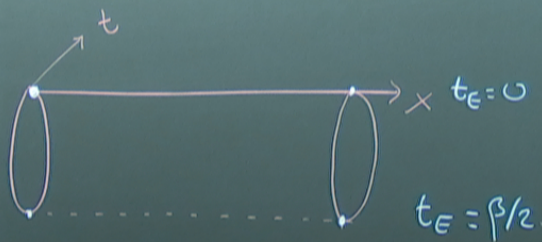
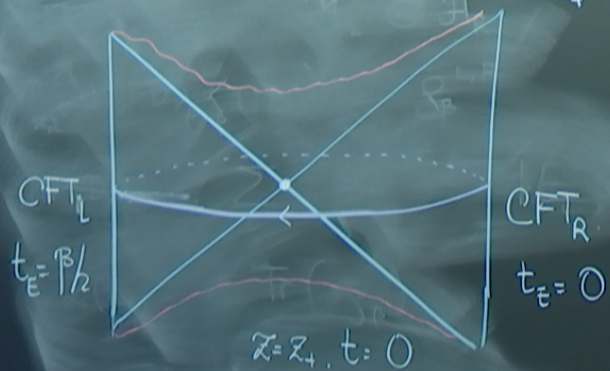
$$f(z) = g(z) = 1 - \frac{z^d}{z_+^d}$$

smoothness:

$$\beta = \frac{4\pi l_{\text{AdS}}^2}{dz_+}$$

$$\rightarrow S(T) = 4\pi C_{\text{eff}} T^{d-1} V_x$$

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



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$$\simeq e^{-I_{\text{grav}}}$$

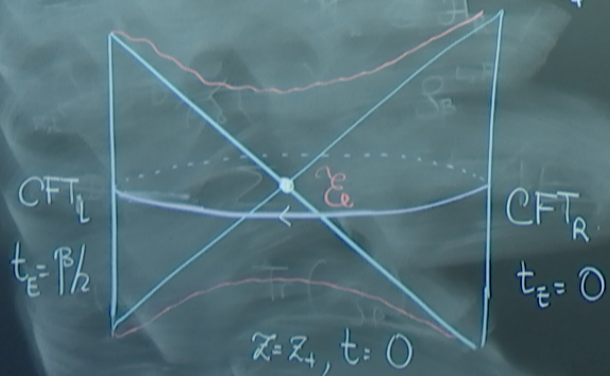
$$C_{\text{eff}} \rightarrow \infty$$

$$S_{\text{grav}} = \frac{1}{16\pi G}$$

$$C_{\text{eff}} =$$

i) CFT_d
equivalent

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$S(T) = S_{\text{Bekenstein-Hawking}} = \frac{\text{Area}(\mathcal{E}_E)}{4 G_N}$$

$\mathcal{E}_E$: bifurcation surface of black hole

$$\left(\frac{\partial}{\partial t} \right)^A = 0 \text{ on } \mathcal{E}_E$$

codim-2 event horizon spacetime $\mathcal{M}_{d+1}$

$$S_{\text{grav}} = \frac{1}{16\pi G_N}$$

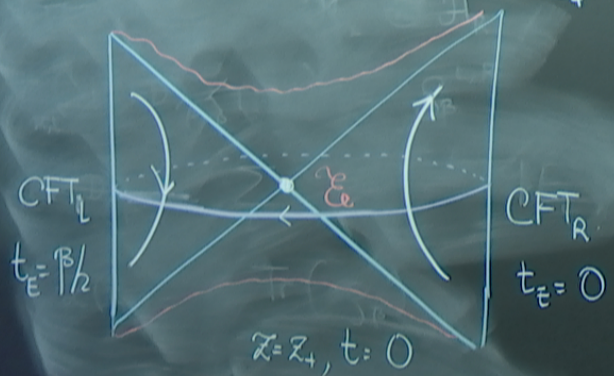
$$C_{\text{eff}} =$$

i) CFT_d

equivalent to $ds^2 = \frac{l_{\text{AdS}}^2}{z^2}$

Einstein's eqns

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$z = z_+, t = 0$$

$$\begin{aligned} (1) &= S_{\text{Bekenstein-Hawking}} \\ &= \frac{\text{Area}(\mathcal{E}_E)}{4 G_N} \end{aligned}$$

$\mathcal{E}_E$: bifurcation surface of black hole

$$\left(\frac{\partial}{\partial t} \right)^A = 0 \text{ on } \mathcal{E}_E$$

codim-2 section spacetime $\mathcal{M}_{d+1}$

$$S_{\text{grav}} = \frac{1}{16\pi G_N}$$

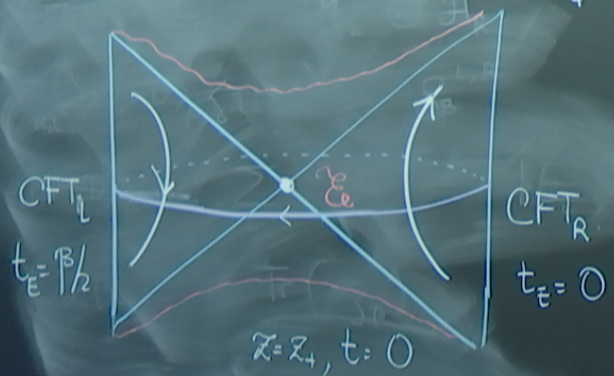
$$C_{\text{eff}} =$$

i) CFT_d

$$\text{equivalent to } ds^2 = \frac{l_{\text{AdS}}^2}{z^2}$$

Einstein's eqns

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$S(T) = S_{\text{Bekenstein-Hawking}} = \frac{\text{Area}(\mathcal{C}_E)}{4 G_N}$$

$\mathcal{C}_E$: bifurcation surface of black hole

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codim-2 in spacetime $\mathcal{M}_{d+1}$

$$S_{\text{grav}} = \frac{1}{16\pi G_N}$$

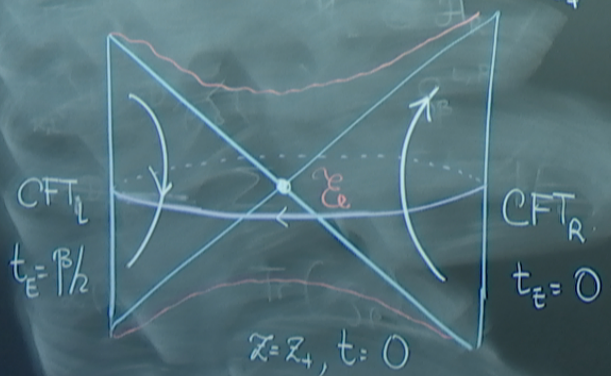
$$C_{\text{eff}} =$$

i) CFT_d

equivalent to $ds^2 = \frac{l_{\text{AdS}}^2}{z^2}$

Einstein's eqns

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$S(T) = S_{\text{Bekenstein-Hawking}} = \frac{\text{Area}(\mathcal{E}_e)}{4 G_N} \ln$$

$\mathcal{E}_e$: bifurcation surface of black hole

$$\left(\frac{\partial}{\partial t} \right)^A = 0 \text{ on } \mathcal{E}_e$$

codim-2 surface in spacetime $\mathcal{M}_{d+1}$

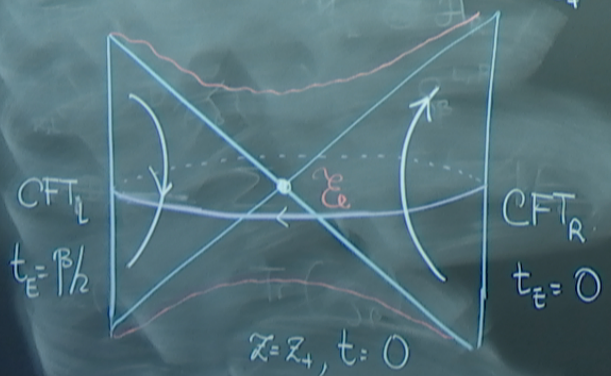
Sgrav

Ceff

i) CFT_d equivalent $ds^2 =$

Einstein's e

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$S(T) = S_{\text{Bekenstein-Hawking}} = \frac{\text{Area}(\mathcal{E}_E)}{4 G_N}$$

$\mathcal{E}_E$: bifurcation surface of black hole.

$$\left(\frac{\partial}{\partial t} \right)^A = 0 \text{ on } \mathcal{E}_E$$

codim-2 surface in spacetime $\mathcal{M}_{d+1}$

$$|TFD\rangle = \sum e^{-\beta E_L/2} |E_L^L E_L^R\rangle$$

$$(H_R \otimes 1_L - 1_R \otimes H_L) |TFD\rangle = 0$$

Sgrav

Ceff

CFT_d

valent

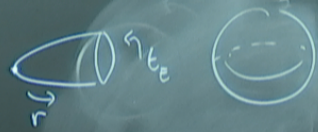
$$= \frac{1}{16\pi G_N} \int F_g (R - 2\Lambda)$$

$$\Lambda = -\frac{d(d-1)}{2\ell_{\text{AdS}}^2}$$

2 possible topologies
(t_E, r, Ω) plane

$$\mathbb{B}_2 \times S^{d-1}$$

$$S^1_\beta \times \mathbb{B}_d$$



To understand which dominates the saddle we compute on-shell

$$S_{\text{grav}} = S_{\text{bulk}} + S_{\text{bdy}}$$

$$\frac{1}{8\pi G} \int \sqrt{-g} R$$

variation

$$\rightarrow S(T) = 4\pi C_{\text{eff}} T^{d-1} V_x$$

$$p = \frac{\ell_{\text{AdS}}^{d-1}}{16\pi G_N}$$

$$\text{on } S^{d-1} \times S^1_\beta @ \beta$$

$$\text{to } \text{ECFT}_d \text{ on } S^{d-1} \times S^1_\beta$$

$$\left[+f(r) dt_E^2 + \frac{dr^2}{g(r)} + r^2 d\Omega^2 \right]$$

eqns $R_{AB} + d g_{AB} = 0$

$$f(r) = g(r) = 1 + \frac{r^2}{\ell_{\text{AdS}}^2}$$

$$f(r) = g(r) = 1 - \frac{z^d}{z_+^d}$$

$$B = \frac{4\pi \ell_{\text{AdS}}^2}{dz_+}$$



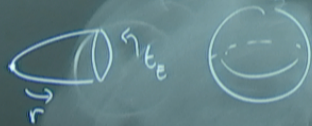
1)

$$\Lambda = -\frac{d(d-1)}{2l_{\text{AdS}}^2}$$

2 possible topologies:

(t, r, Ω) plane

- $\mathbb{B}_2 \times S^{d-1}$



- $S^1_{\beta} \times \mathbb{B}_d$

a) β
 $S^{d-1} \times S^1_{\beta}$
 $\left[\frac{dr^2}{g(r)} + r^2 d\Omega_{d-1}^2 \right]$

Sols:

$S^1_{\beta} \times \mathbb{B}_d$

$f(r) = g(r) = 1 + \frac{r^2}{l_{\text{AdS}}^2}$

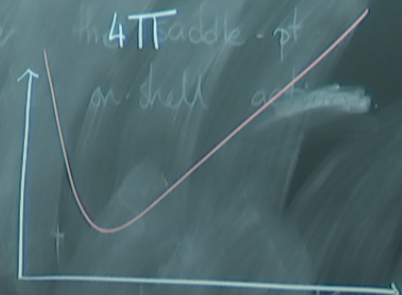
global $E\text{AdS}_{d+1}$

$\mathbb{B}_2 \times S^{d-1}$

$f(r) = g(r) = 1 + \frac{r^2}{l_{\text{AdS}}^2} - \frac{r_+^{d-2}}{r^{d-2}} \left(1 + \frac{r_+^2}{l_{\text{AdS}}^2} \right)$

global Schw AdS_{d+1}

$T = r \frac{f'(r_+)}{4\pi}$



$\frac{r_+}{l_{\text{AdS}}}$

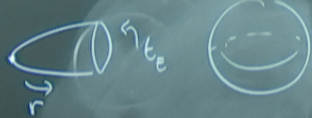
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$$\Lambda = -\frac{d(d-1)}{2l_{\text{AdS}}^2}$$

2 possible topologies:

(t_e, r, Ω) plane

- $\mathbb{B}_2 \times S^{d-1}$



- $S^1_\beta \times \mathbb{B}_d$

2) β
 $S^{d-1} \times S^1_\beta$
 $\left[\frac{dr^2}{g(r)} + r^2 d\Omega_{d-1}^2 \right]$

Solns:

$S^1_\beta \times \mathbb{B}_d$

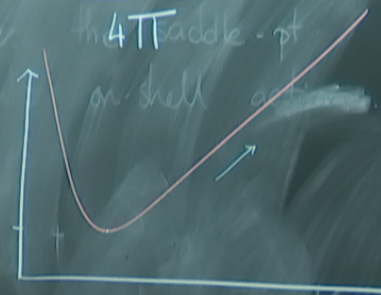
$f(r) = g(r) = 1 + \frac{r^2}{l_{\text{AdS}}^2}$ global EAAdS_{d+1}

$\mathbb{B}_2 \times S^{d-1}$

$f(r) = g(r) = 1 + \frac{r^2}{l_{\text{AdS}}^2} - \frac{r_+^{d-2}}{r^{d-2}} \left(1 + \frac{r_+^2}{l_{\text{AdS}}^2} \right)$

global Schw AdS_{d+1}

$T = r_+ \frac{f'(r_+)}{4\pi}$ soln



→ only saddle for $T < T_c$

$S(T) = 0$

$S(T) \propto \text{Ceff} r_+^{d-1}(T)$

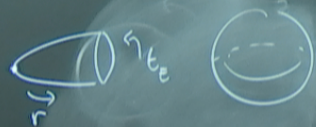
(d-1)
 $\frac{l_{\text{AdS}}^2}{r_+^2}$

2 possible topologies:

(t_e, r, Ω) plane

- $\mathbb{B}_2 \times S^{d-1}$

- $S^1_{\beta} \times \mathbb{B}_d$



S^1_{β}
 Ω_{d-1}

Solns:

$S^1_{\beta} \times \mathbb{B}_d$

$f(r) = g(r) = 1 + \frac{r^2}{l_{\text{AdS}}^2}$

global
 $E\text{AdS}_{d+1}$

→

only saddle for $T < T_c$

$S(T) = \mathcal{O}(1) + \mathcal{O}(1)$

$\mathbb{B}_2 \times S^{d-1}$

$f(r) = g(r) = 1 + \frac{r^2}{l_{\text{AdS}}^2} - \frac{r_+^{d-2}}{r^{d-2}} \left(1 + \frac{r_+^2}{l_{\text{AdS}}^2} \right)$

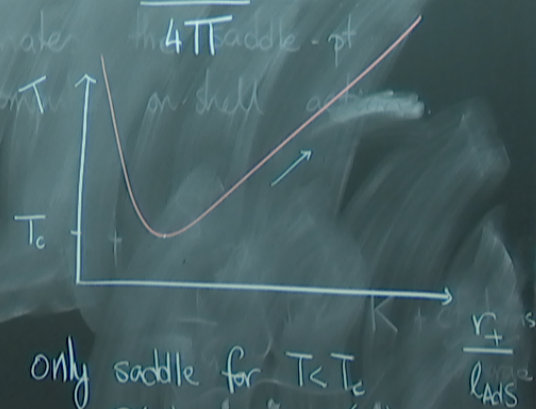
global Schw AdS_{d+1}

→

$S(T) \propto \text{Ceff } r_+^{d-1}(T)$

$T = r_{\text{star}} \frac{f'(r_+)}{4\pi}$

minimizes the 4π saddle-pt
on shell action



$$\left. + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right\}_{t_E}$$

$$S_\beta = C_{\text{eff}} S^{\text{saddle}} + S^{\text{quantum}} + \mathcal{O}\left(\frac{1}{C_{\text{eff}}}\right)$$

CFT_R

$t_E = 0$

Hawking

$\frac{dS}{d\ln}$

$$S_{\text{grav}} = \frac{1}{16\pi G_N} \int F_g (R - 2\Lambda)$$

$$\Lambda = -\frac{d(d-1)}{2l_{\text{AdS}}^2}$$

$$C_{\text{eff}} = \frac{l_{\text{AdS}}^{d-1}}{16\pi G_N}$$

i) CFT_d on $S^{d-1} \times S^1_\beta$ @ β
equivalent to ECFT_d on $S^{d-1} \times S^1_\beta$

$$ds^2 = \left[+f(r) dt_E^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_{d-1}^2 \right]$$

$$\text{Einstein's eqns } R_{AB} + d g_{AB} = 0$$

$$\left. + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$

$$S_{\beta} = C_{\text{eff}} S^{\text{saddle}} + S^{\text{quantum}} + \mathcal{O}\left(\frac{1}{C_{\text{eff}}}\right)$$

S^{quantum} = 1-loop fluctuations about saddle.

CFT_R
 $t_E = 0$

hawking

$$S_{\text{grav}} = \frac{1}{16\pi G_N} \int F_g (R - 2\Lambda)$$

$$\Lambda = -\frac{d(d-1)}{2l_{\text{AdS}}^2}$$

$$C_{\text{eff}} = \frac{l_{\text{AdS}}^{d-1}}{16\pi G_N}$$

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CFT_R

$t_E = 0$

Hawking

$\frac{1}{\epsilon}$

$\frac{1}{N}$

$$S_{\text{grav}} = \frac{1}{16\pi G_N} \int F_g (R - 2\Lambda)$$

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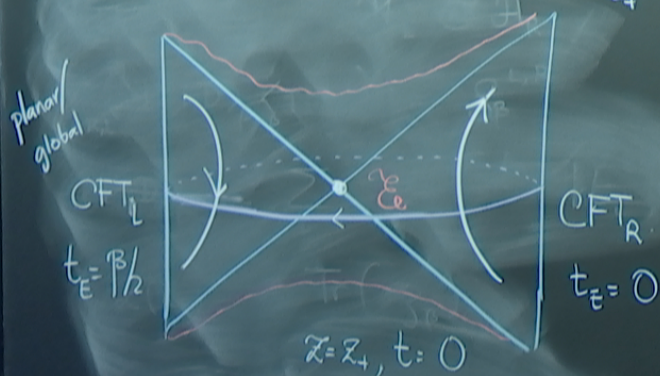
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$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$S(T) = S_{\text{Bekenstein-Hawking}} = \frac{\text{Area}(\gamma_{\xi_0})}{4 G_N}$$

$$S_{\beta} = C_{\text{eff}} S_{\text{saddle}} + S_{\text{quantum}} + \mathcal{O}\left(\frac{1}{C_{\text{eff}}}\right)$$

S_{quantum} = 1-loop fluctuations about saddle.

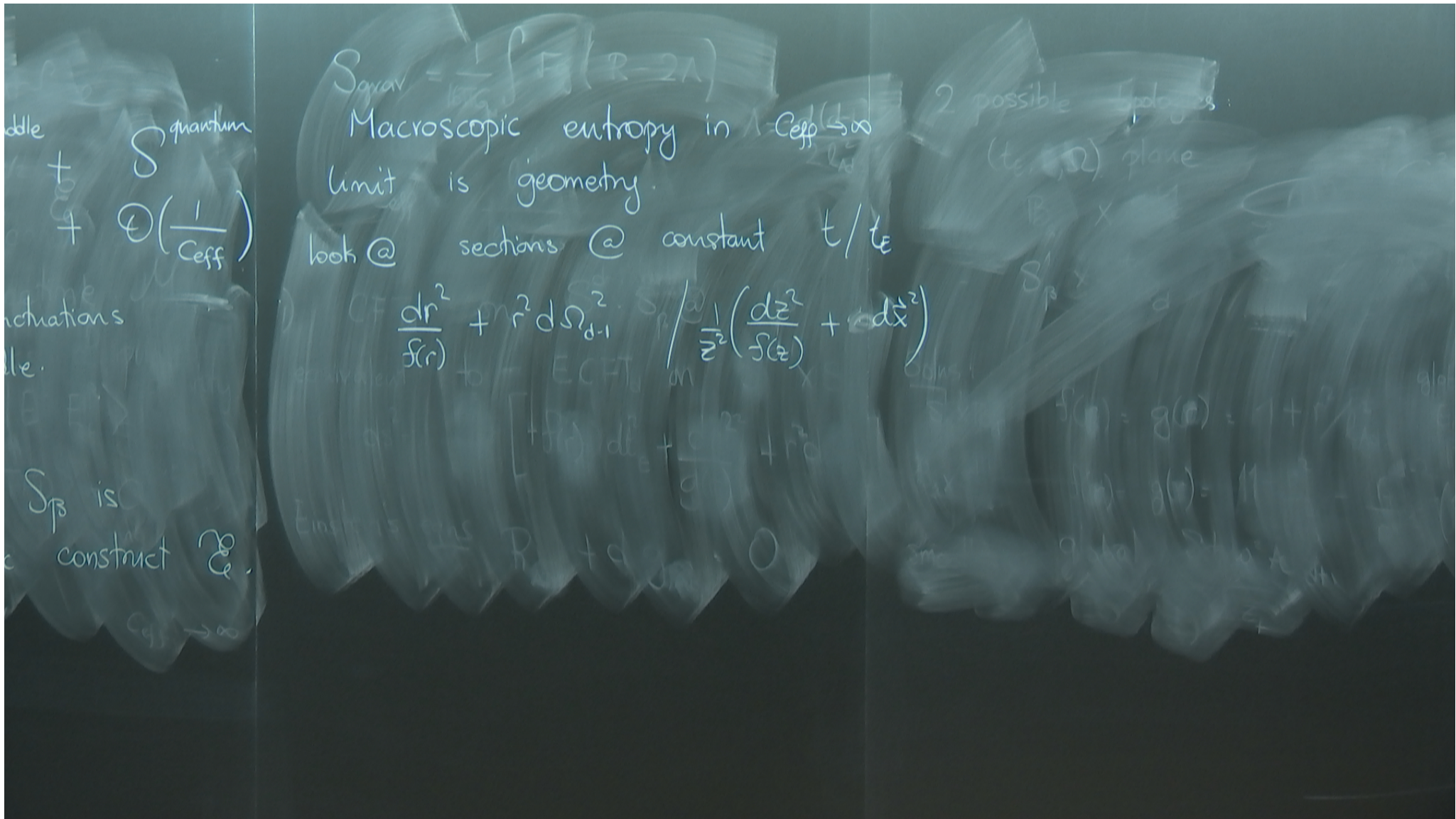
$S_{\beta} = -\text{Tr } S_{\beta} \ln S_{\beta}$
dominant $\mathcal{O}(C_{\text{eff}})$ to S_{β} is contained in a geometric construct γ_{ξ_0} .

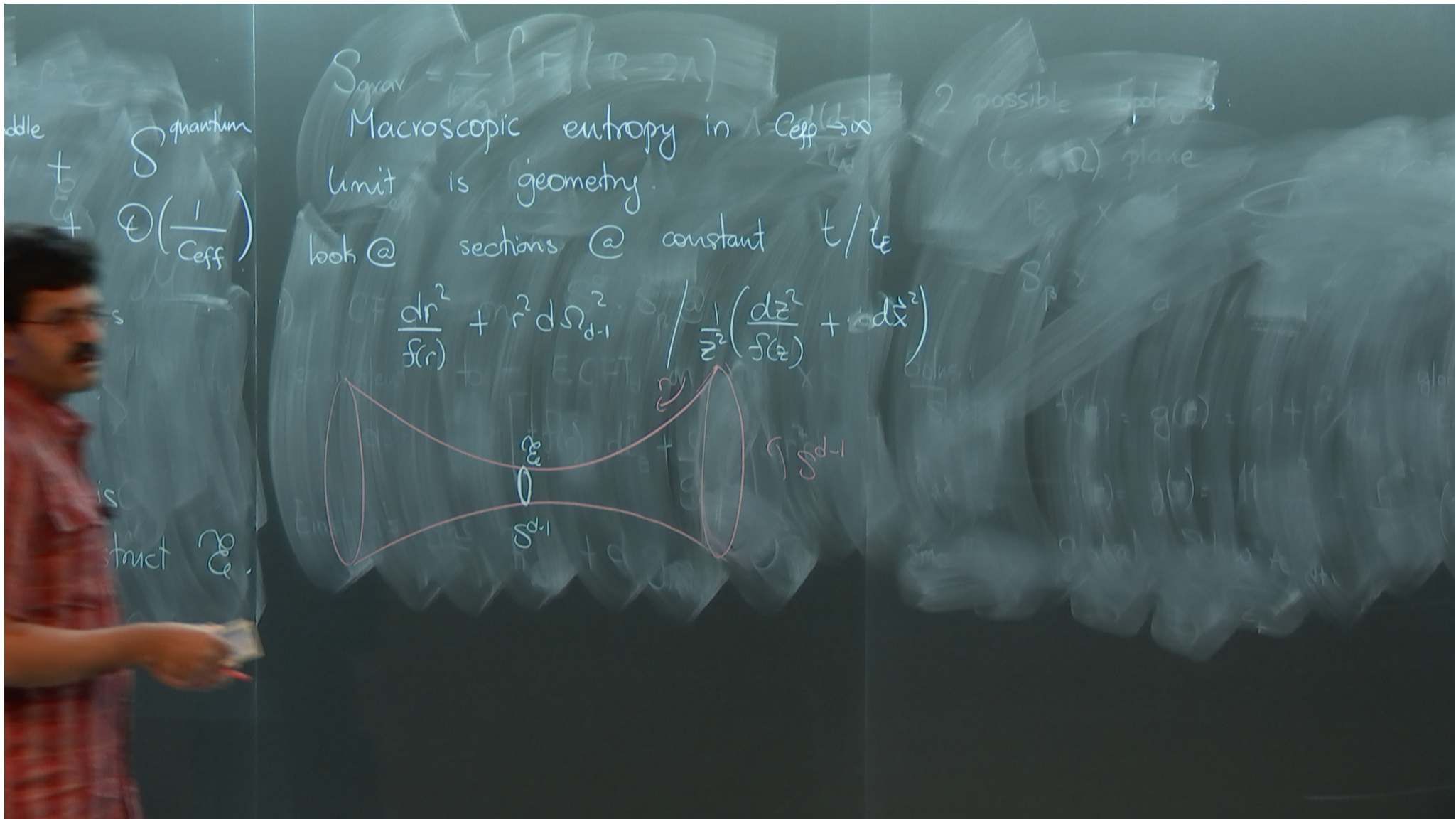
$$S_{\text{grav}} = \frac{1}{16\pi G_N}$$

$$C_{\text{eff}} = \frac{1}{16\pi G_N}$$

i) CFT_d equivalent to $ds^2 =$

Einstein's eqns





$$S_{\text{quantum}} + S_{\text{eff}}$$

Macroscopic entropy in $\hbar \rightarrow 0$ limit is geometry.

$$\frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \Big/ \frac{1}{z^2} \left(\frac{dz^2}{f(z)} + d\tilde{x}^2 \right)$$

S_{quantum}
 $+ \mathcal{O}\left(\frac{1}{C_{\text{eff}}}\right)$

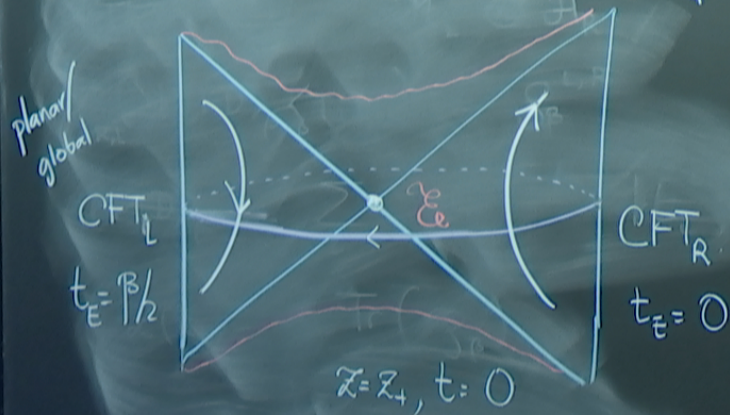
S_{B} is
 construct $\mathcal{E}_e$

$S_{\text{grav}} = -\frac{1}{16\pi G} \int F(R-2\Lambda)$
 Macroscopic entropy in $C_{\text{eff}} \rightarrow \infty$
 limit is geometry.
 look @ sections @ constant t/t_e

$\frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \quad \bigg/ \quad \frac{1}{z^2} \left(\frac{dz^2}{f(z)} + d\vec{x}^2 \right)$

2 possible topologies: codim-2
 $(\mathcal{E}, \Omega)$ place extremal surface.

$$ds^2 = \frac{l_{\text{AdS}}^2}{z^2} \left[- \left(1 - \frac{z^d}{z_+^d} \right) dt^2 + \frac{dz^2}{1 - \frac{z^d}{z_+^d}} + d\vec{x}^2 \right]_{t_E}$$



$$S(T) = S_{\text{Bekenstein-Hawking}}$$

$$= \frac{\text{Area}(\mathcal{E}_0)}{4 G_N} \ln$$

$$S_B = C_{\text{eff}} S_{\text{saddle}} + S_{\text{quantum}} + \mathcal{O}\left(\frac{1}{C_{\text{eff}}}\right)$$

$S_{\text{quantum}} =$ 1-loop fluctuations about saddle.

$S_B = -\text{Tr } S_B \ln S_B$

dominant $\mathcal{O}(C_{\text{eff}})$ to S_B is contained in a geometric construct $\mathcal{E}_0$.

$\frac{1}{4\pi G} \int F(R-2\Lambda)$
 scopic entropy in $A_{\text{eff}} \rightarrow \infty$
 is geometry.

sections @ constant t/t_E
 $+ r^2 d\Omega_{d-1}^2$
 $\frac{1}{z^2} \left(\frac{dz^2}{f(z)} + dx^2 \right)$
 $\mathcal{E}$
 S^{d-1}

2 possible topologies:
 $(\mathcal{E}, \Omega)$ extremal surface.

$\mathcal{E}$ bends in (t, r) plane
 if we send a set (congruence) of
 light-rays away from $\mathcal{E}$
 they locally neither expand nor
 contract.
 $\Theta^\pm = 0$

$T = r \sigma$
 T_c
 only say S

2 possible topologies:
 $\mathcal{E}$ extremal surface. $\text{codim} = 2$

(t, r) plane
 $\mathcal{E}$ bends in (t, r) plane
 if we send a set (congruence) of light-rays away from $\mathcal{E}$
 they locally neither expand nor contract.
 $\Theta^\pm = 0$

$\mathcal{E}$ co-dim -2
 2 normals $n_\pm^A$
 $n_+ \cdot n_+ = 0$
 $n_- \cdot n_- = 0$
 $n_+ \cdot n_- = -1$

$$K_{AB}^\pm = 2 \nabla_{(A} n_{B)}^\pm$$

$$\Theta^\pm = K_{AA}^\pm$$

$$d\vec{x}^2 \Big|_{t_E}$$

Take CFT in some global quantum state $|\mathcal{G}\rangle$.

On CFT_d pick some region $\mathcal{A}$ on a Cauchy slice Σ

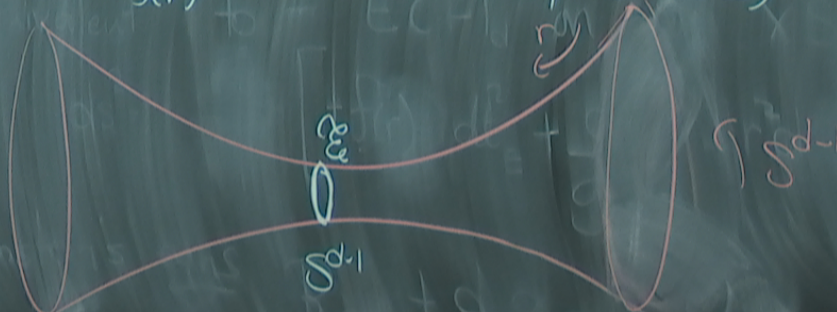


$$S_{\text{grav}} = \frac{1}{16\pi G} \int \sqrt{-g} (R - 2\Lambda)$$

Macroscopic entropy in CFT_d limit is geometry.

look @ sections @ constant t

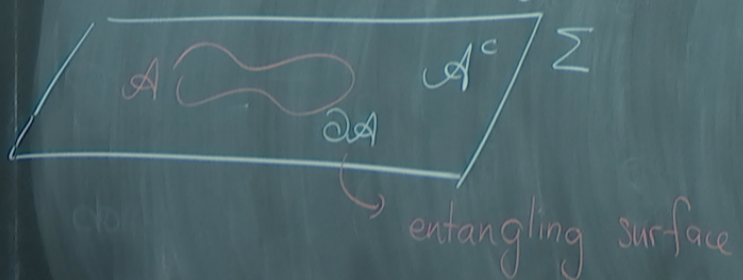
$$\frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \quad \Big/ \quad \frac{1}{z^2} \left(\frac{dz^2}{f(z)} + \right.$$



$$d\vec{x}^2 \Big|_{t_E}$$

Take CFT in some global quantum state ρ .

On CFT_d pick some region A on a Cauchy slice Σ



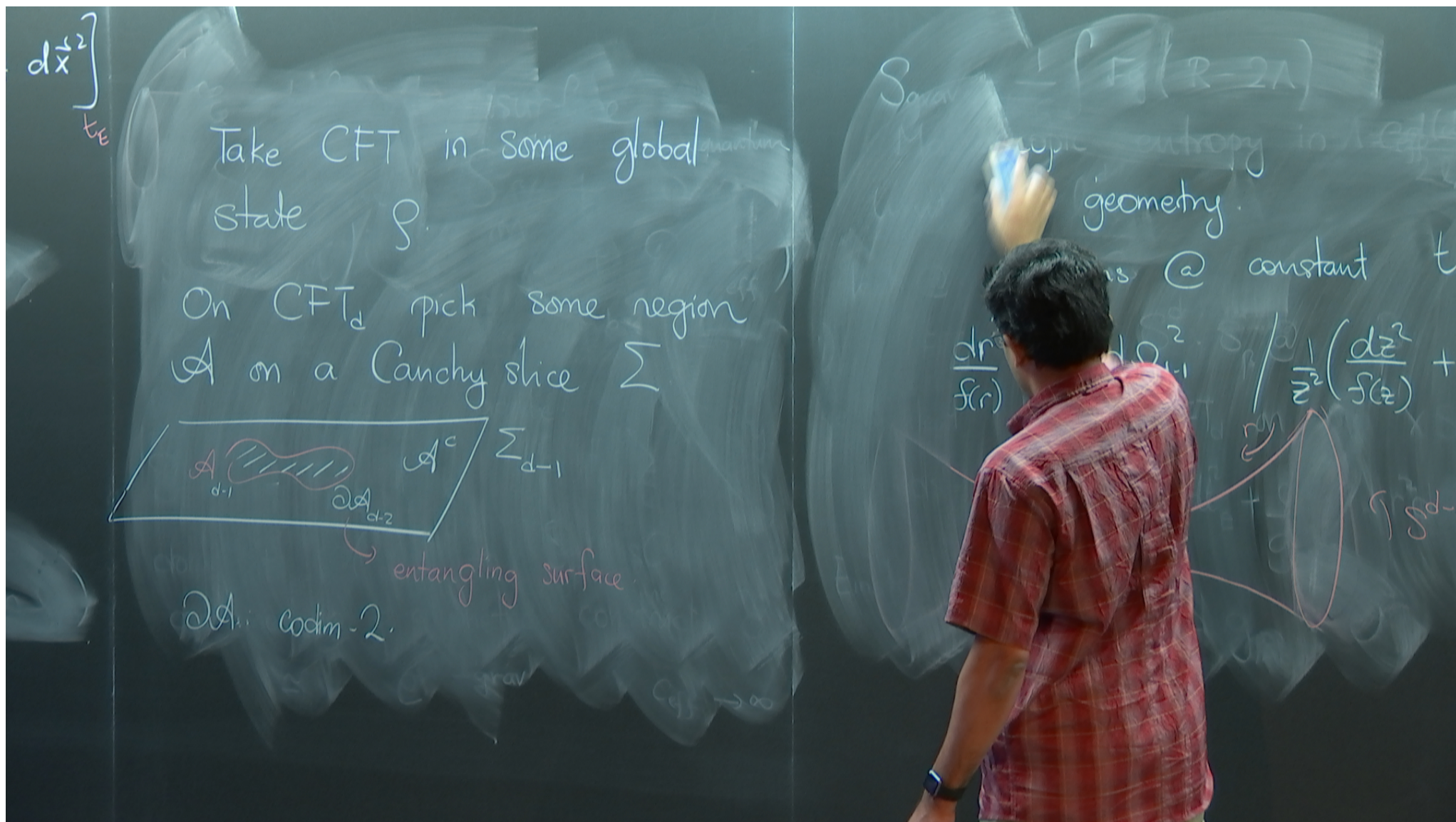
$$S_{\text{grav}} = \frac{1}{16\pi G} \int F(R - 2\Lambda)$$

Macroscopic entropy in C_{eff} limit is geometry.

look @ sections @ constant t

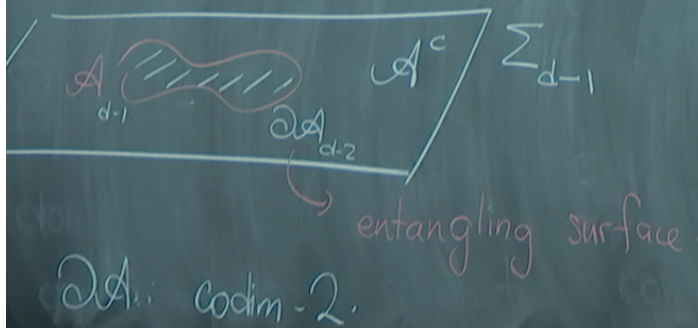
$$\frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \Big/ \frac{1}{z^2} \left(\frac{dz^2}{f(z)} + \dots \right)$$

A diagram illustrating a spacetime geometry. It shows a central vertical axis with a singularity at the bottom labeled S^{d-1} . Two vertical ellipses represent sections at constant time t . The region between them is labeled $\mathcal{E}$. The right ellipse is labeled S^{d-1} .



Take CFT in some global
state ρ .

On CFT_d pick some region
 A on a Cauchy slice Σ



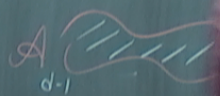
∂A : codim-2.

$$S_A = -\text{Tr}_{\mathcal{H}_A}(\rho_A \log \rho_A)$$

$$S_A = -\text{Tr}_{\mathcal{H}_A}(\rho_A \log \rho_A)$$

Take CFT in some global
state ρ .

On CFT_d some region
 A on a slice Σ



surface

∂A : codim

$C_{\text{FT}} \rightarrow \infty$

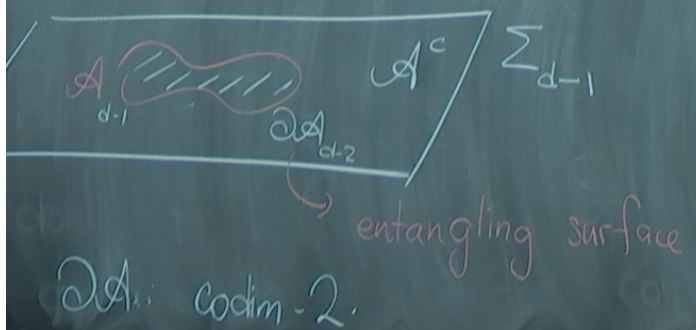
$$S_A = -\text{Tr}_{\mathcal{H}_A}(\rho_A)$$

$$S_A = -\text{Tr}_{\mathcal{H}_A}(\rho_A \log \rho_A)$$

Ryu-Takayanagi '06 (HRT)

Take CFT in some global
state ρ .

On CFT_d pick some region
 A on a Cauchy slice Σ



$$S_A = -\text{Tr}_{A^c}(\rho)$$

$$S_A = -\text{Tr}_A(\rho_A \log \rho_A)$$

Ryu-Takayanagi '06 (HRT)

$$S_A = \frac{\text{Area}(\mathcal{E}_A)}{4G_N}$$

$$= \frac{4\pi c_{\text{eff}} \text{Area}(\mathcal{E}_A)}{4G_N}$$

$$S_A = \text{Tr}_{\mathcal{A}^c}(\rho)$$

$$S_A = - \text{Tr}_A(\rho_A \log \rho_A)$$

Ryu-Takayanagi '06 (HRT)

$$S_A = \frac{\text{Area}(\mathcal{E}_A)}{4G_N}$$

$$= \frac{4\pi c_{\text{eff}} \text{Area}(\mathcal{E}_A)}{4G_N}$$

$\mathcal{E}_A$ extremal surface anchored on ∂A & homologous to it

$\exists \mathcal{R}_A$ (causal diamond)

$\mathcal{E}_A$ lies between ∂A & $\partial \mathcal{R}_A$

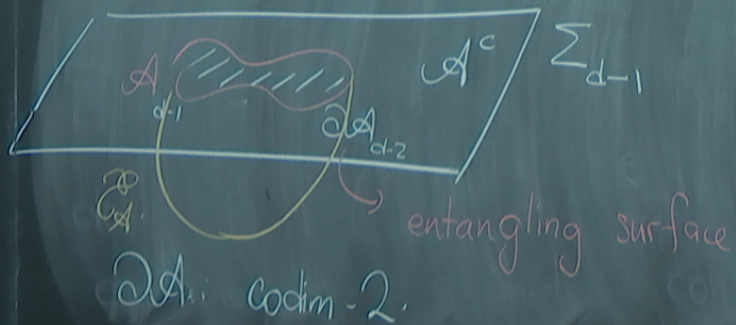
we see $\mathcal{E}_A$ on $\partial \mathcal{R}_A$

hrt - ray

bc

Take CFT in some global state ρ .

On CFT_d pick some region A on a Cauchy slice Σ



$$S_A = -\text{Tr}_{A^c}(\rho)$$

$$S_A = -\text{Tr}_A(\rho \log \rho)$$

Ryu-Takayanagi '06 (HRT)

$$S_A = \frac{\text{Area}(\mathcal{E}_A)}{4G_N}$$

$$= \frac{4\pi c_{\text{eff}} \text{Area}(\mathcal{E}_A)}{4G_N}$$

$$S_A \log S_A$$

(HRT)

$$(\mathcal{E}_A)$$

$$4G_N$$

$$\text{ff Area}(\mathcal{E}_A) +$$

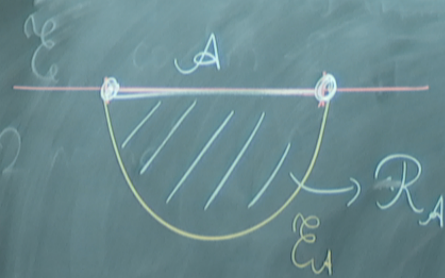
$\mathcal{E}_A$ extremal surface anchored
on ∂A & homologous to it

$\exists \mathcal{R}_A$ (spacelike)

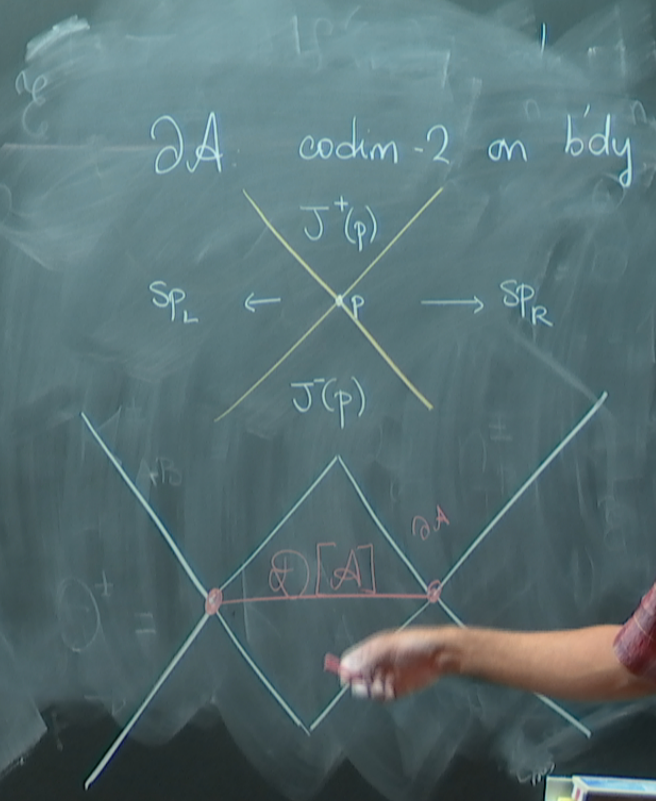
$$\partial \mathcal{R}_A = \mathcal{E}_A \cup A$$

If we send a surface (containing) light-rays away from

quantum mechanically, neither expand nor contract



γ_{E_A} extremal surface anchored
 on ∂A & homologous to it
 $\exists R_A$ (spacelike)
 $\partial R_A = \gamma_{E_A} \cup A$
 If we send a set (containing)
 light-rays away from
 quantum entanglement entropy
 $\text{Area}(\gamma_{E_A}) + S_{\text{quantum}}$



$$d^2 = \left[\frac{dt^2}{1 - \frac{2M}{r}} - dr^2 - r^2 d\Omega_{d-2}^2 \right]$$

Corresponding bulk decomposition

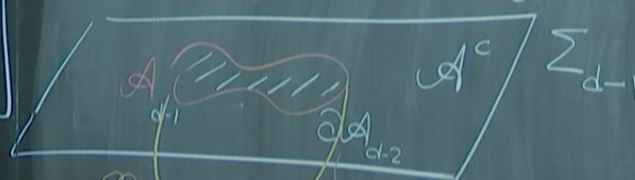
$\mathcal{E}_A$ co. dim - 2

$$\mathcal{M}_{d+1} = \tilde{\mathcal{I}}^+[\mathcal{E}_A] \cup \tilde{\mathcal{I}}^-[\mathcal{E}_A] \cup \underbrace{\mathcal{D}[\mathcal{R}_A] \cup \mathcal{D}[\mathcal{R}_{A^c}]}_{\text{entanglement wedge of } A}$$

entanglement wedge of A

Take CFT in some global state ρ .

On CFT_d pick some region A on a Cauchy slice Σ



∂A : codim - 2.
entangling surface.

$$S_A = -\text{Tr}_{\mathcal{A}^c}(\rho)$$

$$S_A = -\text{Tr}_A(\rho_A \log \rho_A)$$

Ryu-Takayanagi '06 (HRT)

$$S_A = \inf \frac{\text{Area}(\mathcal{E}_A)}{4G_N} +$$

$$= \inf 4\pi c_{\text{eff}} \text{Area}(\mathcal{E}_A) +$$

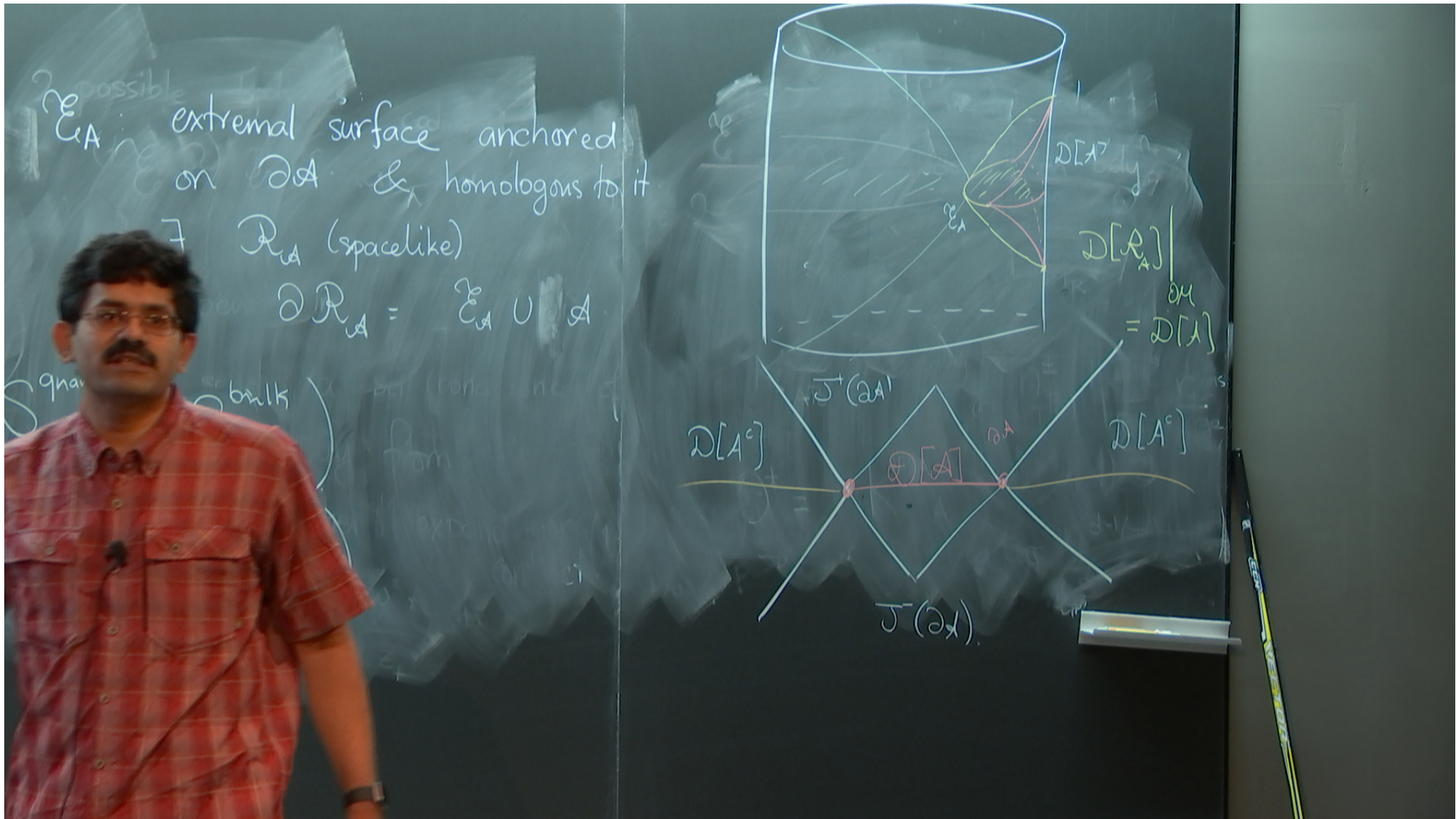
$\mathcal{E}_A$ extremal surface anchored on ∂A & homologous to it

$\exists \mathcal{R}_A$ (spacelike)

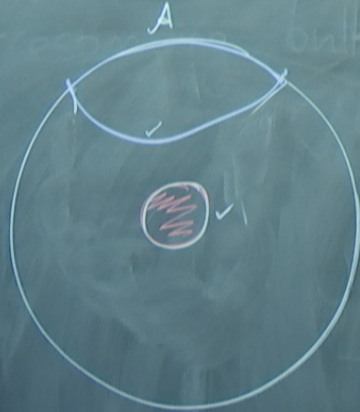
$$\partial \mathcal{R}_A = \mathcal{E}_A \cup A$$

$$S_{\text{quantum}}^{\text{bulk}}(\rho_{\mathcal{R}_A})$$

$$S_{\text{quantum}}^{\text{bulk}}(\rho_{\mathcal{R}_A})$$



$$ds^2 = \frac{1}{z^2} \left[-dt^2 + dx^2 + dz^2 \right]$$



A^c

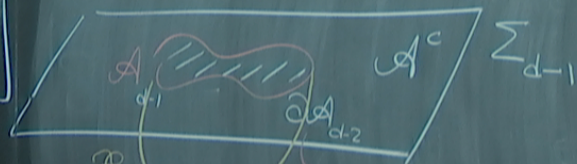
$$S_{A^c} - S_A = S_g$$

Araki-Lieb

$$|S_{A^c} - S_A| \leq S_g$$

Take CFT in some global state g .

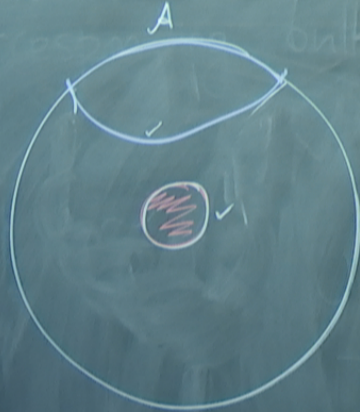
On CFT_d pick some region A on a Cauchy slice Σ



entangling surface

∂A : codim-2.

$$ds^2 = \frac{1}{z^2} \left[\left(\frac{dz}{z} \right)^2 + d\vec{x}^2 \right]$$



A^c

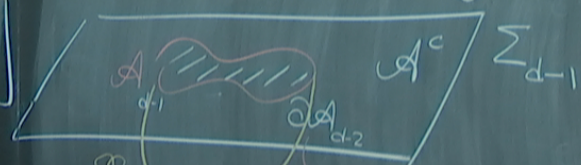
$$S_{A^c} - S_A = S_g$$

Araki-Lieb

$$|S_{A^c} - S_A| \leq S_g$$

Take CFT in some global manifold
state g .

On CFT_d pick some region
 A on a Cauchy slice Σ



∂A_{d-1}

entangling surface

∂A , codim-2.