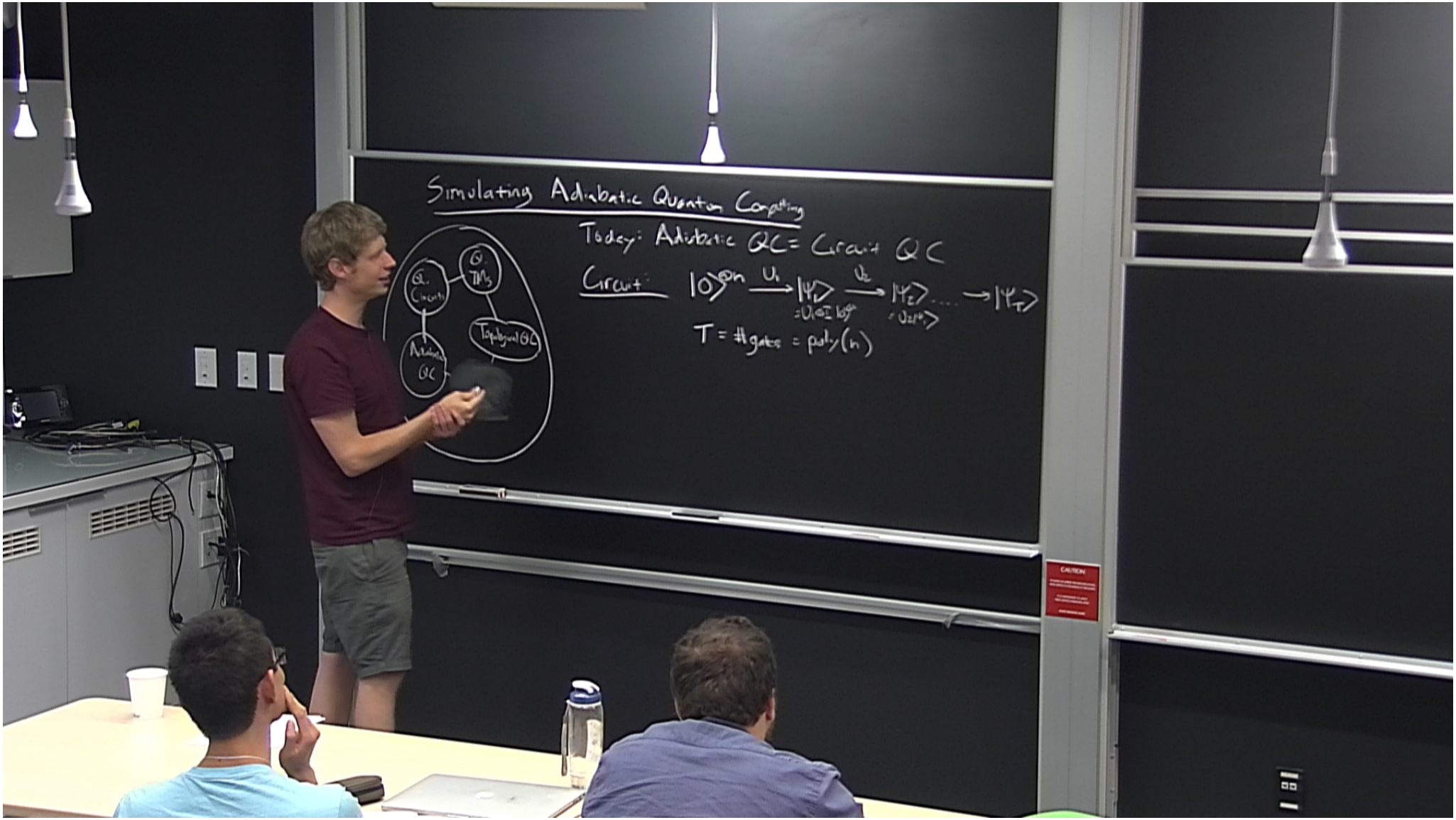
Title: Equivalence of Adiabatic and Circuit Based Quantum Computing

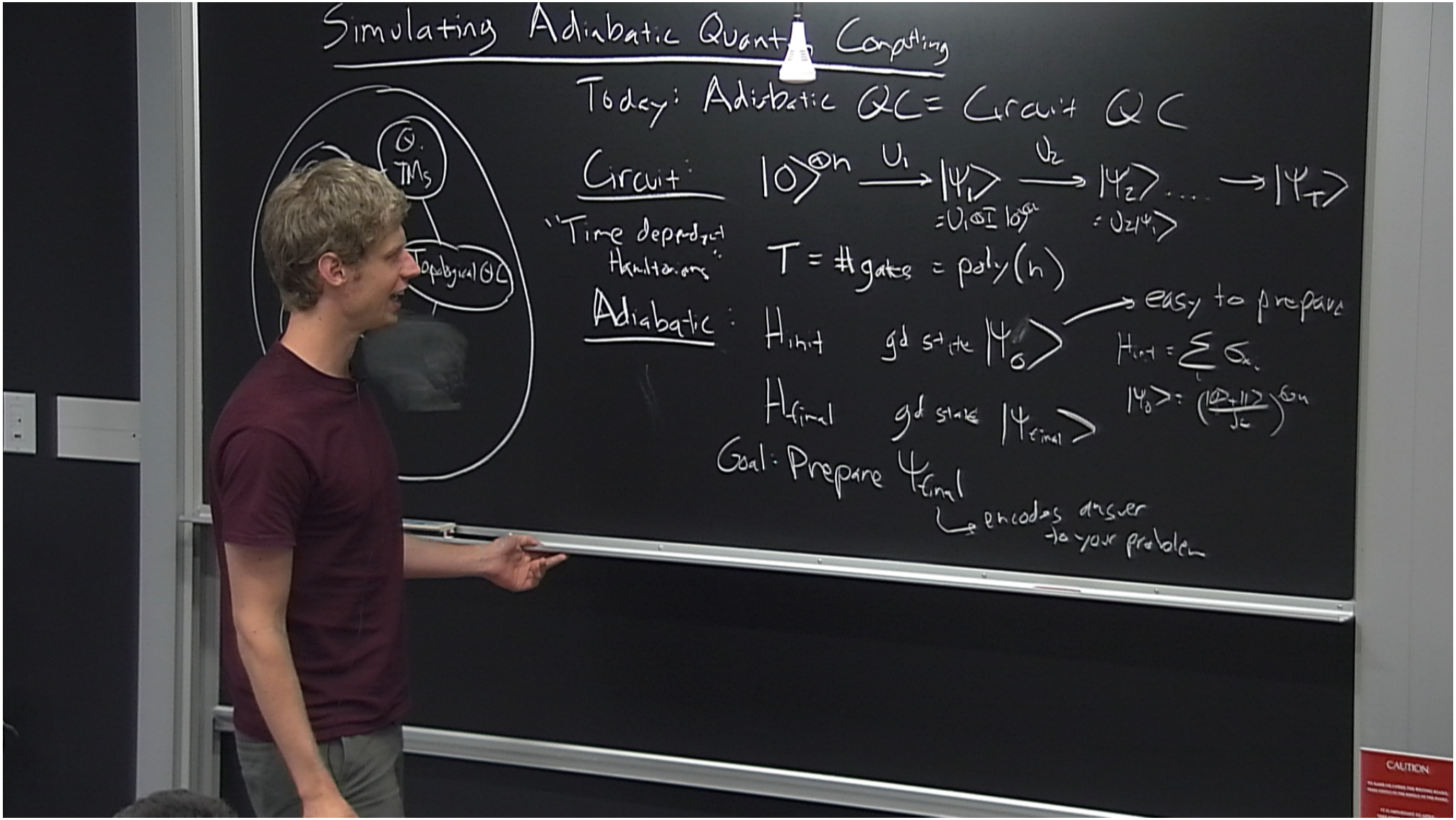
Date: Jul 25, 2016 05:00 PM

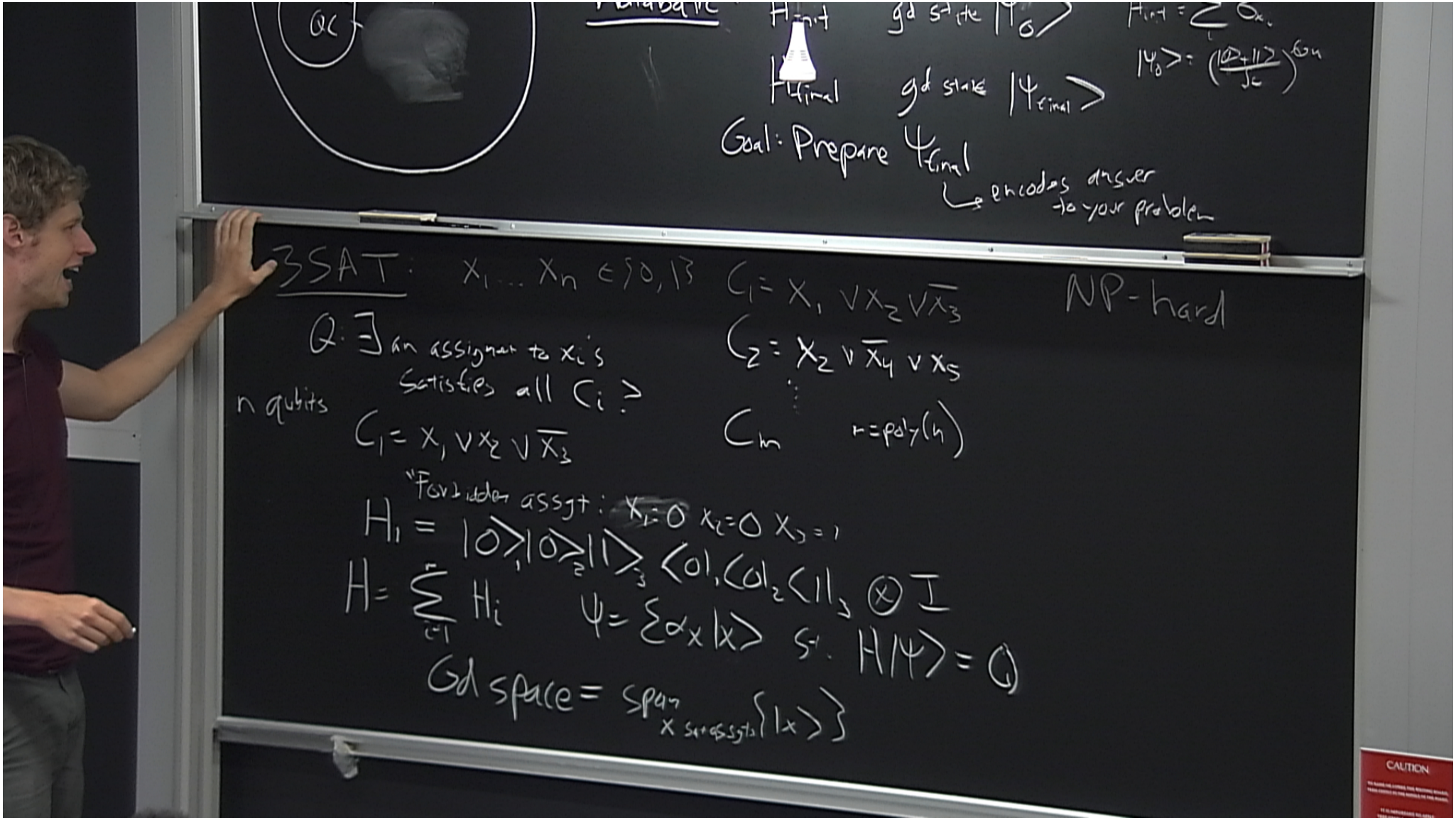
URL: http://pirsa.org/16070039

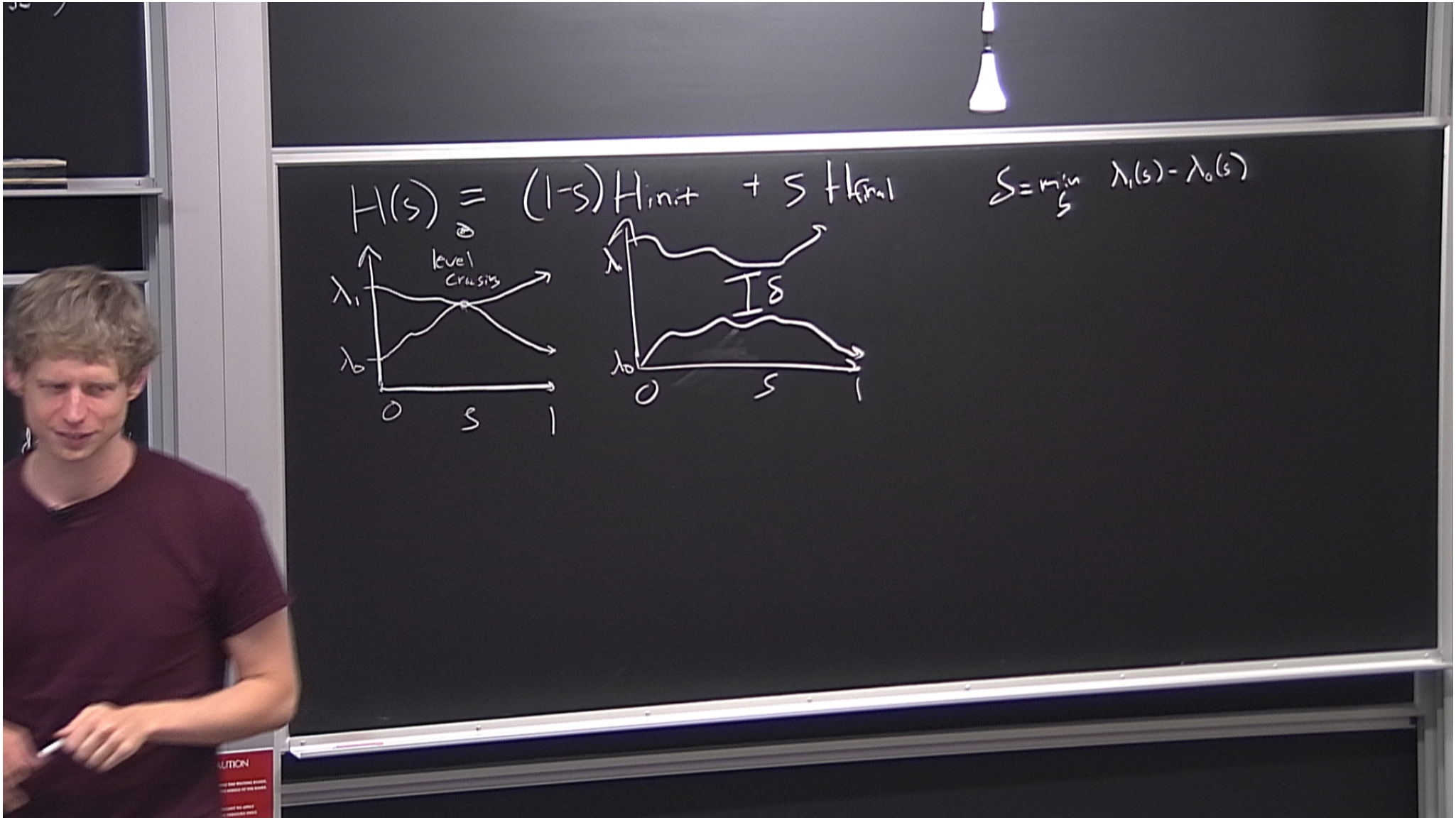
Abstract:


Simulating Adiabatic Quantum Computing
Q. Circuits
Q. TMs
Topological QC
Adiabatic QC
QFA

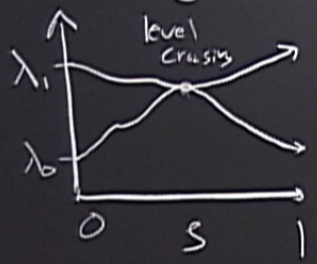

Simulating Adiabatic Quantum Computing
Today: Adiabatic QC = Circuit QC
Circuit
T = #gates = poly(n)
Q. Circuits
Q. TMs
Topological QC
Adiabatic QC

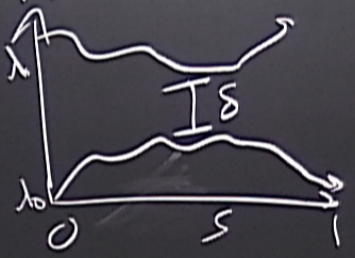

Simulating Adiabatic Quant Computing
Today: Adiabatic QC = Circuit QC
Circuit:
"Time dependent Hamiltonians"
TMs
Topological QC
Adiabatic:
Goal: Prepare
encodes answer to your problem
easy to prepare
CAUTION


QC
Adiabatic
H_{init} gd state $|\psi_0\rangle$
H_{final} gd state $|\psi_{final}\rangle$
$H_{init} = \sum_x |x\rangle\langle x|$
$|\psi_0\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)^{\otimes n}$
Goal: Prepare ψ_{final}
↳ encodes answer to your problem
3SAT: $x_1 \dots x_n \in \{0,1\}$
$C_1 = x_1 \vee x_2 \vee \bar{x}_3$
$C_2 = x_2 \vee \bar{x}_4 \vee x_5$
C_m $m = poly(n)$
NP-hard
Q: ∃ an assignment to x_i's satisfies all C_i?
n qubits
$C_1 = x_1 \vee x_2 \vee \bar{x}_3$
"Forbidden assgt": $x_1=0$ $x_2=0$ $x_3=1$
$H_1 = |0\rangle_1|0\rangle_2|1\rangle_3\langle 0|_1\langle 0|_2\langle 1|_3 \otimes I$
$H = \sum_{i=1}^{m} H_i$ $\psi = \sum \alpha_x |x\rangle$ s.t. $H|\psi\rangle = 0$
Gd space = span$_{x \text{ satisfying assgts}}\{|x\rangle\}$
CAUTION


$H(s) = (1-s)H_{init} + sH_{final}$
$\delta = \min_s \lambda_1(s) - \lambda_0(s)$
level crossing
λ_1
λ_0
0
s
1
$I\delta$

$H(s) = (1-s)H_{init} + sH_{final}$ $\quad s = \min_s \lambda_1(s) - \lambda_0(s)$

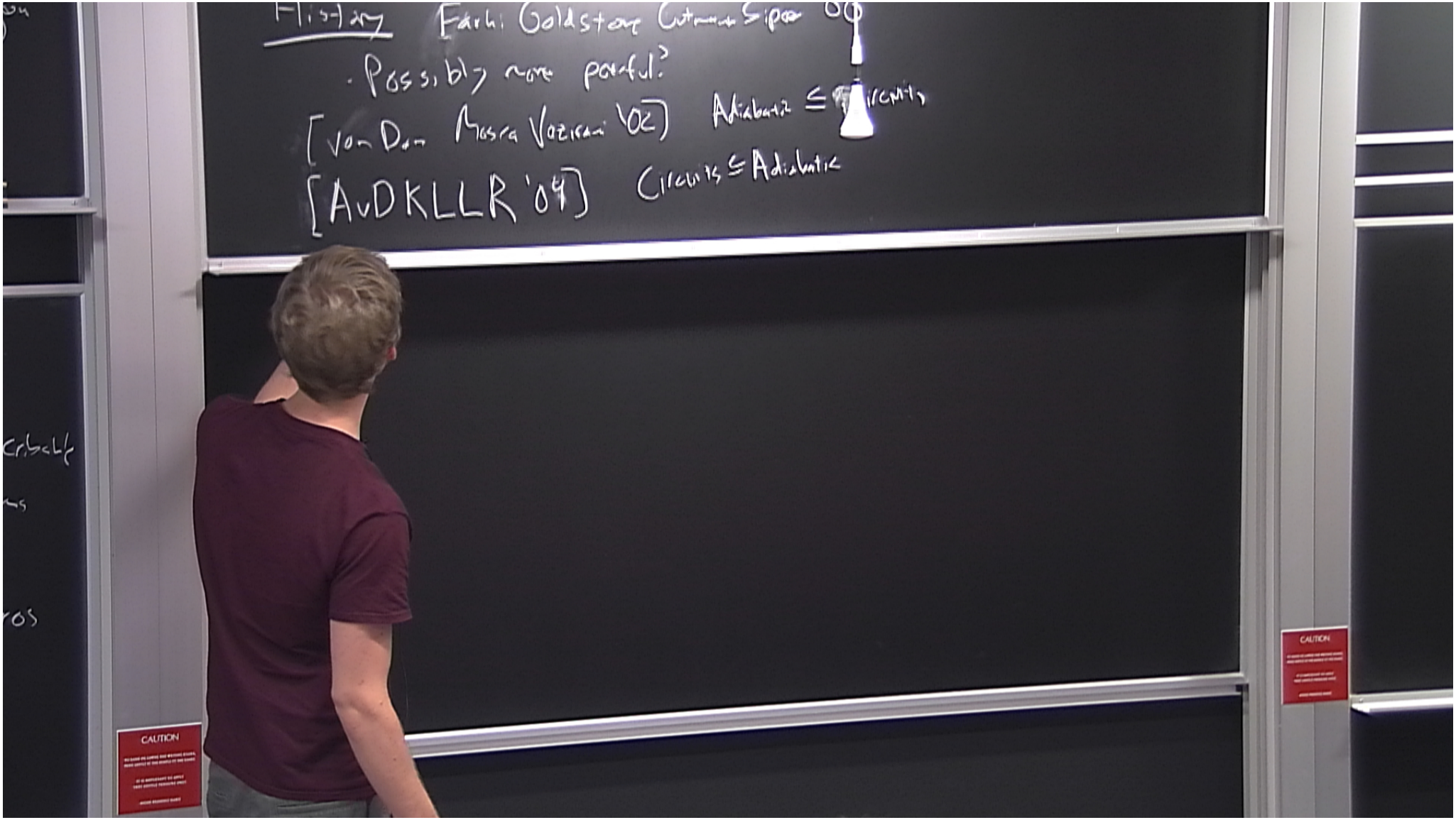


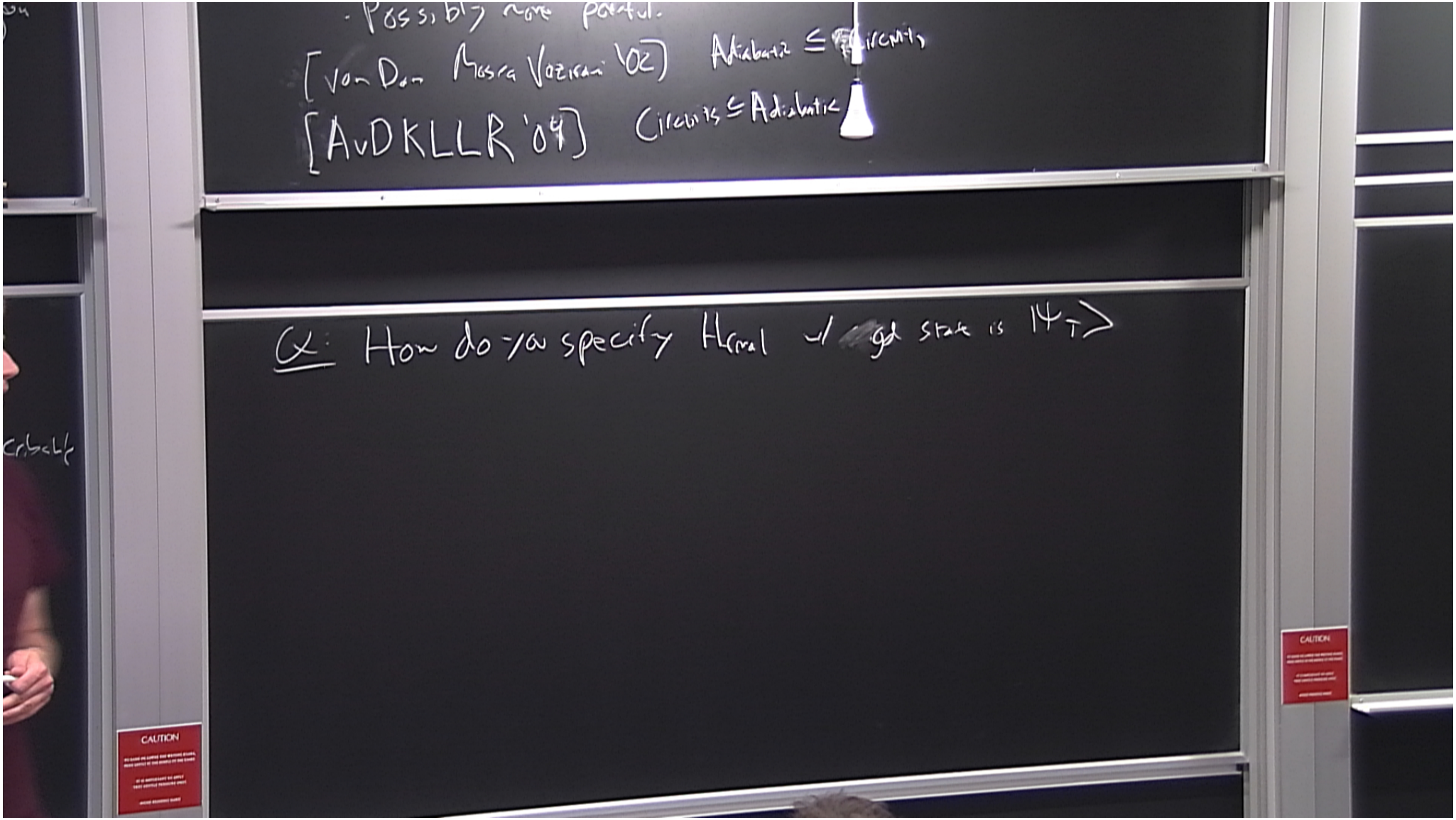


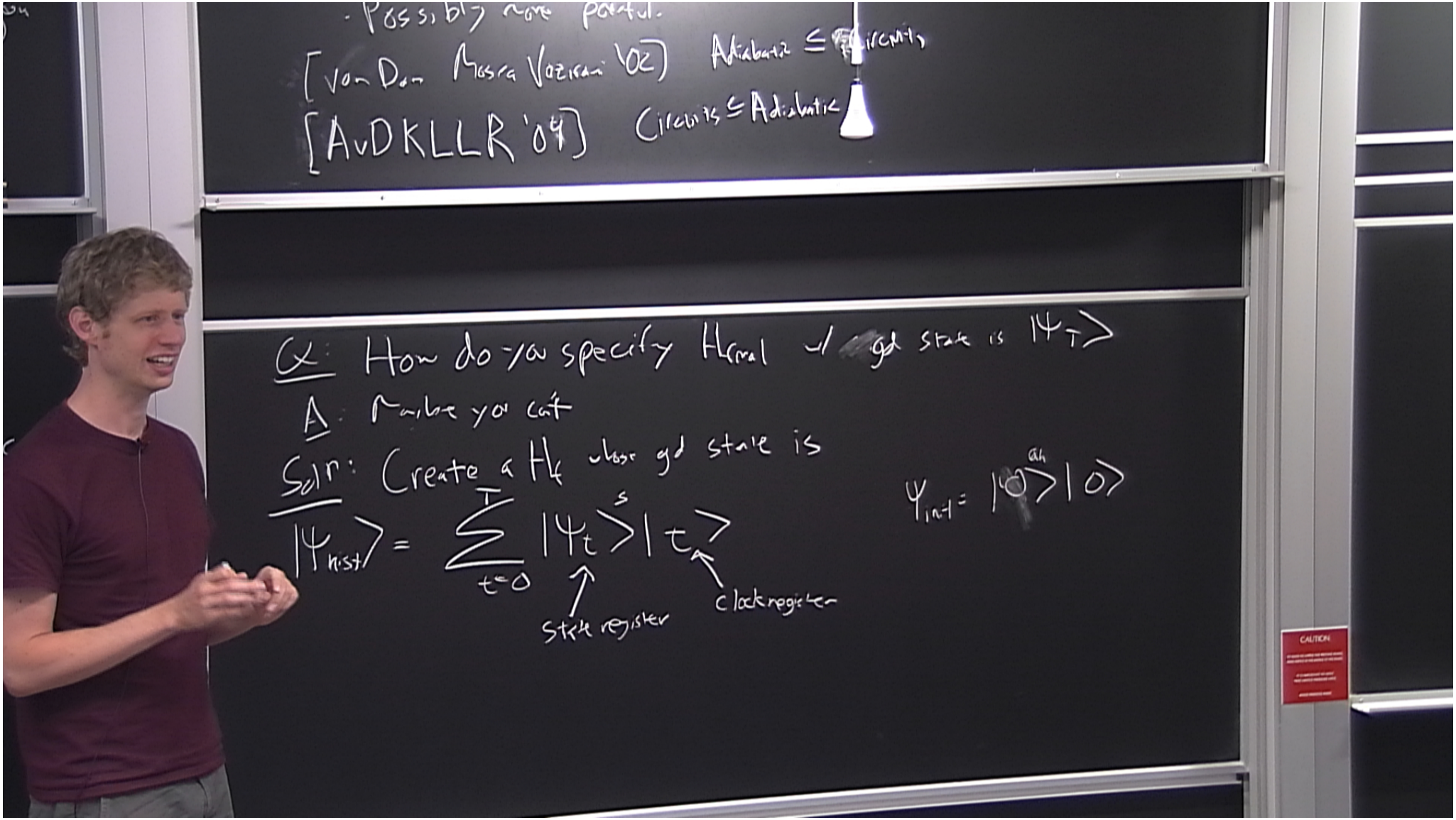
Adiabatic Th:
Vary s from 0 → 1 over time $\sim \frac{1}{\delta^2}$
Stay in gd state whp

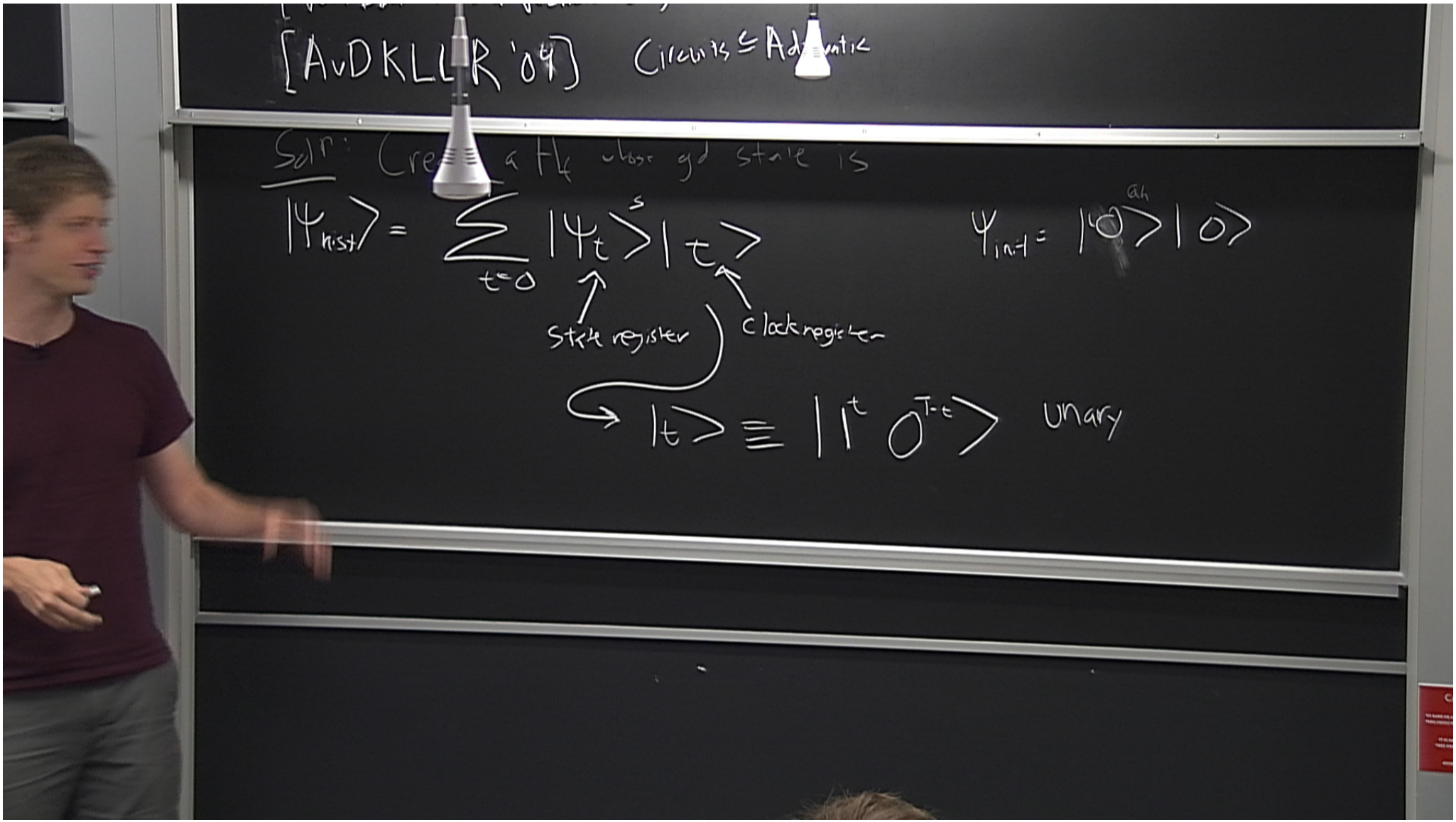
History Farhi Goldstone Gutmann Sipser '00
- Possibly more powerful?

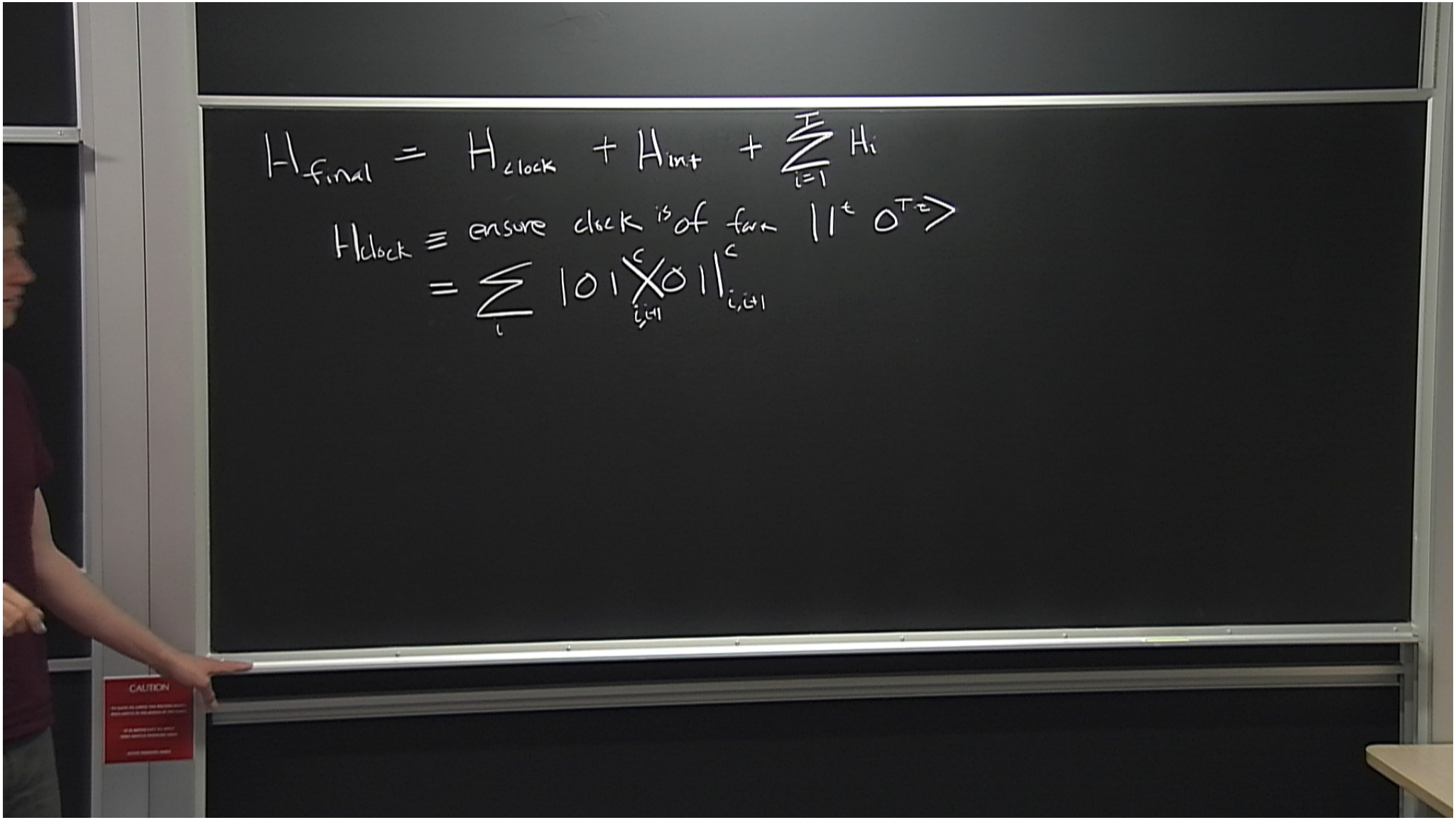
[Van Dam Mosca Vazirani '02] Adiabatic ≤ Circuits

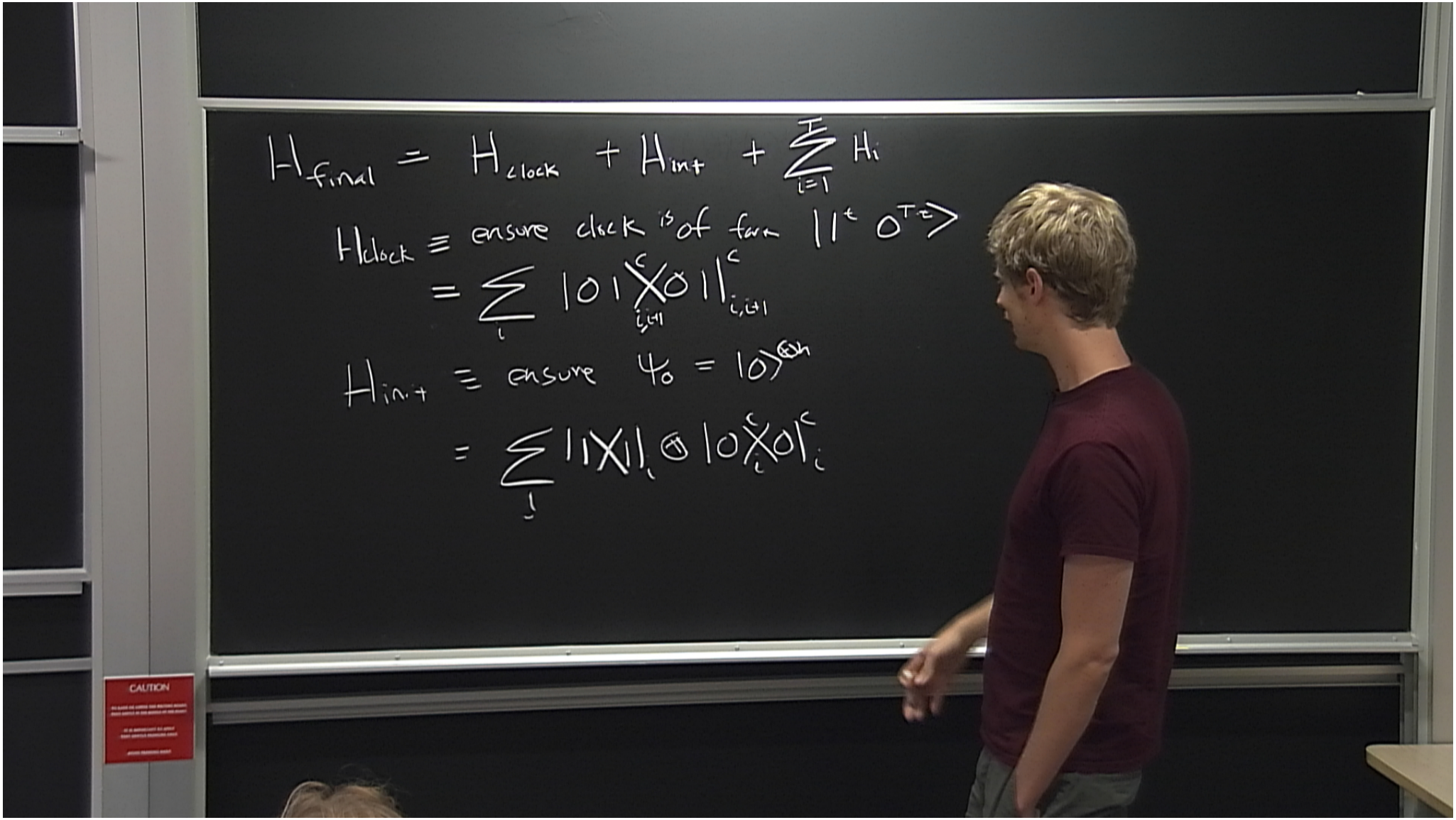

History
Farhi Goldstone
Possibly more powerful?
[vanDam Mosca Vazirani '02] Adiabatic ≤ Circuits
[AvDKLLR '04] Circuits ≤ Adiabatic
CAUTION
CAUTION

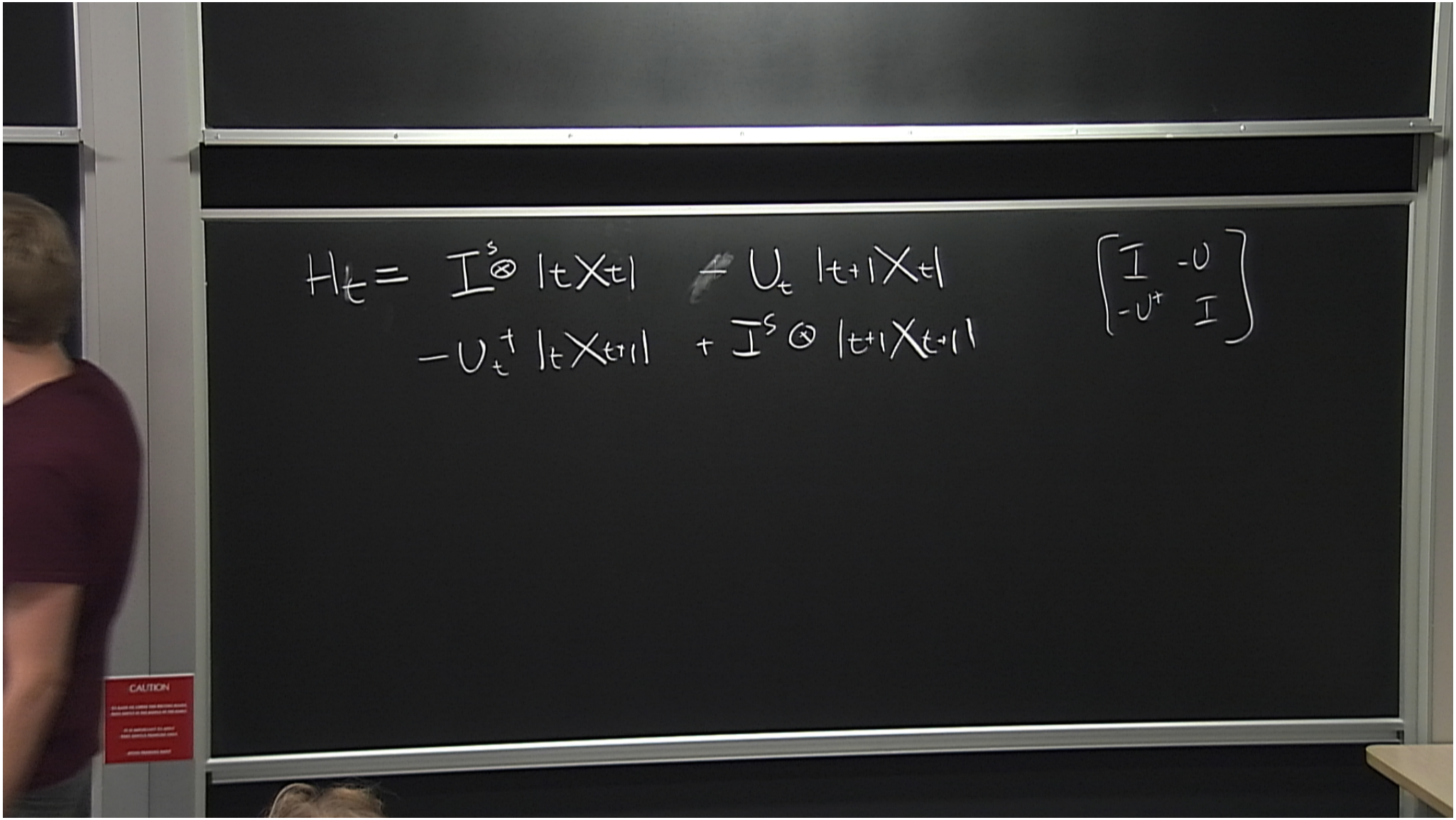

[AvDKLLR '04]
Q: How do you specify Hfinal if ground state is |ψ_T⟩

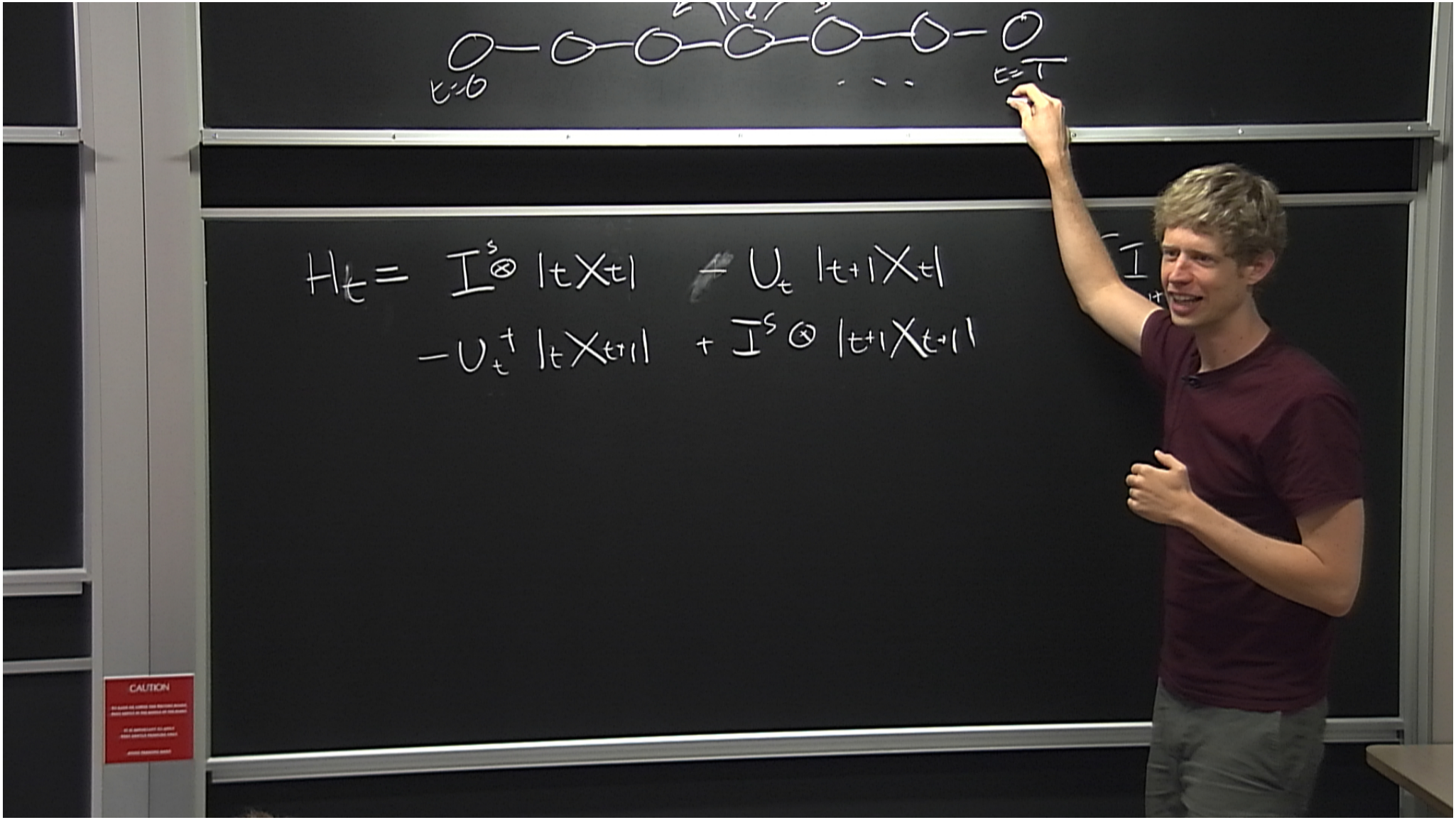

[vanDam Mosca Vazirani '02] Adiabatic ≤ Circuits
[AvDKLLR '04] Circuits ≤ Adiabatic
Q: How do you specify Hfinal w/ gd state is |ψT⟩
A: Maybe you can't
Soln: Create a Hf whose gd state is
|ψhist⟩ = Σ_{t=0}^{T} |ψt⟩^{S} |t⟩
|ψin⟩ = |0⟩^{⊗n} |0⟩
state register
clock register
CAUTION

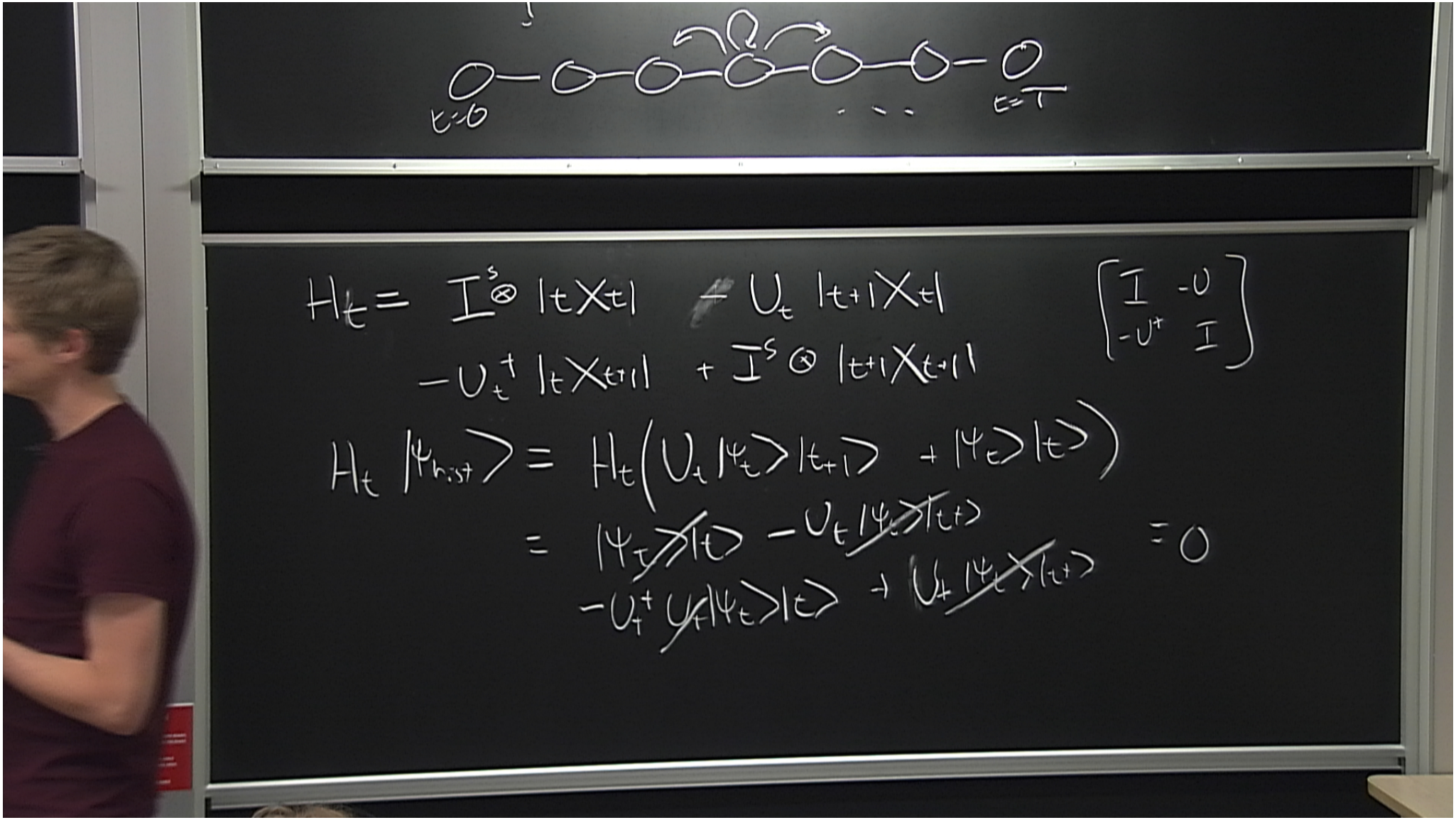

[AvDKLLR '04] Circuits ≤ Ad
Soln: Cre a ft whose gd state is
$|\psi_{hist}\rangle = \sum_{t=0}^{T} |\psi_t\rangle^{S} |t\rangle$
$\psi_{in} = |0\rangle^{\otimes n} |0\rangle$
state register
clock register
$|t\rangle \equiv |1^t 0^{T-t}\rangle$ unary

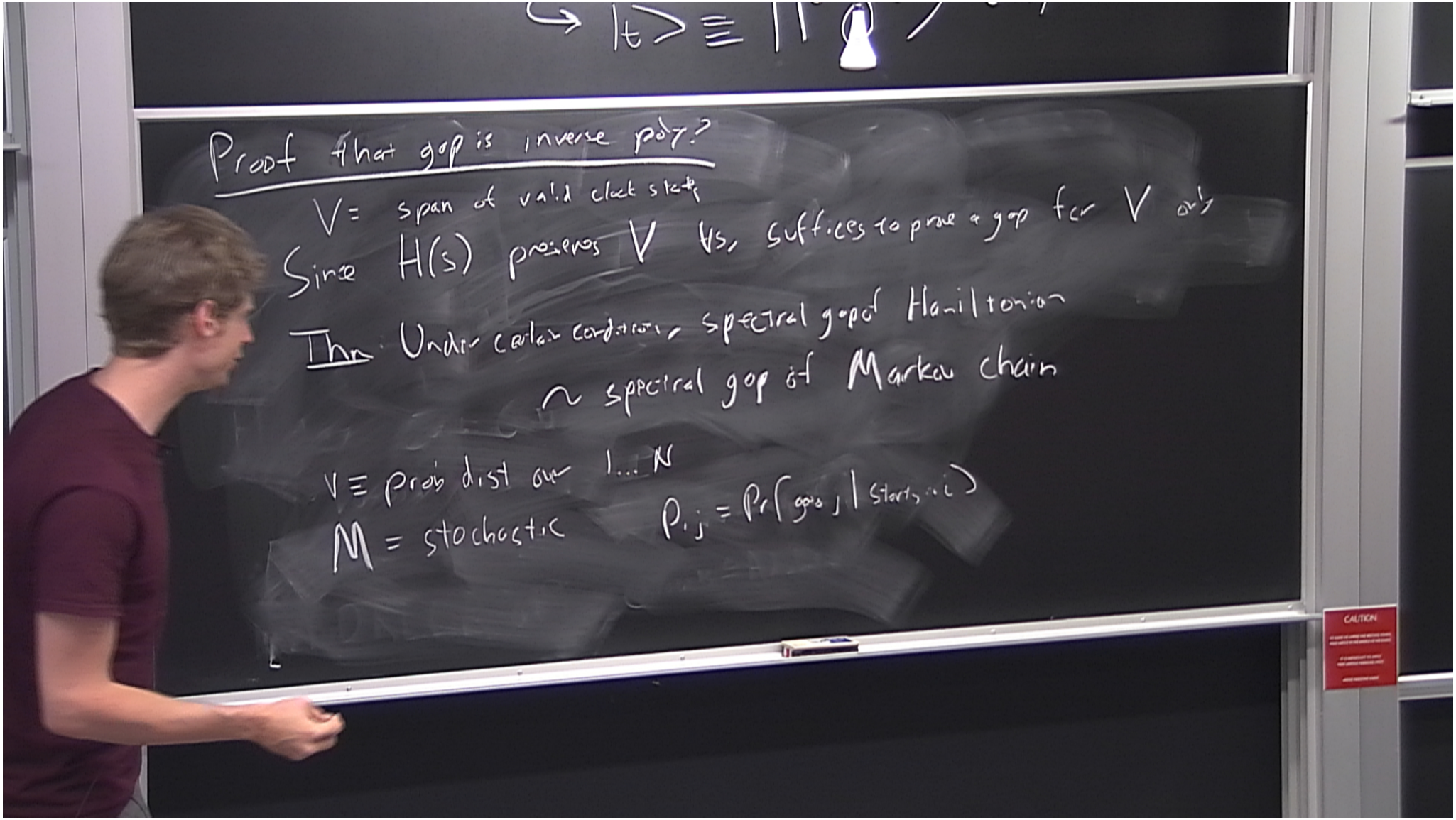

$H_{final} = H_{clock} + H_{int} + \sum_{i=1}^{I} H_i$
$H_{clock} \equiv$ ensure clock is of form $|1^t 0^{T-t}\rangle$
$= \sum_t |01\rangle\langle 01|^c_{i,i+1}$
CAUTION

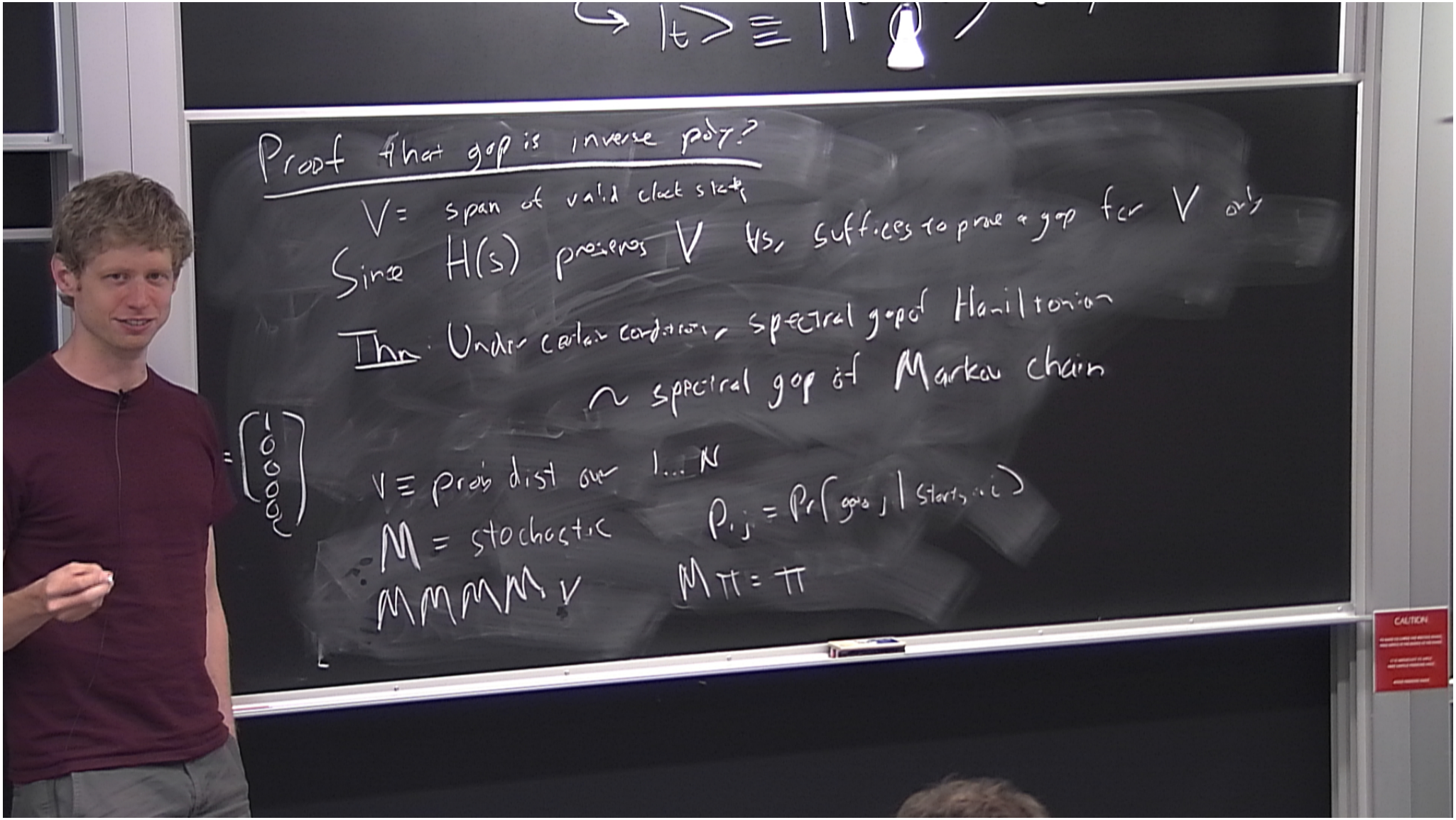

$H_{final} = H_{clock} + H_{int} + \sum_{i=1}^{I} H_i$
$H_{clock} \equiv$ ensure clock is of form $|1^t 0^{T-t}\rangle$
$= \sum_t |01\rangle\langle 01|^C_{i,i+1}$
$H_{int} \equiv$ ensure $\psi_0 = |0\rangle^{\otimes n}$
$= \sum_i |1\rangle\langle 1|_i \otimes |0\rangle\langle 0|^C_i$
CAUTION

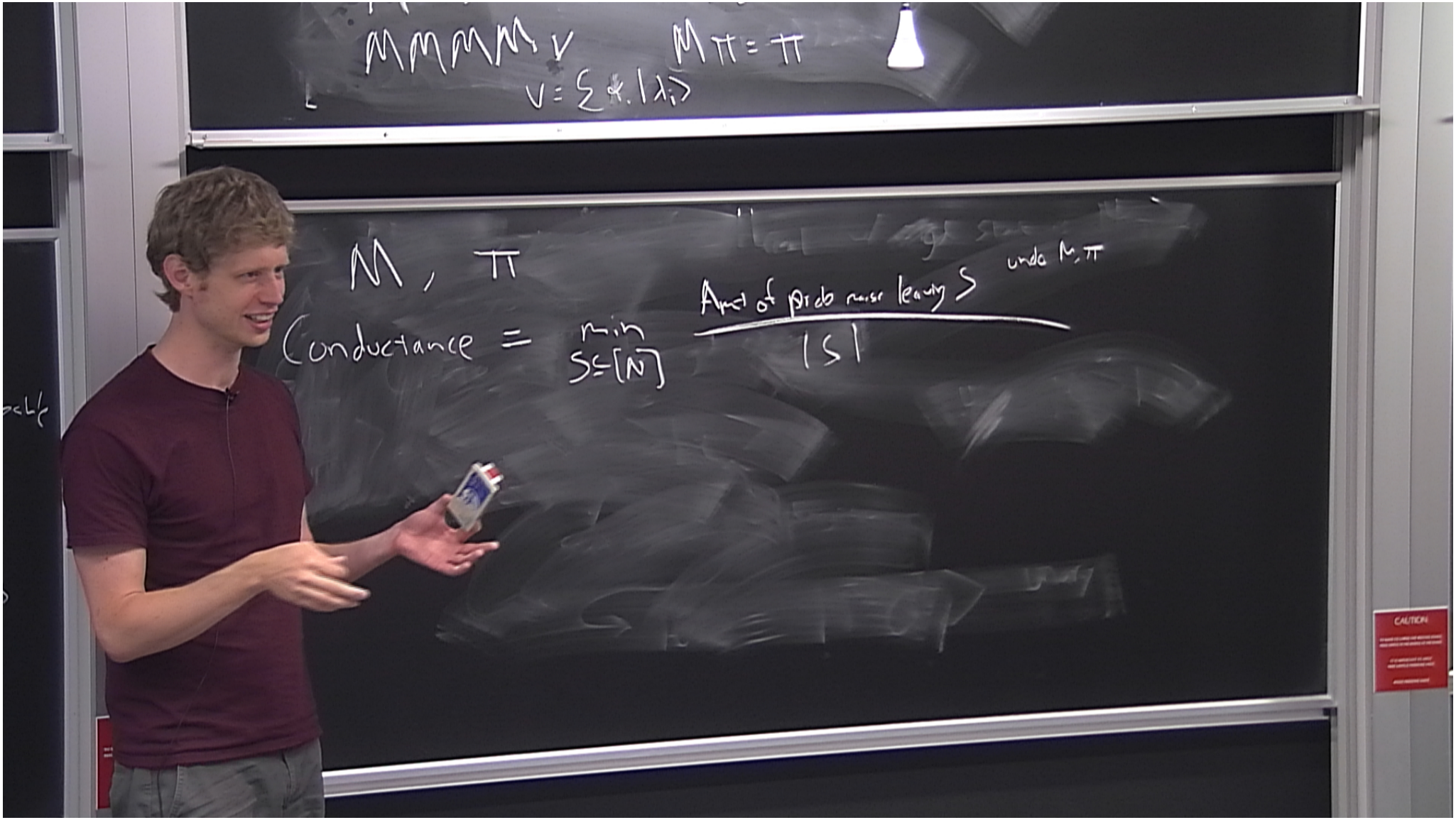

$H_t = I^s \otimes |t\rangle\langle t| - U_t |t+1\rangle\langle t| - U_t^\dagger |t\rangle\langle t+1| + I^s \otimes |t+1\rangle\langle t+1|$
$\begin{bmatrix} I & -U \\ -U^\dagger & I \end{bmatrix}$
CAUTION

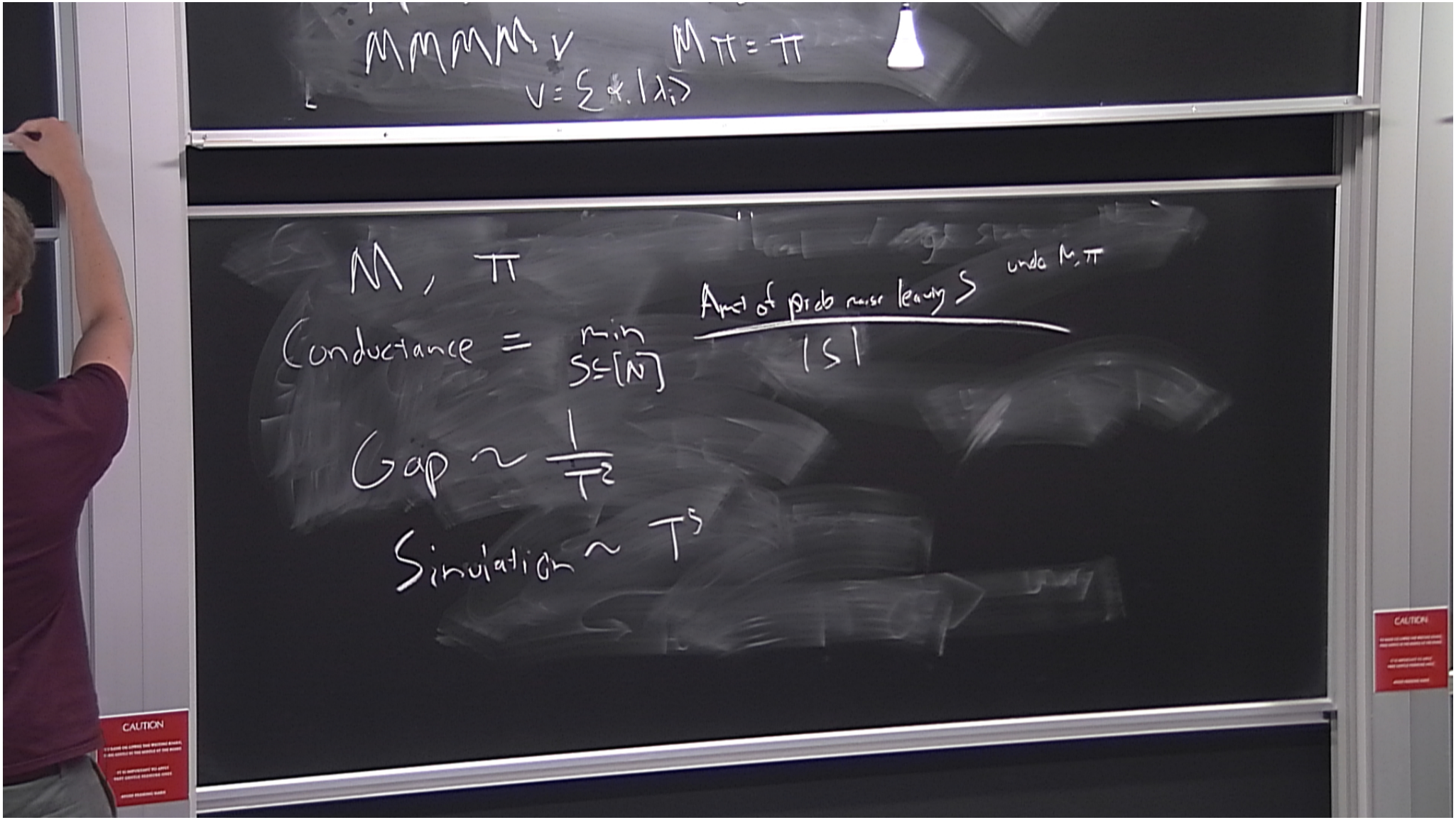

t=0
t=T
$H_t = I^s \otimes |t\rangle\langle t| - U_t |t+1\rangle\langle t| - U_t^\dagger |t\rangle\langle t+1| + I^s \otimes |t+1\rangle\langle t+1|$
CAUTION


t=0
t=T
$H_t = I^s \otimes |t\rangle\langle t| - U_t |t+1\rangle\langle t| - U_t^\dagger |t\rangle\langle t+1| + I^s \otimes |t+1\rangle\langle t+1|$
$\begin{bmatrix} I & -U \\ -U^\dagger & I \end{bmatrix}$
$H_t |\psi_{hist}\rangle = H_t \left(U_t |\psi_t\rangle |t+1\rangle + |\psi_t\rangle |t\rangle \right)$
$= |\psi_T\rangle |0\rangle - U_t |\psi_t\rangle |t+1\rangle - U_t^\dagger U_t |\psi_t\rangle |t\rangle + U_t |\psi_t\rangle |t+1\rangle = 0$


Proof that gap is inverse poly?
V = span of valid clock states
Since H(s) preserves V ∀s, suffices to prove a gap for V only
Thm: Under certain conditions, spectral gap of Hamiltonian
~ spectral gap of Markov chain
V ≡ Prob dist over 1...N
M = stochastic
P_ij = Pr(goes j | starts at i)
CAUTION


Proof that gap is inverse poly?
V = span of valid clock states
Since H(s) preserves V ∀s, suffices to prove a gap for V only
Thm: Under certain conditions, spectral gap of Hamiltonian
~ spectral gap of Markov chain
V ≡ Prob dist over 1...N
M = stochastic
P_ij = Pr(goes j | starts ... i)
MMMMV
MΠ = Π
CAUTION


MMMM, v
Mπ = π
v = Σ α_i |λ_i⟩
M, π
Conductance = min_{S⊆[N]} Amt of prob mass leaving S / |S|
under M, π
CAUTION


MMMM, V
$V = \sum \alpha_i |\lambda_i\rangle$
$M\pi = \pi$
M, π
Conductance = $\min_{S \subseteq [N]}$ Amt of prob mass leaving S / |S| unda M, π
Gap ~ $\frac{1}{T^2}$
Simulation ~ T^5
CAUTION
CAUTION