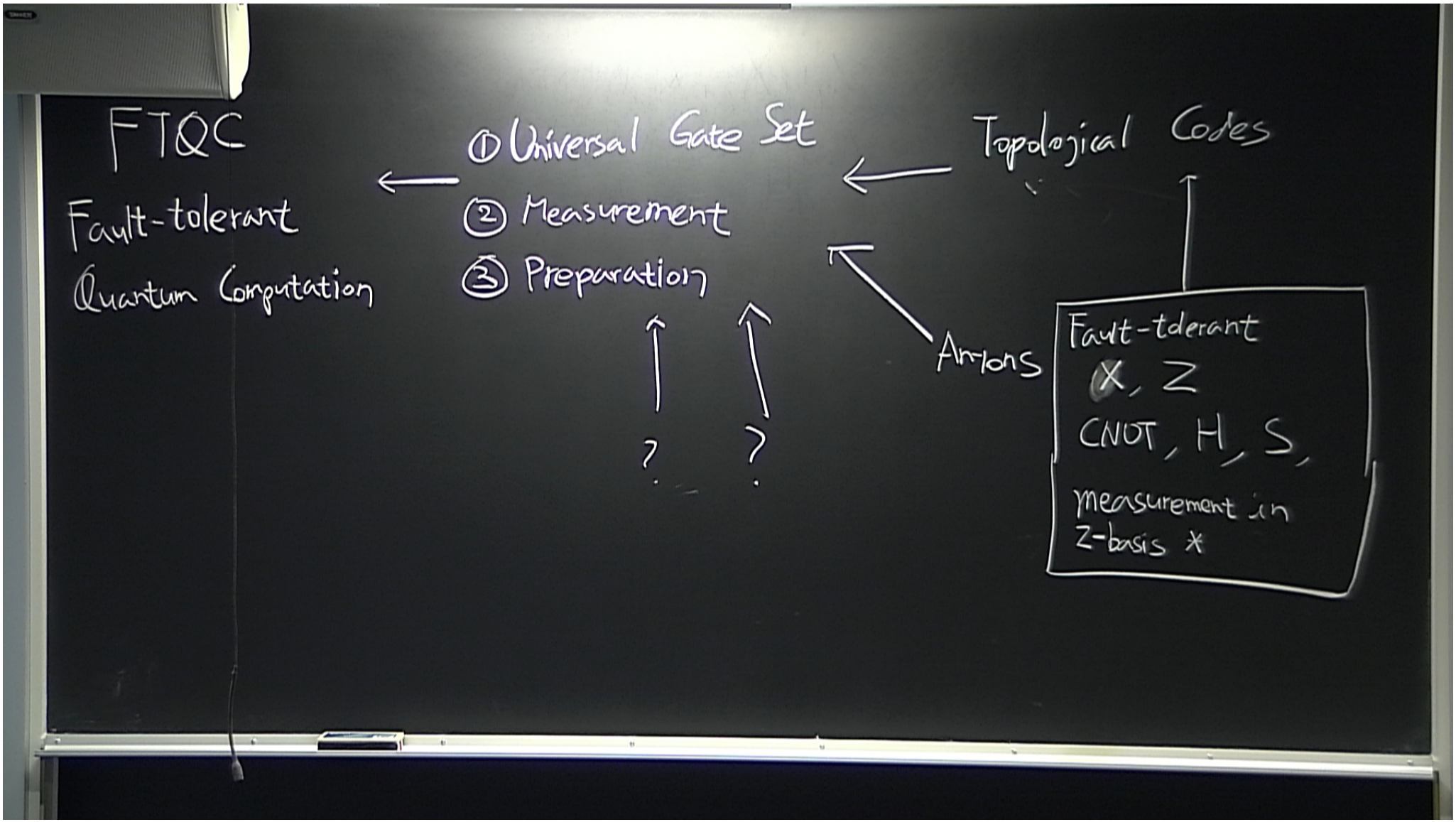
Title: Anyons and Topological Codes

Date: Jul 22, 2016 05:00 PM

URL: http://pirsa.org/16070031

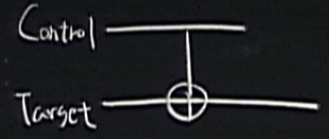
Abstract:


FTQC
Fault-tolerant
Quantum Computation
① Universal Gate Set
② Measurement
③ Preparation
? ?
Topological Codes
Anyons
Fault-tolerant
X, Z
CNOT, H, S,
measurement in
Z-basis *

$P_n = \langle X, Y, Z, I \rangle$

$C_n = \{ G \mid G P_n G^+ \}$

CNOT



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$CNOT_{1\to 2}(X \otimes I) CNOT_{1\to 2} = X \otimes X$

$CNOT_{1\to 2}(I \otimes X) CNOT_{1\to 2} = I \otimes X$

$CNOT_{1\to 2}(Z \otimes I) CNOT_{1\to 2} = Z \otimes I$

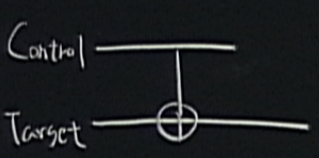
$CNOT_{1\to 2}(I \otimes Z) CNOT_{1\to 2} = Z \otimes Z$

H:

S:

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$CNOT_{1\to 2}(I\otimes Z)CNOT_{1\to 2} = Z\otimes Z$

H: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $HXH = Z$

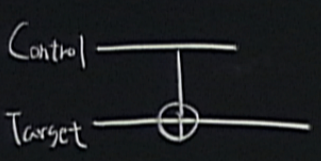
$HZH = X$

S. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ $Y = iXZ$ $HYH = iHXH\,HZH$

$= iZX = -Y$

$C_n = \{G \mid G P_n G^\dagger\}$

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$HXH = Z$

$HZH = X$

$Y = iXZ$

$HYH = iHXH\,HZH$

$= iZX = -Y$

$S: \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

$S^2 = Z$

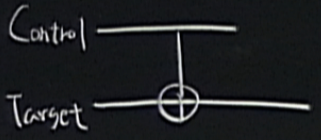
$SXS^\dagger = Y$

$S^\dagger YS = X$

$P_n = \langle X, Y, Z, I \rangle$

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CNOT



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$Y = iXZ$ $\quad HYH = iHXH\,HZH$

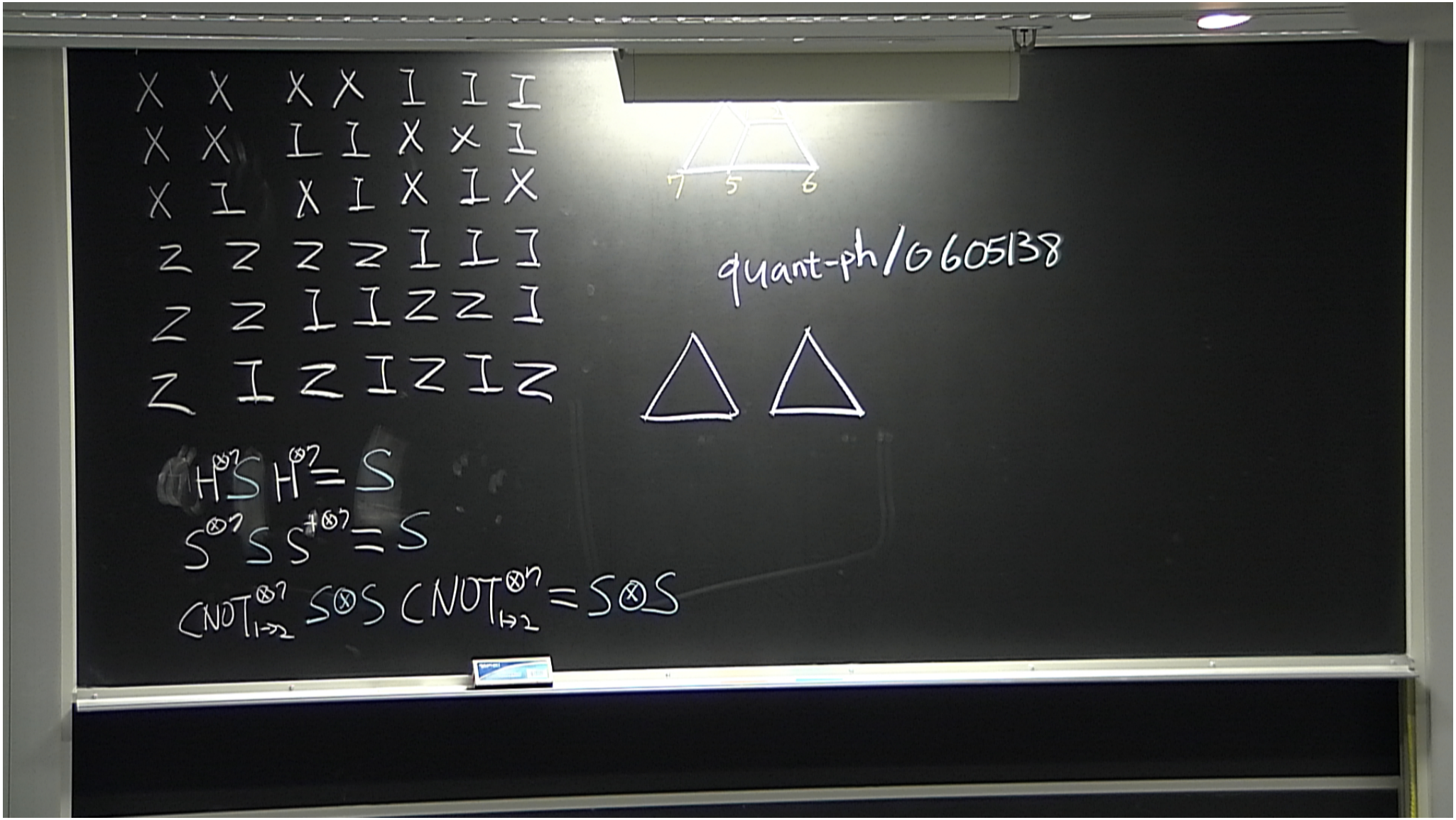
$= iZX = -Y$

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$SXS^\dagger = Y$

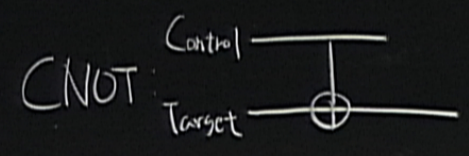
$S^\dagger Y S = X$


X X X X I I I
X X I I X X I
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7 5 6
quant-ph/0605138
H⊗7 S H⊗7 = S
S⊗7 S S†⊗7 = S
CNOT1→2⊗7 S⊗S CNOT1→2⊗7 = S⊗S

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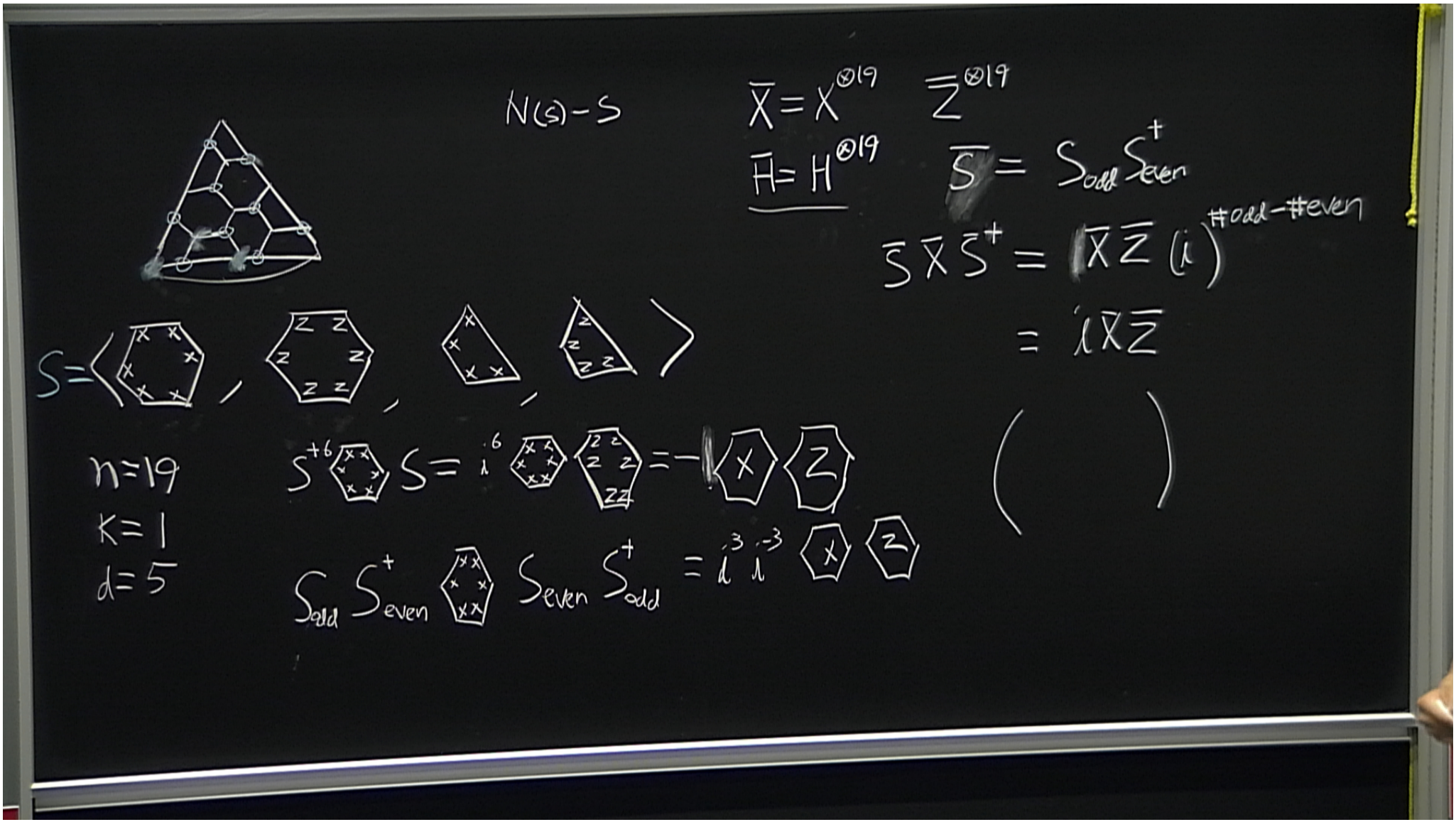
$S^2 = Z$

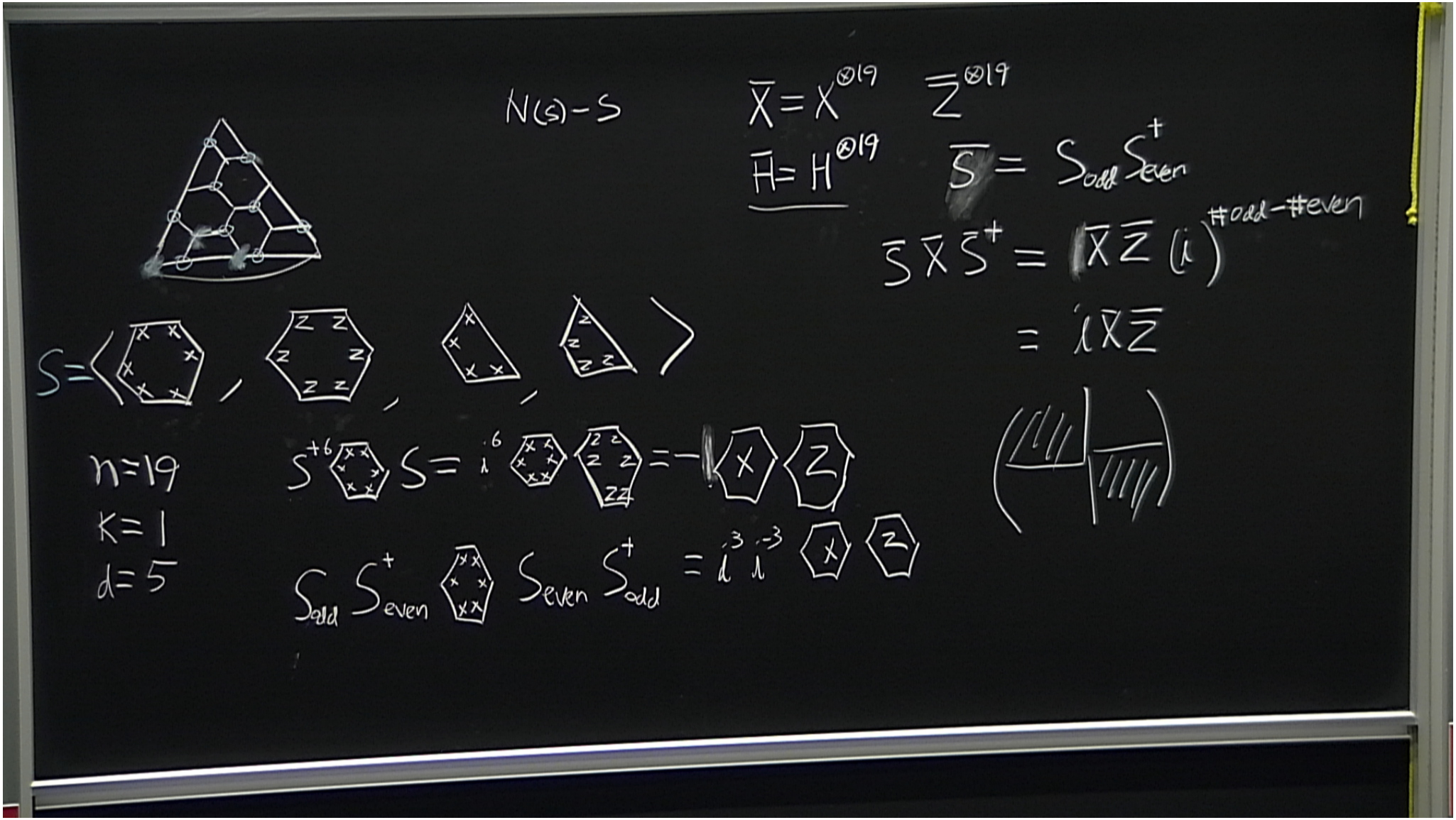
$Y = iXZ \quad HYH = iHXH\,HZH$

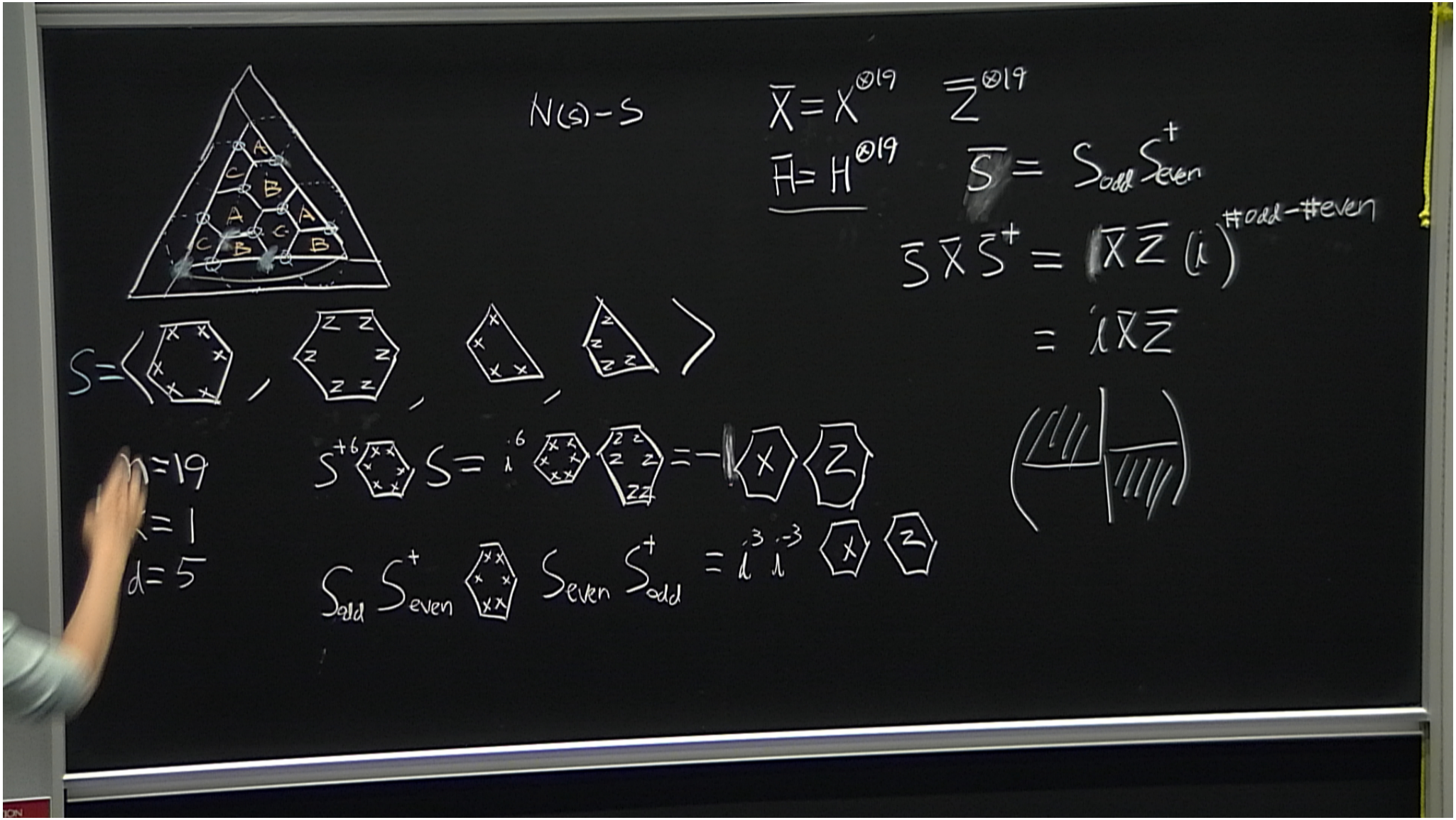
$= iZX = -Y$

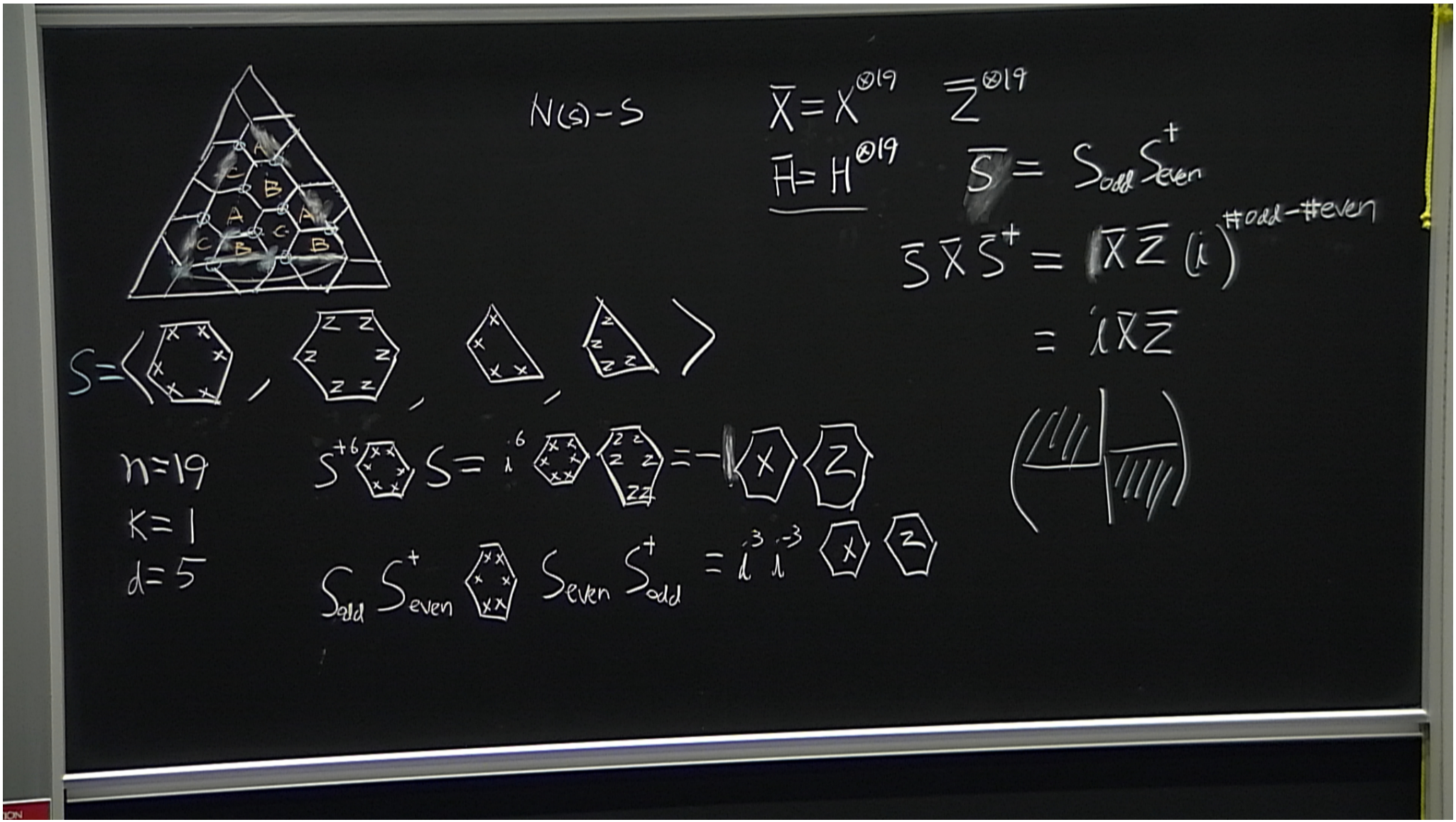
$SXS^\dagger = Y = iXZ$

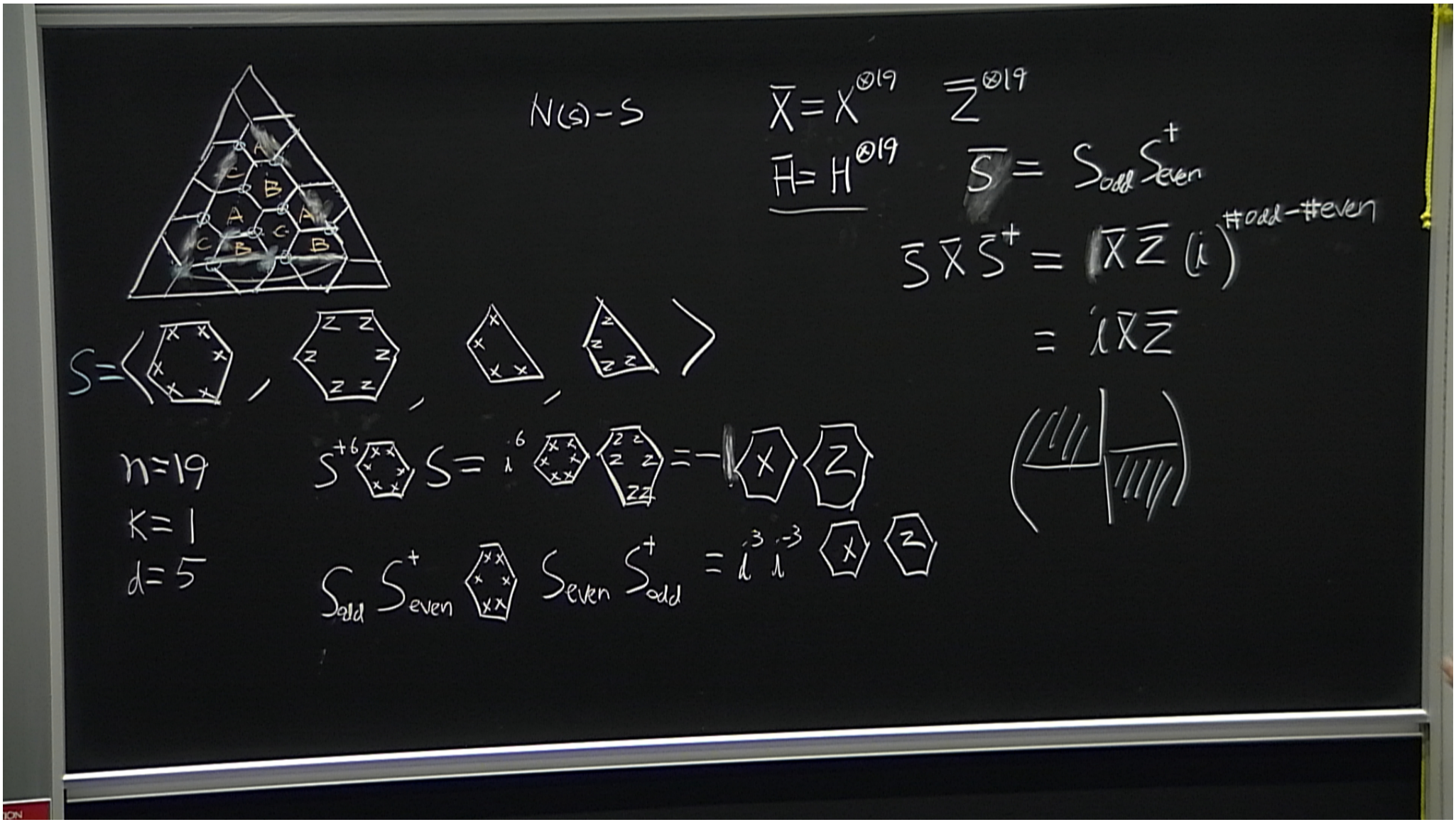
$S^\dagger YS = X$

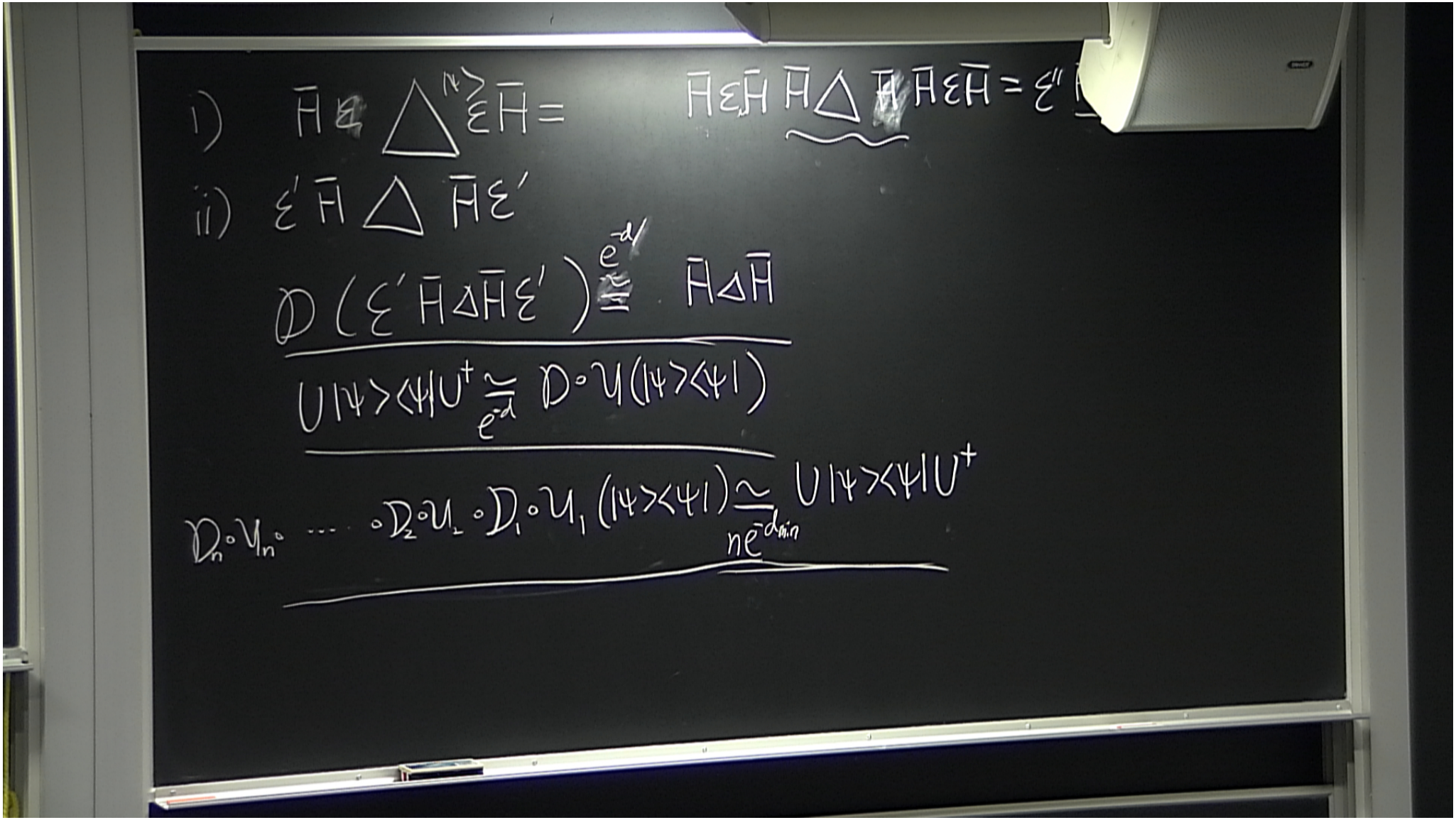

N(S)-S
$\bar{X} = X^{\otimes 19}$ $\bar{Z}^{\otimes 19}$
$\bar{H} = H^{\otimes 19}$
$\bar{S} = S_{odd} S_{even}^{+}$
$\bar{S}\bar{X}\bar{S}^{+} = \bar{X}\bar{Z}(i)^{\#odd - \#even}$
$= i\bar{X}\bar{Z}$
$S = \langle$, , , $\rangle$
$n = 19$
$k = 1$
$d = 5$
$S^{+6} S = i^{6}$ $= -1$
$S_{odd} S_{even}^{+} S_{even} S_{odd}^{+} = i^{3} i^{-3}$

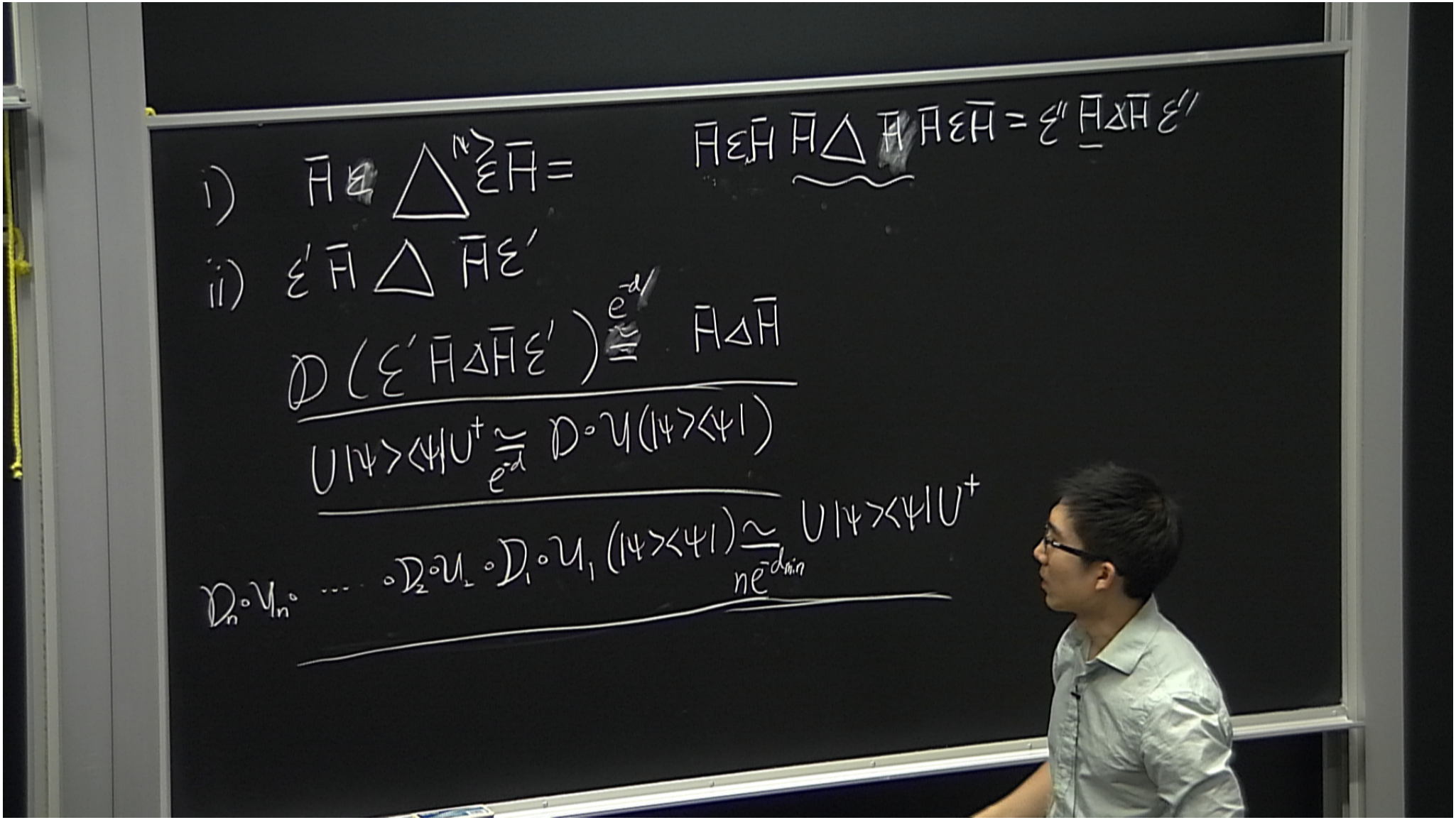

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i) $\bar{\Pi}\mathcal{E}\,\triangle^{(n)}\,\mathcal{E}\bar{\Pi} =$ $\bar{\Pi}\mathcal{E}\bar{\Pi}\,\bar{\Pi}\triangle\bar{\Pi}\,\bar{\Pi}\mathcal{E}\bar{\Pi} = \mathcal{E}''$
ii) $\mathcal{E}'\bar{\Pi}\triangle\bar{\Pi}\mathcal{E}'$
$\mathcal{D}(\mathcal{E}'\bar{\Pi}\triangle\bar{\Pi}\mathcal{E}') \overset{e^{-d}}{\simeq} \bar{\Pi}\triangle\bar{\Pi}$
$U|\psi\rangle\langle\psi|U^\dagger \overset{e^{-\varepsilon}}{\simeq} \mathcal{D}\circ\mathcal{U}(|\psi\rangle\langle\psi|)$
$\mathcal{D}_n\circ\mathcal{U}_n\circ\cdots\circ\mathcal{D}_2\circ\mathcal{U}_2\circ\mathcal{D}_1\circ\mathcal{U}_1(|\psi\rangle\langle\psi|) \underset{ne^{-d_{min}}}{\simeq} U|\psi\rangle\langle\psi|U^\dagger$


i) $\bar{A}\varepsilon\bar{A}\Delta^{(n)}\varepsilon\bar{A} =$
$\bar{A}\varepsilon\bar{A}\,\bar{A}\Delta\bar{A}\,\bar{A}\varepsilon\bar{A} = \varepsilon''\bar{A}\Delta\bar{A}\varepsilon''$
ii) $\varepsilon'\bar{A}\Delta\bar{A}\varepsilon'$
$\mathcal{D}(\varepsilon'\bar{A}\Delta\bar{A}\varepsilon') \overset{e^{-d}}{\simeq} \bar{A}\Delta\bar{A}$
$U|\psi\rangle\langle\psi|U^\dagger \overset{e^{-d}}{\simeq} \mathcal{D}\circ\mathcal{U}(|\psi\rangle\langle\psi|)$
$\mathcal{D}_n\circ\mathcal{U}_n\circ\cdots\circ\mathcal{D}_2\circ\mathcal{U}_2\circ\mathcal{D}_1\circ\mathcal{U}_1(|\psi\rangle\langle\psi|) \underset{ne^{-d_{min}}}{\simeq} U|\psi\rangle\langle\psi|U^\dagger$