Title: Simulation of Quantum Hamiltonians

Date: Jul 23, 2016 09:00 AM

URL: http://pirsa.org/16070034

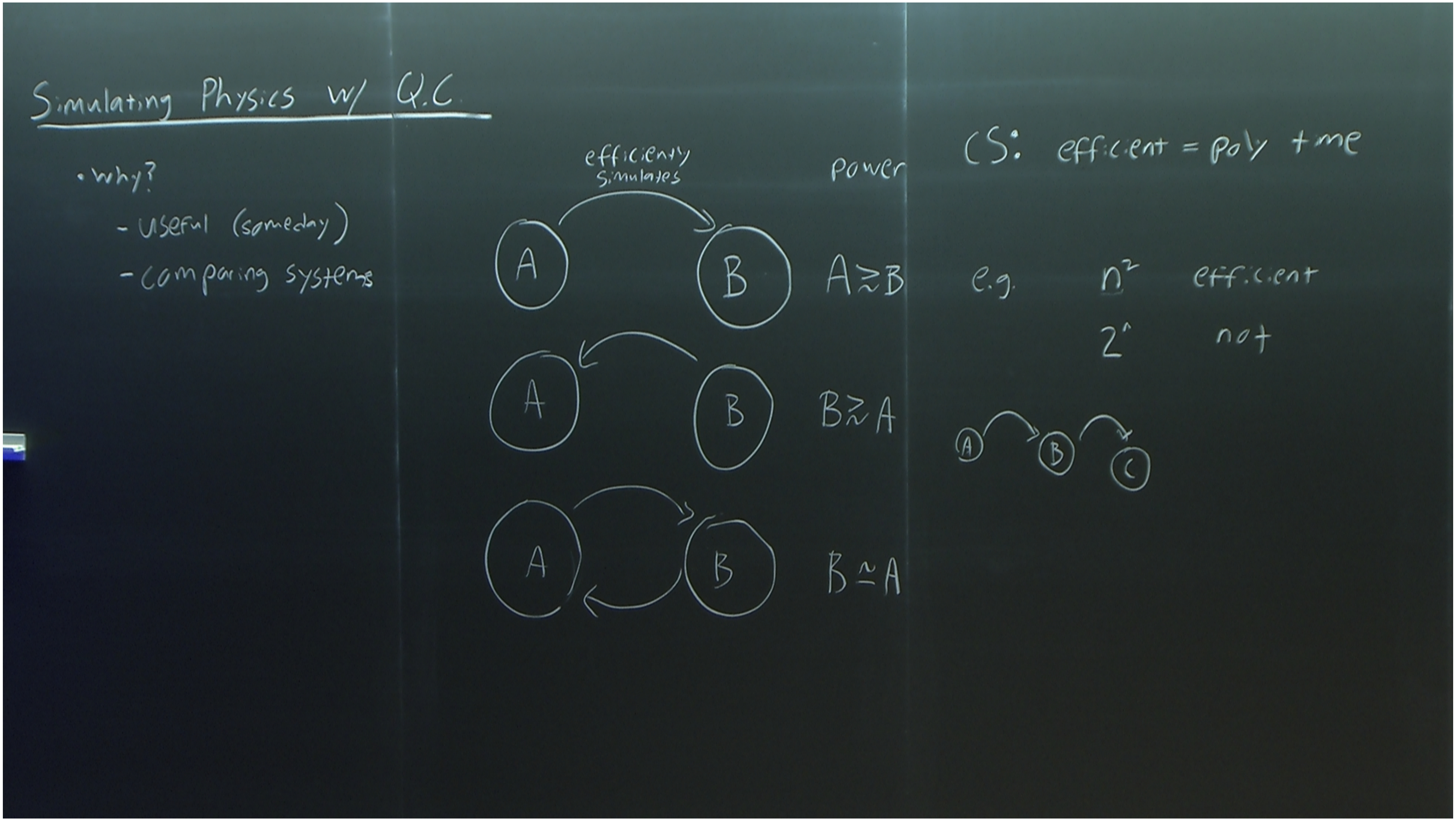
Abstract:

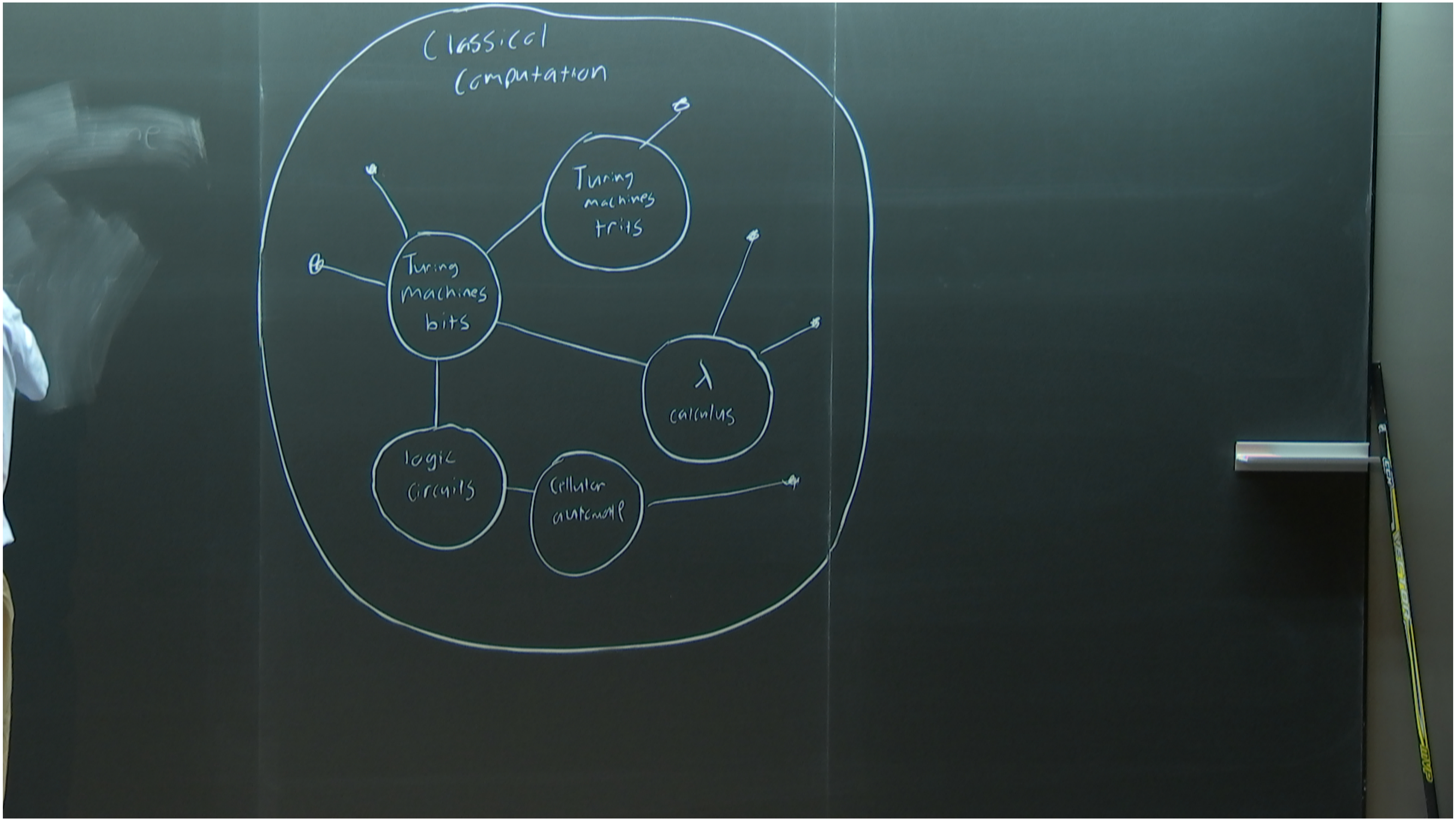
Simulating Physics w/ Q.C.

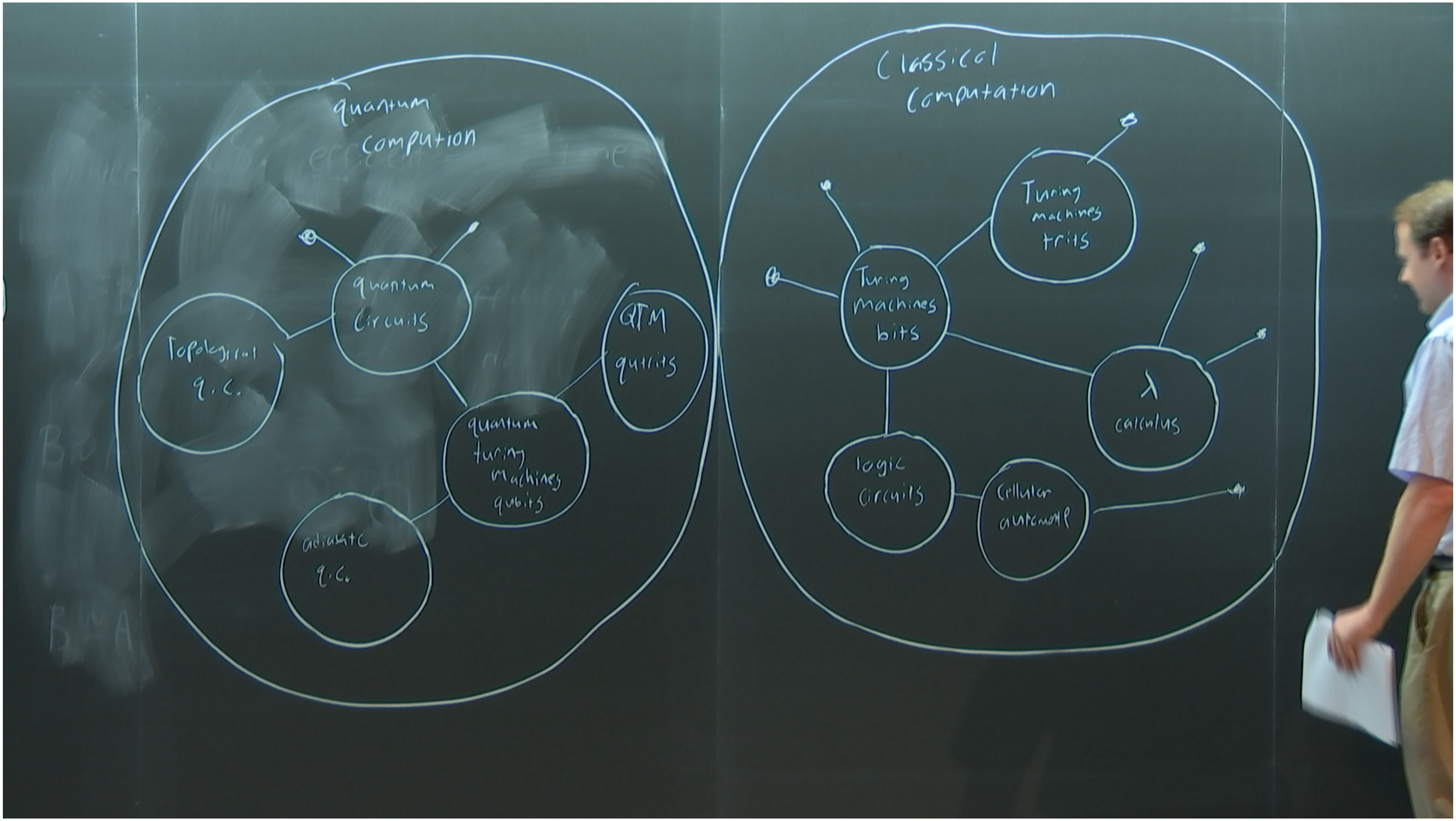
- Why?
  - useful (someday)
  - comparing systems

	power
$A \xrightarrow{\text{efficiently simulates}} B$	$A \gtrsim B$
$A \leftarrow B$	$B \gtrsim A$
$A \rightleftarrows B$	$B \simeq A$

CS: eff


Simulating Physics w/ Q.C.
• why?
- useful (someday)
- comparing systems
efficiently simulates
power
A
B
$A \gtrsim B$
$B \gtrsim A$
$B \simeq A$
CS: efficient = poly time
e.g. n^2 efficient
2^n not
A
B
C


Classical computation
Turing machines trits
Turing machines bits
λ calculus
logic circuits
cellular automata


quantum computation
quantum circuits
Topological q.c.
QTM qutrits
quantum turing machines qubits
adiabatic q.c.
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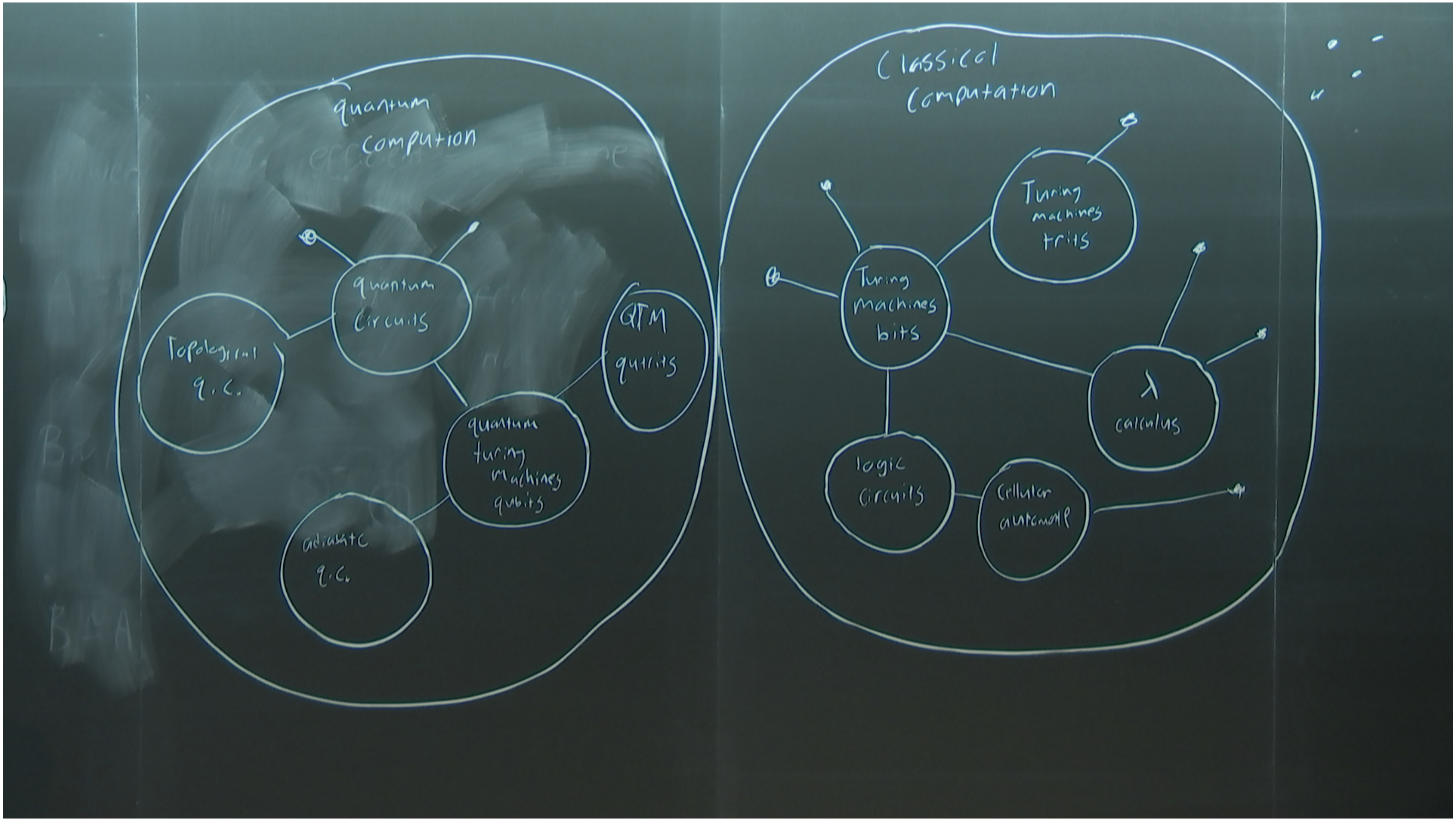
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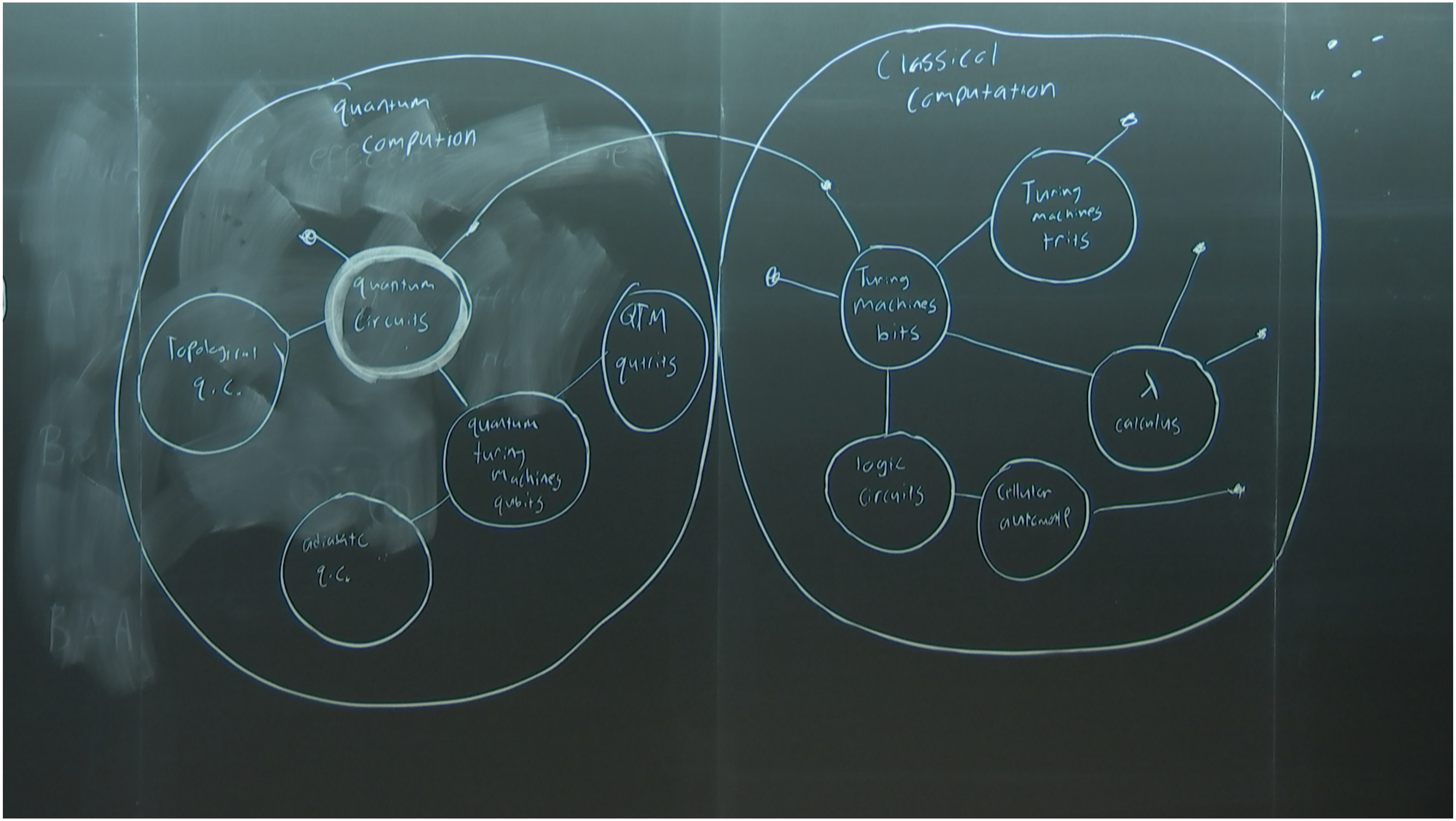
quantum
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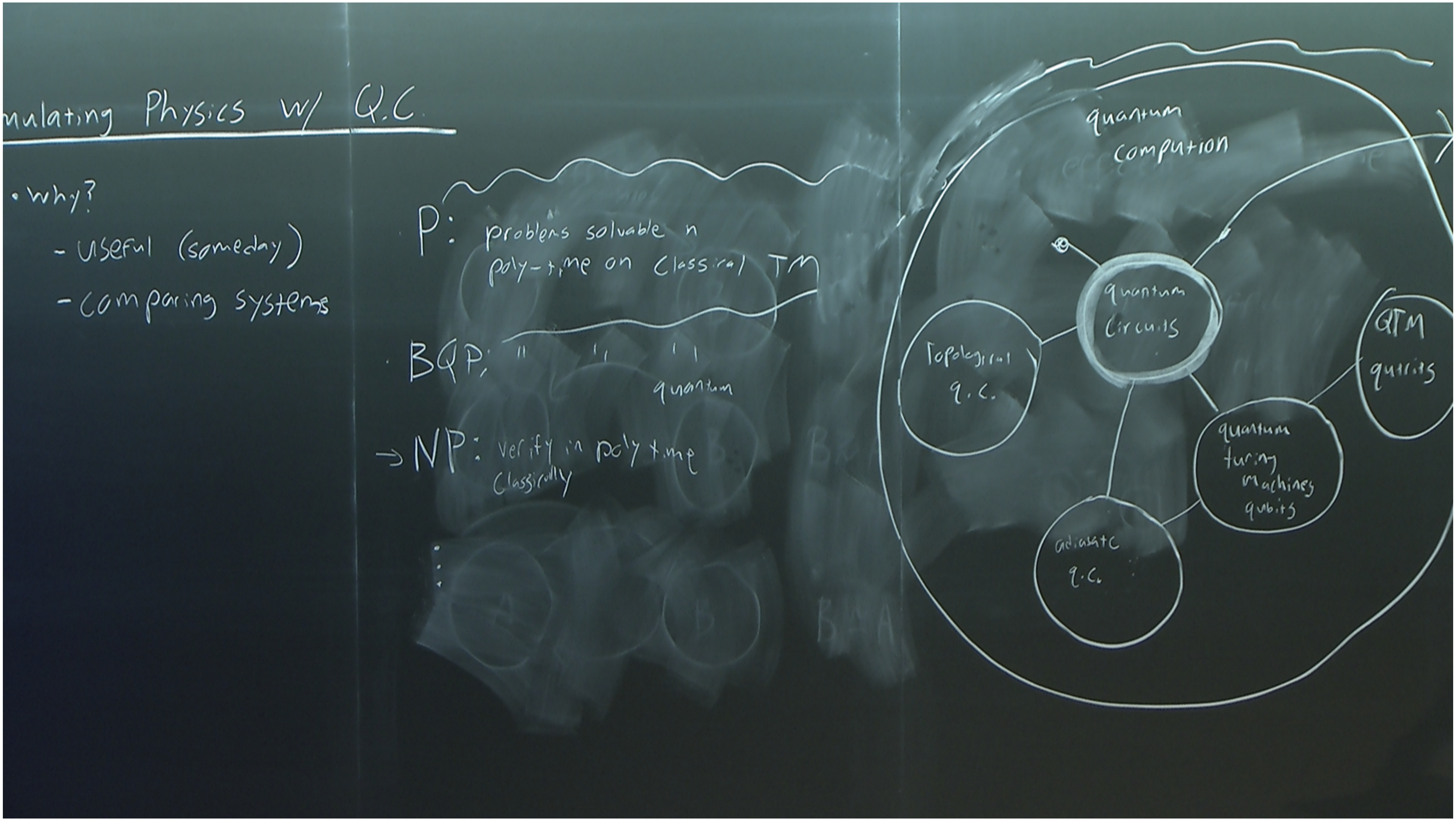
mulating Physics w/ Q.C
• why?
- Useful (someday)
- comparing systems
P: problems solvable n
poly-time on Classical TM
BQP: " " " quantum
→ NP: verify in poly time
Classically
quantum
computation
quantum
Circuits
Topological
q.c.
QTM
qutrits
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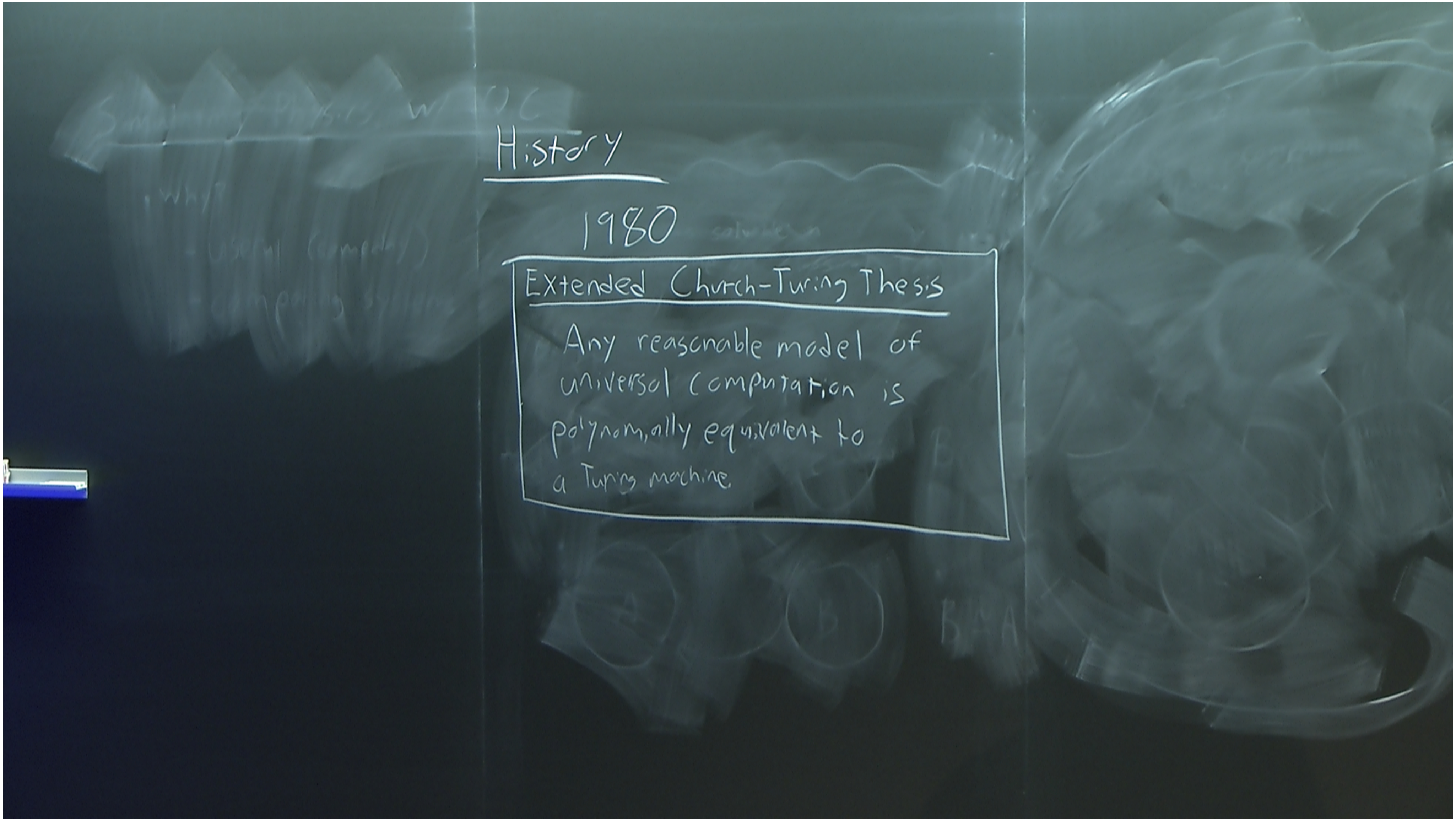
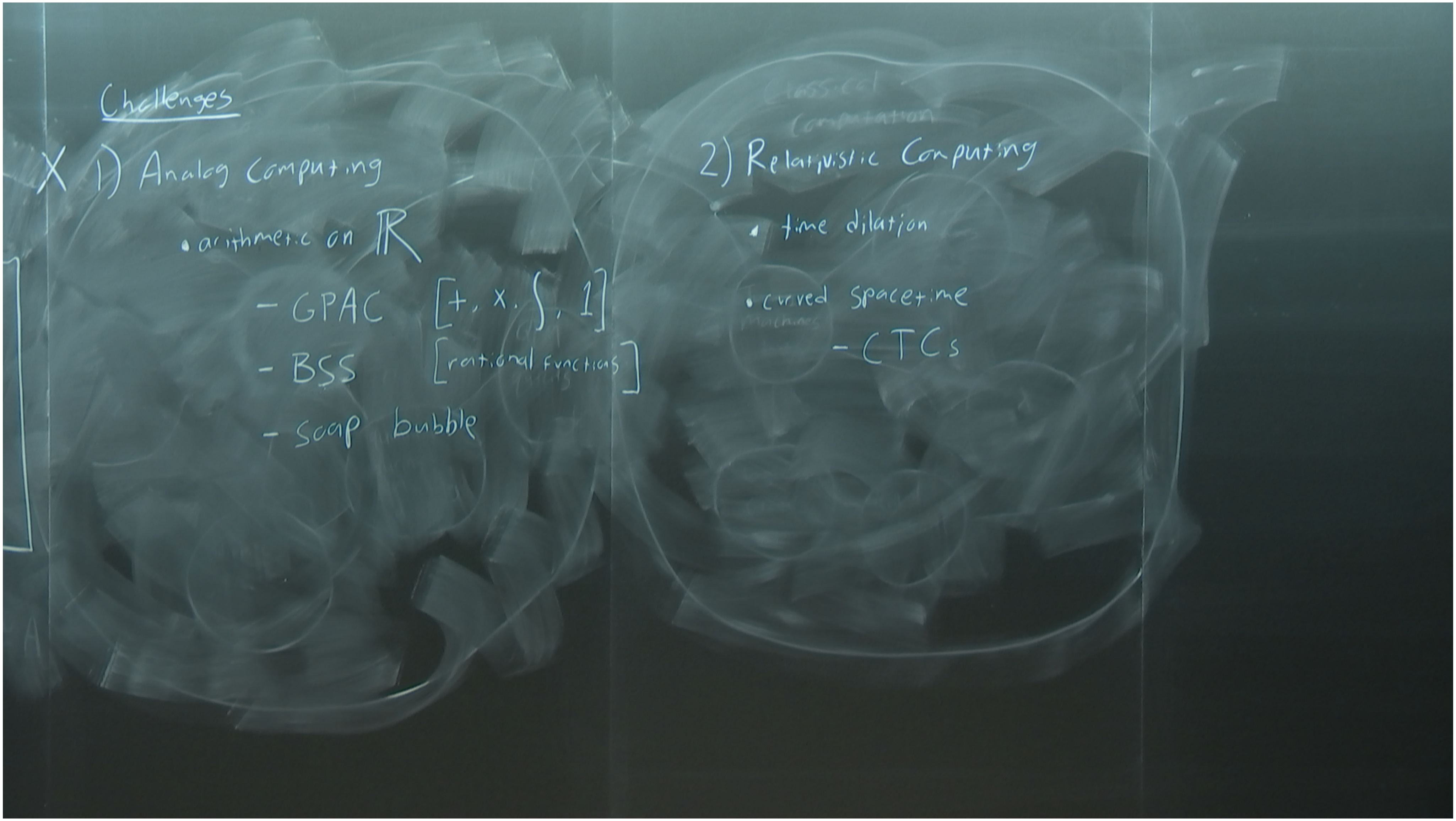

History
1980
Extended Church-Turing Thesis
Any reasonable model of universal computation is polynomially equivalent to a Turing machine.


Challenges
X 1) Analog computing
• arithmetic on ℝ
- GPAC [+, x, ∫, 1]
- BSS [rational functions]
- soap bubble
2) Relativistic Computing
• time dilation
• curved spacetime
- CTCs

History

1980

Extended Church-Turing Thesis

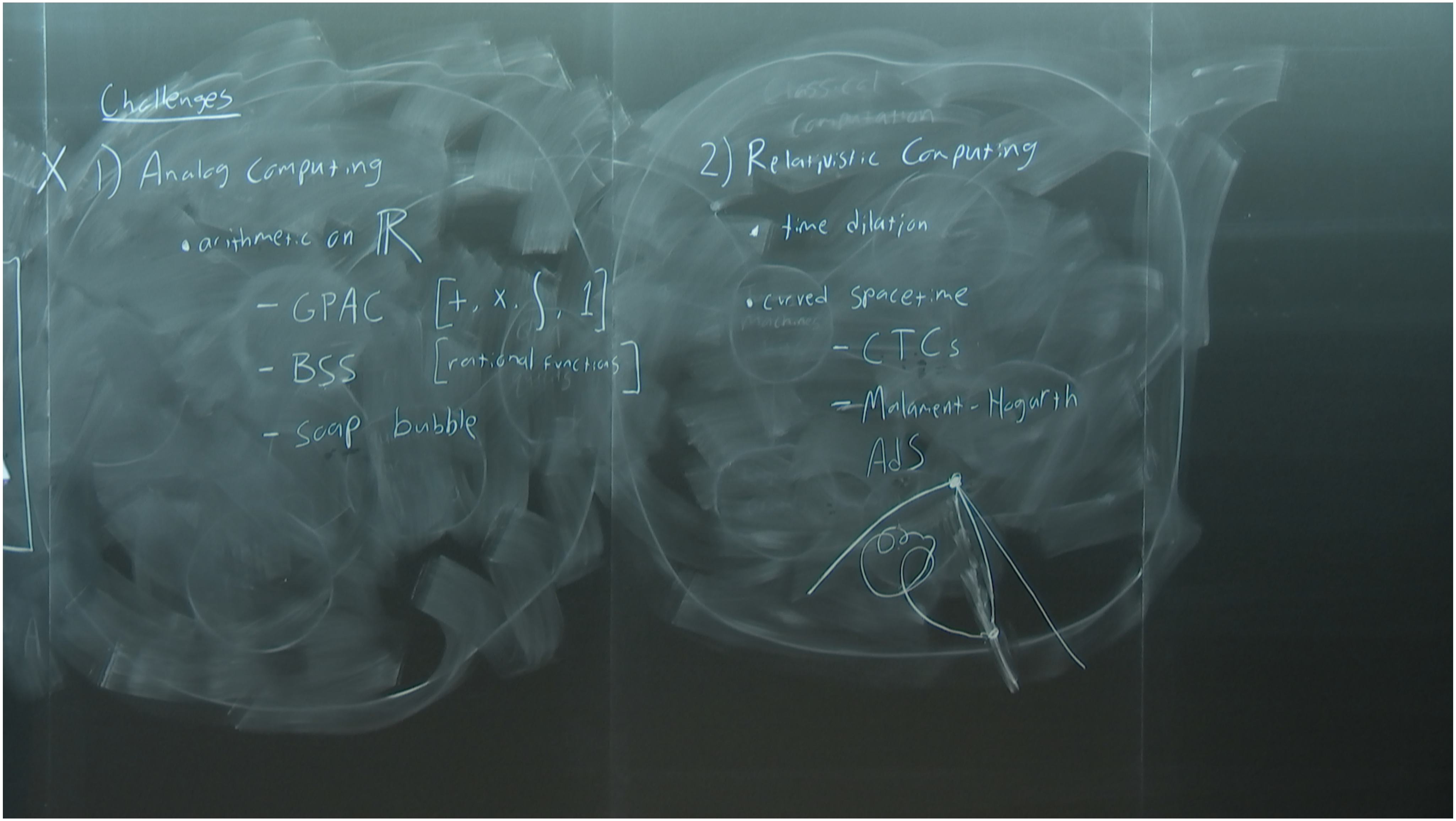
Any reasonable model of universal computation is polynomially equivalent to a Turing machine.

Challenges

X 1) Analog computing

- arithmetic on $\mathbb{R}$
  - GPAC $[+, \times, \int, 1]$
  - BSS [rational functions]
  - soap bubble

2) R


Challenges
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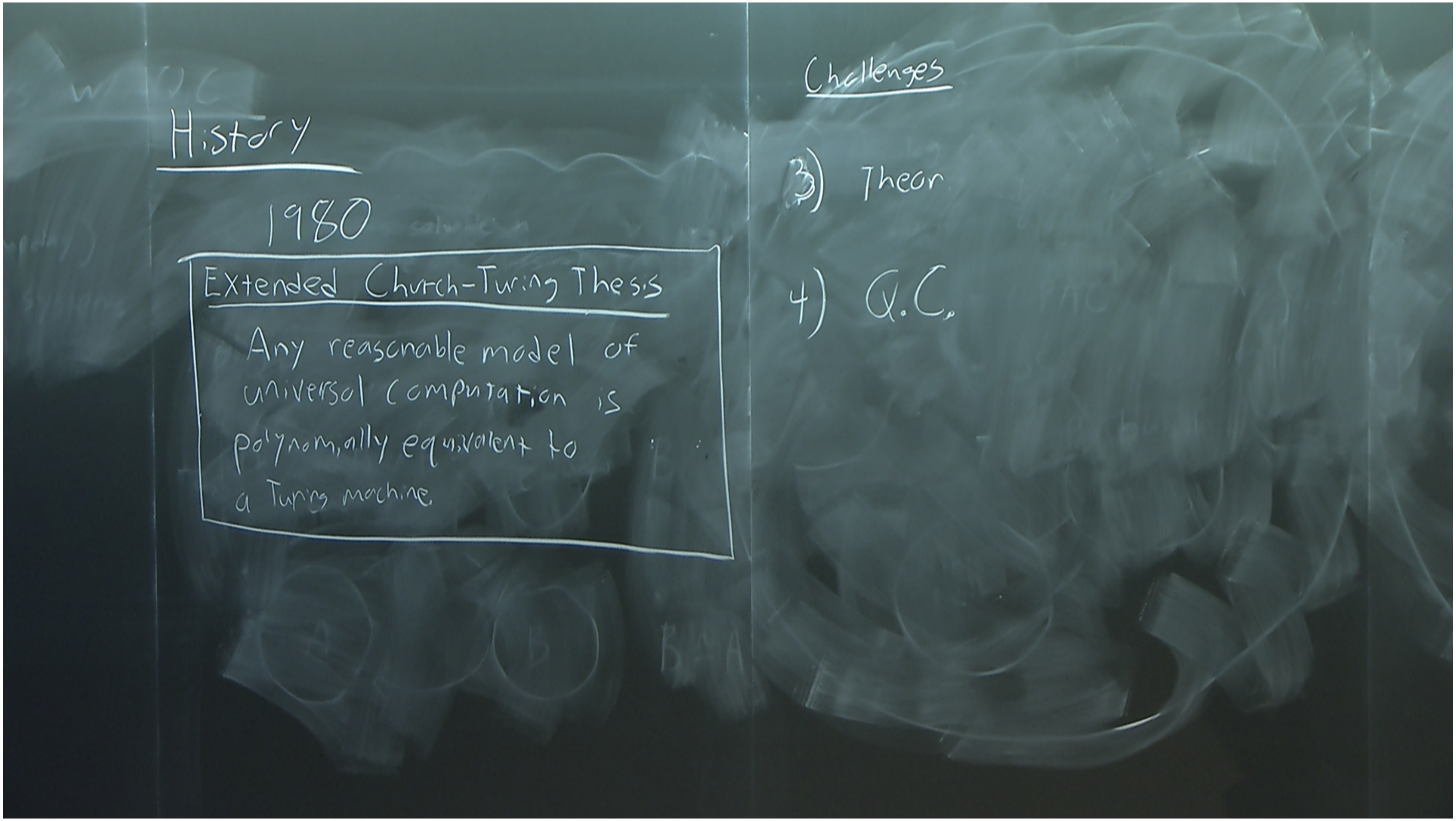
2) Relativistic Computing
• time dilation
• curved spacetime
– CTCs
– Malament - Hogarth
AdS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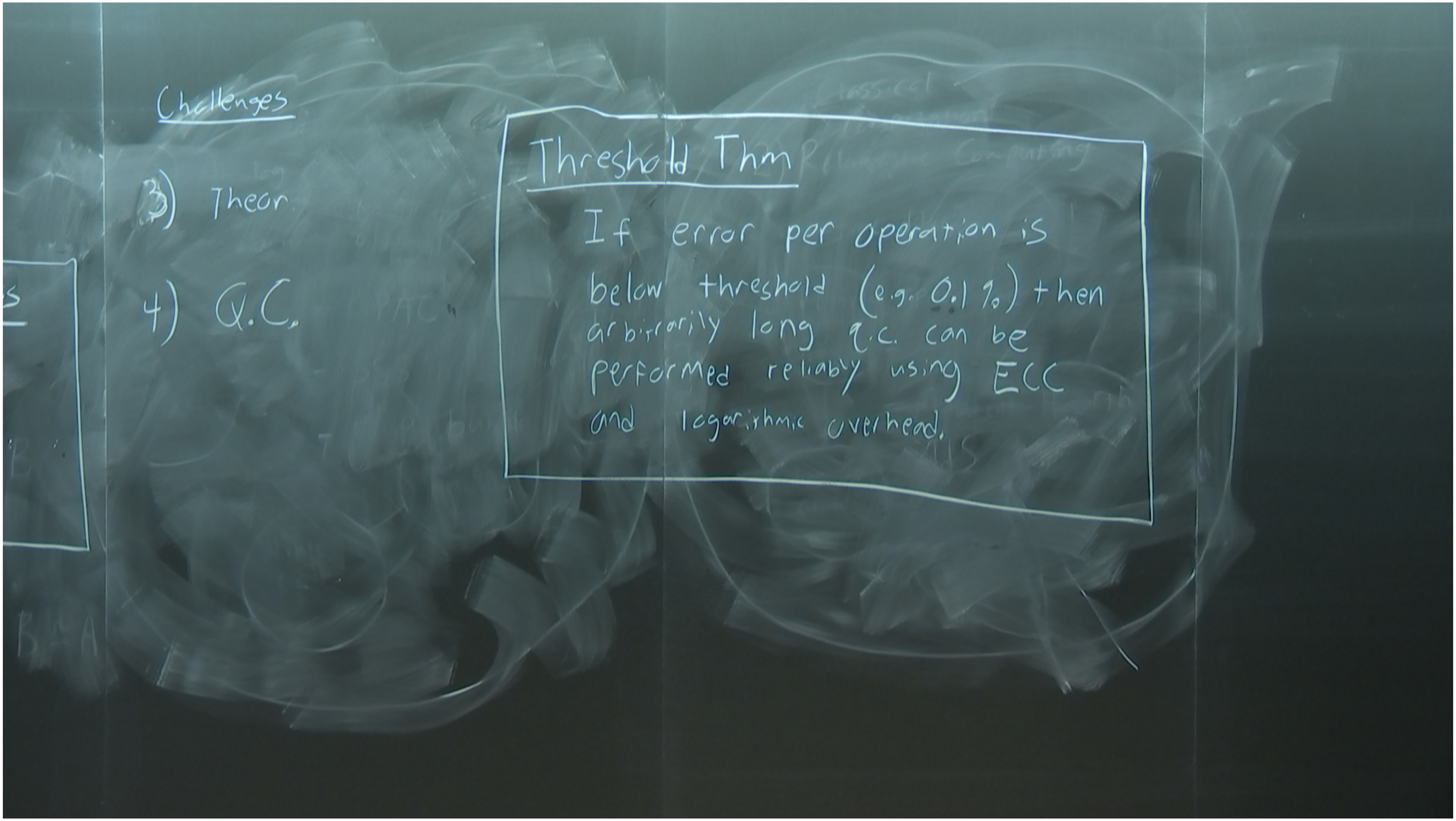
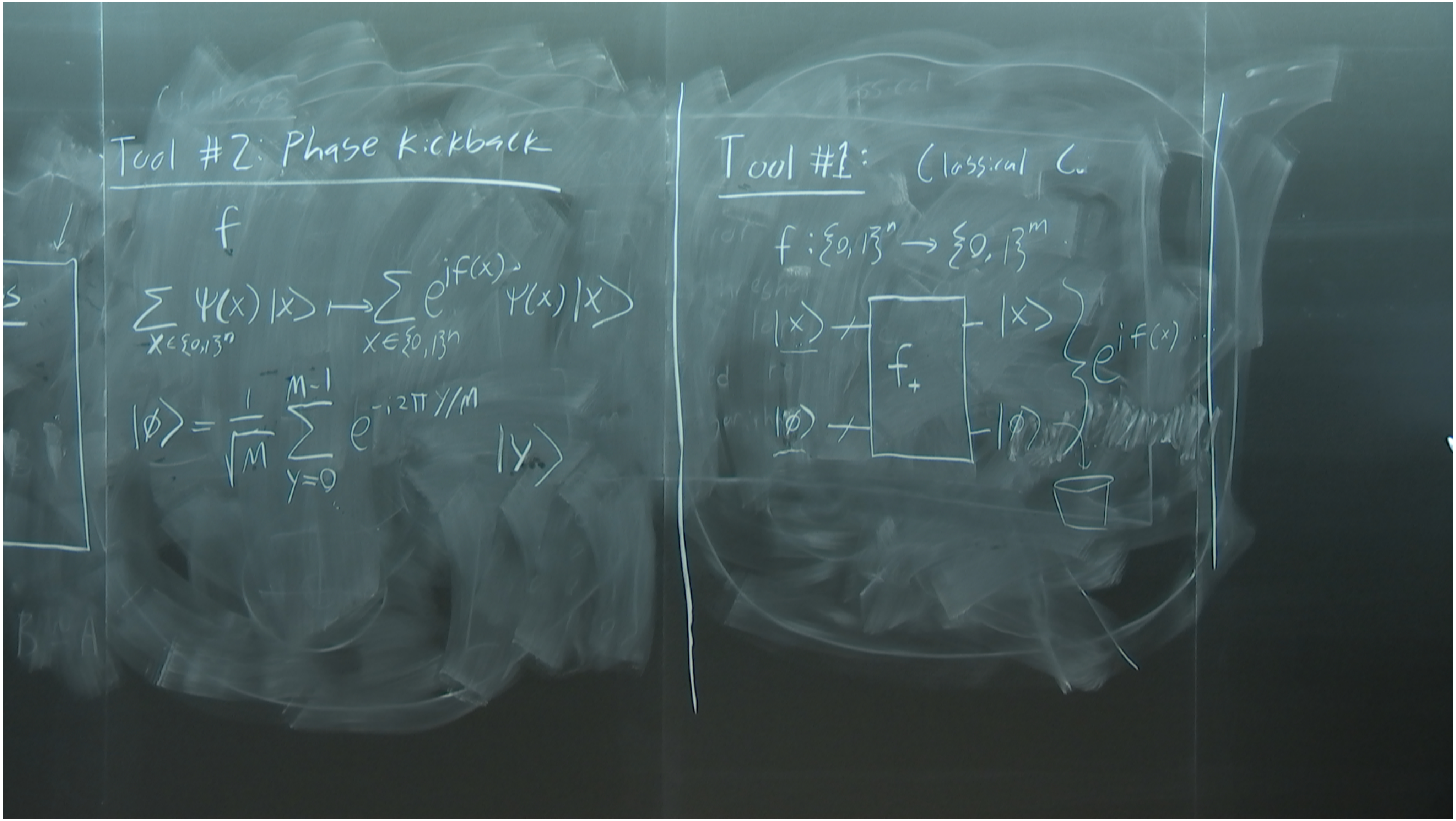
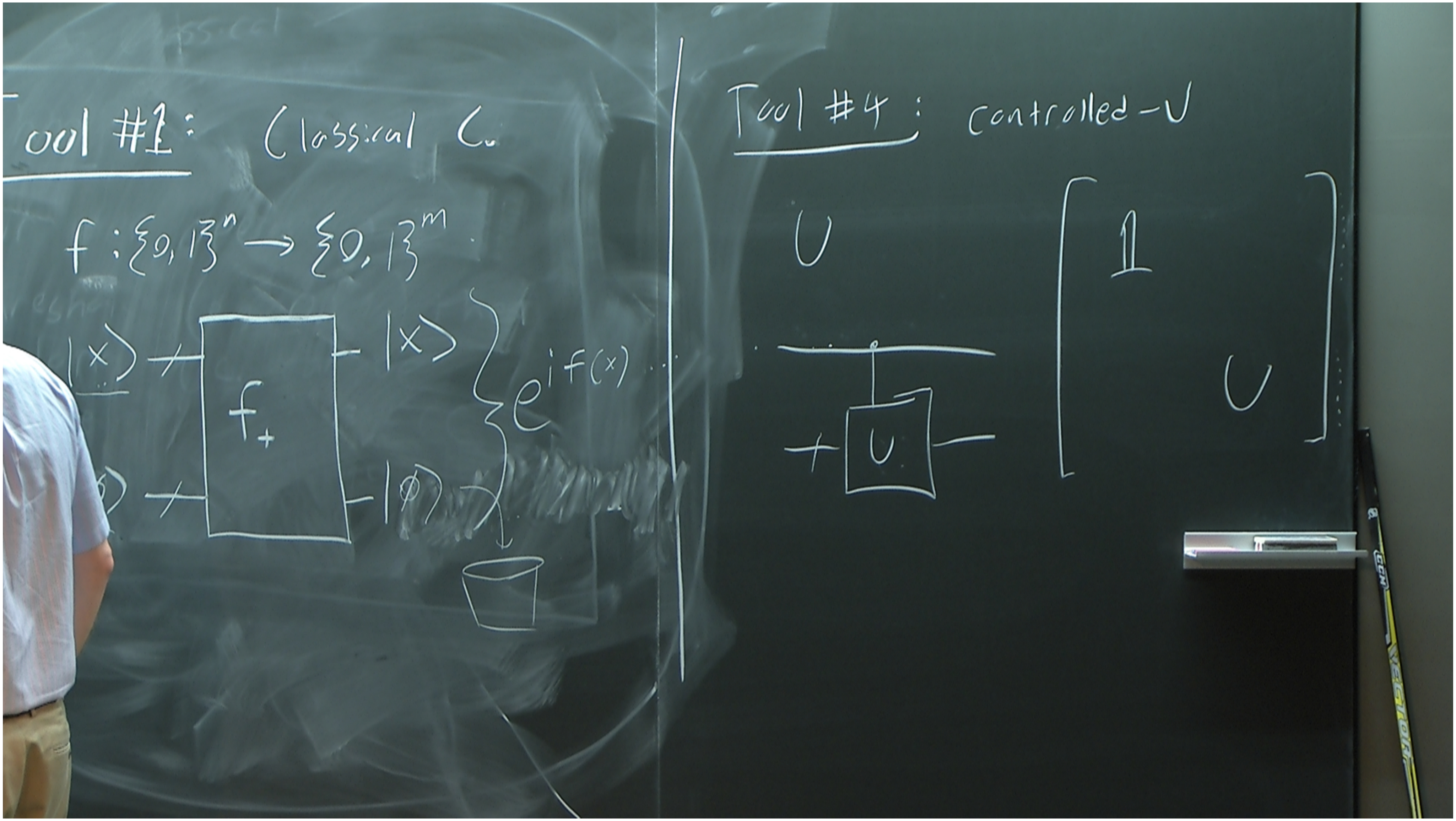
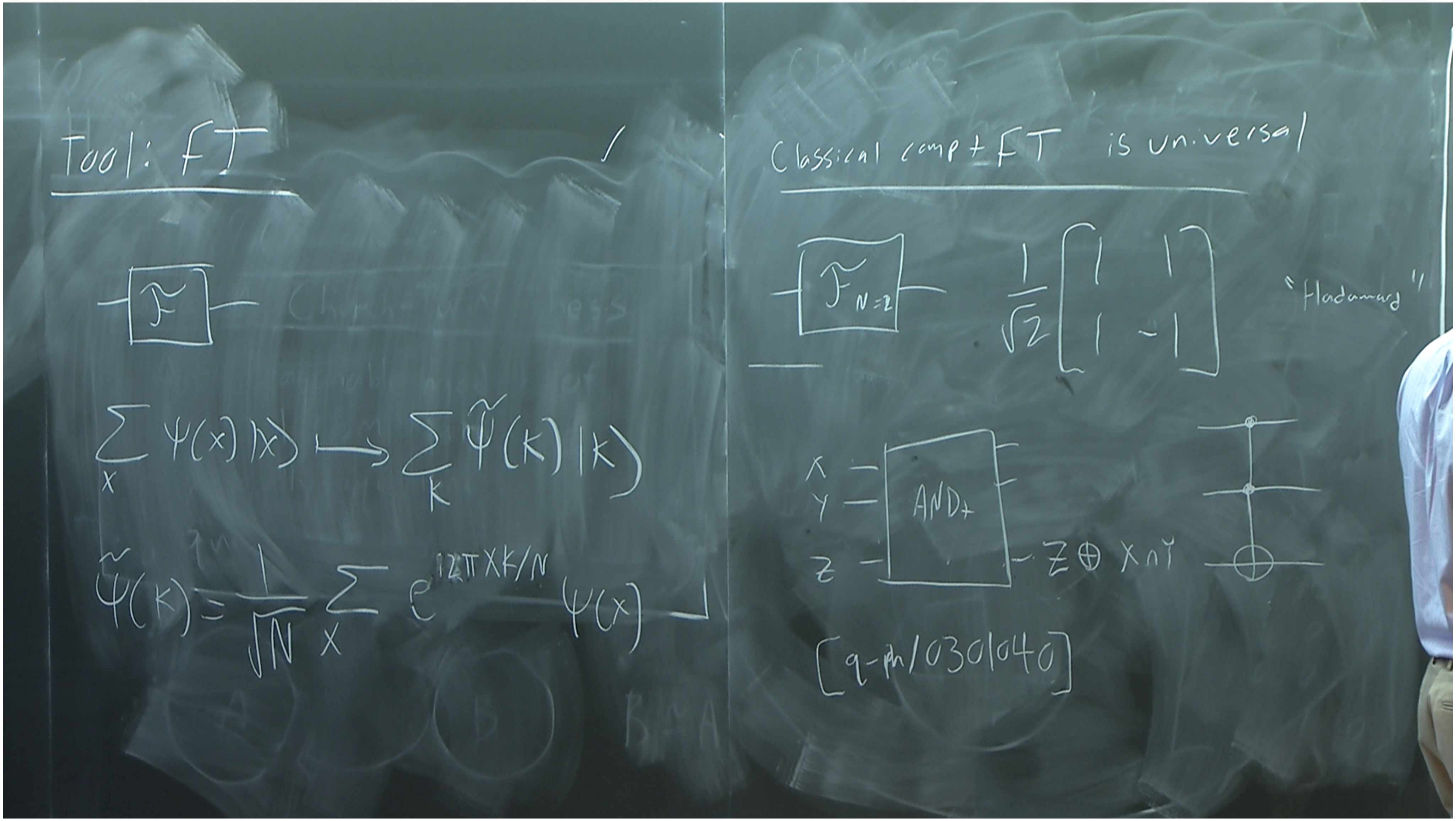
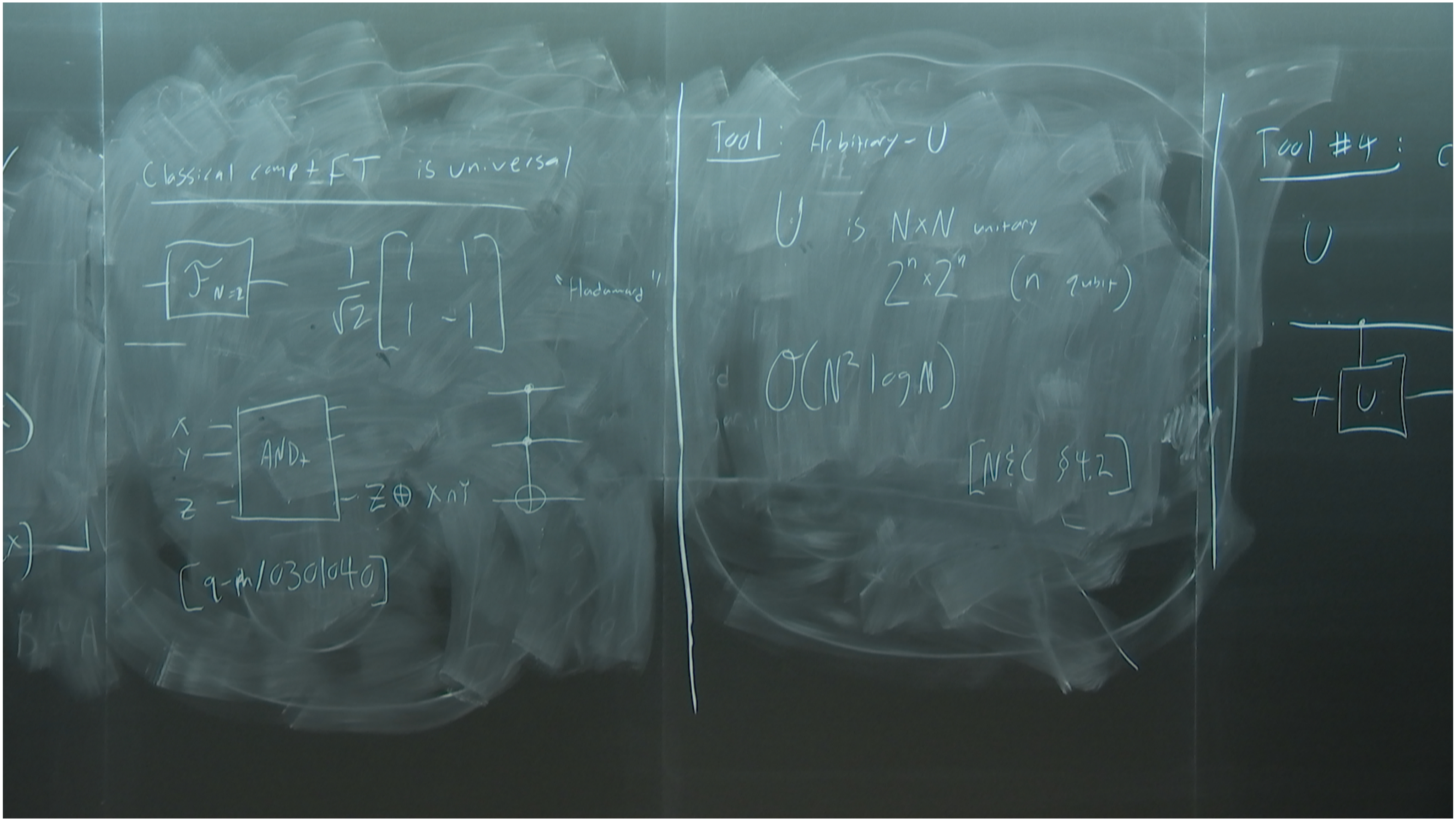
Any reasonable model of universal computation is polynomially equivalent to a Turing machine.
Challenges
3) Theor
4) Q.C.


Challenges
3) Theor
4) Q.C.
Threshold Thm
If error per operation is below threshold (e.g. 0.1%) then arbitrarily long q.c. can be performed reliably using ECC and logarithmic overhead.

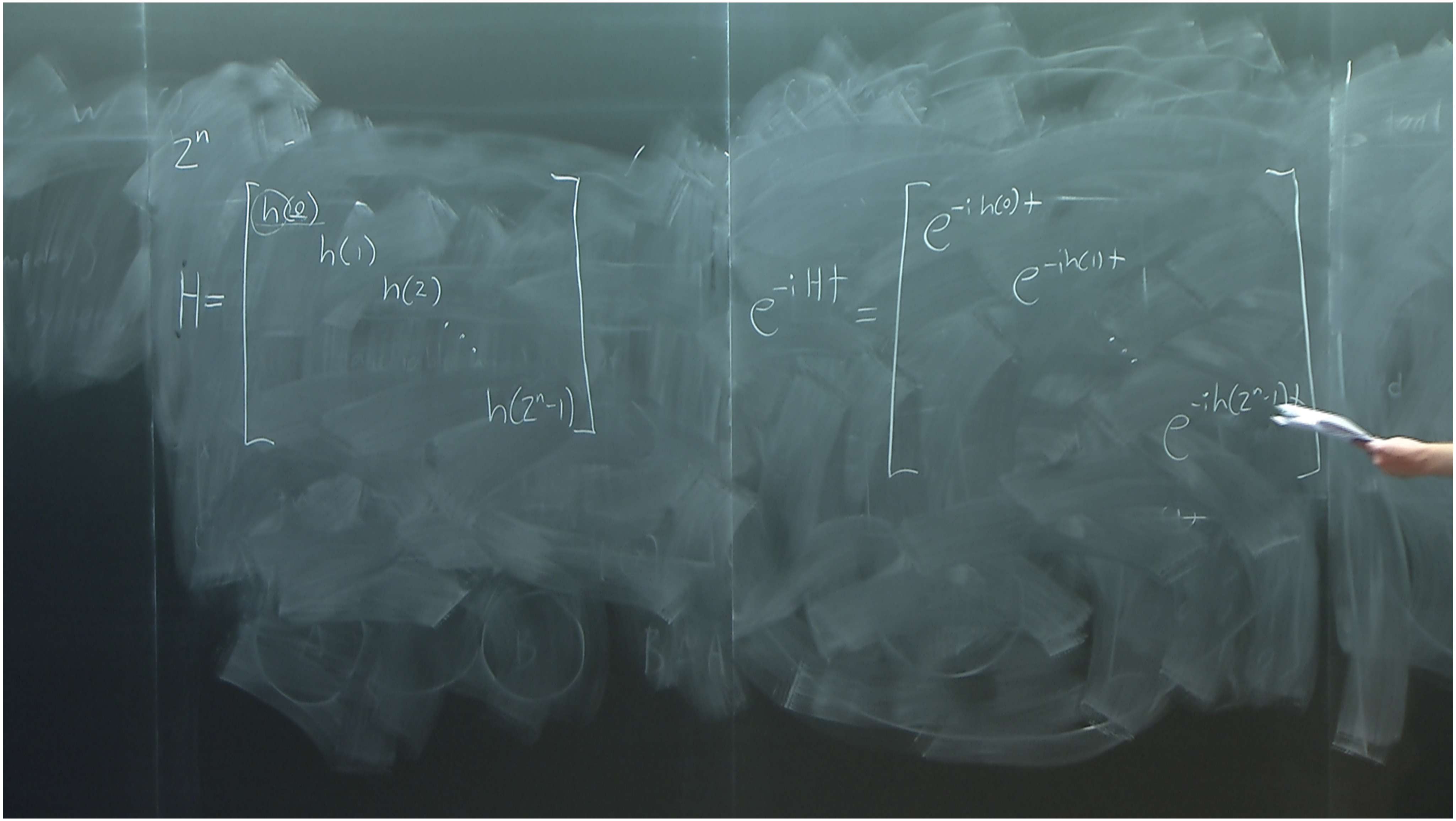

Tool #2: Phase Kickback
f
$\sum_{x\in\{0,1\}^n} \psi(x)|x\rangle \mapsto \sum_{x\in\{0,1\}^n} e^{if(x)} \psi(x)|x\rangle$
$|\phi\rangle = \frac{1}{\sqrt{M}} \sum_{y=0}^{M-1} e^{-i2\pi y/M} |y\rangle$
Tool #1: Classical C
$f:\{0,1\}^n \to \{0,1\}^m$
$|x\rangle$
$|\phi\rangle$
f_+
$|x\rangle$
$|\phi\rangle$
$e^{if(x)}$


Tool #1: Classical C
f : {0,1}^n → {0,1}^m
|x⟩
f+
|x⟩
e^{if(x)}
Tool #4: controlled-U
U
1
U
U

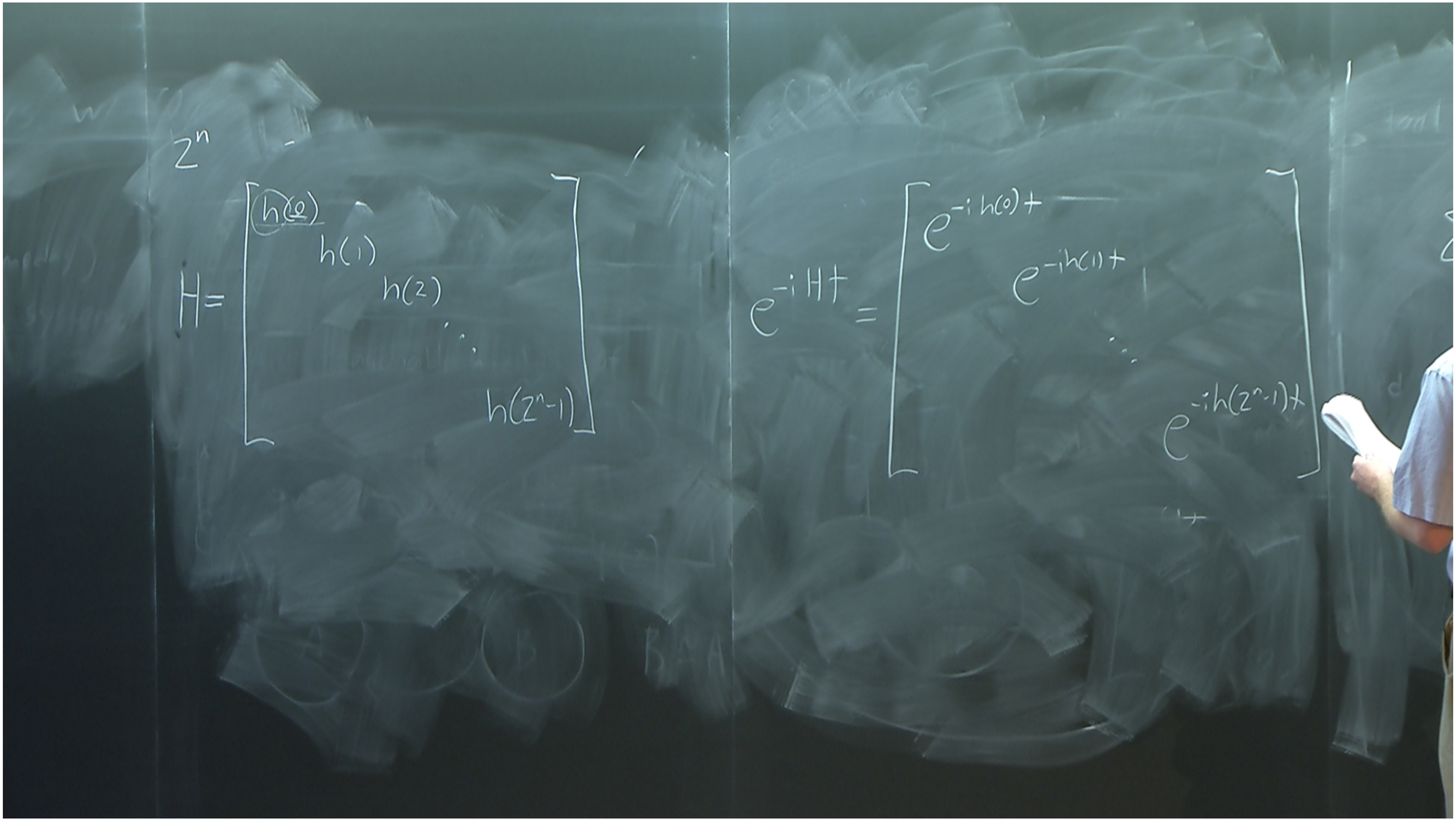

Tool: FT
$\mathcal{F}$
$\sum_x \psi(x)|x\rangle \longmapsto \sum_k \tilde{\psi}(k)|k\rangle$
$\tilde{\psi}(k) = \frac{1}{\sqrt{N}} \sum_x e^{i2\pi xk/N} \psi(x)$
Classical comp + FT is universal
$\mathcal{F}_{N=2}$
$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
"Hadamard"
x
y
z
AND+
$z \oplus x \wedge y$
[q-ph/0301040]



Classical comp + FT is universal
$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
"Hadamard"
AND+
$Z \oplus X \wedge Y$
[q-ph/0301040]
Tool: Arbitrary-U
U is N×N unitary
$2^n \times 2^n$ (n qubit)
$O(N^3 \log N)$
[N&C §4.2]
Tool #4:



2^n
$$H = \begin{bmatrix} h(0) & & & & \\ & h(1) & & & \\ & & h(2) & & \\ & & & \ddots & \\ & & & & h(2^n-1) \end{bmatrix}$$
$$e^{-iHt} = \begin{bmatrix} e^{-ih(0)t} & & & \\ & e^{-ih(1)t} & & \\ & & \ddots & \\ & & & e^{-ih(2^n-1)t} \end{bmatrix}$$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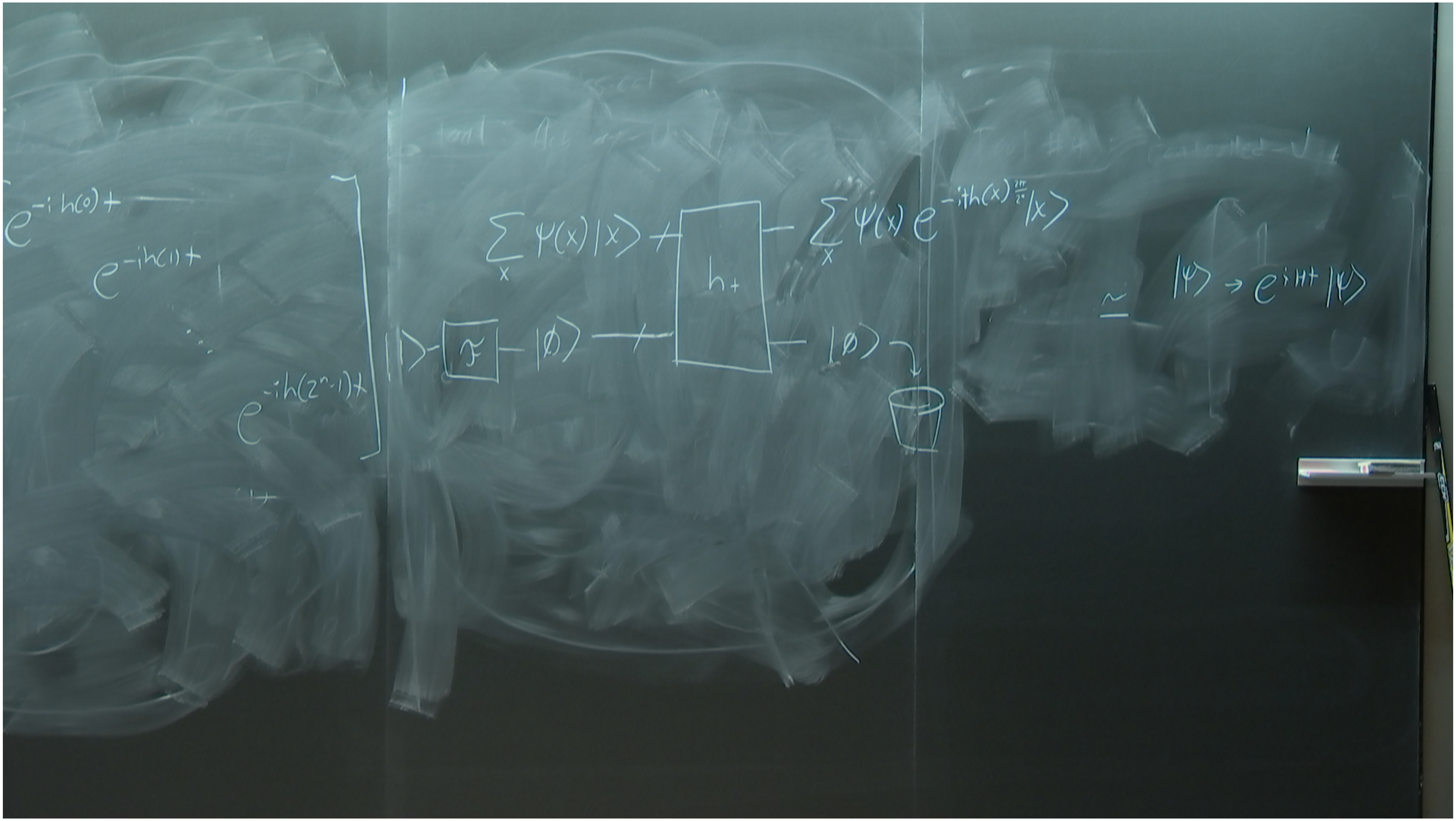
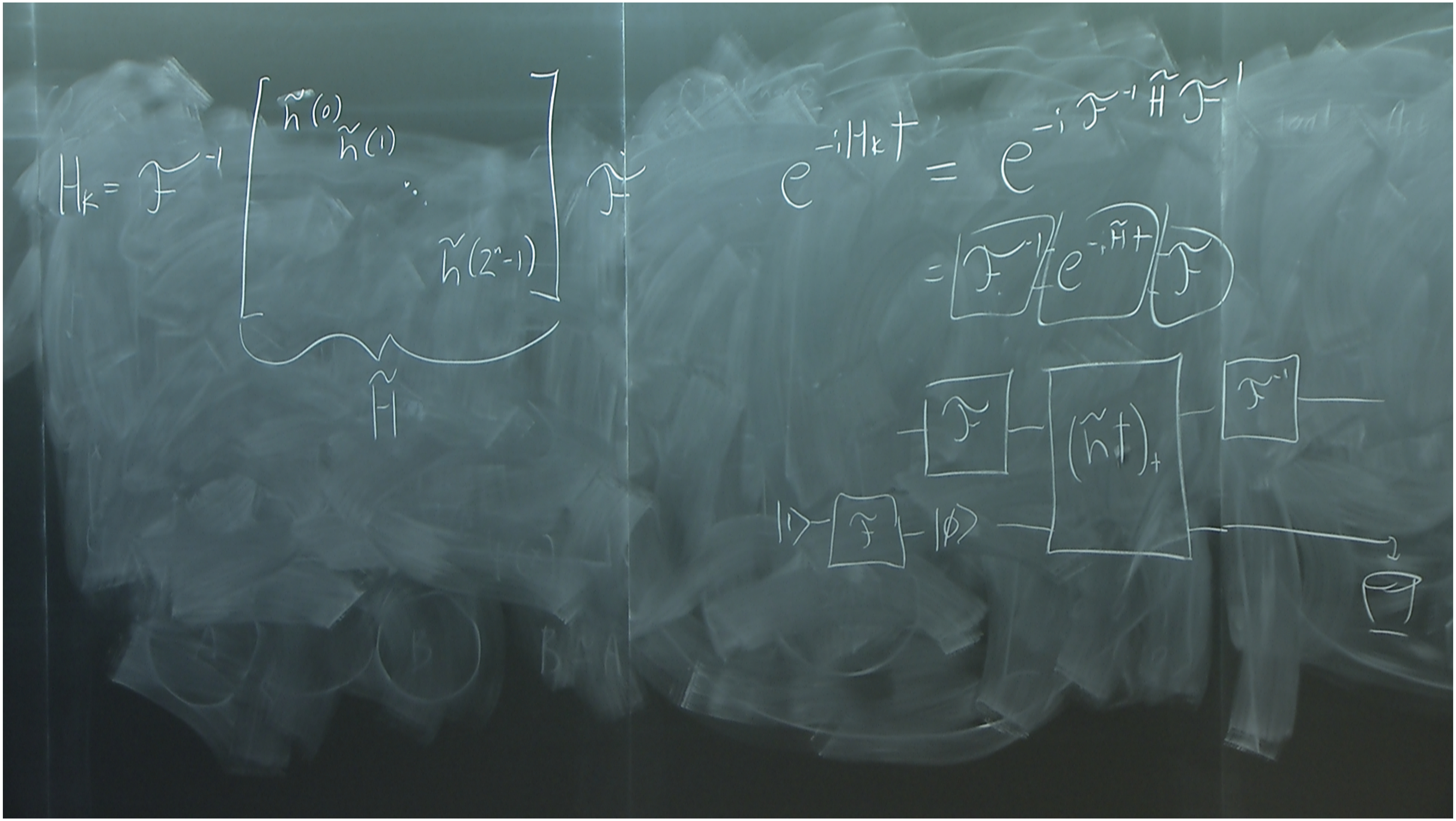
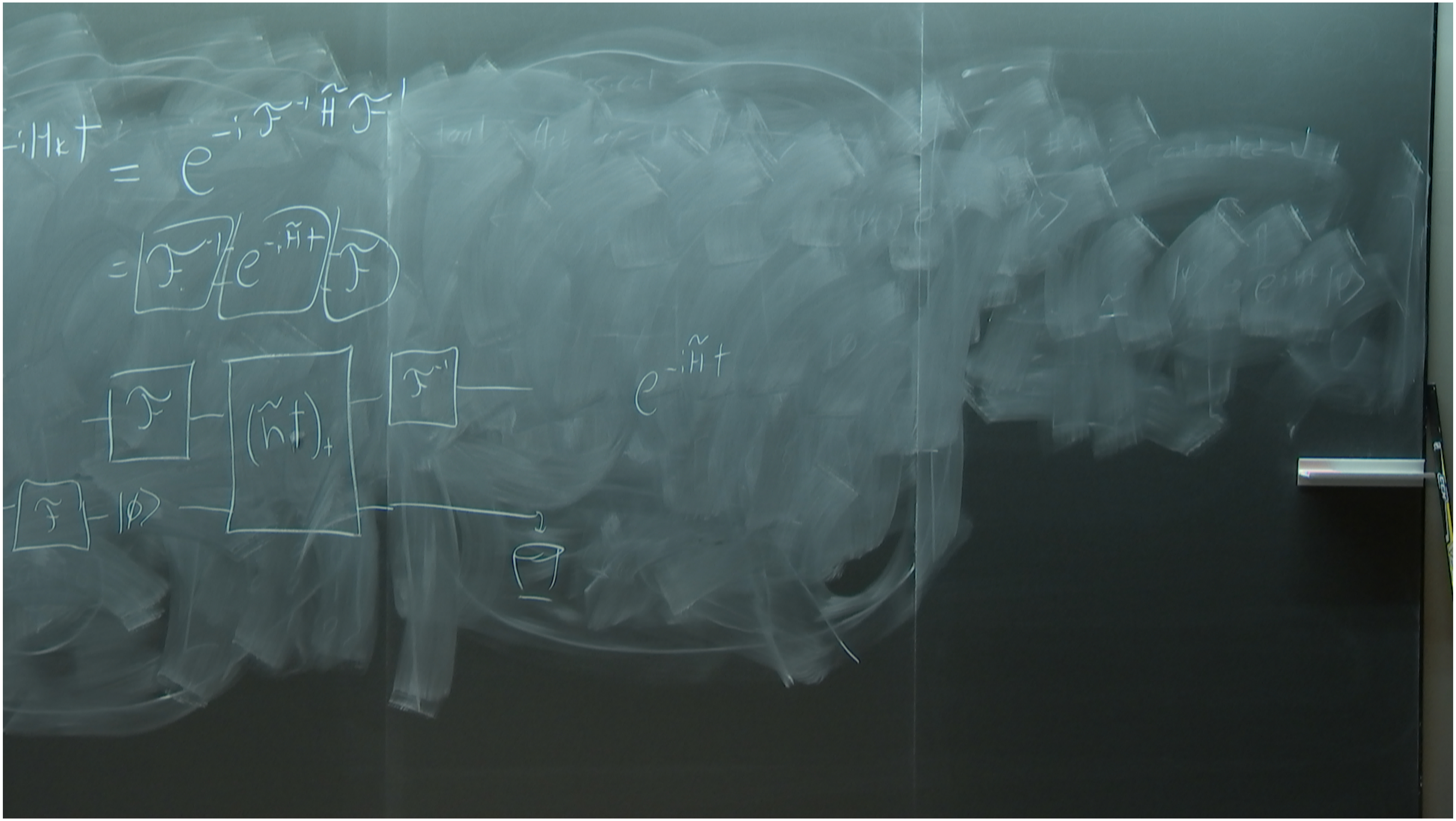


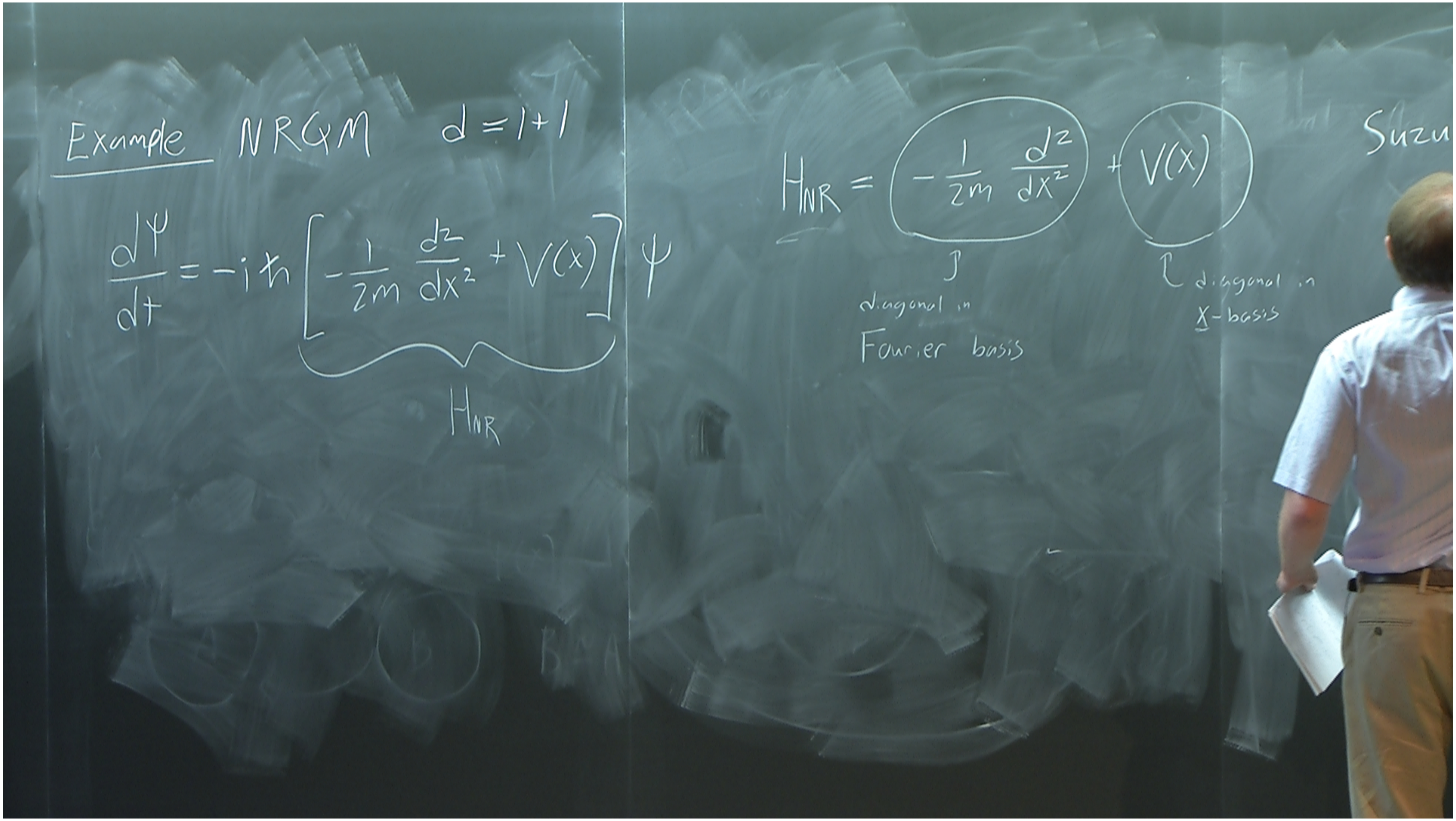
e^{-ih(0)t}
e^{-ih(1)t}
e^{-ih(2^n-1)t}
|1>
|ø>
Σ_x ψ(x)|x>
h_t
Σ_x ψ(x) e^{-ith(x)}|x>
|ø>
|ψ> → e^{iHt}|ψ>



$H_k = \mathcal{F}^{-1} \underbrace{\begin{bmatrix} \tilde{h}(0) & & & \\ & \tilde{h}(1) & & \\ & & \ddots & \\ & & & \tilde{h}(2^n-1) \end{bmatrix}}_{\tilde{H}} \mathcal{F}$
$e^{-iH_k t} = e^{-i\mathcal{F}^{-1}\tilde{H}\mathcal{F}t}$
$= \mathcal{F}^{-1} e^{-i\tilde{H}t} \mathcal{F}$
$\mathcal{F}$
$(\tilde{h}t)_+$
$\mathcal{F}^{-1}$
$|1\rangle$
$\mathcal{F}$
$|\phi\rangle$



e^{-iH̃t}
|ϕ⟩


Example NRQM d = 1+1
$\frac{d\psi}{dt} = -i\hbar \left[-\frac{1}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi$
H_{NR}
$H_{NR} = -\frac{1}{2m} \frac{d^2}{dx^2} + V(x)$
diagonal in Fourier basis
diagonal in X-basis
Suzu

$$R = \left(-\frac{1}{2m}\frac{d^2}{dx^2}\right) + V(x)$$

diagonal in Fourier basis

diagonal in X-basis

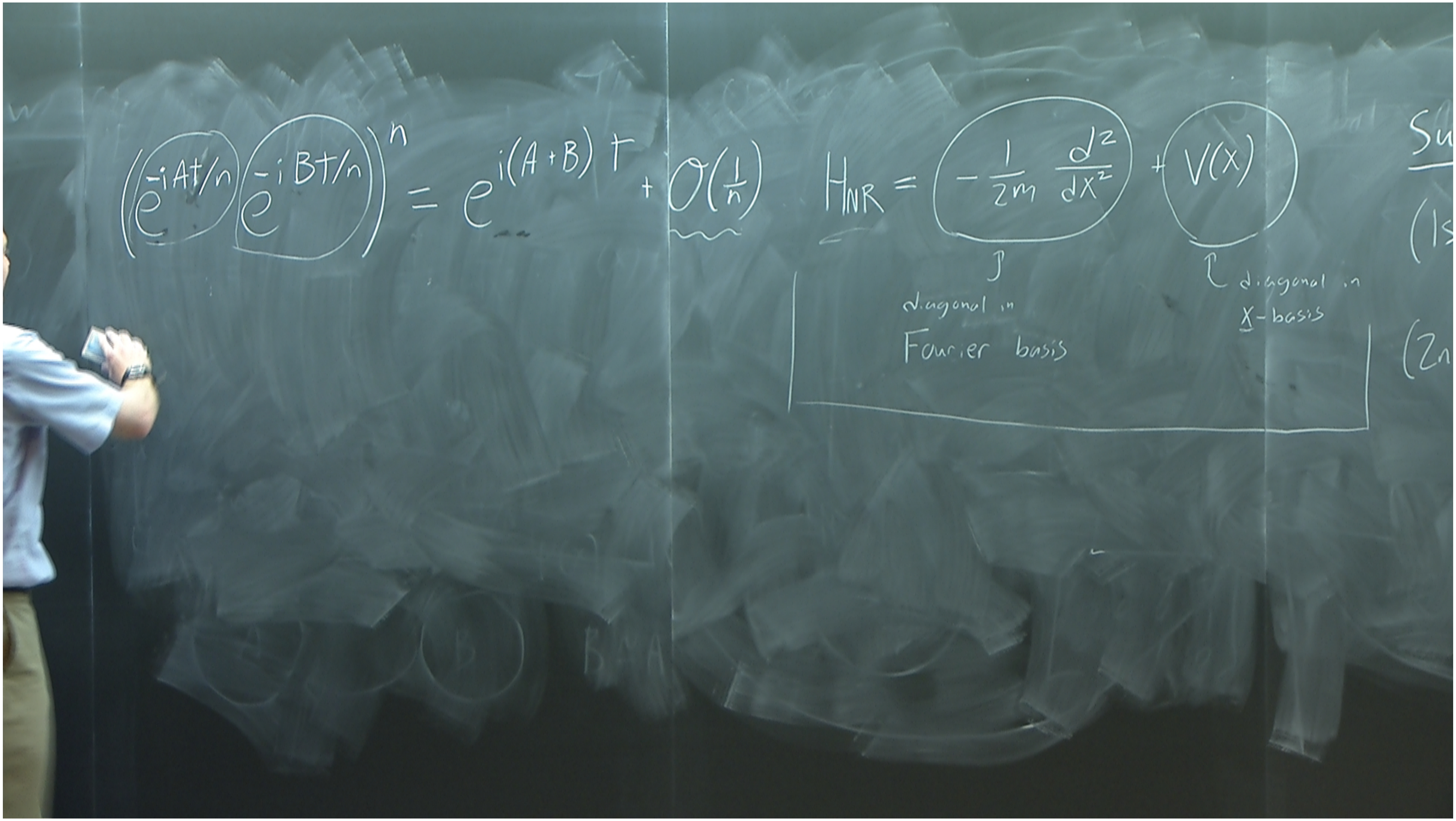
Suzuki-Trotter Formulae

(1st order) $e^{i(A+B)\delta t} = e^{iA\delta t}\, e^{iB\delta t} + \mathcal{O}(\delta t^2)$

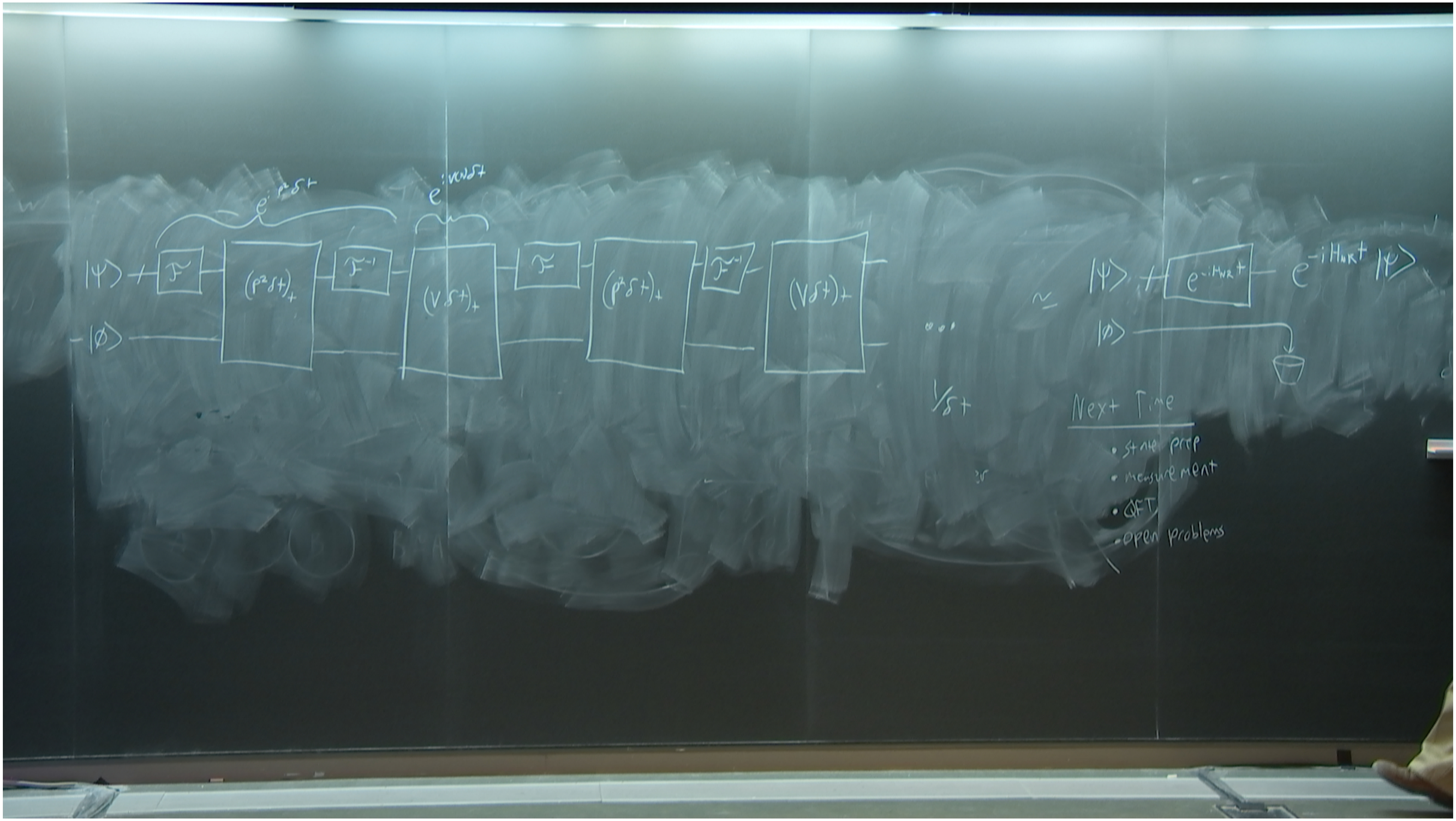
(2nd order) $e^{i(A+B)\delta t} = e^{iA\frac{\delta t}{2}}\, e^{iB\delta t}\, e^{iA\frac{\delta t}{2}} + \mathcal{O}(\delta t^3)$

$\vdots$

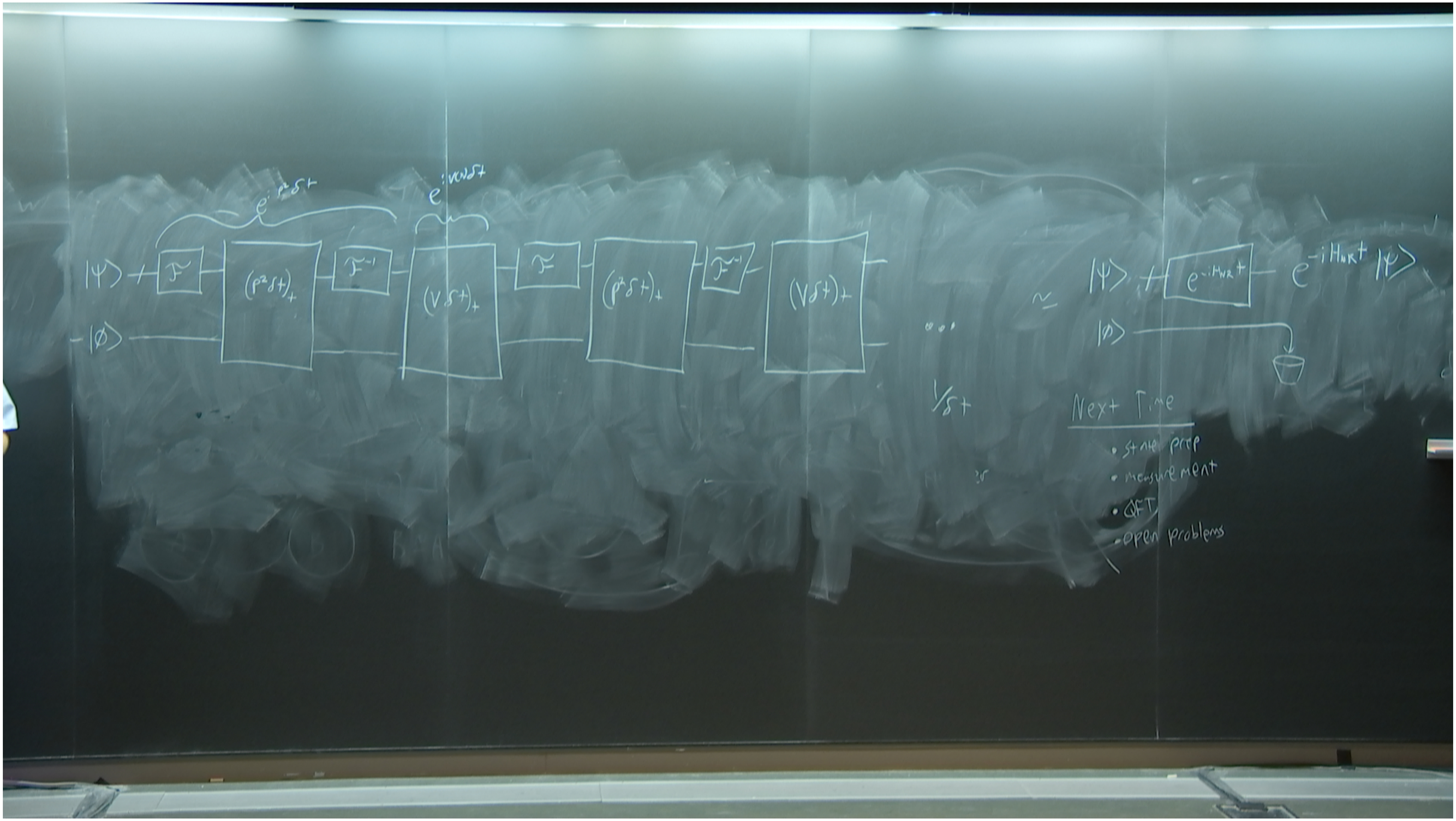
kth order



$\left(e^{-iAt/n} e^{-iBt/n}\right)^n = e^{i(A+B)t} + O\left(\frac{1}{n}\right)$
$H_{NR} = -\frac{1}{2m}\frac{d^2}{dX^2} + V(X)$
diagonal in Fourier basis
diagonal in X-basis



Next Time
• state prep
• measurement
• QFT
• open problems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