Title: AdS/CFT correspondence

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Abstract:

AdS/CFT

Quantum gravity in asymptotically AdS spacetimes is dual to a CFT.

Motivation: look @ dynamics of simple matrix model & see that it has a simple "master field" in a planar limit.

$\Phi^a{}_b$ adj $U(N)$

$\mathcal{L} = \mathrm{Tr}(\Phi^2) + \mathrm{Tr}(\Phi^3) + \mathrm{Tr}(\Phi^4)$

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$$\mathcal{L} = \frac{N}{g^2 N}\left[\mathrm{Tr}(\Phi^2) + \mathrm{Tr}(\Phi^3) + \mathrm{Tr}(\Phi^4)\right]$$

quantum dynamics.

$$\int [\mathcal{D}\Phi]\, e^{\frac{i}{\hbar}\mathcal{L}}$$

Claim: This quantum theory is classical in $N \to \infty$ limit $\lambda = g^2 N$ fixed. w/

$$\hbar_{\text{eff}} = \frac{\hbar}{N}.$$

further, classicality $\Rightarrow$ correlators factors.

ξ

$$\langle \Phi_1 \cdots \Phi_n \rangle \sim \langle \Phi_1 \Phi_2 \rangle \cdots \langle \Phi_{n-1} \Phi_n \rangle$$

Gauge theories like QCD. w/ admit matrix dof simplify in the 't Hooft limit $N \to \infty$. $g^2 N$: fixed

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further, classicality $\Rightarrow$ correlators factors.

$$\mathrm{Tr}(\Phi \cdots \Phi) \quad \text{if} \quad \langle \hat{\Phi}_1 \cdots \hat{\Phi}_n \rangle \sim \langle \hat{\Phi}_1 \hat{\Phi}_2 \rangle \cdots \langle \hat{\Phi}_{n-1} \hat{\Phi}_n \rangle$$

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$\mathcal{D}\Phi]\, e^{\frac{i}{\hbar}\mathcal{L}}$

quantum theory is
$N \to \infty$ limit
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$\hbar_{\text{eff}} = \frac{\hbar}{N}$

further, classicality $\Rightarrow$
correlators factors

$\mathrm{Tr}(\Phi \quad \Phi)$
$=$
$\langle \hat{\Phi}_{n-1} \hat{\Phi}_n \rangle$

$\langle \hat{\Phi}_1 \quad \hat{\Phi}_n \rangle \sim \langle \hat{\Phi}_1 \hat{\Phi}_2 \rangle \quad \langle \hat{\Phi}_{n-1} \hat{\Phi}_n \rangle$

Gauge theories like QCD
w/ admit matrix dof simplify
in the 't Hooft limit $N \to \infty$
$g^2 N$ fixed

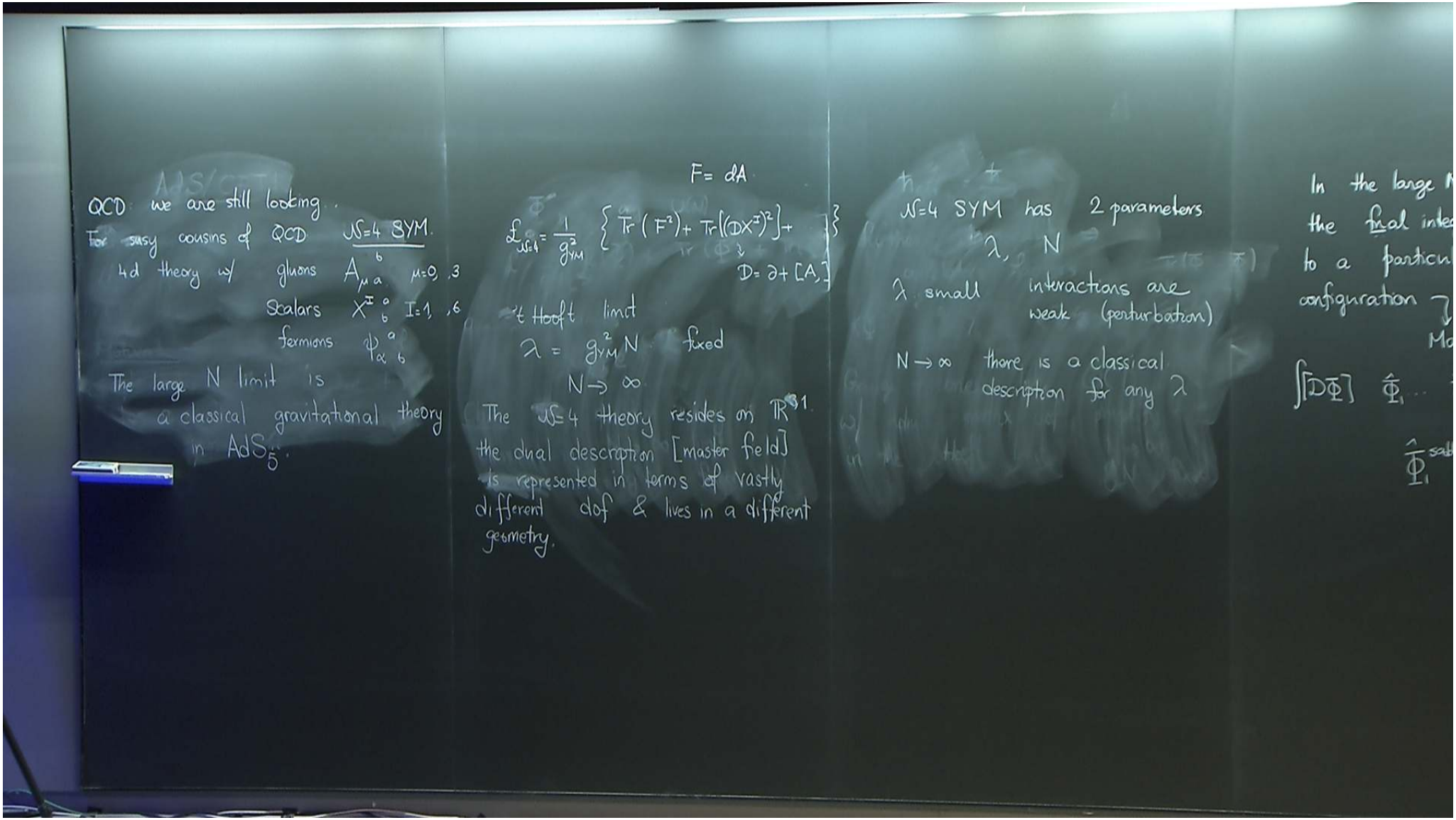
In the large N limit
the final integral localizes
to a particular classical Φ
configuration

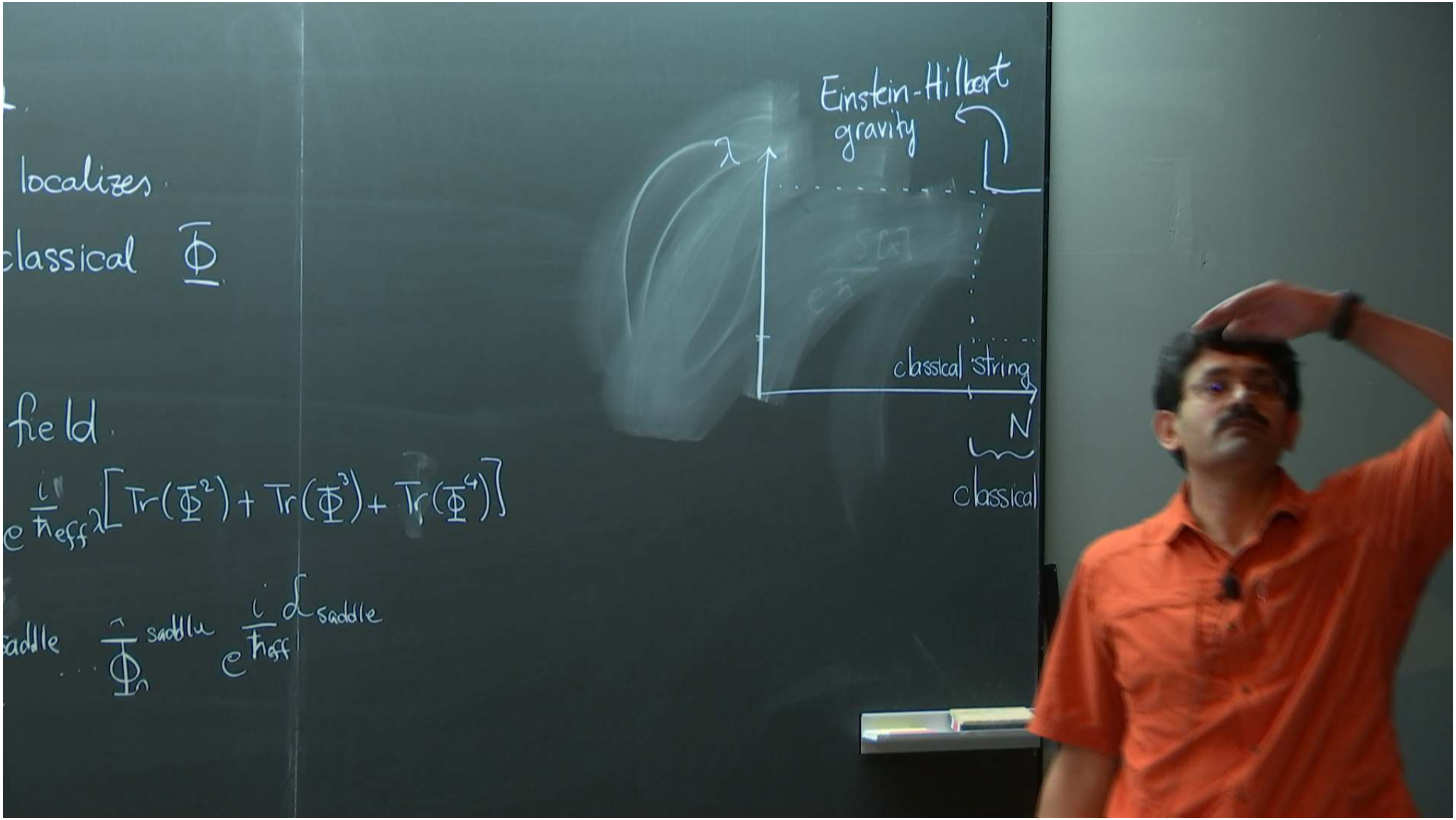
Master field

$$\int [\mathcal{D}\Phi]\ \hat{\Phi}_1 \cdots \hat{\Phi}_n\ e^{\frac{i}{\hbar_{\text{eff}}} \mathcal{L}\left[\mathrm{Tr}(\Phi^2) + \mathrm{Tr}(\Phi^3) + \mathrm{Tr}(\Phi^4)\right]}$$

$$\hat{\Phi}_1^{\text{saddle}}\ \hat{\Phi}_2^{\text{saddle}} \cdots \hat{\Phi}_n^{\text{saddle}}\ e^{\frac{i}{\hbar_{\text{eff}}} \mathcal{L}_{\text{saddle}}}$$

$e^{\frac{i}{\hbar} S[x]}$


QCD we are still looking.
For susy cousins of QCD $\mathcal{N}=4$ SYM.
4d theory w/ gluons $A_{\mu\, a}^{\ b}$ $\mu=0,\dots,3$
Scalars $X^I{}_b^a$ $I=1,\dots,6$
fermions $\psi_{\alpha\, b}^{\ a}$
The large N limit is
a classical gravitational theory
in AdS_5.
$F = dA$
$\mathcal{L}_{\mathcal{N}=4} = \frac{1}{g_{YM}^2}\left\{ \mathrm{Tr}\,(F^2) + \mathrm{Tr}\left[(DX^I)^2\right] + \dots \right\}$
$D = \partial + [A,\ ]$
't Hooft limit
$\lambda = g_{YM}^2 N$ fixed
$N \to \infty$
The $\mathcal{N}=4$ theory resides on $\mathbb{R}^{3,1}$
the dual description [master field]
is represented in terms of vastly
different dof & lives in a different
geometry.
$\mathcal{N}=4$ SYM has 2 parameters
λ, N
λ small interactions are
weak (perturbation)
$N \to \infty$ there is a classical
description for any λ
In the large N
the final inte
to a particu
configuration
$\int [D\Phi]\ \hat{\Phi}_1 \dots$
$\hat{\Phi}_1$


localizes
classical $\bar{\Phi}$
field
$e^{\frac{i}{\hbar_{eff}}\lambda\left[\mathrm{Tr}(\Phi^2)+\mathrm{Tr}(\Phi^3)+\mathrm{Tr}(\Phi^4)\right]}$
saddle $\bar{\Phi}_n$ saddle $e^{\frac{i}{\hbar_{eff}}S_{saddle}}$
Einstein-Hilbert gravity
λ
classical string
N
classical

$F = dA$

$\mathrm{Tr}(F^2) + \mathrm{Tr}[(DX^I)^2] + \ldots$

$D = \partial + [A, \cdot]$

...nct

...N fixed

∞

...heory resides on $\mathbb{R}^{3,1}$

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λ small interactions are weak (perturbation)

$N \to \infty$ there is a classical description for any λ

quantum

$SU(N)$ $\mathcal{N}=4$ SYM (λ, N)

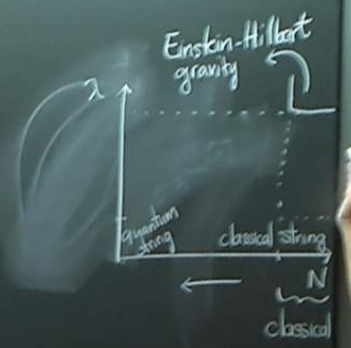
quantum

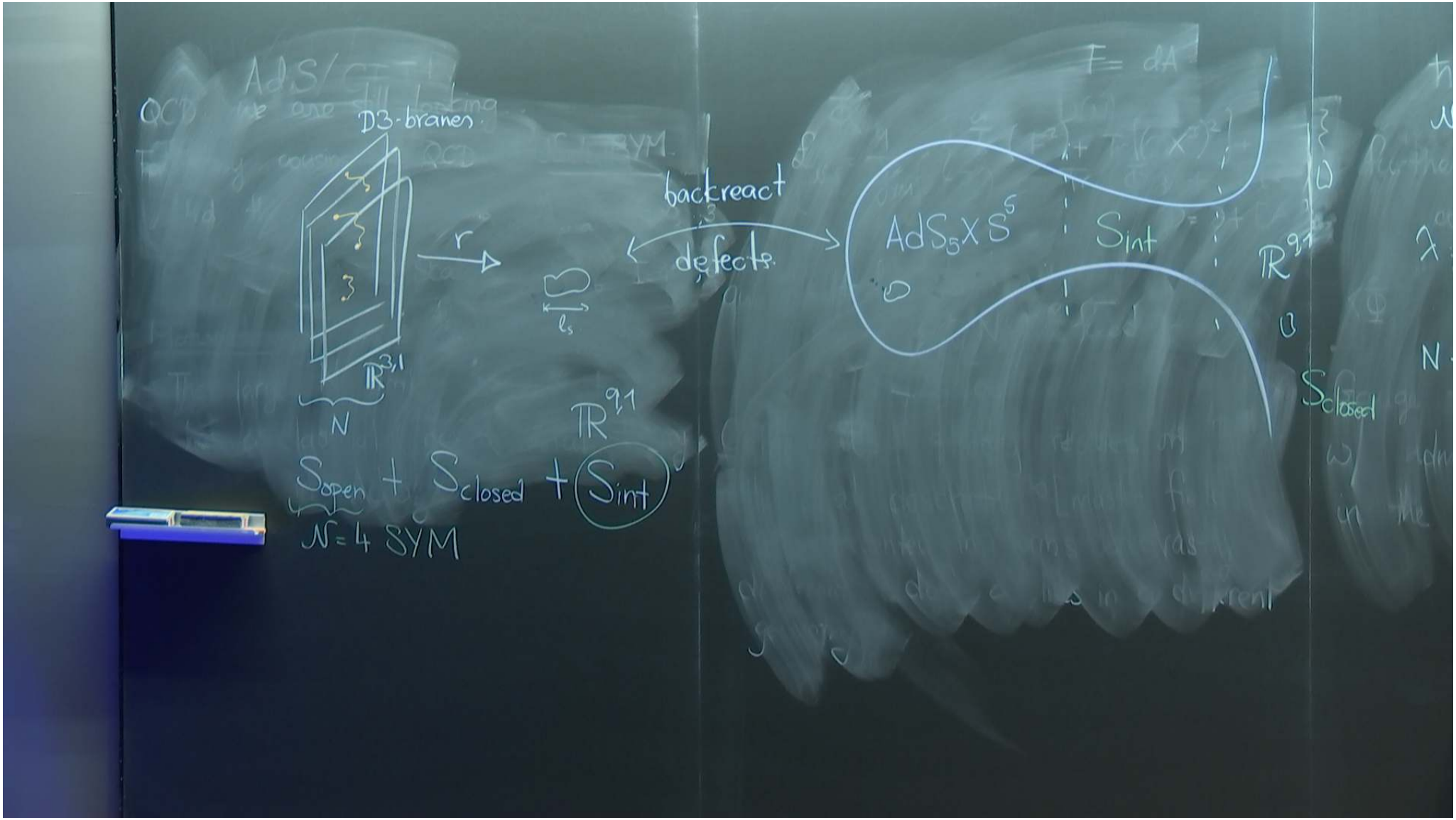
String theory in $AdS_5 \times S^5$

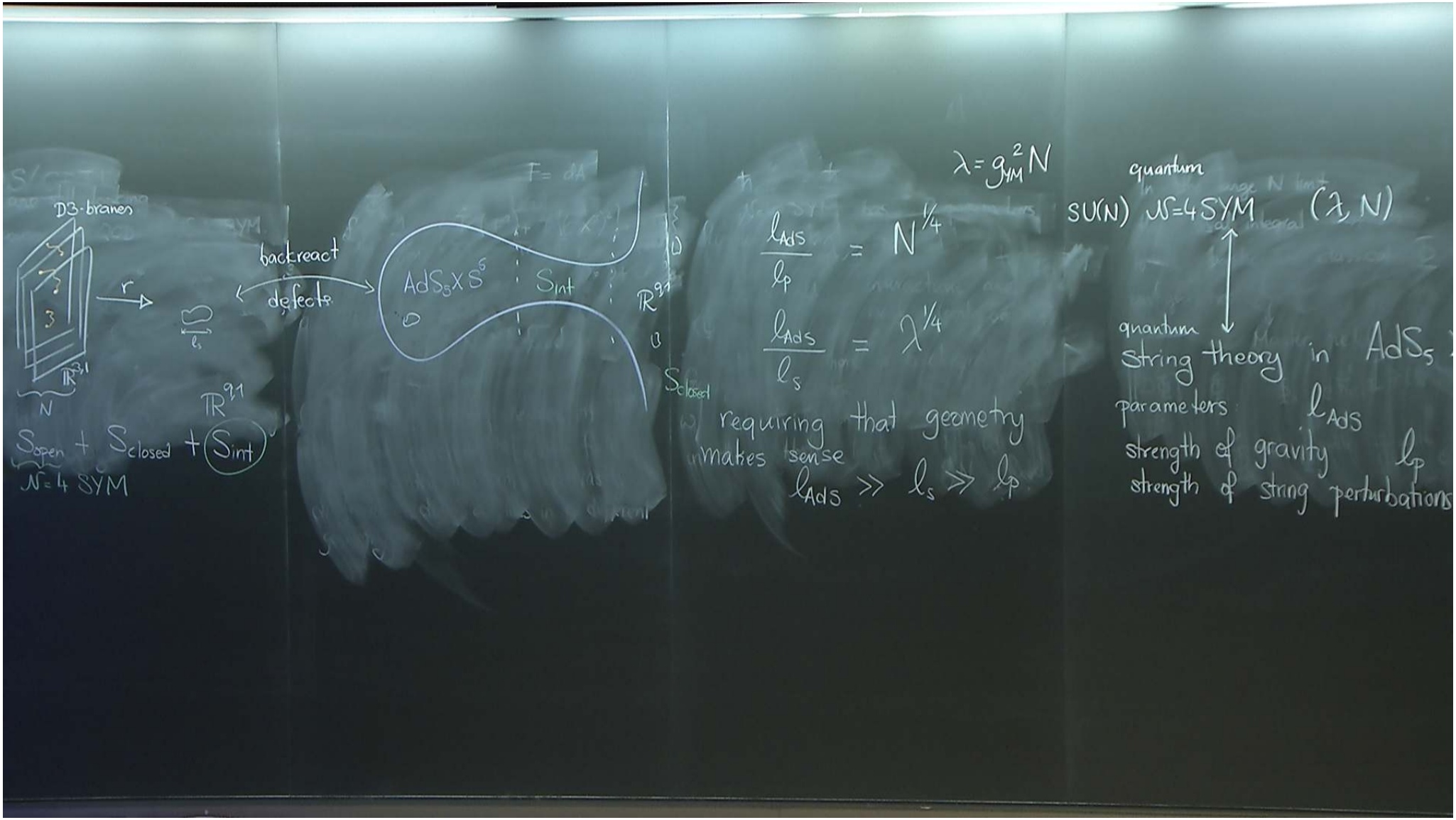
parameters ℓ_{AdS}

strength of gravity ℓ_p (Planck scale)

strength of string perturbations: ℓ_s (string scale)




AdS/
QCD
D3-branes
r
backreact
defects
$AdS_5 \times S^5$
S_{int}
$\mathbb{R}^{3,1}$
$\mathbb{R}^{9,1}$
N
$S_{open} + S_{closed} + S_{int}$
$\mathcal{N}=4$ SYM
S_{closed}
$F = dA$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$\mathcal{N}=4$ SYM
$\lambda = g_{YM}^2 N$
$\frac{l_{AdS}}{l_p} = N^{1/4}$
$\frac{l_{AdS}}{l_s} = \lambda^{1/4}$
requiring that geometry makes sense
$l_{AdS} \gg l_s \gg l_p$
quantum
SU(N) $\mathcal{N}=4$ SYM (λ, N)
quantum
string theory in AdS_5
parameters l_{AdS}
strength of gravity l_p
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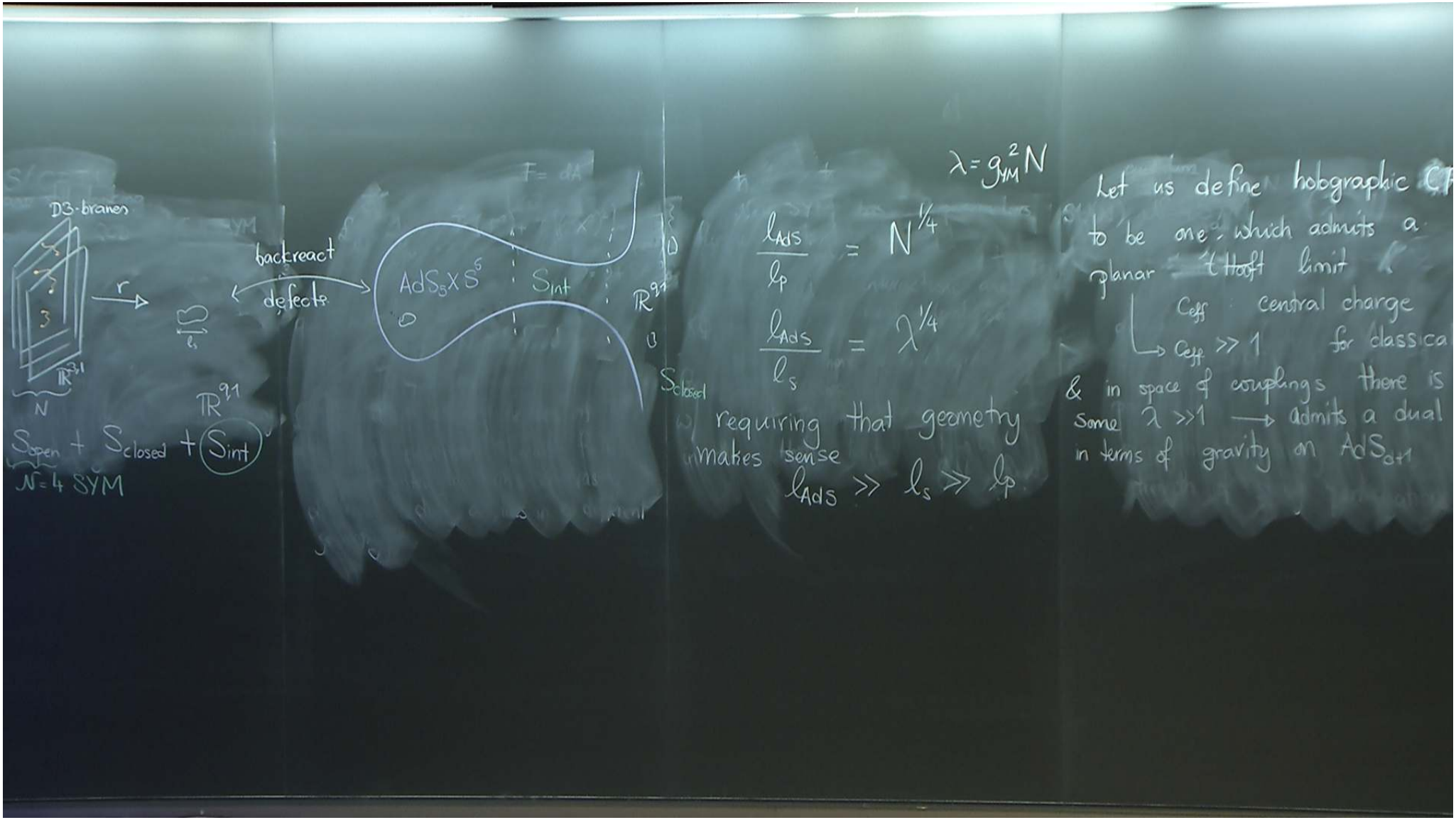
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requiring that geometry makes sense
$\ell_{AdS} \gg \ell_s \gg \ell_p$
Let us define holographic CF
to be one which admits a
planar 't Hooft limit
C_{eff} central charge
$C_{eff} \gg 1$ for classica
& in space of couplings there is
some $\lambda \gg 1$ → admits a dual
in terms of gravity on AdS_{d+1}

$\lambda = g_{YM}^2 N$

Let us define holographic CFT_d to be one which admits a planar 't Hooft limit

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& in space of couplings there is some $\lambda \gg 1$ $\longrightarrow$ admits a dual in terms of gravity on AdS_{d+1}

$$S_{grav} = \frac{1}{16\pi G_N^{(d+1)}} \int d^{d+1}x \sqrt{-g}\left[R - 2\Lambda + \dots\right]$$

$$C_{eff} = \frac{1}{16\pi G_N^{(d+1)}}$$

i) CFT_d should have few low energy states (sparse low lying spectrum)

$\lambda = g_{YM}^2 N$

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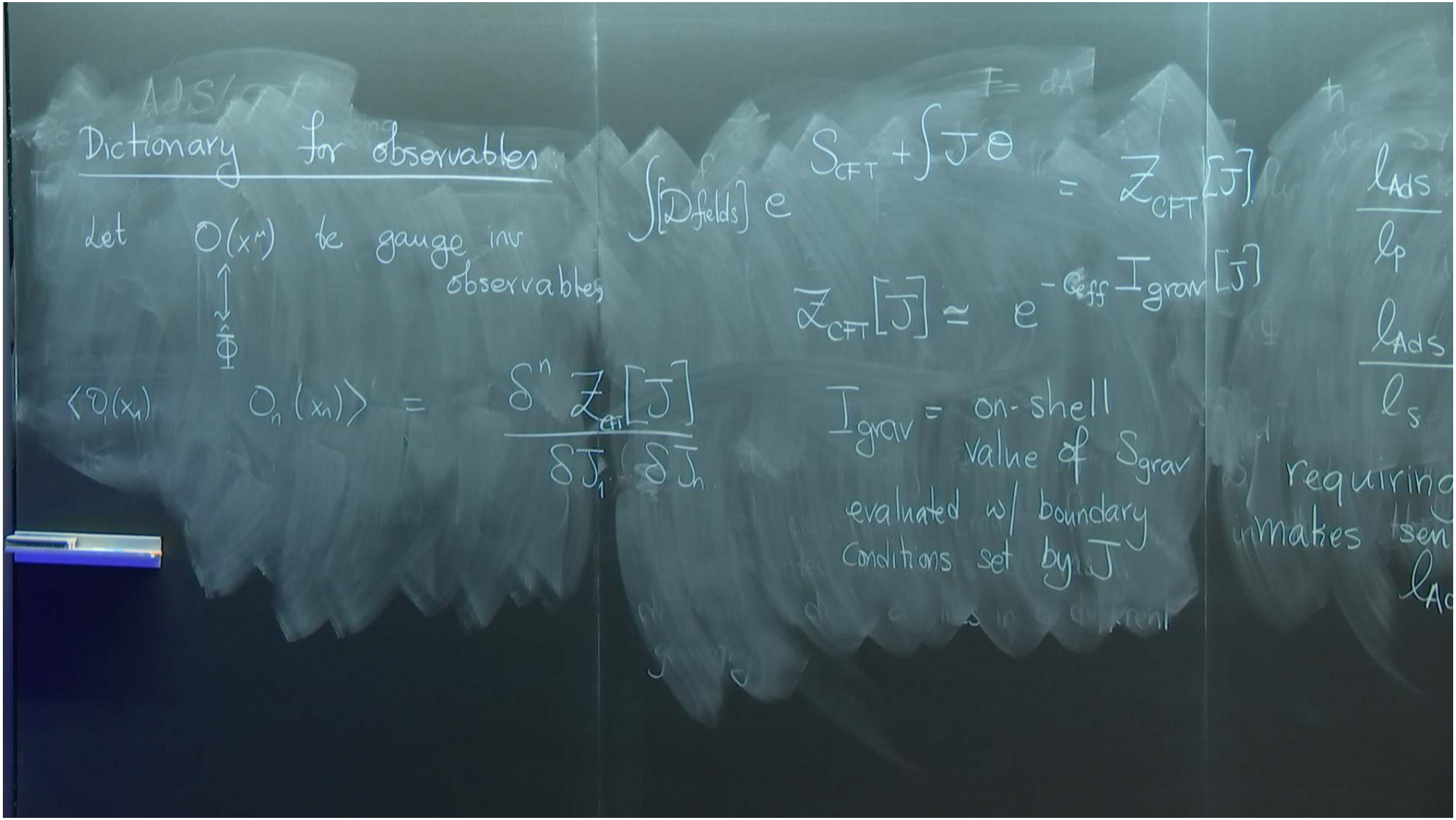
Dictionary for observables:
Let $O(x^m)$ be gauge inv observables
$\Leftrightarrow \hat{\Phi}$
$\langle O_1(x_1) \quad O_n(x_n) \rangle = \frac{\delta^n Z_{CFT}[J]}{\delta J_1 \ldots \delta J_n}$
$\int [\mathcal{D}\text{fields}]\, e^{S_{CFT} + \int J\mathcal{O}} = Z_{CFT}[J]$
$Z_{CFT}[J] \simeq e^{-G_{eff} I_{grav}[J]}$
I_{grav} = on-shell value of S_{grav} evaluated w/ boundary conditions set by J
$F = dA$
$\frac{l_{AdS}}{l_p}$
$\frac{l_{AdS}}{l_s}$
requiring
makes sen