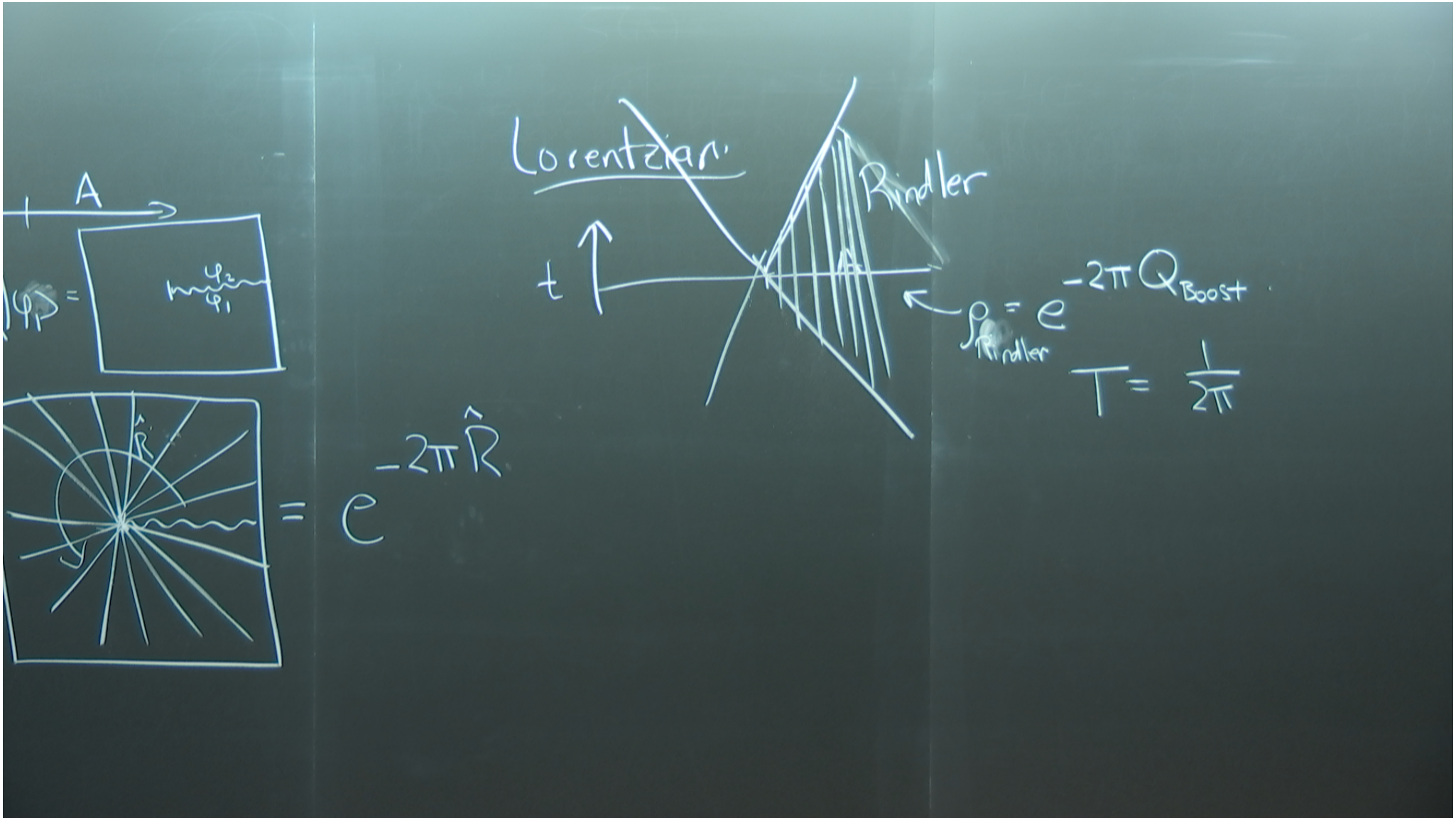
Title: QFT Basics

Date: Jul 21, 2016 09:00 AM

URL: http://pirsa.org/16070024

Abstract:

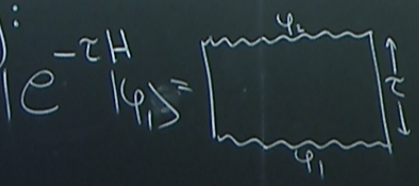

A
Lorentzian:
Rindler
t
$\rho_{Rindler} = e^{-2\pi Q_{Boost}}$
$T = \frac{1}{2\pi}$
$= e^{-2\pi \hat{R}}$

QFT Basics 2 of 2

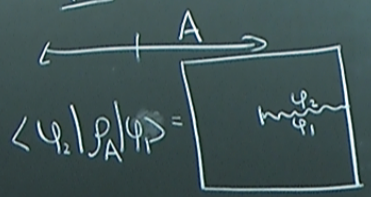
"What is a QFT?"

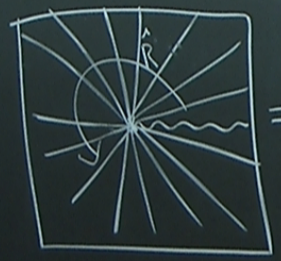
Tuesday:

$\langle \varphi_2 | e^{-\tau H} | \varphi_1 \rangle =$



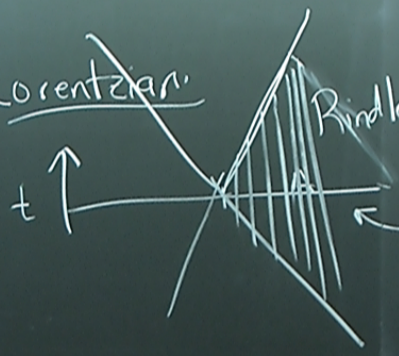
1+1





$= e^{-2\pi \hat{K}}$

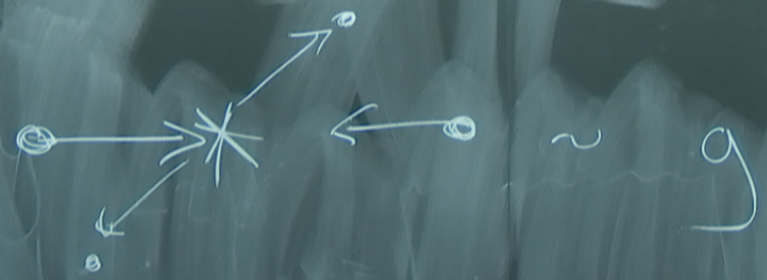
Lorentzian



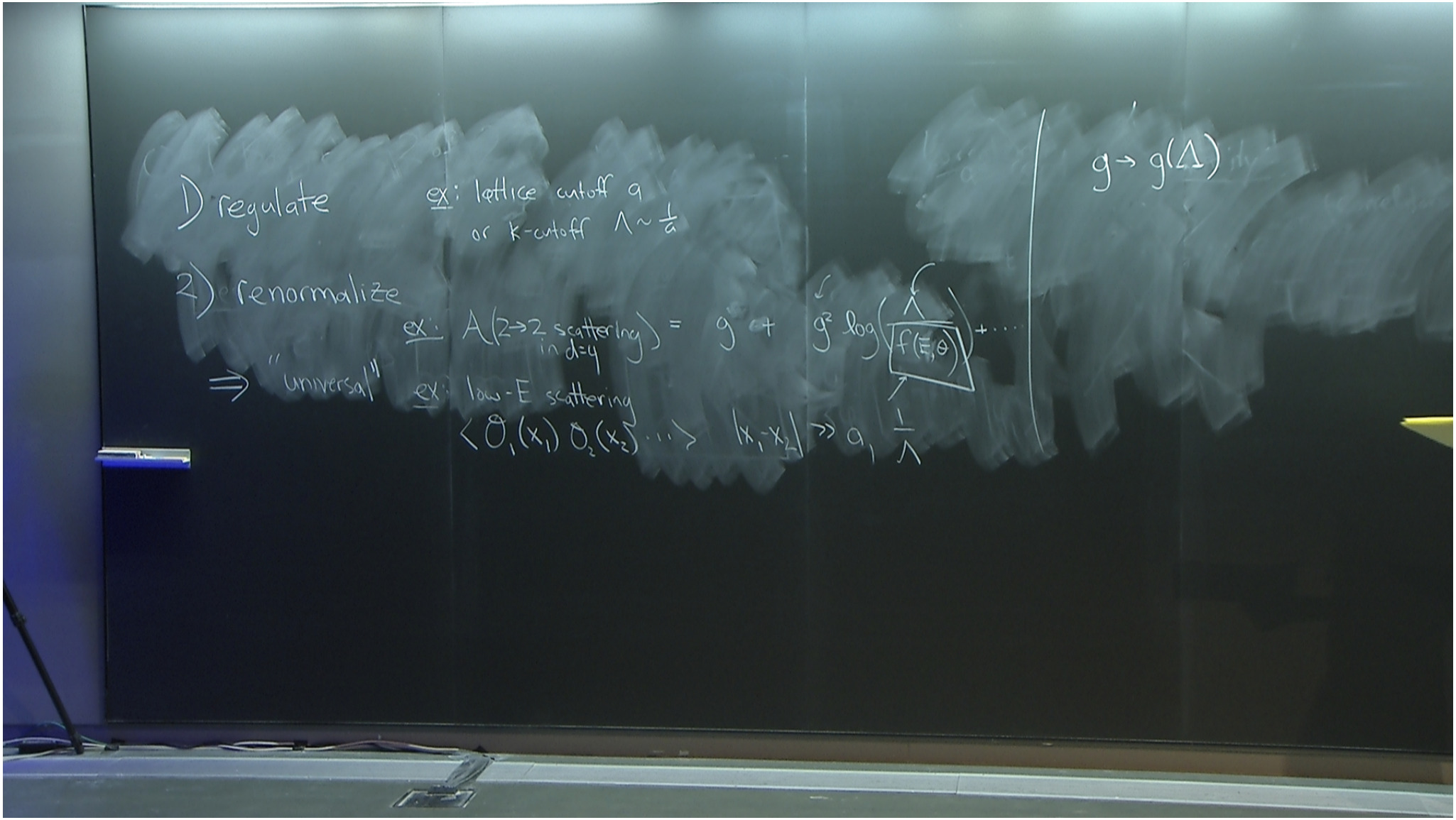
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$T = \frac{1}{2\pi}$

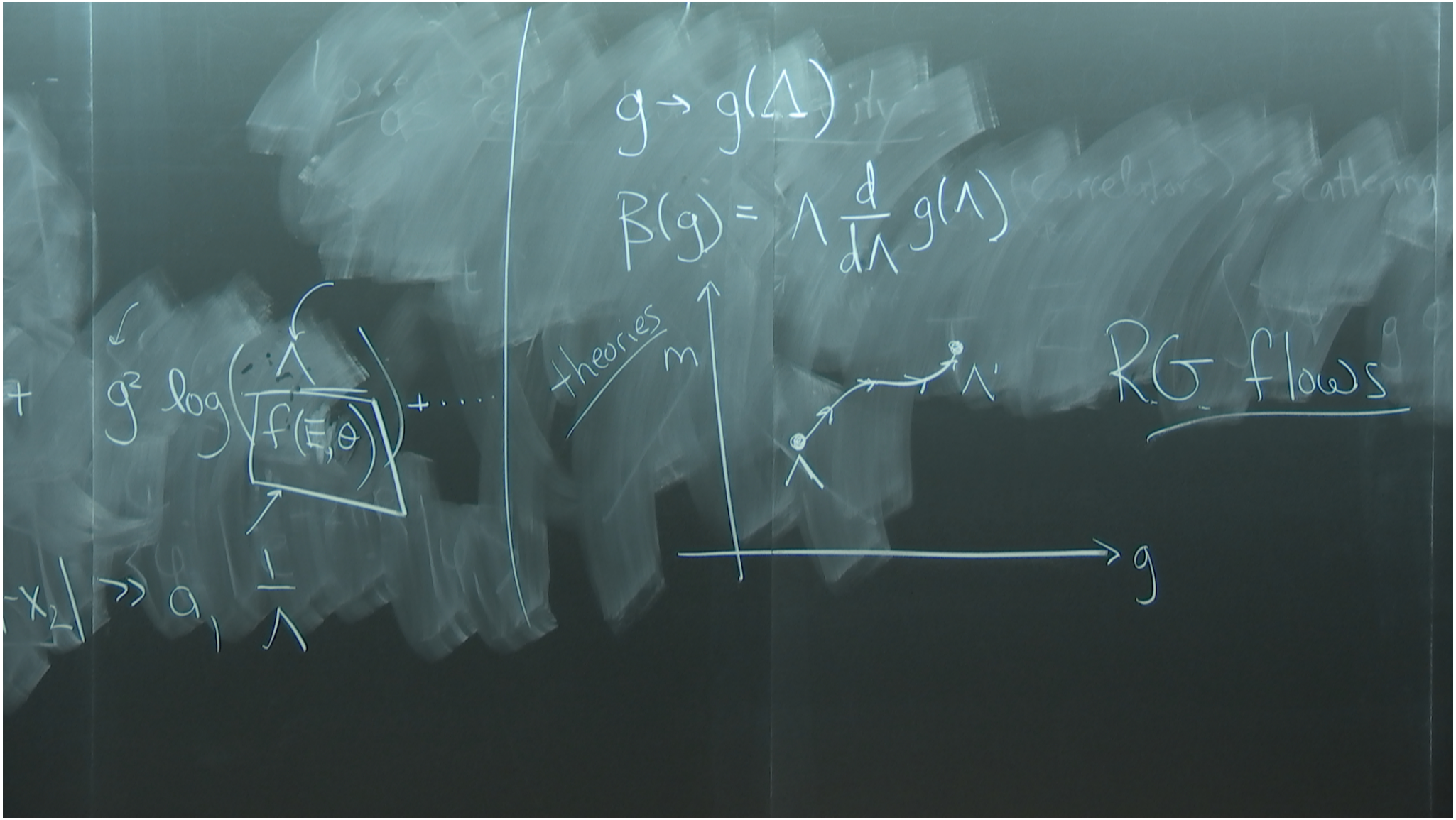
$$S = \int d^d x \left(\partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 + g \phi^4 \right)$$

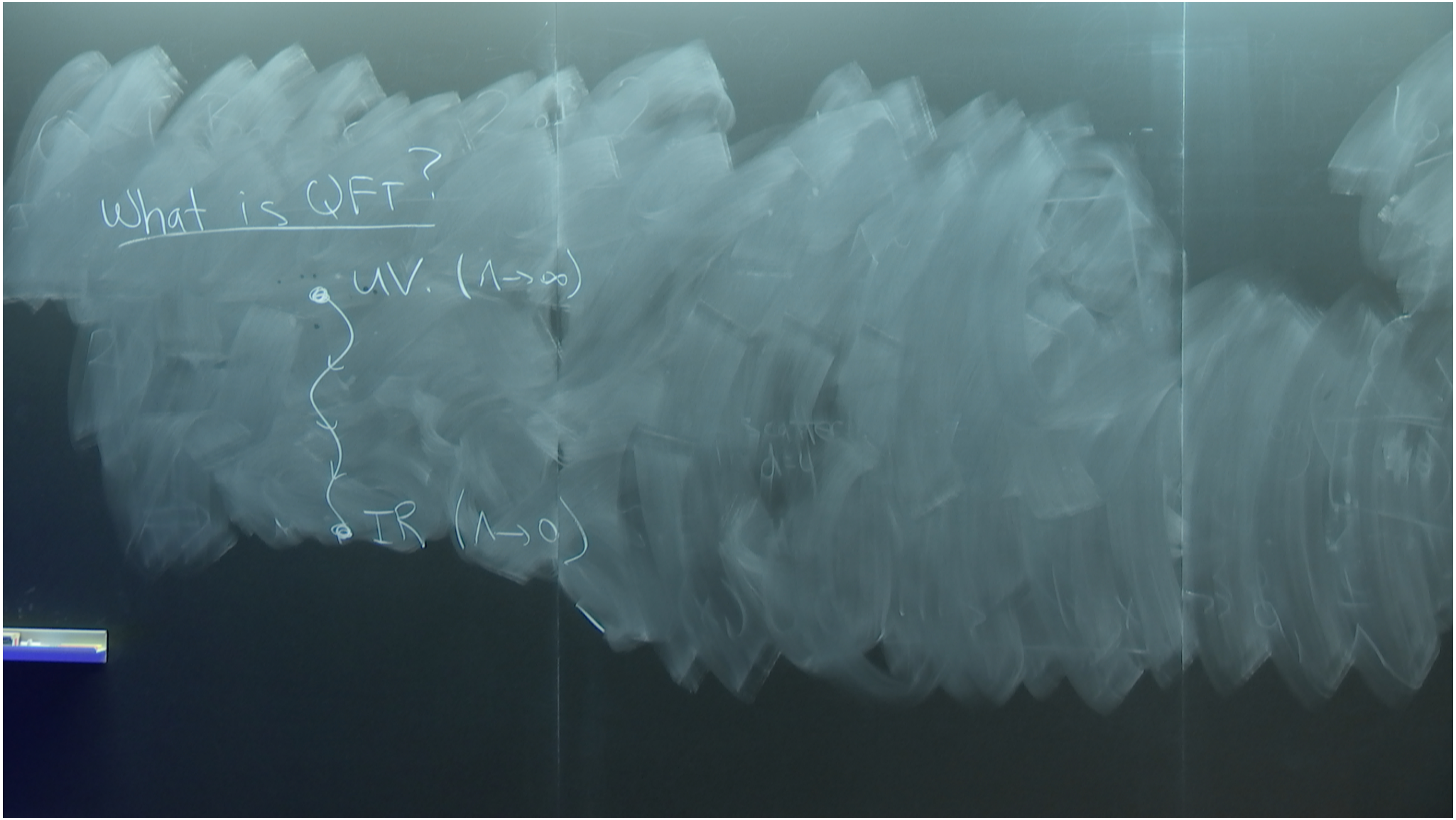


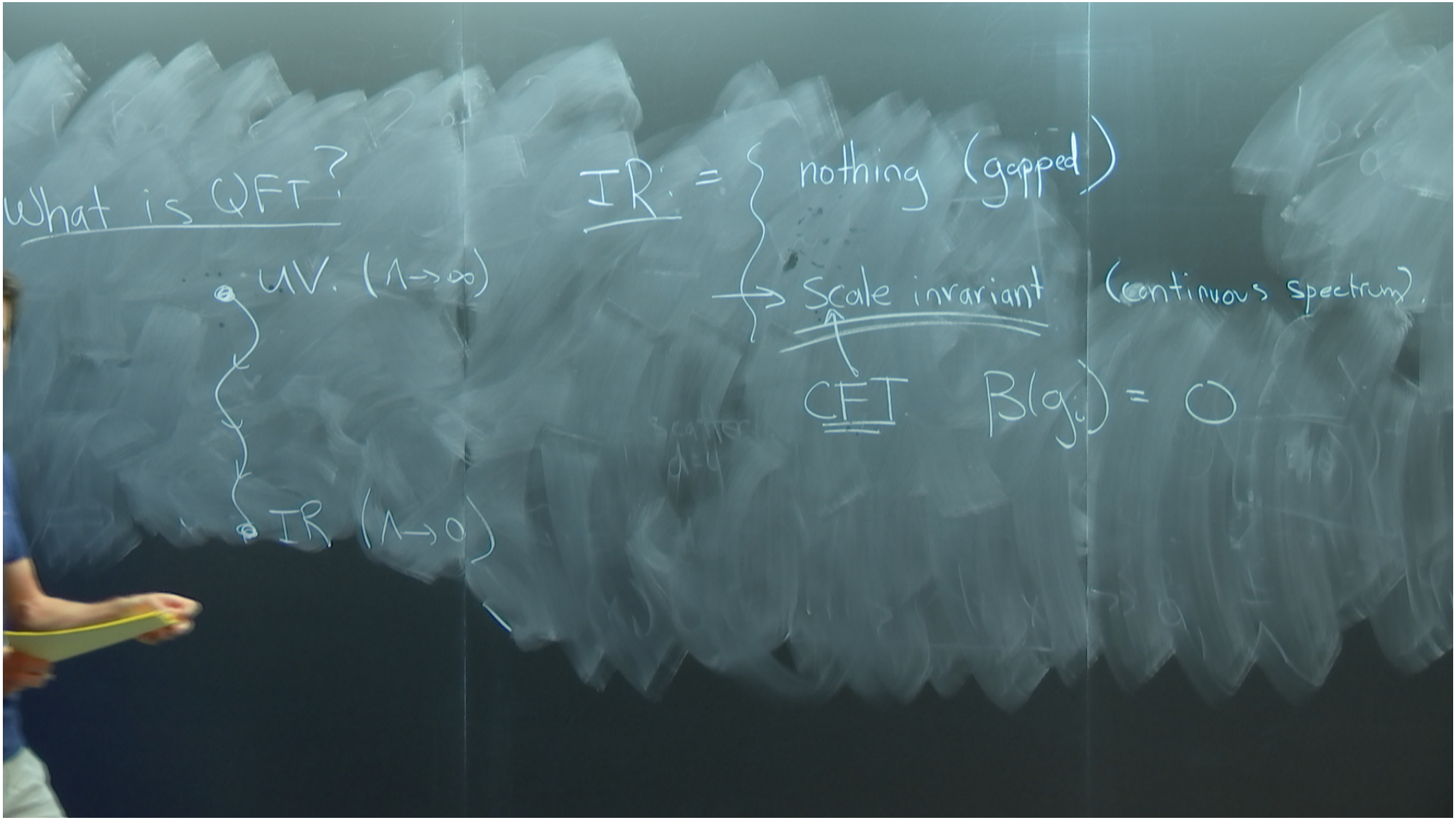
$\sim g$

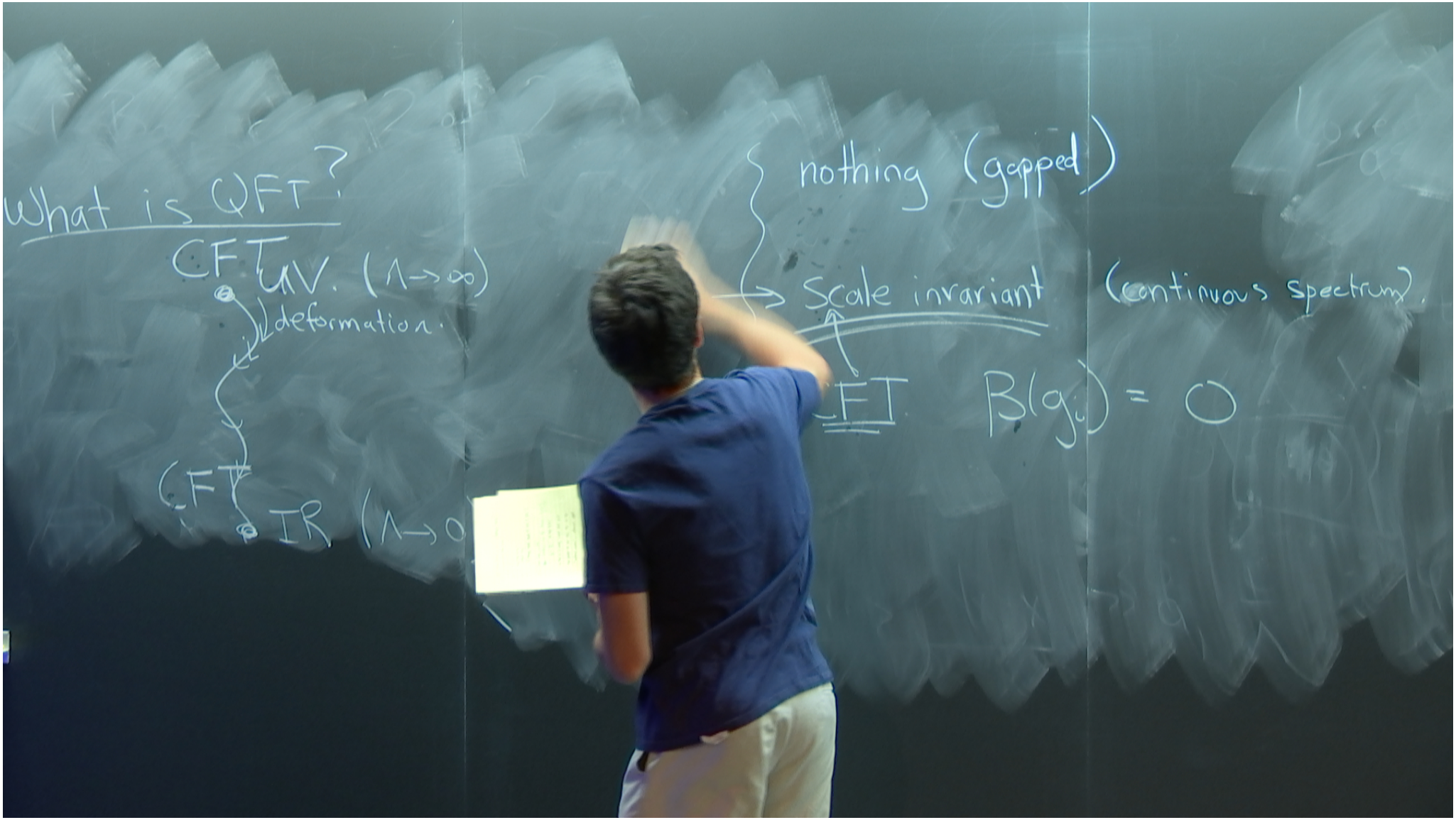

1) regulate
ex: lattice cutoff a
or k-cutoff $\Lambda \sim \frac{1}{a}$
2) renormalize
ex: $A(2\to 2$ scattering in $d=4) = g + g^2 \log\left(\frac{\Lambda}{f(E,\theta)}\right) + \cdots$
$\Rightarrow$ "universal"
ex: low-E scattering
$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \cdots \rangle \quad |x_1 - x_2| \gg a, \frac{1}{\Lambda}$
$g \to g(\Lambda)$

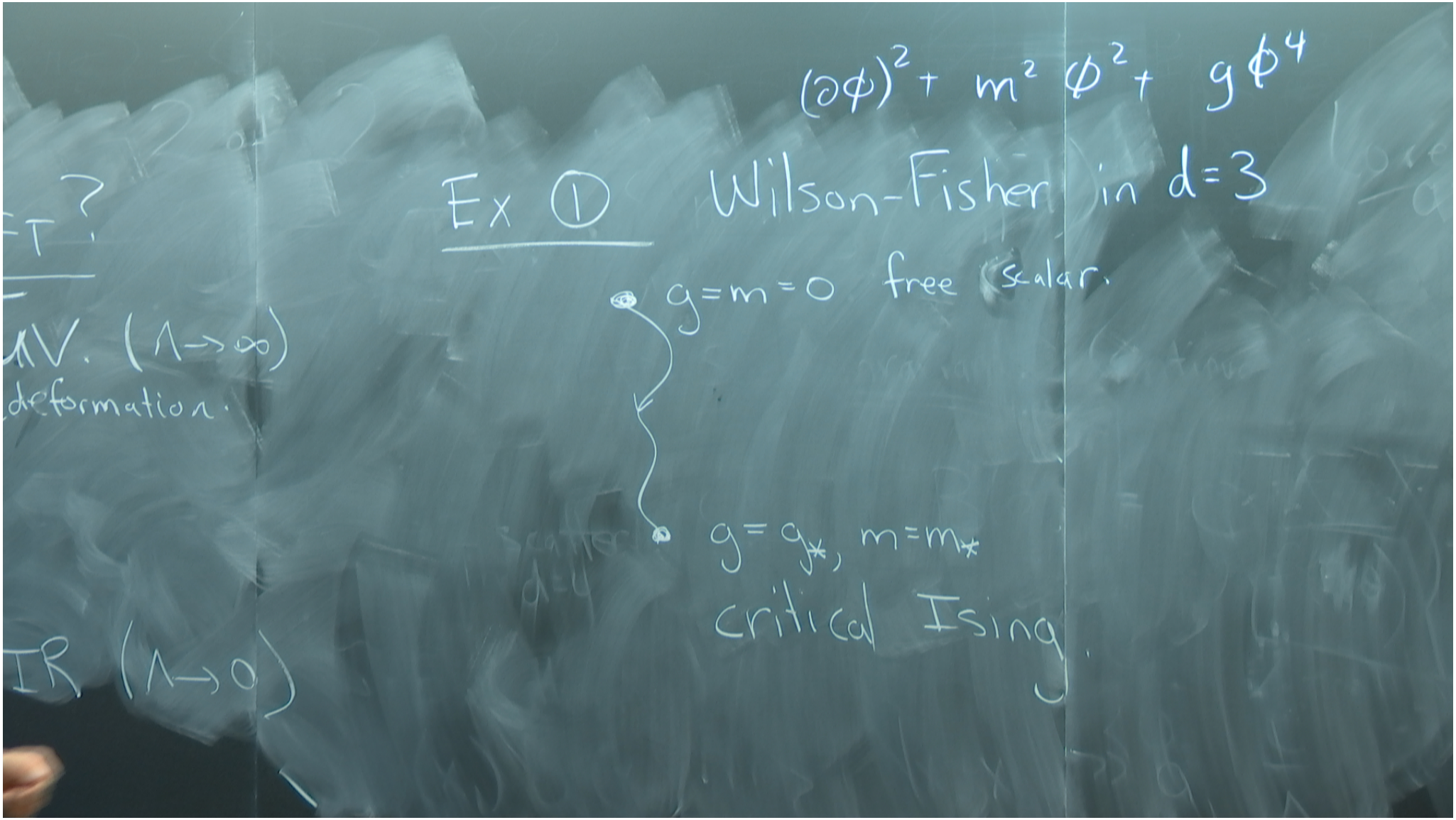


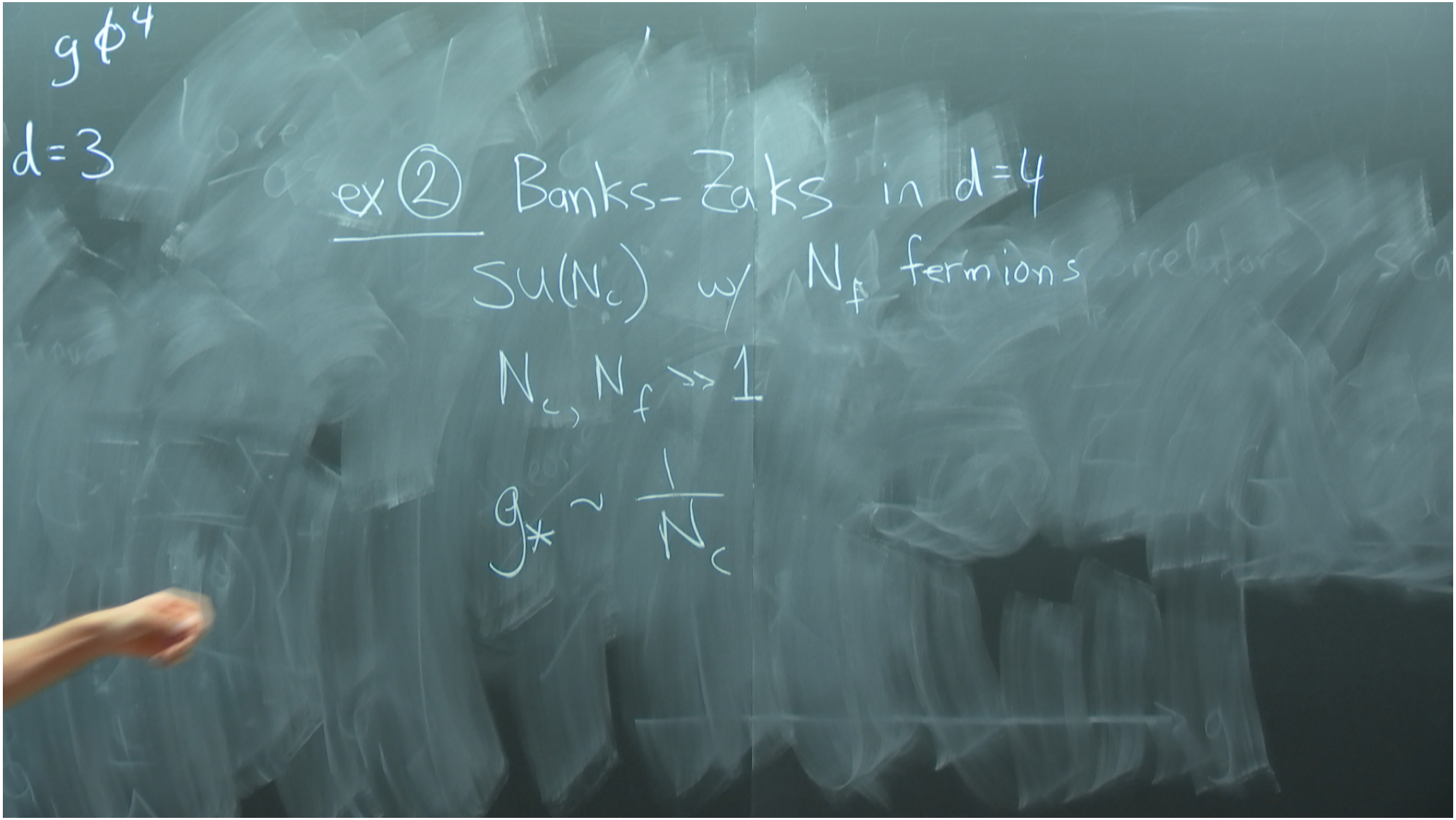

g → g(Λ)
β(g) = Λ d/dΛ g(Λ)
theories
m
g
RG flows
Λ'
f(E,θ)

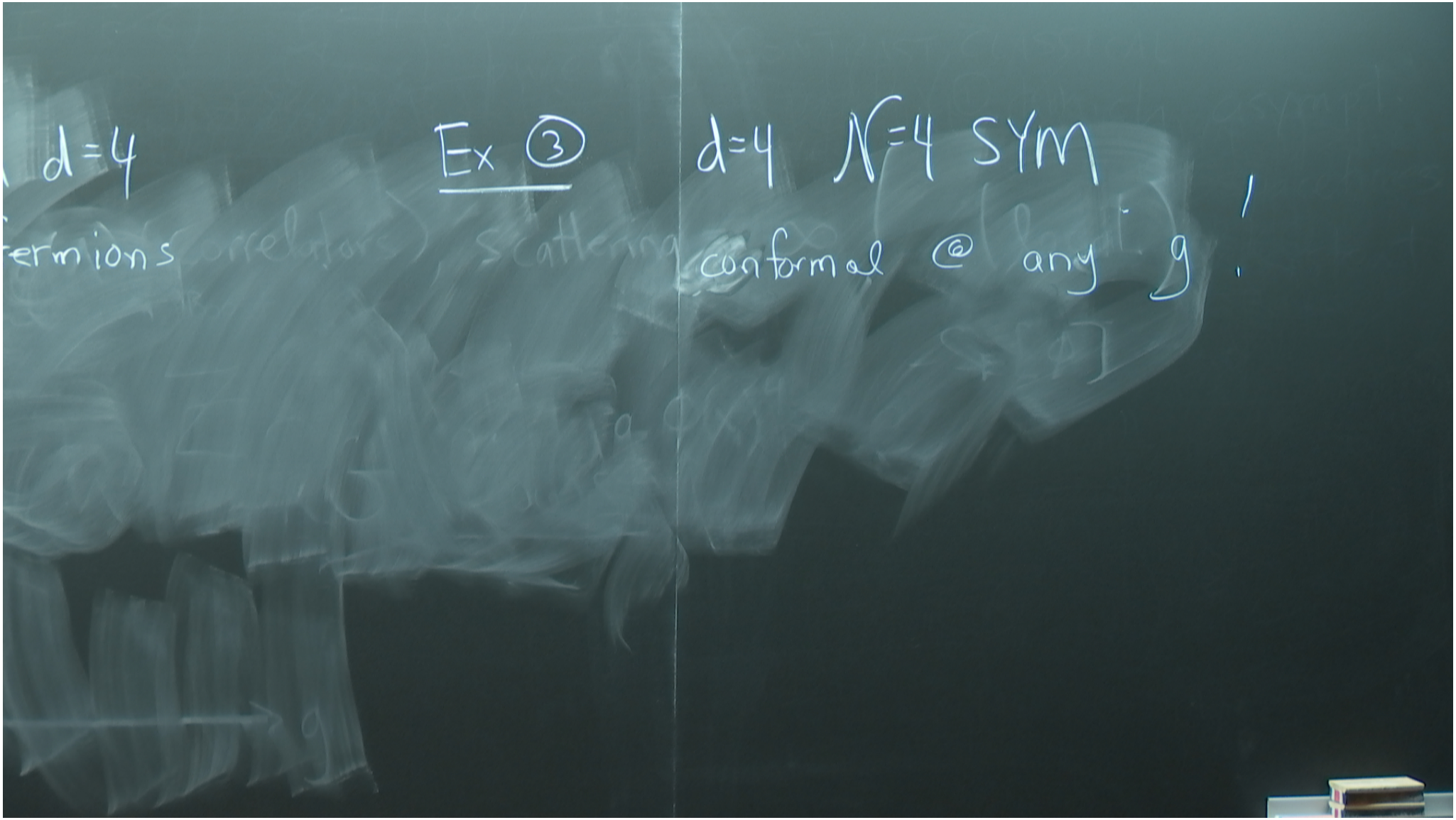

What is QFT?
UV. (Λ→∞)
IR (Λ→0)

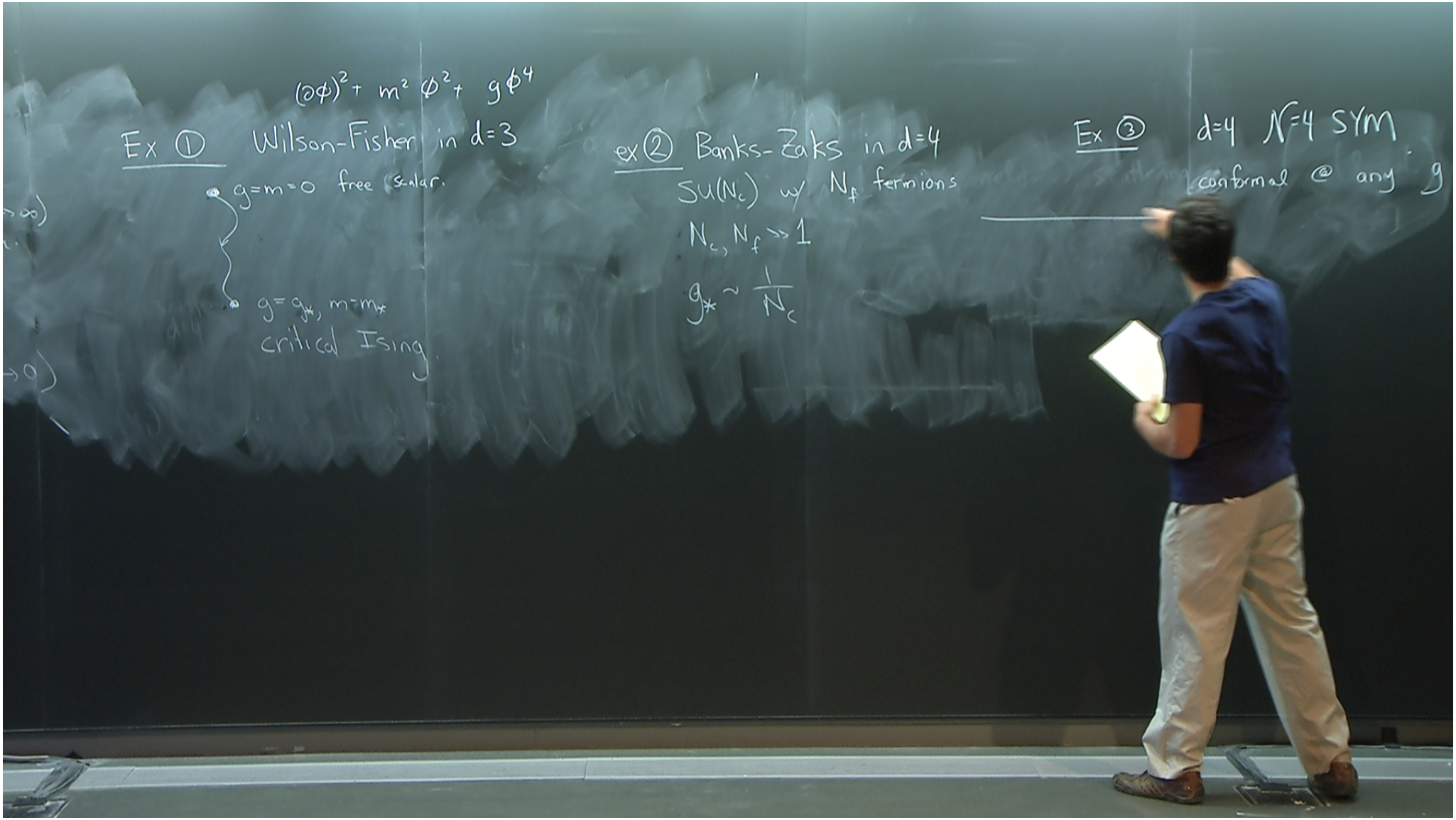

What is QFT?
UV. (Λ→∞)
IR (Λ→0)
IR: =
nothing (gapped)
Scale invariant (continuous spectrum)
CFT
β(g_i) = 0

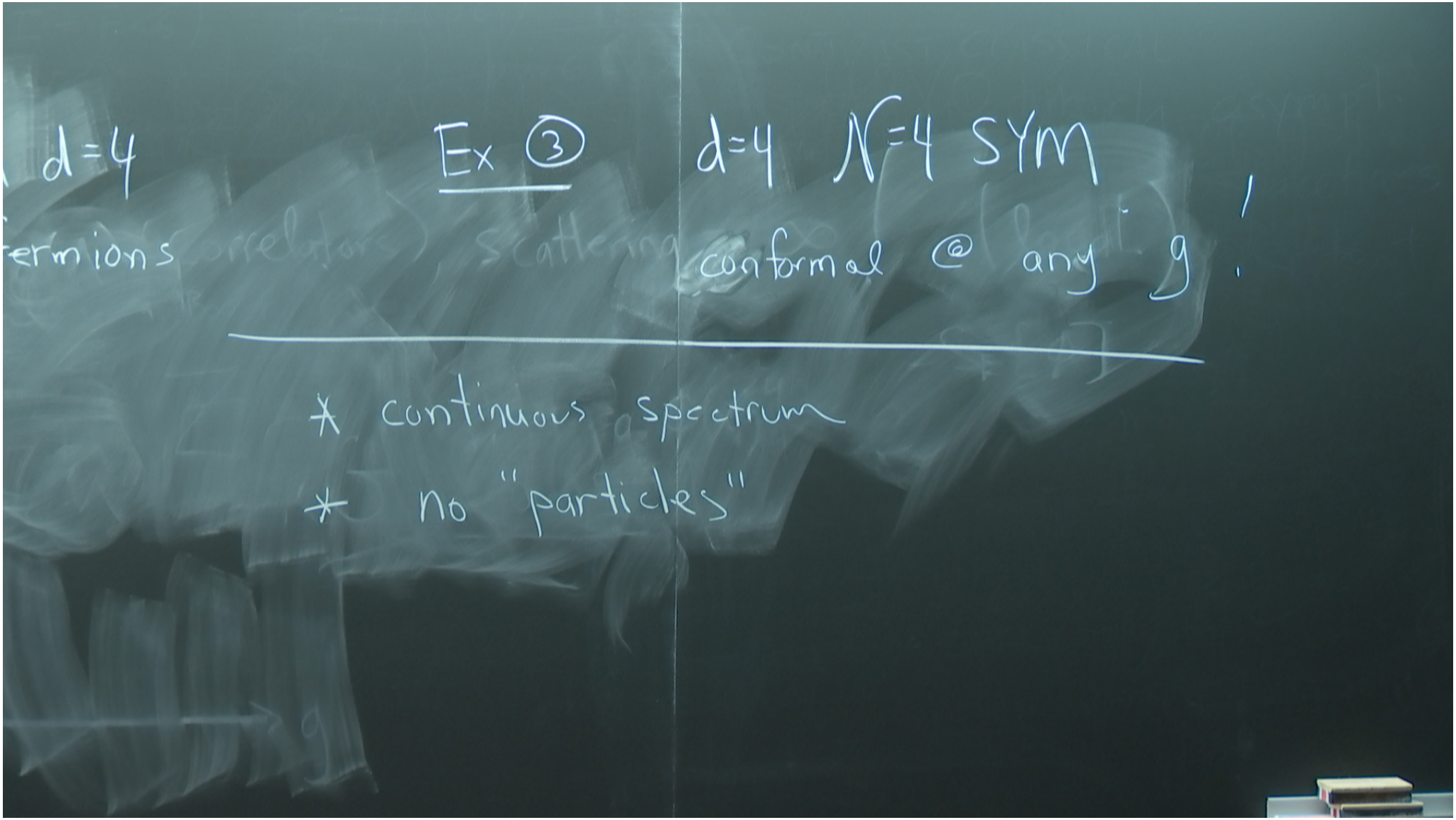

What is QFT?
CFT UV. (Λ→∞)
deformation.
CFT
IR (Λ→0
nothing (gapped)
Scale invariant (continuous spectrum)
CFT
B(g_i) = 0

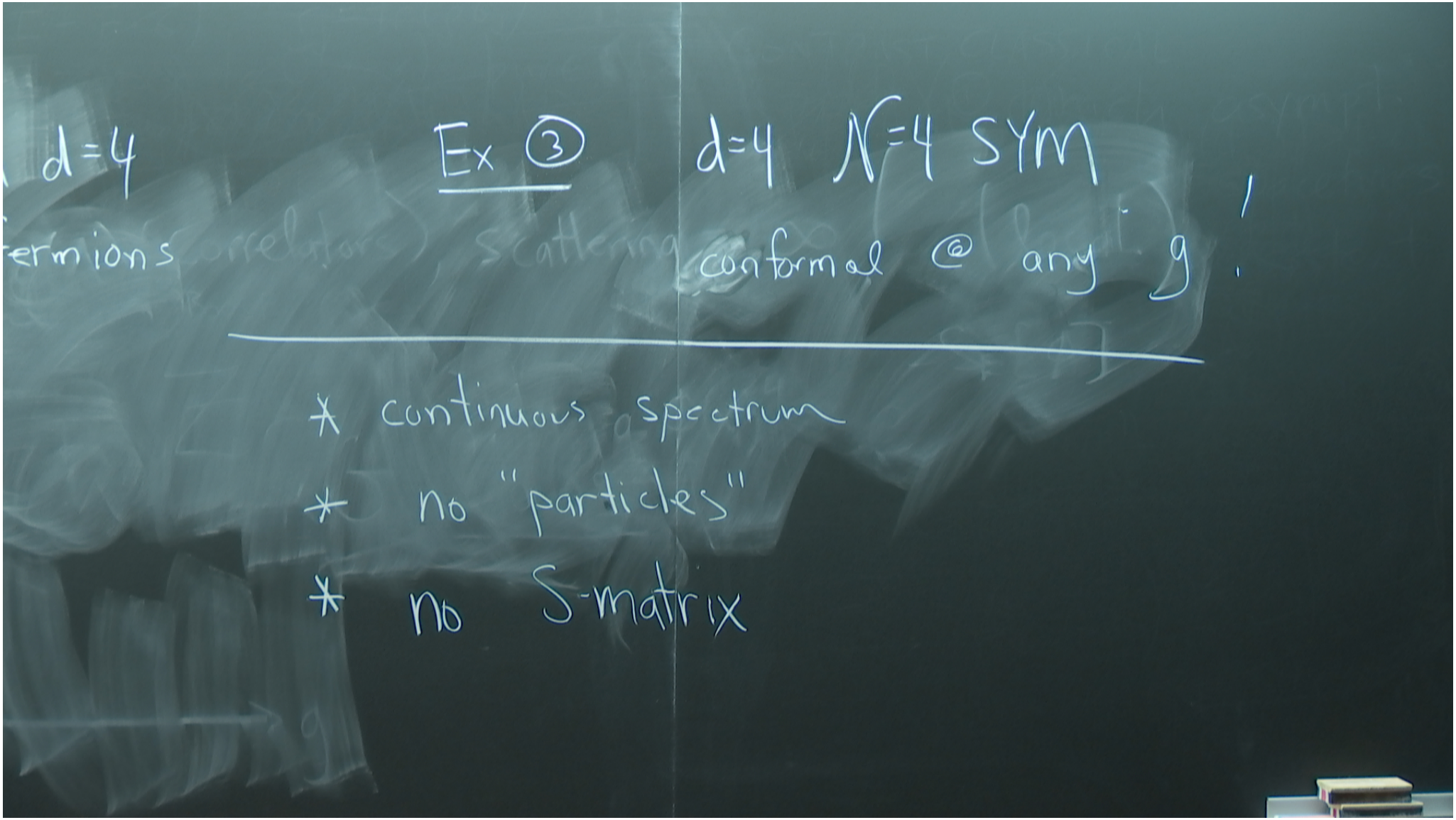

$(\partial\phi)^2 + m^2\phi^2 + g\phi^4$
Ex ① Wilson-Fisher in d=3
g=m=0 free scalar.
$g=g_*,\ m=m_*$
critical Ising.
IR $(\Lambda \to 0)$

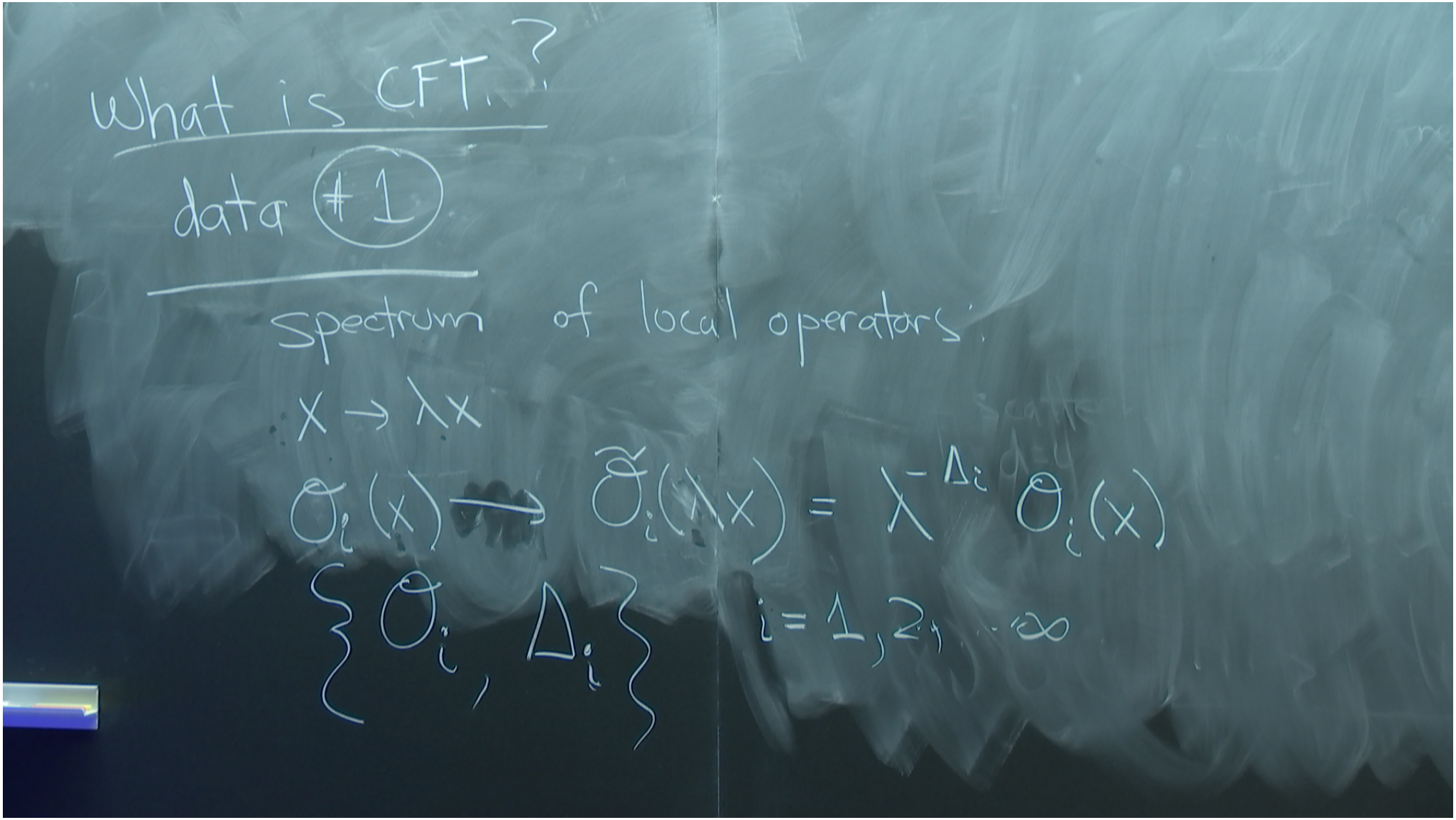

$g\phi^4$
$d=3$
ex ② Banks-Zaks in $d=4$
$SU(N_c)$ w/ N_f fermions
$N_c, N_f \gg 1$
$g_* \sim \frac{1}{N_c}$


d=4
Ex ③
d=4 N=4 SYM
fermions
conformal @ any g !


$(\partial\phi)^2 + m^2\phi^2 + g\phi^4$
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ex② Banks-Zaks in d=4
SU(Nc) w/ Nf fermions
Nc, Nf >> 1
g* ~ 1/Nc
Ex ③ d=4 N=4 SYM
conformal @ any g


d=4
fermions
Ex ③
d=4 N=4 SYM
conformal @ any g !
* continuous spectrum
* no "particles"


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Ex ③
d=4 N=4 SYM
fermions
conformal @ any g !
* continuous spectrum
* no "particles"
* no S-matrix


What is CFT?
data #1
spectrum of local operators:
$x \to \lambda x$
$\mathcal{O}_i(x) \longrightarrow \tilde{\mathcal{O}}_i(\lambda x) = \lambda^{-\Delta_i} \mathcal{O}_i(x)$
$\{\mathcal{O}_i, \Delta_i\}$
$i = 1, 2, \ldots \infty$

$$\langle \mathcal{O}_i(x)\, \mathcal{O}_i(y) \rangle = \frac{1}{|x-y|^{2\Delta_i}}$$

data #2

$$\langle \mathcal{O}_{i_1}(x_1)\, \mathcal{O}_{i_2}(x_2) \cdots \rangle$$

data = "OPE coefficients"

$$\langle \mathcal{O}_i\, \mathcal{O}_j\, \mathcal{O}_k \rangle \sim C_{ijk}$$

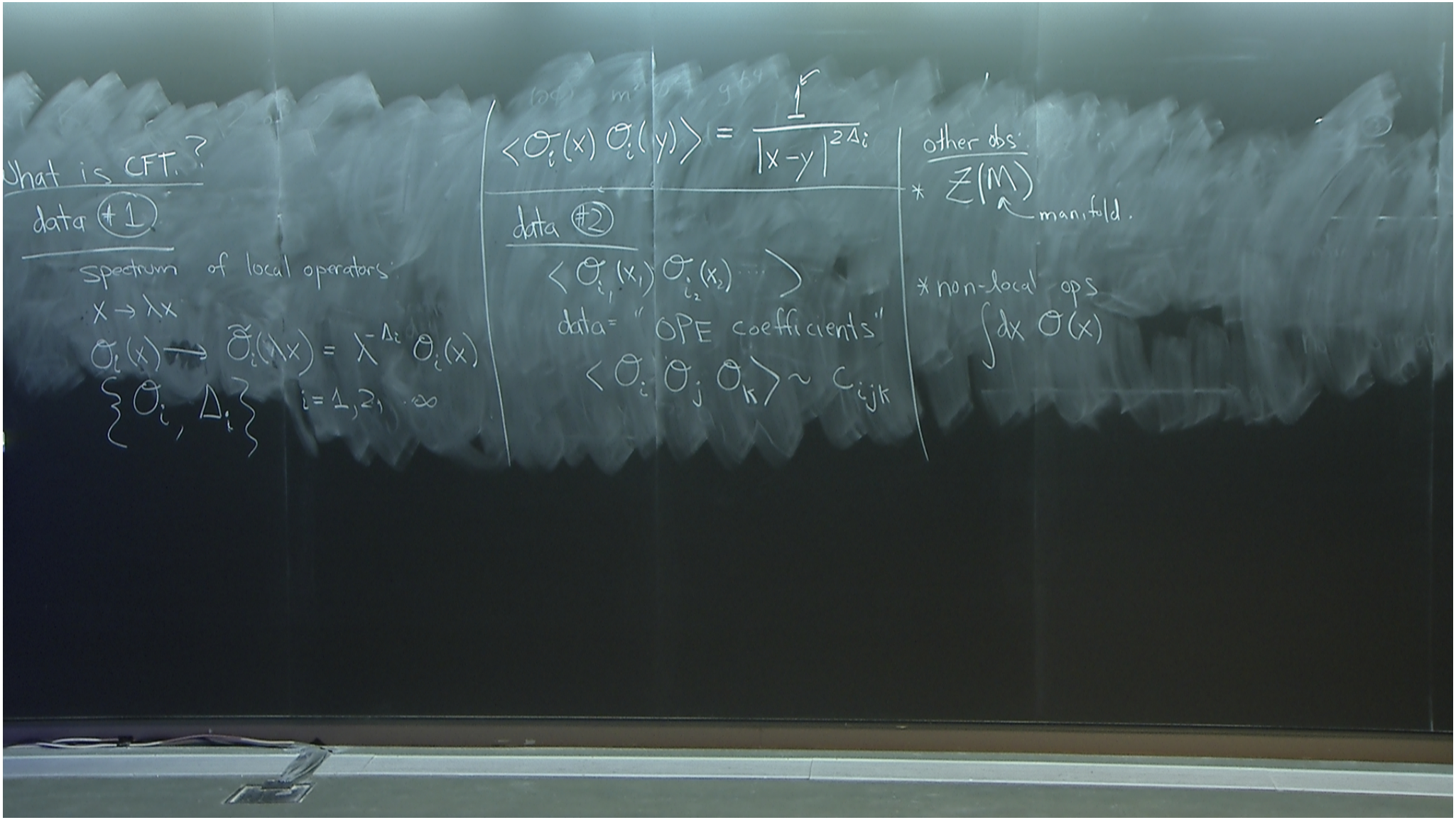
$$\langle \mathcal{O}_i(x)\, \mathcal{O}_i(y) \rangle = \frac{1}{|x-y|^{2\Delta_i}}$$

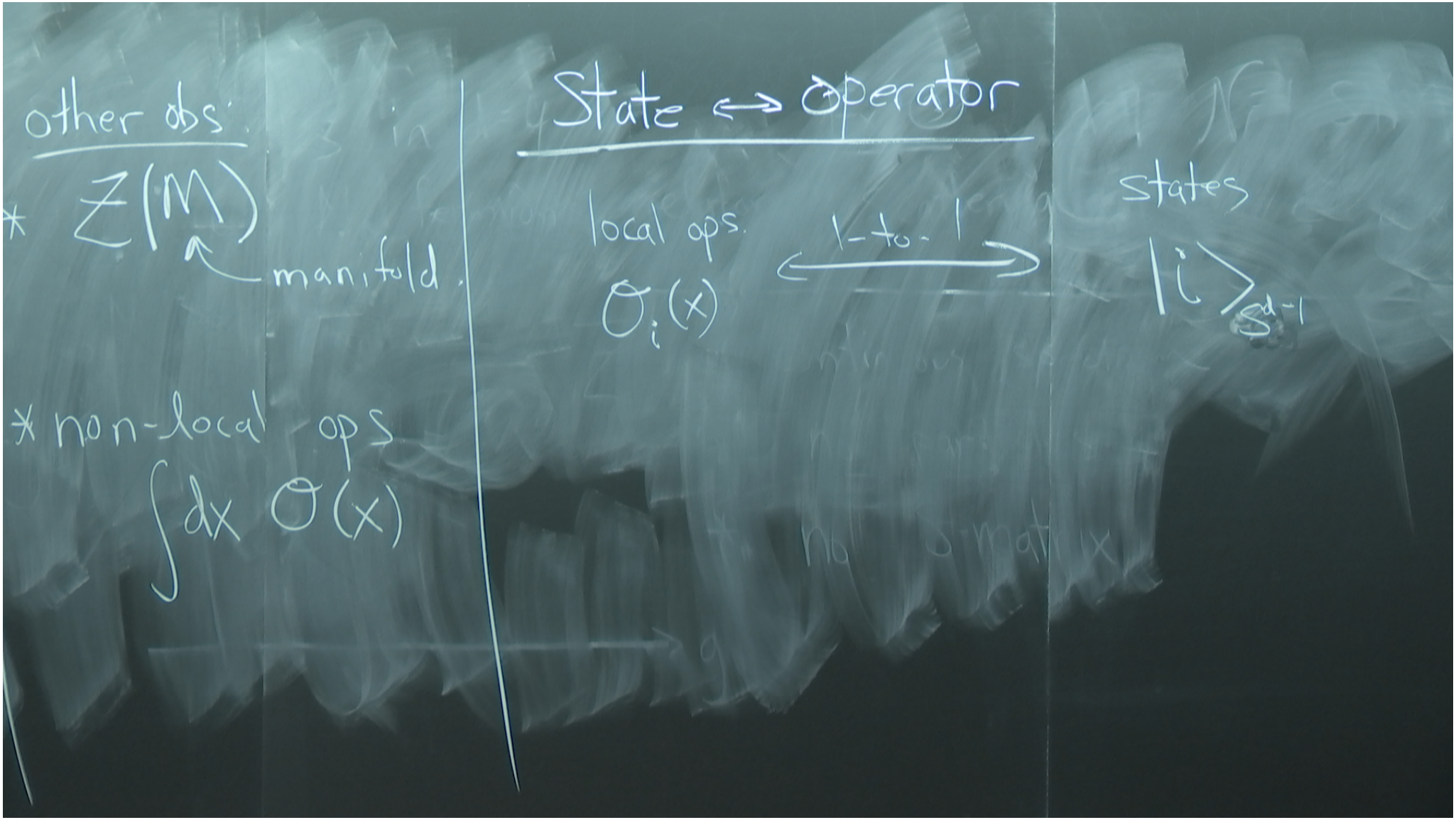
data #2

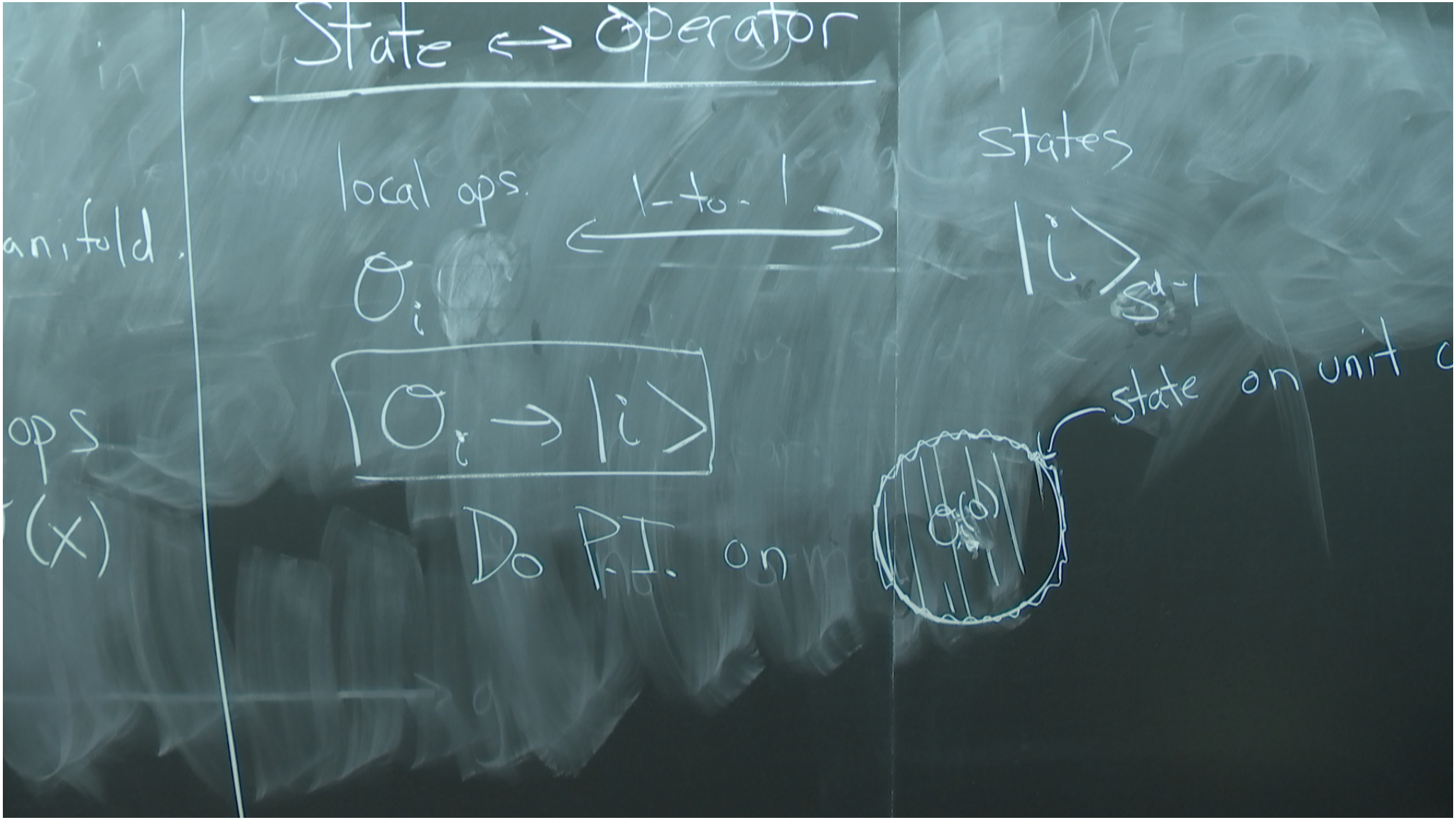
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other obs.
* $Z(M)$ ← manifold.
* non-local ops
$\int dx\, \mathcal{O}(x)$


other obs:
* $Z(M)$
manifold
* non-local ops
$\int dx\, O(x)$
State ↔ operator
local ops.
$O_i(x)$
1-to-1
states
$|i\rangle_{S^{d-1}}$


State ↔ Operator
local ops.
1-to-1
states
O_i
$|i\rangle_{S^{d-1}}$
$O_i \to |i\rangle$
Do P.I. on
state on unit
manifold.
ops
(x)

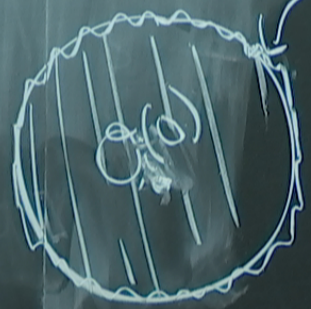
local ops. $\longleftrightarrow$ 1-to-1 $\longrightarrow$ states

O_i $\qquad$ $|i\rangle_{S^{d-1}}$

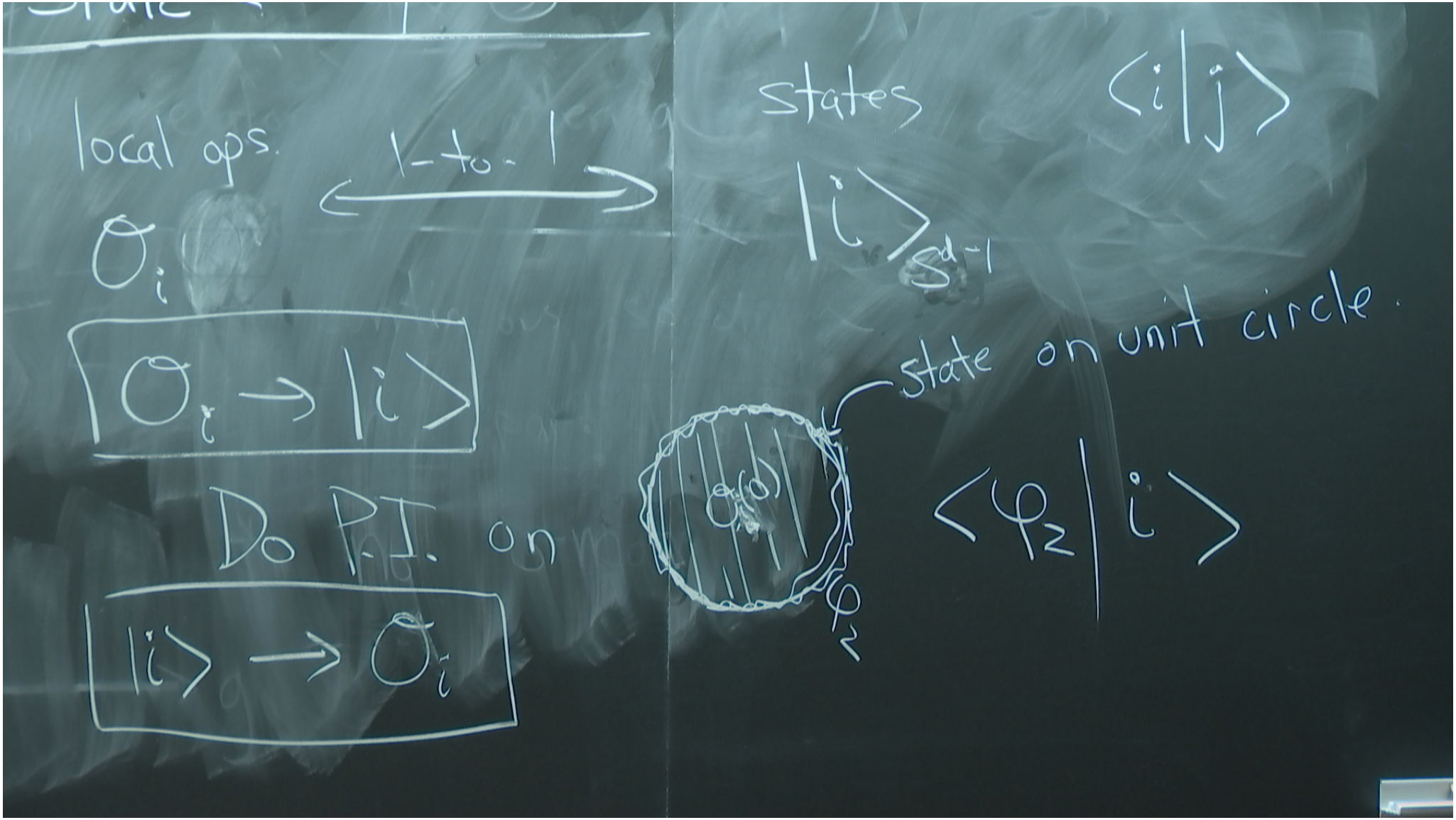
$O_i \to |i\rangle$

Do P.I. on

$|i\rangle \to O_i$



state on unit circle.


local ops.
1-to-1
states
$\langle i|j\rangle$
O_i
$|i\rangle_{S^{d-1}}$
$O_i \to |i\rangle$
state on unit circle.
Do P.I. on
$|i\rangle \to O_i$
$\langle \varphi_2 | i \rangle$

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* non-local ops

$\int dx\, \mathcal{O}(x)$

State ↔ operator

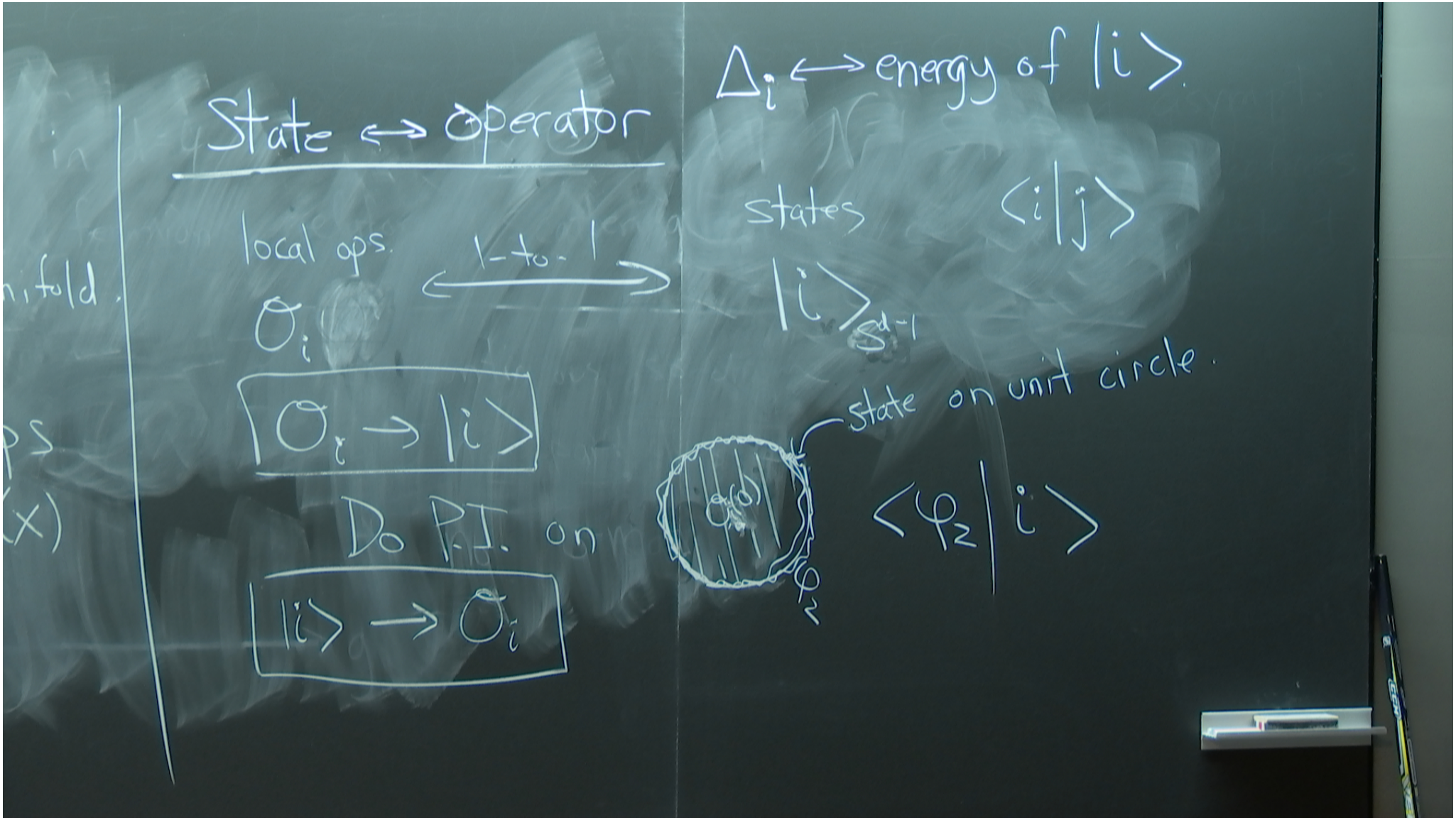
local ops $\mathcal{O}_i$ ⟷ 1-to-1 ⟷ states $|i\rangle_{S^{d-1}}$ $\langle i|j\rangle$

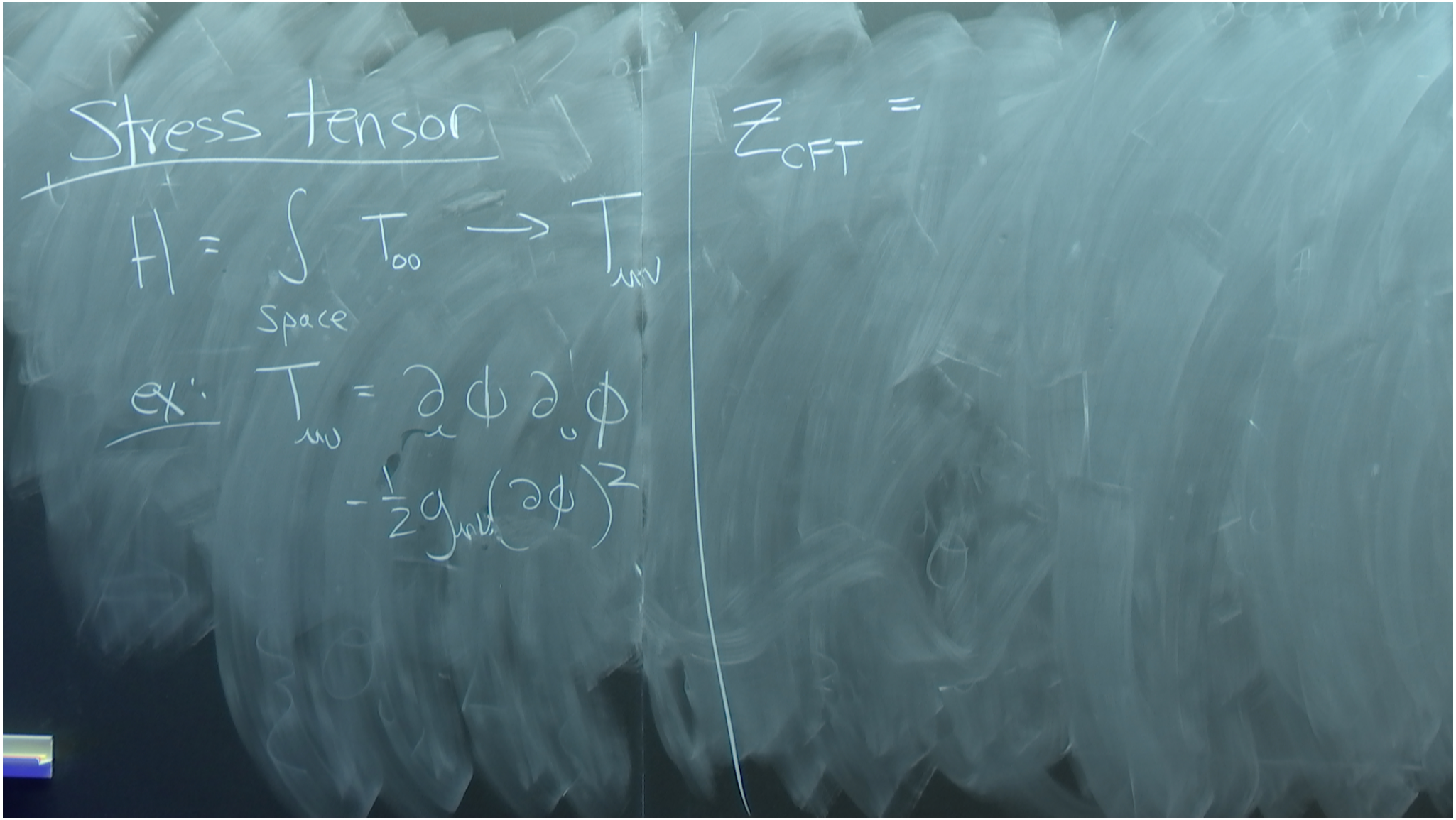
$\mathcal{O}_i \to |i\rangle$

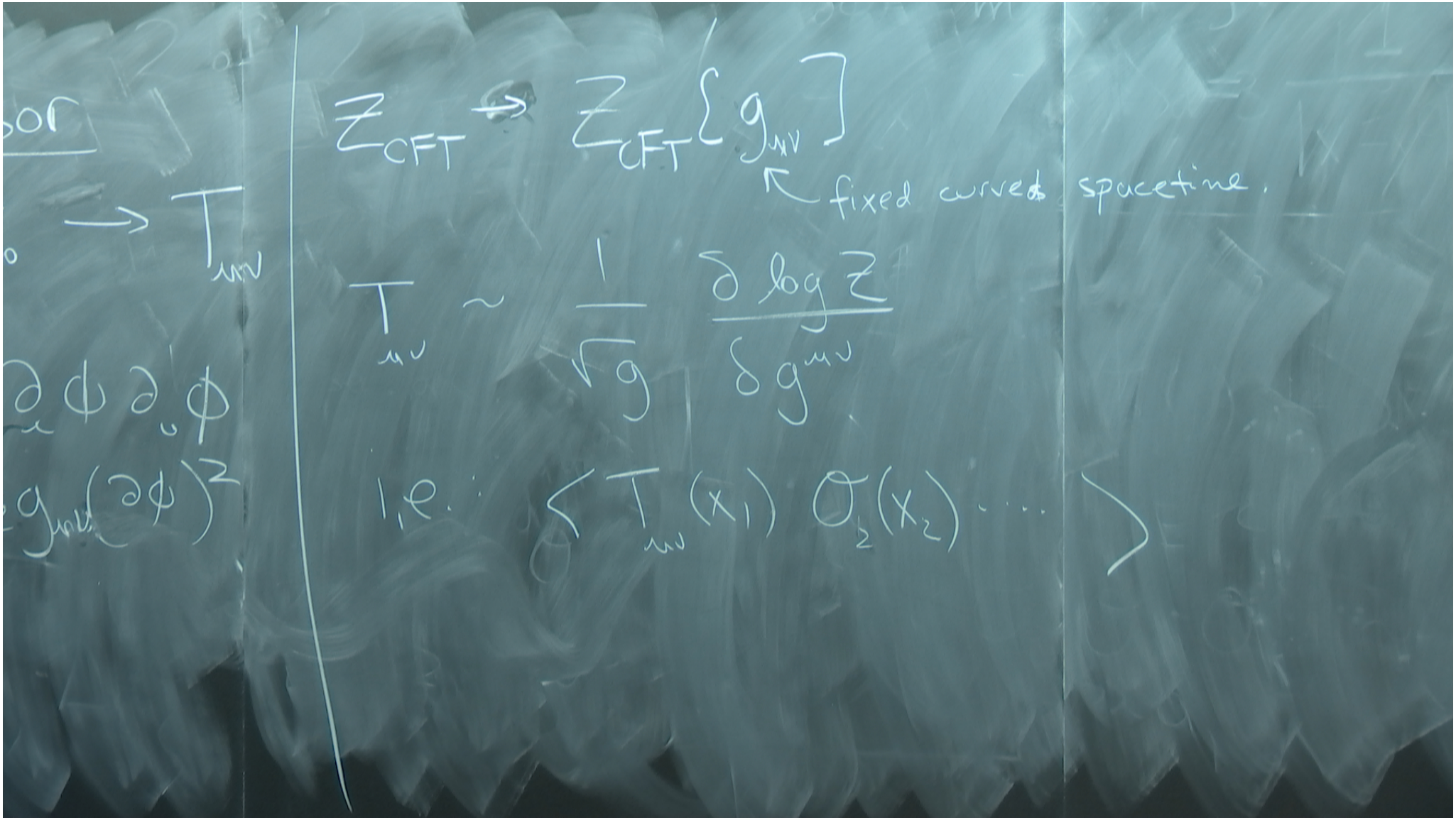
Do P.I. on

$|i\rangle \to \mathcal{O}_i$




State ↔ operator
local ops.
1-to-1
O_i
$O_i \to |i\rangle$
Do P.I. on
$|i\rangle \to O_i$
$\Delta_i \leftrightarrow$ energy of $|i\rangle$.
states
$\langle i|j\rangle$
$|i\rangle_{S^{d-1}}$
state on unit circle.
φ_2
$\langle \varphi_2 | i \rangle$


Stress tensor
$H = \int_{Space} T_{00} \longrightarrow T_{\mu\nu}$
ex: $T_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial \phi)^2$
$Z_{CFT} =$


$Z_{CFT} \Rightarrow Z_{CFT}[g_{\mu\nu}]$
fixed curved spacetime.
$T_{\mu\nu} \sim \frac{1}{\sqrt{g}} \frac{\delta \log Z}{\delta g^{\mu\nu}}$
i.e.: $\langle T_{\mu\nu}(x_1)\, \mathcal{O}_2(x_2) \cdots \rangle$
$\rightarrow T_{\mu\nu}$
$\partial_\mu \phi \partial_\nu \phi$
$g_{\mu\nu}(\partial\phi)^2$

$$Z_{CFT} \Rightarrow Z_{CFT}[g_{\mu\nu}]$$

↖ fixed curved spacetime.

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i.e.: $\langle T_{\mu\nu}(x_1)\, \mathcal{O}_2(x_2) \cdots \rangle_{CFT}$

$$= \frac{\delta}{\delta g^{\mu\nu}(x_1)} \langle \mathcal{O}(x_2) \cdots \rangle_{CFT[g]} \Big|_{g_{\mu\nu} = \delta_{\mu\nu}}$$

*

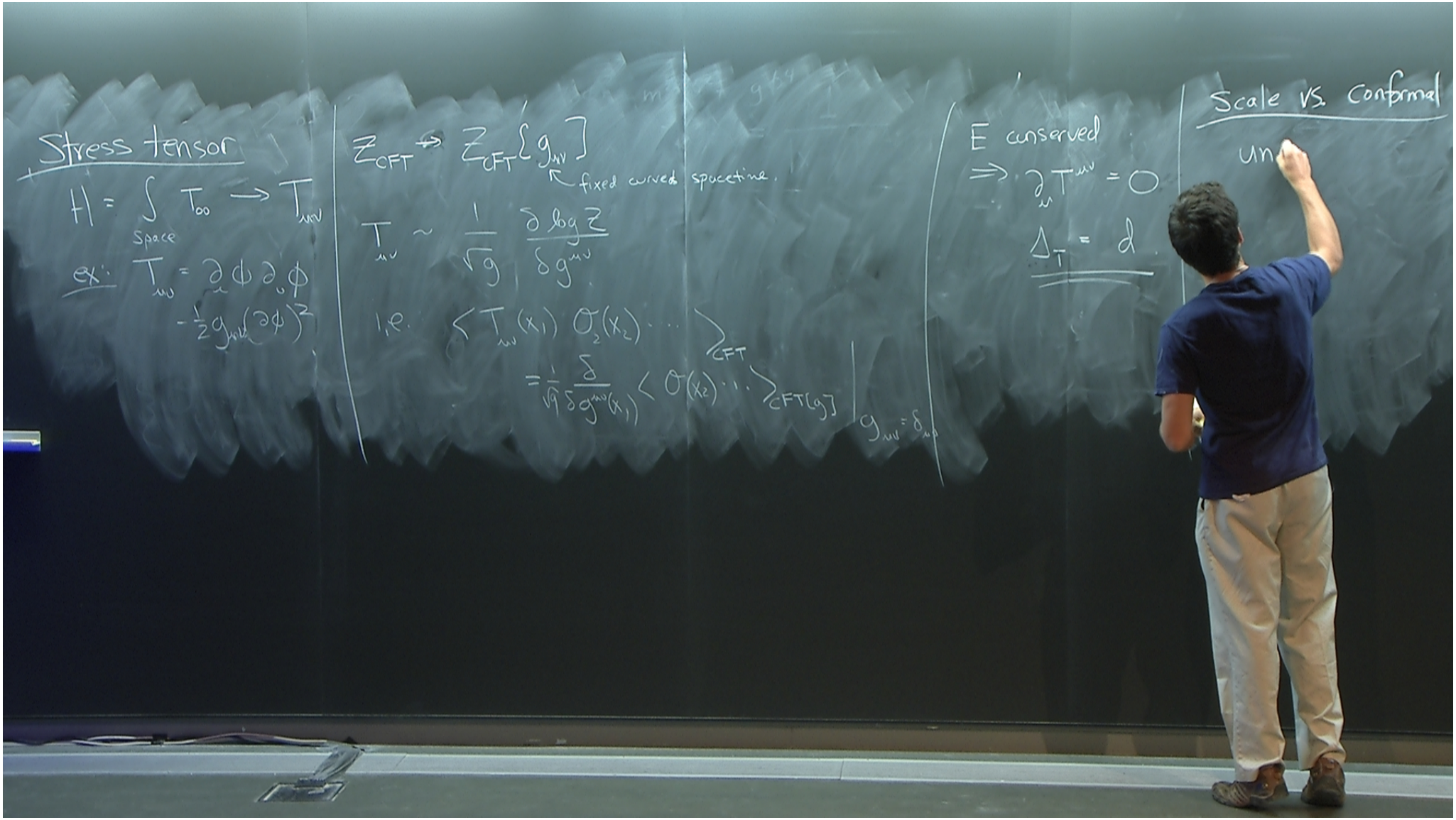
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E conserved
$\Rightarrow \partial_\mu T^{\mu\nu} = 0$
$\Delta_T = d$
Scale vs. Confrmal
un

conserved

$\partial_\mu T^{\mu\nu} = 0$

$\Delta_T = d$

Scale vs. Conformal

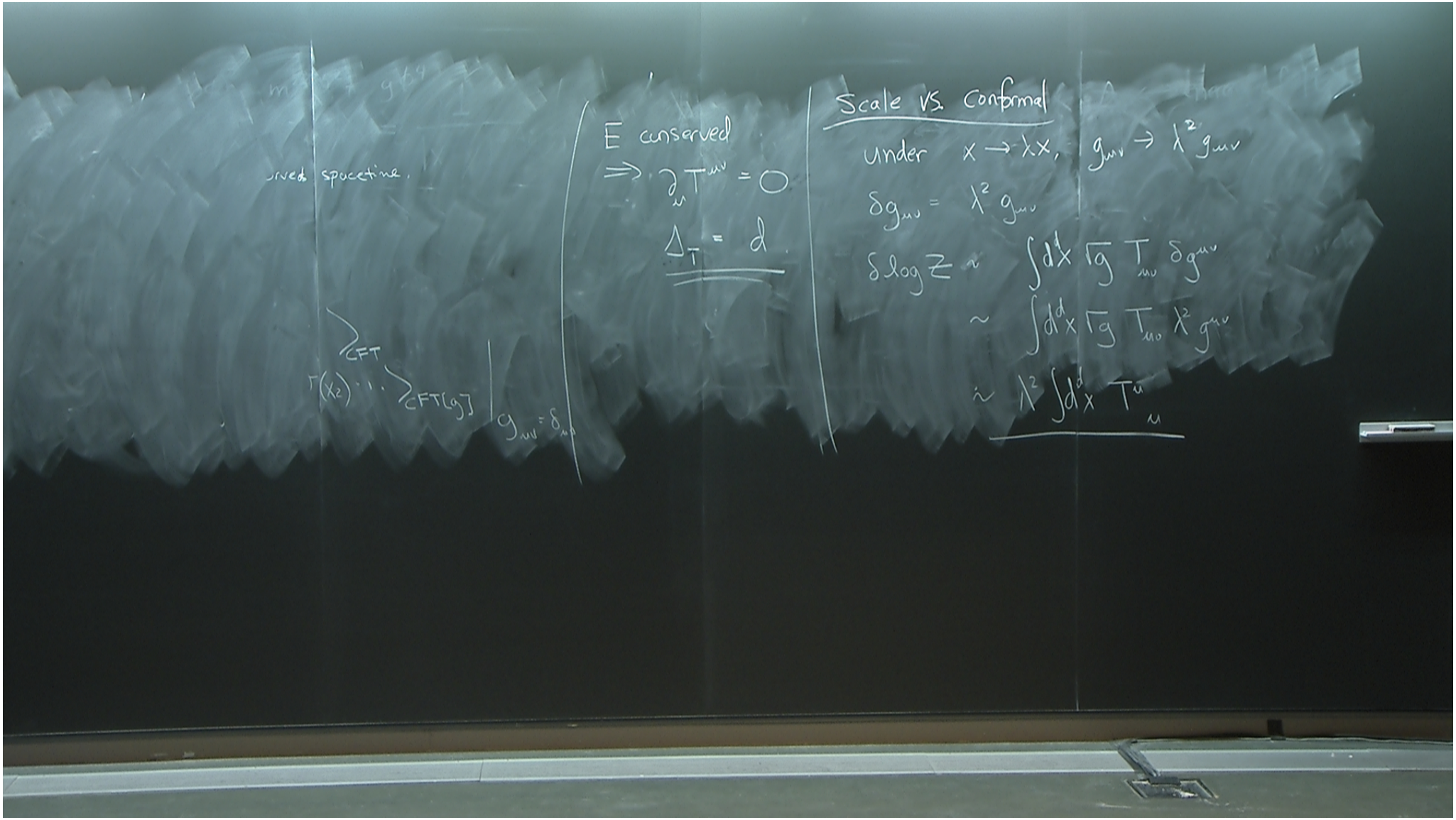
under $x \to \lambda x$, $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$

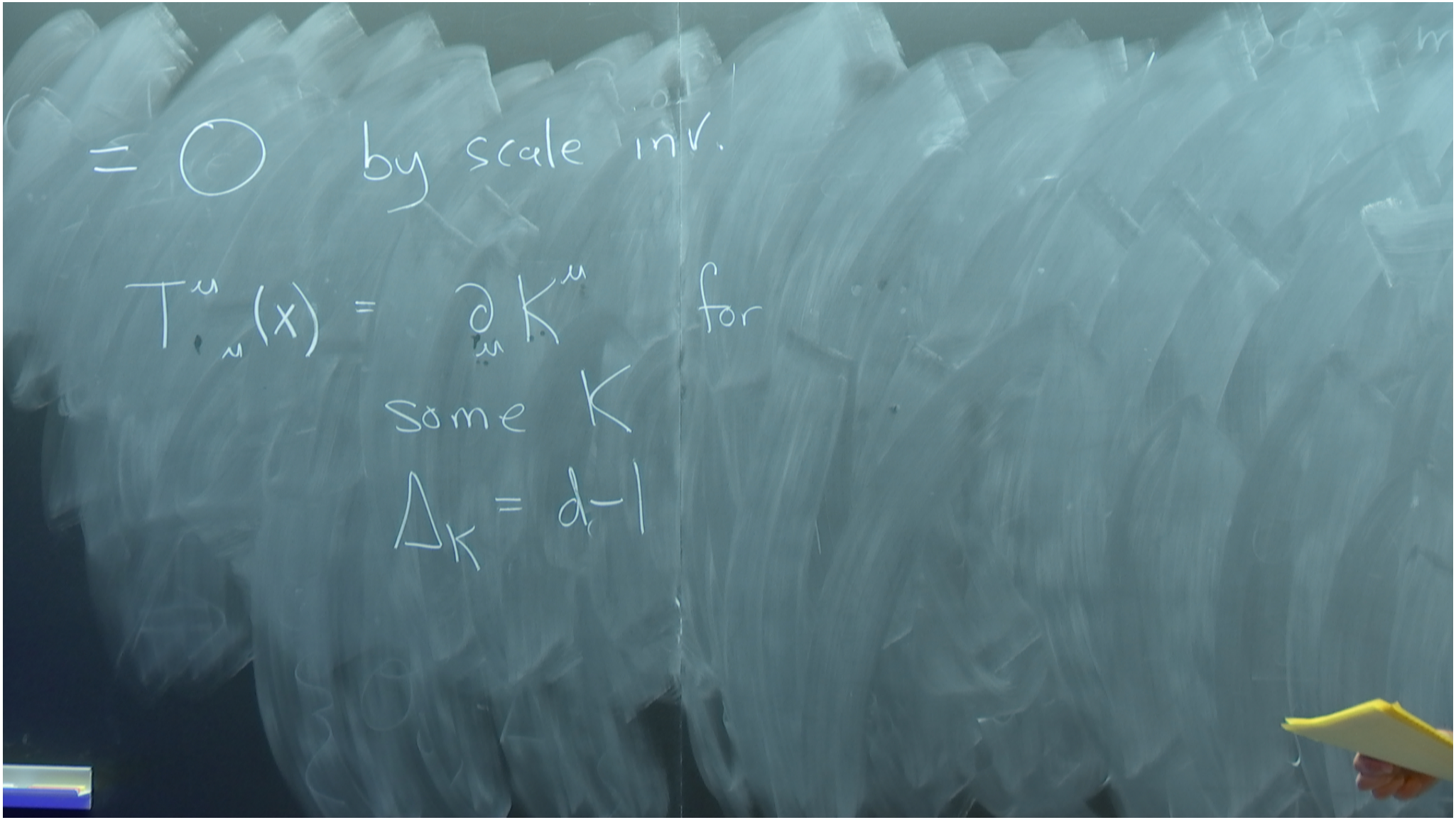
$\delta g_{\mu\nu} = \lambda^2 g_{\mu\nu}$

$$\delta \log Z \sim \int d^d x \sqrt{g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$$

$$\sim \int d^d x \sqrt{g}\, T_{\mu\nu}\, \lambda^2 g^{\mu\nu}$$

$$\sim \lambda^2 \int d^d x\, T^\mu{}_\mu$$


E conserved
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Scale vs. Conformal
under $x \to \lambda x$, $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$
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$\sim \int d^d x \sqrt{g}\, T_{\mu\nu} \lambda^2 g^{\mu\nu}$
$\sim \lambda^2 \int d^d x\, T^\mu{}_\mu$
$g_{\mu\nu} = \delta_{\mu\nu}$


$= 0$ by scale inv.
$T^{\mu}{}_{\mu}(x) = \partial_\mu K^\mu$ for
some K
$\Delta_K = d-1$

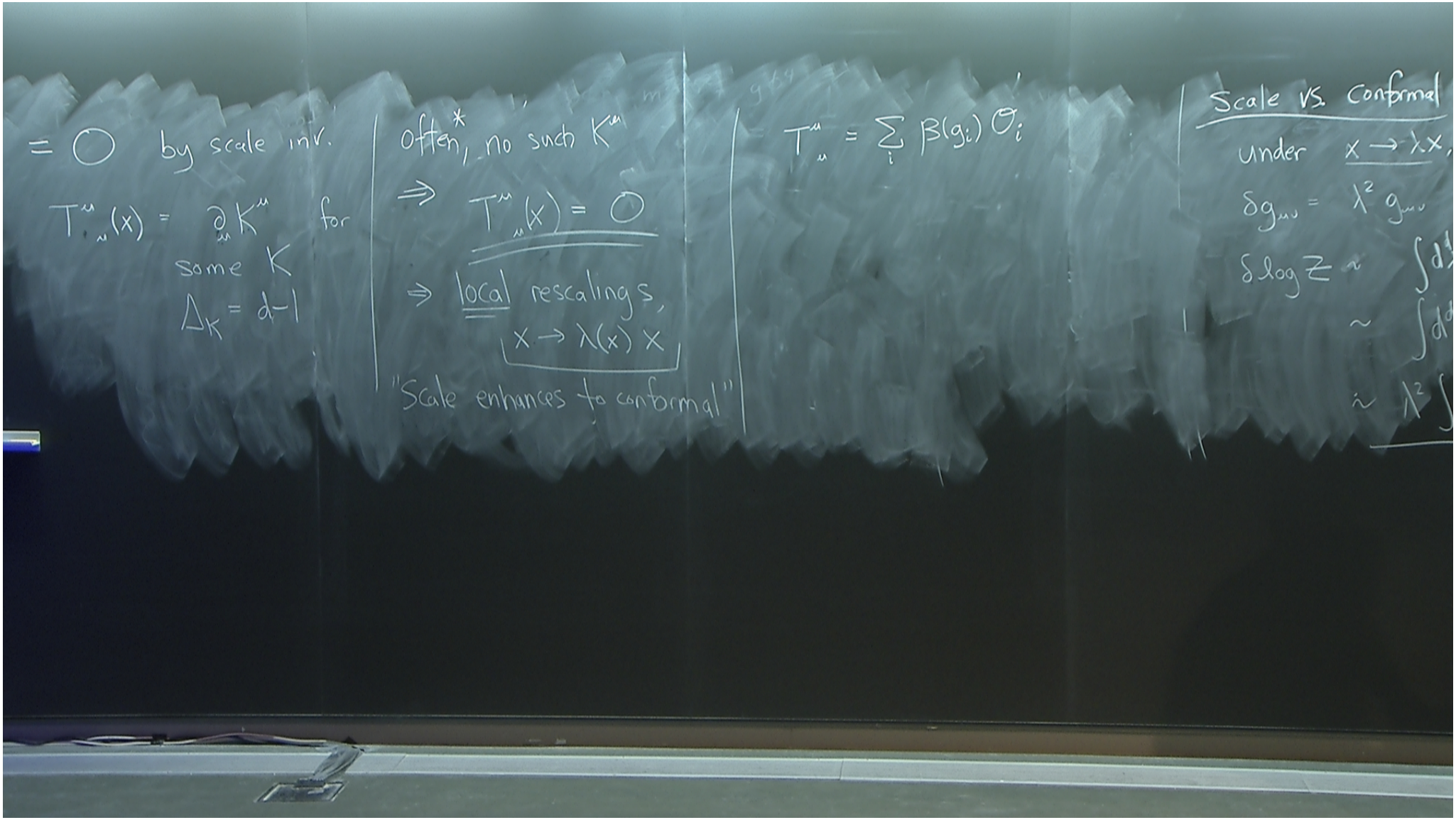
$= 0$ by scale inv.

$T^{\mu}{}_{\mu}(x) = \partial_{\mu} K^{\mu}$ for some K

$\Delta_K = d - 1$

often*, no such K^{μ}

$\Rightarrow$ $T^{\mu}{}_{\mu}(x) = 0$


= 0 by scale inv.
$T^\mu{}_\mu(x) = \partial_\mu K^\mu$ for some K
$\Delta_K = d-1$
Often*, no such K^μ
$\Rightarrow T^\mu{}_\mu(x) = 0$
$\Rightarrow$ local rescalings, $x \to \lambda(x)\, x$
"Scale enhances to conformal"
$T^\mu{}_\mu = \sum_i \beta(g_i)\, \mathcal{O}_i$
Scale vs. Conformal
under $x \to \lambda x$,
$\delta g_{\mu\nu} = \lambda^2 g_{\mu\nu}$
$\delta \log Z \sim \int d^d$
$\sim \int d^d$
$\sim \lambda^2 \int$