

Title: Holographic Correlation Functions

Date: Jul 22, 2016 05:00 PM

URL: <http://pirsa.org/16070033>

Abstract:

Holographic correlation functions

REFERENCES

- Field/operator correspondence: Witten hep-th/9802150
- CFT: Di Francesco, Mathieu, Sénéchal
- Holographic renormalization: deHaro, Skenderis, Solodukhin hep-th/0002230
- AdS/CFT books:
 - Kiritsis
 - Ammon, Erdmenger

* AdS_{d+1} $ds^2_{AdS} = \frac{l_{AdS}^2}{z^2} (dz^2 + dx^2)$

- $l_{AdS} = 1$

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- $z \rightarrow 0$ $\partial AdS : \mathbb{R}^d$

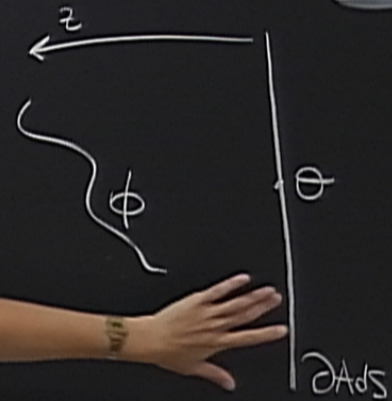
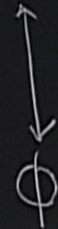
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- $l_{\text{Ads}} = 1$

- $z \rightarrow 0$ $\partial\text{Ads} : \mathbb{R}^d$

* Ads/CFT operator-field

\mathcal{O} in CFT on \mathbb{R}^d



* $AdS_{d+1} \quad ds^2_{AdS} = \frac{l_{AdS}^2}{z^2} (dz^2 + dx^2)$

- $l_{AdS} = 1$

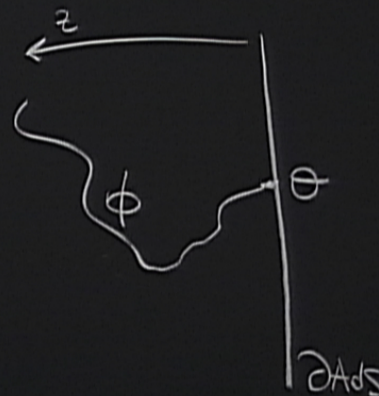
- $z \rightarrow 0 : \partial AdS : \mathbb{R}^d$

* AdS/CFT operator-field map

\mathcal{O} in CFT on \mathbb{R}^d



ϕ w/ BC's: $\phi(z, x) \underset{z \rightarrow 0}{\sim} \phi_*(x)$

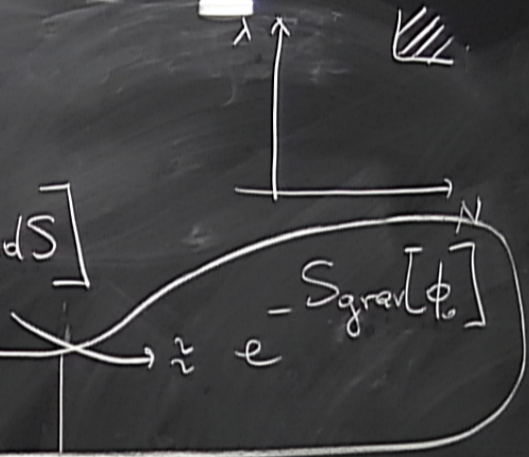


$$\int d^d x \phi_0(x) \mathcal{O}(x)$$

$$Z_{\text{CFT}}[\phi_0] = Z_{\text{grav}}[\phi_0, \partial \text{AdS}]$$

$$\int \mathcal{D}\phi e^{-S + \int d^d x \phi_0(x) \mathcal{O}(x)}$$

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle_{\text{CFT}} = \frac{\delta^n Z_{\text{CFT}}}{\delta \phi_0(x_1) \delta \phi_0(x_n)} \Big|_{\phi_0=0}$$

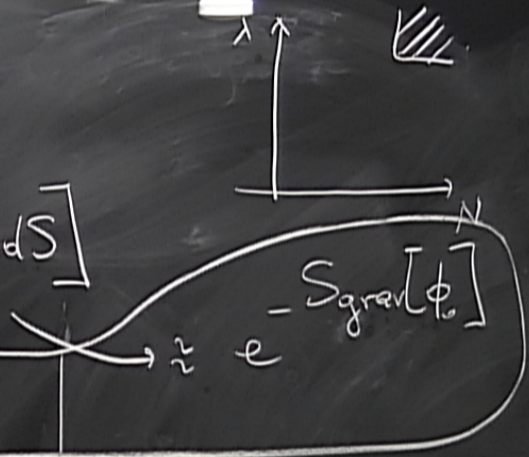


$$\int d^d x \phi_0(x) \mathcal{O}(x)$$

$$Z_{\text{CFT}}[\phi_0] = Z_{\text{grav}}[\phi_0, \partial \text{AdS}]$$

$$\int \mathcal{D}\Phi e^{-S + \int d^d x \phi_0(x) \mathcal{O}(x)}$$

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle_{\text{CFT}} = \frac{\delta^n Z_{\text{CFT}}}{\delta \phi_0(x_1) \delta \phi_0(x_n)} \Big|_{\phi_0=0}$$



$$\int \mathcal{D}\phi \, e^{-S + \int_{\mathbb{R}^d} \phi_0(x) \mathcal{O}(x)}$$

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_{\text{CFT}} = \frac{\delta^n Z_{\text{CFT}}}{\delta \phi_0(x_1) \delta \phi_0(x_n)} \Big|_{\phi_0=0}$$

$$\langle \psi_n(x_1) \delta\phi_n(x_n) | \phi_0 = 0 \rangle$$

* TWO- & THREE POINT FUNCTION in CFT

$$x^\mu \rightarrow \lambda x^\mu$$

CAUTION

$$\langle \psi_n(x_1) \phi_n(x_2) | \phi_0 = 0 \rangle$$

* TWO- & THREE POINT FUNCTION in CFT

$$x^\mu \rightarrow \lambda x^\mu$$

$$\mathcal{O}(x) \rightarrow \lambda^{-\Delta} \mathcal{O}(x)$$

Δ : conformal dimension

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \rangle =$$

$$\left\{ \begin{array}{ll} \frac{\delta_{12}}{|x_1 - x_2|^{2\Delta_1}} & \text{if } \Delta_1 = \Delta_2 \\ 0 & \text{if } \Delta_1 \neq \Delta_2 \end{array} \right.$$

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \rangle =$$

$$\frac{C_{123}}{x_{12}^{\Delta_1 + \Delta_2 - \Delta_3} x_{13}^{\Delta_1 + \Delta_3 - \Delta_2} x_{23}^{\Delta_2 + \Delta_3 - \Delta_1}}$$

$$x_{ij} = |x_i - x_j|$$

CAUTION
BE CAREFUL NOT TO TOUCH THE BOARD
OR TO WRITE ON THE BOARD

$$\langle \sigma(x_1) \sigma(x_2) \rangle_{\text{vacuum}}$$

$\partial w / \Delta$

$\longleftrightarrow \phi$

MASSIVE

in

$E A S_{d+1}$

$$z^2 \partial_z^2 \tilde{\phi} - (d-1)z \partial_z \tilde{\phi} + z^2 \partial_x^2 \tilde{\phi} - m^2 \tilde{\phi} = 0 \quad \left| \quad \phi(z, x) = \int \frac{dq}{(2\pi)^d} \phi(z, q) e^{iqx}$$

$$\nu = \sqrt{\frac{d^2}{4} + m^2} \quad |q| = \sqrt{q^2, \delta^2} \geq 0$$

$z \rightarrow \infty$: regularity $\Rightarrow B_q = 0$

$z \rightarrow 0$: $\tilde{\phi}(z, q) \sim A_q z^{\frac{d}{2} - \nu}$

$\phi(z, x) \sim \phi_0(z) z^{\frac{d}{2} - \nu}$

$$\int d^d x \phi_0(x) \theta(x)$$

$$x \rightarrow \lambda x$$

$$\frac{d}{d\lambda} \sim \lambda^{-\Delta}$$

$$= \sqrt{\frac{d^2}{4} + m^2}$$

$$(x, z) \rightarrow (\lambda x, \lambda z)$$

$$* \phi_0(\lambda x) = \lambda^{\nu - \frac{d}{2}} \phi_0(x)$$

$$\int d^d x \phi_0(x) \partial(x)$$

$$\sim \lambda^d \cdot \lambda^{\nu - \frac{d}{2}} \sim \lambda^{-\Delta}$$

$$d - \Delta \equiv \frac{d}{2} - \sqrt{\frac{d^2}{4} + m^2} \Rightarrow \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

$$m^2 = \Delta(\Delta - d)$$

* $\Delta > d$ IRRELEVANT DEF $m^2 > 0$

* $\Delta = d$ MARGINAL $m^2 = 0$

* $\Delta < d$ RELEVANT $m^2 < 0$

$x \rightarrow \lambda x$

BF bound: AdS is stable

$$\text{if } m^2 \geq -\frac{d^2}{4}$$

$$\Rightarrow \text{Euclidean } \Delta \geq \frac{d}{2}$$

CAUTION

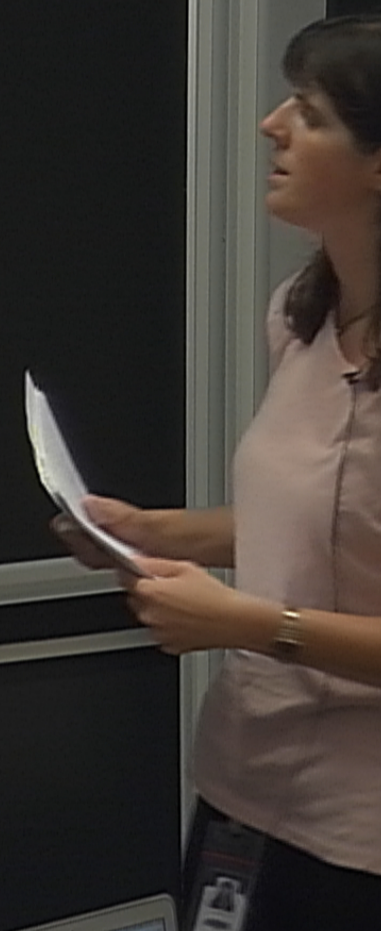
DO NOT STAND ON TOP OF THE BOARD.
IT IS NECESSARY TO ASK
YOUR INSTRUCTOR PERMISSION FIRST.

CAUTION

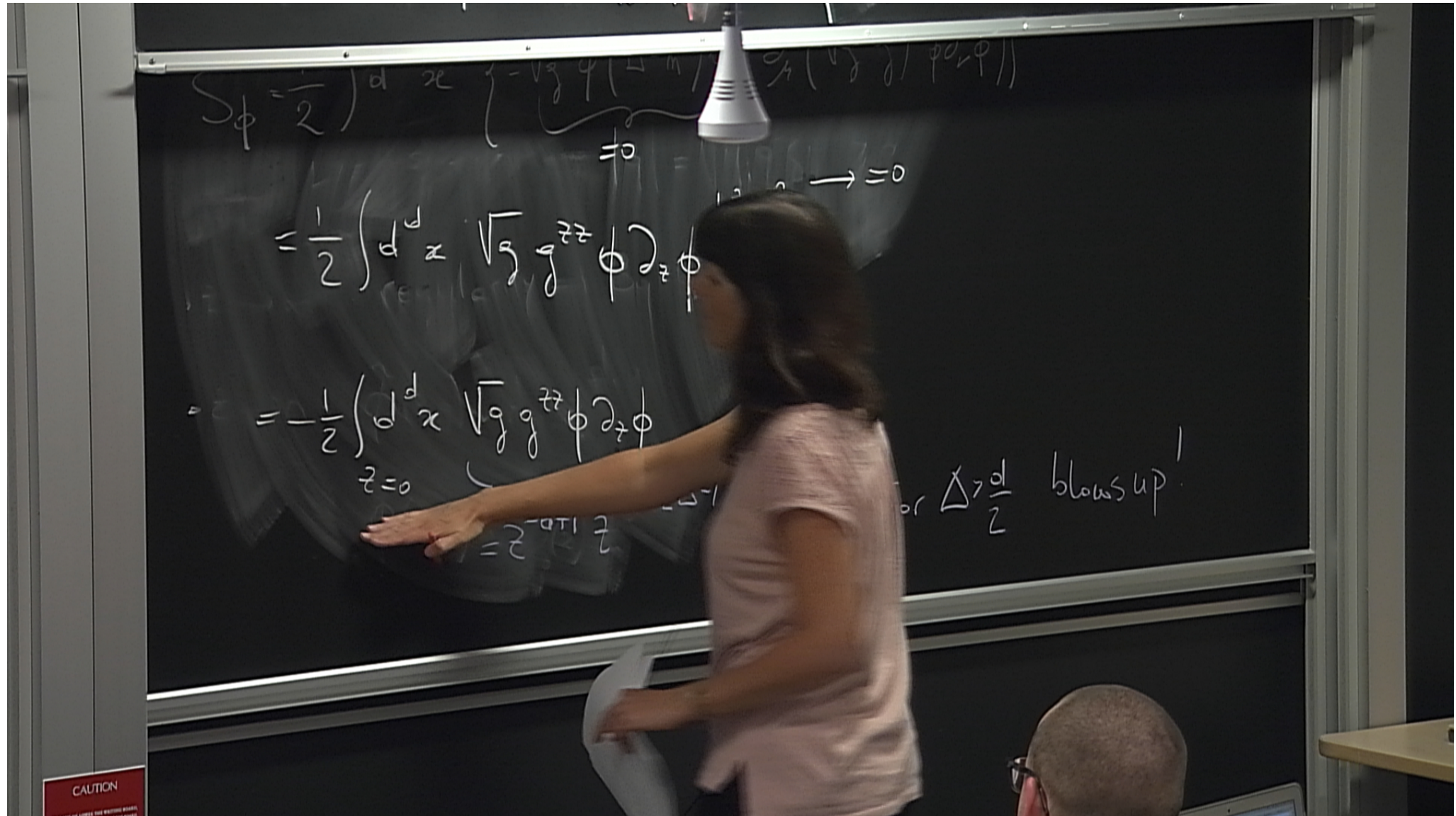
$\delta\phi = 2 \dots$
 $= 0$

$$= \frac{1}{2} \int d^d x \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \Big|_{z=0}^{z=\infty} \rightarrow = 0$$

$$= -\frac{1}{2} \int_{z=0} d^d x \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi$$



CAUTION
DO NOT TOUCH THE CHALKBOARD



$$S_\phi = \frac{1}{2} \int d^d x \sqrt{g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$= \frac{1}{2} \int_{z=0}^{z=\infty} d^d x \sqrt{g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \Big|_{z=0}^{z=\infty} \rightarrow = 0$$

$$= -\frac{1}{2} \int_{z=0} d^d x \sqrt{g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$= z^{-d+1} z^{2d-2\Delta-1} = z^{d-2\Delta} \text{ for } \Delta > \frac{d}{2} \text{ 'blows up'}$$

$z = \epsilon$ REGULATOR

$$|\phi_0 = 0|$$

(7)

0, 10

$z = \varepsilon$

$$K(z, x; x') \sim z^{d-\Delta} \int^d (x-x') + \frac{C_\Delta z^\Delta}{|x-x'|^{2\Delta}}$$

$$\varepsilon^{d-2\Delta} \int^d (x_1-x_2) + \frac{C_\Delta}{|x_1-x_2|^{2\Delta}} d$$

CAUTION

DO NOT STAND ON TOP OF THE BOARD.
IT IS NECESSARY TO ASK
YOUR INSTRUCTOR BEFORE
USING ERASERS.

CAUTION

$|\phi_0\rangle$

$$= -\frac{1}{2} \int d^d x_1 d^d x_2 \phi_0(x_1) \phi_0(x_2) \int d^d x \tau K(z, x, x_1) \partial_z K(z, x, x_2)$$

$z = \epsilon$

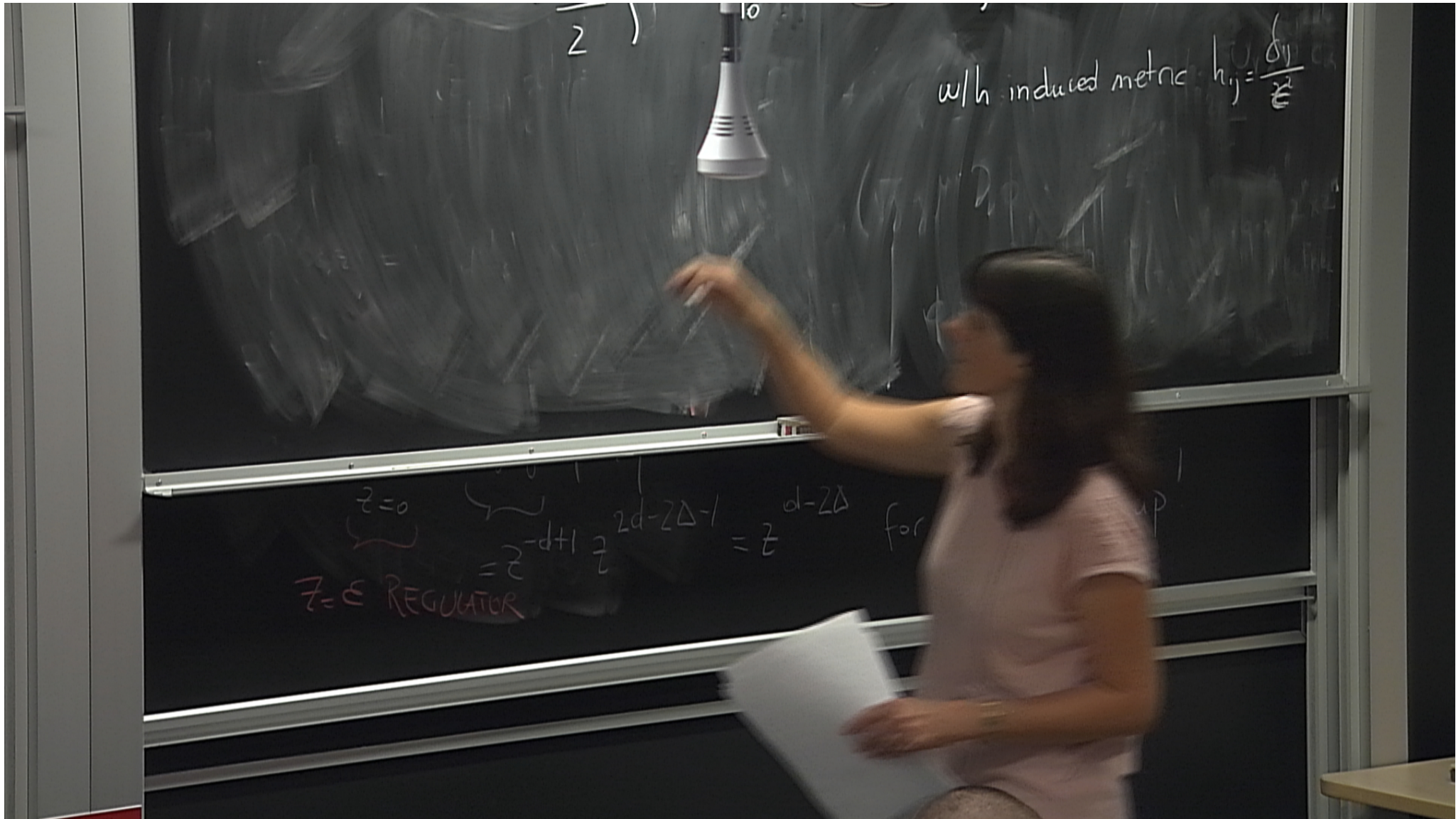
$$K(z, x, x_1) \sim z^{d-\Delta} \int^d (x-x_1) + \frac{C_\Delta z^\Delta}{|x-x_1|^{2\Delta}} + \dots$$

$$\sim \epsilon^{d-2\Delta} \int^d (x_1-x_2) + \frac{C_\Delta}{|x_1-x_2|^{2\Delta}} + \dots$$

CAUTION

BE CAREFUL NOT TO TOUCH THE BOARD
IF IT IS NECESSARY TO ADJUST
YOUR CHAIR OR DESK, PLEASE
ASK A TA OR TAUGHTER

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$$\lim_{\epsilon \rightarrow 0} S_{\text{ren}} = \frac{1}{2} (d-2\Delta) C_{\Delta} \int_{d^d x_1, d^d x_2} \frac{\phi_0(x_1) \phi_0(x_2)}{|x_1 - x_2|^{2\Delta}}$$

w/h induced metric $h_{ij} = \frac{\delta_{ij}}{\epsilon^2}$

$z=0$
 $z^{-d+1} z^{2d-2\Delta-1} = z^{d-2\Delta}$ for $\Delta > \frac{d}{2}$ blows up!
 $z = \epsilon$ REGULATOR

$$\lim_{\epsilon \rightarrow 0} S_{\text{ren}} = \frac{1}{z} (d-2\Delta) C_{\Delta} \int d^d x_1 d^d x_2 \frac{\phi_0(x_1) \phi_0(x_2)}{|x_1 - x_2|^{2\Delta}}$$

w/h induced metric $h_{ij} = \frac{\delta_{ij}}{\epsilon^2}$

$$\langle \Theta(x_1) \Theta(x_2) \rangle \approx \frac{1}{\text{logarithmic } |x_1 - x_2|^{2\Delta}}$$

$z=0$
 $z^{-d+1} z^{2d-2\Delta-1} = z^{d-2\Delta}$ for $\Delta > \frac{d}{2}$ blows up!
 $z = \epsilon$ REGULATOR

$\partial \phi^{-2}$

$=0$

$$= \frac{1}{2} \int_{z=0}^{z=\infty} d^d x \sqrt{g} g^{zz} \phi \partial_z \phi \Big|_{z=0}^{z=\infty} \rightarrow =0$$

$$= -\frac{1}{2} \int_{z=0} d^d x \sqrt{g} g^{zz} \phi \partial_z \phi$$

$z=0$

z^{-d+1}

$z^{d-2\Delta-1}$

$= z^{d-2\Delta}$

for $\Delta > \frac{d}{2}$ blows up!

$\epsilon = \epsilon$ REGULATOR

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$\bar{z} = \epsilon$ REGULATOR

$$\langle \mathcal{O}(x) \rangle_{\text{source}} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$$

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CAUTION

BE CAREFUL NOT TO TOUCH THE BOARD
IF IT IS NECESSARY TO ADJUST
THE BOARD, PLEASE CONTACT THE
TECHNICAL SUPPORT

* $\langle \mathcal{O}(x) \rangle_{\text{source}} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$

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CAUTION
DO NOT TOUCH THE BOARD
OR THE SURFACE OF THE BOARD
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* $\langle O(x) \rangle_{\text{source}} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$

* HOLOGRAPHIC RENORMALIZATION

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* $\langle \mathcal{O}(x) \rangle_{\text{Source}} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$

* Holographic Renormalization

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CAUTION

BE CAREFUL NOT TO TOUCH THE BOARD.
IT IS IMPORTANT TO KEEP
YOUR DISTANCE FROM THE BOARD.
PLEASE RESPECT THE BOARD.

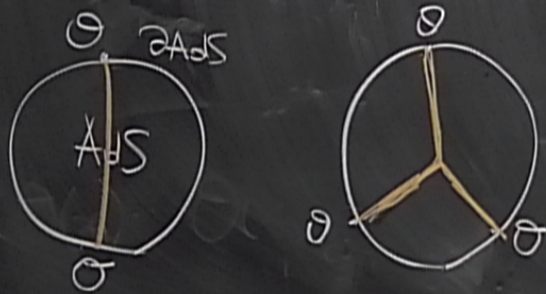
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$$* \langle \mathcal{O}(x) \rangle_{\text{Source}} \sim \int d^d x' \frac{\phi_0(x')}{|x-x'|^{2\Delta}}$$

* HOLOGRAPHIC RENORMALIZATION

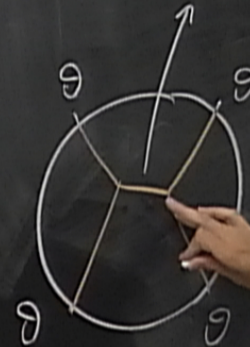
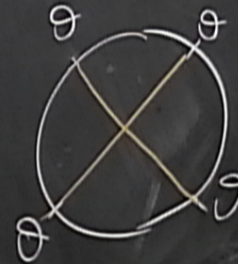
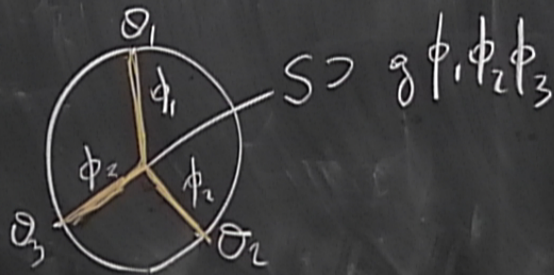
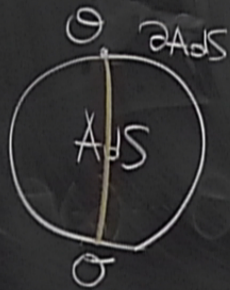
* WITTEN DIAGRAMS



* $\langle O(x) \rangle_{\text{Source}} \sim \int d\alpha \frac{1}{|x-x'|^{2\Delta}}$

* HOLOGRAPHIC RENORMALIZATION

* WITTEN DIAGRAMS



CAUTION

PLEASE DO NOT TOUCH THE WHITEBOARD.
 IT IS IMPORTANT TO KEEP THE WHITEBOARD CLEAN.
 IF YOU HAVE ANY QUESTIONS, PLEASE ASK THE LECTURER.

CAUTION

* $\langle O(x) \rangle_{\text{Source}} \sim \frac{1}{|x-x'|^{2\Delta}}$

* HOLOGRAPHIC RENORMALIZATION

* WITTEN DIAGRAMS

