Title: AdS/CFT correspondence

Date: Jul 25, 2016 11:00 AM

URL: http://pirsa.org/16070037

Abstract:

$CFT_d \longleftrightarrow$ String / quantum gravity

c_{eff}, λ $\qquad$ $AdS_{d+1} \times \chi$

$c_{eff} \gg 1, \quad \lambda \gg 1$ $\qquad$ Einstein - Hilbert gravity

$$\int d^{d+1}x \sqrt{-g}\left[R - 2\Lambda + \cdots\right]$$

$$\underset{CFT}{W[J]} = -\log \left\langle e^{\int J \cdot \mathcal{O}} \right\rangle_{CFT}$$

$$= -\log \mathcal{Z}_{string}\Big|_{\partial = J}$$

$$\simeq I_{grav}\Big|_{\partial = J}$$

y

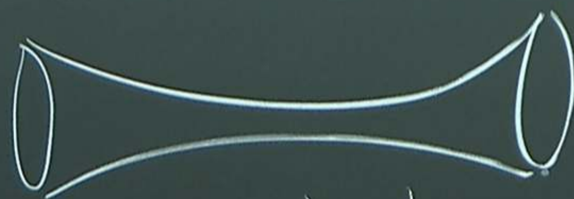
$$-X_{-1} \;\; X_0 \;\; \sum_{i=1} \;\; X_i \;\; = \;\; \ell$$

metric on $\mathbb{R}^{d,2}$

$$ds^2 = -\,dX_{-1}^2 - dX_0^2 + \sum_i dX_i^2$$

ravity

$\Lambda + \cdots\,]$



Intrinsic coordinates

global

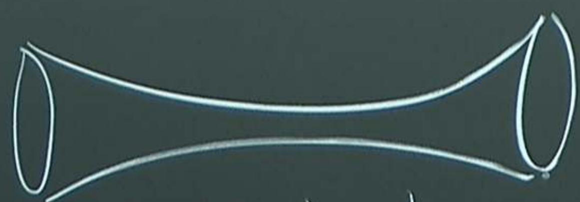
$$ds^2_{d+1} = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{\left(1+r^2/\ell^2\right)} + r^2 d\Omega^2_{d-1}$$

$$d\Omega^2_{d-1} = d\theta_1^2 + \sin^2\theta_1\, d\Omega^2_{d-2}$$

$$\left.\begin{array}{l} AdS_{d+1} : \text{hyperboloid in } \mathbb{R}^{d,2} \\ -X_{-1}^2 - X_0^2 + \sum_{i=1}^{d} X_i^2 = -\ell^2 \end{array}\right\} \quad SO(d,2)$$

metric on $\mathbb{R}^{d,2}$

$$ds^2 = -dX_{-1}^2 - dX_0^2 + \sum_i dX_i^2$$



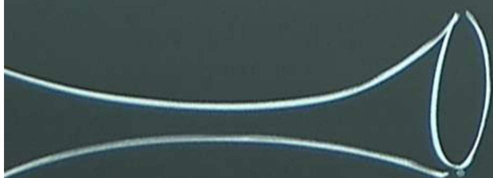
<u>Intrinsic coordinates</u>:

global $$ds_{d+1}^2 = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{(1+r^2/\ell^2)} + r^2 d\Omega_{d-1}^2$$

$$-X_{-1}^2 - X_0^2 + \sum_{i=1} X_i^2 = -\ell^2$$

c on $\mathbb{R}^{d,2}$

$$ds^2 = -dX_{-1}^2 - dX_0^2 + \sum_i dX_i^2$$

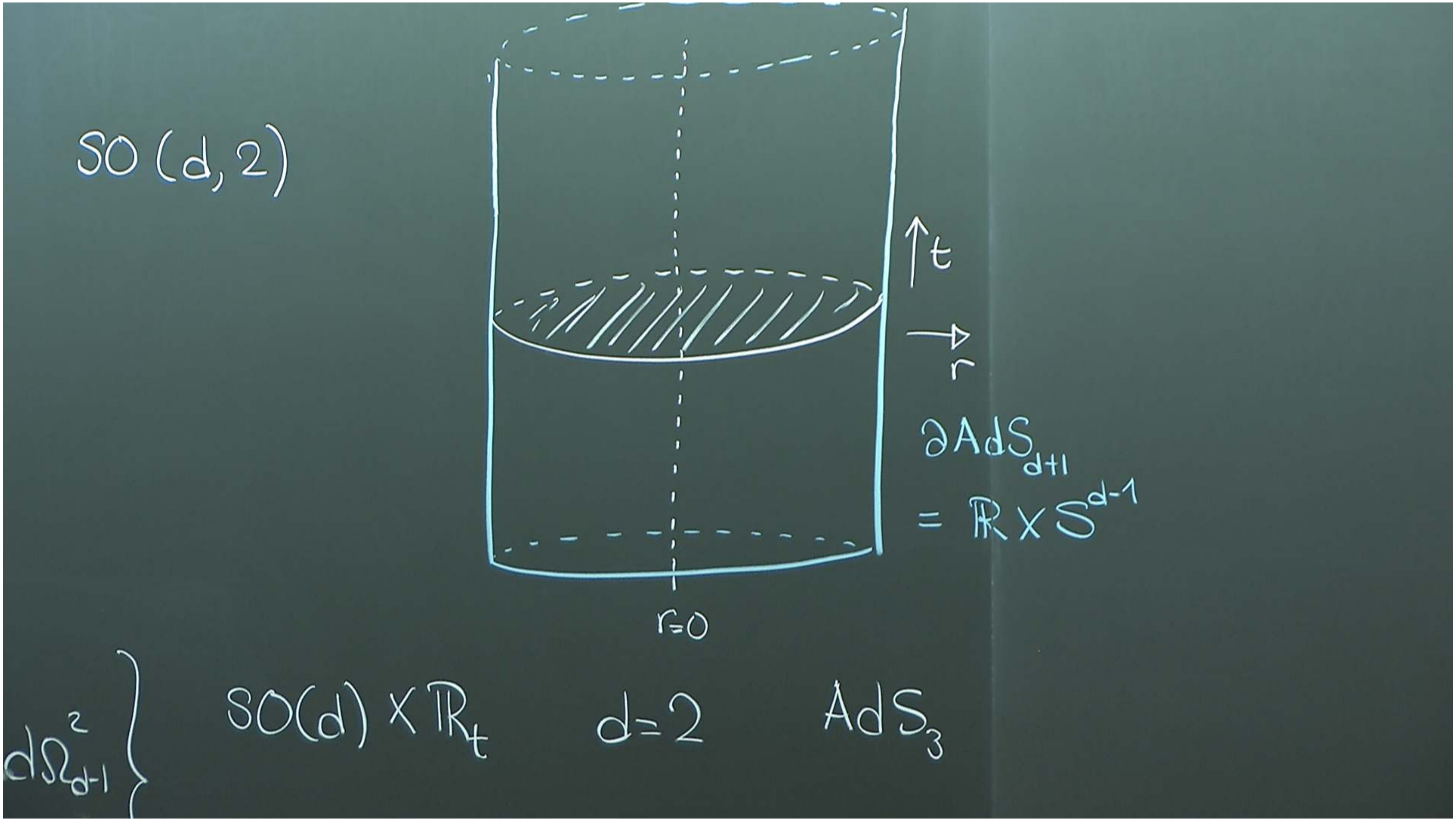


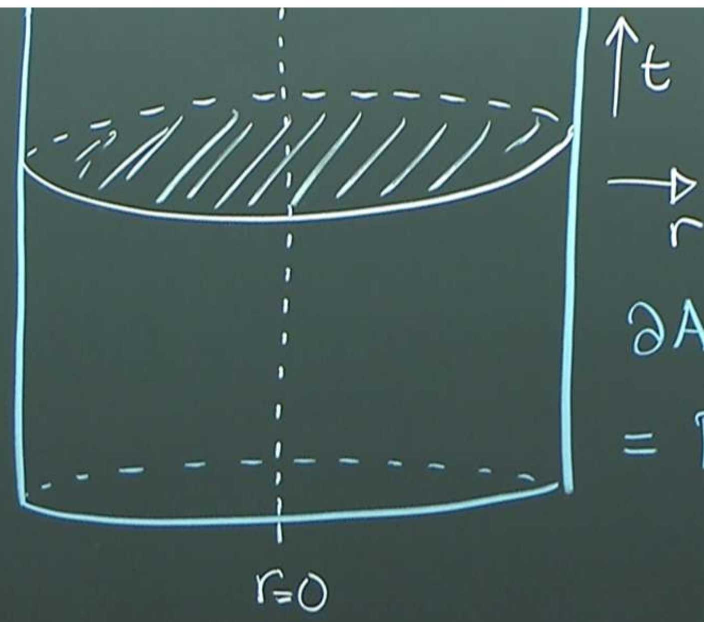
coordinates

$$ds^2_{d+1} = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{(1+r^2/\ell^2)} + r^2 d\Omega^2_{d-1}$$

$SO(d) \times \mathbb{R}_t$

$$= d\theta_1^2 + \sin^2\theta_1 \, d\Omega^2_{d-2}$$


$SO(d,2)$
t
r
∂AdS_{d+1}
$= \mathbb{R} \times S^{d-1}$
$r=0$
$d\Omega^2_{d-1}$
$SO(d) \times \mathbb{R}_t$
$d=2$
AdS_3

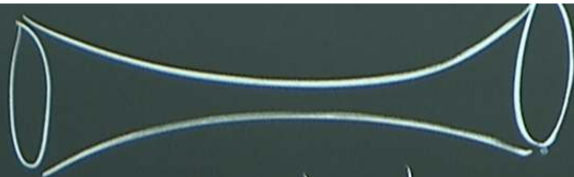


$$\partial AdS_{d+1} = \mathbb{R} \times S^{d-1}$$

$$d\Omega^2_{d-1} \}$$

$$SO(d) \times \mathbb{R}_t \qquad d=2 \qquad AdS_3$$

$$ds_2^2 = \frac{dr^2}{1+r^2/\ell^2} + r^2 d\varphi^2$$



Intrinsic coordinates:

global $ds^2_{d+1} = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{(1+r^2/\ell^2)} + r^2 d\Omega^2_{d-1} \Big\}$ $SO(d) \times$

$$d\Omega^2_{d-1} = d\theta_1^2 + \sin^2\theta_1 \, d\Omega^2_{d-2}$$

Useful for studying CFT_d on $\mathbb{R} \times S^{d-1}$: ESU_d

AdS_{d+1} : hyperboloid in $\mathbb{R}^{d,2}$

$$-X_{-1}^2 - X_0^2 + \sum_{i=1}^{d} X_i^2 = -\ell^2$$

metric on $\mathbb{R}^{d,2}$

$$ds^2 = -dX_{-1}^2 - dX_0^2 + \sum_i dX_i^2$$

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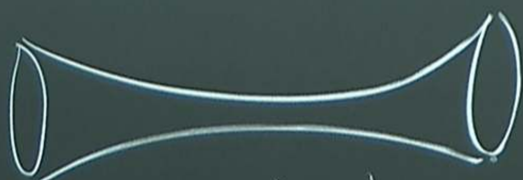
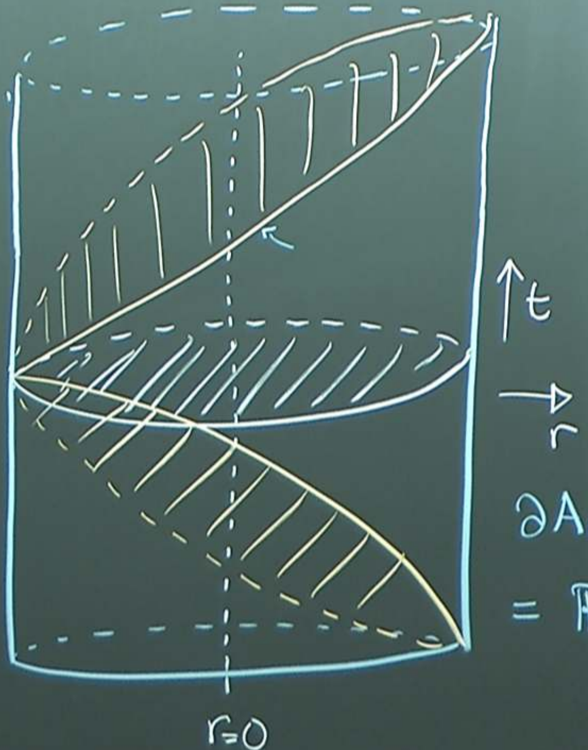


$$\left.\begin{matrix} SO(d,2) \\ \cup \\ SO(d) \times SO(2) \\ \wr \\ \mathbb{R} \end{matrix}\right\}$$

Intrinsic coordinates

global $\quad ds_{d+1}^2 = -\left(1+\frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{(1+r^2/\ell^2)} + r^2 d\Omega_{d-1}^2 \Big\} \quad SO(d) \times \mathbb{R}_t$

$$d\Omega_{d-1}^2 = d\theta_1^2 + \sin^2\theta_1 \, d\Omega_{d-2}^2$$



$\partial AdS_{d+1} = \mathbb{R} \times S^{d-1}$

$\times \mathbb{R}_t \qquad d=2 \qquad AdS_3$

$$ds_2^2 = \frac{dr^2}{1+r^2/\ell^2} + r^2 d\varphi^2$$

Poincaré coordinates:

$$ds^2 = \frac{\ell^2}{z^2}\left[-dt^2 + dz^2 + d\vec{x}_{d-1}^{\,2}\right]$$

explicit $SO(d-1,1)$

useful for CFT_d on $\mathbb{R}^{d-1,1}$

$z \to \infty$ Poincaré horizon

$z \to 0$ Boundary

é coordinates:

$$= \frac{\ell^2}{z^2}\left[-dt^2 + dz^2 + d\vec{x}^{\,2}_{d-1}\right]$$

explicit $SO(d-1,1)$

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$z \to \infty$ Poincaré horizon

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Operators in CFT $\mathcal{O}$:

- gauge inv. $\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})$

$\mathrm{Tr}(X^I X^J)$

- carry quantum #s

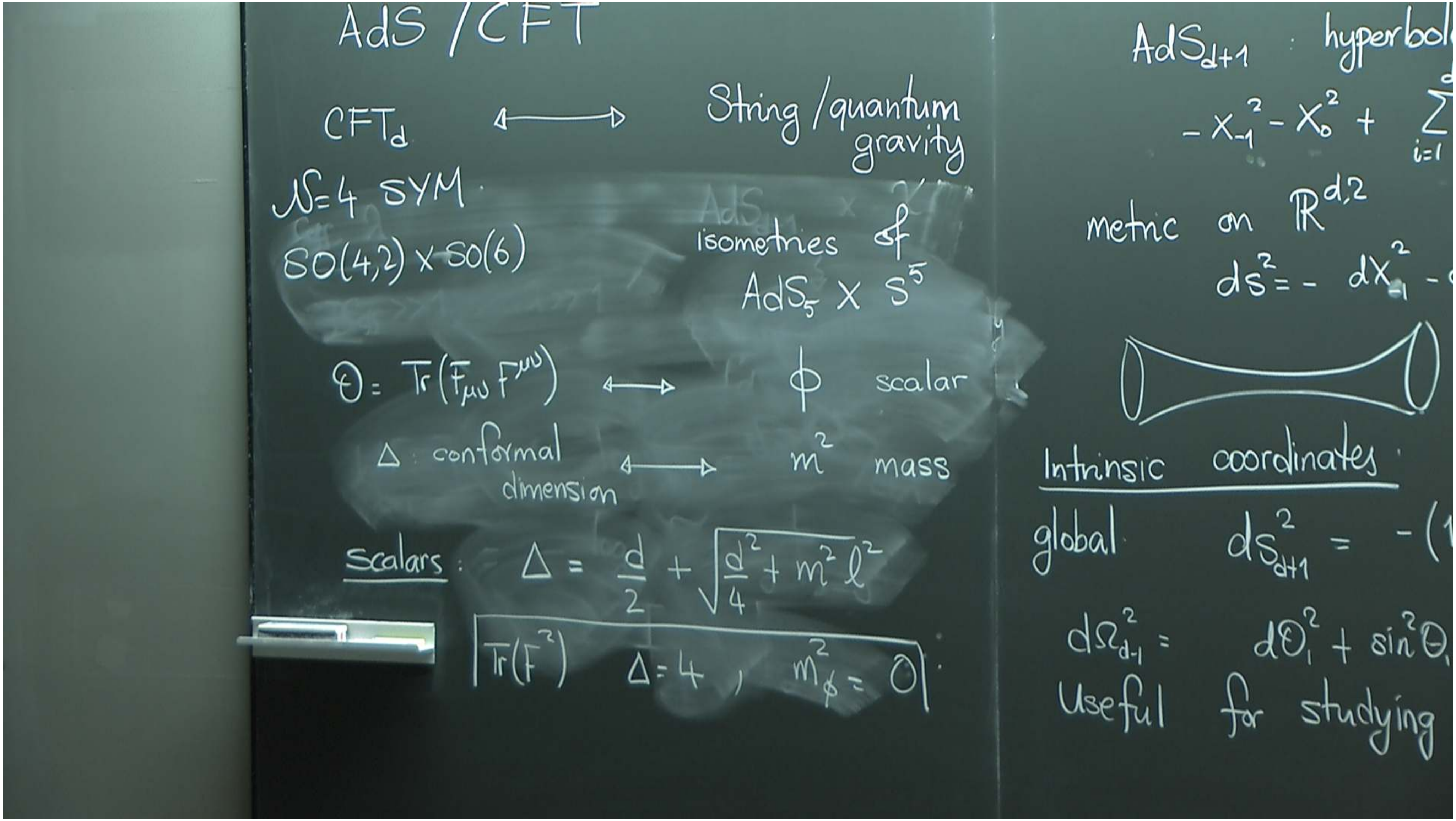
$$SO(d,2) \times R_{int}$$

$$\downarrow$$

quantum #s in dual description.

$SO(d,2)$: isometry group of AdS_{d+1}

R_{int} : isometry group X


AdS / CFT
CFT$_d$ ⟷ String / quantum gravity
$\mathcal{N}=4$ SYM
SO(4,2) × SO(6)
isometries of AdS$_5$ × S^5
$\Theta = \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ ⟷ ϕ scalar
Δ : conformal dimension ⟷ m^2 mass
Scalars : $\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2\ell^2}$
$\mathrm{Tr}(F^2)$ $\Delta = 4$, $m_\phi^2 = 0$
AdS$_{d+1}$: hyperbol
$-X_{-1}^2 - X_0^2 + \sum_{i=1}^{d}$
metric on $\mathbb{R}^{d,2}$
$ds^2 = -\,dX_{-1}^2 -$
Intrinsic coordinates
global $ds^2_{d+1} = -($
$d\Omega^2_{d-1} = d\Theta_1^2 + \sin^2\Theta_1$
Useful for studying

String / quantum gravity

isometries of $AdS_5 \times S^5$

ϕ scalar

$\longrightarrow$ m^2 mass

$+\sqrt{\frac{d^2}{4} + m^2 \ell^2}$

$\boxed{= 4 \ , \ m_\phi^2 = 0}$

vectors: $J^\mu \longleftrightarrow A_A$

$\nabla_\mu J^\mu = 0$

global symmetry current

massless gauge field.

bulk gauge invariance

$\longrightarrow$ b'dy global sym.

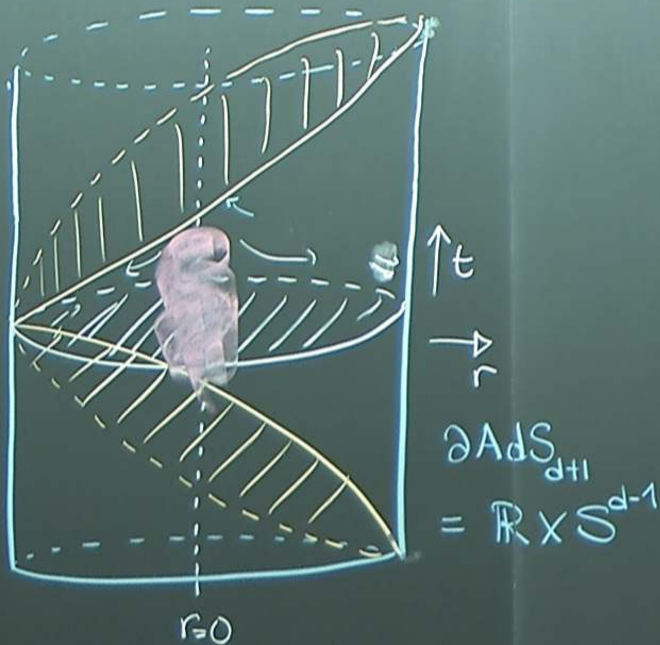
boundary gauge sym is invisible in the bulk:

$\mathbb{R}^{d,2}$

SO(d, 2)

SO(d) x SO

SO(



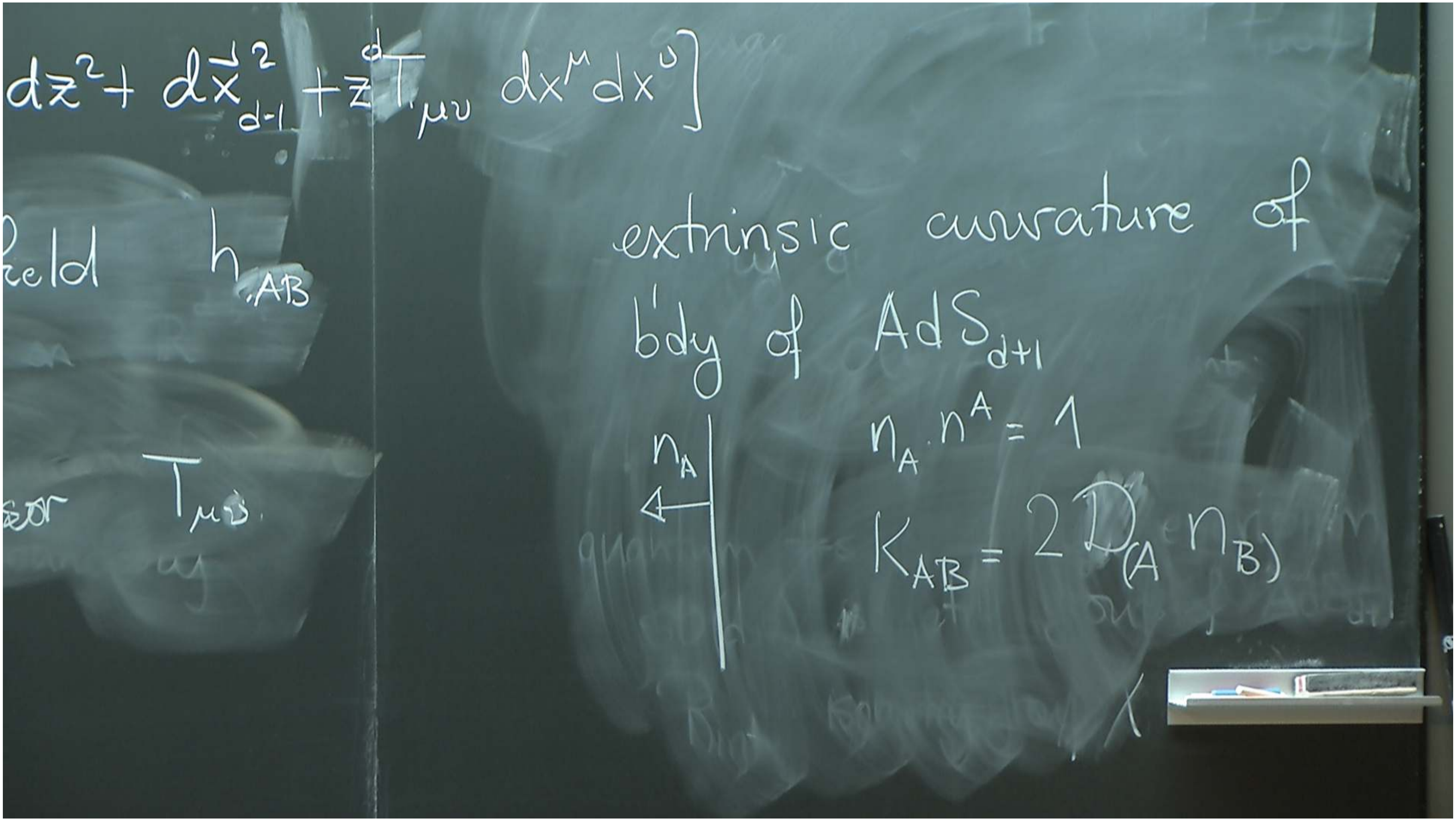
Poincaré coordinates:

$$ds^2 = \frac{\ell^2}{z^2}\left[-dt^2 + dz^2 + d\vec{x}^{\,2}_{d-1} + z^d T_{\mu\nu}\, dx^\mu dx^\nu\right]$$

spin-2 massless field h_{AB}

$\downarrow$

energy momentum tensor $T_{\mu\nu}$


$dz^2 + d\vec{x}^2_{d-1} + z^d T_{\mu\nu} dx^\mu dx^\nu]$
eld h_{AB}
sor $T_{\mu\nu}$
extrinsic curvature of
bdy of AdS_{d+1}
n_A
$n_A \cdot n^A = 1$
$K_{AB} = 2 D_{(A} n_{B)}$

$\partial AdS_{d+1} = \mathbb{R} \times S^{d-1}$

spin-2 massless field h_{AB}

$\downarrow$

energy momentum tensor $T_{\mu\nu}$

$$T^{\mu\nu} = K^{\mu\nu} - K\gamma^{\mu\nu} + \cdots$$

Radial direction: z / r.

is

scale in CFT

Look @ Poincaré geometry

$t \to t\Lambda$

$\vec{x} \to \vec{x}\Lambda$

Poincaré metric: inv. $z \to \Lambda z$

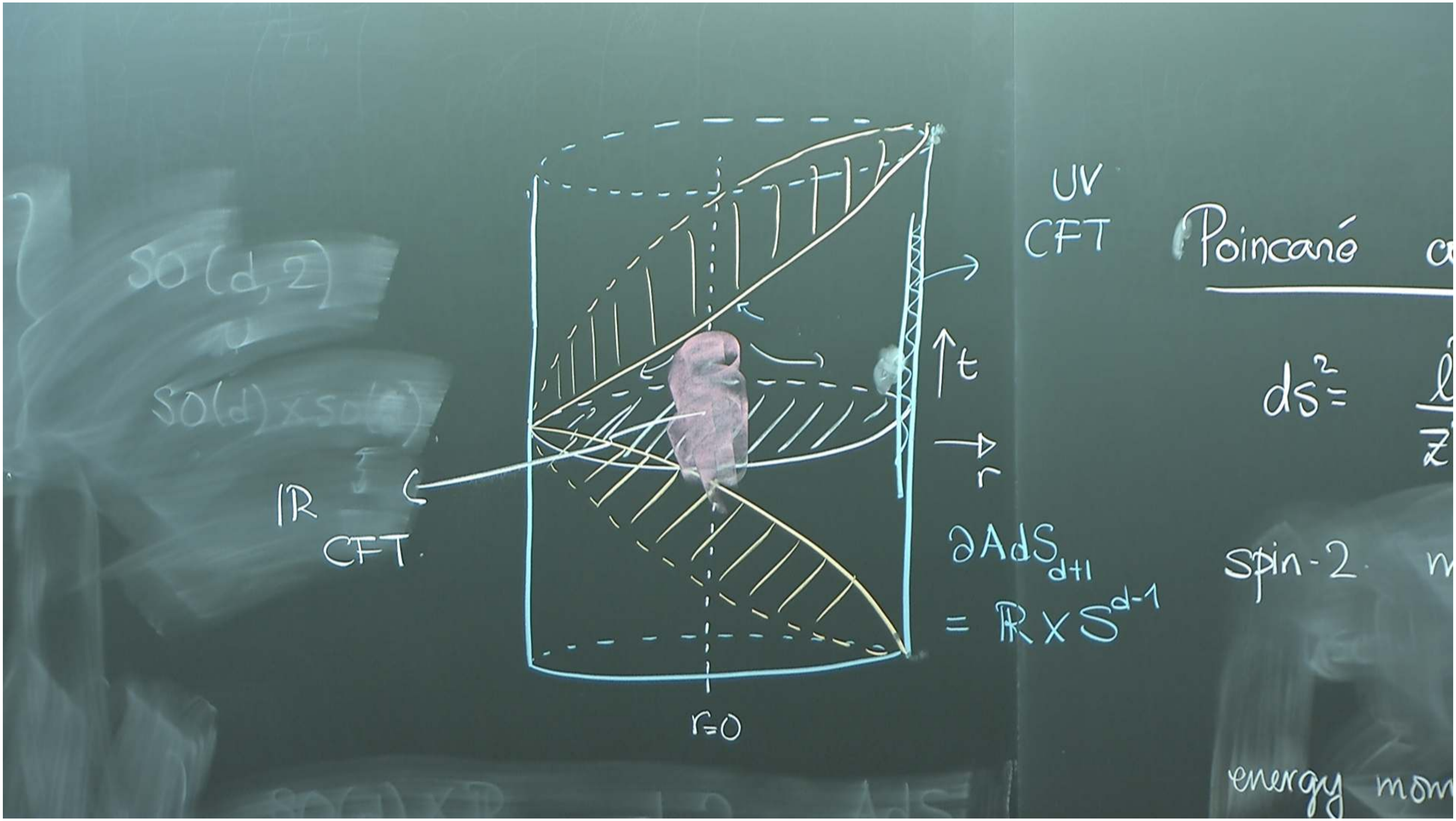
vecto

globa

bu

boun

bulk


UV
CFT
Poincaré
IR
CFT
t
r
∂AdS_{d+1}
$= \mathbb{R} \times S^{d-1}$
r=0
$ds^2 =$
spin-2
energy

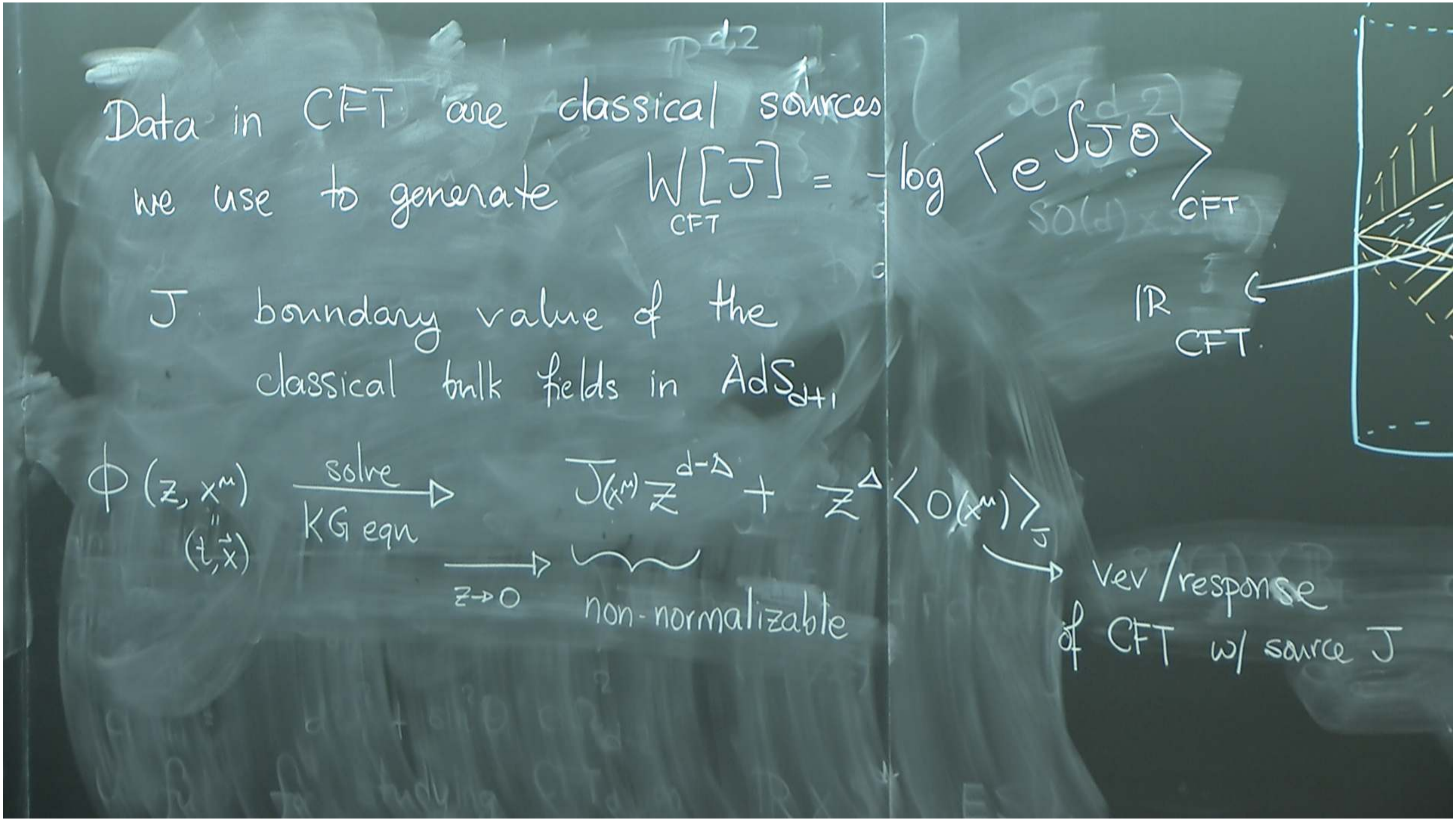
Data in CFT are classical sources we use to generate $W_{CFT}[J]$

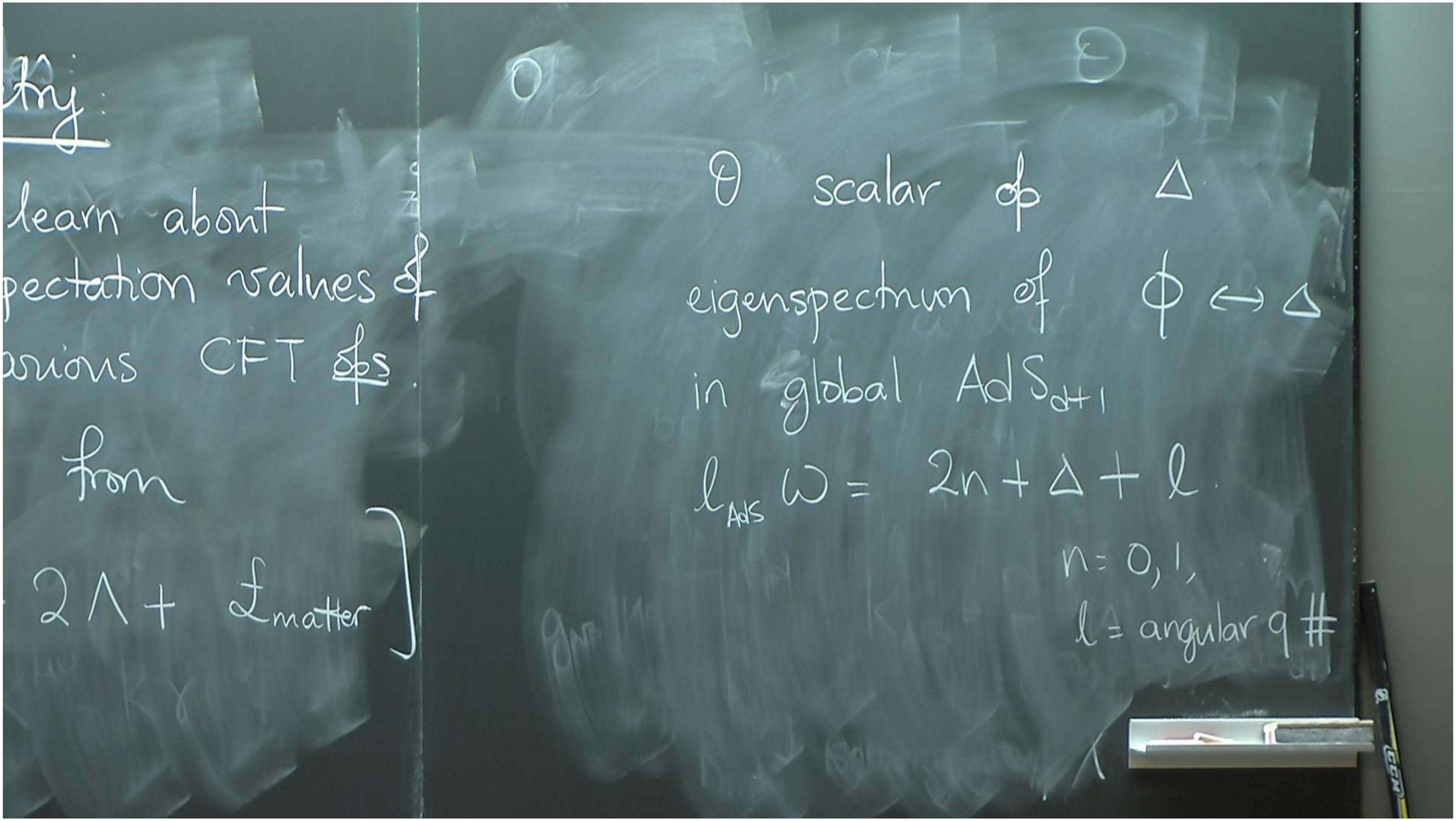
J: boundary value of the classical bulk fields in AdS_{d+1}

$$\phi(z, x^\mu) \xrightarrow[\text{KG eqn}]{\text{solve}} J(x^\mu)\, z^{d-\Delta} + z^{\Delta} \langle O(x^\mu) \rangle$$

$$x^\mu = (t, \vec{x})$$

$$\xrightarrow[z \to 0]{}$$


Data in CFT are classical sources
we use to generate $W_{CFT}[J] = -\log \langle e^{\int J O} \rangle_{CFT}$
J boundary value of the classical bulk fields in AdS_{d+1}
$\phi(z, x^\mu) \xrightarrow[\text{KG eqn}]{\text{solve}} J(x^\mu) z^{d-\Delta} + z^{\Delta} \langle O(x^\mu) \rangle_J$
$(t, \vec{x})$
$z \to 0$
non-normalizable
vev / response of CFT w/ source J
IR CFT.



try:
learn about
pectation values &
arious CFT ops
from
2Λ + ℒmatter
O scalar op Δ
eigenspectrum of φ ↔ Δ
in global AdSd+1
lAdS ω = 2n + Δ + l
n = 0, 1,
l = angular q #

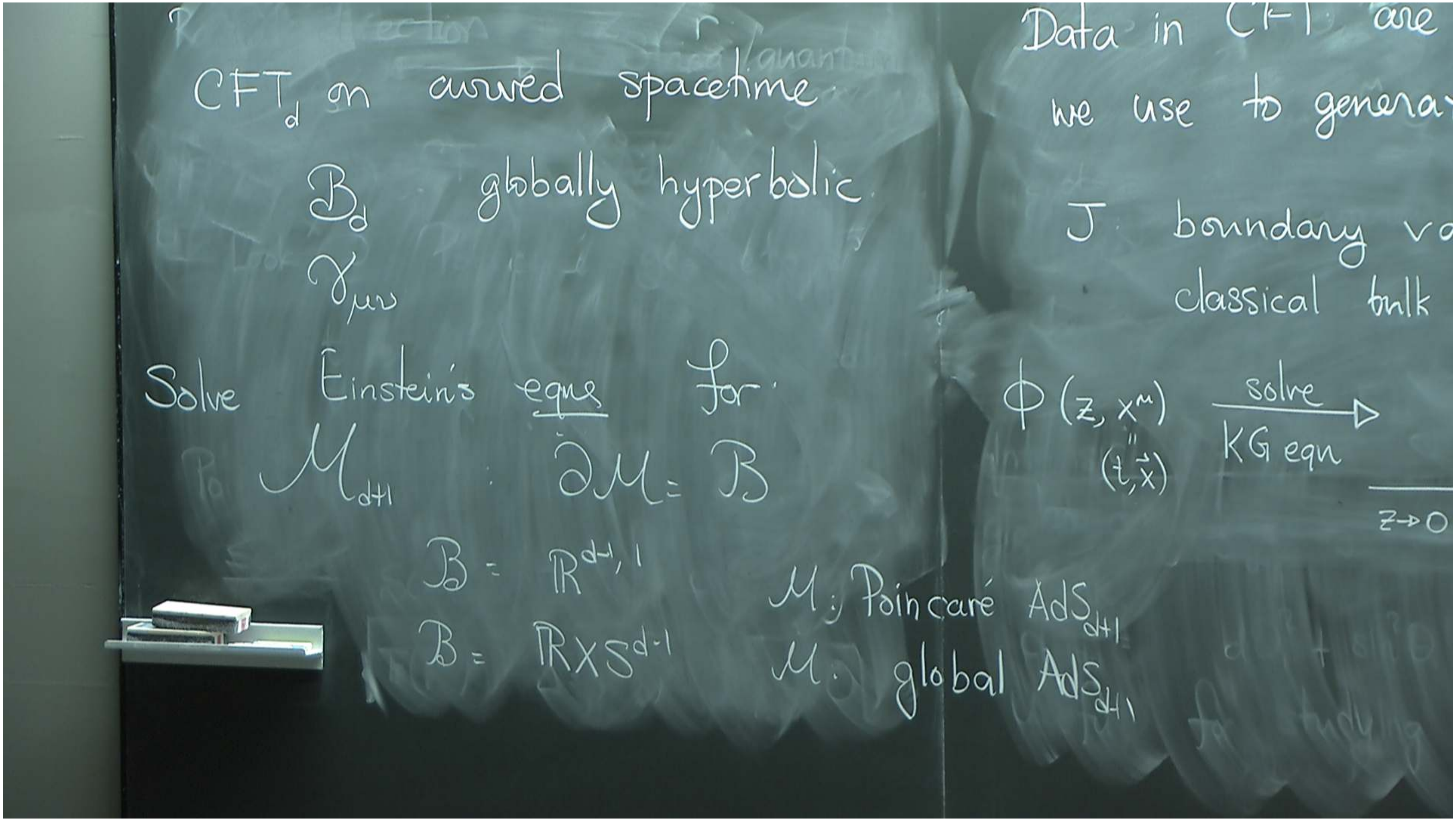
expectation values of
various CFT ops.

Solve the equations from

$$\frac{1}{16\pi G}\int d^{d+1}x\sqrt{-g}\left[R-2\Lambda+\mathcal{L}_{matter}\right]$$

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_{d-1}^2$$

$\phi(r)$



CFT$_d$ on curved spacetime
$\mathcal{B}_d$ globally hyperbolic
$\gamma_{\mu\nu}$
Solve Einstein's eqns for
$\mathcal{M}_{d+1}$: $\partial\mathcal{M} = \mathcal{B}$
$\mathcal{B} = \mathbb{R}^{d-1,1}$
$\mathcal{M}$: Poincaré AdS$_{d+1}$
$\mathcal{B} = \mathbb{R} \times S^{d-1}$
$\mathcal{M}$: global AdS$_{d+1}$
Data in CFT are
we use to genera
J : boundary va
classical bulk
$\phi(z, x^\mu)$
$(t, \vec{x})$
solve
KG eqn
$z \to 0$

Thermal / Gibbs density matrix for CFT_d.

$$\rho = e^{-\beta H}$$

$$Z(\beta) = \mathrm{Tr}\left(e^{-\beta H}\right)$$

QFT_d @ temperature $T = \frac{1}{\beta}$

$\updownarrow$

d Euclidean statistical model w/ $t_E \sim t_E + \beta$

bbs density matrix

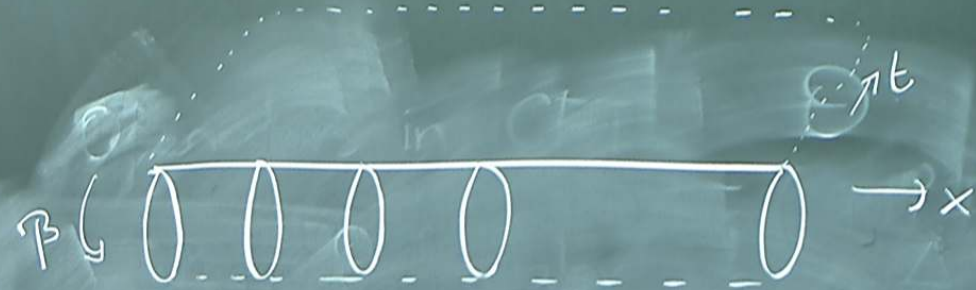
$= e^{-\beta H}$

$= \mathrm{Tr}(e^{-\beta H})$

mperature $T = \frac{1}{\beta}$

$\updownarrow$

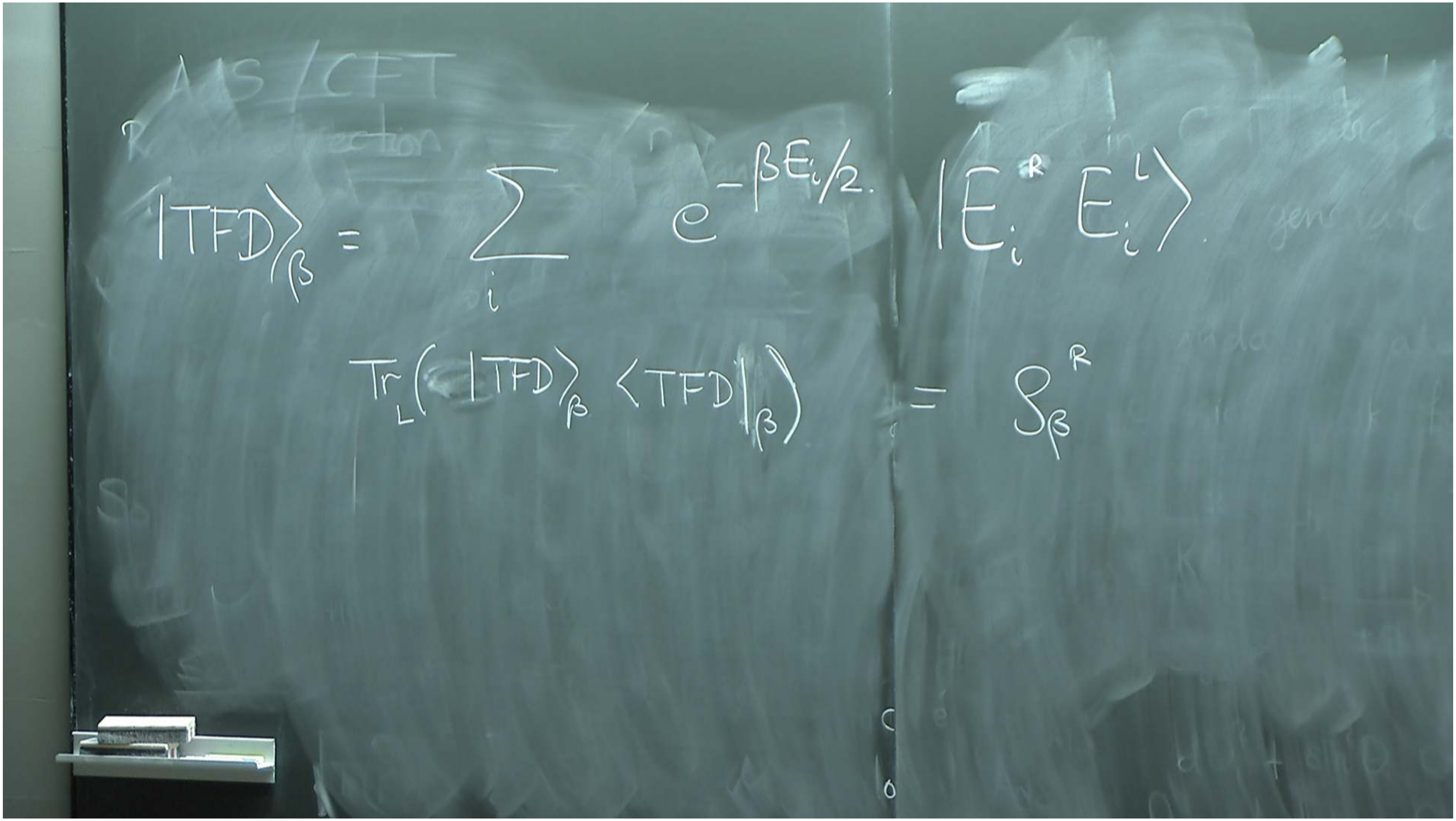
model w/ $t_E \sim t_E + \beta$



$$\rho_\beta = \sum_i e^{-\beta E_i} |E_i\rangle\langle E_i|$$

Purify this ρ_β to $|TFD\rangle_\beta$

in $\mathcal{H}_{CFT} \otimes \mathcal{H}_{CFT}$

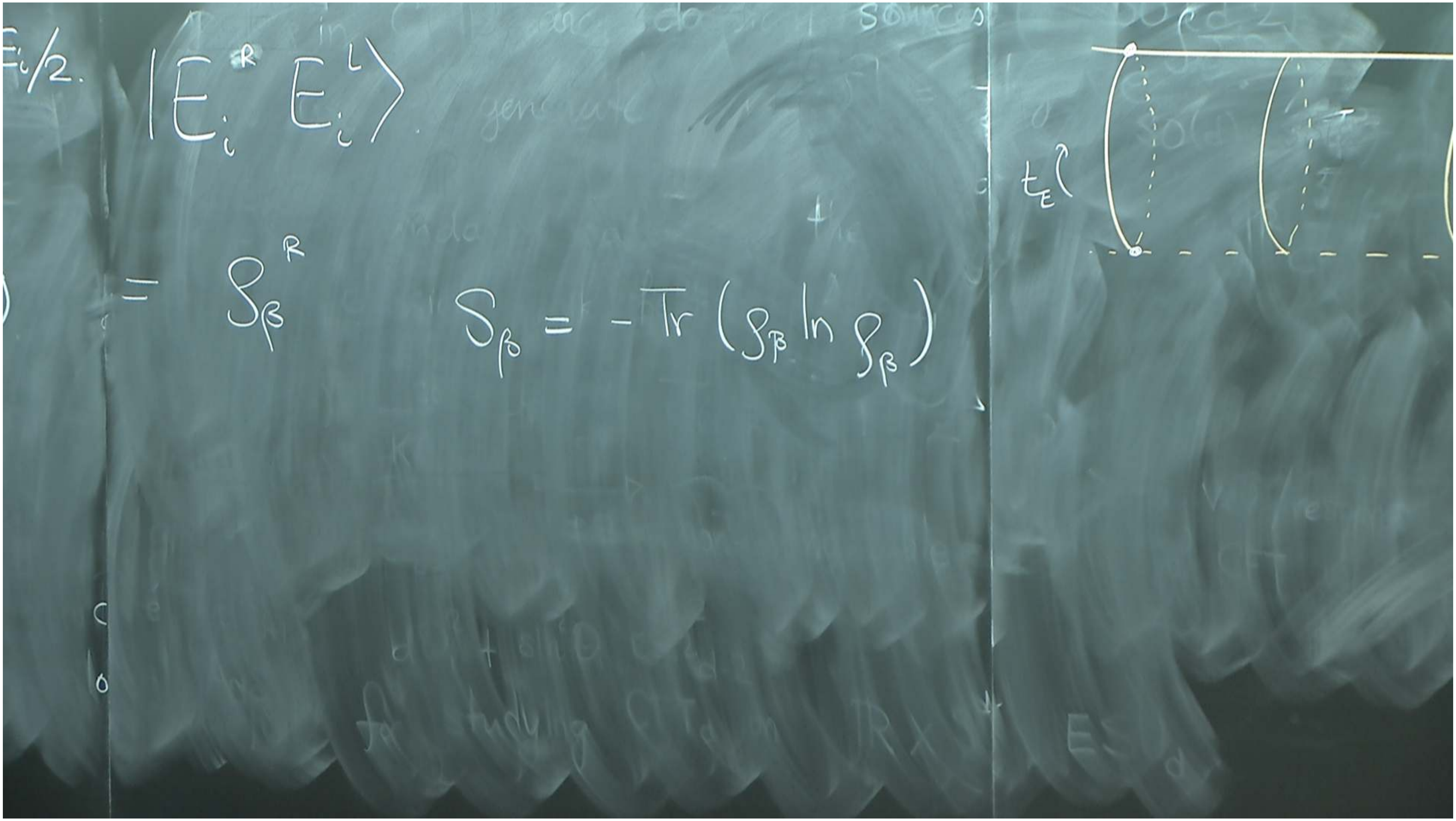

$|TFD\rangle_\beta = \sum_i e^{-\beta E_i/2} |E_i^R E_i^L\rangle$
$Tr_L(|TFD\rangle_\beta \langle TFD|_\beta) = \rho_\beta^R$

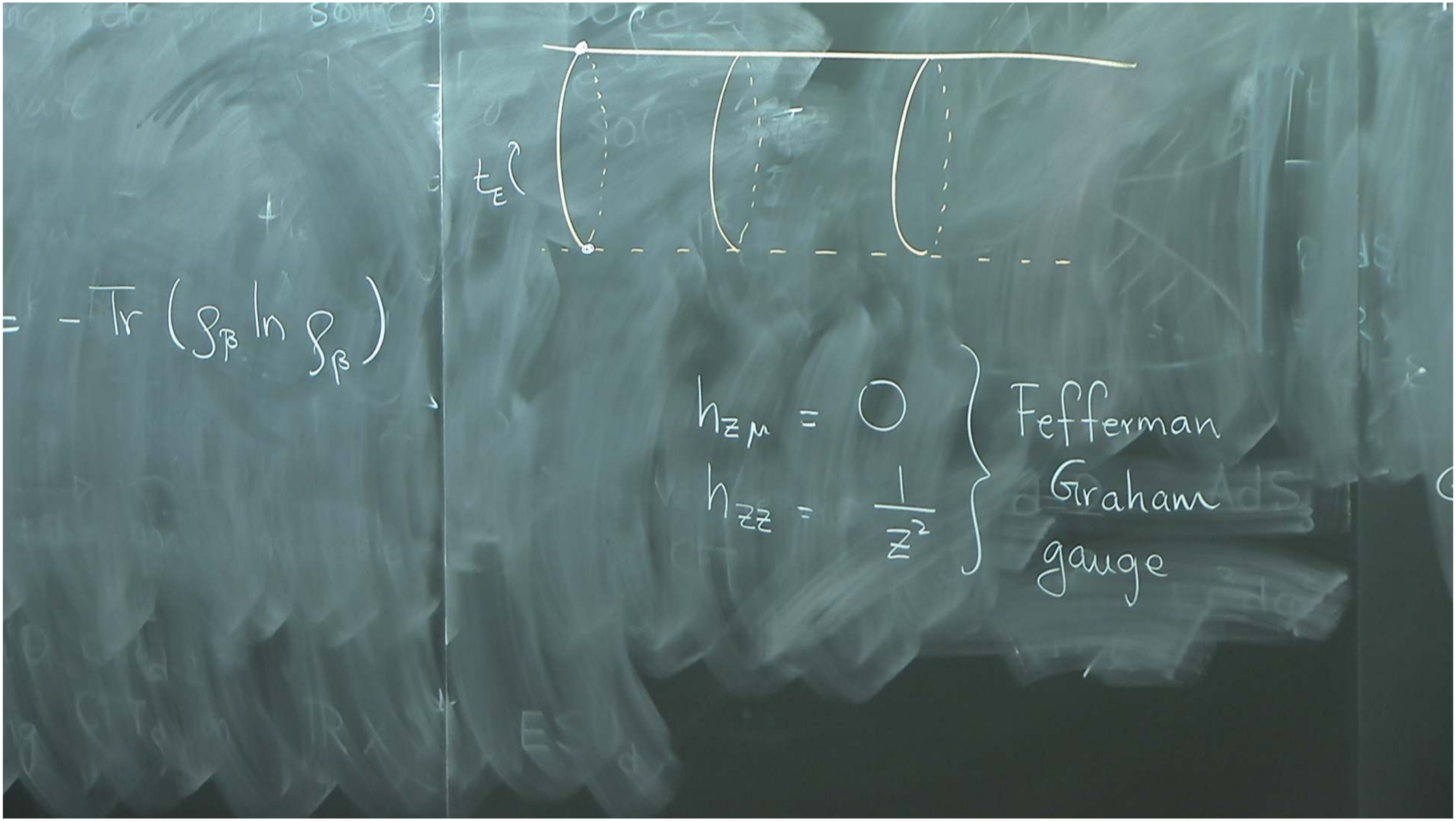
$$|TFD\rangle_\beta = \sum_i e^{-\beta E_i/2} |E_i^R E_i^L\rangle$$

$$\mathrm{Tr}_L\left(|TFD\rangle_\beta \langle TFD|_\beta\right) = \rho_\beta^R$$

$$H = \left(H_R \otimes \mathbb{1}_L - \mathbb{1}_R \otimes H_L\right)$$

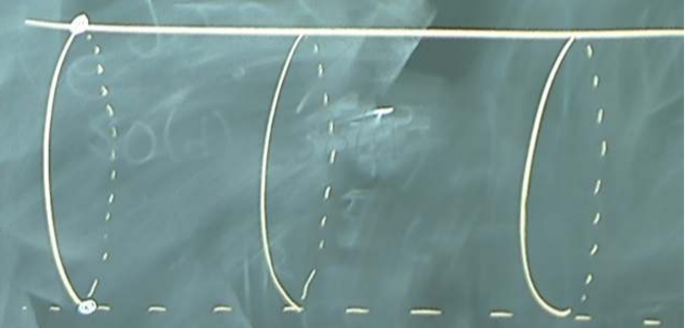
$$H|TFD\rangle_\beta = 0$$


$E_i/2.$
$|E_i^R \, E_i^L\rangle$
$= \rho_\beta^R$
$S_\beta = -\mathrm{Tr}(\rho_\beta \ln \rho_\beta)$
t_E


t_E
$= -\mathrm{Tr}\,(\rho_\beta \ln \rho_\beta)$
$h_{z\mu} = 0$
$h_{zz} = \frac{1}{z^2}$
Fefferman
Graham
gauge

$$|E_i^R\, E_i^L\rangle$$

t_E



$$= \rho_\beta^R \qquad S_\beta = -\mathrm{Tr}\left(\rho_\beta \ln \rho_\beta\right)$$

$$\left.\begin{aligned} h_{z\mu} &= 0 \\ h_{zz} &= \frac{1}{z^2} \end{aligned}\right\}$$

$$\boxed{T^{\mu\nu} = \#\left[K^{\mu\nu} - K\gamma^{\mu\nu} - (d-1)\gamma^{\mu\nu} + \frac{1}{2(d-2)}\left[{}^{\gamma}R^{\mu\nu} - \frac{1}{2}\,{}^{\gamma}R\,\gamma^{\mu\nu}\right]\right]}$$