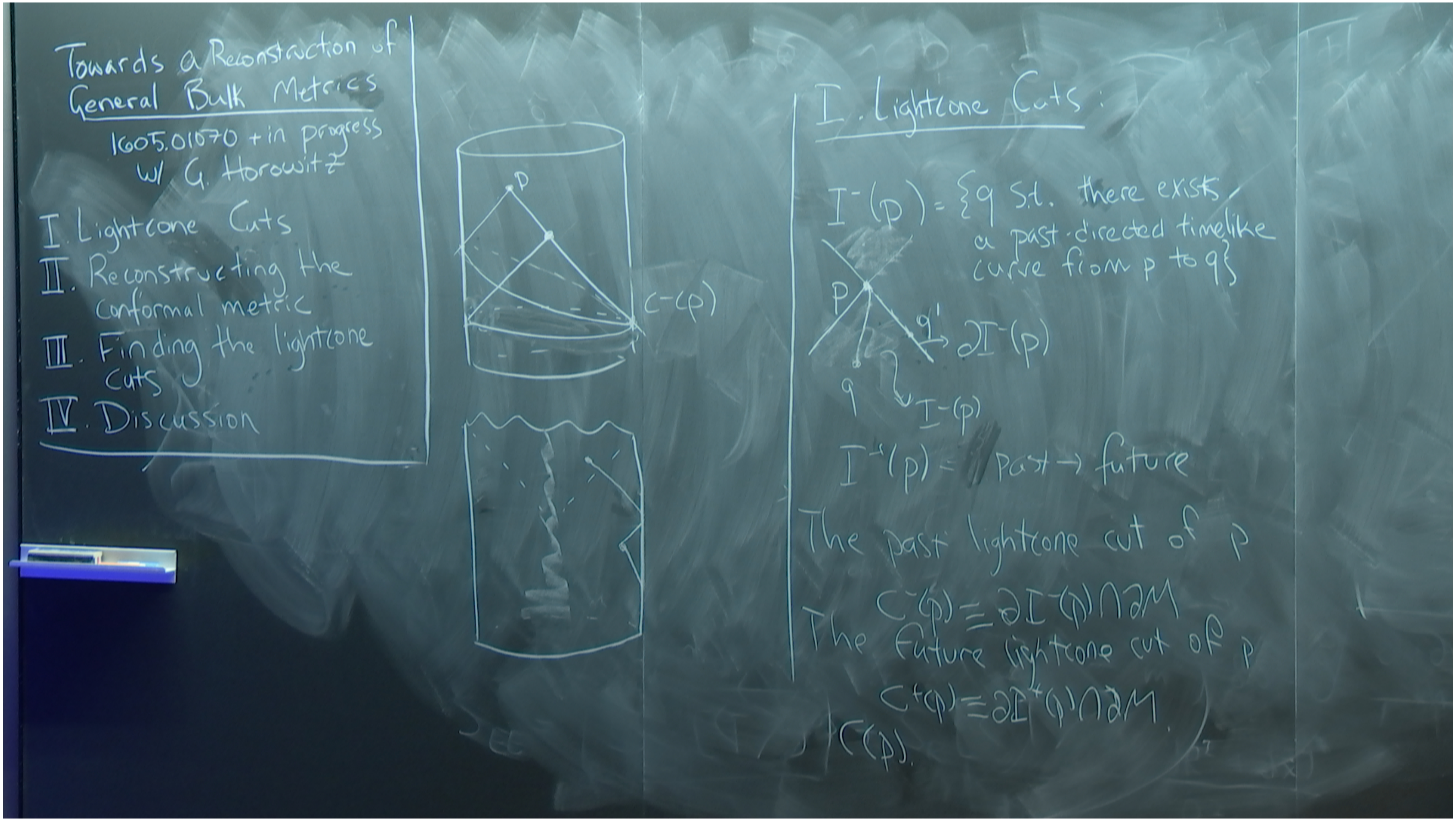
Title: Towards a Reconstruction of General Bulk Metrics

Date: Jul 22, 2016 05:00 PM

URL: http://pirsa.org/16070032

Abstract: I will describe a procedure for reconstructing the metric of a general holographic spacetime (up to an overall conformal factor) from distinguished spatial slices - â€œlight-cone cutsâ€• - of the conformal boundary. This reconstruction can be applied to bulk points in causal contact with the boundary. I will also discuss a prescription for obtaining the light-cone cuts from divergences of correlators in the dual field theory.


Towards a Reconstruction of General Bulk Metrics
1605.01070 + in progress w/ G. Horowitz
I. Lightcone Cuts
II. Reconstructing the conformal metric
III. Finding the lightcone cuts
IV. Discussion
p
C⁻(p)
I. Lightcone Cuts:
I⁻(p) = {q s.t. there exists a past-directed timelike curve from p to q}
p
q'
∂I⁻(p)
q
I⁻(p)
I⁺(p) = past → future
The past lightcone cut of p
C⁻(p) ≡ ∂I⁻(p) ∩ ∂M
The future lightcone cut of p
C⁺(p) ≡ ∂I⁺(p) ∩ ∂M
C(p)

ructure"

e except a

(1) Cut-Point Correspondence

(a) $C(p) = C(q)$ iff $p = q$

(b) If p can receive signals from ∂M

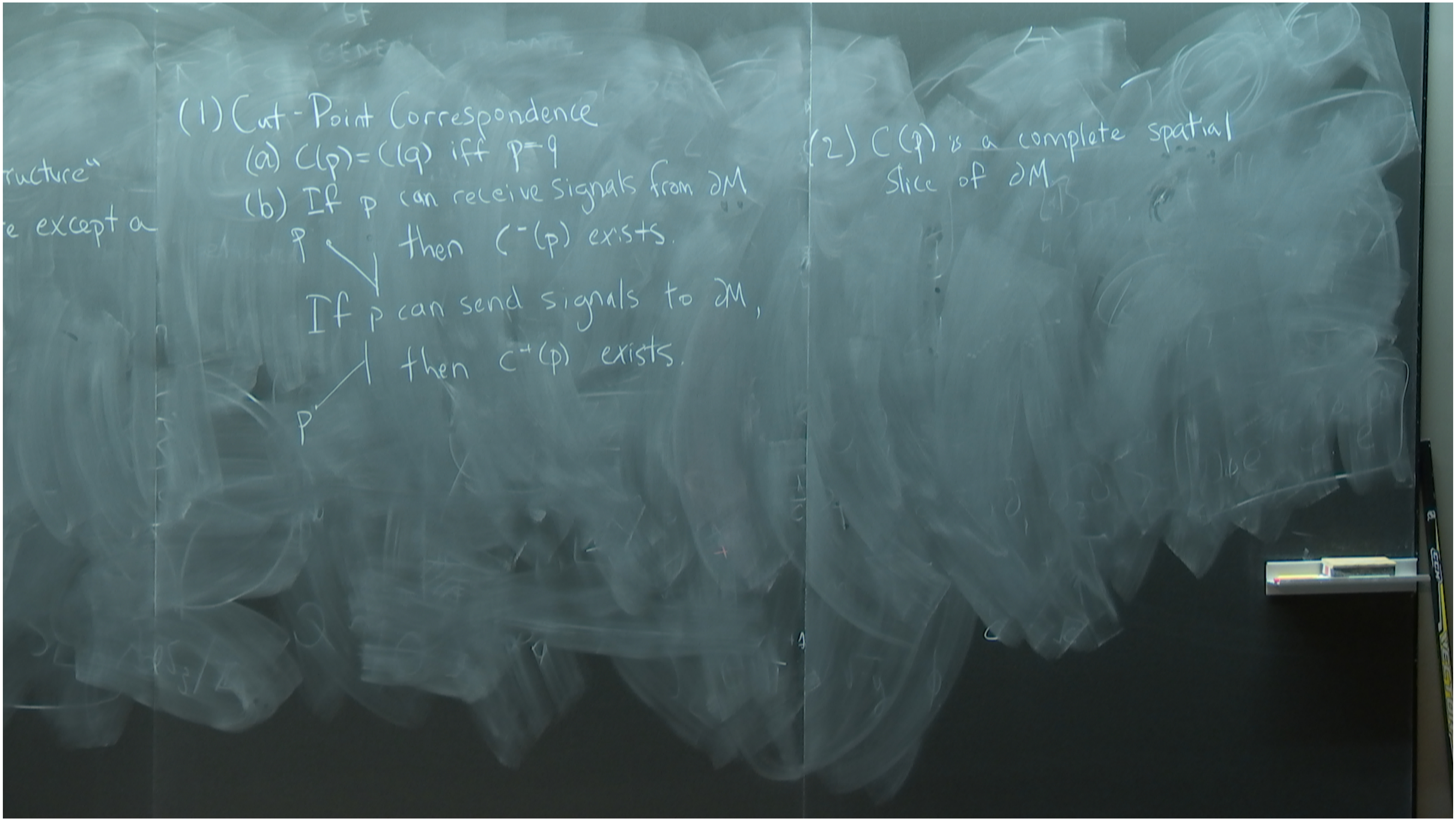
p then $C^-(p)$ exists.

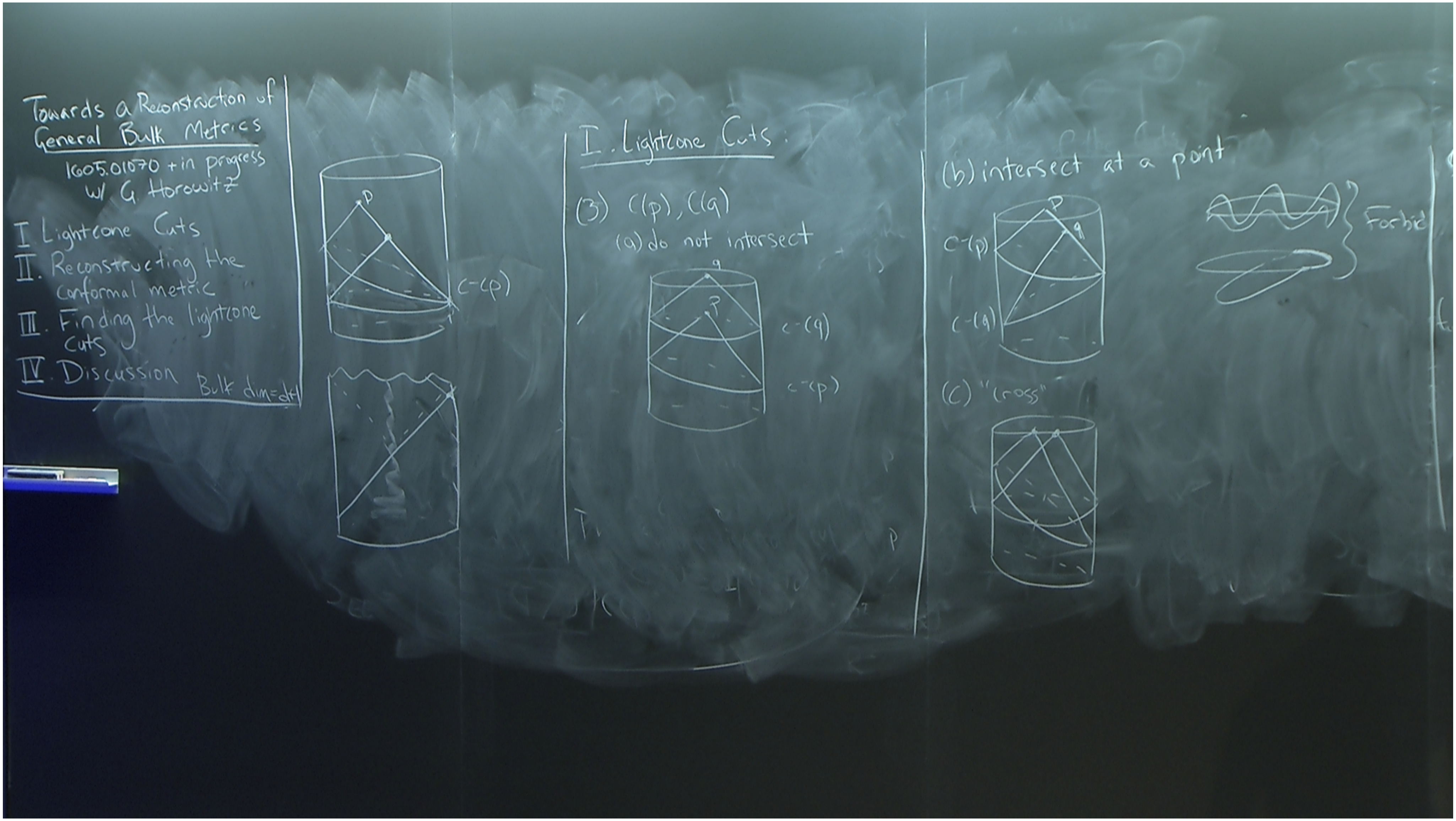
If p can send signals to ∂M,

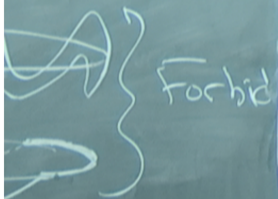
then $C^+(p)$ exists.

p

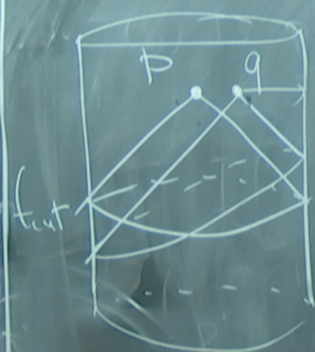
(2) $C(p)$ is a complete spatial slice of ∂M


(1) Cut-Point Correspondence
(a) $C(p) = C(q)$ iff $p = q$
(b) If p can receive signals from ∂M
then $C^-(p)$ exists.
If p can send signals to ∂M,
then $C^+(p)$ exists.
(2) $C(p)$ is a complete spatial
slice of ∂M


Towards a Reconstruction of General Bulk Metrics
1605.01070 + in progress
w/ G Horowitz
I Lightcone Cuts
II. Reconstructing the conformal metric
III. Finding the lightcone cuts
IV. Discussion
Bulk dim=d+1
p
C^-(p)
I. Lightcone Cuts:
(3) C(p), C(q)
(a) do not intersect
q
p
C^-(q)
C^-(p)
(b) intersect at a point
p
q
C^-(p)
C^-(q)
(c) "cross"
Forbid



e.g. Cuts of AdS_{d+1}



$$ds^2_{bulk} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega^2_{d-1}$$

$$ds^2_{bdy} = -dt^2 + d\Omega^2_{d-1}$$

$p: \; r=0, \; t=t_0$

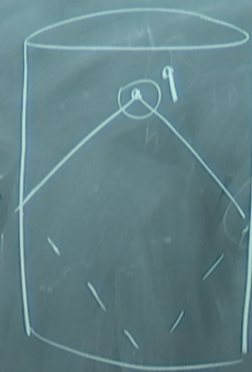
$$t_{cut} = t_0 \pm \frac{\pi}{2}$$

$q: \; t=t_0, \; r=r_0$ along direction X

$$\tan\left[t_{cut} - t_0\right] = \pm \frac{\sqrt{1+r_0^2\sin^2\theta}}{r_0\cos\theta}$$

$r_0 \to 0$

$r_0 \to \infty \qquad t_{cut} = t_0 \pm \theta$



(1) p $\{l_i\}_{i=1}^{d+1}$ are null, indep vectors at p

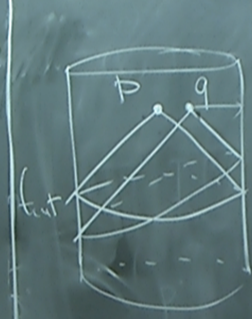
$l_i \cdot l_i = 0$

basis at p $\quad l_i \cdot l_j = ?$

$$\eta_k = \sum_i M_{ki} l_i$$

$$0 = \eta_k \cdot \eta_k = \sum_{i,j} M_{ki} M_{kj} \underbrace{l_i \cdot l_j}_{\text{conformal metric}}$$

e.g. Cuts of AdS_{d+1}



$$ds^2_{bulk} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega^2_{d-1}$$

$$ds^2_{bdy} = -dt^2 + d\Omega^2_{d-1}$$

p: $r=0, t=t_0$

$$t_{cut} = t_0 \pm \frac{\pi}{2}$$

q: $t=t_0$, $r=r_0$ along direction X

$$\tan[t_{cut} - t_0] = \pm \frac{\sqrt{1+r_0^2 \sin^2\theta}}{r_0 \cos\theta}$$

$r_0 \to 0$

$r_0 \to \infty$ $\quad t_{cut} = t_0 \pm \theta$

(1)  $\{l_i\}_{i=1}^{d+1}$ are null, indep vectors at p

$l_i \cdot l_i = 0$

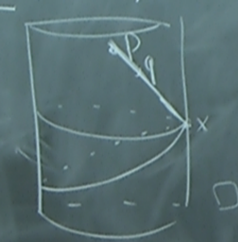
basis at p $\quad l_i \cdot l_j = ?$

$$\eta_k = \sum_i M_{ki}\, l_i$$

$$0 = \eta_k \cdot \eta_k = \sum_{i,j} M_{ki} M_{kj} \underbrace{l_i \cdot l_j}_{\text{conformal metric}}$$

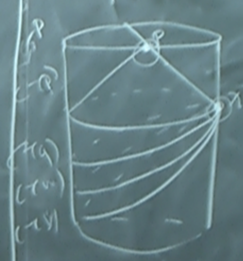
Thm: If $C(p)$, $C(q)$ intersect at one point x and they are C^1 at x, then p and q are null-sep

Pf (sketch).

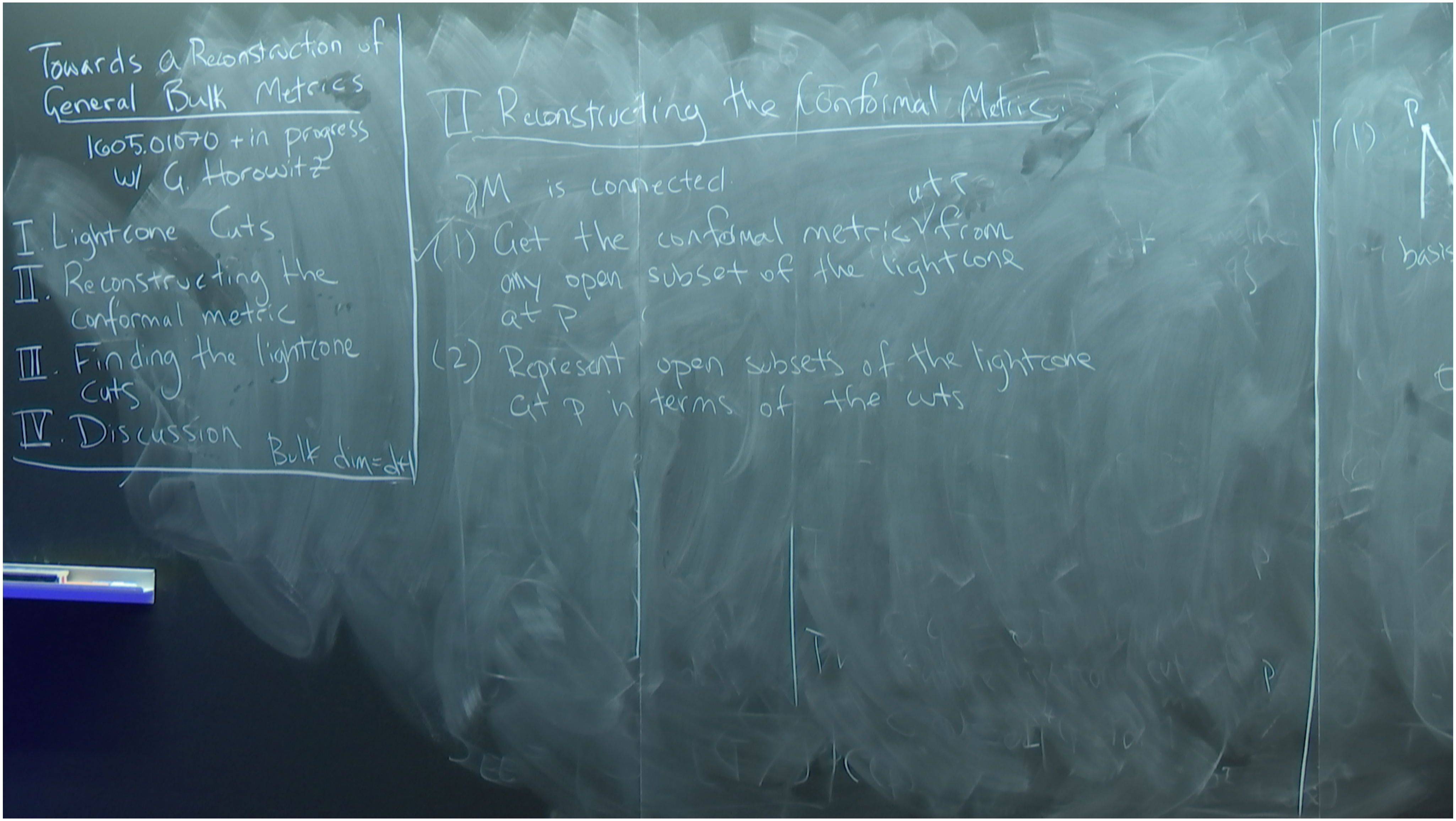


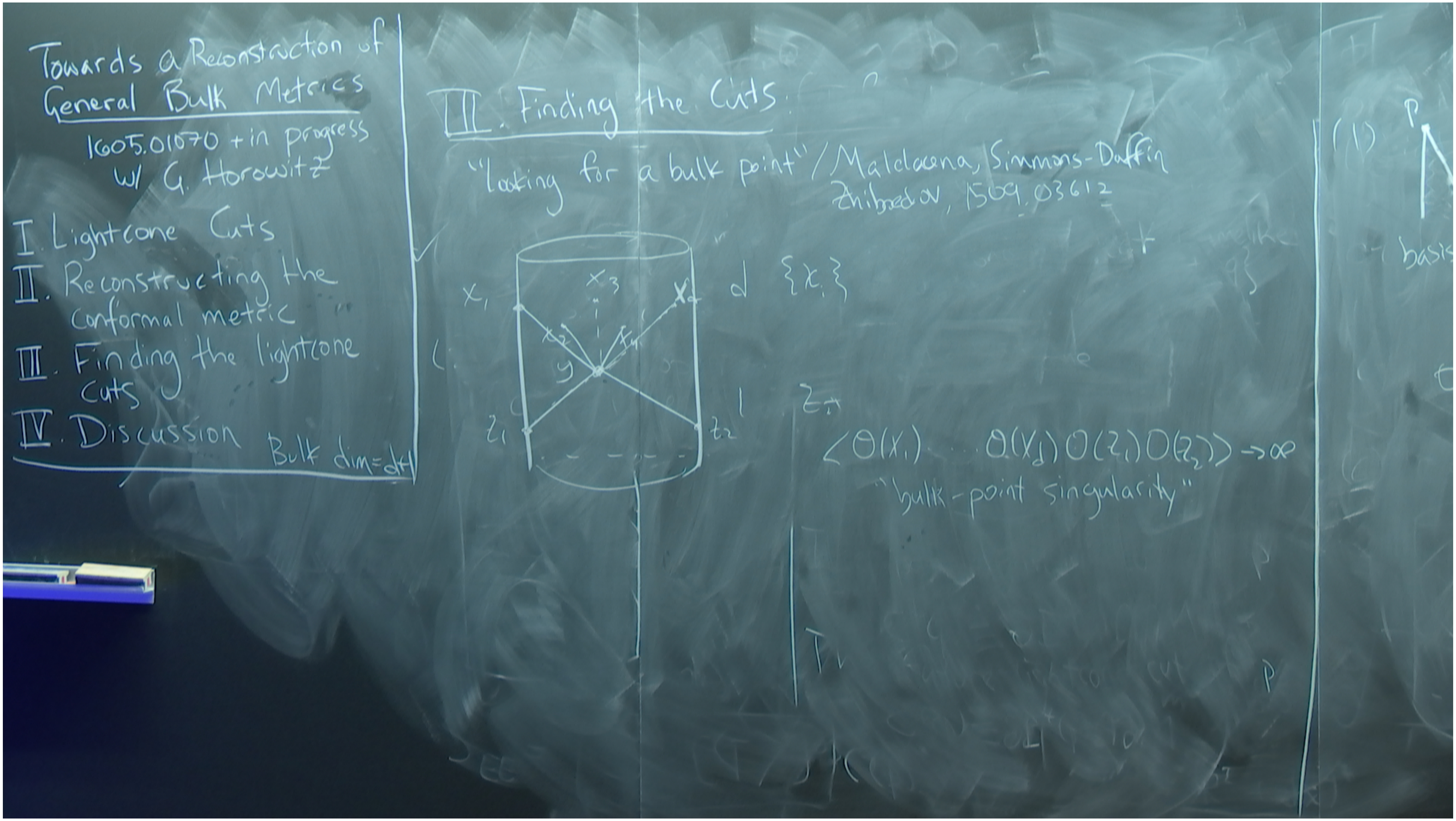
$\square$

M



$\mathcal{M}$ = space of cuts


Towards a Reconstruction of General Bulk Metrics
1605.01070 + in progress
w/ G. Horowitz
I. Lightcone Cuts
II. Reconstructing the conformal metric
III. Finding the lightcone cuts
IV. Discussion
Bulk dim=d+1
II. Reconstructing the Conformal Metric
∂M is connected
(1) Get the conformal metric at p from any open subset of the lightcone at p
(2) Represent open subsets of the lightcone at p in terms of the cuts


Towards a Reconstruction of General Bulk Metrics
1605.01070 + in progress
w/ G. Horowitz
I. Lightcone Cuts
II. Reconstructing the conformal metric
III. Finding the lightcone cuts
IV. Discussion
Bulk dim = d+1
III. Finding the Cuts
"Looking for a bulk point" / Maldacena, Simmons-Duffin, Zhiboedov, 1509.03612
x_1
x_3
x_d
x_2
x_4
y
z_1
z_2
d $\{x_i\}$
$\langle O(x_1) \ldots O(x_d) O(z_1) O(z_2) \rangle \to \infty$
"bulk-point singularity"
basis

Simmons-Duffin
...09.03612

$\mathcal{O}(x_{d+1})$

$\langle \mathcal{O}(x_1) \mathcal{O}(z_1) \mathcal{O}(z_2) \rangle \to \infty$

"...oint singularity"

$\boxed{\text{IV}}$ Discussion:

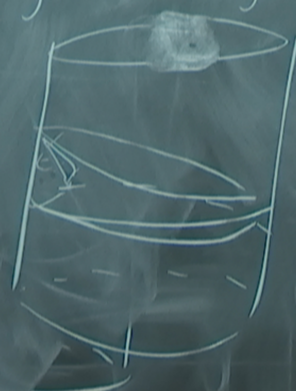
(a) Conformal factor?

If analytic, yes!

Get from entanglement?

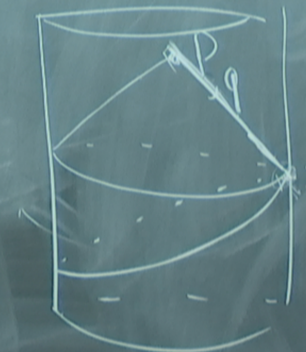
(b) Lightcone cuts outside wedge?

(c) Subregion/Subregion duality

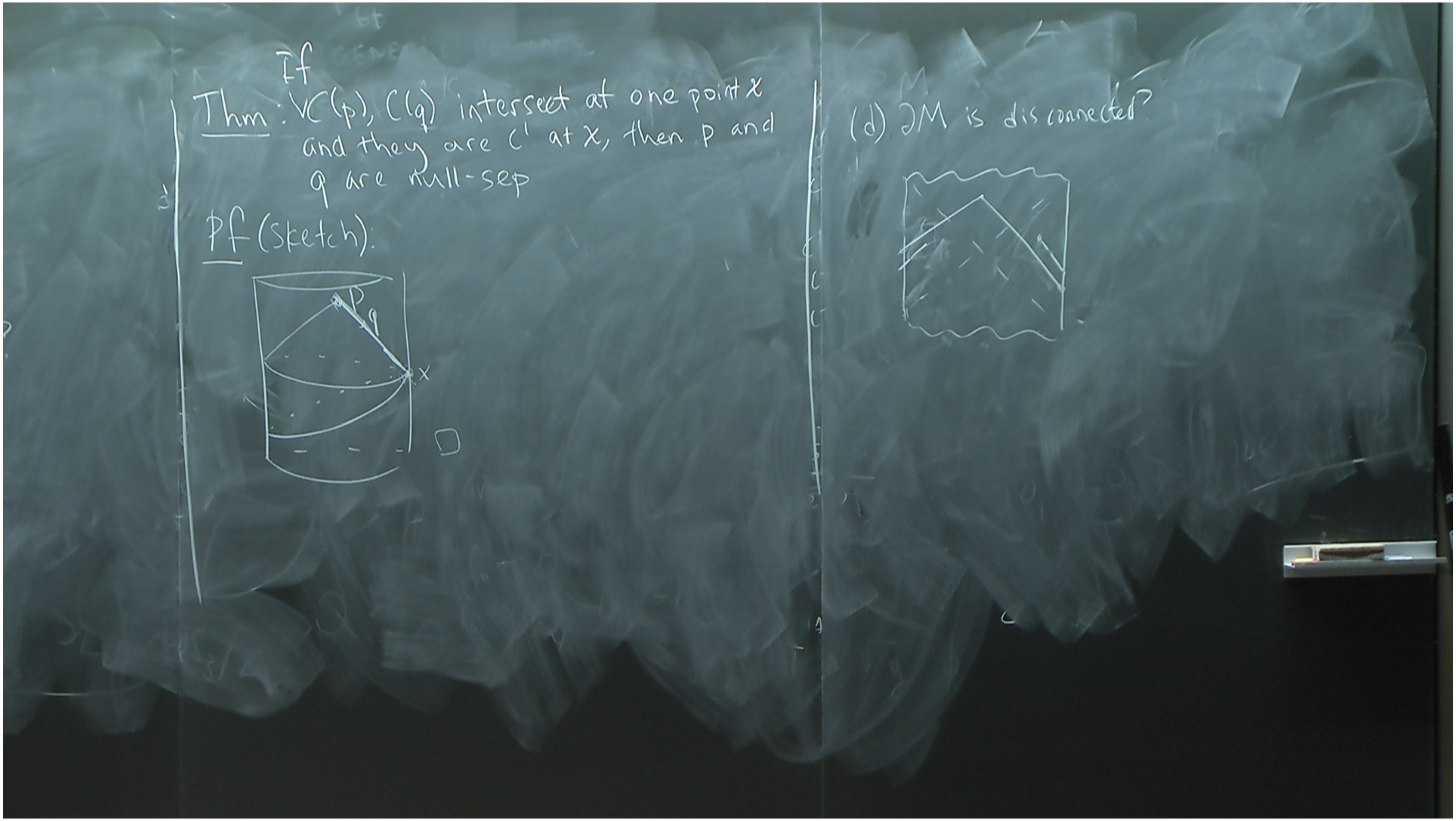


Thm: $\forall C(p), C(q)$ ...
and they ...
q are null ...

Pf (Sketch):



$\square$


Thm: If $\dot{V}C(p)$, $C(q)$ intersect at one point x
and they are C^1 at x, then p and
q are null-sep
Pf (sketch):
p
q
x
(d) ∂M is disconnected?