Title: Quantum Shannon Theory

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URL: http://pirsa.org/16070041

Abstract:

QUANTUM SHANNON TH'Y

INTERP $S(\rho)$

- HOW TO: COMPRESS
  - COMM $-\boxed{\mathcal{N}^{\otimes n}}-$
  - DISTILL $\rho_{AB}^{\otimes n} \rightsquigarrow |\Phi\rangle$
- OPERATIONAL
- ASYMPTOTIC
- COMPUTATION FREE

$\log = \log_2$

...NON TH'Y

...SS

$-|N^{\otimes n}|-$

$\rho_{AB}^{\otimes n} \leadsto [|\Phi\rangle$

$\log = \log_2$

...EE

INTERP $S(\rho)$

- $S(\rho)$ CONTROLS EFFECTIVE H.S. DIM

$\rho^{\otimes n}$ $\quad \exists\, \Pi_\epsilon^n$ s.t. $\mathrm{tr}\,\rho^{\otimes n}\,\Pi_\epsilon^n > 1-\epsilon$

$\mathrm{rk}\,\Pi_\epsilon^n \leq 2^{nS(\rho)+o(n)}$

OPTIMAL.

~~STRONGER~~: $|\varphi_\rho\rangle_{RA}$ PURIF OF ρ

$$ {}_{RA}\langle \varphi_\rho|^{\otimes n}\left(I_R \otimes \Pi_\epsilon^n\right)|\varphi_\rho\rangle_{RA}^{\otimes n} > 1-\epsilon $$

L.

U — T — $\frac{1}{n}\log\dim T \Leftrightarrow S(\rho)$

$\bar{T}$ — $\approx|0\ldots0\rangle$

$T\otimes\bar{T}$

TO CONSTRUCT $\hat{\Pi}_\epsilon$?

E LAW OF LARGE #'S IN

IGEN BASIS OF $\rho^{\otimes n}$

2) USE SYMMETRY S_n.

FOCUS LECTURE.

3) IN PHYSICS, CARE ABOUT $n=1$.

e.g. 1+1D CFT.

1401.1540. $\leftarrow$ END LECTURE.

2) USE SYMMETRY S_n.

FOCUS LECTURE.

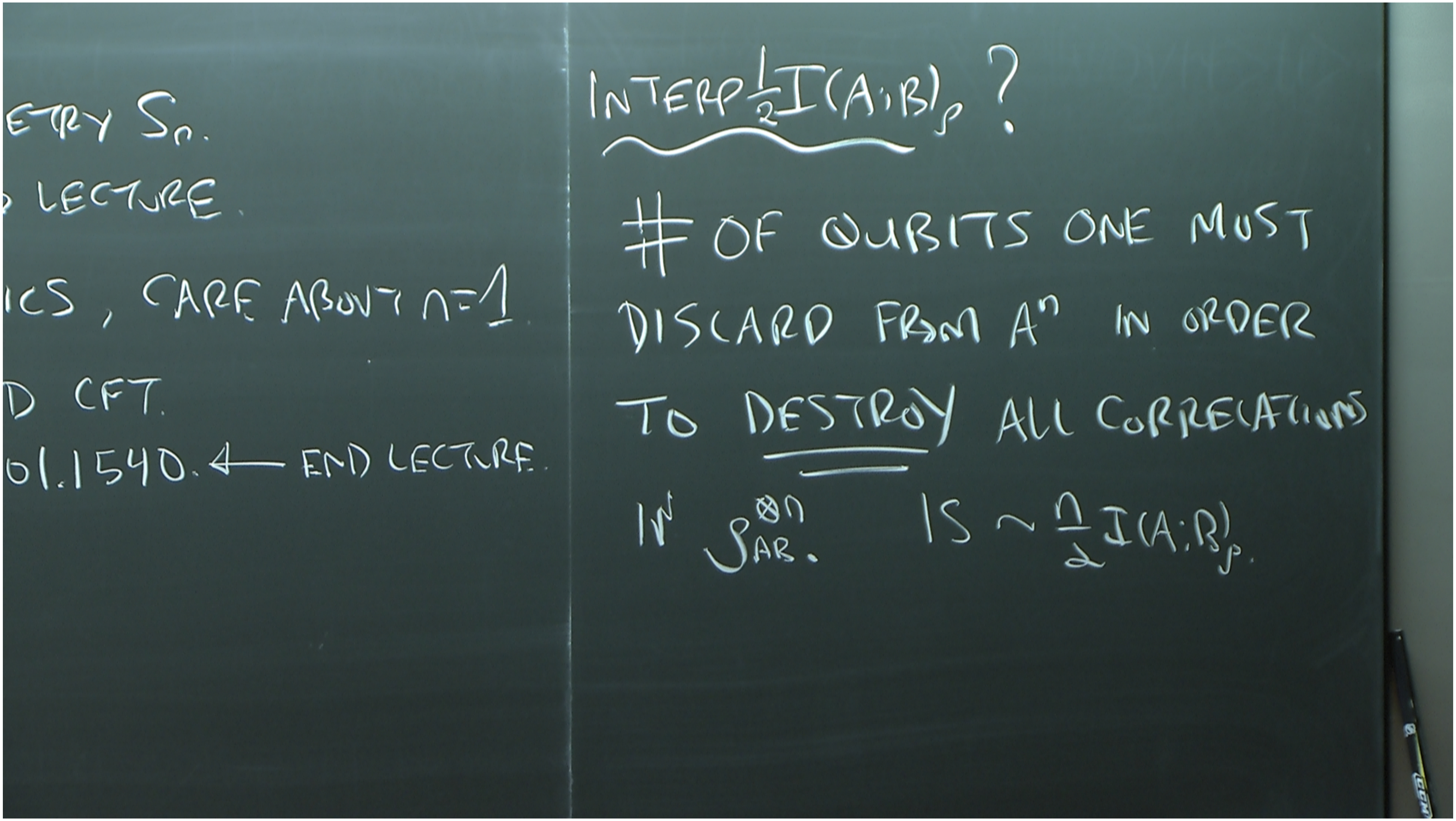
3) IN PHYSICS, CARE ABOUT $n=1$

e.g. 1+1D CFT

1401.1540. ← END LECTURE.

INTERP $I(A;B)_\rho$?

OF QUBITS ONE MUST DISCARD FROM A^n IN ORDER TO **DESTROY** ALL CORRELATIONS IN $\rho_{AB}^{\otimes n}$.


ETRY S_n.
LECTURE
ICS, CARE ABOUT $n=1$
D CFT.
01.1540. ← END LECTURE.
INTERP $\frac{1}{2}I(A;B)_\rho$?
OF QUBITS ONE MUST
DISCARD FROM A^n IN ORDER
TO DESTROY ALL CORRELATIONS
IN $\rho_{AB}^{\otimes n}$ IS $\sim \frac{n}{2} I(A;B)_\rho$.

$S(A)_\varphi = S(B)_\varphi \quad S(AB)_\varphi = 0$

$$\boxed{\frac{1}{2} I(A:B)_\varphi = S(A)_\varphi}$$

RESS $A^n \longrightarrow T \otimes T$

$\uparrow \sim nS(A)_\rho$

RD) T.

2) $\rho_{AB} = \frac{1}{d} \sum_{x=1}^{d} |x\rangle\langle x|_A \otimes |x\rangle\langle x|_B$

$$S(A)_\rho = \log d = S(B)_\rho = S(AB)_\rho$$

$$\frac{1}{2} I(A:B)_\rho = \frac{1}{2} \log d.$$

STRATEGY OF (1) FAILS.

• CHANGE BASIS. $d=4$

$A = A_1 \otimes A_2$ 2 QUBITS

$$|x\rangle \mapsto |\Phi_x\rangle_{A_1A_2} = (\sigma_x \otimes I)|\Phi_0\rangle$$

$$|\Phi_0\rangle = |00\rangle + |11\rangle$$

2) $\rho_{AB} = \frac{1}{d} \sum_{x=1}^{d} |x\rangle\langle x|_A \otimes |x\rangle\langle x|_B$

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$\mathrm{tr}_{A_1} |\Phi_x\rangle\langle\Phi_x|_{A_1A_2} = \frac{I}{2}$

- ALICE DISCARDS $A_1 \leftarrow 1$ QUBIT

$\frac{1}{2}\log d = \frac{1}{2}\log 4 = 1$

GENERAL CASE

1) COMPRESS $A^n \rightarrow T \otimes T$

2) APPLY RANDOM UNITARY TO T.

$x=1$

$\ldots g\, d = S(B)_\rho = S(AB)_\rho$

$\frac{1}{2}\log d$.

… OF (i) FAILS.

…E BASIS. $d=4$

$\ldots_1 \otimes A_2$ 2 QUBITS

$|\Phi_x\rangle_{A_1A_2} = (\sigma_x \otimes I)|\Phi_0\rangle$

$|\Phi_0\rangle = |00\rangle + |11\rangle$

$tr_{A_1} |\Phi_x\rangle\langle\Phi_x|_{A_1A_2} = \frac{I}{2}$.

- ALICE DISCARDS $A_1 \leftarrow 1$ QUBIT

$\frac{1}{2}\log d = \frac{1}{2}\log 4 = 1$

GENERAL CASE

1) COMPRESS $A^n \rightarrow T \otimes \bar{T}$.
2) APPLY RANDOM UNITARY TO T.
3) DISCARD $\sim nI(A;B)_\rho$ QUBITS FROM T.

2) USE S…

F…

3) IN PH…

e.g. …

$\frac{I}{2}$.

S $A, \leftarrow 1$ QUBIT

$= 1$

$\to T \otimes T$.

UNITARY

$(A;B)_\rho / 2$.

ASIDE: RECALL FROM QI BASICS PSET.

IF $|\psi\rangle_{XY}$ & $|\xi\rangle_{XY}$ SAT. $\psi_X = \xi_X$

THEN $\exists U_Y$ S.T.

$|\xi\rangle_{XY} = (I_X \otimes U_Y) |\psi\rangle_{XY}$

INTERP $\frac{1}{2} I(A;B)_\rho$

OF QUBIT

DISCARD FROM

TO DESTROY

IN $\rho_{AB}^{\otimes n}$. IS

$\frac{I}{2}$.

S A, ← 1 QUBIT

$= 1$

⇒ TOT

UNITARY

$(A;B)_\rho / 2$.

M I

ASIDE: RECALL FROM QI BASICS PSET.

IF $|\psi\rangle_{XY}$ & $|\xi\rangle_{XY}$ SAT. $\psi_X = \xi_X$

THEN $\exists\, U_Y$ S.T.

$$|\xi\rangle_{XY} = (I_X \otimes U_Y)|\psi\rangle_{XY}$$

$$\Longrightarrow F(\rho_X, \sigma_X) = \max_{U_Y} \left| \langle \psi_\rho | (I_X \otimes U_Y) | \xi_\sigma \rangle \right|$$

FIXED PURIFS.

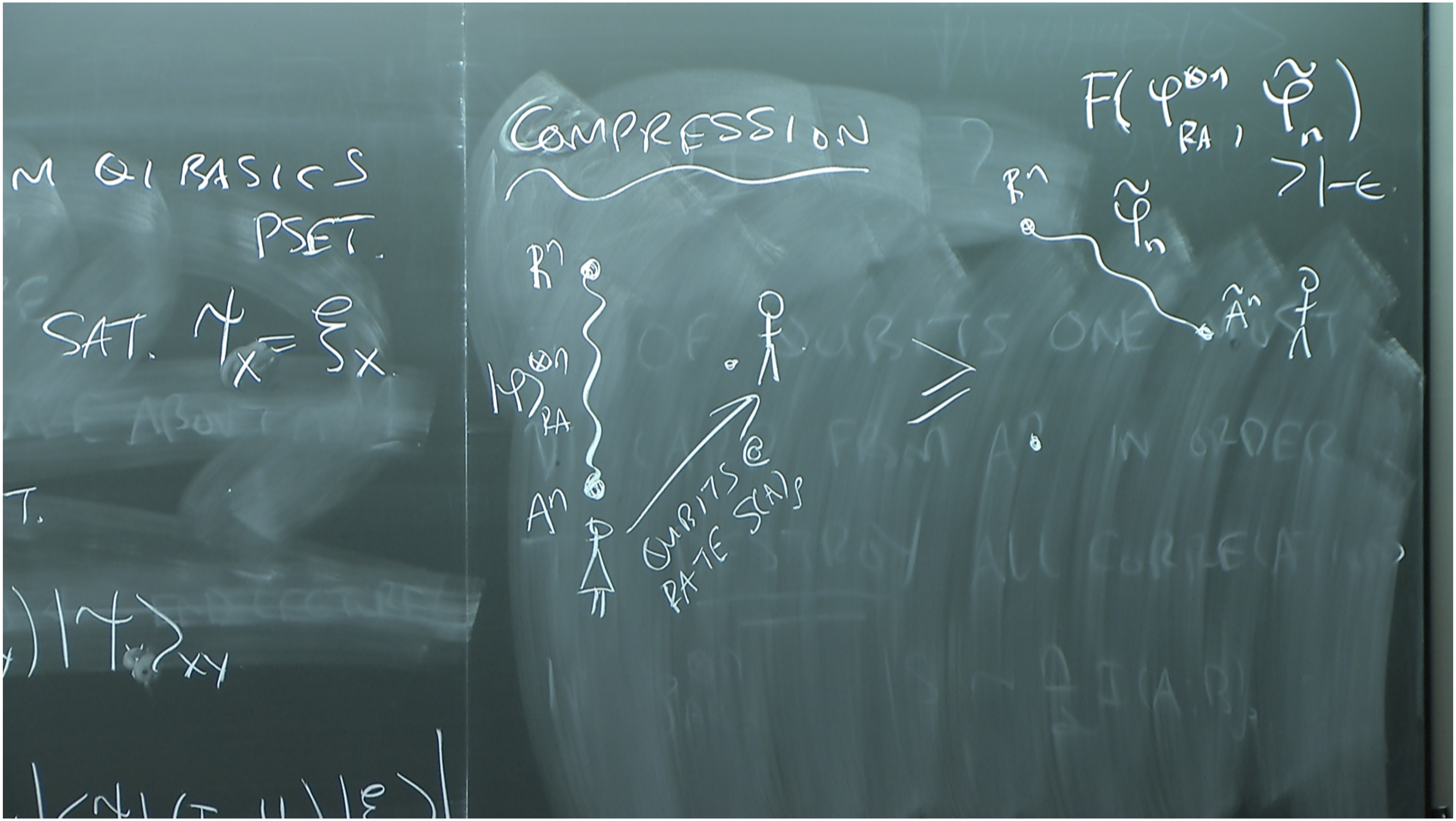
INTERP $\frac{1}{2} I(A;B)_\rho$

OF QUBI

DISCARD FROM

TO DESTROY

IN $\rho_{AB}^{\otimes n}$. IS


M QI BASICS
PSET.
SAT.
COMPRESSION
F(φ⊗n RA , φ̃n)
>1-ε
QUBITS @
RATE S(A)ρ

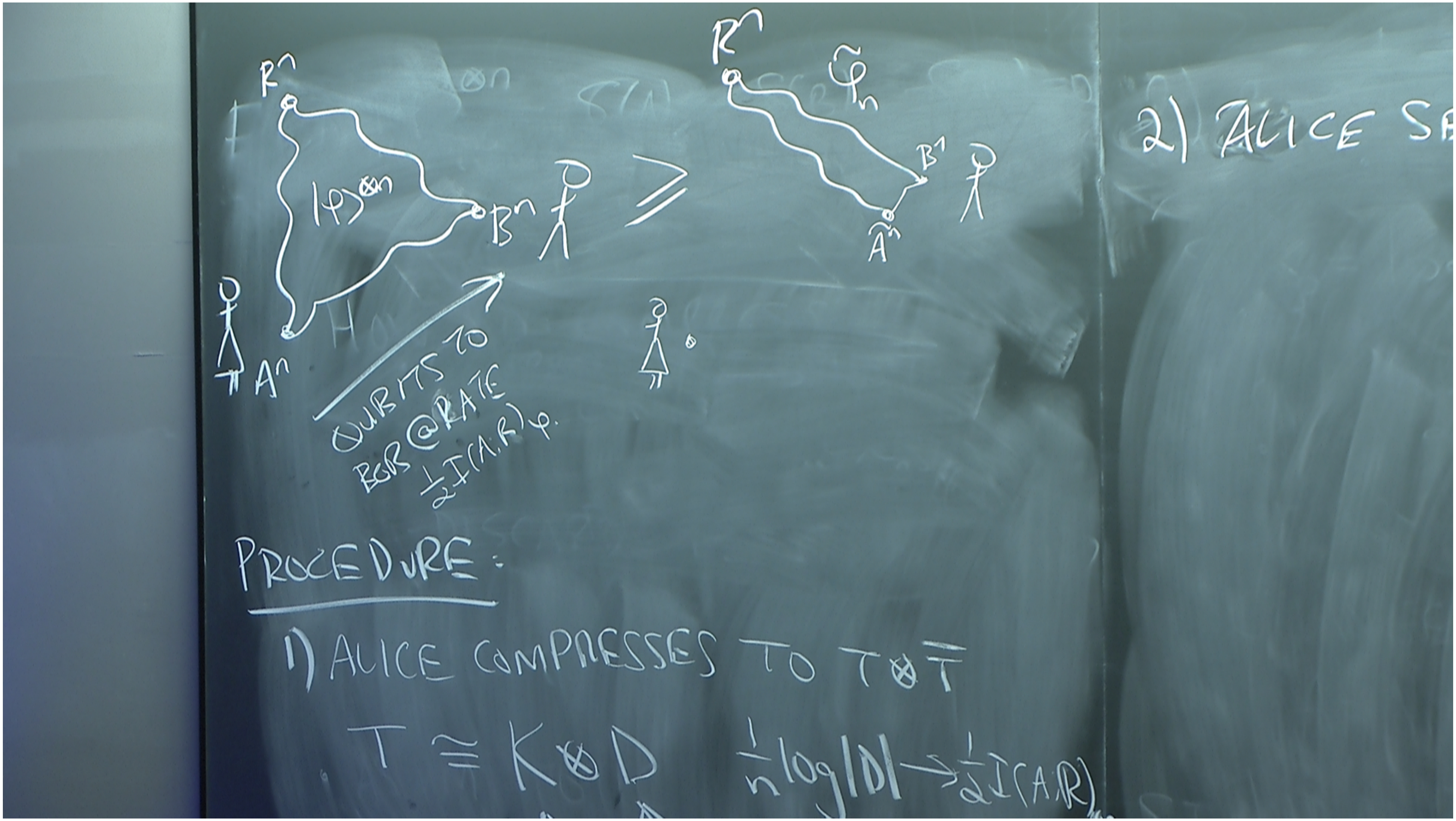
PROCEDURE:

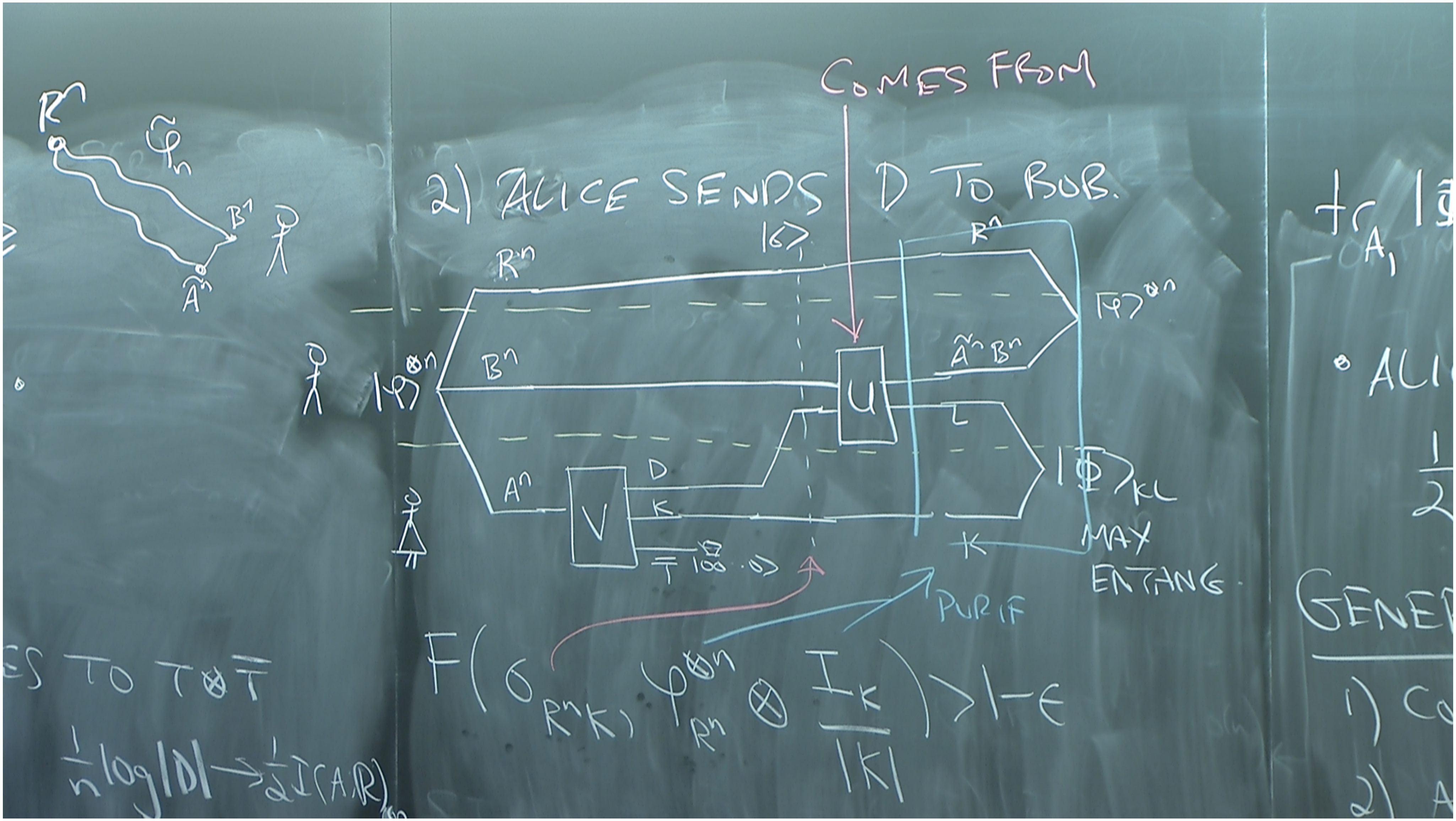
1) ALICE COMPRESSES TO $T \otimes \bar{T}$

$$T \cong K \otimes D \qquad \frac{1}{n}\log|D| \to \frac{1}{2} I(A;R)_{\rho}$$

K: keep, D: discard

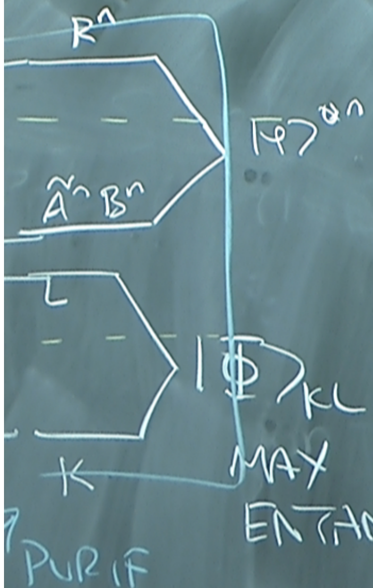
(ie Alice destroys $R^n \otimes A^n$ CORRELATIONS)


R^n
$|\varphi\rangle^{\otimes n}$
B^n
A^n
QUBITS TO
BOB @ RATE
$\frac{1}{2} I(A;R)_\varphi$
$\geq$
R^n
$\tilde{\varphi}_n$
B^n
$\hat{A}^n$
PROCEDURE:
1) ALICE COMPRESSES TO $T \otimes \bar{T}$
$T \cong K \otimes D$
$\frac{1}{n} \log|D| \to \frac{1}{2} I(A;R)$
2) ALICE SE


COMES FROM
2) ALICE SENDS D TO BOB.
MAX ENTANG.
PURIF
GENE
ES TO T⊗T

ES FROM (*)

TO BOB.



$) > 1 - \epsilon$

HOW MANY EBITS

$$\frac{1}{n}\log|T| \sim S(A)_\varphi$$

$$\frac{1}{n}\log|K| \sim S(A)_\varphi - \frac{1}{2}I(A;R)_\varphi$$

$$= \frac{1}{2}I(A;B)_\varphi \quad \text{SINCE } |\varphi\rangle_{RAB}$$

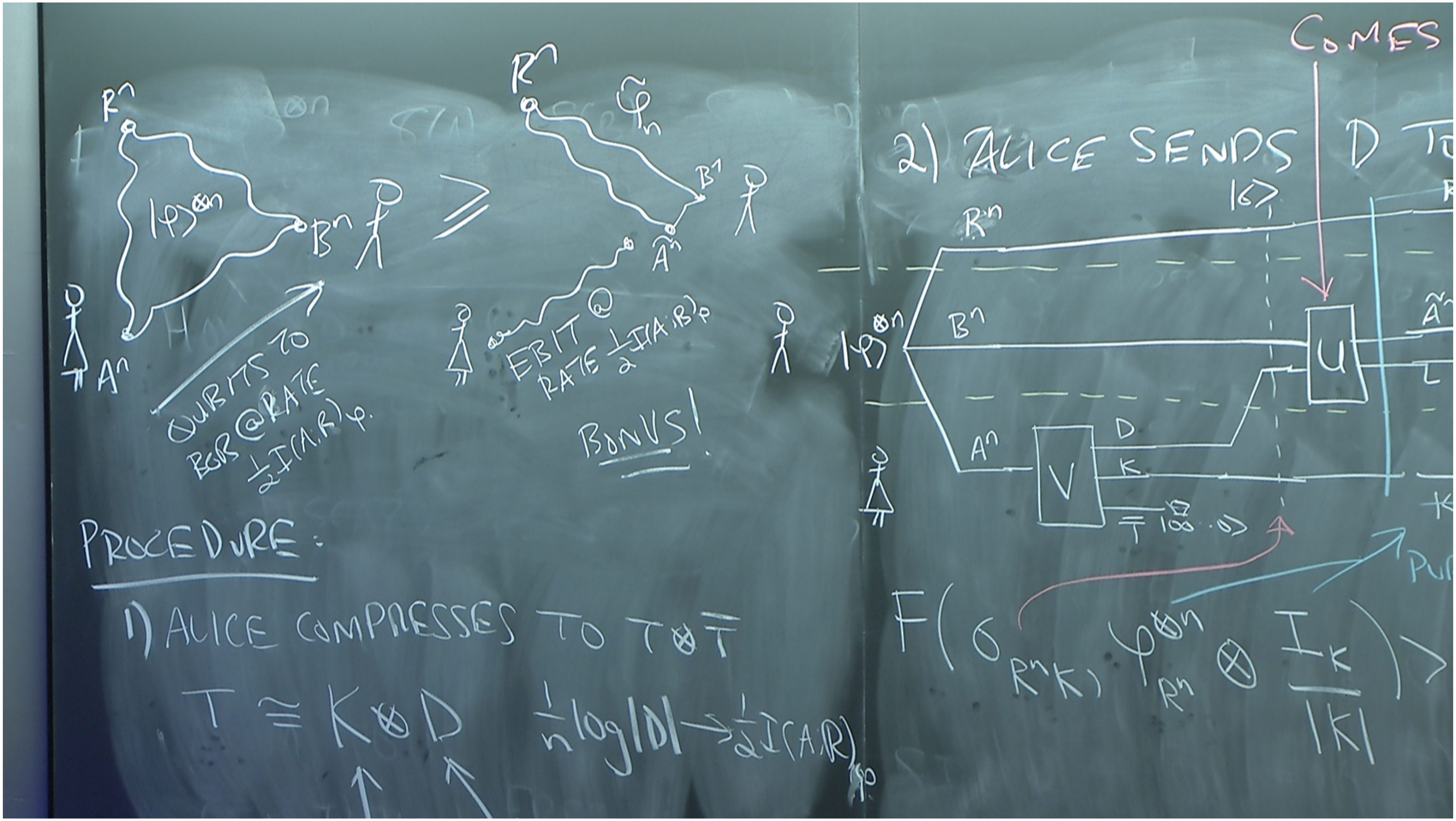
ASIDE:

IF $|\psi$

THEN

$|\varrho\rangle_{S(xy}$

$\Longrightarrow$ F(



R^n
$|\varphi\rangle^{\otimes n}$
B^n
A^n
QUBITS TO BOB @ RATE $\frac{1}{2}I(A;R)_\varphi$
$\geq$
R^n
$\tilde{\varphi}_n$
B^n
$\tilde{A}^n$
EBIT @ RATE $\frac{1}{2}I(A;B)_\varphi$
BONUS!
PROCEDURE:
1) ALICE COMPRESSES TO $T \otimes \bar{T}$
$T \cong K \otimes D$
$\frac{1}{n}\log|D| \to \frac{1}{2}I(A;R)_\varphi$
COMES
2) ALICE SENDS D TO
$|\sigma\rangle$
R^n
B^n
$|\varphi\rangle^{\otimes n}$
A^n
V
U
D
K
$\tilde{A}^n$
L
$\bar{T}$
$|00\cdots0\rangle$
$F(\sigma_{R^nK}, \varphi^{\otimes n}_{R^n} \otimes \frac{I_K}{|K|}) >$

HOW MANY EBITS

$$\frac{1}{n}\log|T| \sim S(A)_\varphi.$$

$$\frac{1}{n}\log|K| \sim S(A)_\varphi - \frac{1}{2}I(A;R)_\varphi$$

$$= \frac{1}{2}I(A;B)_\varphi \quad \text{SINCE } |\varphi\rangle_{RAB}.$$

HOW TO $\varphi_{AB}^{\otimes n} \xrightarrow{\text{LOCC}} -S(A|B)_\varphi$ EBITS

TP: $[qq] + 2[c\to c] \geq [q\to q]$

Bellpair — 2 cbits $A\to B$ — 1 qubit $A\to B$

$$\langle \varphi_{AB}\rangle + \frac{1}{2}I(A;R)_\varphi [q\to q] \geq \frac{1}{2}I(A;B)_\varphi [qq]$$

SUB IN.

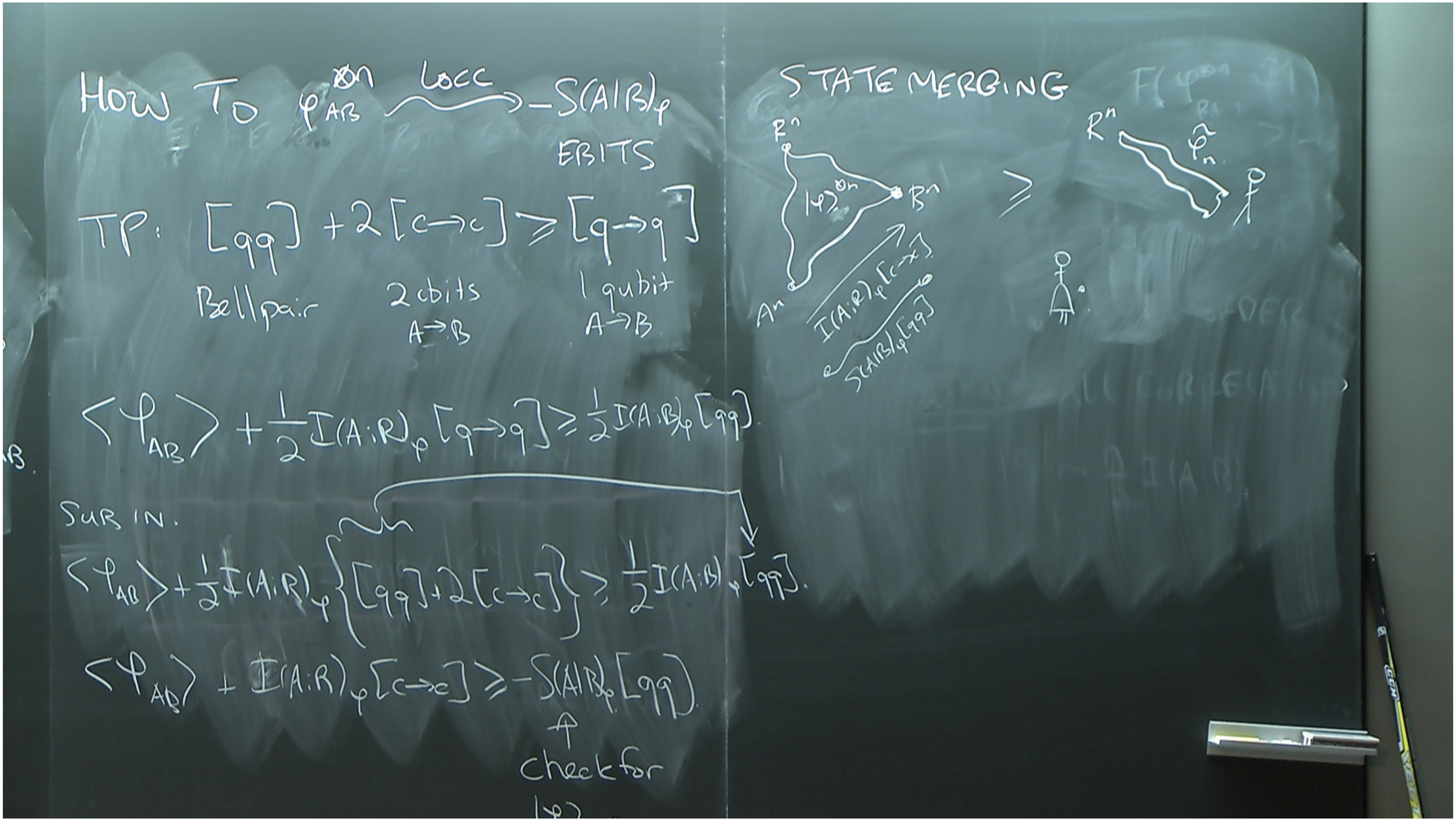
$$\langle \varphi_{AB}\rangle + \frac{1}{2}I(A;R)_\varphi \left\{[qq] + 2[c\to c]\right\} \geq \frac{1}{2}I(A;B)_\varphi [qq].$$

$$\langle \varphi_{AB}\rangle + I(A;R)_\varphi [c\to c] \geq -S(A|B)_\varphi [qq].$$

Check for $|\varphi\rangle_{ABR}$.

Con

R^n $|\varphi\rangle_{RA}^{\otimes n}$ A^n


HOW TO $\varphi_{AB}^{\otimes n} \xrightarrow{\text{LOCC}} -S(A|B)_\varphi$ EBITS
TP: $[qq] + 2[c\to c] \geq [q\to q]$
Bell pair
2 cbits $A\to B$
1 qubit $A\to B$
$\langle \varphi_{AB} \rangle + \frac{1}{2} I(A;R)_\varphi [q\to q] \geq \frac{1}{2} I(A;B)_\varphi [qq]$
SUB IN.
$\langle \varphi_{AB} \rangle + \frac{1}{2} I(A;R)_\varphi \{[qq] + 2[c\to c]\} \geq \frac{1}{2} I(A;B)_\varphi [qq]$
$\langle \varphi_{AB} \rangle + I(A;R)_\varphi [c\to c] \geq -S(A|B)_\varphi [qq]$
Check for $|\varphi\rangle$
STATE MERGING
R^n
$|\varphi\rangle^{\otimes n}$
B^n
A^n
$I(A;R)_\varphi [c\to c]$
$S(A|B)_\varphi [qq]$
$\geq$
R^n
$\hat{\varphi}_n$

$$\varphi^{\otimes n}_{AB} \xrightarrow{\text{LOCC}} -S(A|B)_\varphi \text{ EBITS}$$

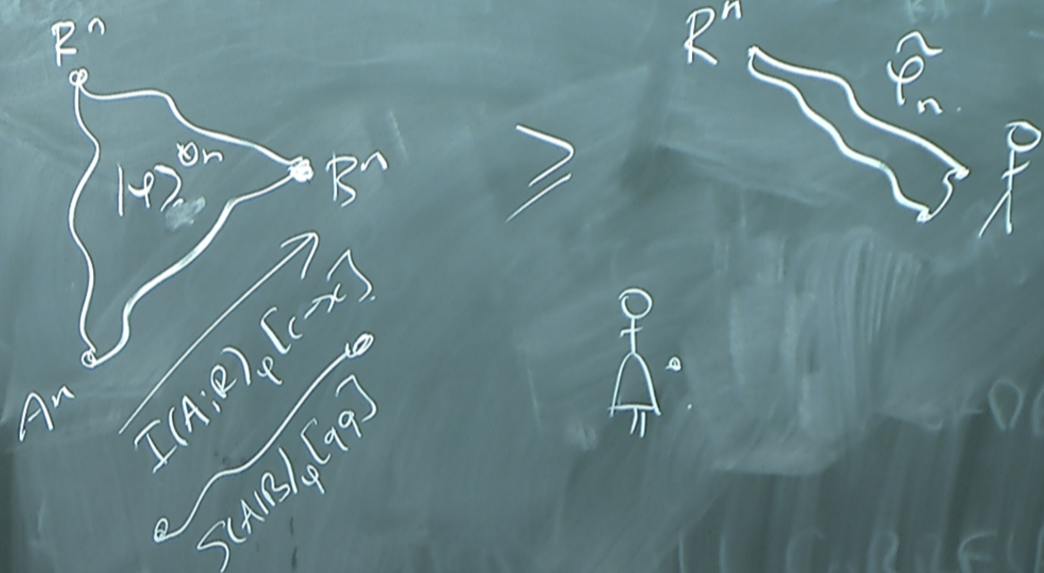
$$+2[c\to c] \geq [q \to q]$$

2 cbits $A \to B$ — 1 qubit $A \to B$

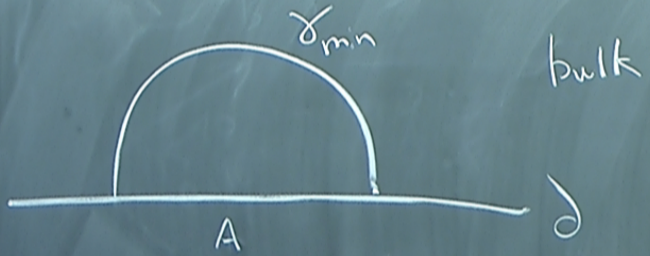
$$I(A;R)_\varphi [q\to q] \geq \tfrac{1}{2} I(A;B)_\varphi [qq].$$

$$\{[qq] + 2[c\to c]\} \geq \tfrac{1}{2} I(A;B)_\varphi [qq].$$

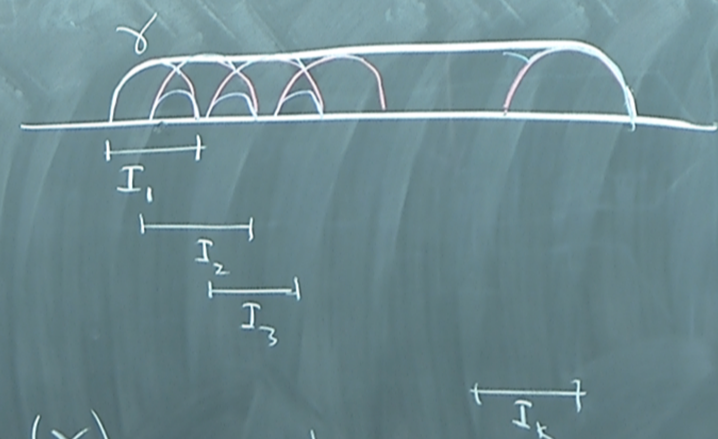
STATE MERGING



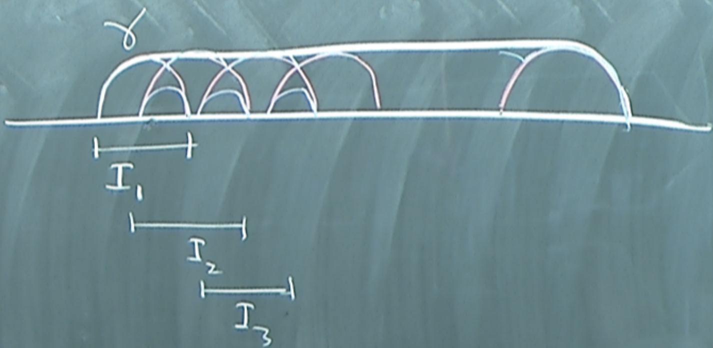
HOLE-OGRAPHIC ENTROPY



RT $$S(A) = \frac{\text{len}(\gamma_{min})}{4G_N}.$$



$$\frac{\text{len}(\gamma)}{4G_N} = \lim_{k\to\infty} \sum_{j=1}^{k} \left[S(I_j) - S(I_j \cap I_{j+1}) \right]$$



I_k

$$\frac{\mathrm{len}(\gamma)}{4G_N} = \lim_{k\to\infty} \sum_{j=1}^{k} \Big[\underbrace{S(I_j) - S(I_j \cap I_{j-1})}_{S(A_j|B_j)} \Big]$$

I_{j-1}

I_j

$B_j = I_j \cap I_{j-1}$ $\quad A_j$

SPS ALICE HOLDS $A = \bigcup_j I_j$ AND WISHES TO MERGE A TO BOB USING STREAMING PROTOCOL IN WHICH @ j^{th} STEP, SHE HAS ACCESS ONLY TO I_j.

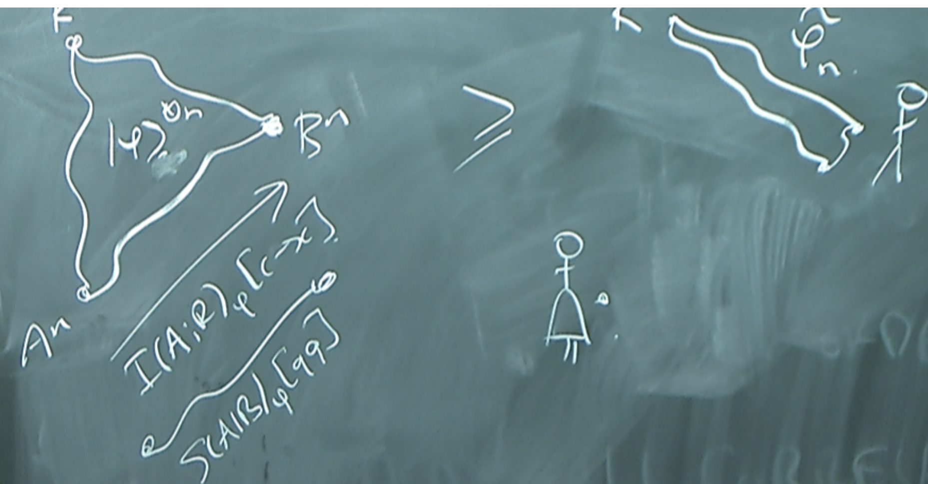
<u>TH'M</u>: MIN EBIT COST IS

$$\frac{\mathrm{len}(\gamma)}{4G_N}.$$

EBITS

$2[c\to c] \geq [q\to q]$

2 cbits $A\to B$ 1 qubit $A\to B$



$;R)_\varphi [q\to q] \geq \frac{1}{2} I(A;B)_\varphi [qq].$

$[qq] + 2[c\to c]\} \geq \frac{1}{2} I(A;B)_\varphi [qq].$

$[c\to c] \geq -S(A|B)_\varphi [qq].$

$$-I_3(A;B;C) = I(A:BC) - I(A;B) - I(A;C) \geq 0.$$