Title: Holographic Correlation Functions

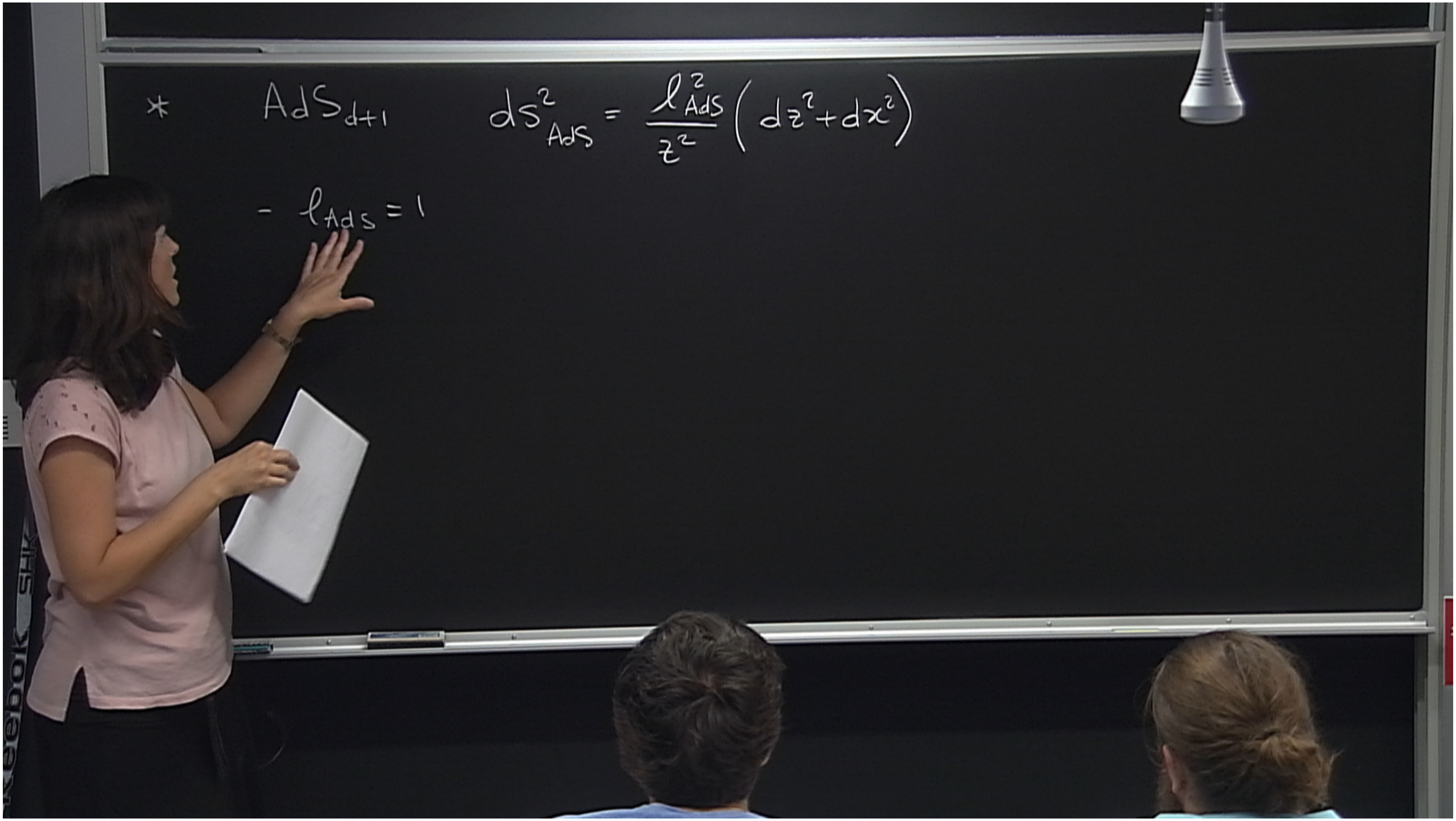
Date: Jul 22, 2016 05:00 PM

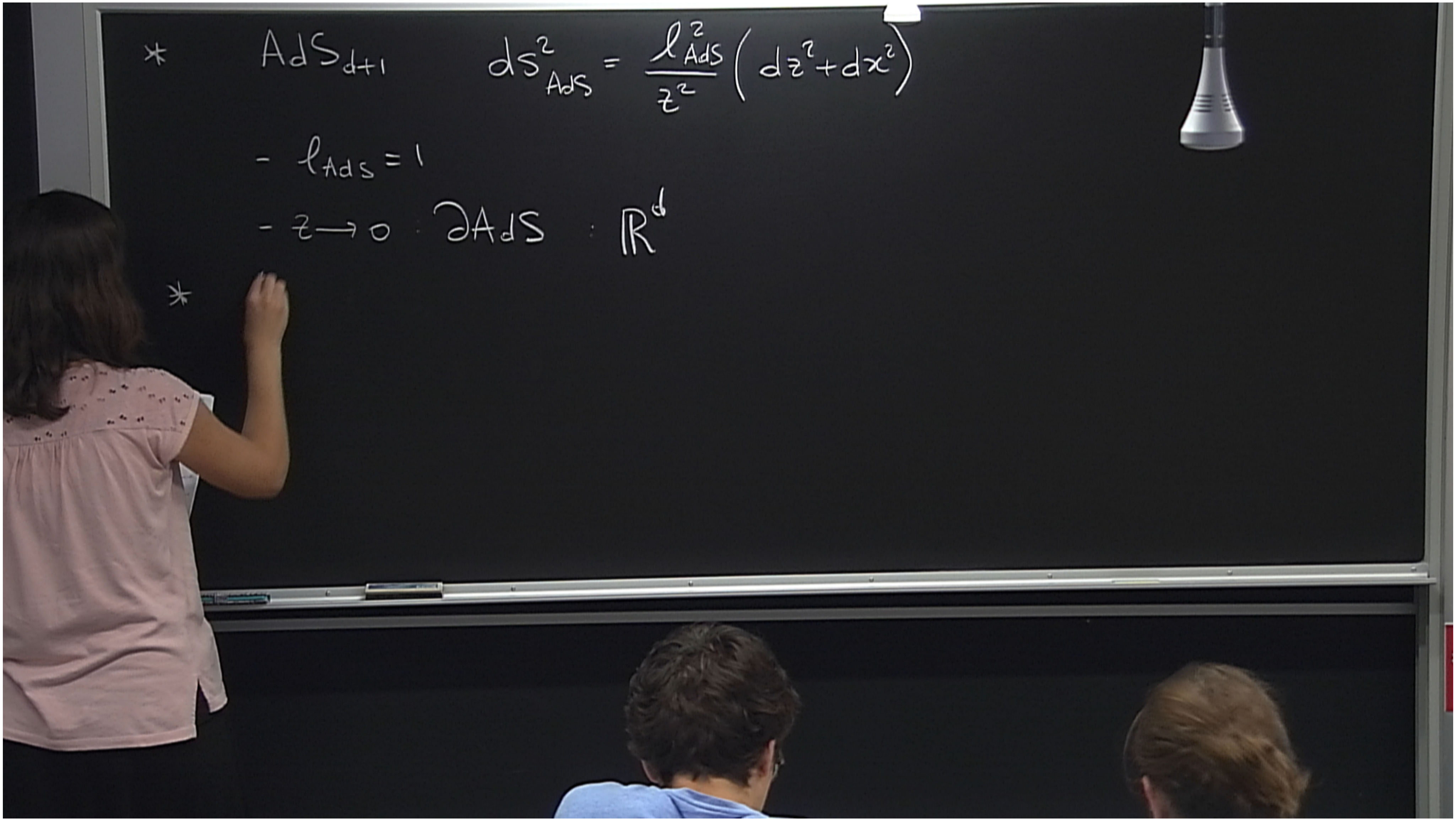
URL: http://pirsa.org/16070033

Abstract:

Holographic correlation functions

- REFERENCES.
  - Field/operator correspondence: Witten hep-th/9802150
  - CFT: Di Francesco, Mathieu, Sénéchal
  - Holographic renormalization: de Haro, Skenderis, Solodukhin hep-th/0002230
  - AdS/CFT books: Kiritsis
    Ammon, Erdmenger


* AdS_{d+1} $ds^2_{AdS} = \frac{\ell^2_{AdS}}{z^2}\left(dz^2 + dx^2\right)$
- $\ell_{AdS} = 1$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- $z \to 0$: ∂AdS : $\mathbb{R}^d$
*


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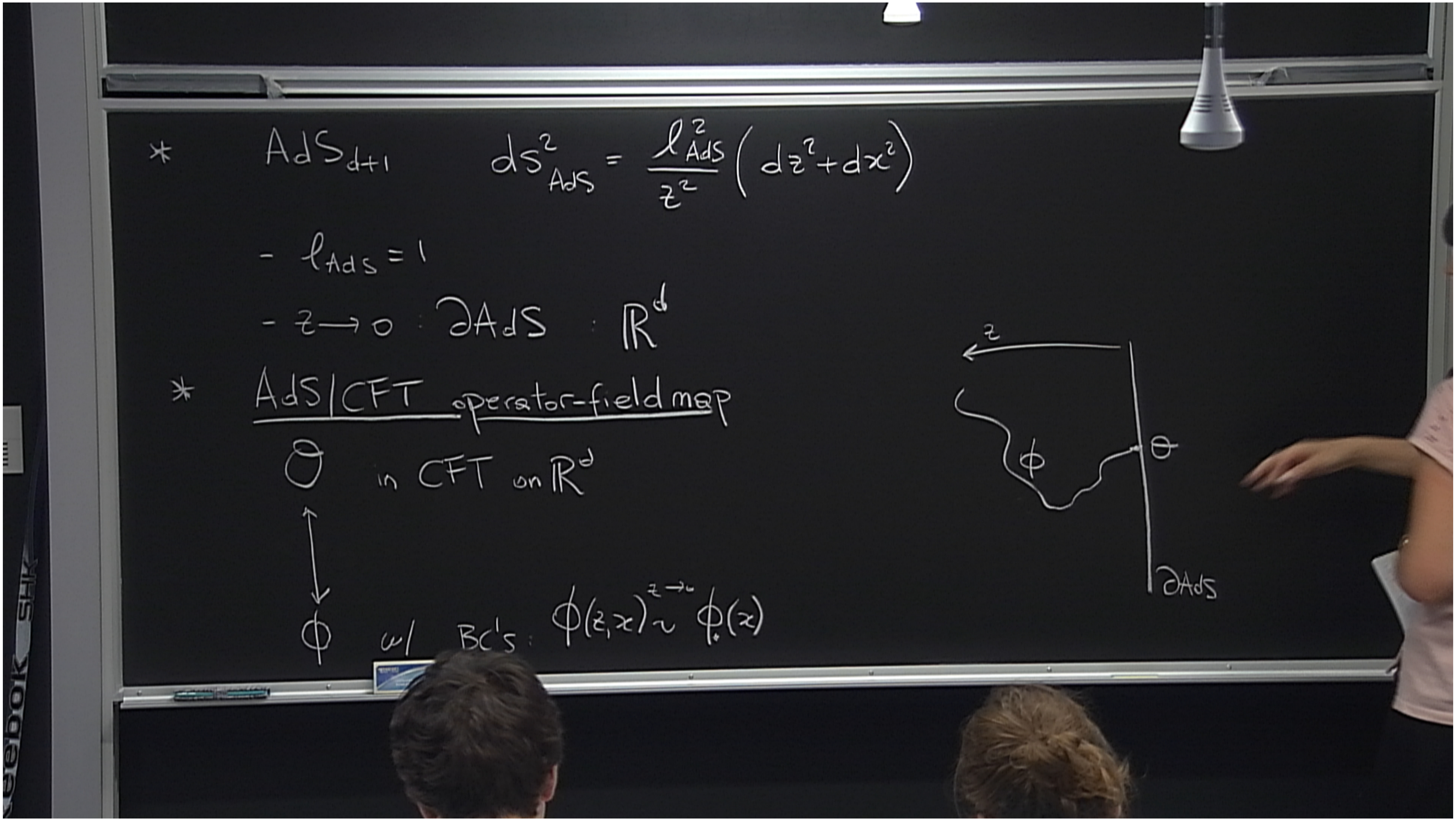
* AdS/CFT operator-fiel
$\mathcal{O}$ in CFT on $\mathbb{R}^d$
ϕ
z
ϕ
$\mathcal{O}$
∂AdS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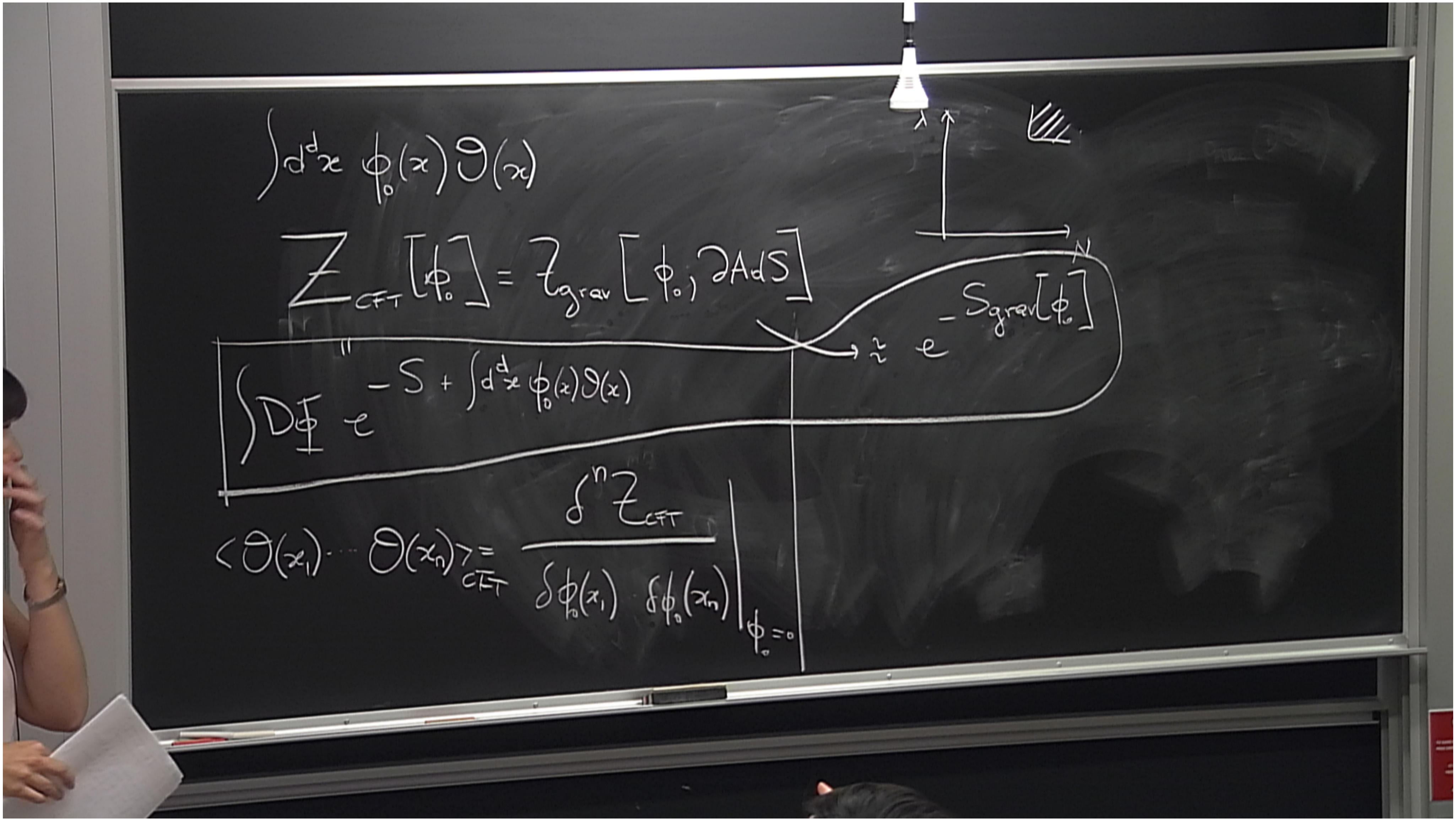
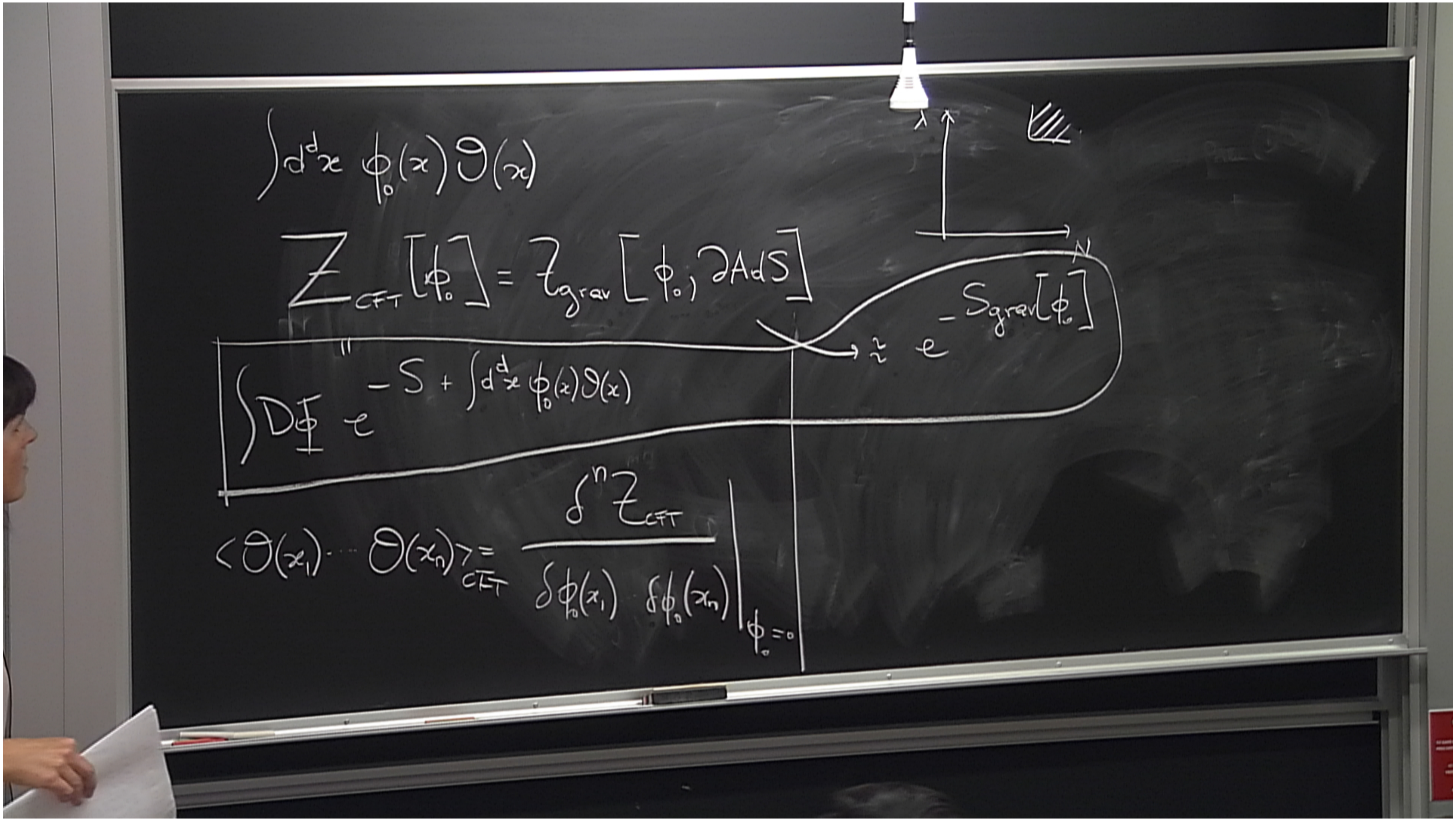
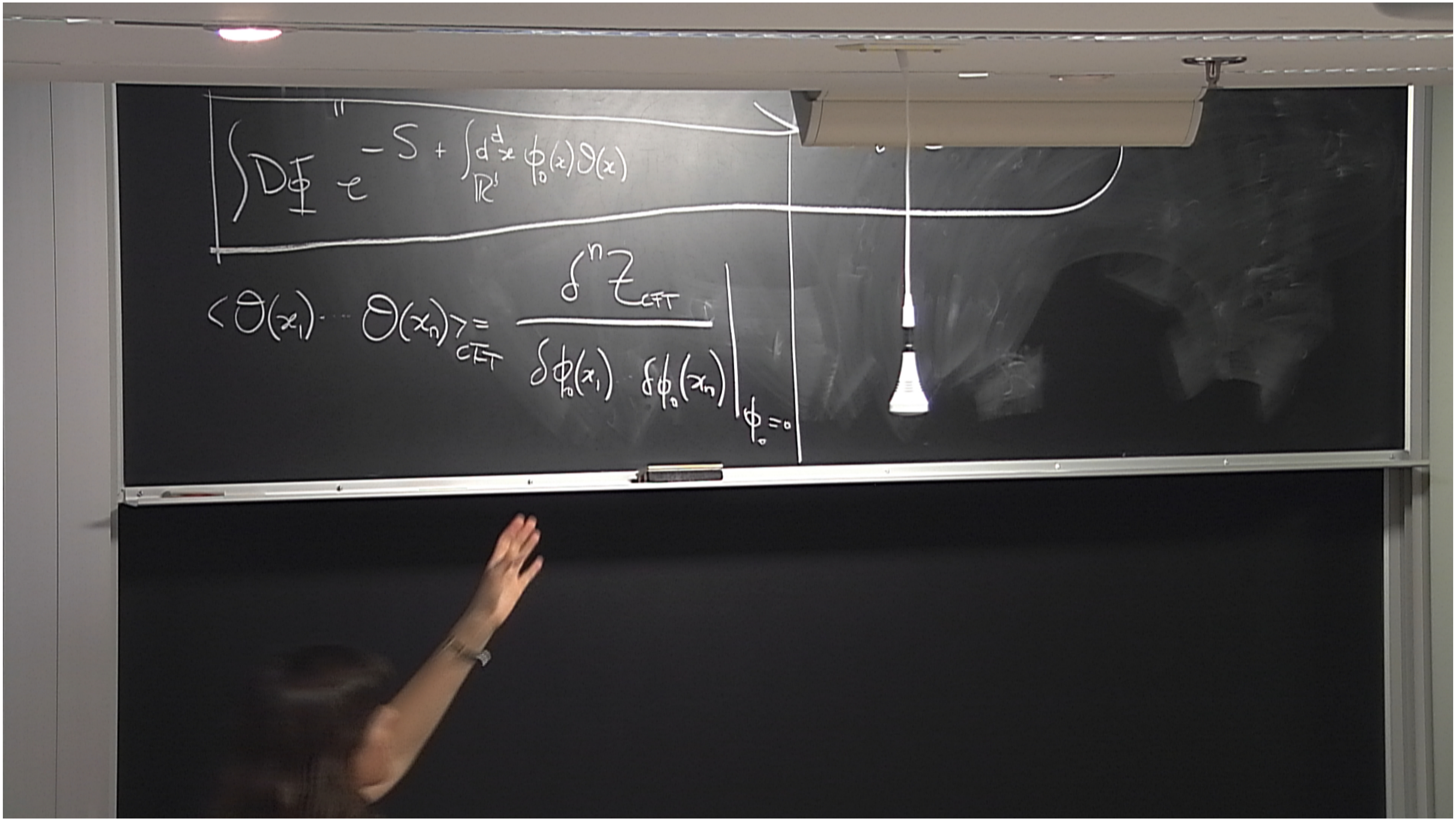
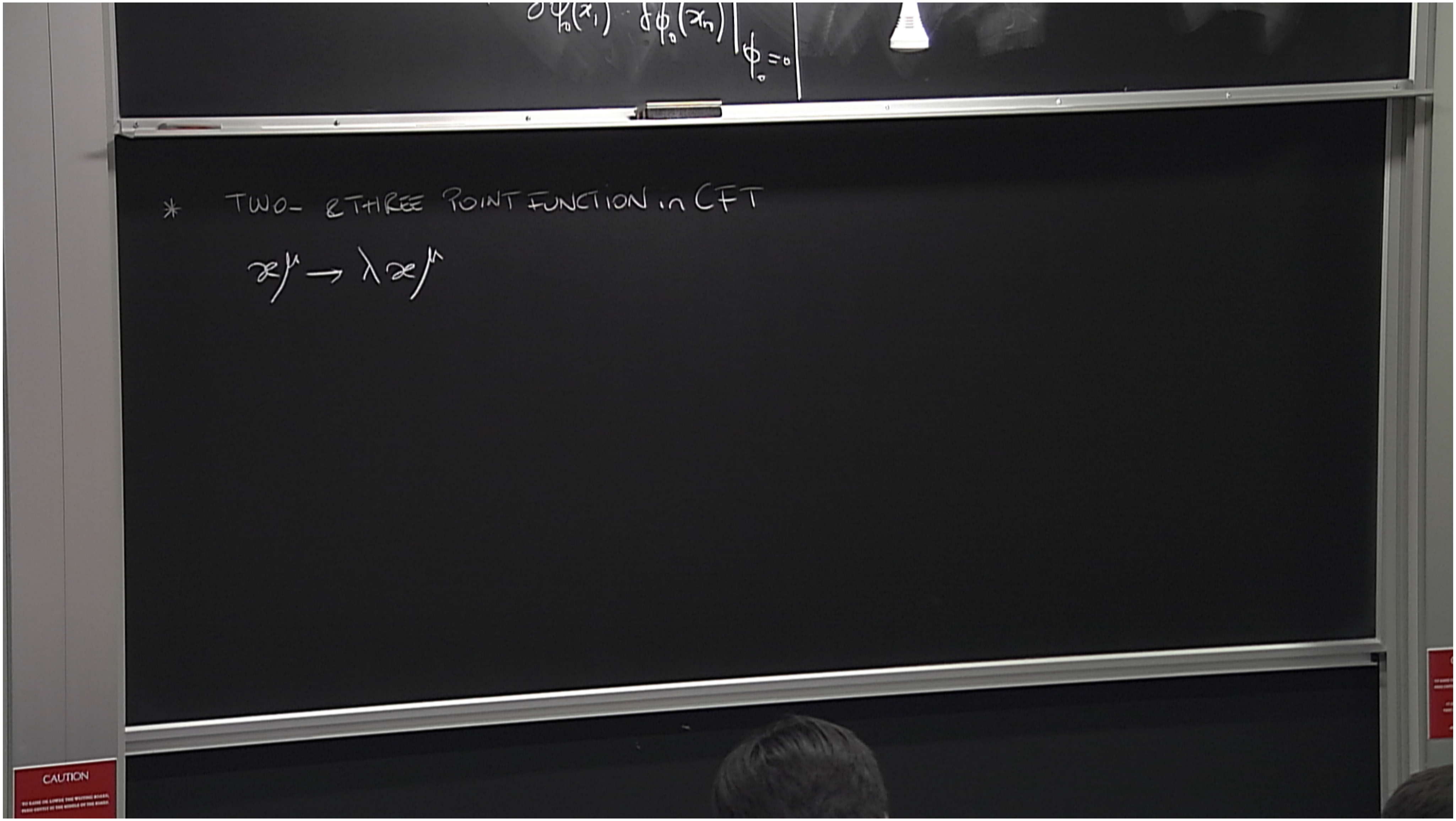
- $\ell_{AdS} = 1$
- $z \to 0$: ∂AdS : $\mathbb{R}^d$
* AdS/CFT operator-field map
$\mathcal{O}$ in CFT on $\mathbb{R}^d$
ϕ w/ BC's: $\phi(z,x) \overset{z\to 0}{\sim} \phi_0(x)$
z
ϕ
$\mathcal{O}$
∂AdS


$\int d^dx\, \phi_0(x)\mathcal{O}(x)$
$Z_{CFT}[\phi_0] = Z_{grav}[\phi_0, \partial AdS]$
$\simeq e^{-S_{grav}[\phi_0]}$
$\int D\Phi\, e^{-S + \int d^dx\, \phi_0(x)\mathcal{O}(x)}$
$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle_{CFT} = \frac{\delta^n Z_{CFT}}{\delta\phi_0(x_1)\cdots\delta\phi_0(x_n)}\Big|_{\phi_0=0}$
λ
N



$\int d^dx\, \phi_0(x)\mathcal{O}(x)$
$Z_{CFT}[\phi_0] = Z_{grav}[\phi_0, \partial AdS]$
$\simeq e^{-S_{grav}[\phi_0]}$
$= \int D\Phi\, e^{-S + \int d^dx\, \phi_0(x)\mathcal{O}(x)}$
$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n)\rangle_{CFT} = \left.\frac{\delta^n Z_{CFT}}{\delta\phi_0(x_1) \cdots \delta\phi_0(x_n)}\right|_{\phi_0 = 0}$
λ
N


$\int D\Phi\, e^{-S + \int_{\mathbb{R}^d} d^d x\, \phi_0(x) \mathcal{O}(x)}$
$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_{CFT} = \frac{\delta^n Z_{CFT}}{\delta \phi_0(x_1) \cdots \delta \phi_0(x_n)} \Big|_{\phi_0 = 0}$


$\delta\phi_0(x_1) \cdots \delta\phi_0(x_n)\Big|_{\phi_0=0}$
* TWO- & THREE POINT FUNCTION in CFT
$x^\mu \to \lambda x^\mu$
CAUTION

$$\delta\phi_0(x_1)\cdots\delta\phi_0(x_n)\Big|_{\phi_0=0}$$

* TWO- & THREE POINT FUNCTION in CFT

$$x^\mu \to \lambda x^\mu$$

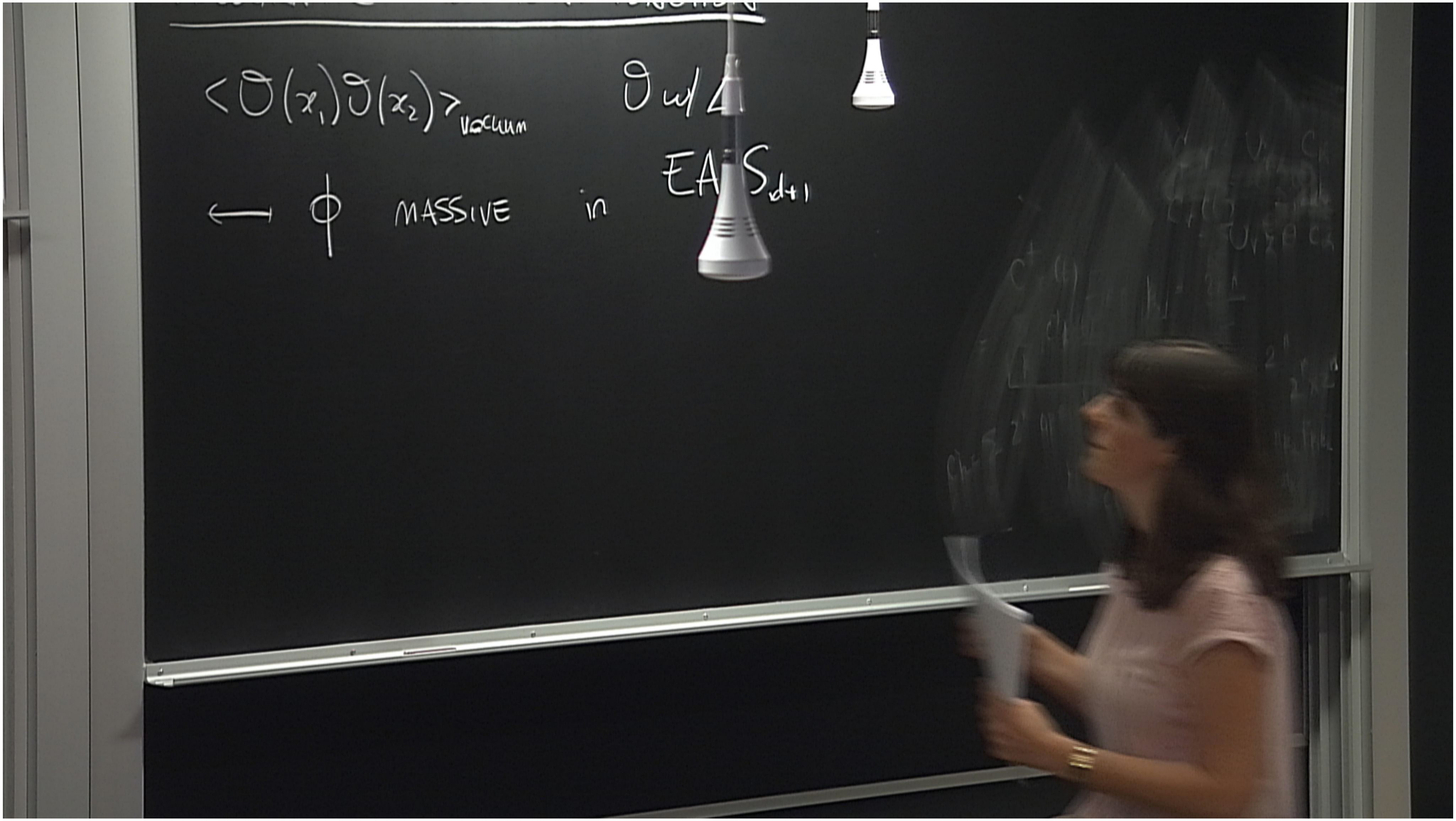
$$\mathcal{O}(x) \longrightarrow \lambda^{-\Delta}\mathcal{O}(x)$$

Δ : conformal dimension

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\rangle = \begin{cases} \dfrac{\delta_{12}}{|x_1-x_2|^{2\Delta_1}} & \text{if } \Delta_1=\Delta_2 \\[2ex] 0 & \text{if } \Delta_1\neq\Delta_2 \end{cases}$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\,x_{13}^{\Delta_1+\Delta_3-\Delta_2}\,x_{23}^{\Delta_2+\Delta_3-\Delta_1}}$$

$$x_{ij} = |x_i - x_j|$$



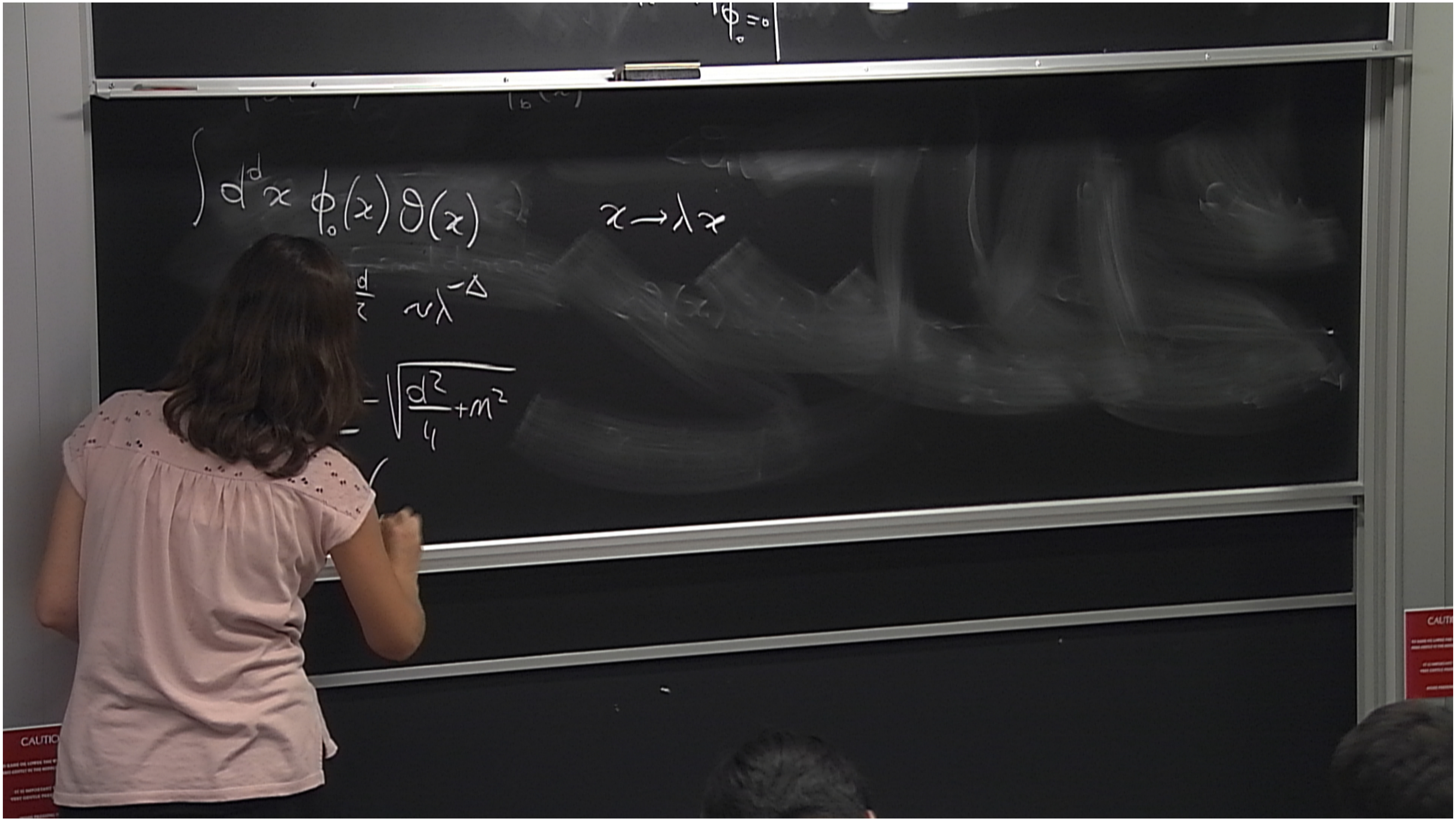
$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_{\text{vacuum}}$
$\mathcal{O}$ w/ Δ
$\longleftarrow \phi$ MASSIVE in $EAdS_{d+1}$

$$z^2\partial_z^2\tilde\phi - (d-1)z\partial_z\tilde\phi + z^2\underbrace{\partial_x^2\tilde\phi}_{-q^2} - m^2\tilde\phi = 0 \qquad \phi(z,x) = \int\frac{d^dq}{(2\pi)^d}\,\tilde\phi(z,q)\,e$$

$$\nu \equiv \sqrt{\frac{d^2}{4}+m^2} \qquad |q| = \sqrt{q_i q_j \delta^{ij}} \geq 0$$

- $z\to\infty$: regularity $\Rightarrow B_q = 0$
- $z\to 0$: $\tilde\phi(z,q) \sim A_q\, z^{\frac{d}{2}-\nu}$

$$\phi(z,x) \sim \phi_0(x)\, z^{\frac{d}{2}-\nu}$$


∫ d^d x φ₀(x) O(x)
x → λx
~λ^{-Δ}
√(d²/4 + m²)

$\phi_0 = 0$

$(x, z) \longrightarrow (\lambda x, \lambda z)$

$$\phi_0(\lambda x) = \lambda^{\nu - \frac{d}{2}} \phi_0(x)$$

$$\int d^d x \, \phi_0(x) \mathcal{O}(x) \qquad x \rightarrow \lambda x$$

$$\sim \lambda^d \cdot \lambda^{\nu - \frac{d}{2}} \sim \lambda^{-\Delta}$$

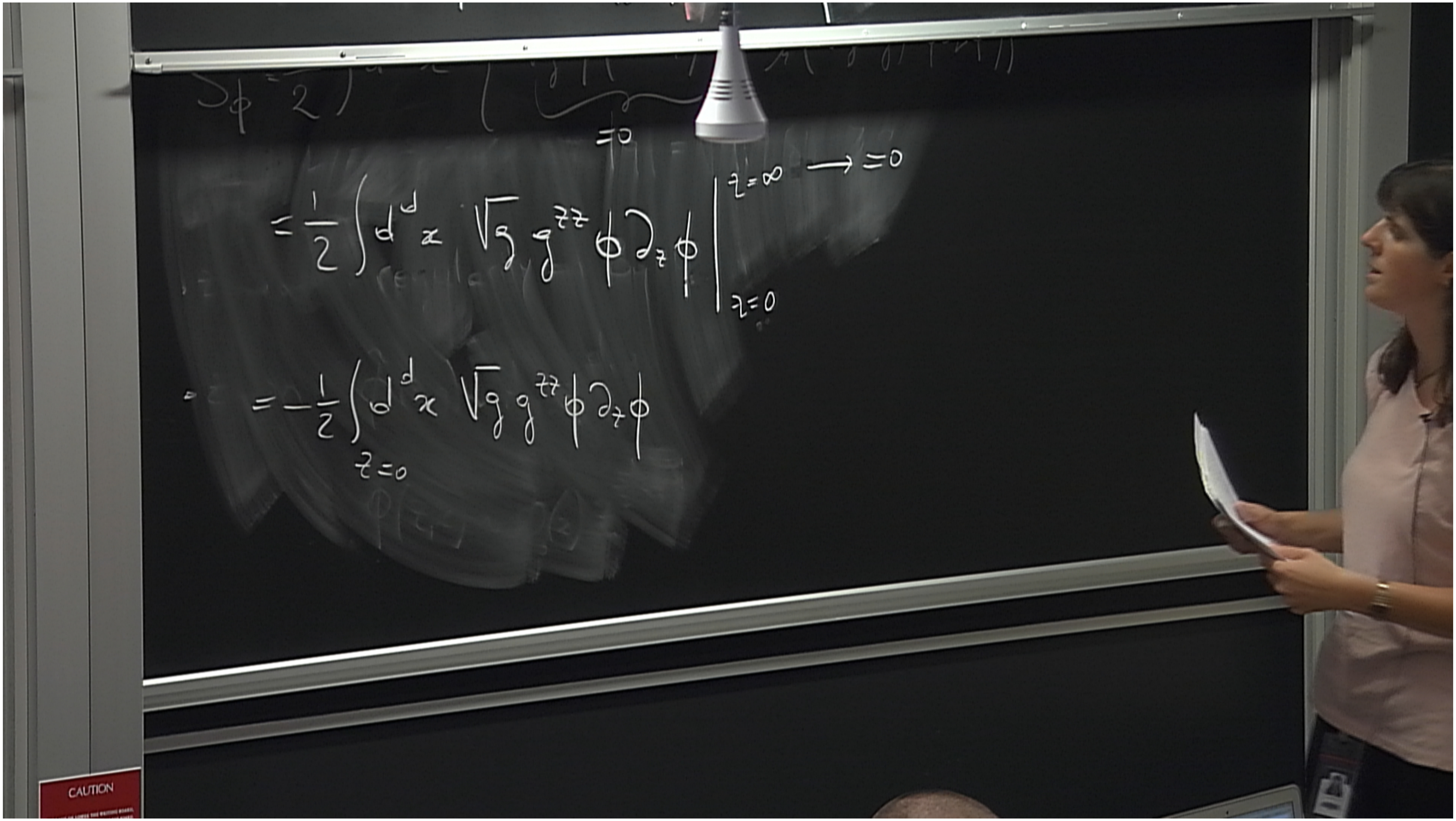
$$d - \Delta \equiv \frac{d}{2} - \sqrt{\frac{d^2}{4} + m^2} \Longrightarrow \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

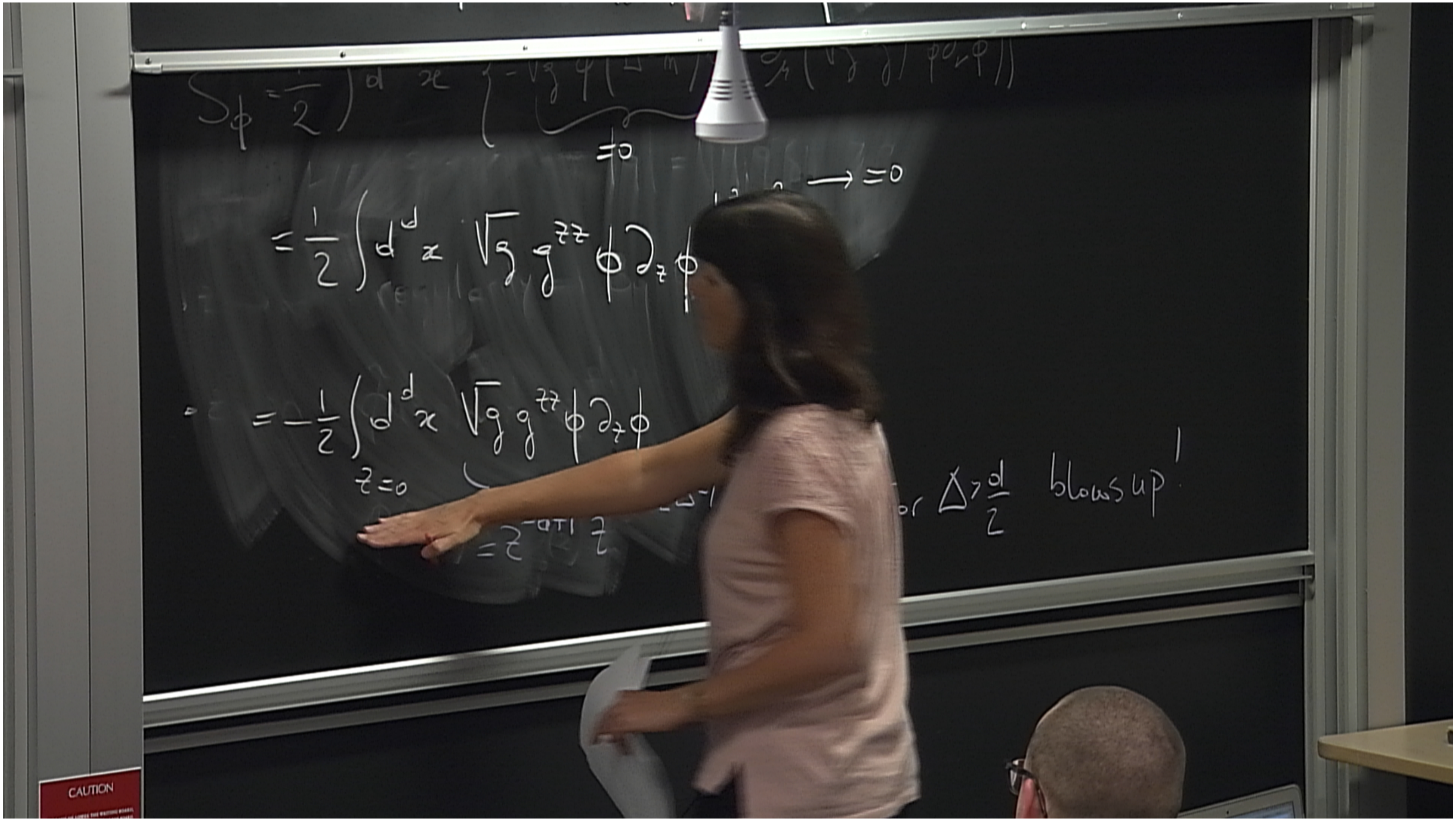
$$m^2 = \Delta(\Delta - d)$$

* $\Delta > d$ IRRELEVANT DEF $m^2 > 0$
* $\Delta = d$: MARGINAL $m^2 = 0$
* $\Delta < d$: RELEVANT $m^2 < 0$

BF bound: AdS is stable if $m^2 \geq -\frac{d^2}{4}$

$\Longrightarrow$ Euclidean $\Delta \geq \frac{d}{2}$


$=0$
$= \frac{1}{2} \int d^d x \sqrt{g}\, g^{zz} \phi \partial_z \phi \Big|_{z=0}^{z=\infty} \longrightarrow =0$
$= -\frac{1}{2} \int_{z=0} d^d x \sqrt{g}\, g^{zz} \phi \partial_z \phi$
CAUTION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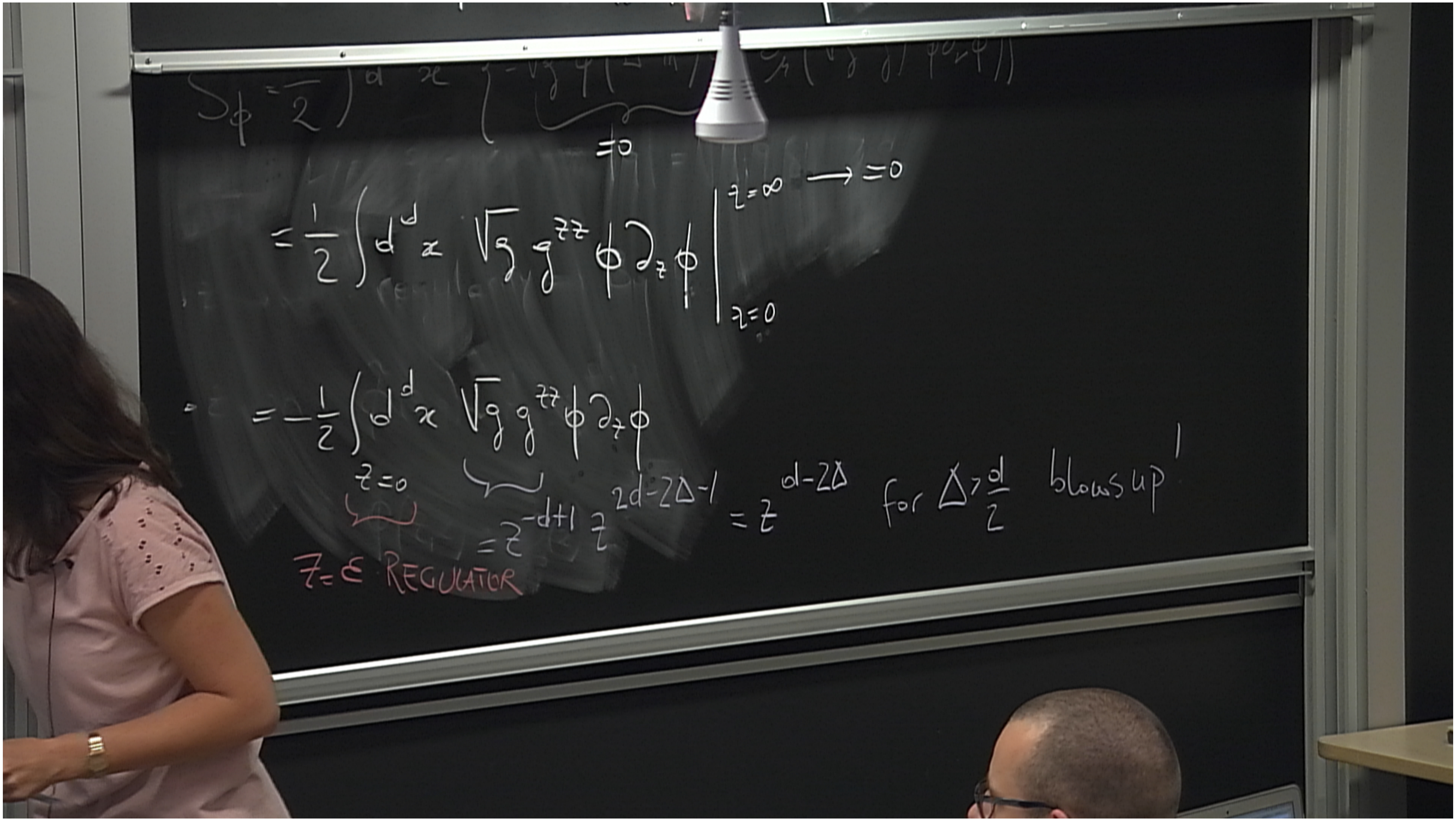
for $\Delta > \frac{d}{2}$ blows up!
CAUTION


=0
$z=\infty \longrightarrow =0$
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$z=0$
$= z^{-d+1} z^{2d-2\Delta-1} = z^{d-2\Delta}$
for $\Delta > \frac{d}{2}$ blows up!
Z=ε REGULATOR

$$K(z,x;x') \sim z^{d-\Delta}\,\delta^d(x-x') + \frac{C_\Delta z^\Delta}{|x-x'|^{2\Delta}} + \cdots$$

$$z = \epsilon$$

$$\epsilon^{d-2\Delta}\,\delta^d(x_1-x_2) + \frac{C_\Delta}{|x_1-x_2|^{2\Delta}}\; d$$

$\phi_0 = 0$

$$= -\frac{1}{2}\int d^d x_1\, d^d x_2\, \phi_0(x_1)\phi_0(x_2) \int d^d x\, z^{1-d} K(z,x,x_1)\, \partial_z K(z,x,x_2)\Big|_{z=\epsilon}$$

$$K(z,x;x') \sim z^{d-\Delta}\delta^d(x-x') + \frac{C_\Delta z^\Delta}{|x-x'|^{2\Delta}} + \cdots$$

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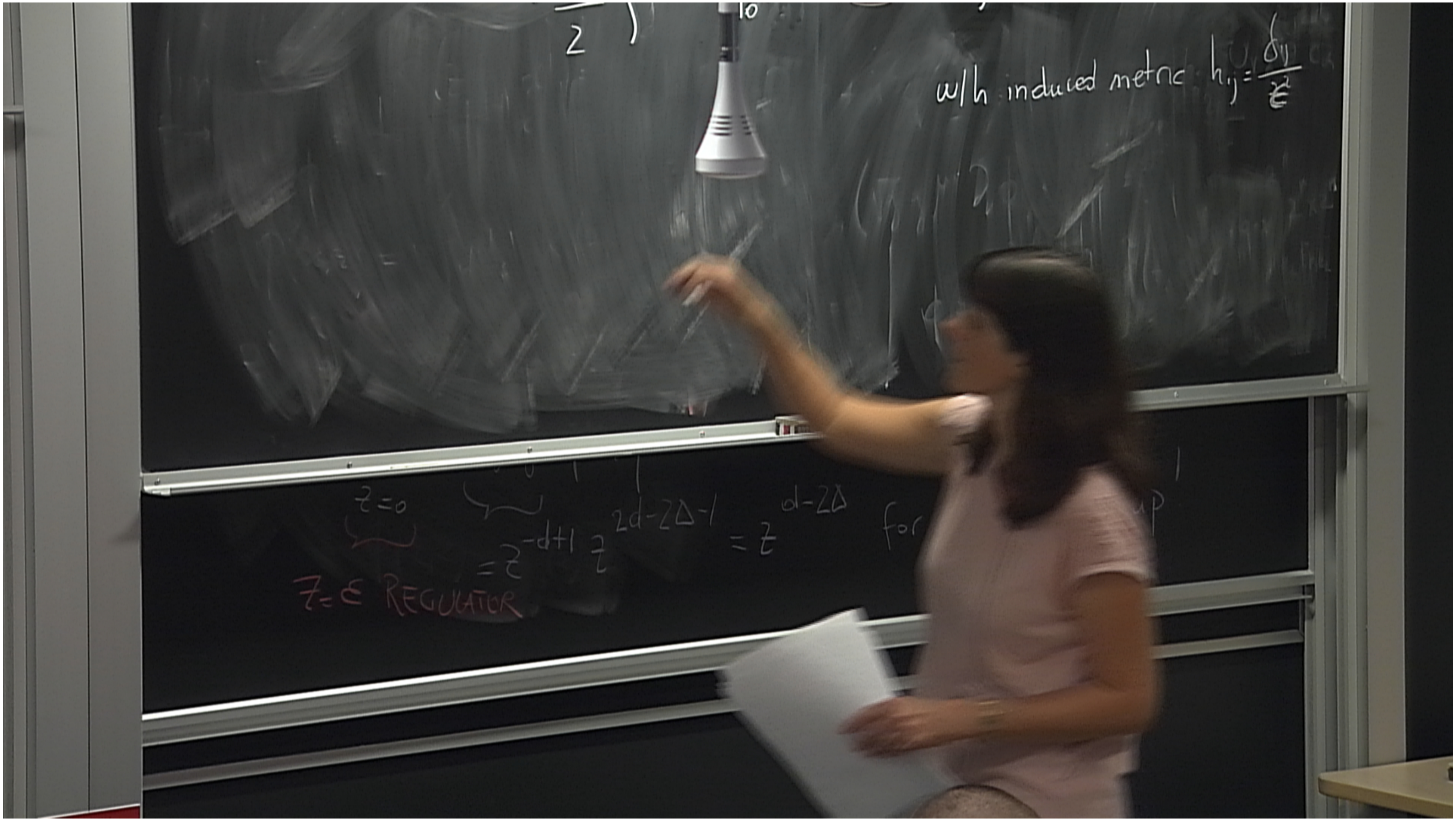
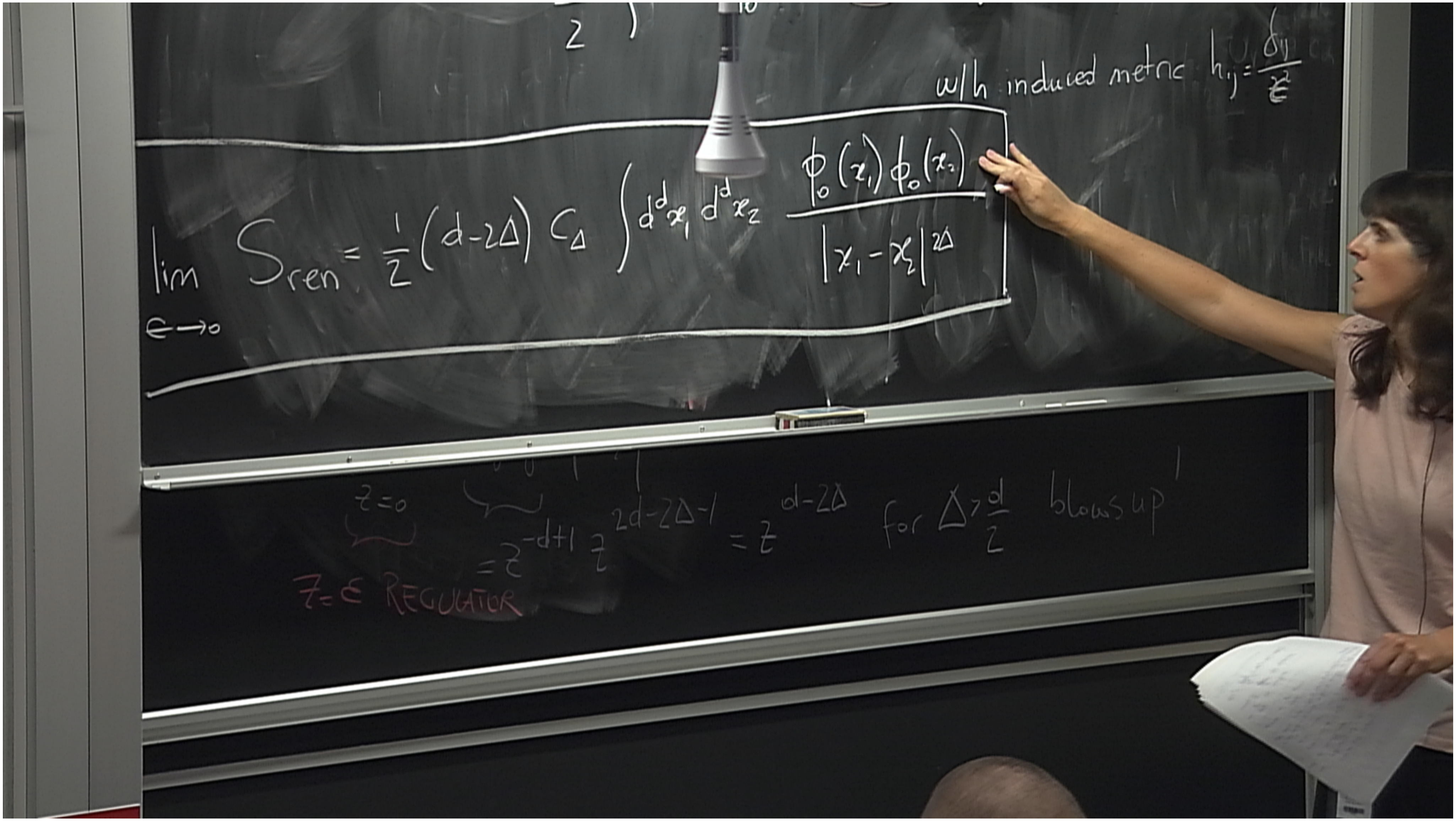

w/h induced metric $h_{ij} = \frac{\delta_{ij}}{\epsilon^2}$
$z=0$
$= z^{-d+1}\, z^{2d-2\Delta-1} = z^{d-2\Delta}$
for
$Z = \epsilon$ REGULATOR


w/h induced metric $h_{ij} = \frac{\delta_{ij}}{\epsilon^2}$
$\lim_{\epsilon \to 0} S_{ren} = \frac{1}{2}(d-2\Delta)\, C_\Delta \int d^d x_1\, d^d x_2 \frac{\phi_0(x_1)\phi_0(x_2)}{|x_1 - x_2|^{2\Delta}}$
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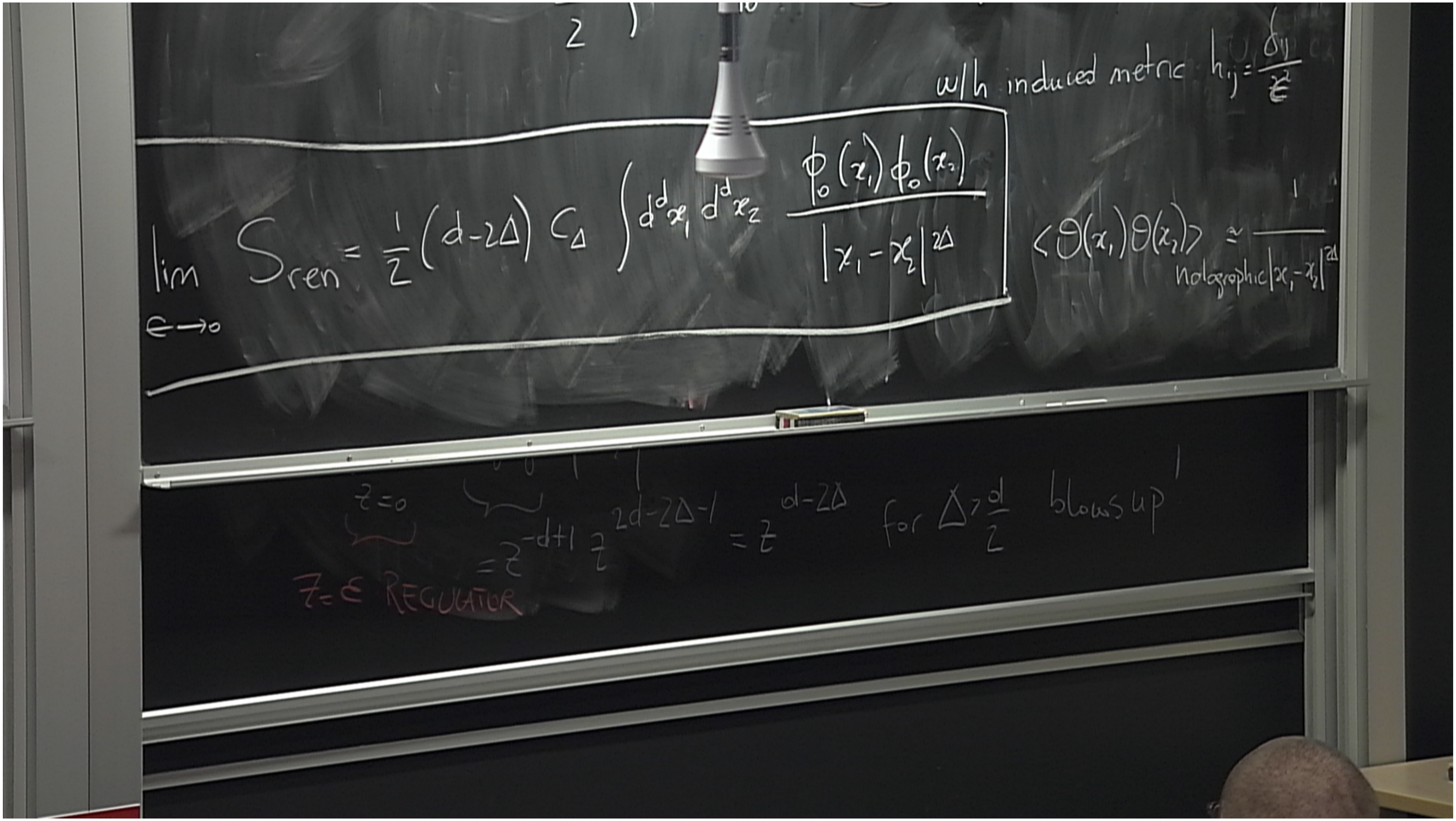
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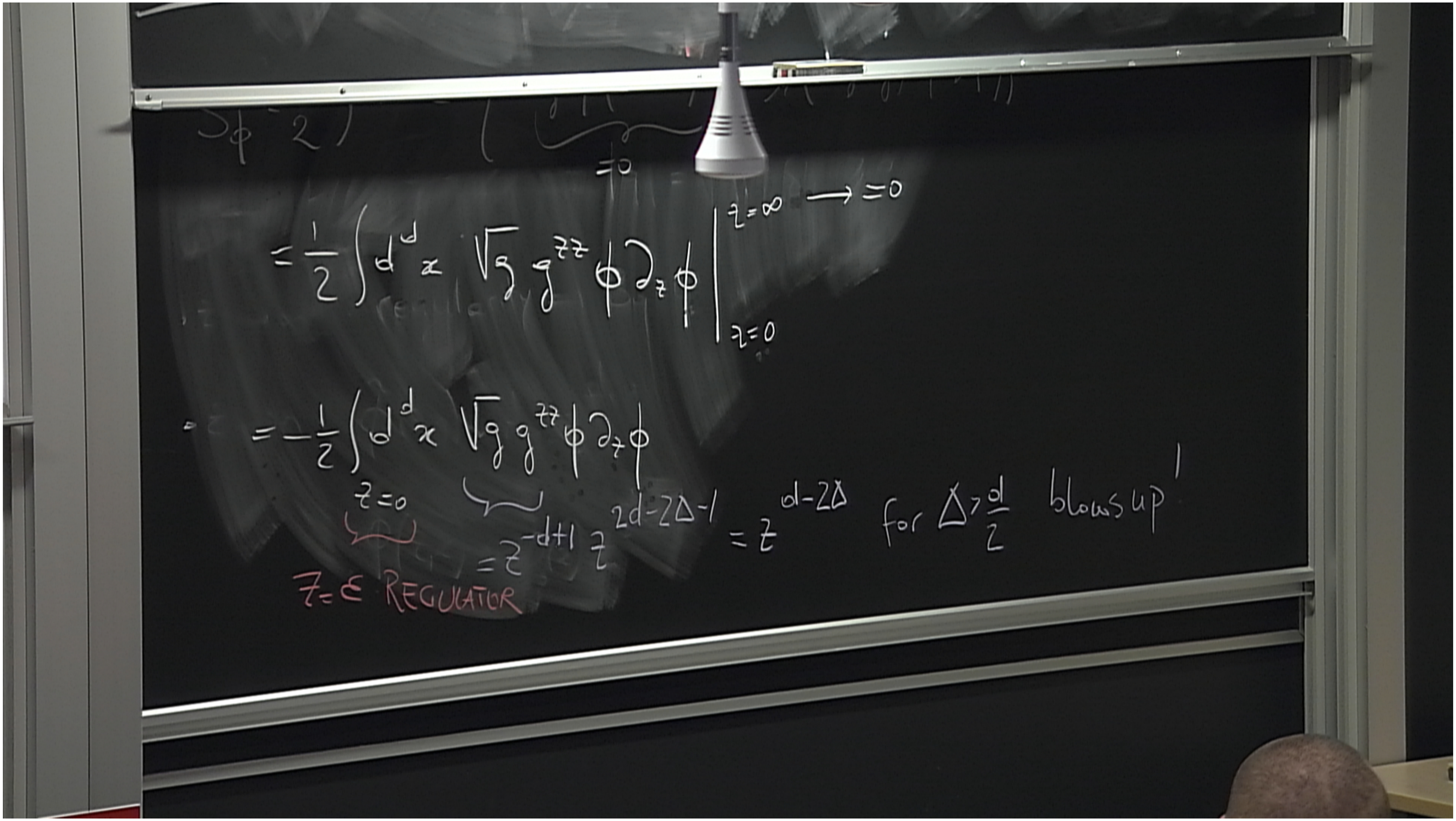
$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_{\text{holographic}} \simeq \frac{1}{|x_1 - x_2|^{2\Delta}}$
$z=0$
$= z^{-d+1} z^{2d-2\Delta-1} = z^{d-2\Delta}$ for $\Delta > \frac{d}{2}$ blows up!
$Z = \epsilon$ REGULATOR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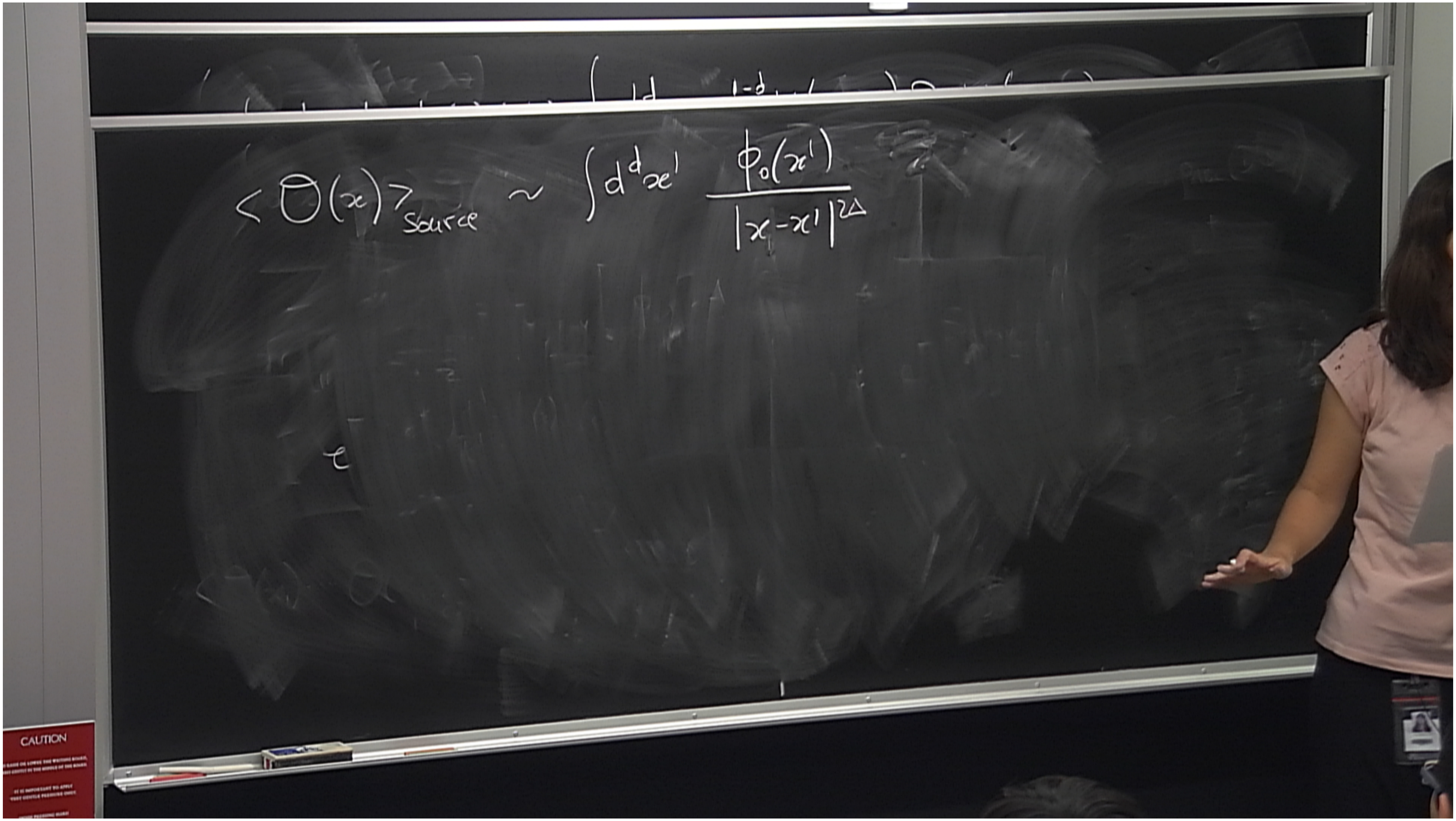
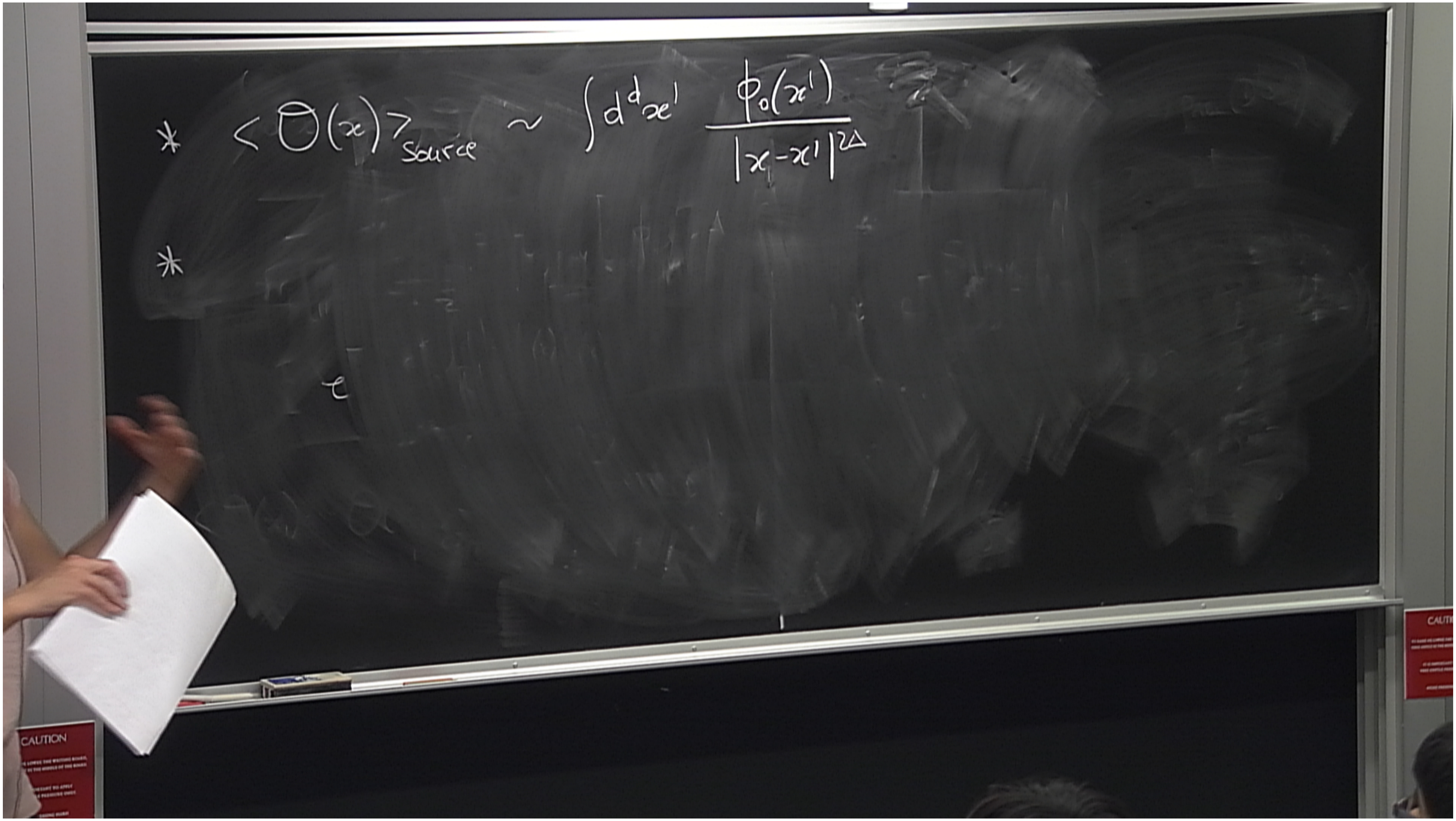
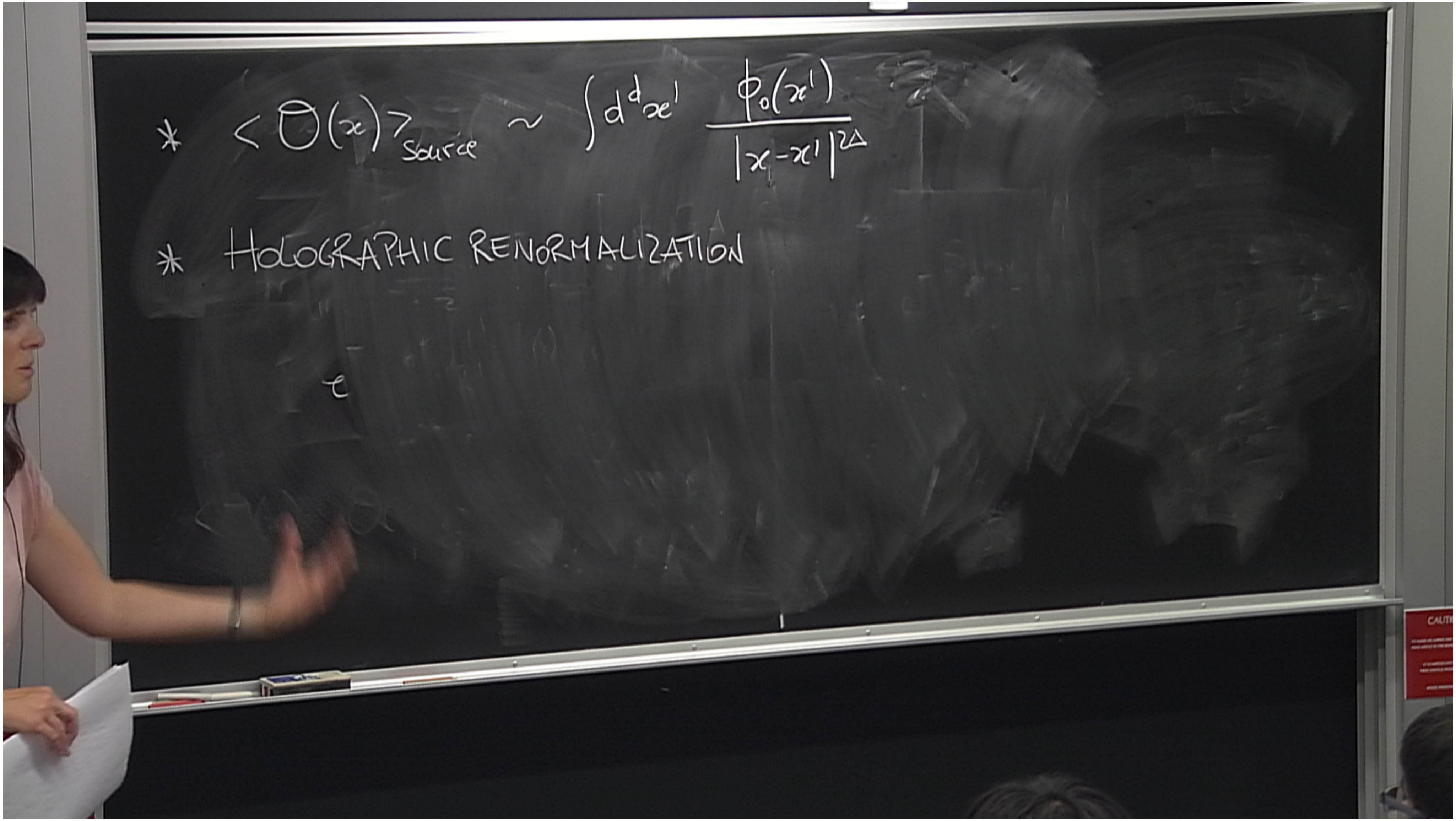
$= z^{d-2\Delta}$ for $\Delta > \frac{d}{2}$ blows up!


$\langle \mathcal{O}(x) \rangle_{\text{source}} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$
CAUTION


$* \quad \langle \mathcal{O}(x) \rangle_{\text{source}} \sim \int d^d x' \, \frac{\phi_0(x')}{|x - x'|^{2\Delta}}$
*
CAUTION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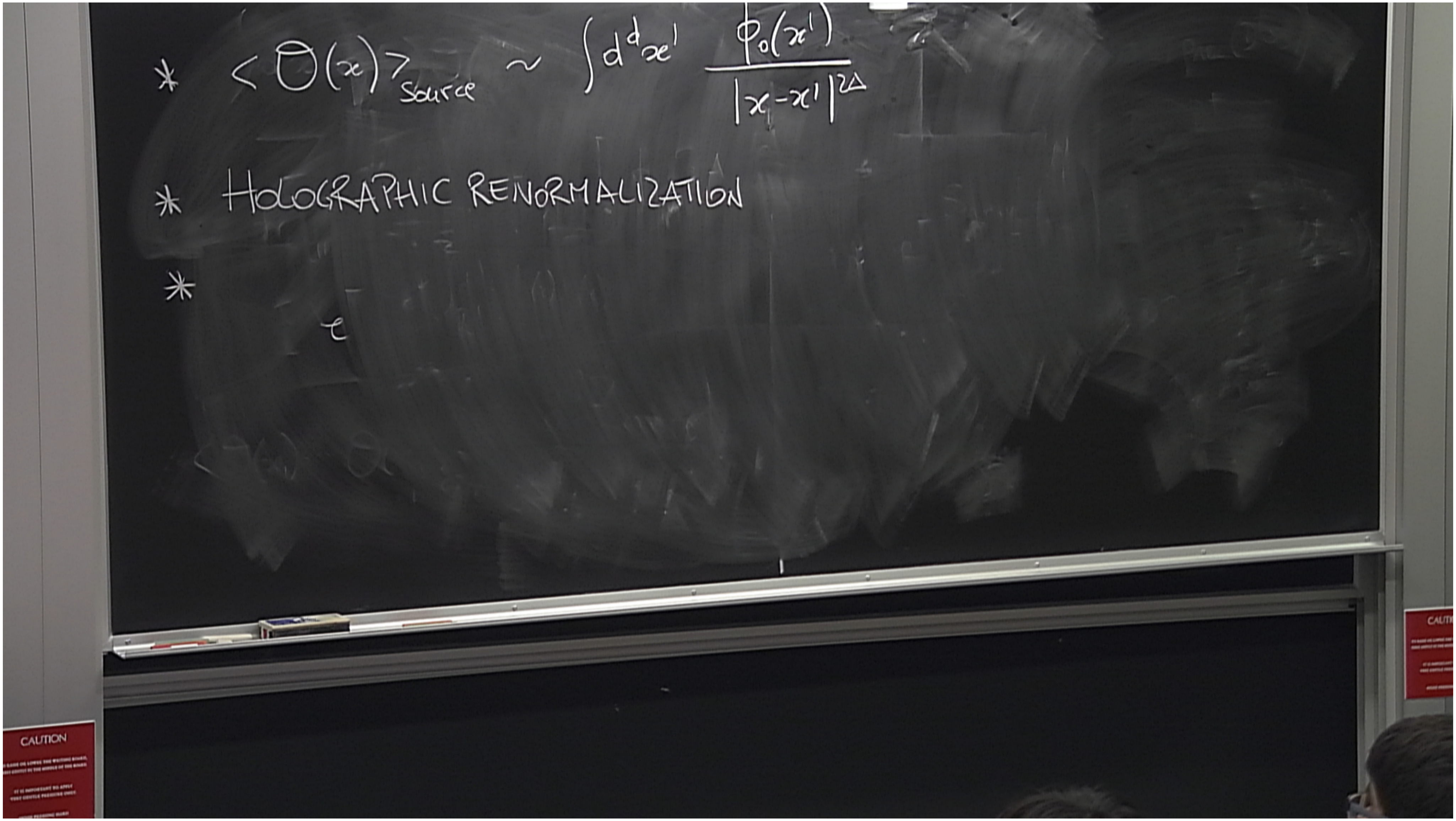
* HOLOGRAPHIC RENORMALIZATION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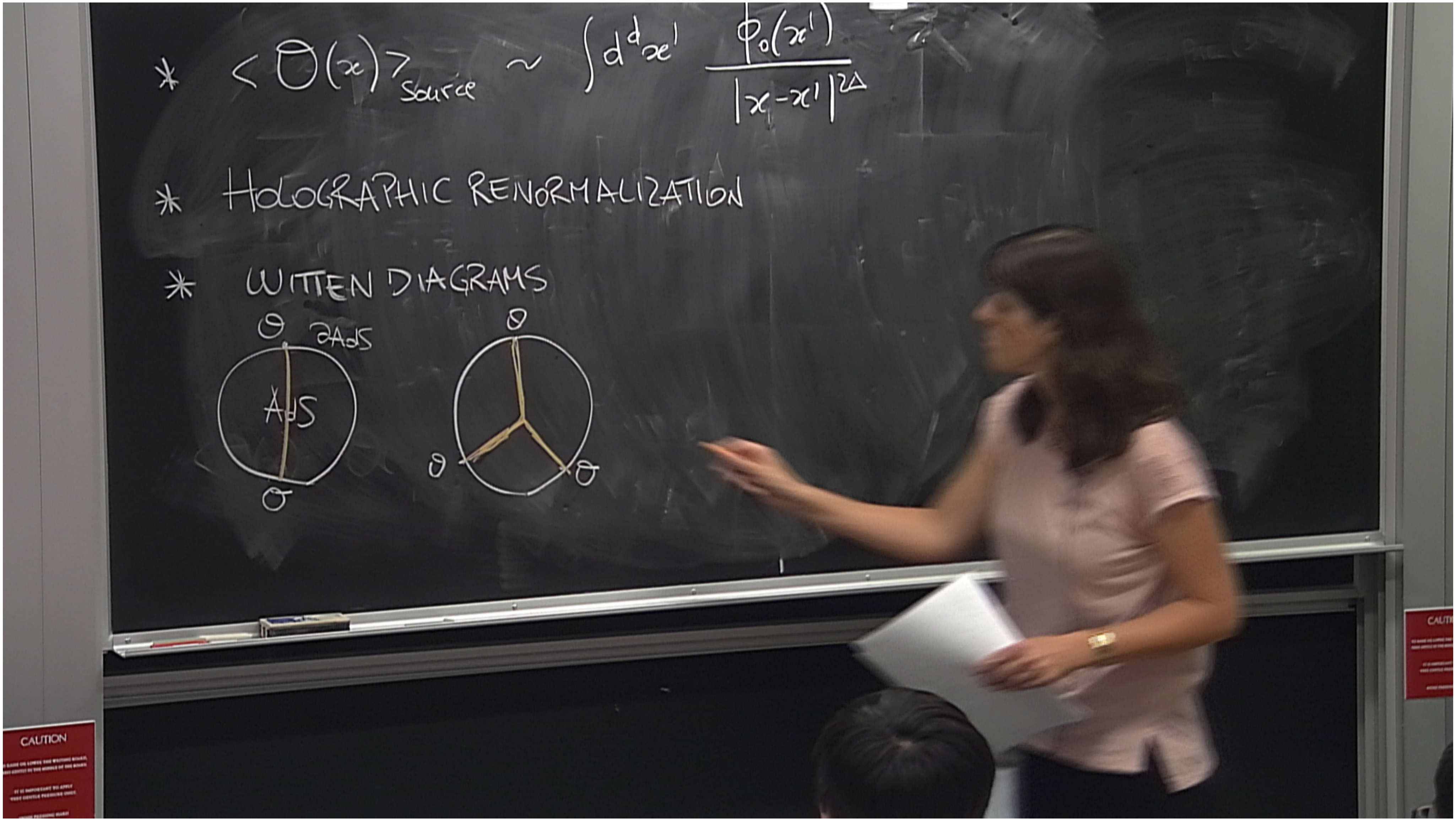
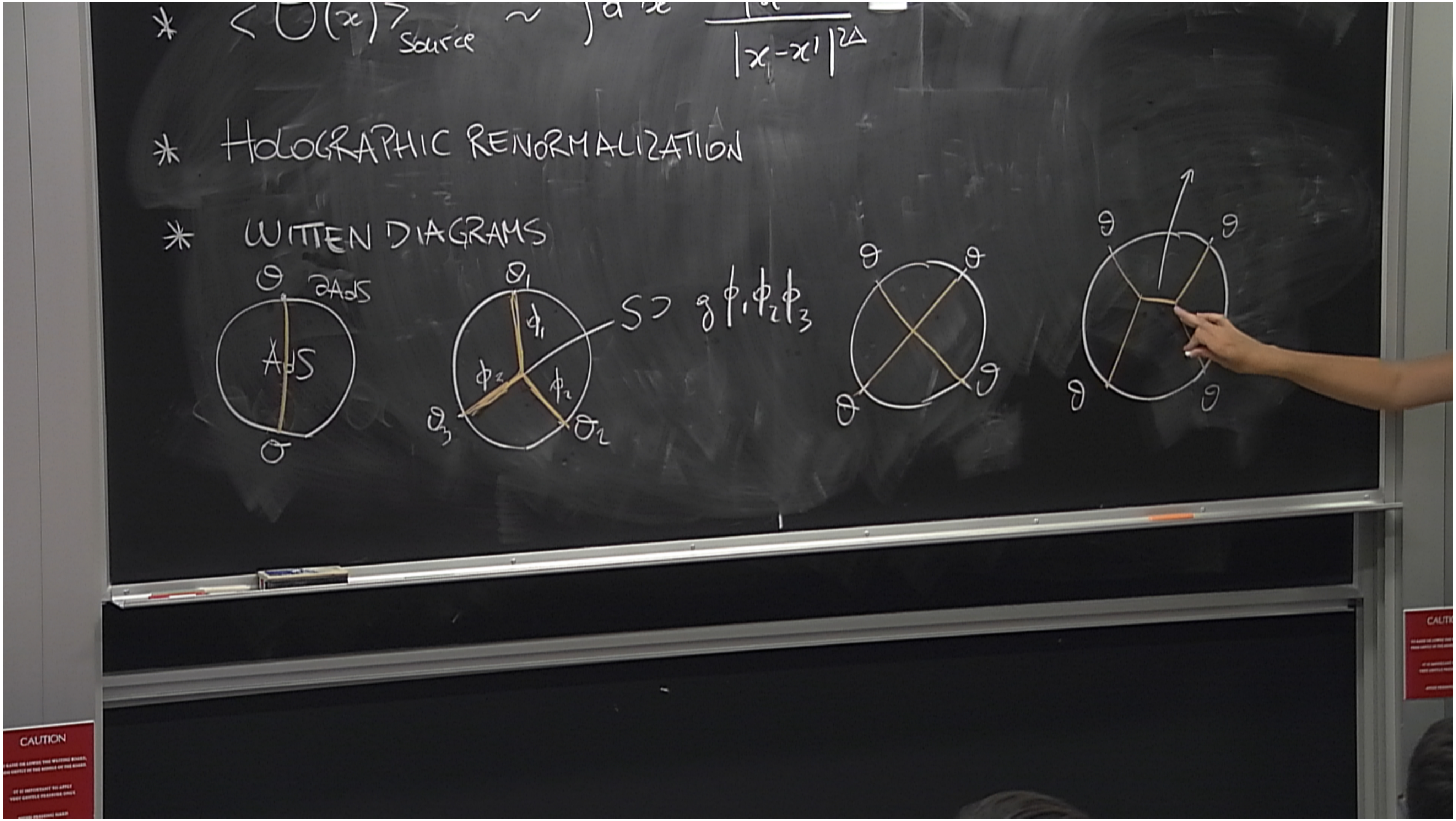
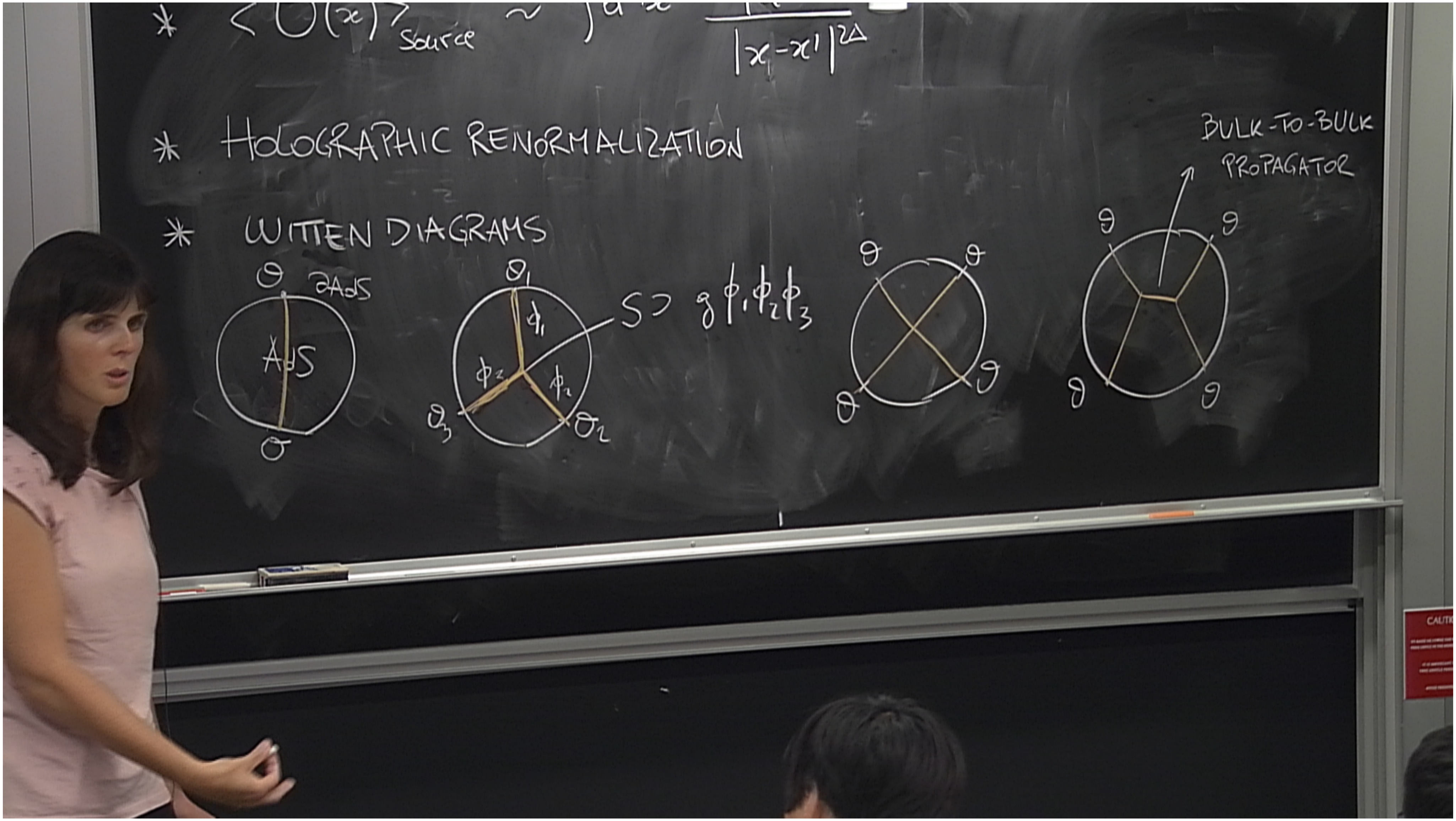
* HOLOGRAPHIC RENORMALIZATION
*
CAUTION


$* \quad \langle \mathcal{O}(x) \rangle_{source} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$
* HOLOGRAPHIC RENORMALIZATION
* WITTEN DIAGRAMS
$\mathcal{O}$
∂AdS
AdS
$\mathcal{O}$
$\mathcal{O}$
$\mathcal{O}$
$\mathcal{O}$
CAUTION


Source
|x - x'|^{2Δ}
* HOLOGRAPHIC RENORMALIZATION
* WITTEN DIAGRAMS
∂AdS
AdS
S ⊃ g φ1 φ2 φ3
CAUTION


* $\langle O(x) \rangle_{source}$
$|x-x'|^{2\Delta}$
* HOLOGRAPHIC RENORMALIZATION
* WITTEN DIAGRAMS
O
∂AdS
AdS
O
O_1
ϕ_1
ϕ_2
ϕ_2
O_3
O_2
$S \supset g\phi_1\phi_2\phi_3$
BULK-TO-BULK
PROPAGATOR