Title: Simulation of Quantum Hamiltonians

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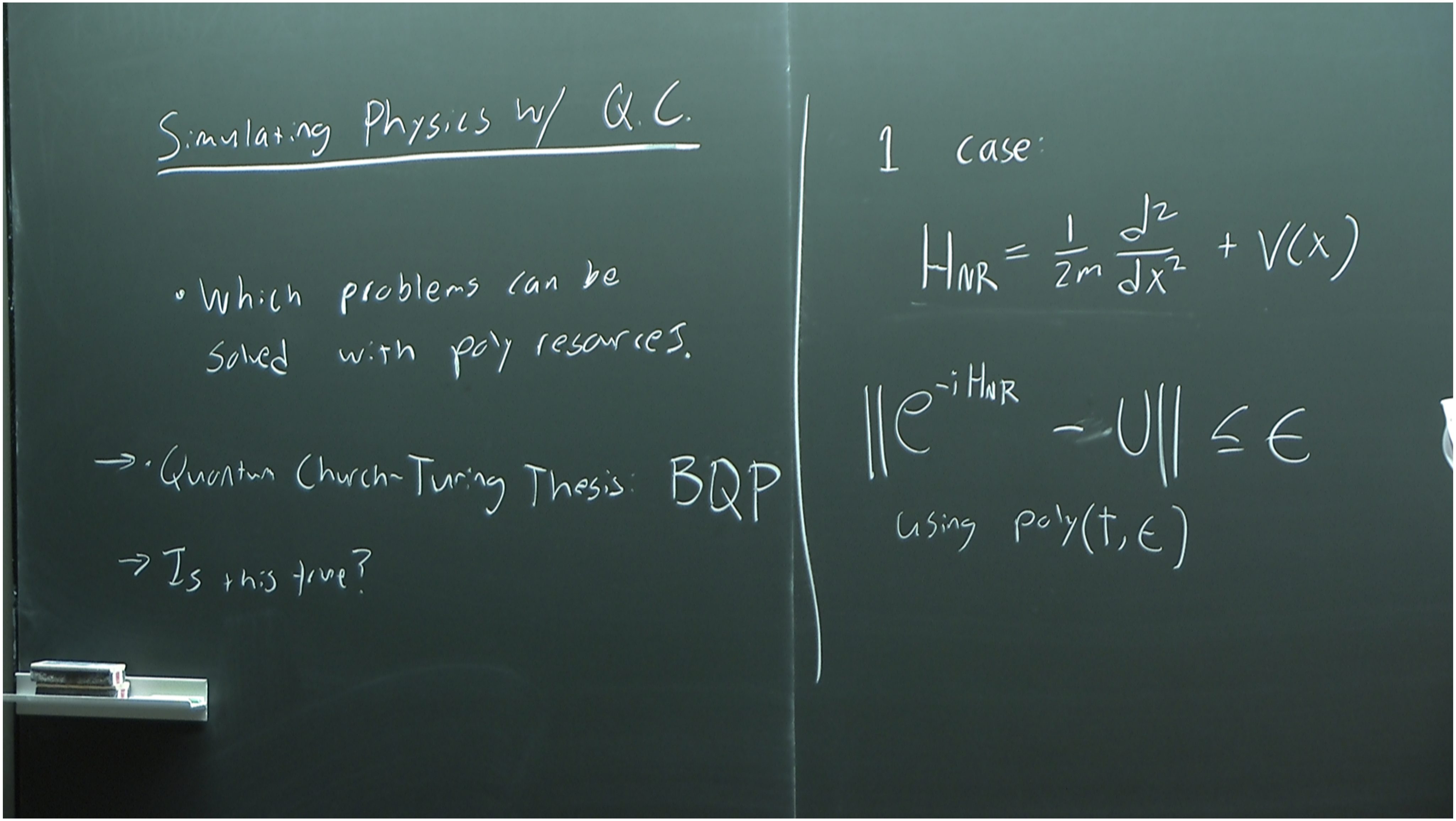
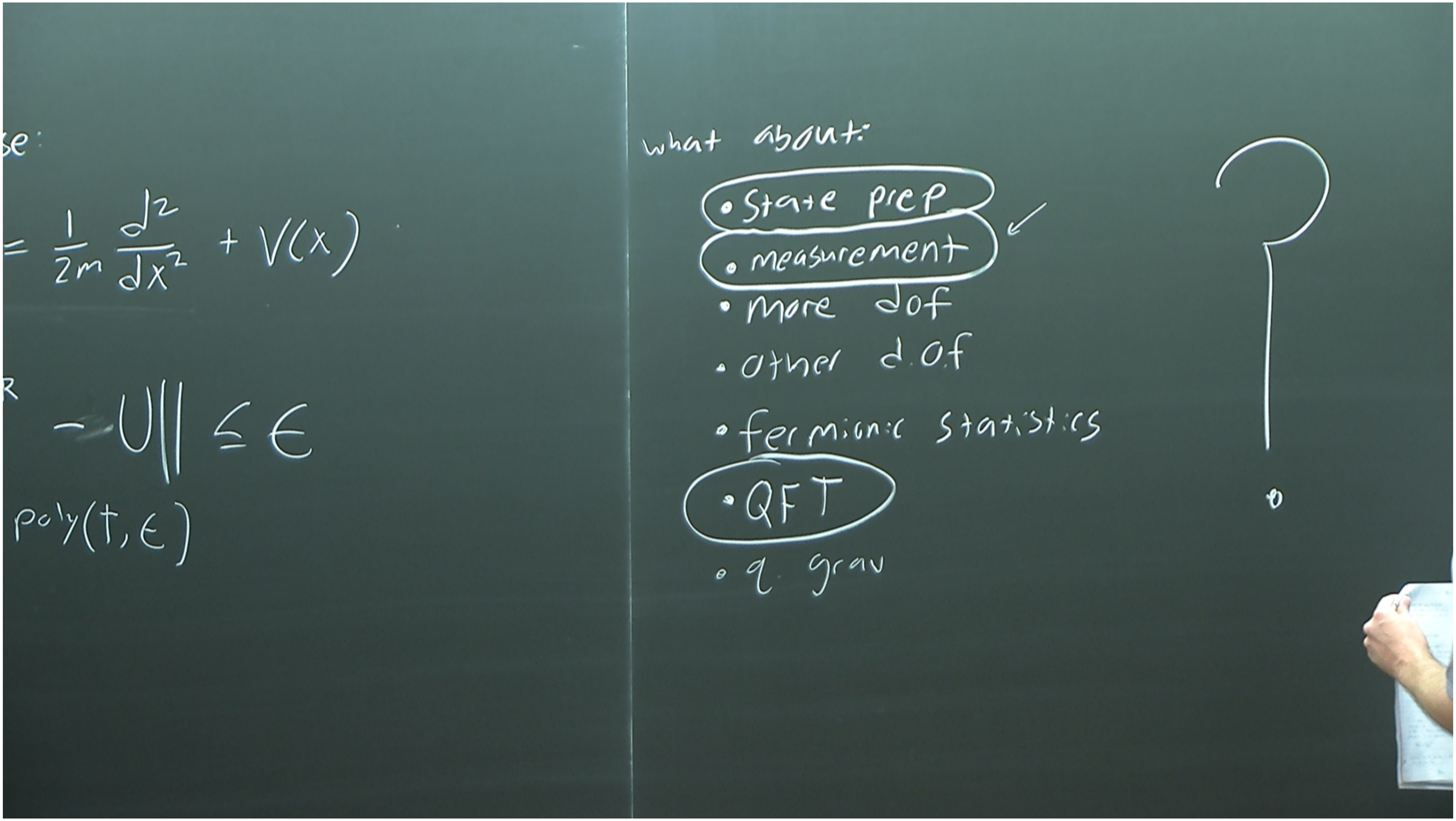
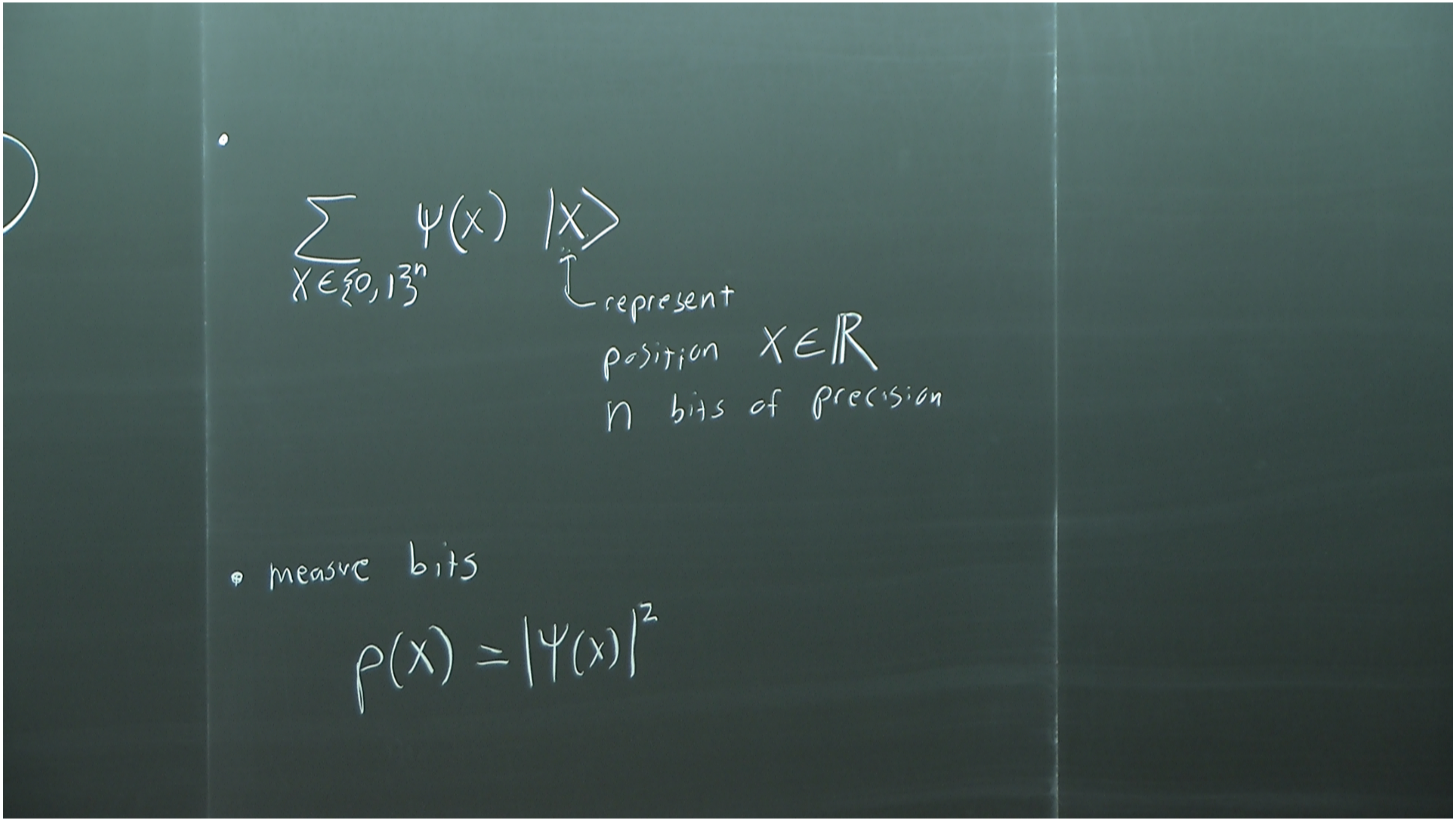
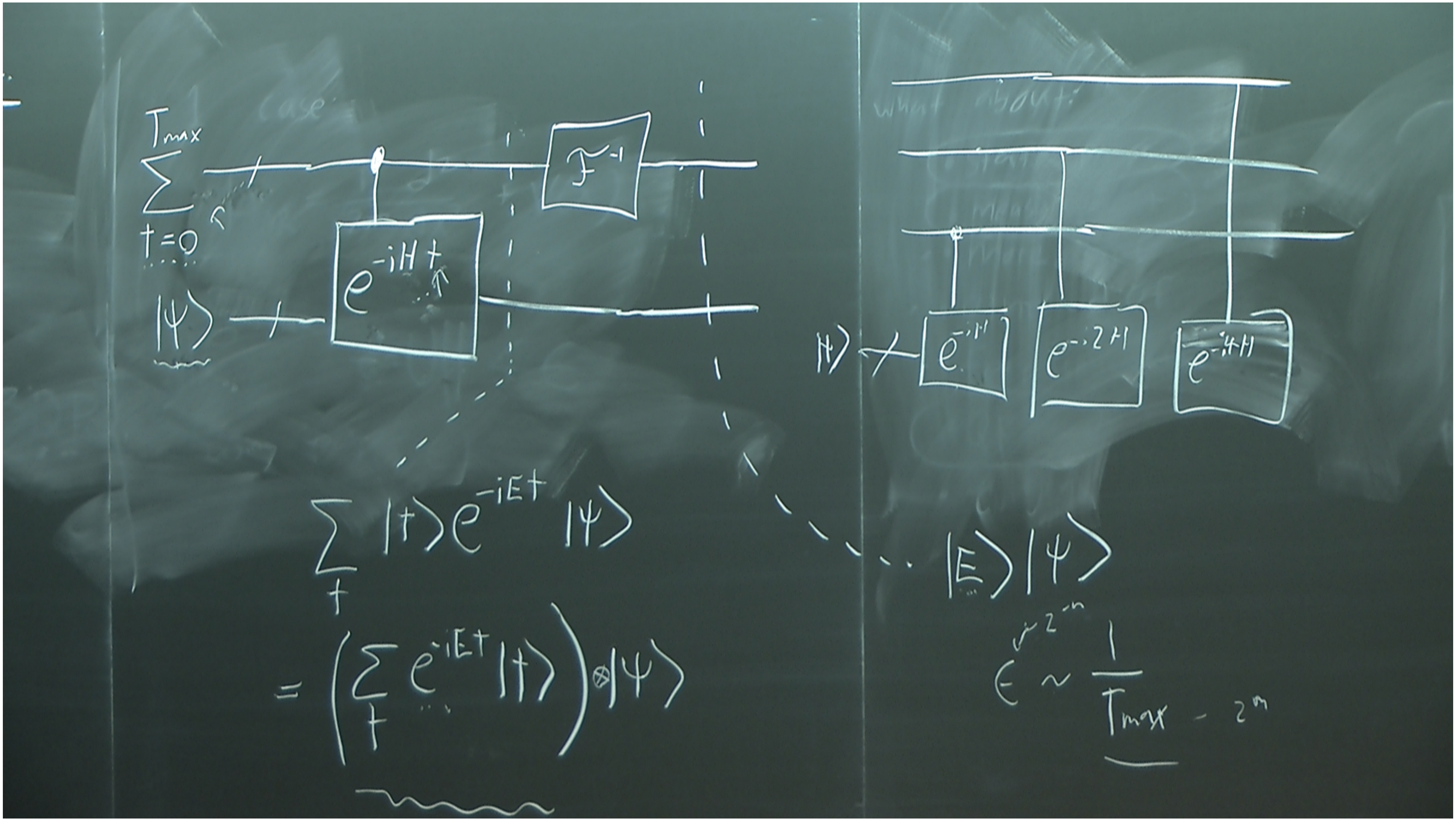
URL: http://pirsa.org/16070036

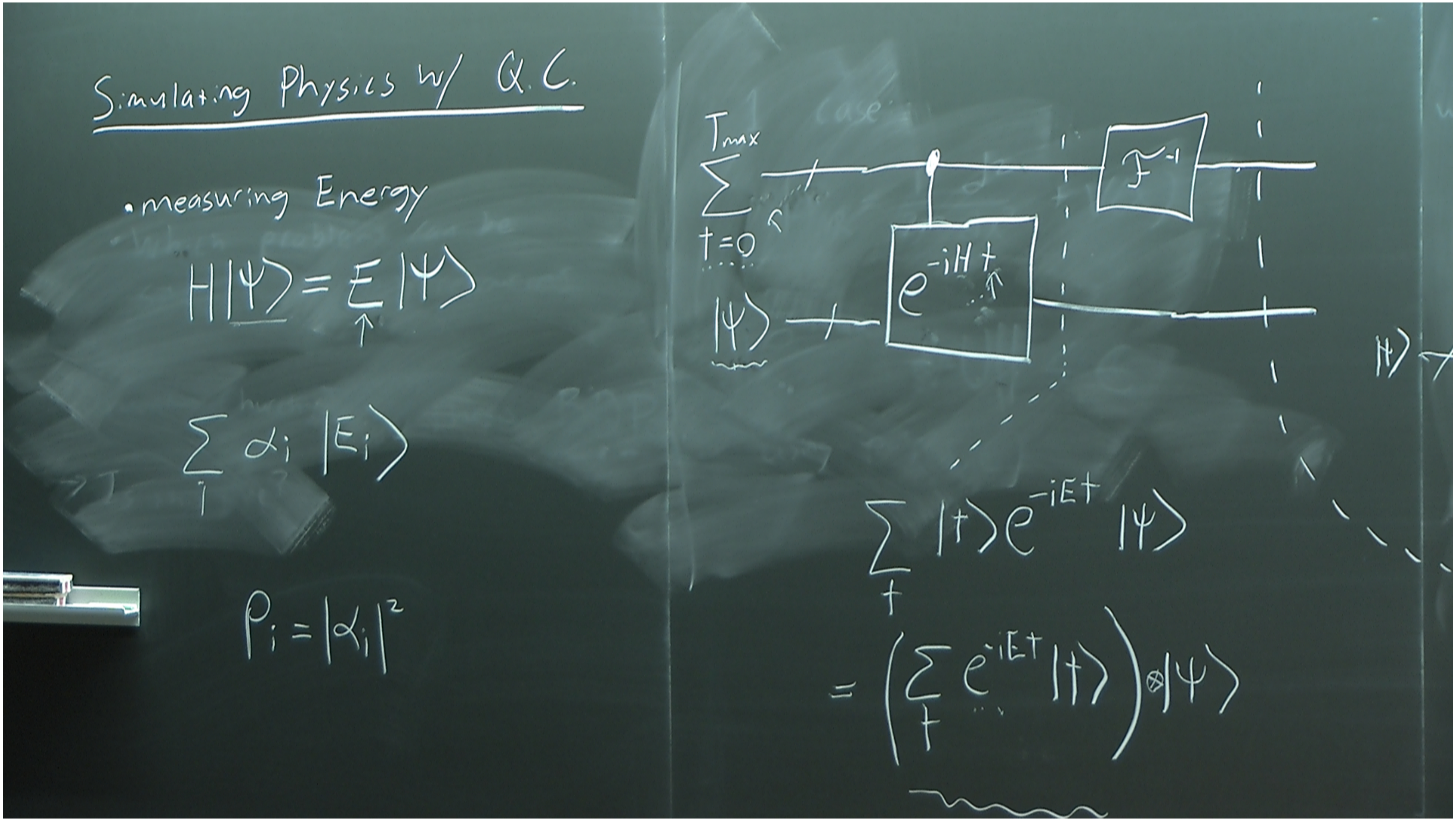
Abstract:

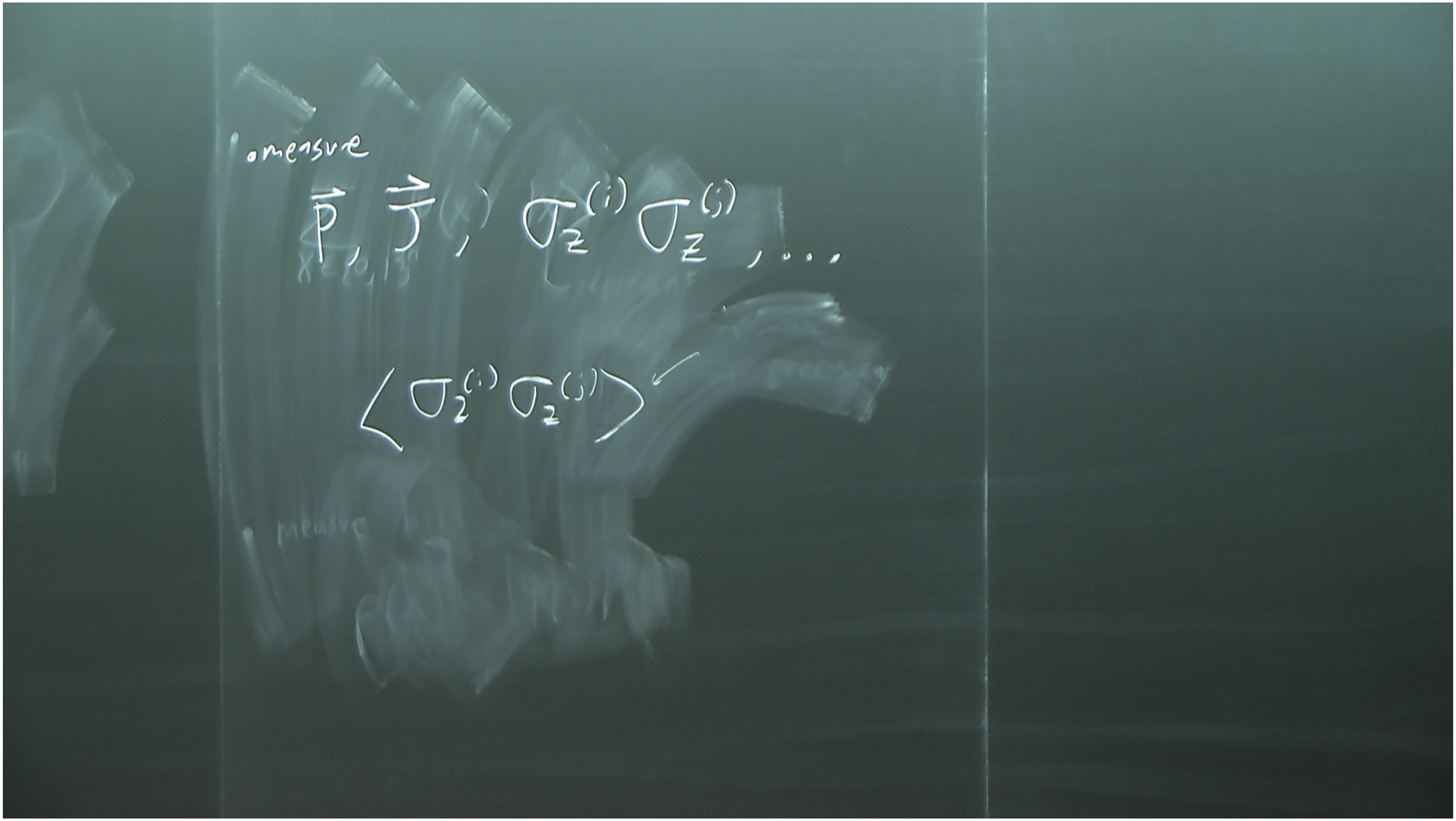

Simulating Physics w/ Q.C.
• Which problems can be solved with poly resources.
→ • Quantum Church-Turing Thesis: BQP
→ Is this true?
1 case:
$H_{NR} = \frac{1}{2m}\frac{d^2}{dx^2} + V(x)$
$\| e^{-iH_{NR}} - U \| \leq \epsilon$
using poly(t, ε)

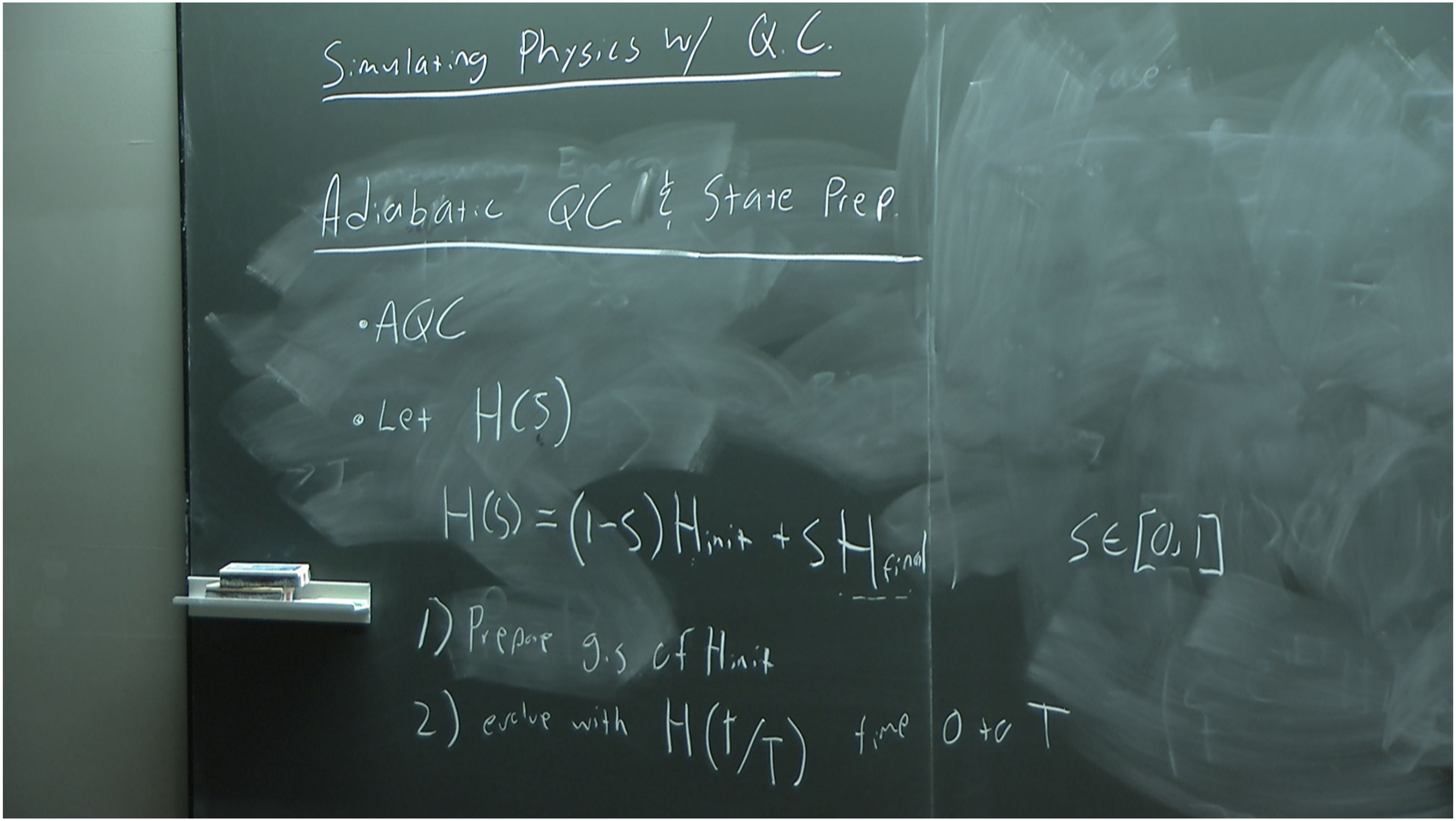

se:
= 1/2m d²/dx² + V(x)
R
− U|| ≤ ε
poly(t, ε)
what about:
• State prep
• measurement
• more dof
• other d.o.f
• fermionic statistics
• QFT
• q. grav
?

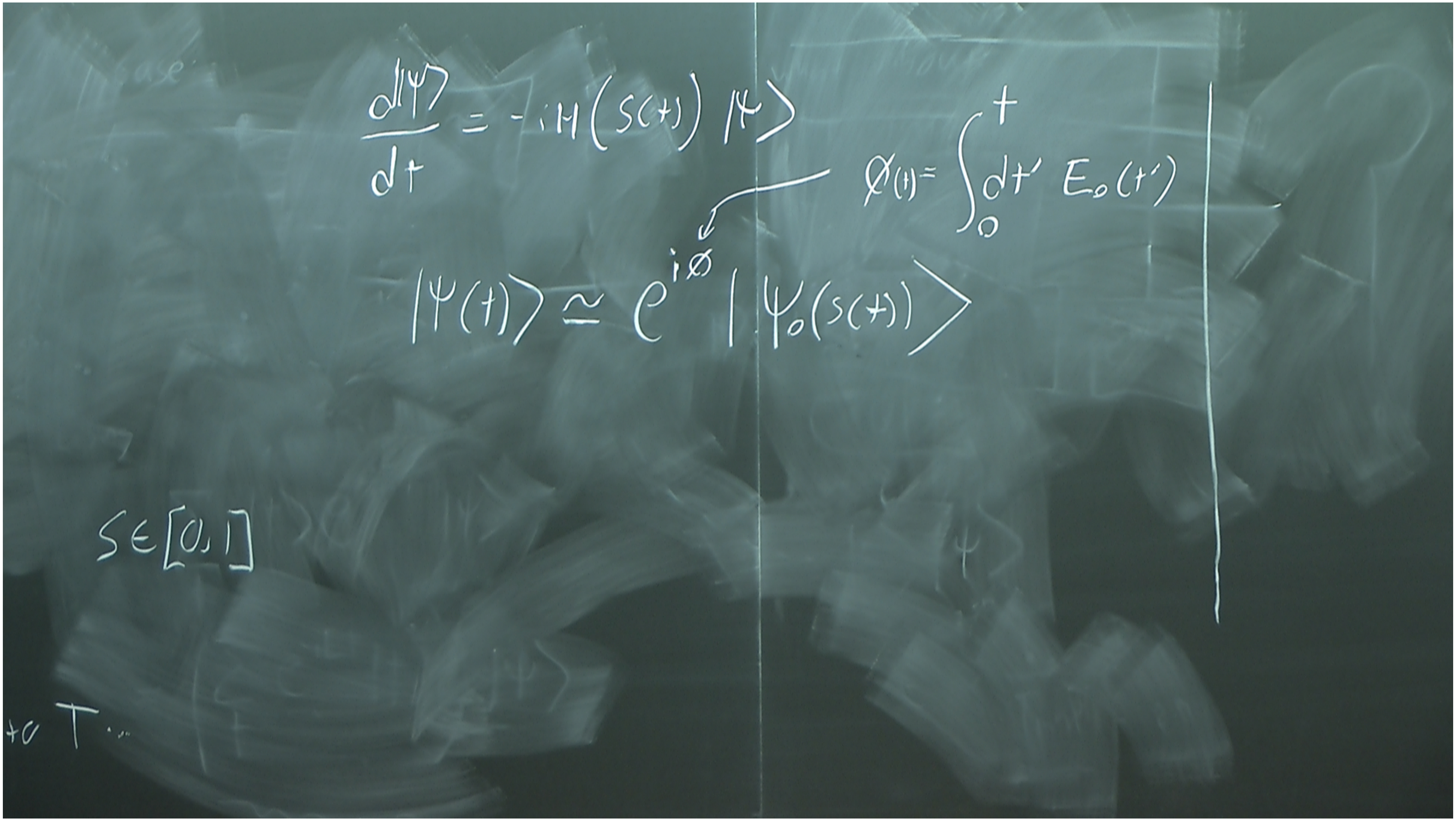

$\sum_{x \in \{0,1\}^n} \psi(x) |x\rangle$
represent
position $x \in \mathbb{R}$
n bits of precision
• measure bits
$p(x) = |\psi(x)|^2$


$\sum_{t=0}^{T_{max}}$
$|\psi\rangle$
e^{-iHt}
$\mathcal{F}^{-1}$
$\sum_t |t\rangle e^{-iEt} |\psi\rangle$
$= \left(\sum_t e^{-iEt}|t\rangle\right) \otimes |\psi\rangle$
$|E\rangle|\psi\rangle$
$\epsilon \sim \frac{1}{T_{max}}$
e^{-iH}
e^{-i2H}
e^{-i4H}

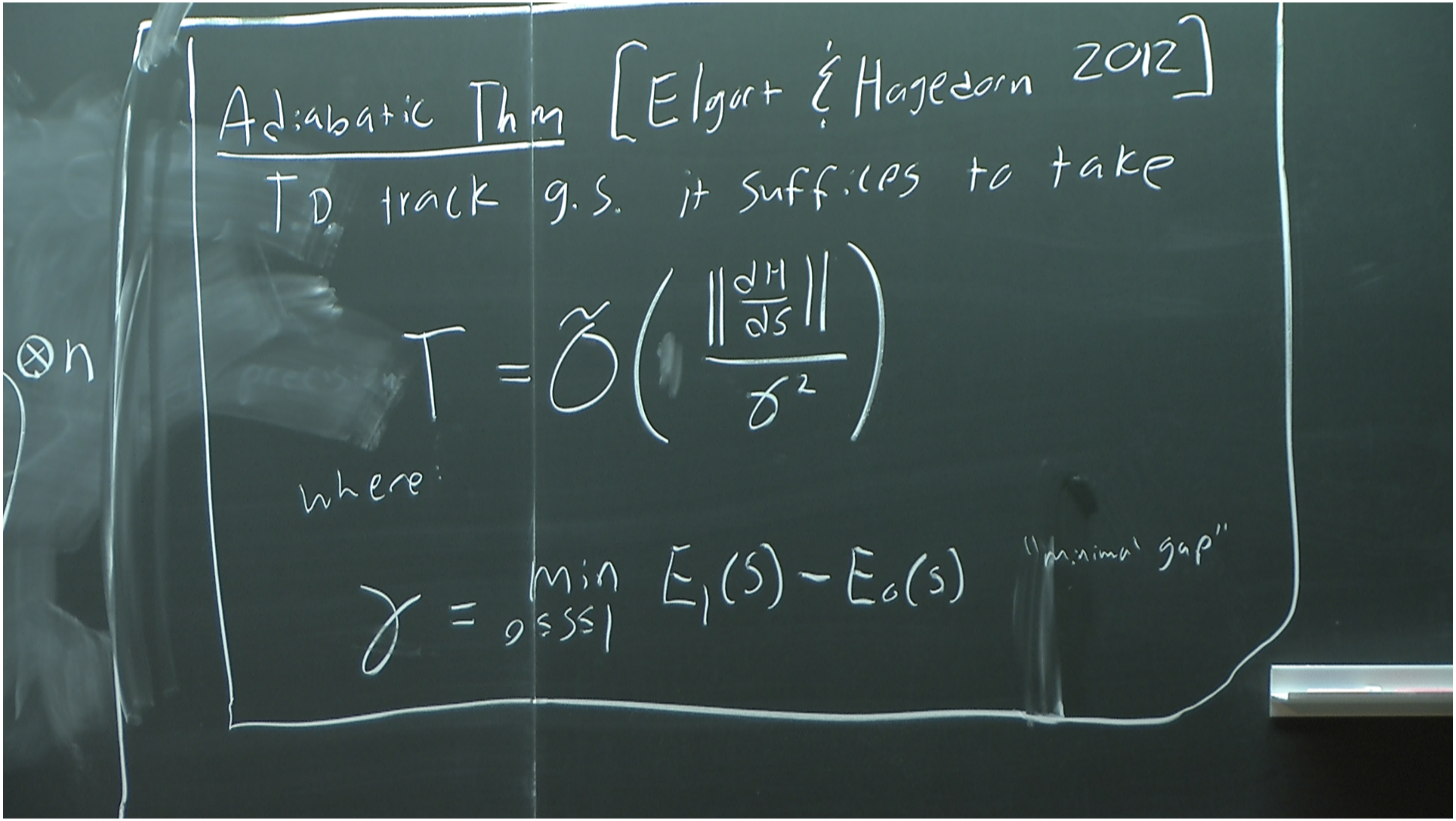

Simulating Physics w/ Q.C.
• measuring Energy
H|ψ⟩ = E|ψ⟩
∑_i α_i |E_i⟩
p_i = |α_i|²
T_max
∑
t=0
e^{-iHt}
|ψ⟩
F^{-1}
∑_t |t⟩e^{-iEt} |ψ⟩
= (∑_t e^{-iEt} |t⟩) ⊗ |ψ⟩

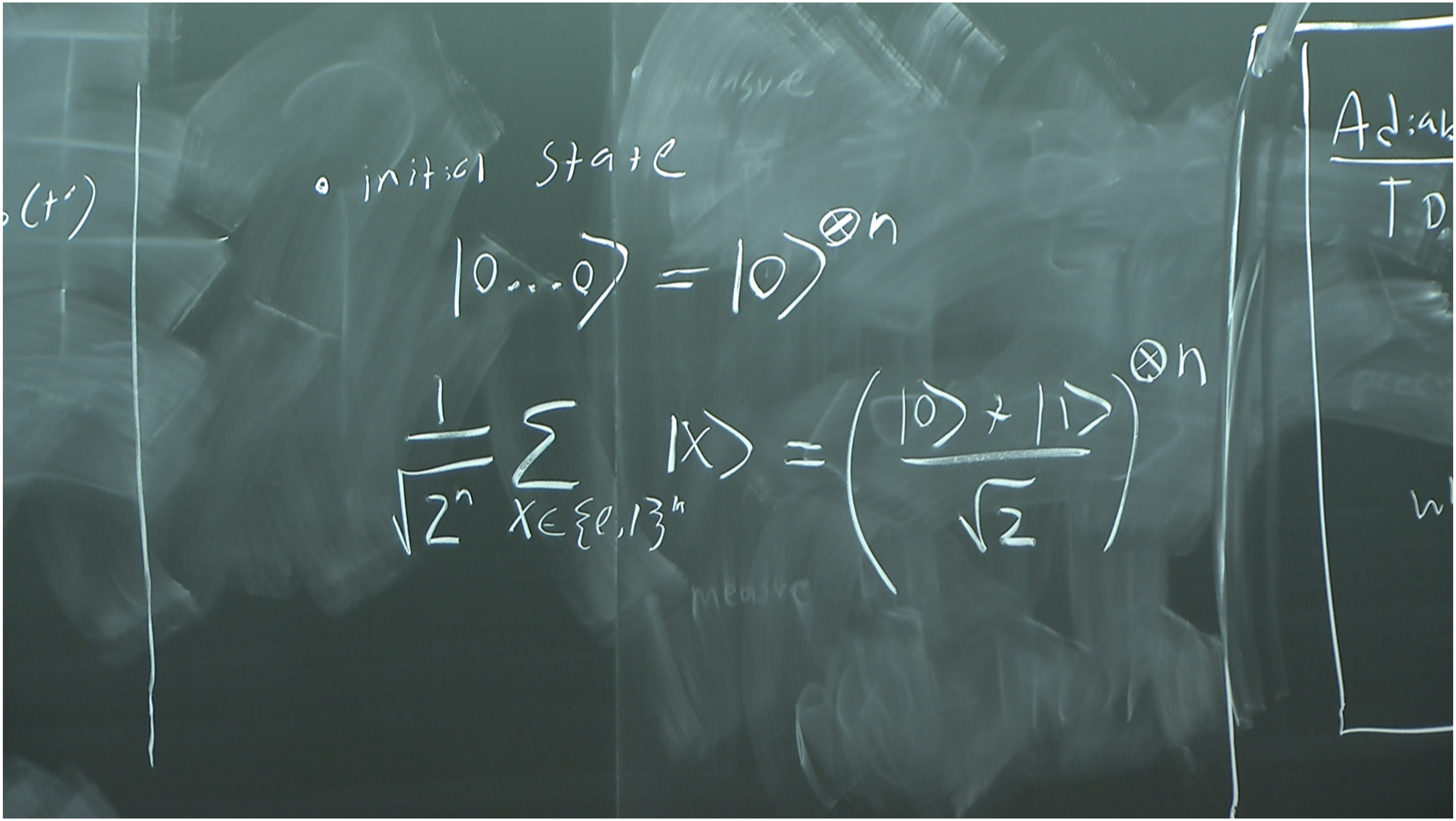

• measure
$\vec{P}, \vec{J}; \sigma_z^{(i)} \sigma_z^{(j)}, \dots$
$\langle \sigma_z^{(i)} \sigma_z^{(j)} \rangle$


Simulating Physics w/ Q.C.
Adiabatic QC & State Prep.
• AQC
• Let H(s)
H(s) = (1-s) Hinit + s Hfinal
s ∈ [0, 1]
1) Prepare g.s of Hinit
2) evolve with H(t/T) time 0 to T



$\frac{d|\psi\rangle}{dt} = -iH(s(t))\,|\psi\rangle$
$\phi(t) = \int_0^t dt'\, E_0(t')$
$|\psi(t)\rangle \simeq e^{i\phi}\,|\psi_0(s(t))\rangle$
$s \in [0,1]$


Adiabatic Thm [Elgart & Hagedorn 2012]
To track g.s. it suffices to take
$T = \tilde{O}\left(\frac{\| \frac{dH}{ds} \|}{\gamma^2} \right)$
where:
$\gamma = \min_{0 \leq s \leq 1} E_1(s) - E_0(s)$
"minimal gap"
$\otimes n$



• initial state
|0...0⟩ = |0⟩^⊗n
1/√2^n Σ_{x∈{0,1}^n} |x⟩ = ((|0⟩ + |1⟩)/√2)^⊗n

- restrict to k-qubit interactions

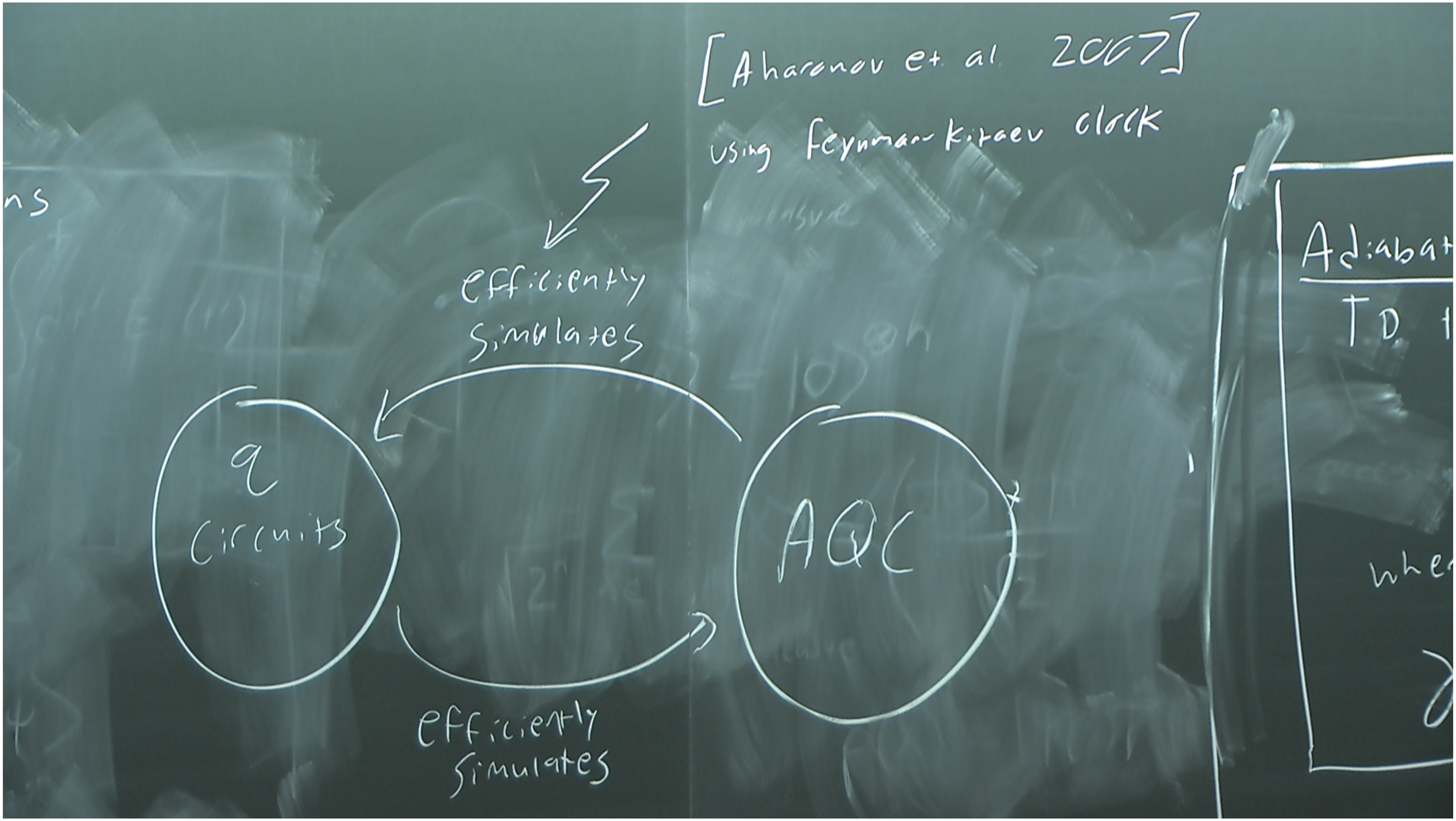
$$k=2 \begin{cases} \sigma_x^{(i)} \\ \sigma_z^{(i)} \sigma_z^{(j)} \end{cases}$$

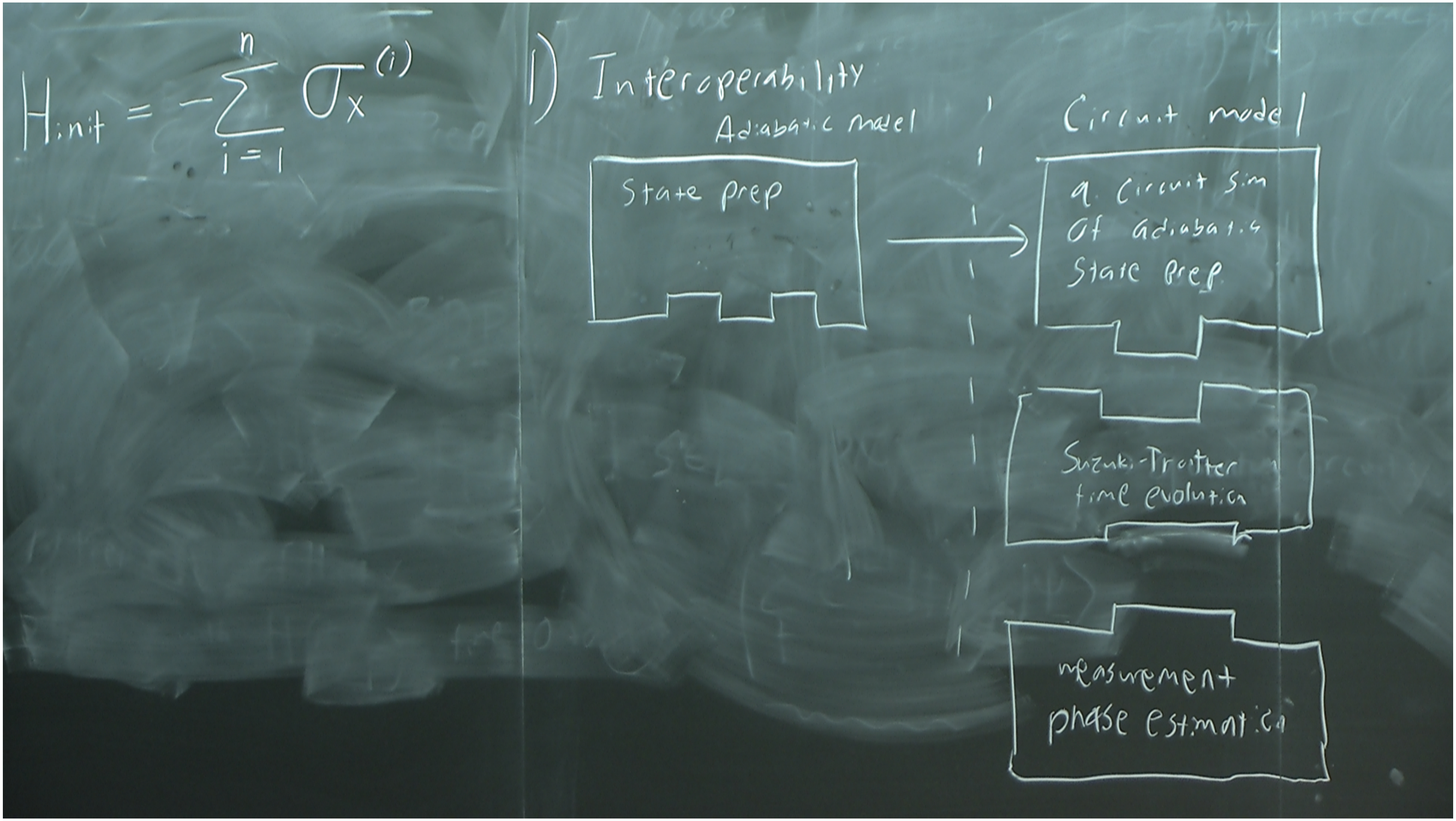
- for any constant $k \geq 2$

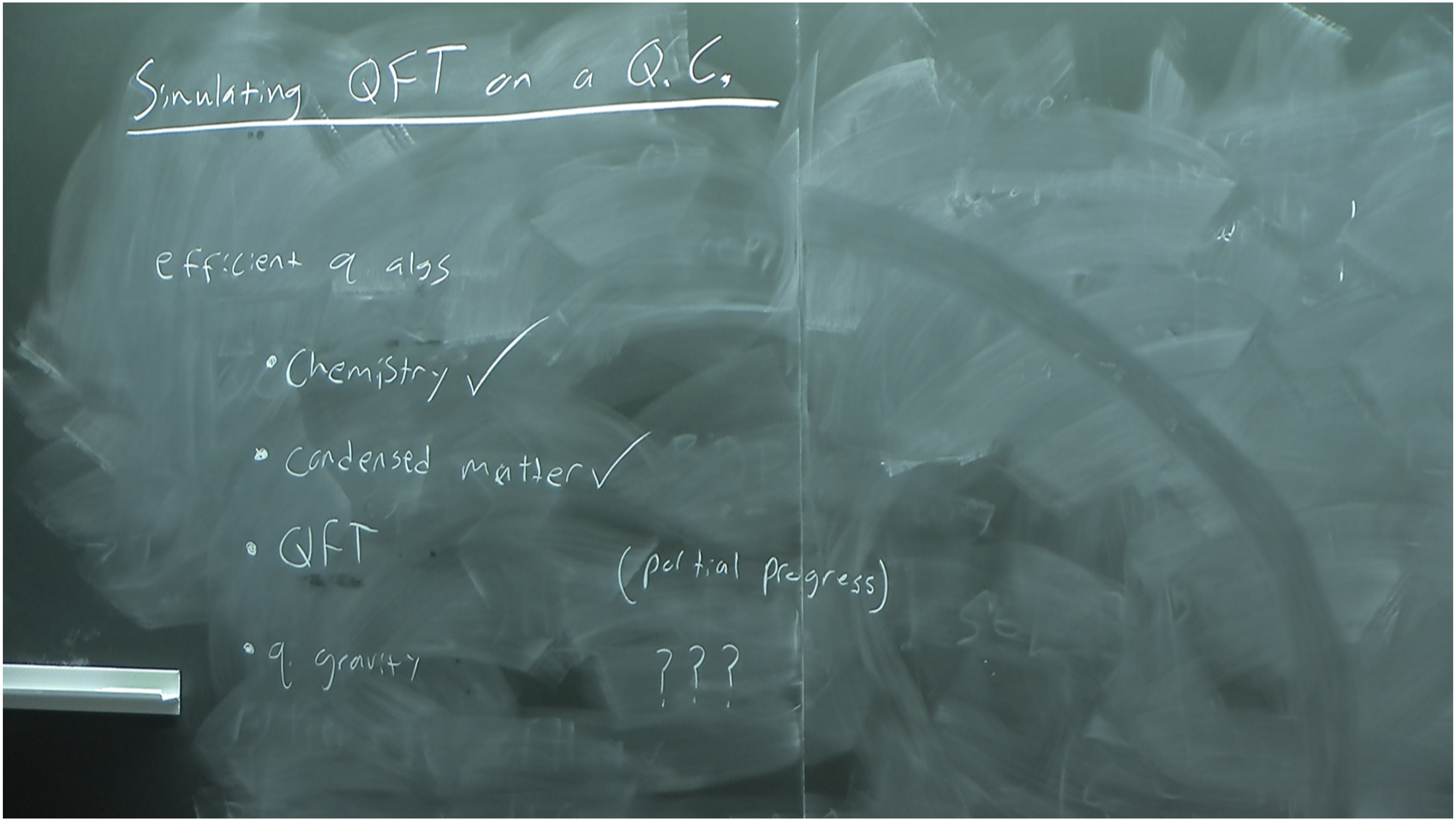
AQC $\leftrightarrow$ quantum circuits

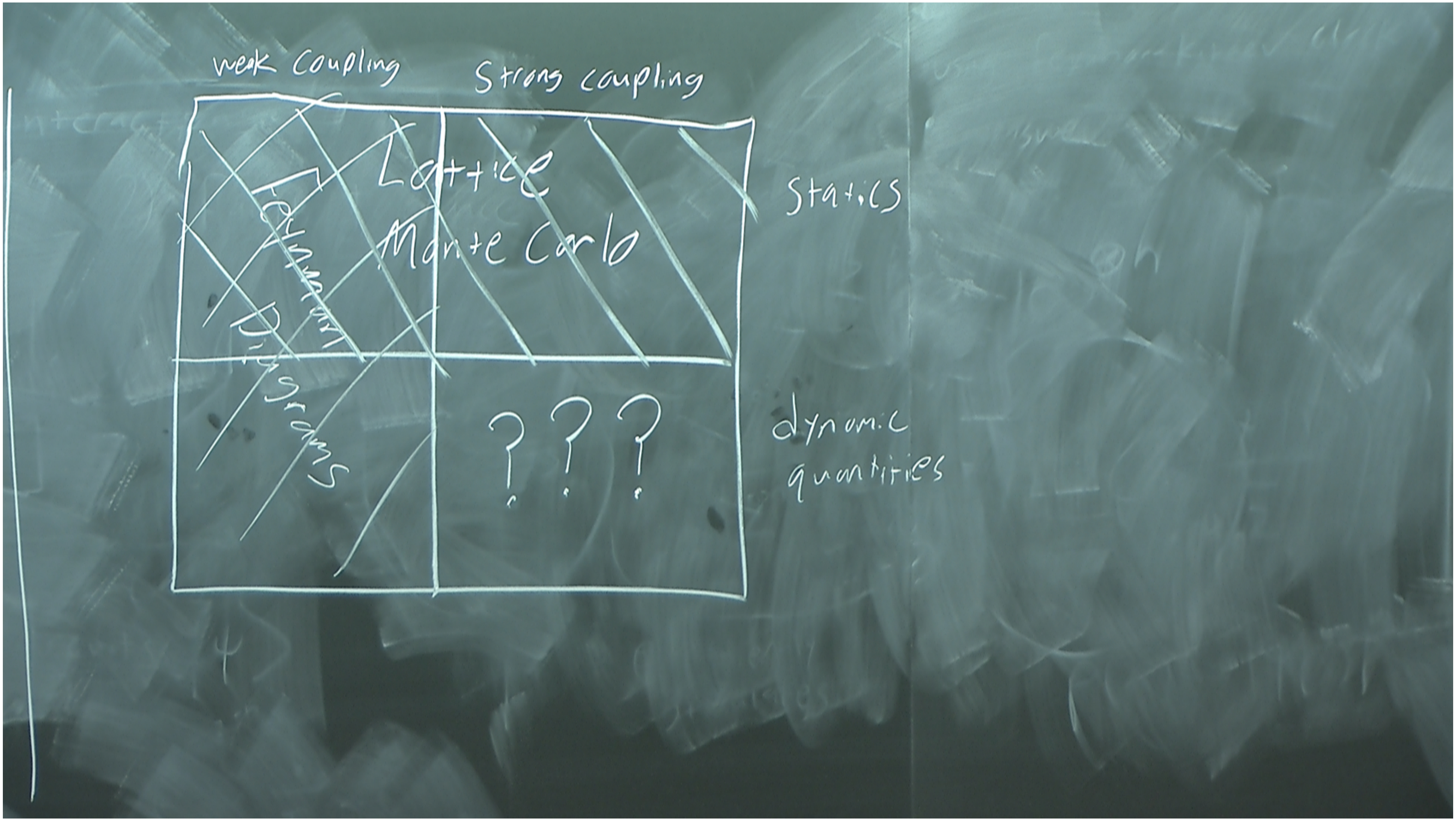
$[0,1]$

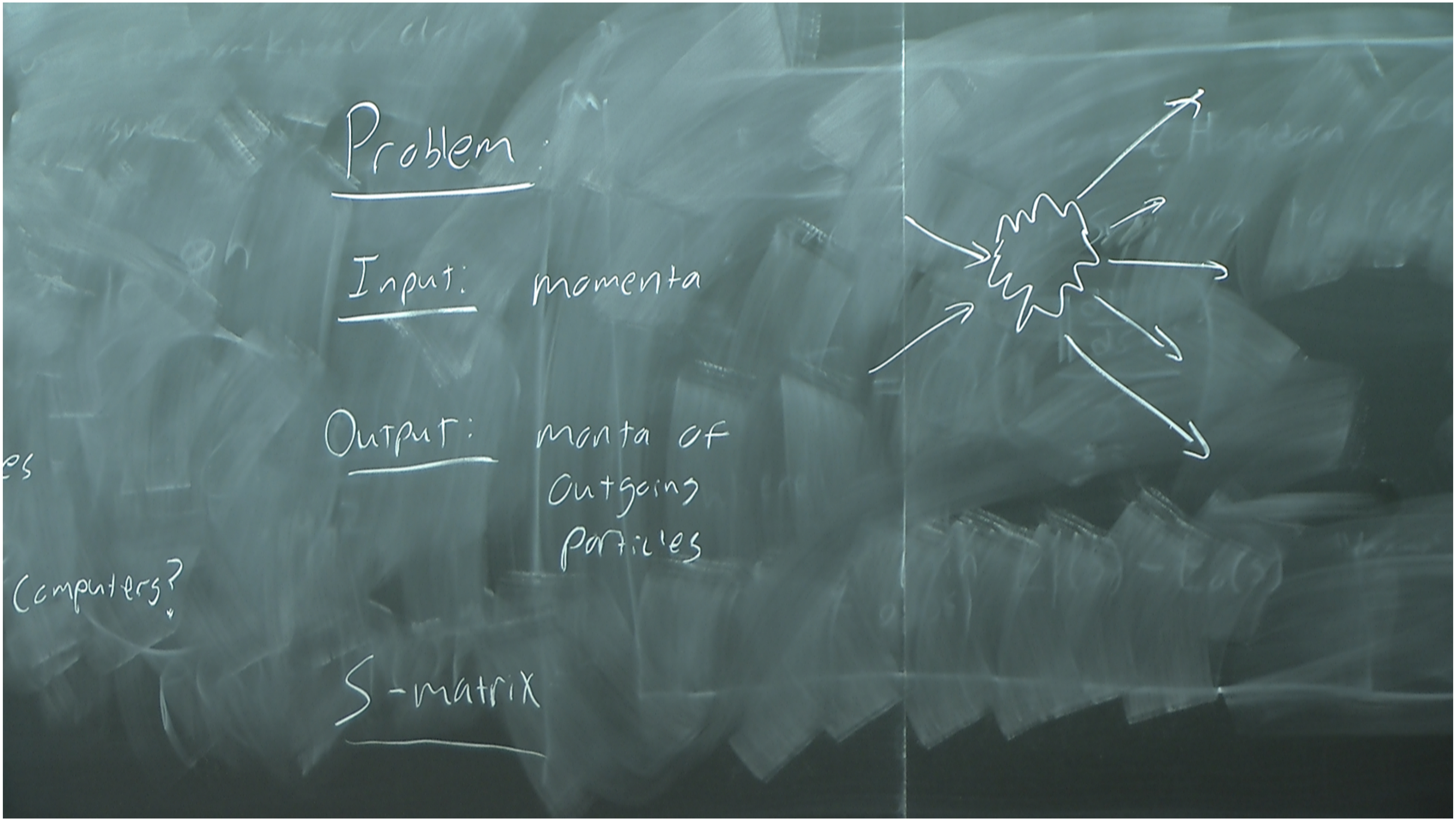
- ini

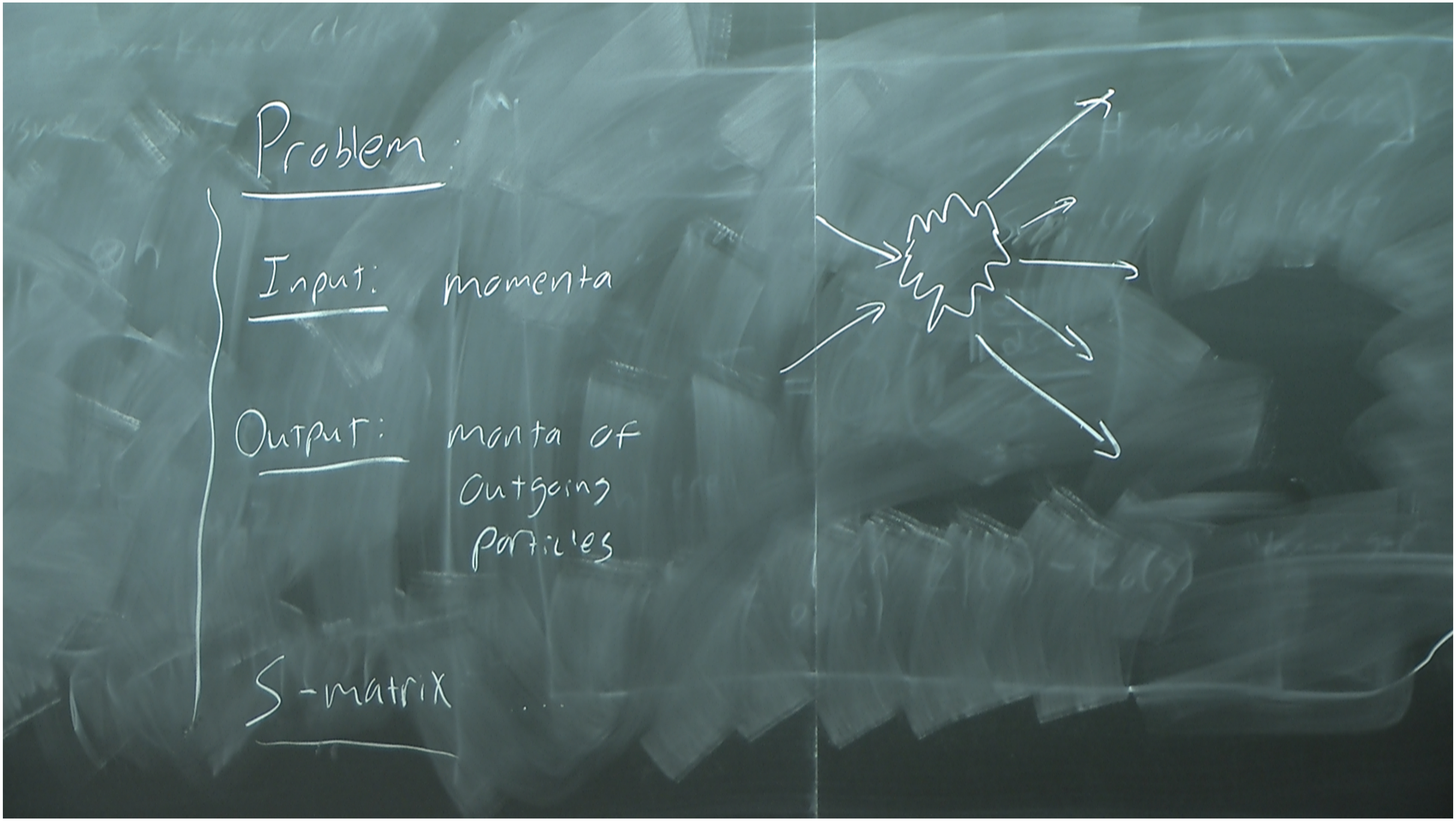

[Aharonov et. al 2007]
using Feynman-Kitaev clock
efficiently simulates
q circuits
AQC
efficiently simulates
Adiabat
T.D.
whe


$H_{init} = -\sum_{i=1}^{n} \sigma_x^{(i)}$
1) Interoperability
Adiabatic model
Circuit model
State prep
q. Circuit sim of adiabatic state prep
Suzuki-Trotter time evolution
measurement phase estimation


Simulating QFT on a Q.C.
efficient q algs
• Chemistry ✓
• condensed matter ✓
• QFT (partial progress)
• q. gravity ???


weak coupling
Strong coupling
Lattice
Monte Carlo
Feynman
Diagrams
statics
???
dynamic
quantities


Problem:
Input: momenta
Output: monta of
Outgoing
particles
Computers?
S-matrix


Problem:
Input: momenta
Output: monta of
Outgoing
particles
S-matrix ....

Simulating QFT on a Q.C.

ϕ^4 - theory

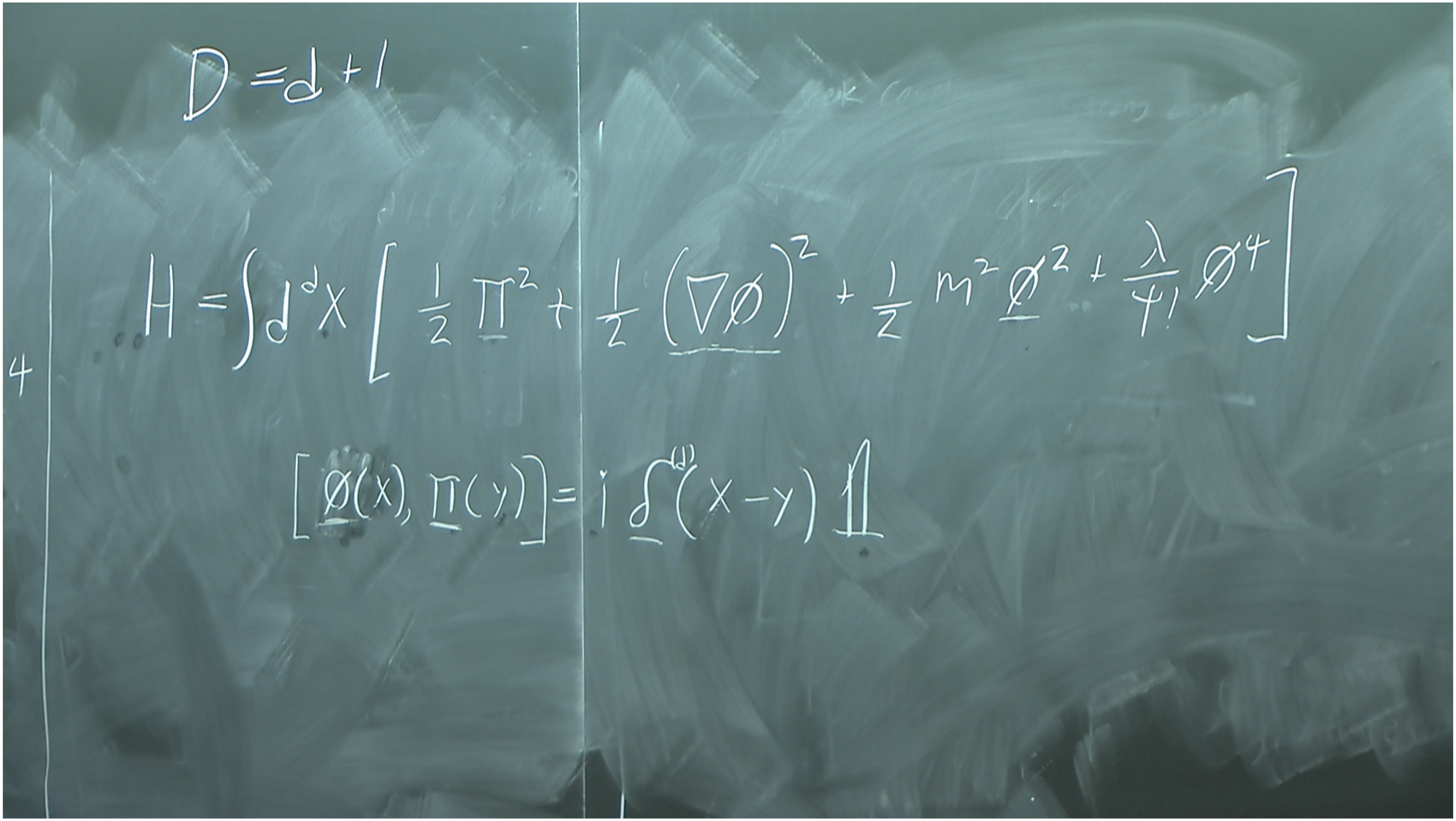
$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi\, \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

Simulating QFT on a Q.C.

ϕ^4- theory

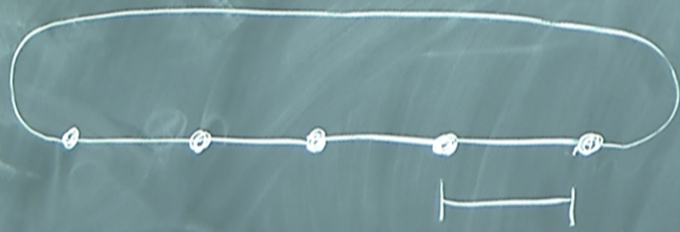
$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi\, \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

$$\int \mathcal{D}\phi\, e^{-i\int d^d x\, \mathcal{L}}$$


$D = d + 1$
4
$H = \int d^d x \left[\frac{1}{2} \pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$
$[\phi(x), \pi(y)] = i \delta^{(d)}(x - y) \mathbb{1}$

$$|\Psi(t)\rangle = \int d\phi_1 \cdots \int d\phi_N \; \Psi(\phi_1 \ldots \phi_N, t) \, |\phi_1, \ldots, \phi_N\rangle$$

Ω

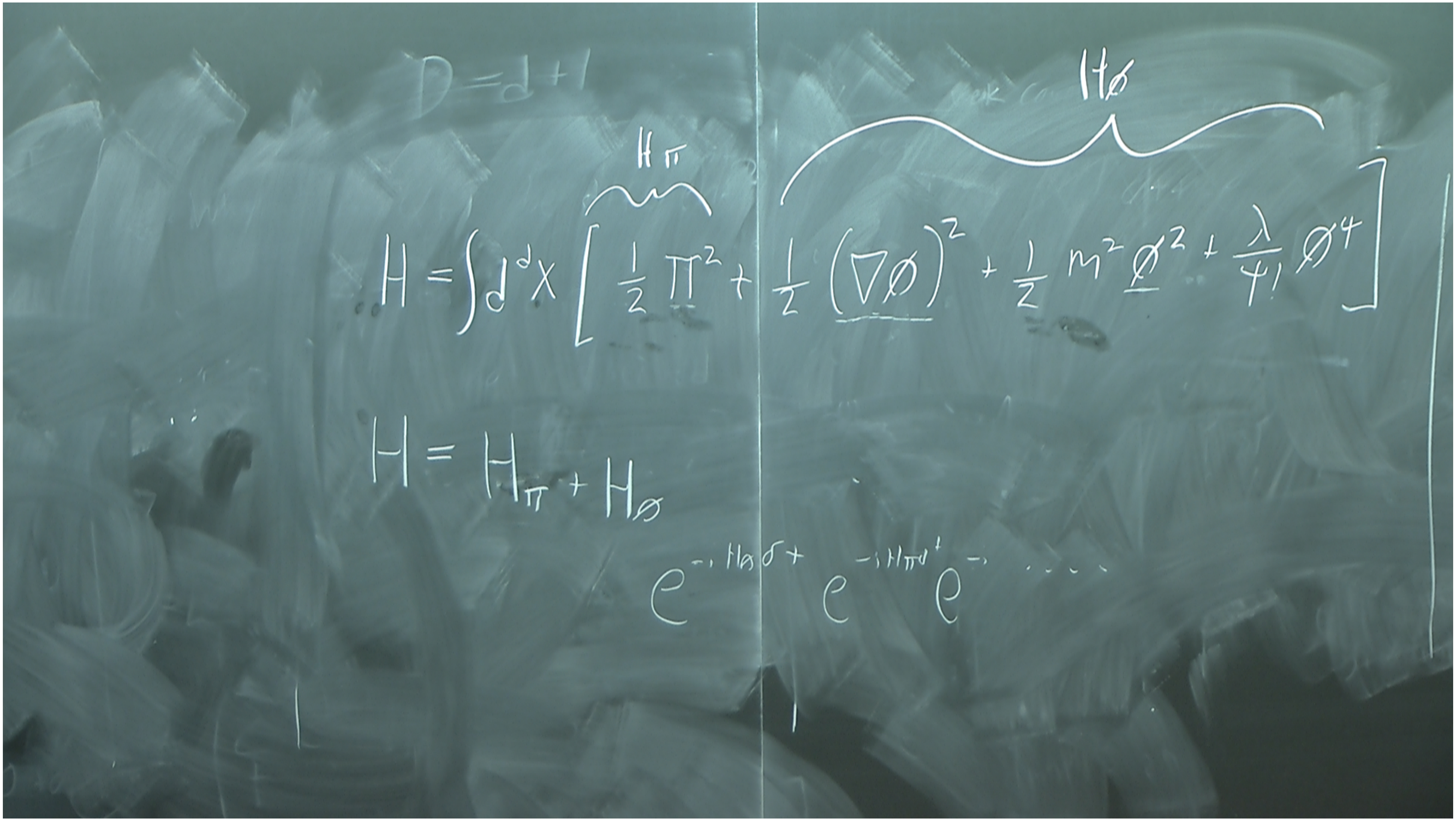


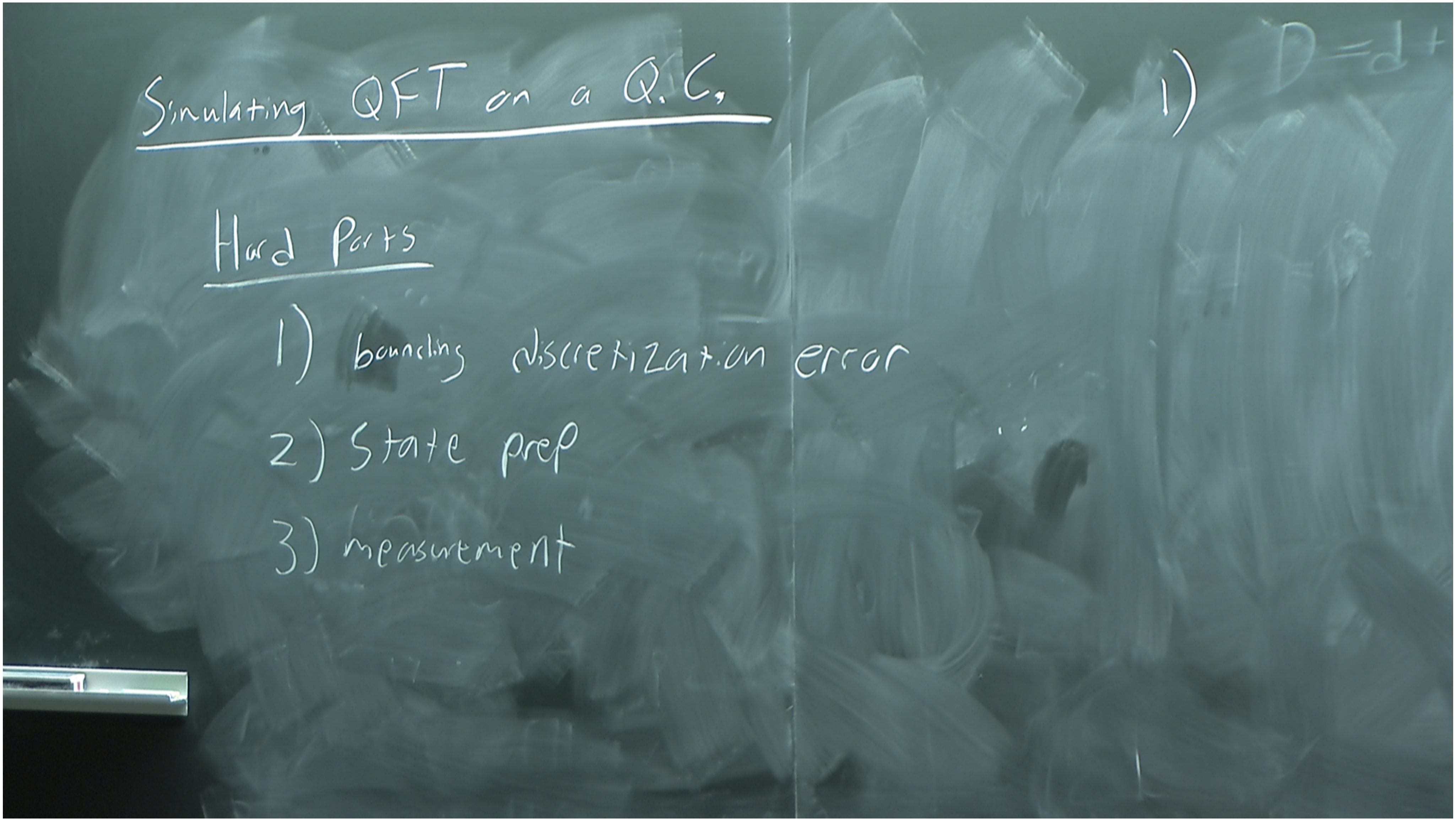
a lattice spacing

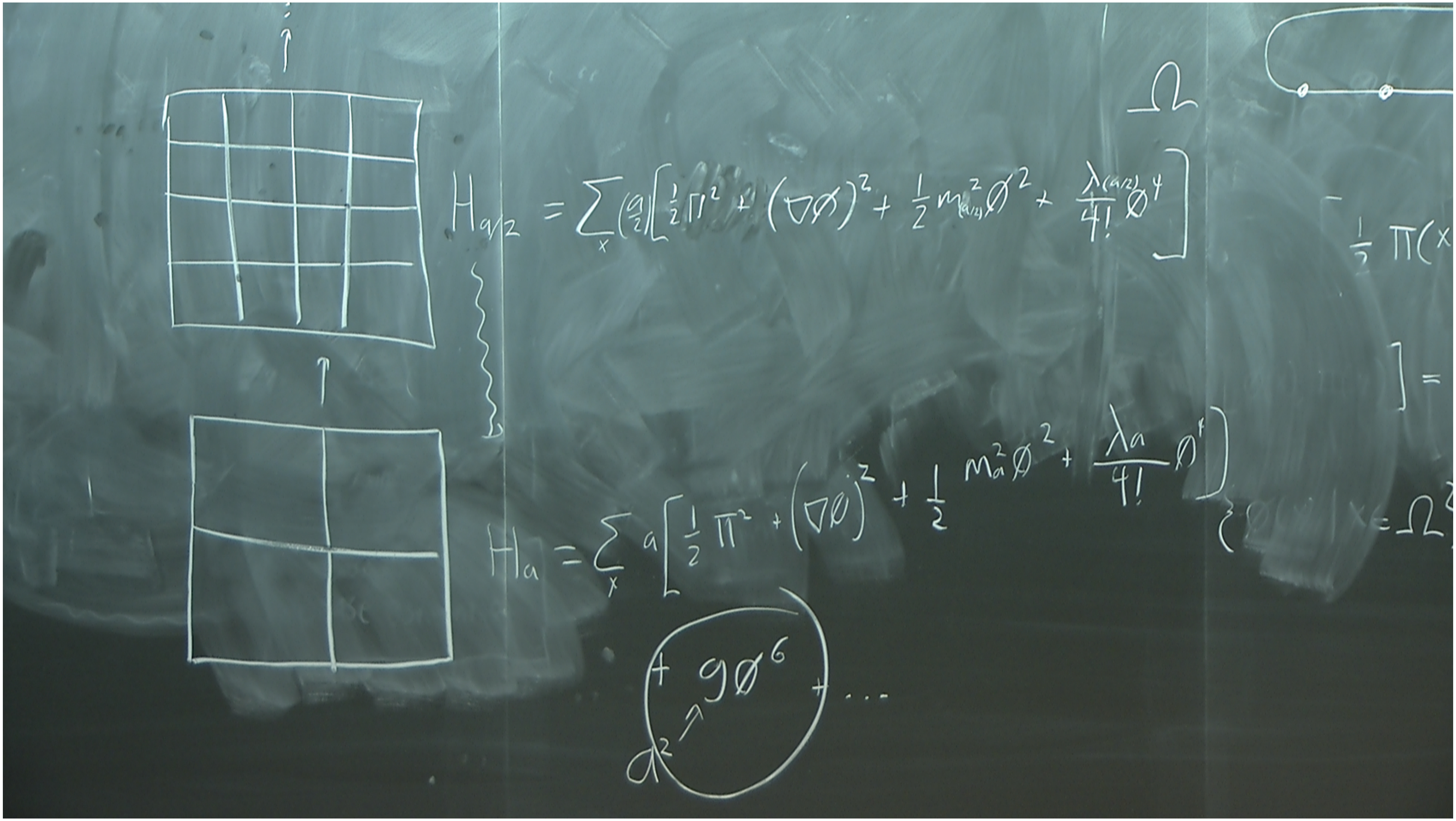
$$H = \sum_{x \in \Omega} a \left[\frac{1}{2} \pi(x)^2 + \frac{1}{2} \left(\frac{\phi(x+a) - \phi(x)}{a} \right)^2 + \frac{1}{2} m_0^2 \phi(x)^2 + \frac{\lambda_0}{4!} \phi(x)^4 \right]$$

$$[\phi(x), \pi(y)] = \frac{i}{a} \delta_{x,y} \mathbb{1}$$

$$\{ \phi(x) \mid x \in \Omega \}$$


$D = d + 1$
H_π
H_ϕ
$H = \int d^d x \left[\frac{1}{2} \pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$
$H = H_\pi + H_\phi$
$e^{-i H_\phi \delta t} \, e^{-i H_\pi \delta t} \, e^{-i} \ldots$


Simulating QFT on a Q.C.
Hard Parts
1) bounding discretization error
2) State prep
3) measurement
1)


Ω
$H_{a/2} = \sum_x \left(\frac{a}{2}\right)\left[\frac{1}{2}\Pi^2 + (\nabla\phi)^2 + \frac{1}{2}m^2_{(a/2)}\phi^2 + \frac{\lambda_{(a/2)}}{4!}\phi^4\right]$
$H_a = \sum_x a\left[\frac{1}{2}\Pi^2 + (\nabla\phi)^2 + \frac{1}{2}m_a^2\phi^2 + \frac{\lambda_a}{4!}\phi^4\right]$
$+ g\phi^6 + \ldots$
a^2
$\frac{1}{2}\Pi(x$

$$\Pi = \begin{bmatrix} \Pi_1 \\ \vdots \\ \Pi_N \end{bmatrix} \qquad \phi = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix}$$

$$H = \Pi^T \mathbb{1} \Pi + \phi^T M \phi$$

$$H = \sum_j \omega_j \left(\tilde{\Pi}_j^2 + \tilde{\phi}_j^2 \right)$$

$$\Omega$$

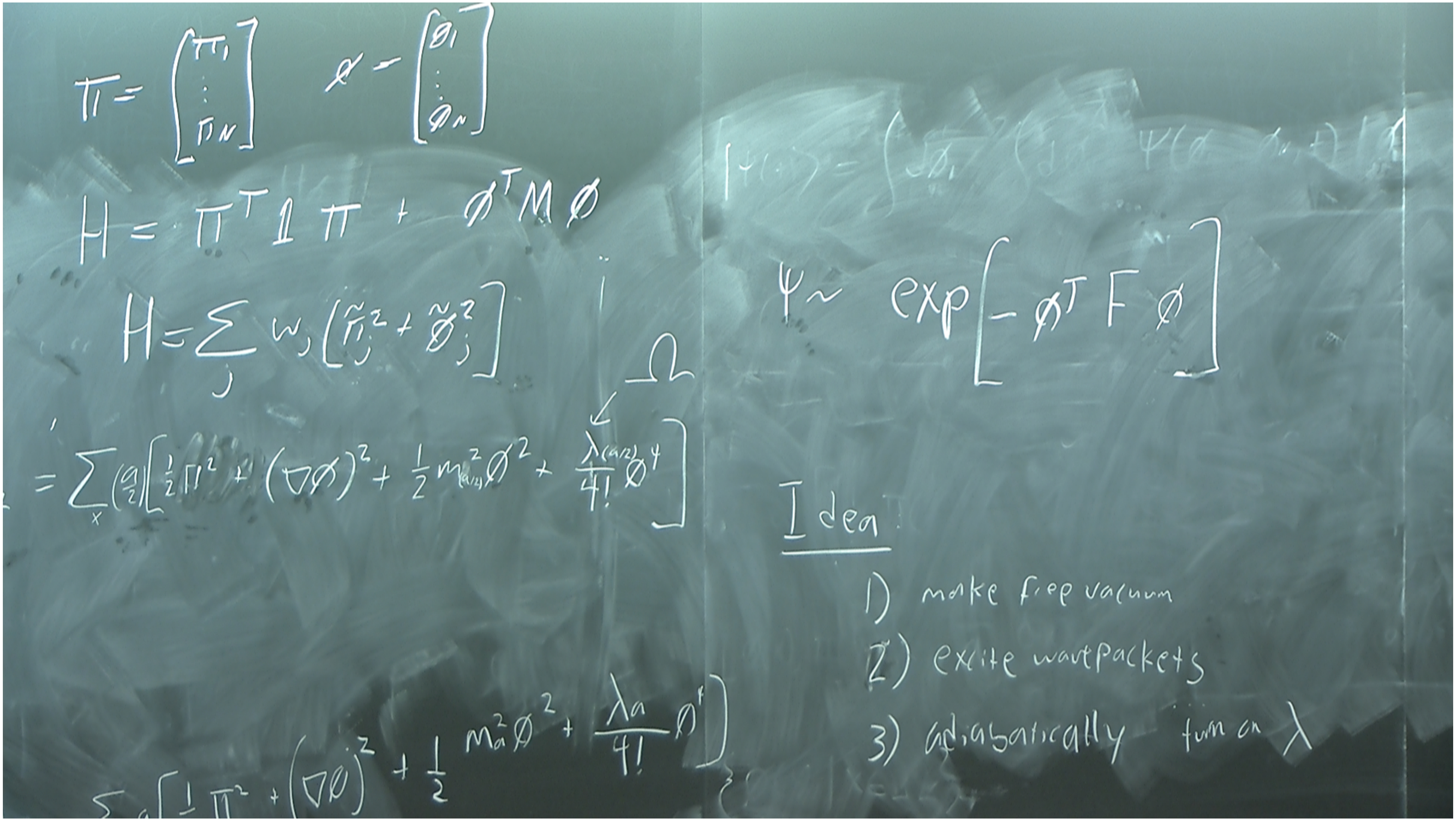
$$= \sum_x \left(\frac{a}{2}\right) \left[\frac{1}{2}\Pi^2 + (\nabla\phi)^2 + \frac{1}{2} m_{(a/2)}^2 \phi^2 + \frac{\lambda_{(a/2)}}{4!}\phi^4 \right]$$

$$\sum a \left[\frac{1}{2}\Pi^2 + (\nabla\phi)^2 + \frac{1}{2} m_a^2 \phi^2 + \frac{\lambda_a}{4!}\phi^4 \right]$$

$$\Psi \sim \exp\left[-\phi^T F \phi \right]$$

Idea

1) make free vacuum
2) excite wavepackets
3) adiabatically turn on λ


$\Pi = \begin{bmatrix} \Pi_1 \\ \vdots \\ \Pi_N \end{bmatrix}$ $\quad \phi = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix}$
$H = \Pi^T \mathbb{1} \Pi + \phi^T M \phi$
$H = \sum_j \omega_j \left(\tilde{\Pi}_j^2 + \tilde{\phi}_j^2 \right)$
Ω
$= \sum_x \left(\frac{a}{2}\right) \left[\frac{1}{2}\Pi^2 + (\nabla\phi)^2 + \frac{1}{2} m_{(a/2)}^2 \phi^2 + \frac{\lambda_{(a/2)}}{4!}\phi^4 \right]$
$\sum a \left[\frac{1}{2}\Pi^2 + (\nabla\phi)^2 + \frac{1}{2} m_a^2 \phi^2 + \frac{\lambda_a}{4!}\phi^4 \right]$
$\Psi \sim \exp\left[-\phi^T F \phi \right]$
Idea
1) make free vacuum
2) excite wavepackets
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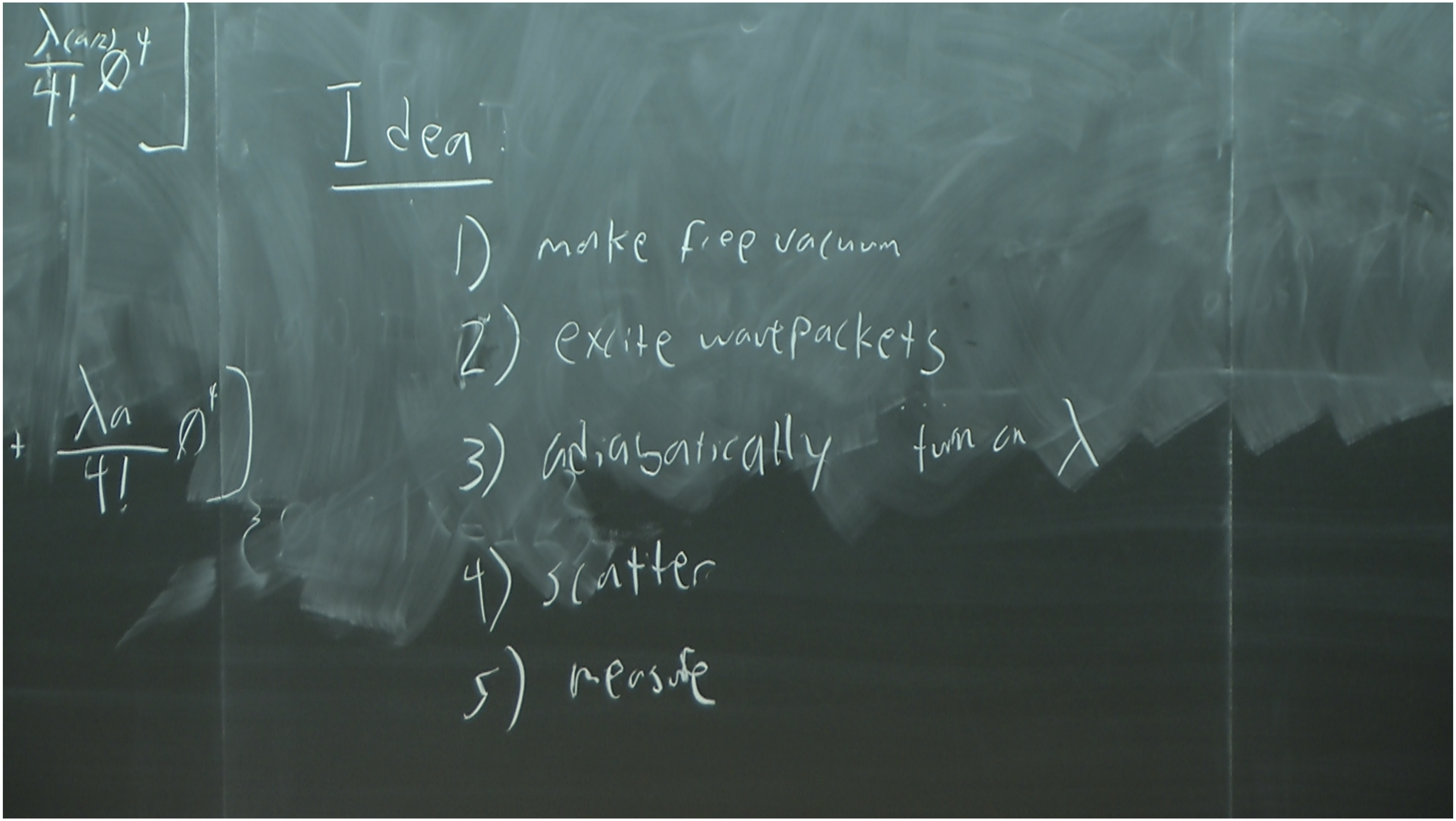
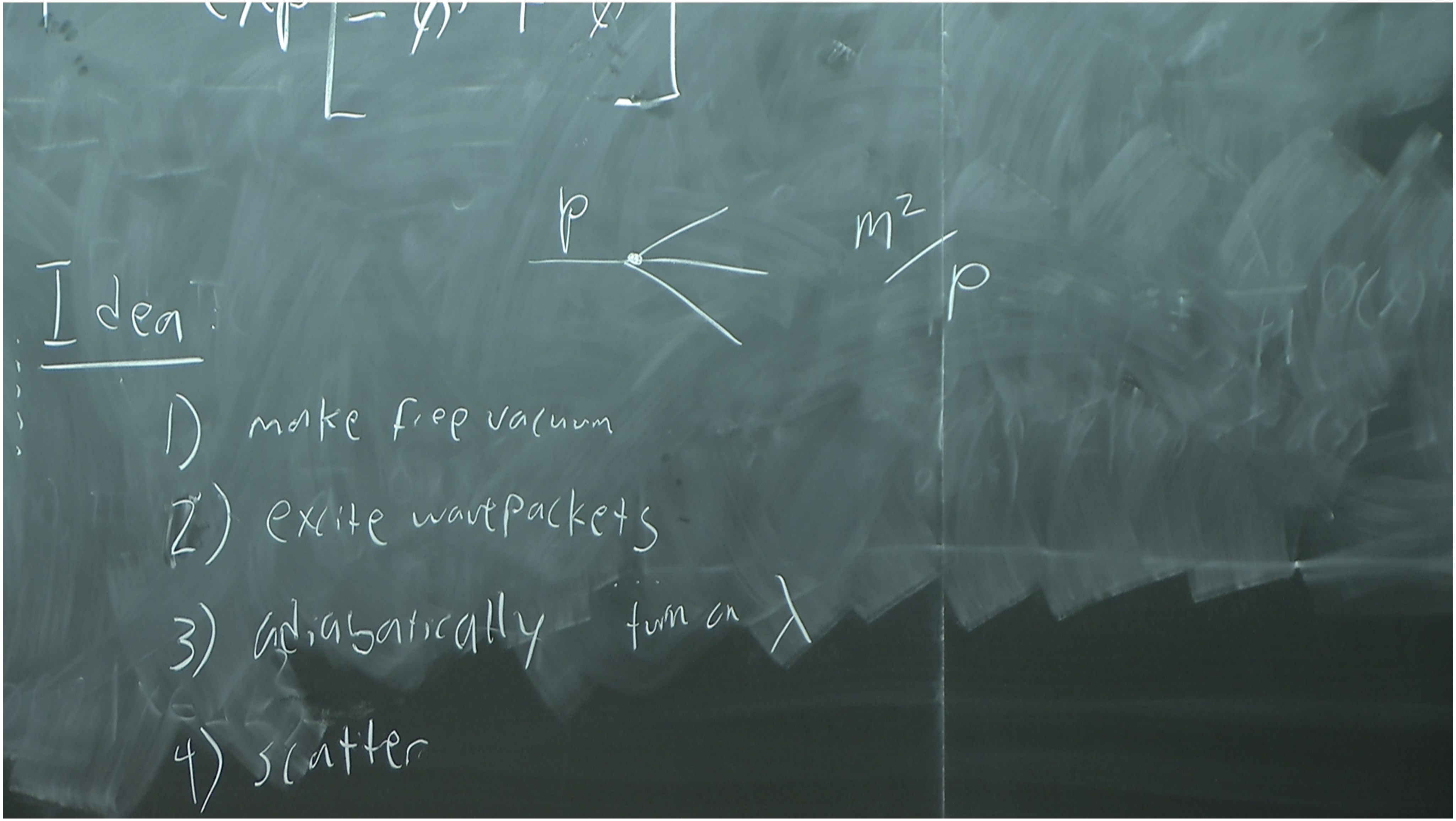
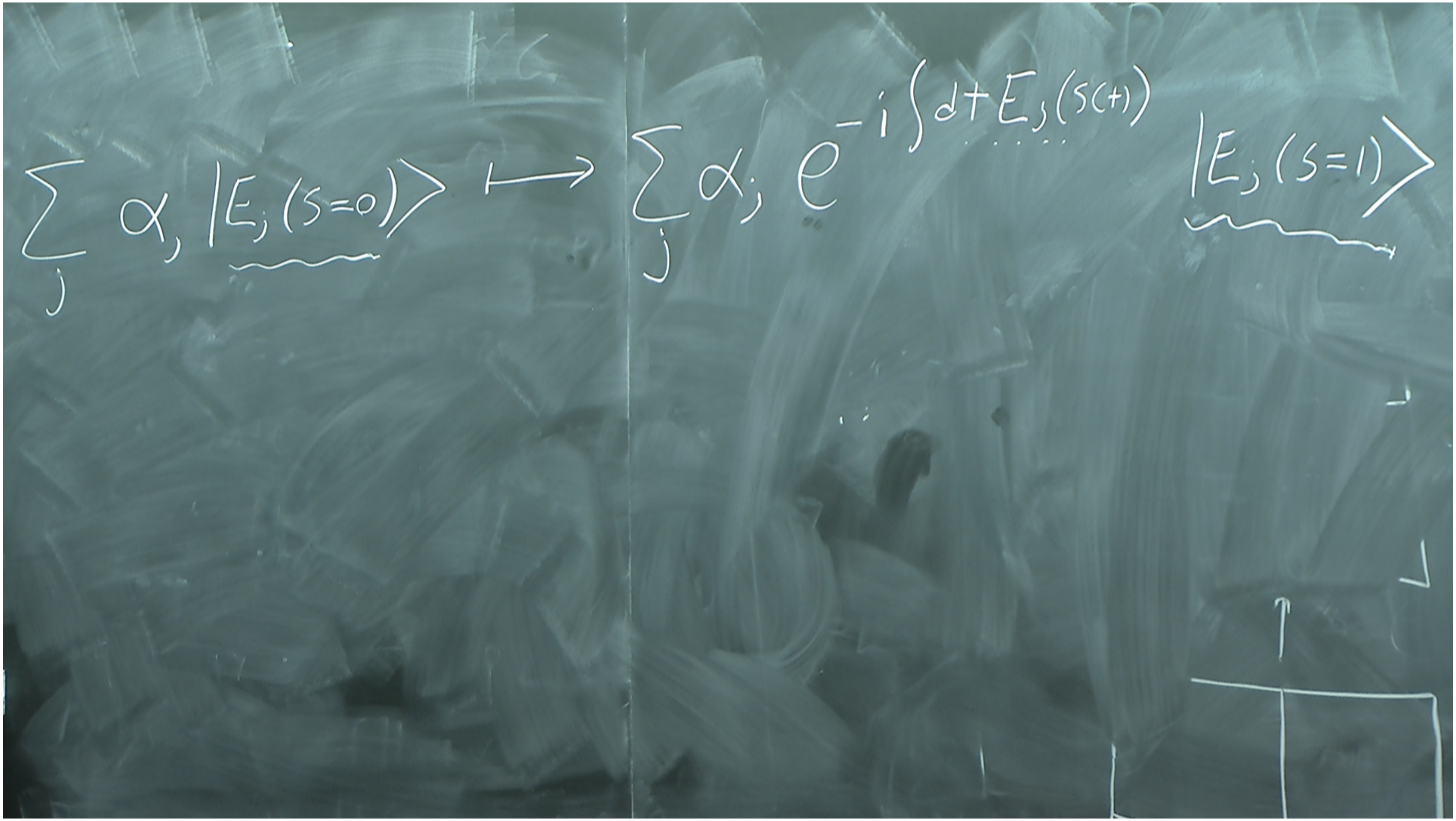
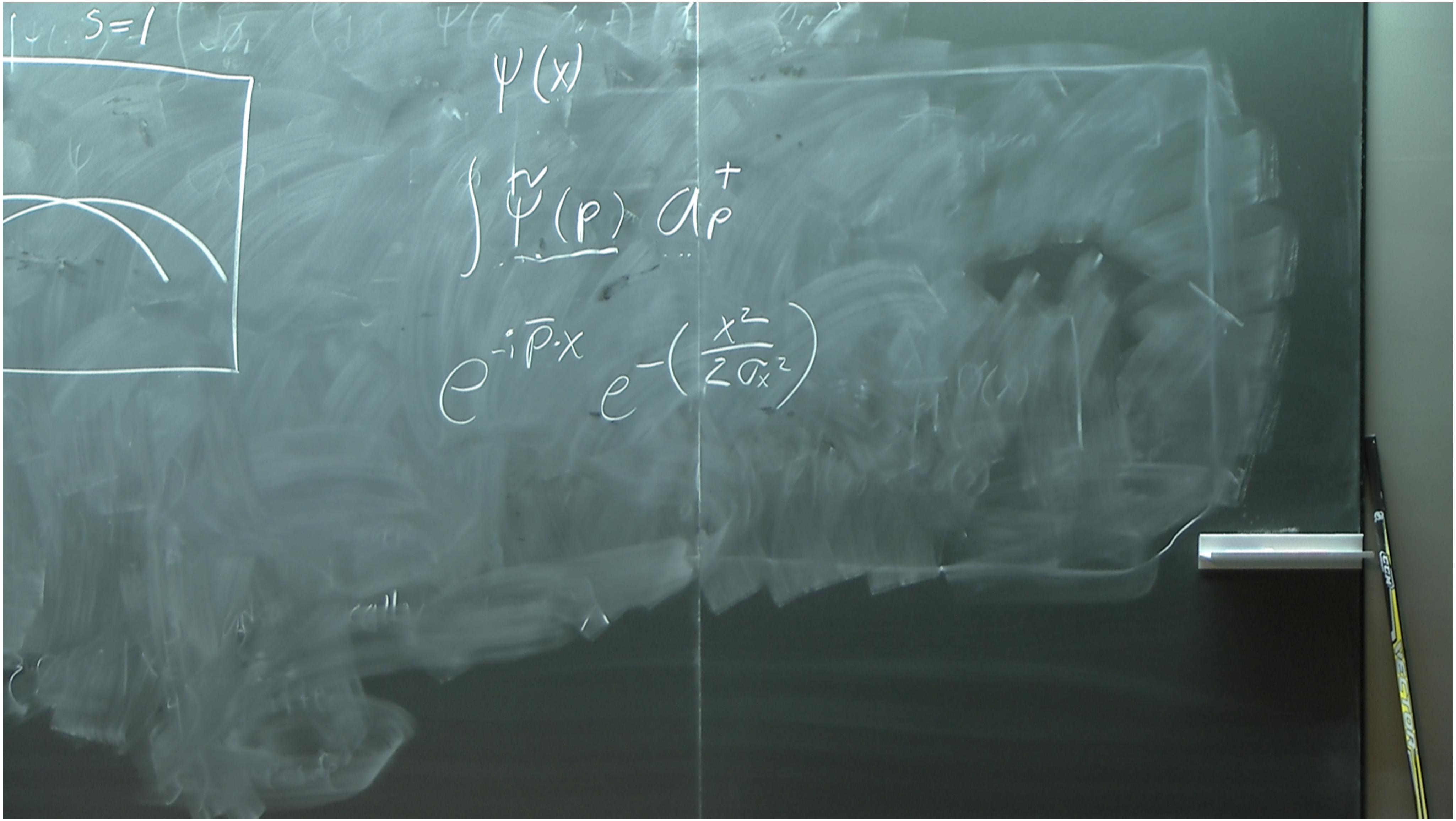

$\frac{\lambda(a/2)}{4!}\phi^4$
$+\frac{\lambda a}{4!}\phi^4$
Idea:
1) make free vacuum
2) excite wavepackets
3) adiabatically turn on λ
4) scatter
5) measure


p
m²/p
Idea:
1) make free vacuum
2) excite wavepackets
3) adiabatically turn on λ
4) scatter


$\sum_j \alpha_j |E_j(s=0)\rangle \longmapsto \sum_j \alpha_j e^{-i\int dt\, E_j(s(t))} |E_j(s=1)\rangle$


S=1
$\psi(x)$
$\int \tilde{\psi}(p)\, a_p^{+}$
$e^{-i\bar{p}\cdot x}\; e^{-\left(\frac{x^2}{2\sigma_x^2}\right)}$