Title: Entanglement in QFT

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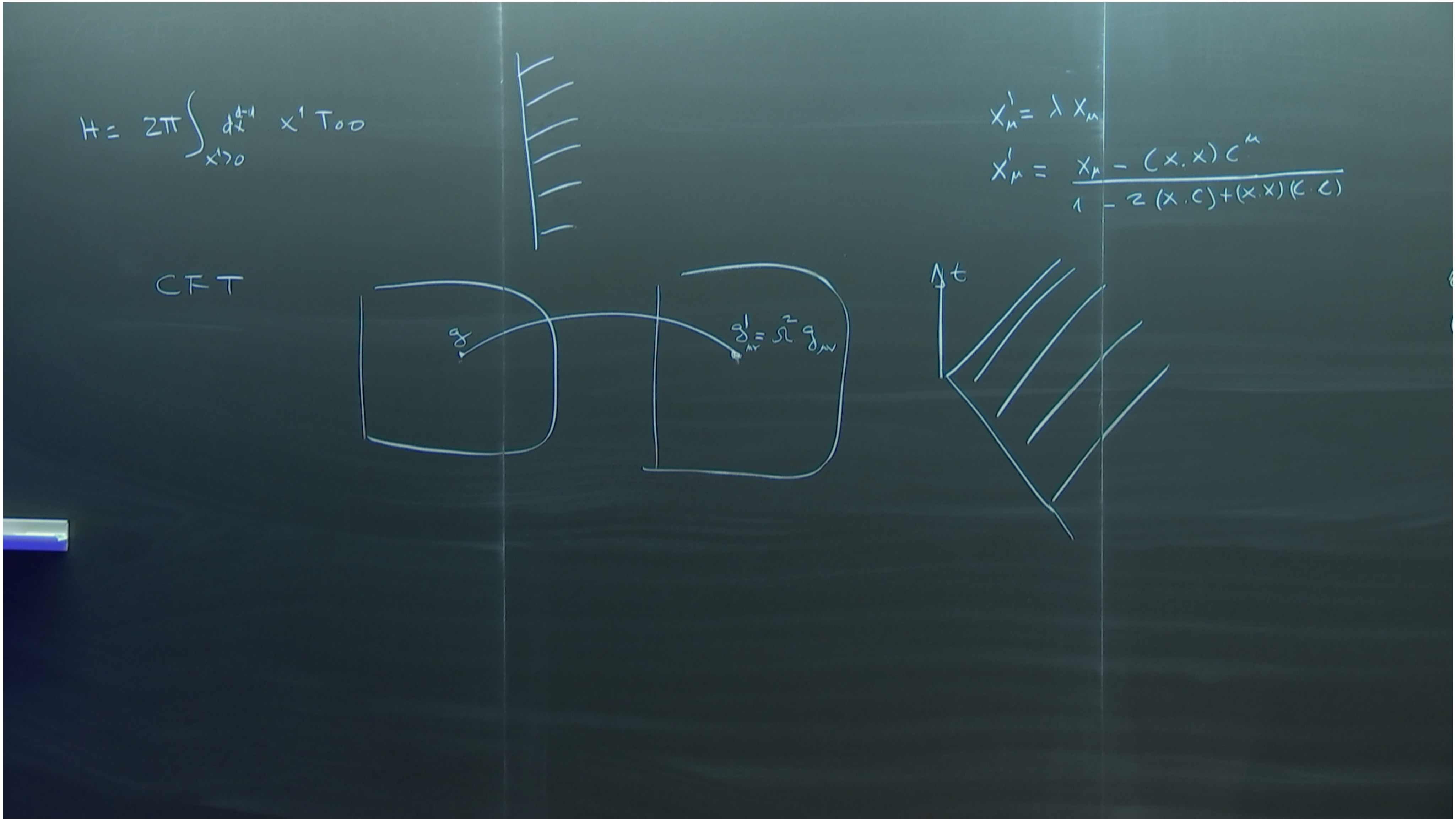
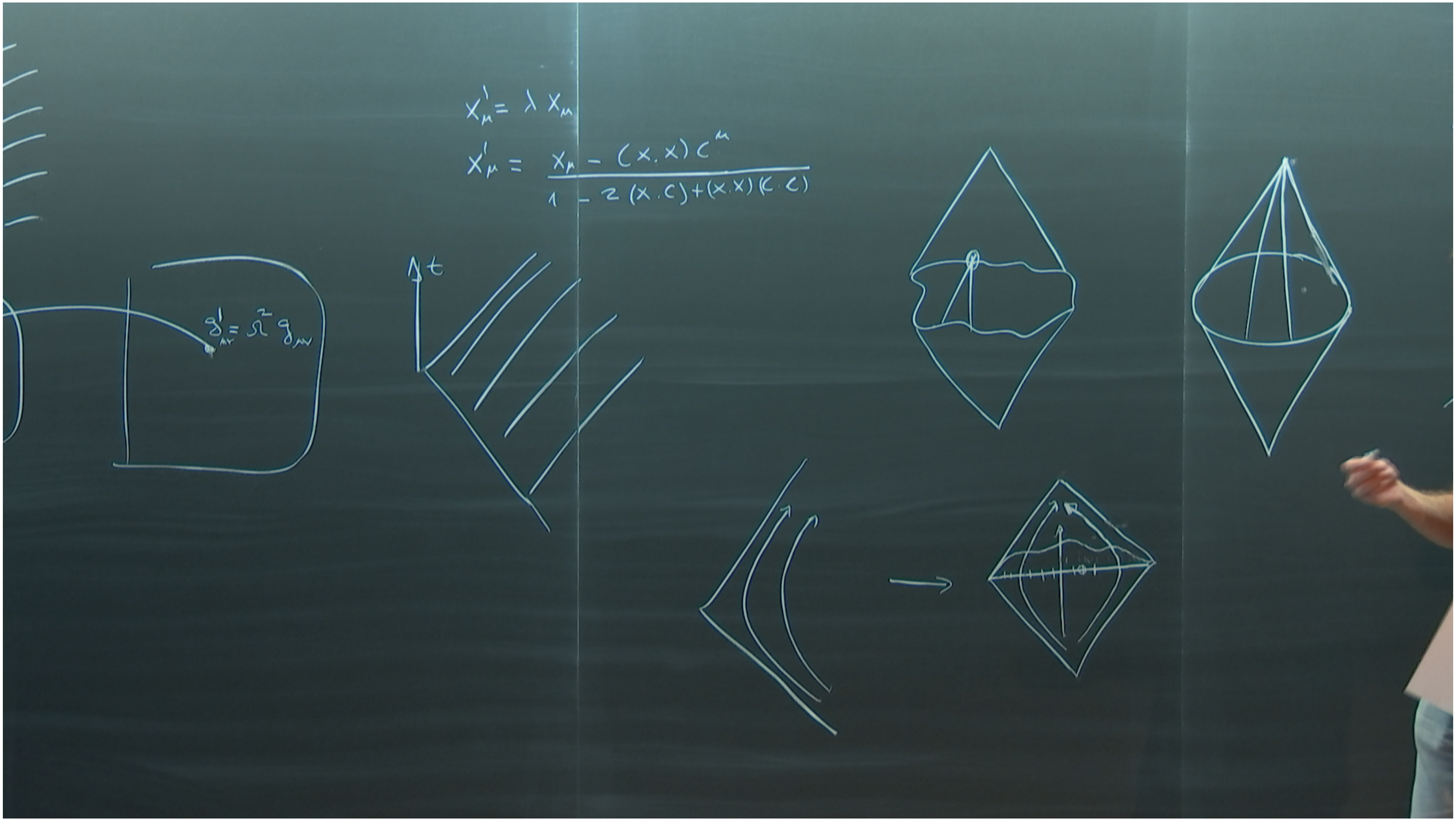
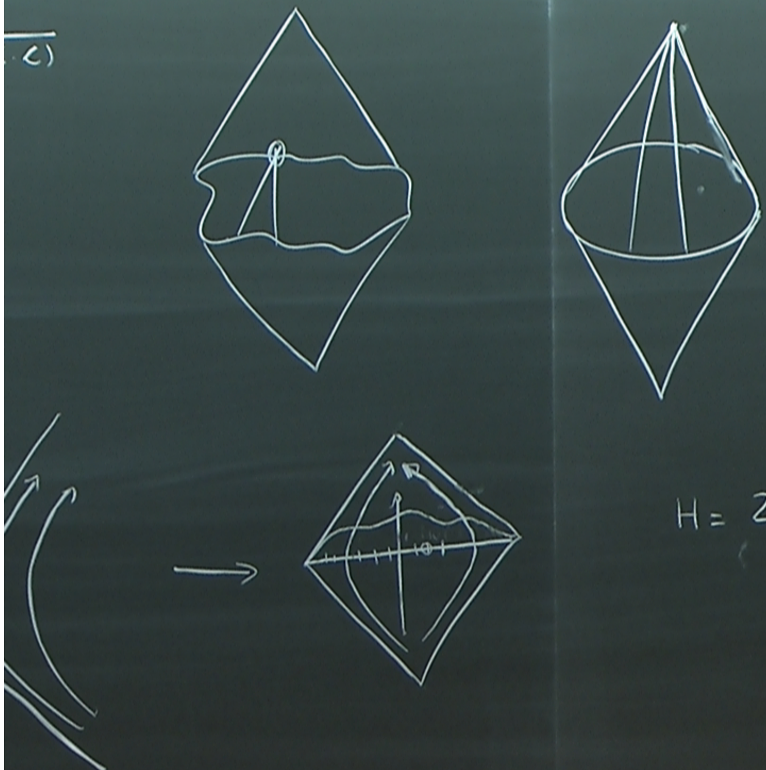
Abstract:


H = 2π ∫ x¹>0 d^{d-1}x x¹ T00
CFT
g
g'μν = Ω² gμν
x'μ = λ xμ
x'μ = (xμ − (x.x) c^μ) / (1 − 2 (x.c) + (x.x)(c.c))
t

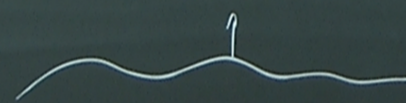

x'μ = λ xμ
x'μ = xμ − (x.x) c^μ / 1 − 2(x.c)+(x.x)(c.c)
g'μν = Ω² gμν
t



$$T^\mu{}_\mu = 0 \qquad \partial_\mu T^\mu{}_\nu = 0$$

$$J^\mu = T^{\mu\nu} a_\nu + T^{\mu\nu}\omega_{\nu\alpha} x^\alpha + c\, T^{\mu\nu} x_\nu + d\, T^{\mu\nu}\left(c_\nu x^2 - 2 x_\nu (c\cdot x)\right)$$

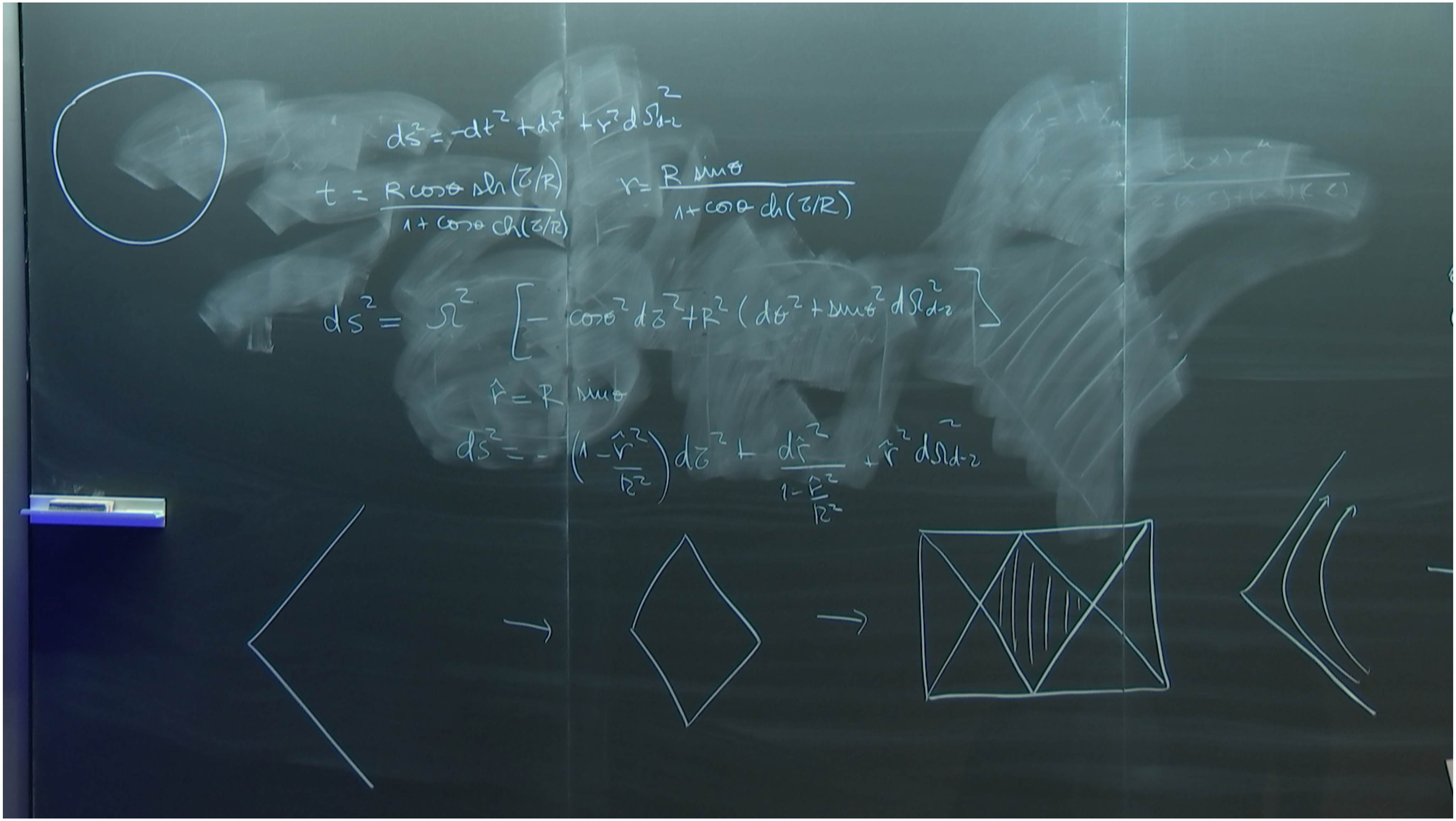
$$\partial_\mu J^\mu = 0$$

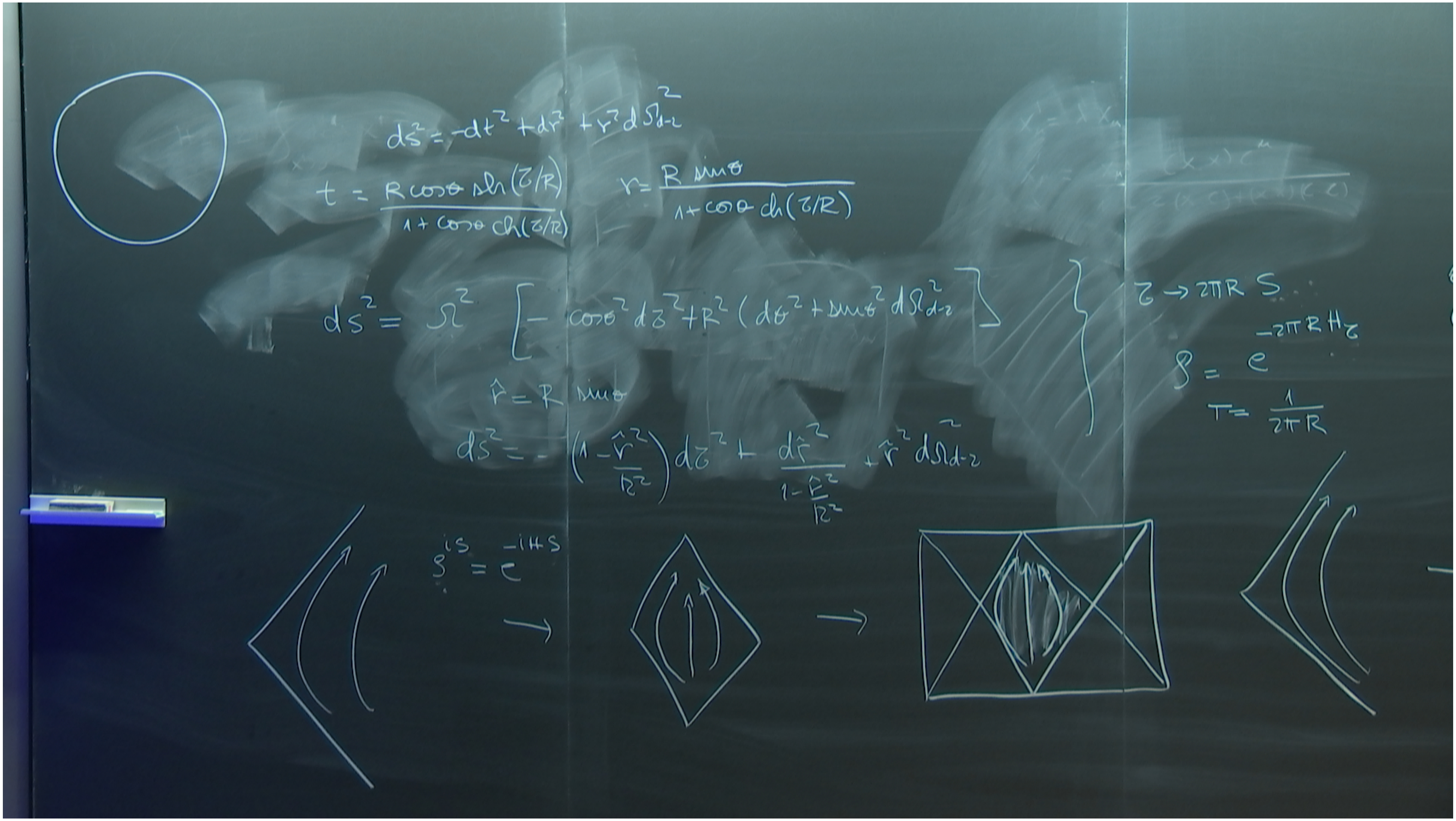


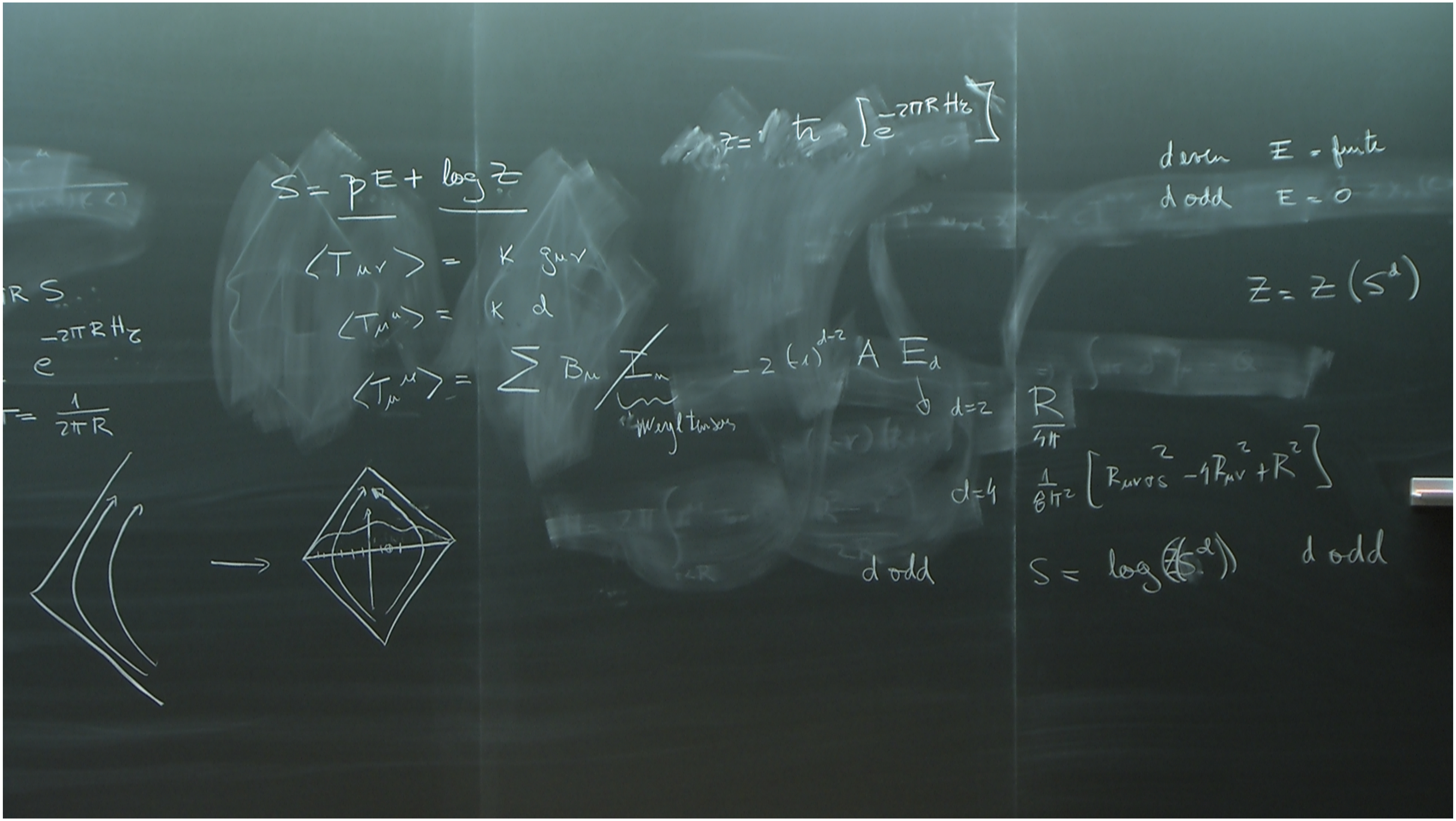
$$\Rightarrow \int d\sigma\, J^\mu \eta_\mu = Q$$

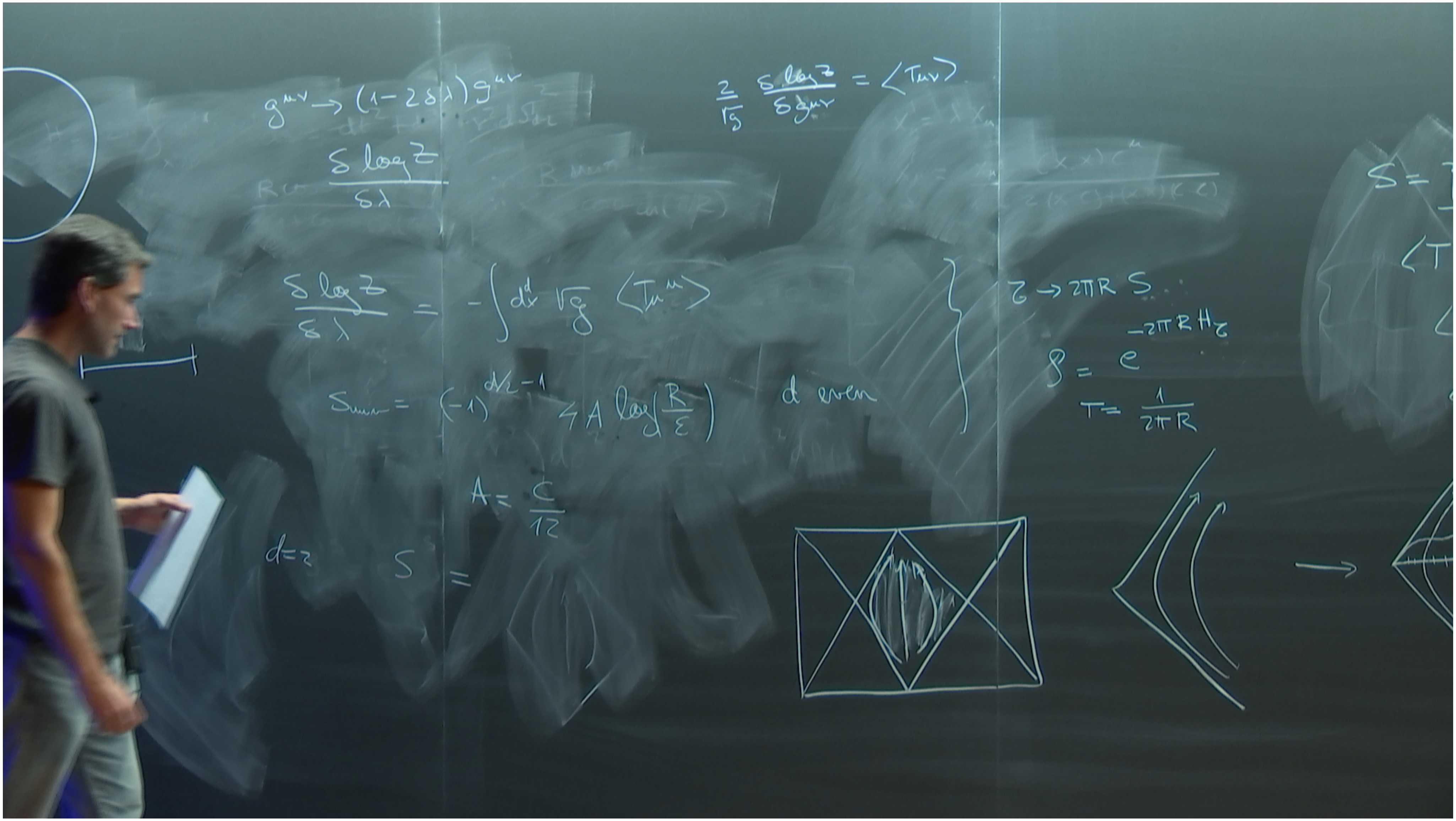
$$(R-r)(R+r)$$

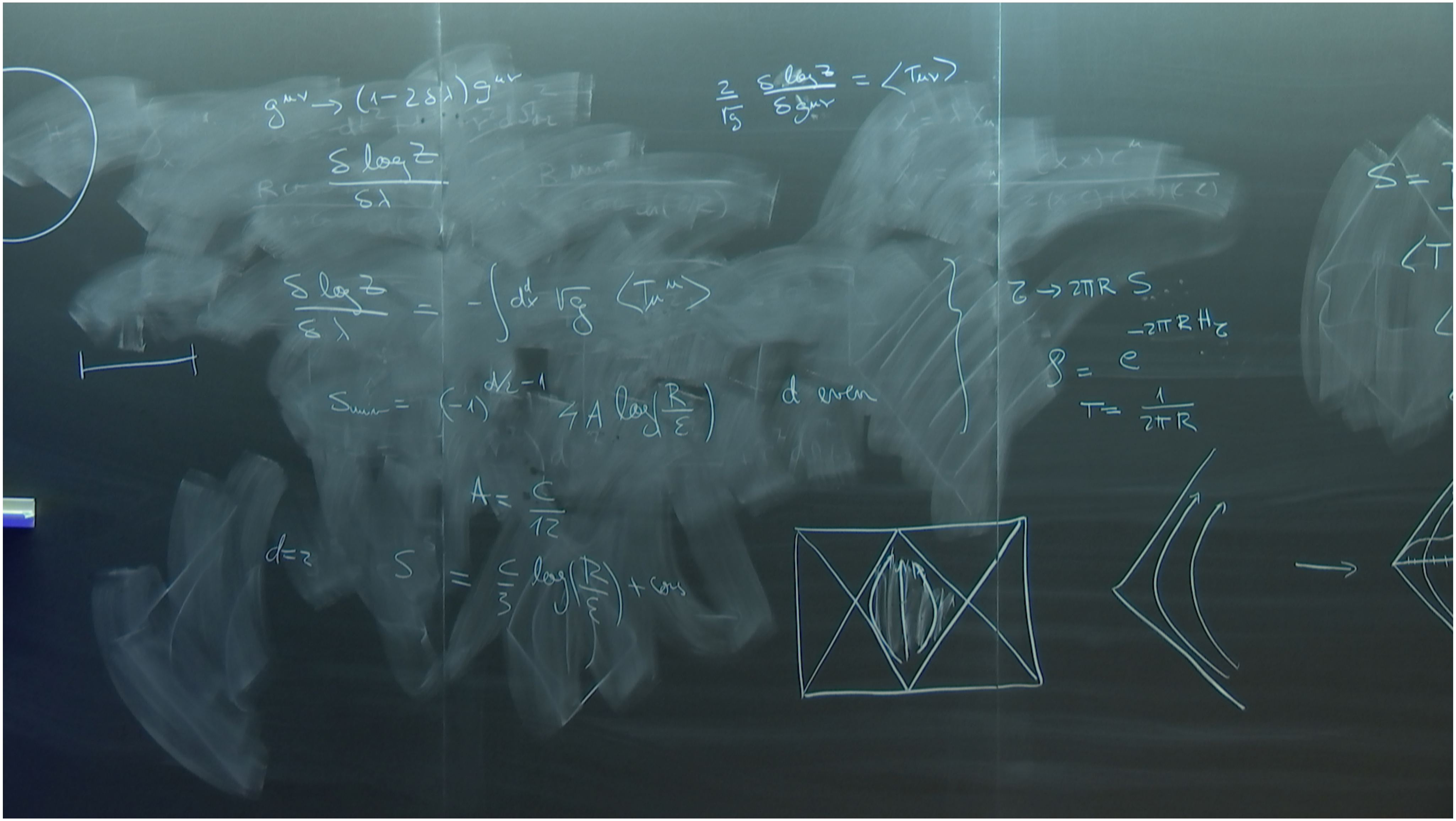
$$H = 2\pi \int_{r<R} d^{d-1}x\; T_{00}(x) \left(\frac{R^2 - r^2}{2R}\right)$$

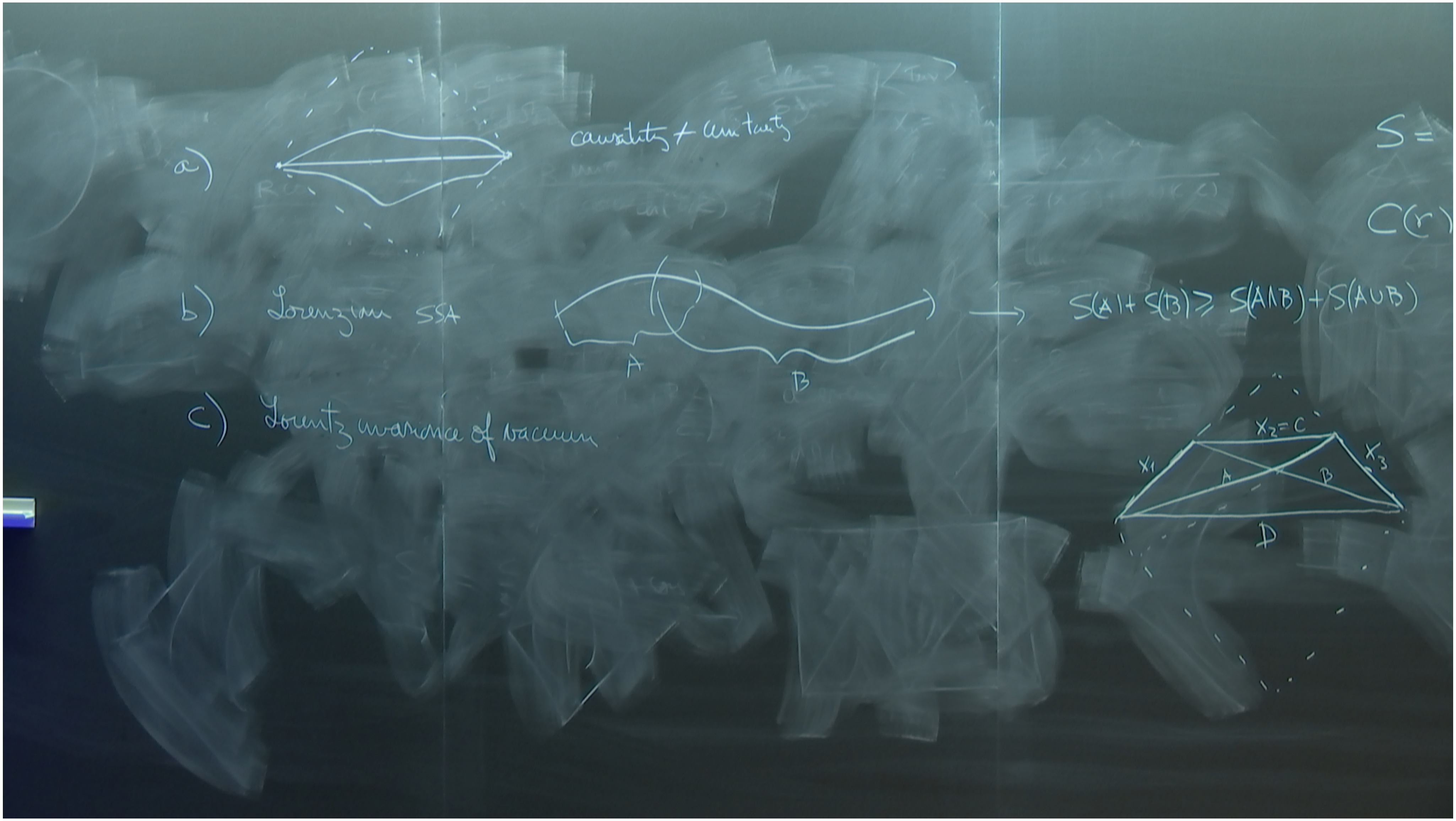

ds^2 = -dt^2 + dr^2 + r^2 dΩ^2_{d-2}
t = R cosθ sh(τ/R) / (1 + cosθ ch(τ/R))
r = R sinθ / (1 + cosθ ch(τ/R))
ds^2 = Ω^2 [- cos^2θ dτ^2 + R^2 (dθ^2 + sin^2θ dΩ^2_{d-2})]
r̂ = R sinθ
ds^2 = - (1 - r̂^2/R^2) dτ^2 + dr̂^2/(1 - r̂^2/R^2) + r̂^2 dΩ^2_{d-2}

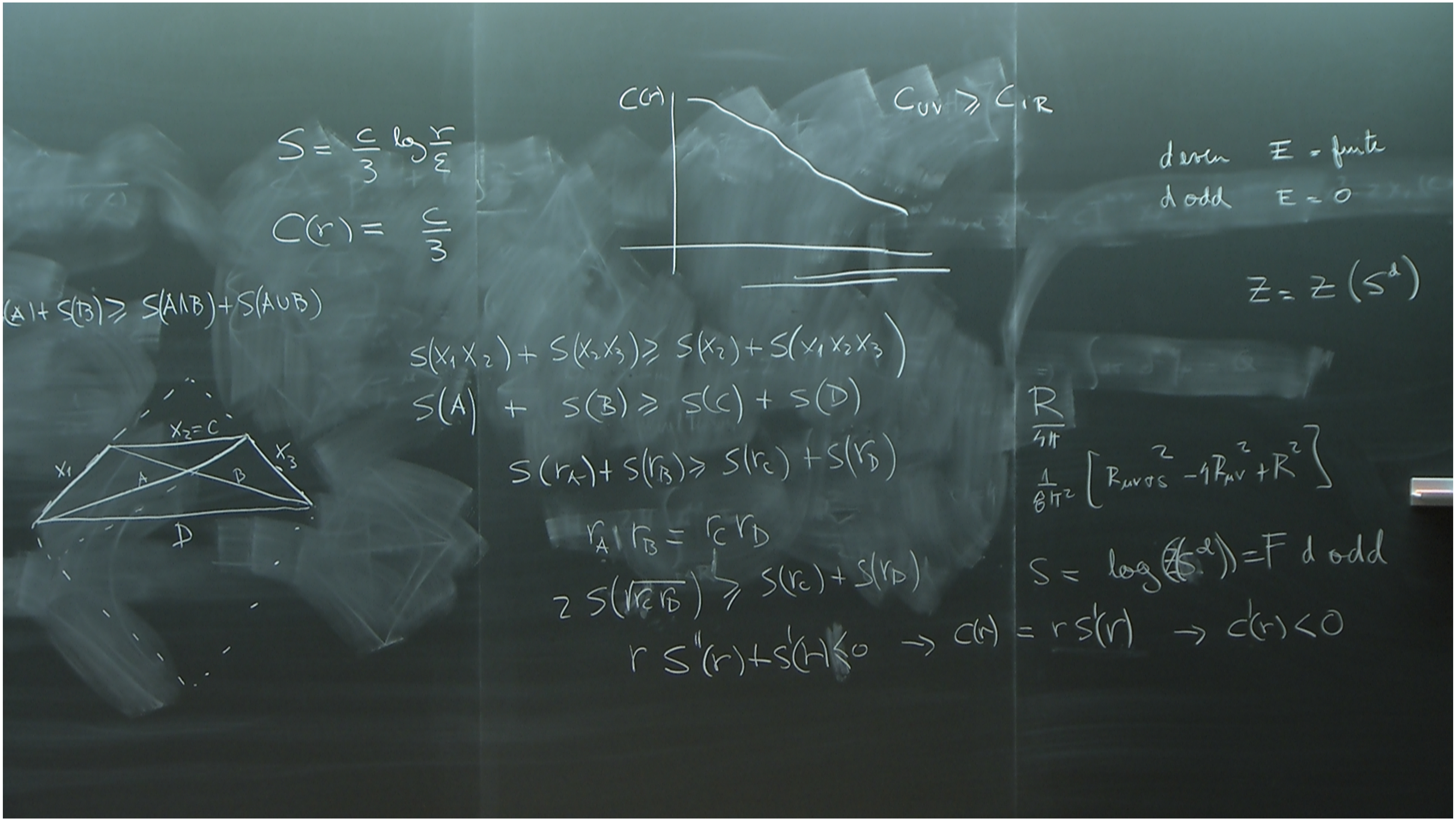

$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2$
$t = \frac{R \cos\theta \, sh(\tau/R)}{1 + \cos\theta \, ch(\tau/R)}$
$r = \frac{R \sin\theta}{1 + \cos\theta \, ch(\tau/R)}$
$ds^2 = \Omega^2 \left[-\cos\theta^2 d\tau^2 + R^2 (d\theta^2 + \sin\theta^2 d\Omega_{d-2}^2) \right]$
$\hat{r} = R \sin\theta$
$ds^2 = -\left(1 - \frac{\hat{r}^2}{R^2}\right) d\tau^2 + \frac{d\hat{r}^2}{1 - \frac{\hat{r}^2}{R^2}} + \hat{r}^2 d\Omega_{d-2}^2$
$\tau \to 2\pi R \, S$
$\rho = e^{-2\pi R H_\tau}$
$T = \frac{1}{2\pi R}$
$g^{is} = e^{-iHs}$


S = βE + log Z
⟨T_μν⟩ = K g_μν
⟨T_μ^μ⟩ = K d
⟨T_μ^μ⟩ = Σ B_n I_n − 2(−1)^(d−2) A E_d
Weyl tensors
Z = tr [e^(−2πR H_τ)]
d even E = finite
d odd E = 0
Z = Z(S^d)
d = 2 R/4π
d = 4 1/8π² [R_μνρσ² − 4R_μν² + R²]
d odd
S = log(Z(S^d)) d odd
e^(−2πR H_τ)
T = 1/2πR

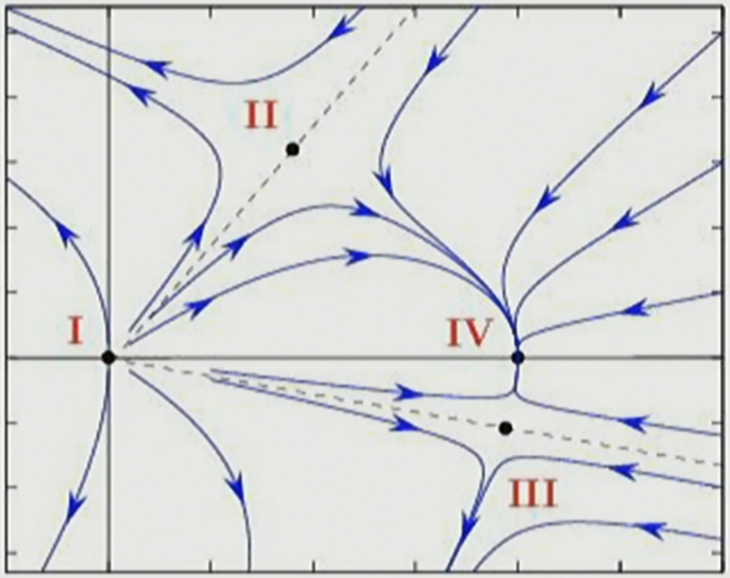

g^{μν} → (1 − 2δλ) g^{μν}
δ log Z / δλ
2/√g δ log Z / δg_{μν} = ⟨T_{μν}⟩
δ log Z / δλ = − ∫ d^d x √g ⟨T_μ^μ⟩
S_univ = (−1)^{d/2 − 1} 4 A log(R/ε)
d even
A = c/12
d=2
S =
τ → 2πR S
ρ = e^{−2πR H_τ}
T = 1/(2πR)


d even
d=2


a)
causality + unitarity
b) Lorenzian SSA
A
B
S(A)+S(B) ≥ S(A∩B)+S(A∪B)
c) Lorentz invariance of vacuum
S =
C(r)
X2=C
X1
X3
A
B
D


S = c/3 log r/ε
C(r) = c/3
C(r)
C_UV ≥ C_IR
d even E = finite
d odd E = 0
Z = Z(S^d)
S(A) + S(B) ≥ S(A∩B) + S(A∪B)
S(X1 X2) + S(X2 X3) ≥ S(X2) + S(X1 X2 X3)
S(A) + S(B) ≥ S(C) + S(D)
S(rA) + S(rB) ≥ S(rC) + S(rD)
rA rB = rC rD
2 S(√(rC rD)) ≥ S(rC) + S(rD)
r S''(r) + S'(r) ≤ 0 → C(r) = r S'(r) → C'(r) < 0
X1
X2 = C
X3
A
B
D
R/4π
1/8π² [R_μνσs² − 4R_μν² + R²]
S = log(Z(S^d)) = F d odd

Renormalization group flow in the space of QFT



$$\tau \frac{dg_i}{d\tau} = \beta_i(\{g(\tau)\})$$

Change in the physics with scale through the change of coupling constants with the RG flow. At fixed points there is scale invariance: the theory looks the same at all scales. The RG flow Interpolates between UV (short distance) to IR (large distance) fix points.

Are there any general constraints on these RG flows?

C-theorem in more dimensions?

Proposal for even dimensions: coefficient of the Euler density term in the trace anomaly at the fixed point, Cardy (1988).

d=2 $\langle \Theta \rangle = -cR/12 \longrightarrow c = -\frac{3}{\pi}\int_{S^2} \langle \Theta \rangle \sqrt{g}\, d^2x$ $\Theta(x) = T^\mu_\mu(x)$

general d $\langle \Theta(x) \rangle = \frac{(-1)^{d/2}}{2} a_d E(x) + \text{other polynomials of order d/2 in the curvatura tensor}$

d=4: $E(x) \sim R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

$\longrightarrow C = (-1)^{d/2} a_d \int_{S^d} d^d x \sqrt{g} \langle \Theta \rangle$

As θ meassures the variation of the effective action under scaling this number is proportional to the logarithmically divergent term in log Z on a d-dimensional sphere

Proved by Komargodski and Schwimmer for d=4 (2011) (a-theorem) using unitarity of the S-Matrix.

Odd dimensions? No trace anomaly in odd dimensions

Myers-Sinha (2010) **Holographic c-theorems**

$$ds^2 = e^{2A(r)}(-dt^2 + d\vec{x}^2_{d-1}) + dr^2 \qquad A(r) = \text{cons} \;\; r \qquad \text{at fixed points (AdS space)}$$

Higher curvature gravity lagrangians:
a(r) function of A(r) and coupling constants
$a(r) = a^* = \text{constant}$ at fixed points

$$a'(r) \sim (T^t_t - T^r_r) \geq 0$$

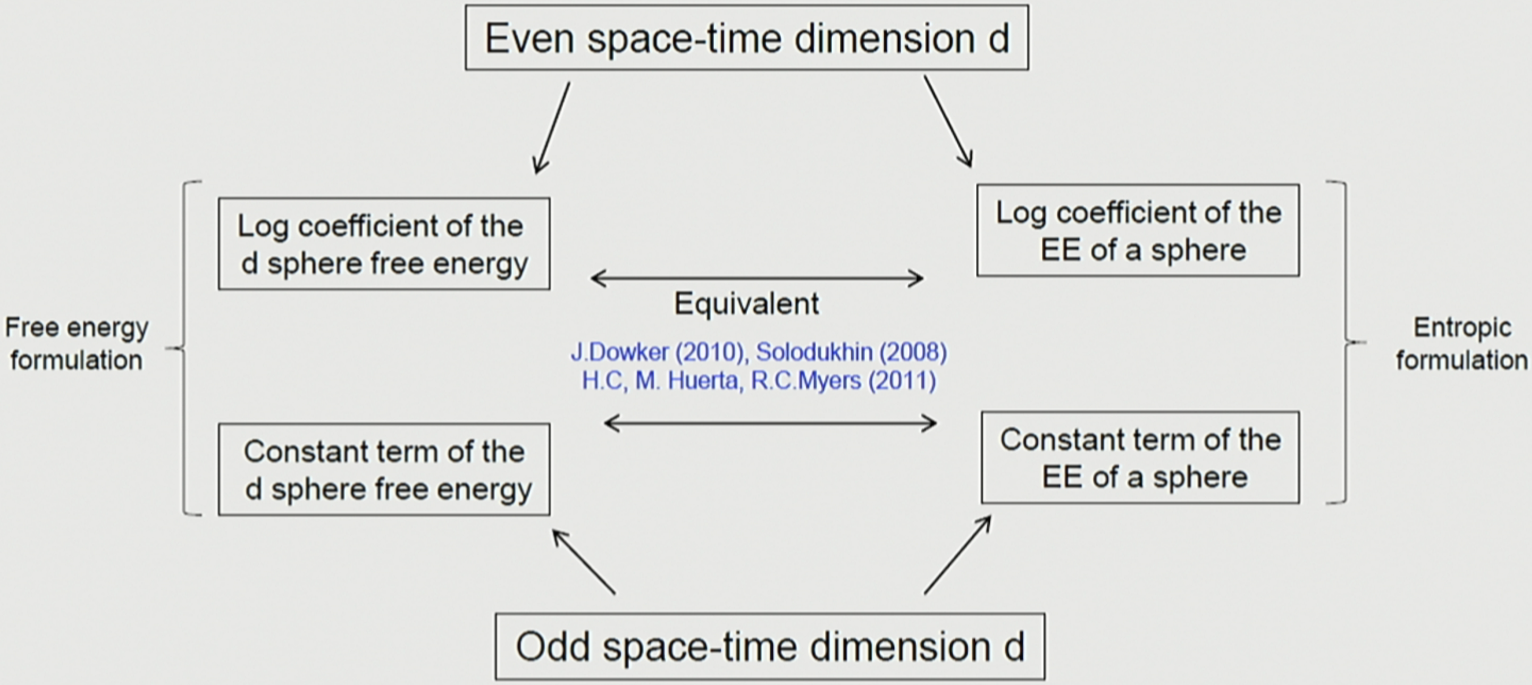
null energy condition

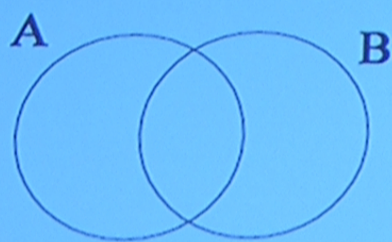
$a^*_{uv} \geq a^*_{ir}$ QFT interpretation: For even spacetime dimensions a^* is the coefficient of the Euler term in the trace anomaly (coincides with Cardy proposal for the c-theorem)

For odd dimensions the constant term of the sphere entanglement entropy is proportional to a^* (by interpreting entanglement entropy in the boundary as BH entropy in the bulk)

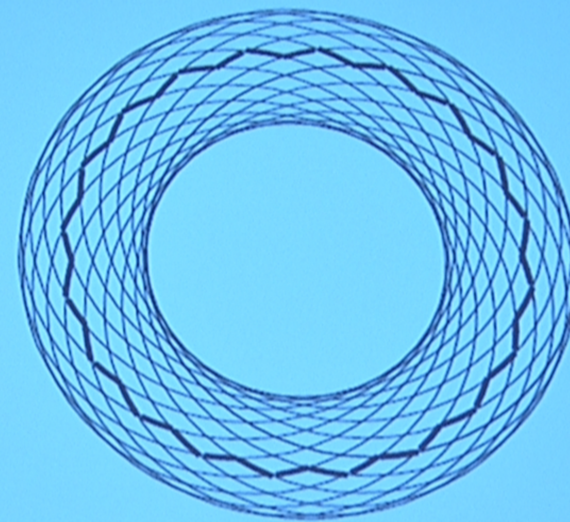
F-theorem (Jafferis, Klebanov, Pufu, Safdi (2011)): propose finite term in the free energy F=–log(Z) of a three sphere decreases between fix points under RG. Non trivial tests for supersymmetric and non-susy theories (Explicit computations of F for interacting theories by localization)

Relation between EE of spatial (d-2) entangling sphere / partition function on euclidean d-sphere





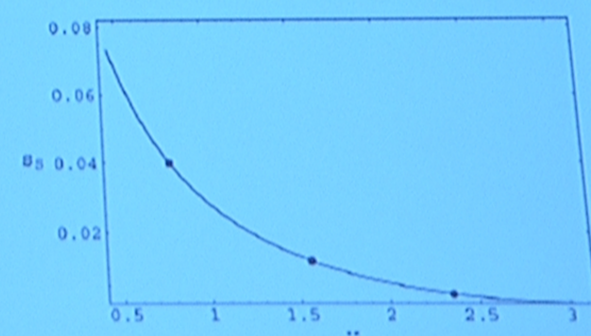
Two problems: different shapes and log divergent angle contributions. Use many rotated regions for first problem



From SSA:

$$\sum_i S(X_i) \geq S(\cup_i X_i) + S(\cup_{\{ij\}}(X_i \cap X_j)) + S(\cup_{\{ijk\}}(X_i \cap X_j \cap X_k)) + \ldots + S(\cap_i X_i)$$

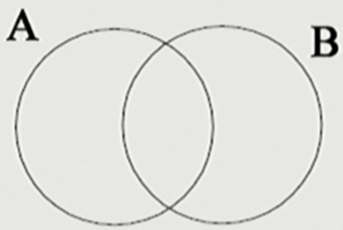
Log divergent terms cannot appear for «angles» on a null plane since the feature does not have any local geometric measure



Coefficient of the logarithmically divergent term for a free scalar field

C-theorem in d=3 from SSA for circles

H.C, M. Huerta 2012



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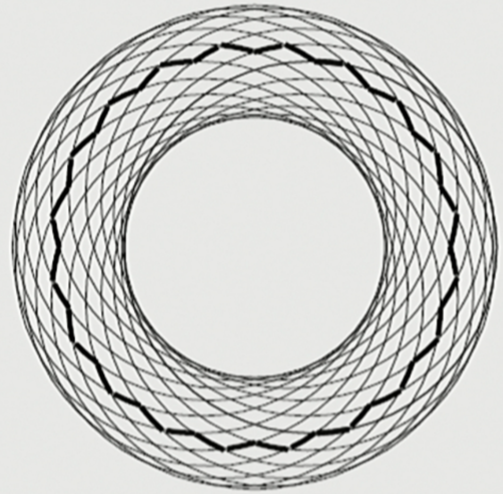

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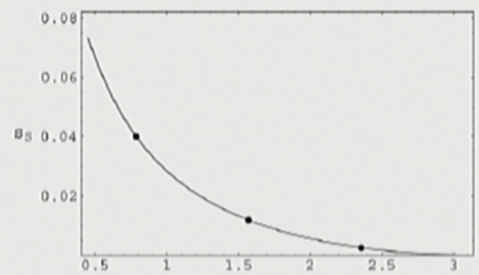
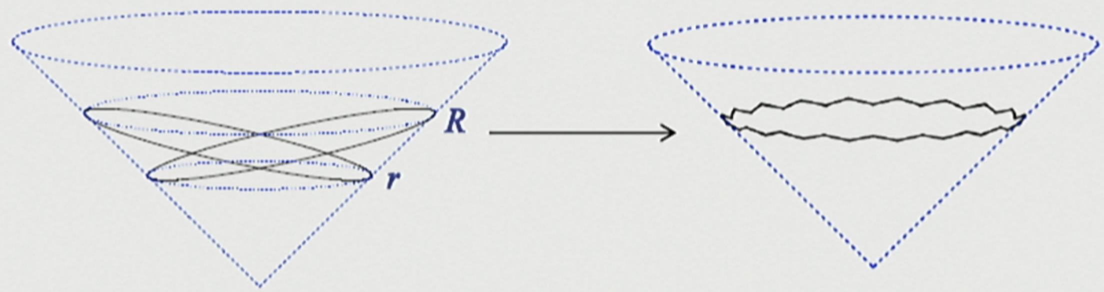


Coefficient of the logarithmically divergent term for a free scalar field



$$S(\sqrt{Rr}) \geq \frac{1}{\pi}\int_0^{\pi} dz\, S\left(\frac{2rR}{R+r-(R-r)\cos(z)}\right) \Longrightarrow S'' \leq 0$$

$$c_0(r) = rS'(r) - S(r) \Rightarrow c_0(r)' \leq 0$$

Dimensionless and decreasing
C-function proposed by H.Liu and M. Mezei (2012)
Based on holografic and QFT analysis

At fixed points $S(R) = c_1 R - c_0$

$c_0(r) = c_0$ Is the constant term of the entropy of the circle

Running of area term: allways decreases towards the infrared

At fix points $S(R) = R\left(\frac{k_1}{\epsilon} + k_0\right) - c_0$

Away from fix points $S''(R) < 0$

$$
\begin{aligned}
c_0^{UV} - c_0^{IR} &= -\int_0^\infty dR\, R\, S''(R) \geq 0\,, \\
\mu = k_0^{IR} - k_0^{UV} &= \int_0^\infty dR\, S''(R) \leq 0\,.
\end{aligned}
$$

Sum rule for variation of area term

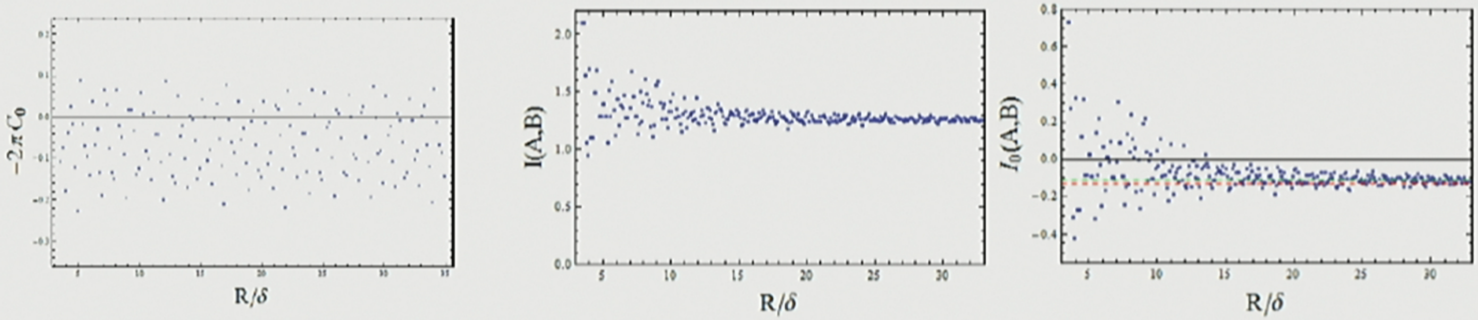
V.Rosenhaus,M.Smolkin (2014),
H.C., D.Mazzitelli,E.Teste (2014)

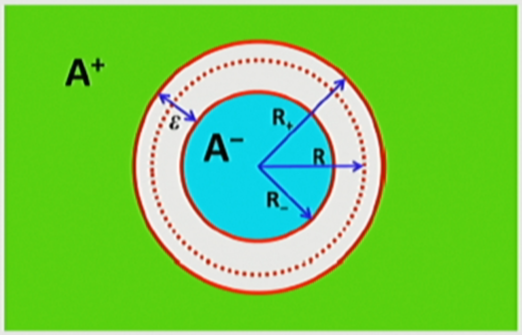
$$\mu = -\frac{\pi}{6}\int d^3x\; x^2\, \langle 0|\Theta(0)\Theta(x)|0\rangle$$

Area term drives constant term.
Implies c_0 necesarilly changes with RG running, as in Zamolodchikov's theorem

Subtleties in defining C_0 at fix points: Mutual information clarify this issue

H.C., M. Huerta, R.C. Myers, A. Yale





$$I(A,B) = S(A) + S(B) - S(A \cup B)$$

The local divergences cancel in I(A,B) which is finite and well defined in QFT

Mutual information as a geometric regulator for EE: all coefficients on the expansion are universal and well defined

$$S(R) = \left(\frac{a}{\delta} + b\right) R - C_0 \rightarrow I(A^+, A^-) = 2\left(\frac{\tilde{a}}{\epsilon} + \tilde{b}\right) R - 2C_0$$

Locality+symmetry argument

(similar to Liu-Mezei 2012, Grover, Turner,Vishwanath 2011)

C charge well defined through mutual information
IR , UV values depend only on the CFT
This is a physical quantity calculable with any regularization, including lattice
The constant term coincides with the one in the entropy of a circle for «good enough» regularizations.

Entropic proof in more dimensions?

Symmetric configuration of boosted spheres in the limit of large number of spheres

Divergent terms do not cancel, trihedral angles, curved dihedral angles. Wiggly spheres not converge to smooth spheres: mismatch between curvatures.

More generally:

a) Strong subadditivity always gives inequalities for second derivatives
b) This inequality should give C'<0. Then C is constructed with S and S'
c) C has to be cutoff independent. But at fix points

$$S(r) = c_2 \frac{r^2}{\epsilon^2} + c_{\log} \log(r/\epsilon) + c_0$$

It is not possible to extract the coeff. of the logarithmic term with S and S'.

New inequalties for the entropy? Some posibilities have been discarded holografically (H.Liu and M. Mezei (2012)) c-theorem in 1+1 and 2+1 dimensions for relativistic theories have been found using entanglement entropy and strong subadditivity. No proof has been found yet for 2+1 that does not use entanglement entropy. (Difficult to construct C_0 from correlators if it contains topological information. How to uniquely define the theory on the sphere from the one in flat space?)

Why a c-theorem should exist?: loss of d.o.f along RG not a good reason (in a direct way). It can be a relativistic QFT theorem as CPT, spin-statistics, etc., or there is a deeper information theoretical explanation (suggested by entanglement entropy)

Is C a measure of «number of field degrees of freedom»?
C is not an anomaly in d=3. It is a small universal non local term in a divergent entanglement entropy. It is very different from a «number of field degrees of freedom»: Topological theories with no local degree of freedom can have a large C (topological entanglement entropy)! C does measure some form of entanglement that is lost under renormalization, but what kind of entanglement?

Is there some loss of information interpretation?
Even if the theorem applies to an entropic quantity, there is no known interpretation in terms of some loss of information. Understanding this could tell us whether there is a version of the theorem that extends beyond relativistic theories.