Title: The Sachdev-Ye-Kitaev model and AdS2/CFT1

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URL: http://pirsa.org/16070040

Abstract: The application of holography to fundamental problems in quantum gravity has been hindered by the lack of a solvable model. However, building on work by Sachdev and Ye, Kitaev has proposed a solvable QM system as a dual to an AdS2 black hole. I will discuss the model and its possible bulk interpretation.

ks online)

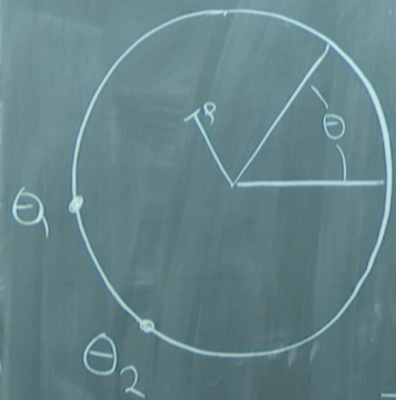
bin Yang

enhaus

/ Yoon

f

NAIVE EXP. FOR AdS_2/CFT_1



$$ds^2 = d\rho^2 + \sinh^2\rho \, d\theta^2$$

WANTED:

1) large N classical fields

2) conformally invariant

$$\langle \mathcal{O}(\theta_1)\,\mathcal{O}(\theta_2)\rangle = \left(\frac{1}{\sin\frac{\theta_{12}}{2}}\right)^{2\Delta}$$

Θ^2

3) large S at low T

4) higher pt

four pt. $\begin{cases} \to \text{chaos} \\ \to \text{sparse} \end{cases}$

fields

ant

$\left(\frac{1}{}\right)^{2\Delta}$

aos

rse

SYK

N Majorana $\psi_a \qquad a = 1, \dots, N$

$$\psi_a \psi_b + \psi_b \psi_a = \delta_{ab} \qquad \dim \quad 2^{N/2}$$

$$H = \sum_{a<b<c<d} J_{abcd}\, \psi_a \psi_b \psi_c \psi_d$$

$$\langle J_{abcd} J_{a'b'c'd'} \rangle = \delta_{aa'} \cdots \delta_{dd'} \frac{J^2}{N^3} 3!$$

aos

rse

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$$\langle J_{abcd}J_{a'b'c'd'}\rangle=\delta_{aa'}\cdots\delta_{dd'}\frac{J^2\,3!}{N^3}$$

$$N\gg\beta J\gg 1$$

aos

rse

REFERENCES

93 Sachdev / Ye

98 Parcollet / Georges

10 Sachdev

14/15 Kitaev (KITP talks online)

16 Maldacena / DS

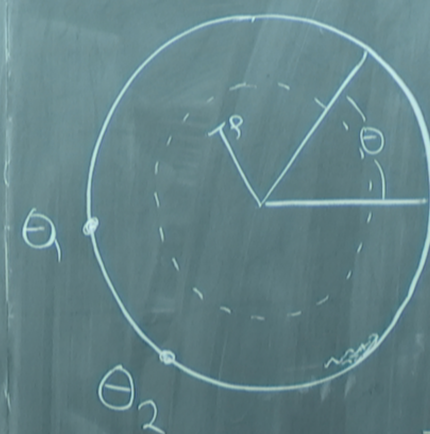
Maldacena / DS / Zhenbin Yang

See also: Polchinski / Rosenhaus

Jevicki / Suzuki / Yoon

Annninos / Denef

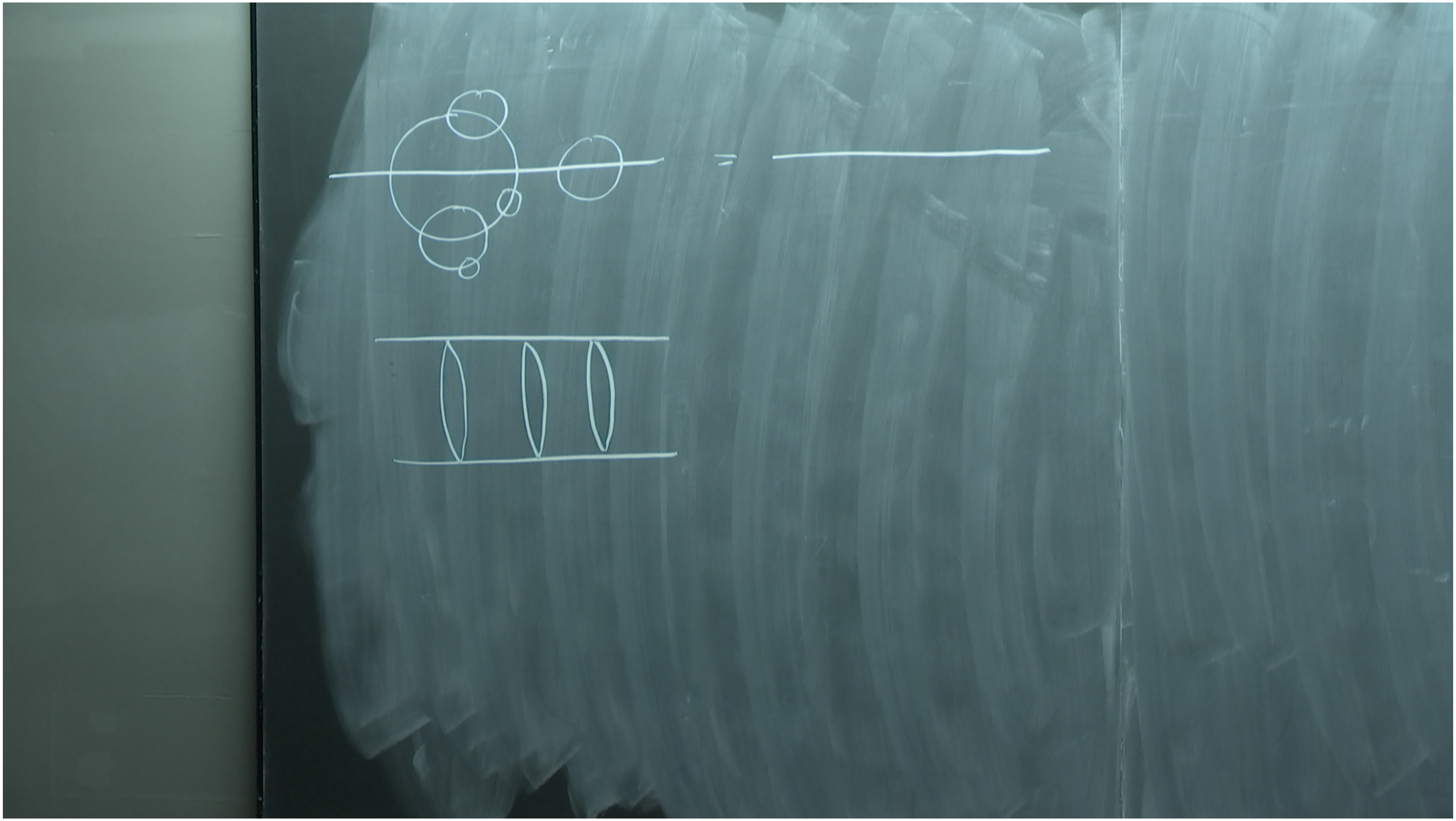
NAIVE EXP.



W

1) la

2) c



$$Z(J_{abcd}) = \int \mathcal{D}\psi_a \exp\left\{ -\int \psi_a \partial_\tau \psi_a + J_{abcd}\, \psi_a \psi_b \psi_c \psi_d \right\}$$

$$\int dJ_{abcd}\; e^{-\frac{J_{abcd}^2}{2 \frac{J^2 3!}{N^3}}}\; Z(J_{abcd})$$

$$= \int \mathcal{D}\psi_a \exp\left\{ -\int \psi_a \partial_\tau \psi_a + \frac{3!\, J^2}{N^3} \iint \psi_a \psi_b \psi_c \psi_d(\tau)\, \psi_a \psi_b \psi_c \psi_d(\tilde{\tau}) \right\}$$

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$$= \frac{J^2}{4}\iint \left(\frac{\psi_a(\tau)\,\psi_a(\tilde{\tau})}{N}\right)^4$$

$$G(\tau,\tilde{\tau}) \text{ "=" } \frac{\psi_a(\tau)\,\psi_a(\tilde{\tau})}{N} \qquad \Sigma(\tau,\tilde{\tau})$$

$$= \int \mathcal{D}\Sigma\,\mathcal{D}G\,\mathcal{D}\psi_a \, \exp\left\{ -\int \psi_a \partial_\tau \psi_a + N \iint \Sigma(\tau,\tilde{\tau})\left[G(\tau,\tilde{\tau}) - \frac{\psi_a(\tau)\,\psi_a(\tilde{\tau})}{N} \right] + \frac{J^2}{4} N \, G(\tau,\tilde{\tau})^4 \right\}$$

$$= \int \mathcal{D}\Sigma\,\mathcal{D}G \, \exp\left\{ -N \, I[\Sigma, G] \right\}$$

$$I = -\frac{1}{2} \log \det(\partial_\tau - \Sigma) + \frac{1}{2} \iint \left[\Sigma(\tau,\tilde{\tau})\, G(\tau,\tilde{\tau}) - \frac{J^2}{4} G(\tau,\tilde{\tau})^4 \right]$$

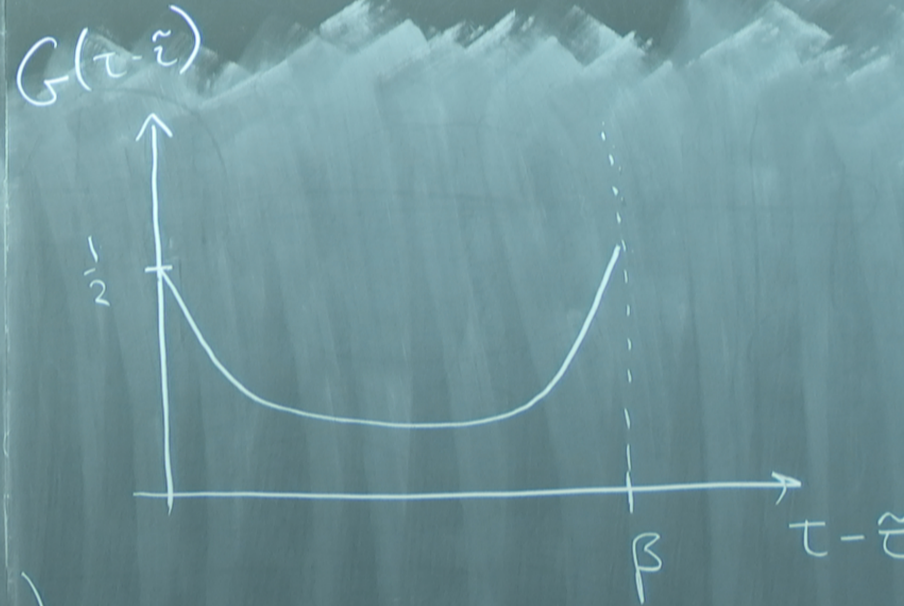
SADDLE PT:

1) EOM

$$\Sigma(\tau,\tilde{\tau}) = J^2\, G(\tau,\tilde{\tau})^3$$

$$G(\tau,\tilde{\tau}) = \left(\partial_\tau - \Sigma\right)^{-1}(\tau,\tilde{\tau})$$

$$G(\omega) = \frac{1}{-i\omega - \Sigma(\omega)}$$



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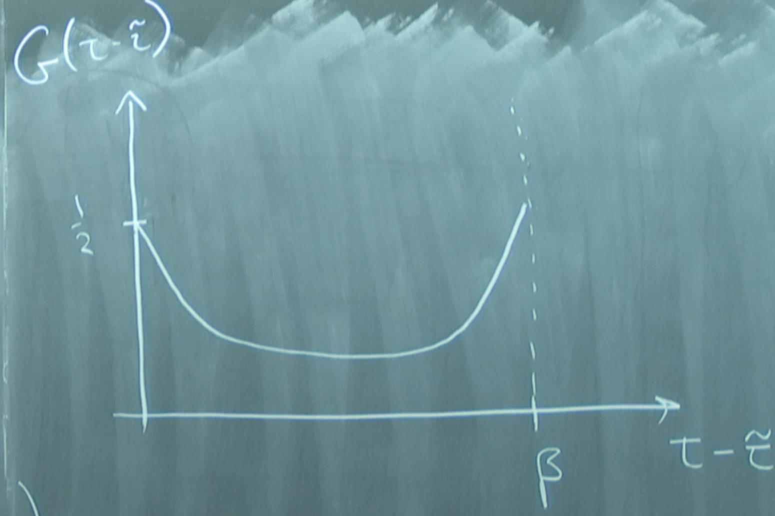

$$G(\tau-\tilde{\tau}) = \frac{b}{\left(\sin\frac{\pi(\tau-\tilde{\tau})}{\beta}\right)^{2\Delta}} \qquad \Delta = \frac{1}{2}$$

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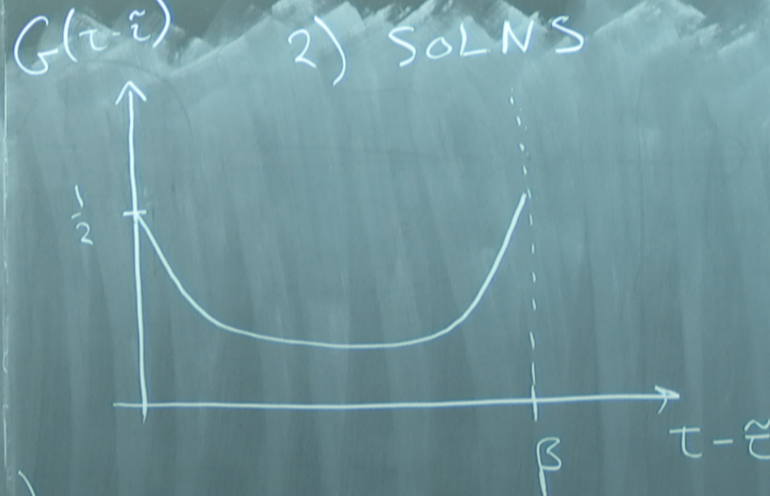
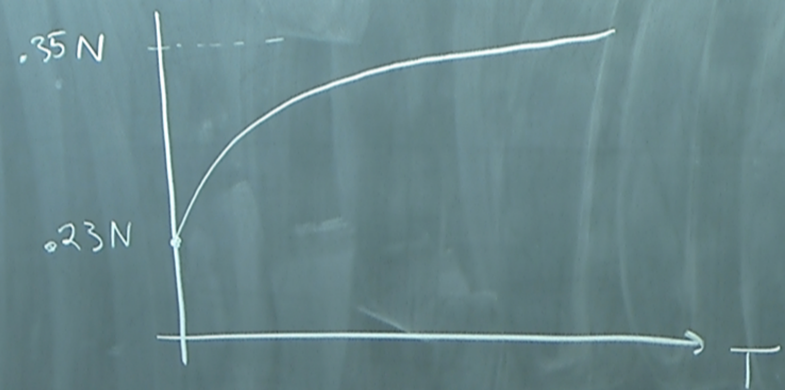
2) SOLNS



$$G(\tau-\tilde{\tau}) = \frac{b}{\left(\sin\frac{\pi(\tau-\tilde{\tau})}{\beta}\right)^{2\Delta}} \qquad \Delta = \frac{1}{4}$$

3) F.E.

$$\log Z = -N I(G_*, \Sigma_*)$$



$$I = -\frac{1}{2}\log\det(\partial_\tau - \Sigma) + \frac{1}{2}\iint\Big[\Sigma(\tau,\tau')G(\tau,\tau')$$

4) REPAR. INV.

$$G_f = f'(\tau)^{\Delta} f'(\tilde{\tau})^{\Delta} G_*(f(\tau), f(\tilde{\tau}))$$

$$-\frac{1}{2} \log \det(\partial_\tau - \Sigma) + \frac{1}{2} \iint \left[\Sigma(\tau,\tilde{\tau}) G(\tau,\tilde{\tau}) - \frac{J^2}{4} G(\tau,\tilde{\tau})^4 \right]$$

FLUCTUATIONS

$$G = G_* + g$$

$$\Sigma = \Sigma_* + \sigma$$

$$I(g) = I_* + \iint g(\tau_1 \tau_2)\, Q(\tau_1 \tau_2 ; \tau_3 \tau_4)\, g(\tau_3 \tau_4)$$

$$+ \ldots$$

$$Q(G_*, \Sigma_*) \qquad Q = 1 - K$$

FLUCTUATIONS

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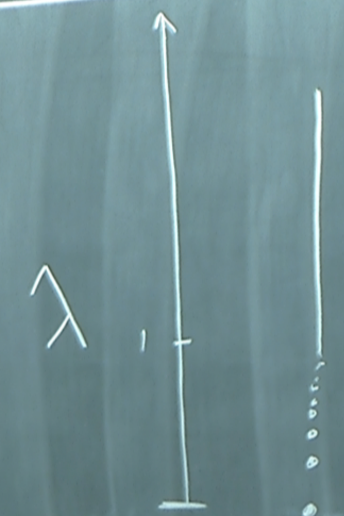
$$Q(G_*, \Sigma_*) \qquad Q = 1 - K$$

SPECTRUM



$$\left\langle \frac{\psi_a(\tau_1)\,\psi_a(\tau_2)}{N} \quad \frac{\psi_b(\tau_3)\,\psi_b(\tau_4)}{N} \right\rangle_{conn} = \left\langle g(\tau_1, \tau_2)\; g(\tau_3 \tau_4) \right\rangle$$

$$I = -\frac{1}{2} \log$$

$$\left\langle \frac{\psi_a(\tau_1)\,\psi_a(\tau_2)}{N} \; \frac{\psi_b(\tau_3)\,\psi_b(\tau_4)}{N} \right\rangle_{\text{conn}} = \left\langle g(\tau_1,\tau_2)\; g(\tau_3\tau_4) \right\rangle$$

$$= \frac{1}{N}\; Q^{-1}(\tau_1\tau_2;\tau_3\tau_4)$$

$$= \frac{1}{N} \oint \frac{\langle \tau_1\tau_2 | \lambda \rangle \langle \lambda | \tau_3\tau_4 \rangle}{\lambda}$$

$$\left\langle \frac{\psi_a(\tau_1)\,\psi_a(\tau_2)}{N} \; \frac{\psi_b(\tau_3)\,\psi_b(\tau_4)}{N} \right\rangle_{\text{conn}} = \langle g(\tau_1, \tau_2)\; g(\tau_3 \tau_4) \rangle$$

$$= \frac{1}{N} \; Q^{-1}(\tau_1 \tau_2 ; \tau_3 \tau_4)$$

$$= \frac{1}{N} \oint \frac{\langle \tau_1 \tau_2 | \lambda \rangle \langle \lambda | \tau_3 \tau_4 \rangle}{\lambda}$$

$$= \frac{1}{N} \sum_h c_h^2 \; z^h \; {}_2F_1(h, h, 2h, z)$$

$$\left\langle \frac{\psi_a(\tau_1)\,\psi_a(\tau_2)}{N} \; \frac{\psi_b(\tau_3)\,\psi_b(\tau_4)}{N} \right\rangle_{\text{conn}} = \left\langle g(\tau_1, \tau_2)\; g(\tau_3 \tau_4) \right\rangle$$

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$$+ \infty$$

ZERO MODES

$$\lambda = 0 + \frac{\#\, n}{\beta J} - \frac{\tilde{\#}\, n^2}{(\beta J)^2} + \dots$$

i) dominant

ii) ~~conformal~~

iii) chaos bound

ZERO MODES

$$\lambda = \bigcirc + \frac{\# \, n}{\beta J} - \frac{\tilde{\#} \, n^2}{(\beta J)^2} + \ldots$$

i) dominant

ii) ~~conformal~~

iii) chaos bound

iv) energy fluctuations

v) agrees w/ dilaton gravity!

EFFECTIVE ACTION FOR RFP

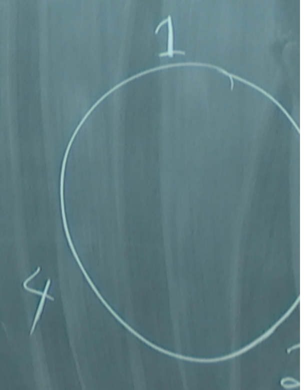
$$G = f'(\tau_1)^{\Delta} f'(\tau_2)^{\Delta} G_c\left(f(\tau_1), f(\tau_2)\right)$$

$$f = \tau + \varepsilon(\tau)$$

$$I = \frac{1}{\beta J} \int (\varepsilon'')^2 - (\varepsilon')^2 \, d\tau$$

$$\rightarrow \frac{1}{\beta J} \int \mathrm{Sch}(f, \tau) \, d\tau$$

$$\Big\langle \frac{\psi_a(\tau_1)\psi}{N}$$



SUMMARY

- "low-tension string" in dual

- enhanced sector ~~conformal~~ agrees w/ dilaton gravity