

Title: AdS/CFT correspondence

Date: Jul 22, 2016 11:00 AM

URL: <http://pirsa.org/16070030>

Abstract:

AdS/CFT

Quantum gravity in asymptotically AdS spacetimes is dual to a CFT.

Motivation: look @ dynamics of simple matrix model & see that it has a simple "master field" in a planar limit.

$$\Phi^a_b \quad \text{adj} \quad U(N)$$
$$\mathcal{L} = \text{Tr}(\Phi^2) + \text{Tr}(\Phi^3) + \text{Tr}(\Phi^4)$$

Φ^a_b adj $U(N)$

$$\mathcal{L} = \frac{N}{g^2 N} \left[\text{Tr}(\Phi^2) + \text{Tr}(\Phi^3) + \text{Tr}(\Phi^4) \right]$$

quantum dynamics

$$\int [D\Phi] e^{\frac{i}{\hbar} \mathcal{L}}$$

Claim: This quantum theory is
classical in $N \rightarrow \infty$ limit
 $\lambda = g^2 N$ fixed. w/

$$\hbar_{\text{eff}} = \frac{\hbar}{N}$$

further, classicality $\Rightarrow$
correlators factor.

$$\langle \Phi_1 \dots \Phi_n \rangle \sim \langle \Phi_1 \Phi_2 \rangle \dots \langle \Phi_{n-1} \Phi_n \rangle$$

Gauge theories like QCD.
w/ admit matrix dof simplify
in the 't Hooft limit $N \rightarrow \infty$
 $g^2 N$ fixed

$$\Phi^a_b \quad \text{adj} \quad U(N)$$

$$\mathcal{L} = \frac{N}{g^2 N} \left[\text{Tr}(\Phi^2) + \text{Tr}(\Phi^3) + \text{Tr}(\Phi^4) \right]$$

quantum dynamics

$$\int [D\Phi] e^{\frac{i}{\hbar} \mathcal{L}}$$

Claim: This quantum theory is
classical in $N \rightarrow \infty$ limit
 $\lambda = g^2 N$ fixed. w/

$$\hbar_{\text{eff}} = \frac{\hbar}{N}$$

further, classicality $\Rightarrow$
correlators factors

$$\langle \hat{\Phi}_1 \dots \hat{\Phi}_n \rangle \sim \langle \hat{\Phi}_1 \hat{\Phi}_2 \rangle \dots \langle \hat{\Phi}_{n-1} \hat{\Phi}_n \rangle$$

$\parallel$
 $\text{Tr}(\Phi \dots \Phi)$

Gauge theories like QCD.
w/ admit matrix dof simplify
in the 't Hooft limit $N \rightarrow \infty$
 $g^2 N$ fixed

$$\text{adj } U(N) \\ \text{Tr}(\Phi^2) + \text{Tr}(\Phi^3) + \text{Tr}(\Phi^4)$$

$$\text{dynamics} \\ \int \mathcal{D}\Phi e^{\frac{i}{\hbar} \mathcal{L}}$$

quantum theory is
 $N \rightarrow \infty$ limit
 $\lambda = g^2 N$ fixed. w/

$$\hbar_{\text{eff}} = \frac{\hbar}{N}$$

further, classicality $\Rightarrow$
 correlators factors

$$\langle \hat{\Phi}_1 \hat{\Phi}_n \rangle \sim \langle \hat{\Phi}_1 \hat{\Phi}_1 \rangle \langle \hat{\Phi}_n \hat{\Phi}_n \rangle \quad \text{Tr}(\Phi \Phi)$$

Gauge theories like QCD.
 w/ admit matrix dof simplify
 in the 'tHooft limit $N \rightarrow \infty$
 $g^2 N$ fixed

In the large N limit
 the full integral localizes
 to a particular classical Φ
 configuration

Master field

$$\int \mathcal{D}\Phi \hat{\Phi}_1 \dots \hat{\Phi}_n e^{\frac{i}{\hbar_{\text{eff}}} [\text{Tr}(\Phi^2) + \text{Tr}(\Phi^3) + \text{Tr}(\Phi^4)]}$$

$$\hat{\Phi}_1^{\text{saddle}} \hat{\Phi}_2^{\text{saddle}} \hat{\Phi}_n^{\text{saddle}} e^{\frac{i}{\hbar_{\text{eff}}} \mathcal{L}_{\text{saddle}}}$$



QCD: we are still looking.

For susy cousins of QCD $\mathcal{N}=4$ SYM.
 4d theory w/ gluons $A_{\mu a}$ $\mu=0,3$
 Scalars X^I_a $I=1,6$
 fermions $\psi_{\alpha a}$

The large N limit is
 a classical gravitational theory
 in AdS_5 .

$$F = dA$$

$$\mathcal{L}_{\mathcal{N}=4} = \frac{1}{g_{YM}^2} \left\{ \text{Tr}(F^2) + \text{Tr}[(DX^I)^2] + \dots \right\}$$

$$D = \partial + [A, \cdot]$$

't Hooft limit

$$\lambda = g_{YM}^2 N \text{ fixed}$$

$$N \rightarrow \infty$$

The $\mathcal{N}=4$ theory resides on $\mathbb{R}^{3,1}$.
 the dual description [master field]
 is represented in terms of vastly
 different dof & lives in a different
 geometry.

$\mathcal{N}=4$ SYM has 2 parameters

$$\lambda, N$$

λ small interactions are
 weak (perturbation)

$N \rightarrow \infty$ there is a classical
 description for any λ

In the large N
 the final inter
 to a particu
 configuration

$$\int [D\Phi] \hat{\Phi}_1 \dots$$

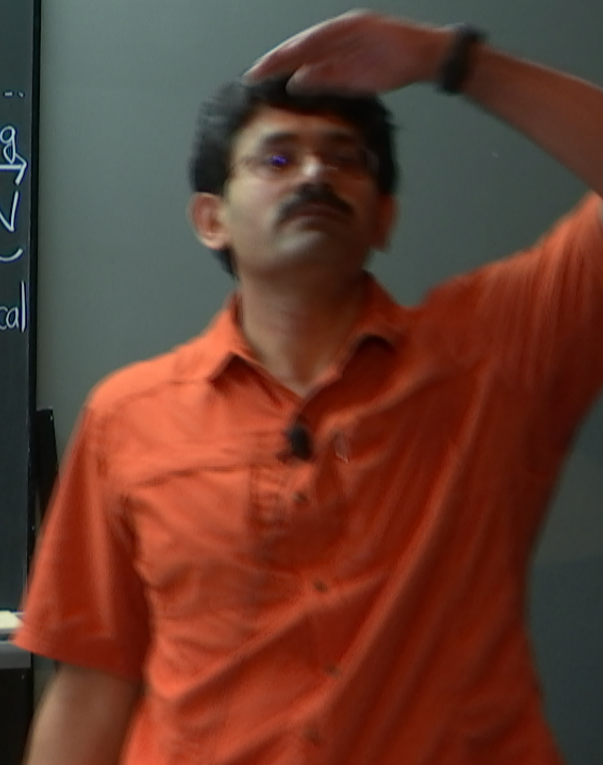
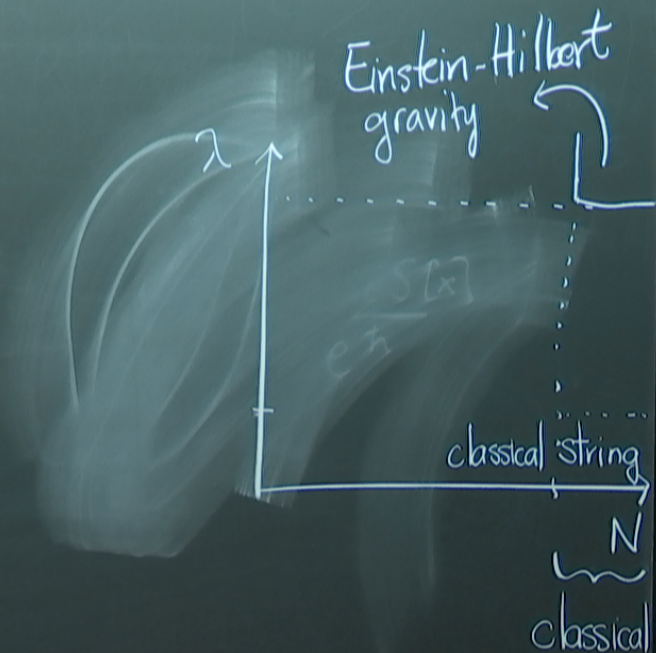
$$\frac{1}{\Phi_1} \text{ sat}$$

localizes
classical Φ

field

$$e^{\frac{i}{\hbar_{\text{eff}}} \lambda [\text{Tr}(\Phi^2) + \text{Tr}(\Phi^3) + \text{Tr}(\Phi^4)]}$$

saddle $\frac{1}{\hbar_{\text{eff}}} \int_{\text{saddle}} \mathcal{L}_{\text{saddle}}$



$$F = dA$$

$$\text{Tr}(F^2) + \text{Tr}((DX)^2) + \dots$$

$$D = \partial + [A, \cdot]$$

met
N fixed
 ∞

theory resides on $\mathbb{R}^{3,1}$
in terms of master field
& lives in a different

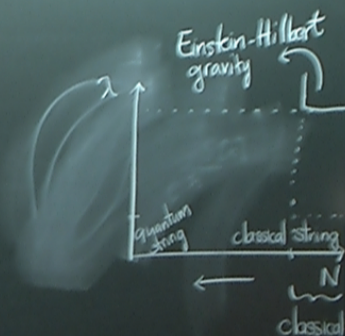
$\mathcal{N}=4$ SYM has 2 parameters λ, N
 λ small interactions are weak (perturbation)

$N \rightarrow \infty$ there is a classical description for any λ

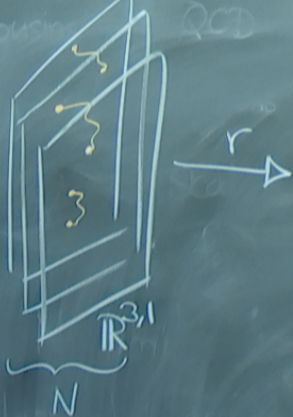
quantum
in large N limit
by integral
classical
SU(N) $\mathcal{N}=4$ SYM (λ, N)

quantum
String theory in $AdS_5 \times S^5$
parameters l_{AdS}

strength of gravity l_p (Planck scale)
strength of string perturbations: l_s (string scale)

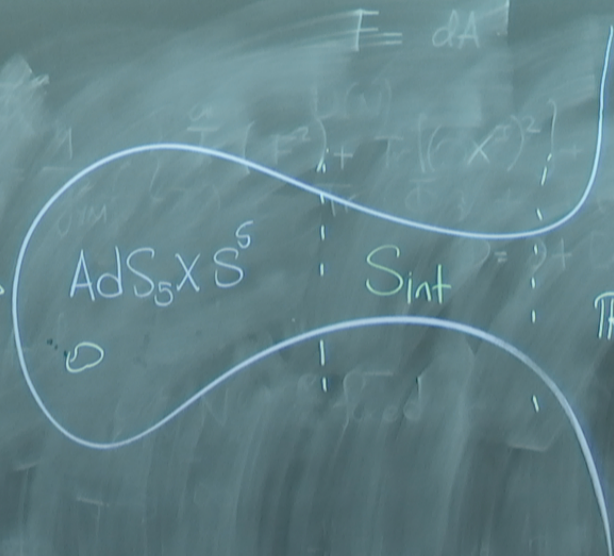


AdS/CFT
QCD we are still looking for
D3-branes
QCD

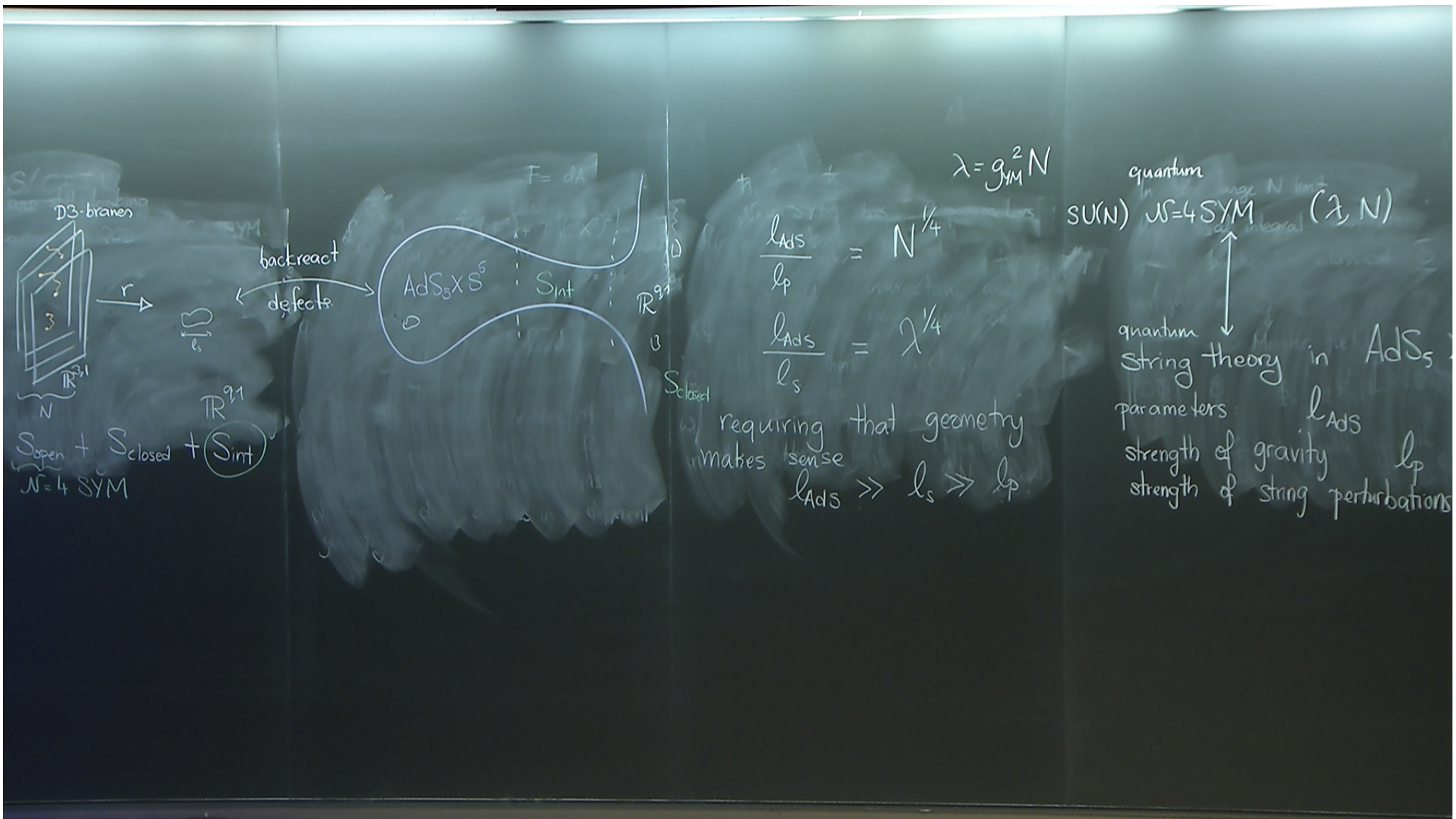


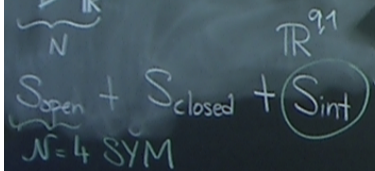
$S_{\text{open}} + S_{\text{closed}} + S_{\text{int}}$
 $\mathcal{N}=4$ SYM

backreact
defects



$F = dA$
 $\mathcal{L} = -\frac{1}{4} F^2 + \frac{1}{2} (D_\mu X^a)^2$
 $\mathbb{R}^{9,1}$
 S_{closed}
in the





backreact
defects.

$$AdS_5 \times S^5$$

Sint

R⁹¹

Sclosed

requiring that geometry makes sense
 $l_{\text{AdS}} \gg l_s \gg l_p$

$$\lambda = g_{YM}^2 N$$

Let us define $\text{hobographic } C^1$
to be one which admits a
planar ℓ^1 limit

C_{eff} : central charge
 $C_{eff} > 1$ for classical

& in space of couplings there is some $\lambda \gg 1 \rightarrow$ admits a dual in terms of gravity on AdS_{d+1}

$$\lambda = g_{YM}^2 N$$

Let us define N holographic CFT to be one, which admits a planar 't Hooft limit

C_{eff} : central charge
 $\rightarrow C_{eff} \gg 1$ for classicality

& in space of couplings there is some $\lambda \gg 1 \rightarrow$ admits a dual in terms of gravity on AdS_{d+1}

(strength of string interactions)

$$S_{grav} = \frac{1}{16\pi G_N^{(d+1)}} \int d^{d+1}x \sqrt{-g} [R - 2\Lambda + \dots]$$

$$C_{eff} = \frac{1}{16\pi G_N^{(d+1)}}$$

i) CFT should have few low energy states (sparse low lying spectrum)

$$N = g_{YM}^2 N$$

Let us define N holographic CFT to be one, which admits a planar 't Hooft limit

C_{eff} : central charge
 $\rightarrow C_{eff} \gg 1$ for classicality

& in space of couplings there is some $\lambda \gg 1 \rightarrow$ admits a dual in terms of gravity on AdS_{d+1}

$$S_{grav} = \frac{1}{16\pi G_N^{(d+1)}} \int d^{d+1}x \sqrt{-g} [R - 2\Lambda + \dots]$$

$$C_{eff} = \frac{1}{16\pi G_N^{(d+1)}}$$

i) CFT should have few low energy states (sparse low lying spectrum)

AdS/CFT

Dictionary for observables:

Let $O(x_n)$ be gauge inv
observables

$\uparrow$
 $\hat{\Phi}$

$$\langle O_i(x_i) O_n(x_n) \rangle = \frac{\delta^n Z_{\text{CFT}}[J]}{\delta J_i \delta J_n}$$

$$\int [D\text{fields}] e^{S_{\text{CFT}} + \int J \Theta} = Z_{\text{CFT}}[J]$$

$$Z_{\text{CFT}}[J] \simeq e^{-c_{\text{eff}} I_{\text{grav}}[J]}$$

I_{grav} = on-shell
value of S_{grav}
evaluated w/ boundary
conditions set by J

$\frac{l_{\text{AdS}}}{l_p}$
 $\frac{l_{\text{AdS}}}{l_s}$
requiring
makes sense
 l_{AdS}