Title: Holographic complexity by design

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URL: http://pirsa.org/16070023

Abstract:

Holographic complexity
(by design)

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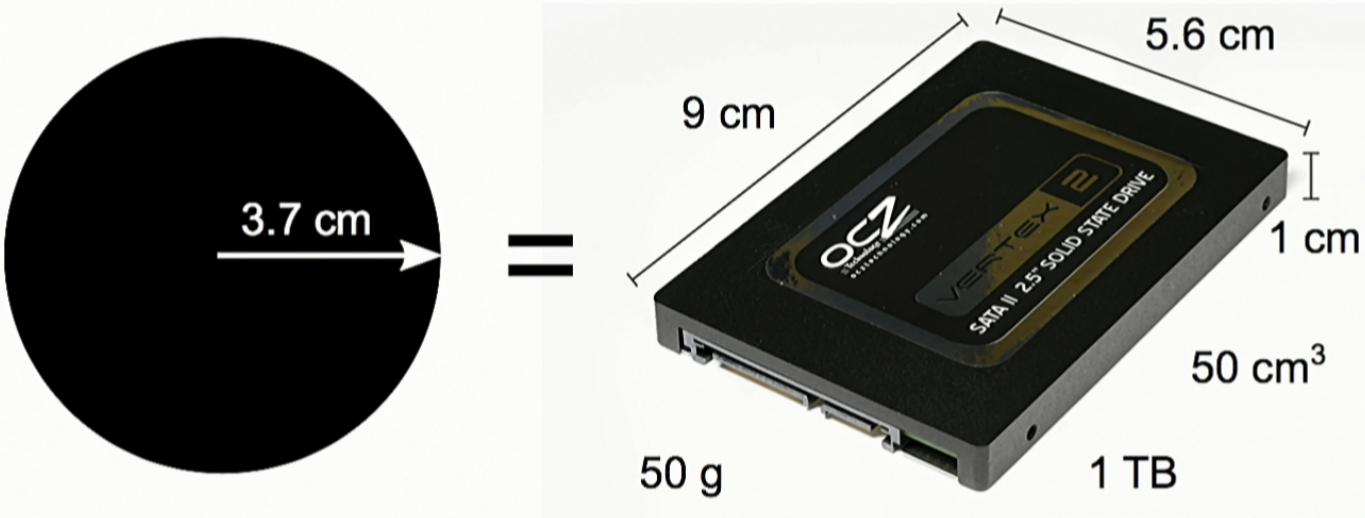
July 20, 2016

arXiv:1509.07876 with Adam Brown, Brian Swingle, Leonard Susskind, and Ying Zhao.
arXiv:1512.04993 with Adam Brown, Brian Swingle, Leonard Susskind, and Ying Zhao.
quant-ph/1608.????? with Beni Yoshida.

Conference summary: 1975



$$R = 2G_N M \qquad S_{BH} = \frac{A}{4G_N}$$

Conference summary: 1997

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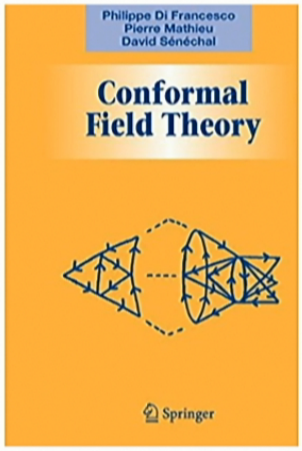

$$AdS = CFT$$

Conference summary: now

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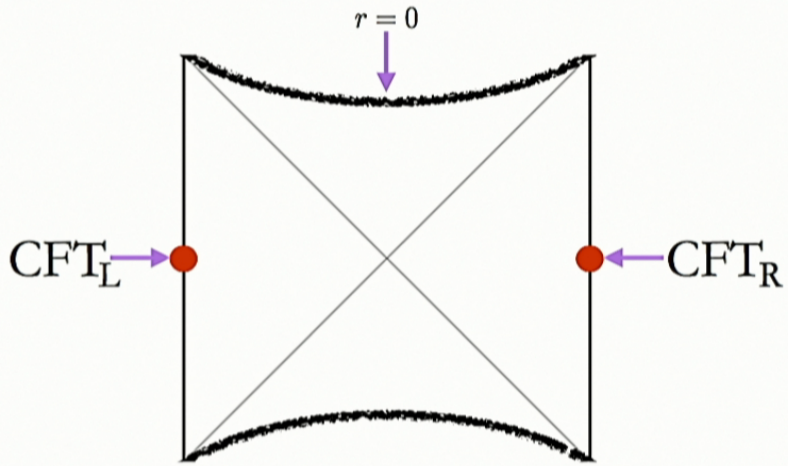
$$S_X = \min_x \frac{A(x)}{4G_N}\bigg|_{\partial X = \partial x}$$

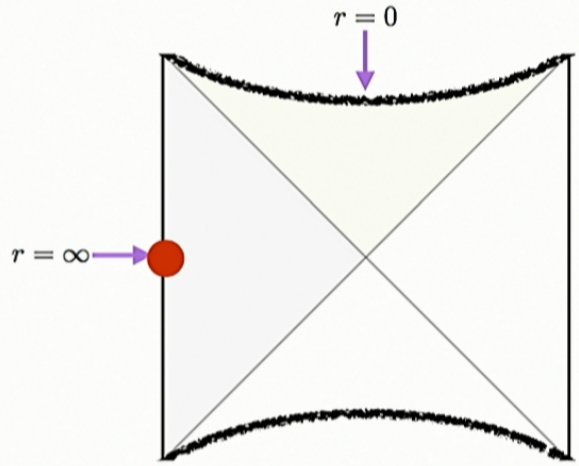
$$S_X = -\mathrm{tr}\left\{\rho_x \log \rho_x\right\}$$

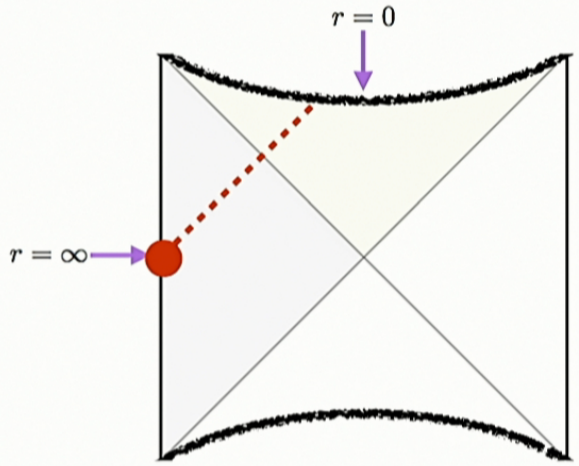


$$|\mathrm{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$$

Question:

- What clock ticks for the longest time in AdS?


$r = 0$
$r = \infty$


$r = 0$
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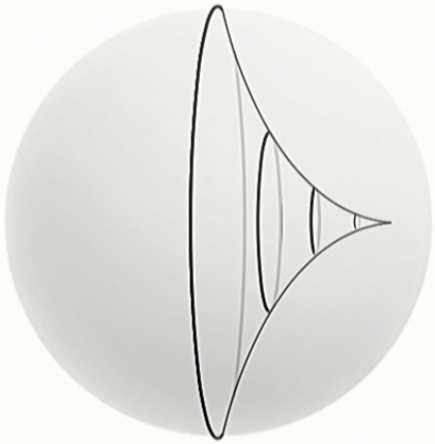
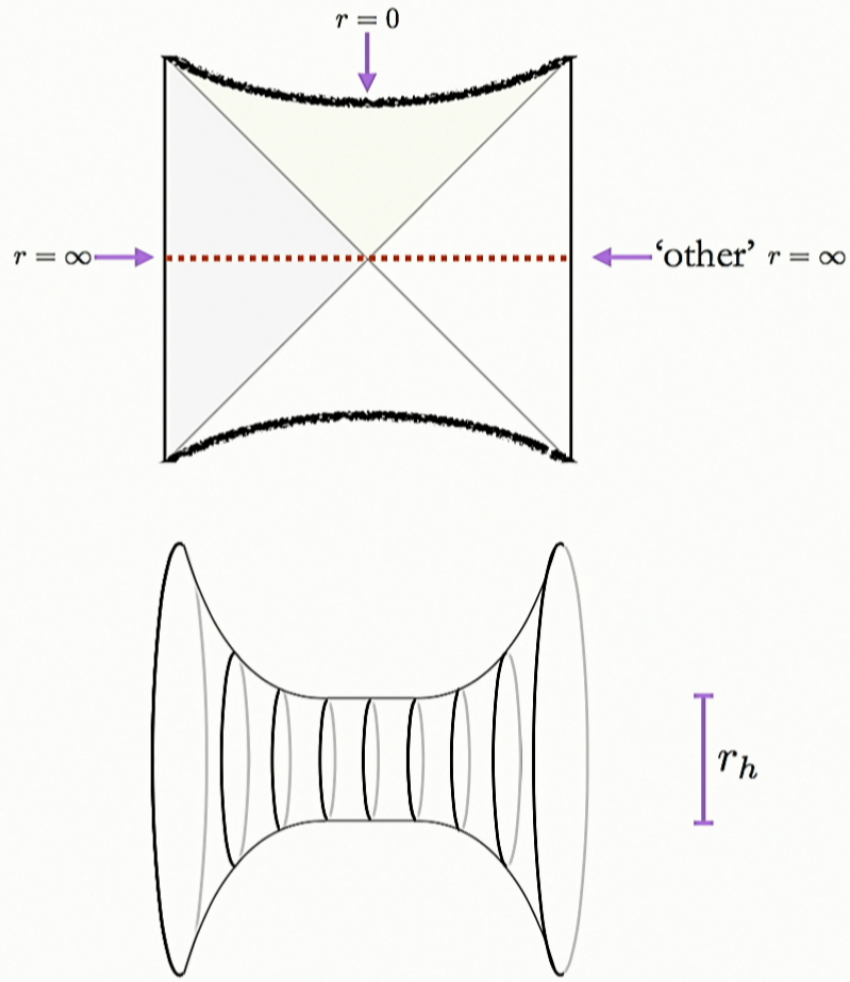
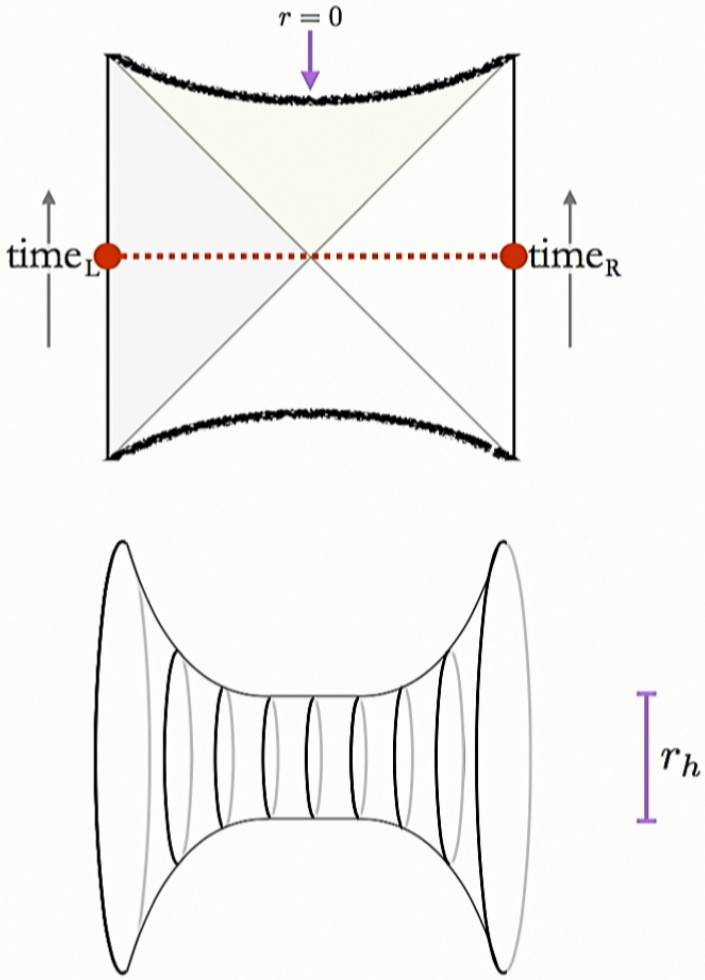
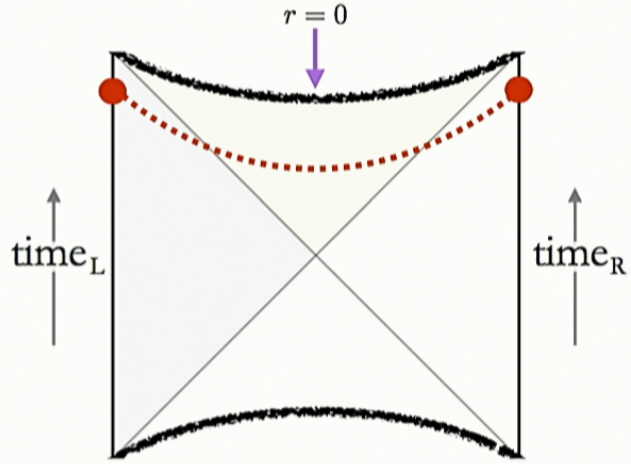
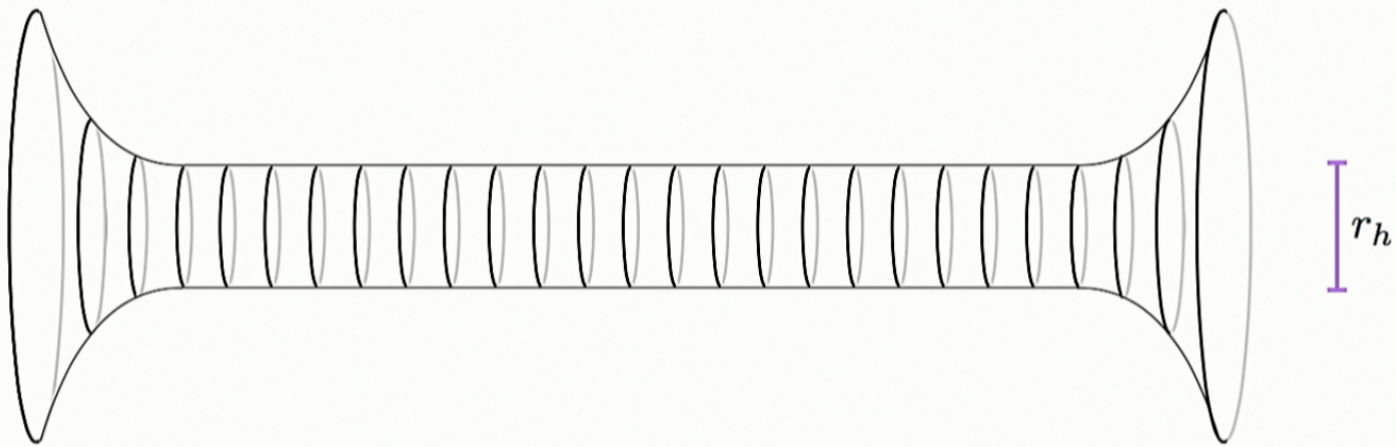



$r = 0$
$r = \infty$
'other' $r = \infty$
r_h


$r = 0$
time_L
time_R
r_h

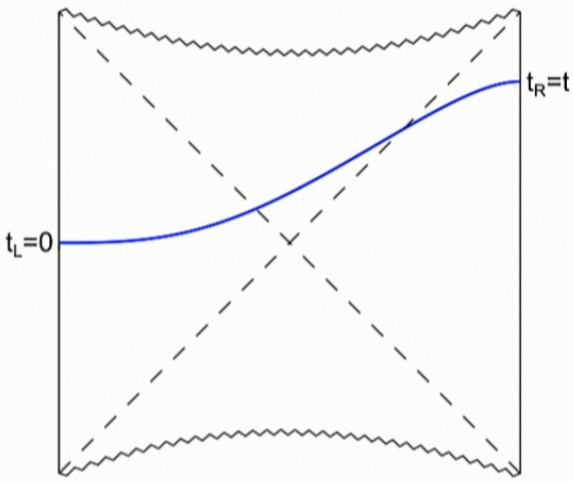


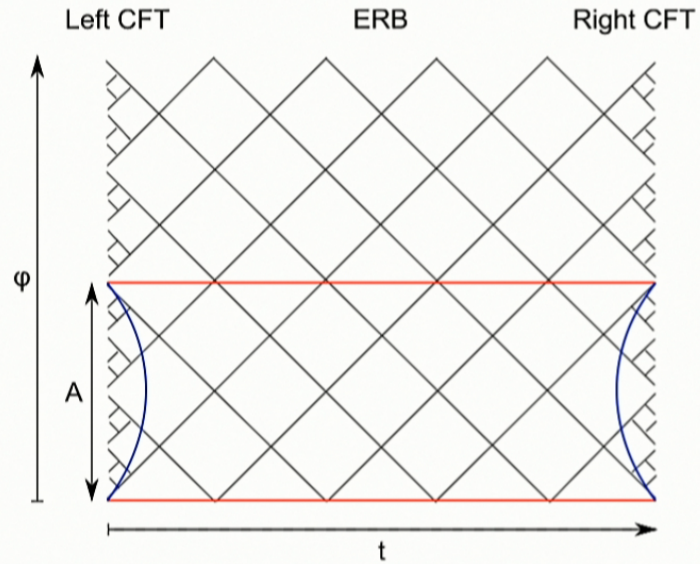


wormhole length $\sim t_L + t_R$

Entanglement entropy?

[Hartman/Maldacena] showed that the entanglement entropy of a subregion of linear size L of the thermofield double state grows linearly with time until saturating at time $O(L)$.





$$S_A \sim t, \qquad t < L,$$
$$S_A \sim L, \qquad t > L.$$

Question:

- What clock ticks for the longest time in a CFT?

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COMPLEXITY?

computational gate complexity of a quantum state

e.g. $|\psi(t_L, t_R)\rangle = \sum_i e^{-\beta E_i/2 + iE_i(t_L+t_R)} |E_i\rangle_L |E_i\rangle_R$

DEFINITION? starting in a reference state

e.g. $|\mathrm{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$

how many fundamental gates

e.g. unitaries each of which

act only on two-qubits

are needed to make the state

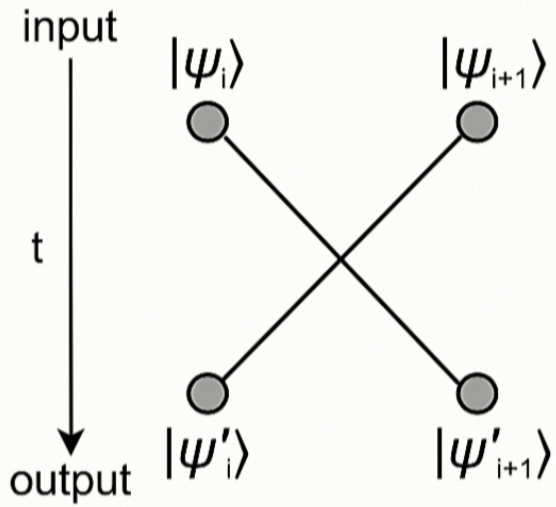
e.g. to within an accuracy ϵ

- can be exponentially large (Hilbert Space is huge)
- expected to grow linearly (at early times)

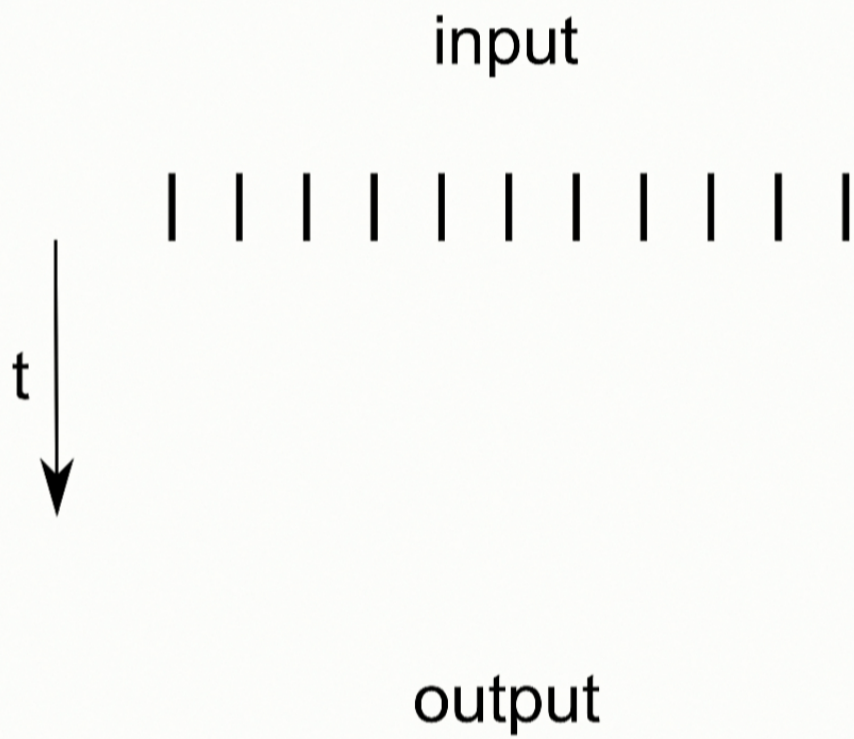
Complexity of time evolution

Discretize $U(t) = e^{-iHt}$ with a set of 2-body unitaries, or gates

$$U^{(i,i+1)} = \sum_{jk} u_{jk}^{(i,i+1)} |j\rangle\langle k| \approx e^{-ih_{i,i+1}\delta}$$



Complexity of time evolution: $e^{-0iH\delta}$

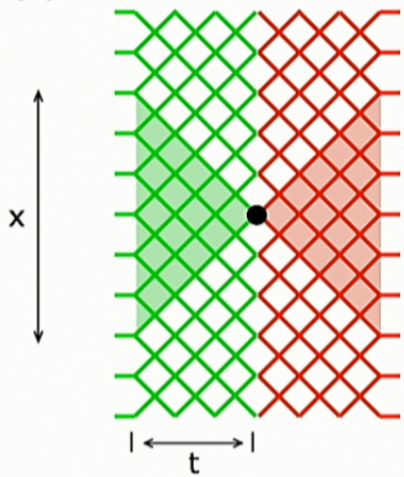


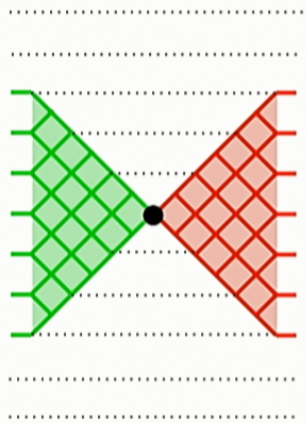
complexity ~ size of wormhole?

EVIDENCE:
- both expected to grow linearly (at early times)
- can be exponentially large (max out at same time)

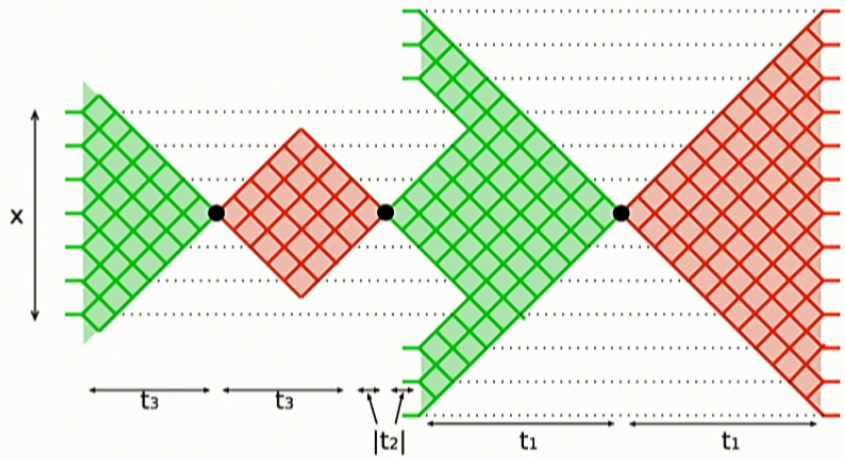
Complexity means minimal

$$W(t) = e^{-Ht}\, W\, e^{Ht}$$





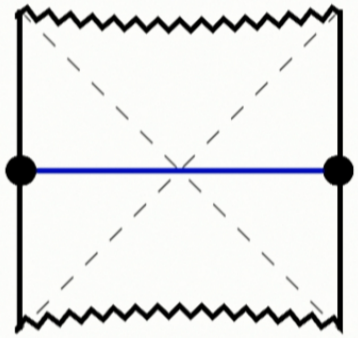
Complexity means minimal



$$W(t_3)W(t_2)W(t_1)$$

Black holes and the butterfly effect

Start with the thermofield double state.

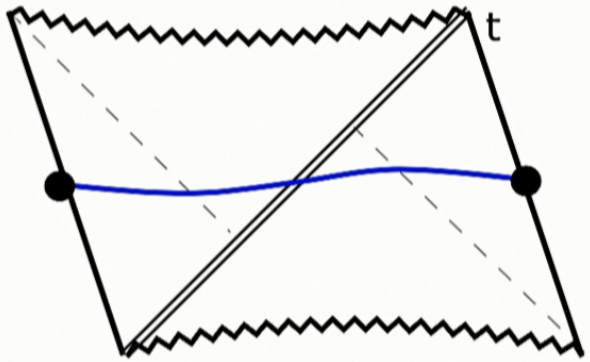


$|TFD\rangle$

[Shenker/Stanford][1000]

Black holes and the butterfly effect

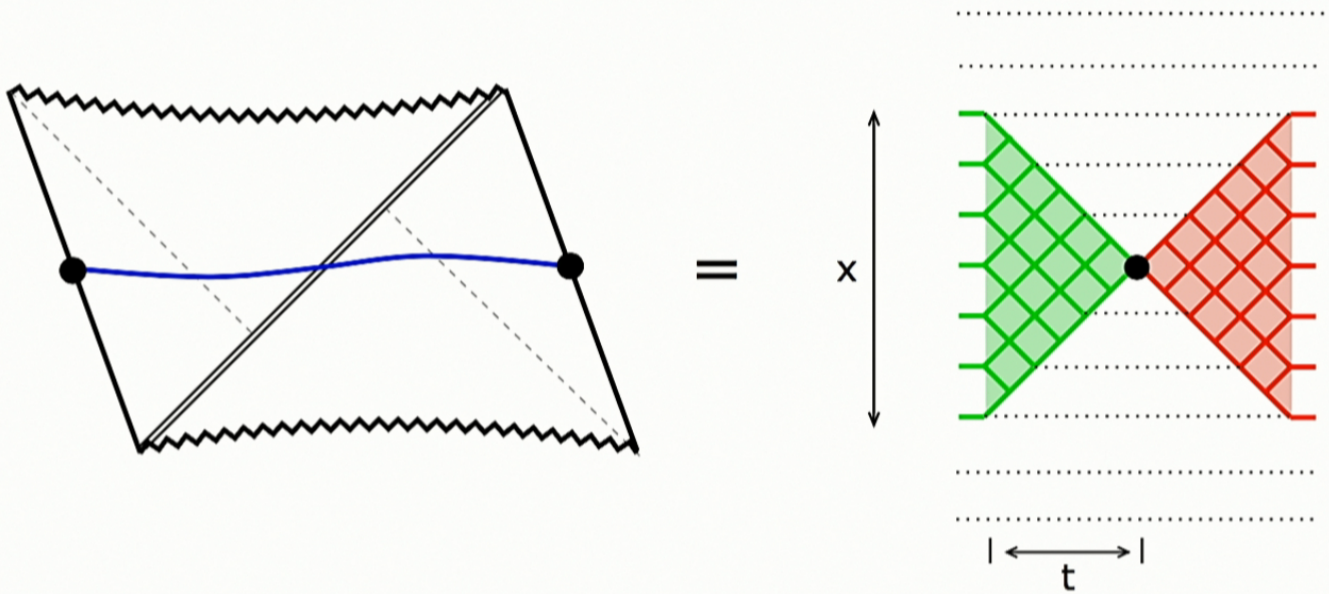
Start with the thermofield double state. Evolve to time t on the right boundary, act with an operator W at position x. Finally, evolve back to time $t_R = 0$.



$$e^{iHt}\, W\, e^{-iHt} |TFD\rangle = W(t) |TFD\rangle$$

[Shenker/Stanford][1000]

Cancellations in the ERB



[Stanford/Susskind and DR/Stanford/Susskind]

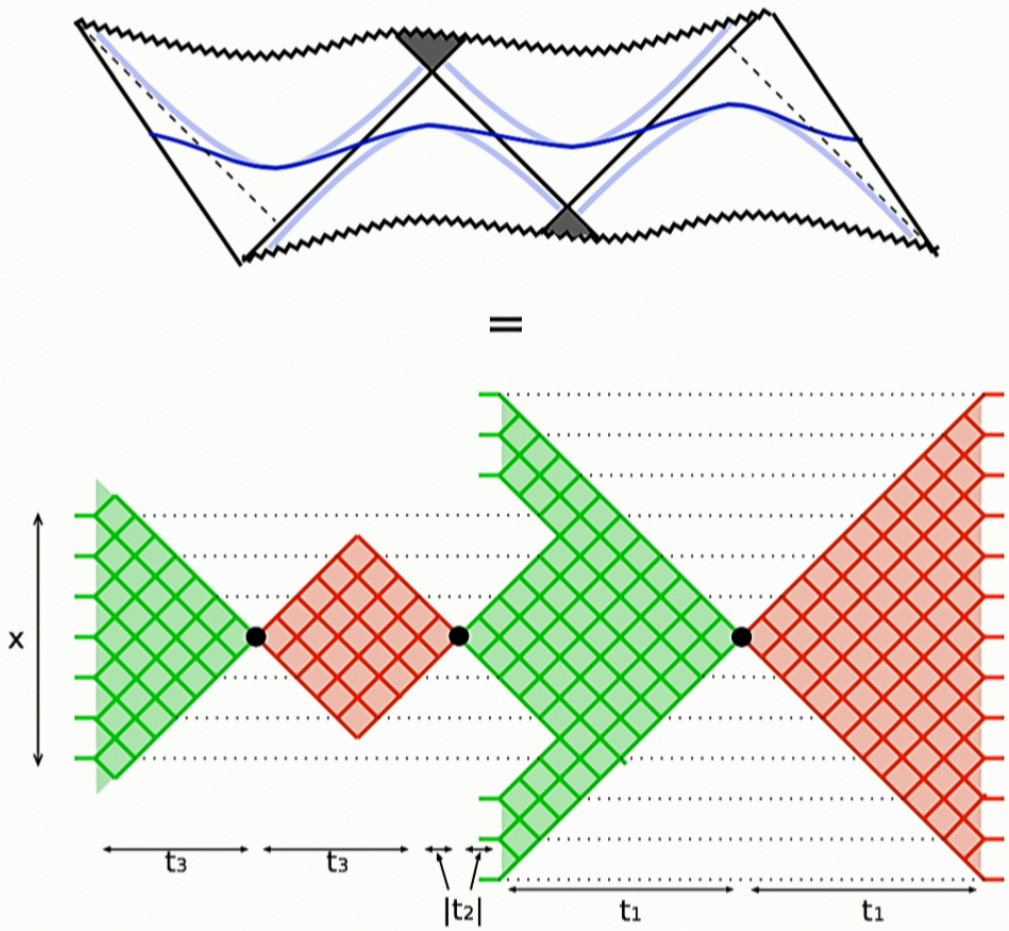
complexity ~ size of wormhole?

EVIDENCE:

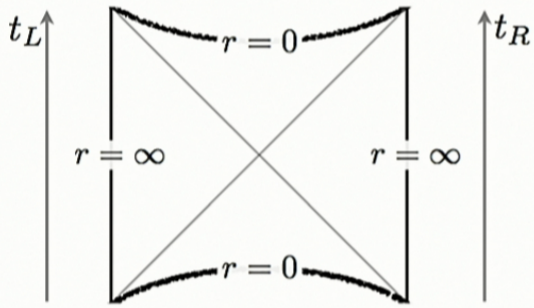
- both expected to grow linearly (at early times)
- can be exponentially large (max out at same time)
- perturb black hole with boundary operator
  - in CFT, leads to increase in complexity (chaos)
  - in wormhole, leads to shockwave, increases size

    the two increases match
    including delicate cancellations!
- single/multiple/localized perturbations

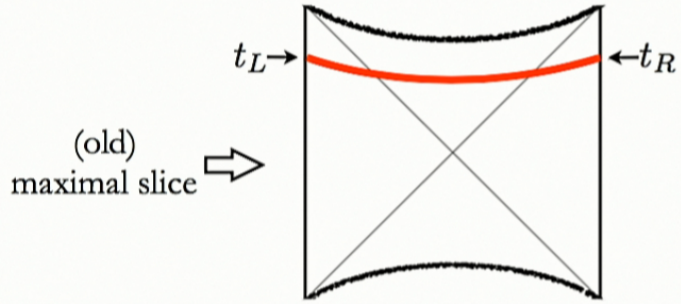
Multiple cancellations in the ERB



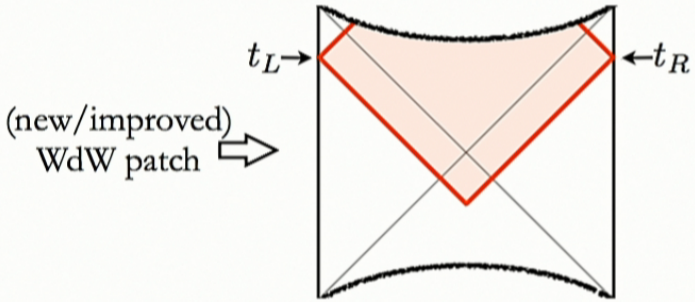
how to characterize size of wormhole?



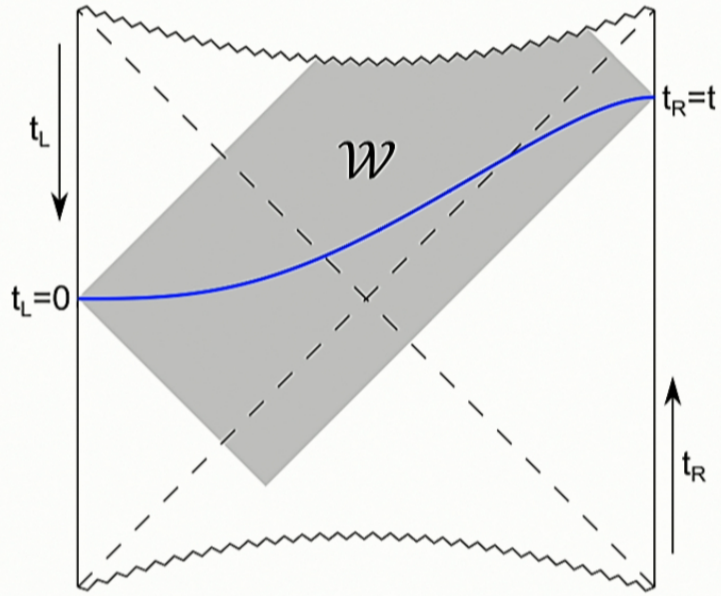
$\frac{1}{G\ell_{\mathrm{AdS}}}$ **volume**



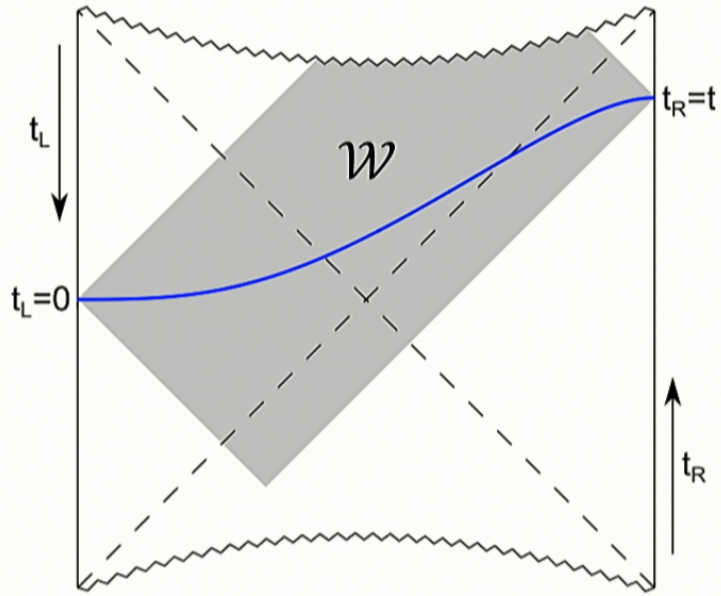
$\frac{1}{\hbar}$ **action**



Wheeler-DeWitt patch

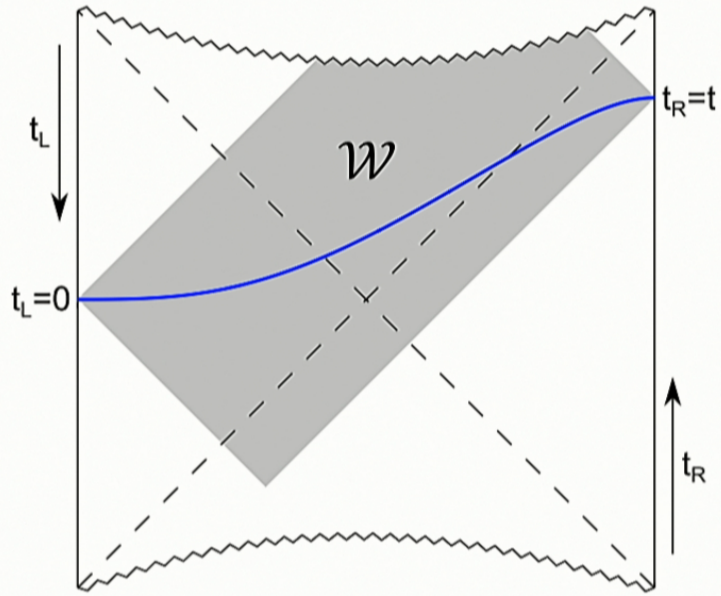


Conjecture



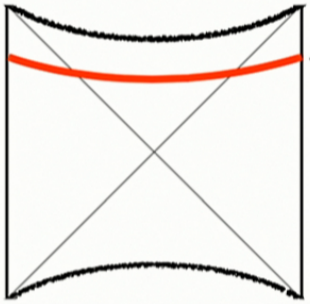
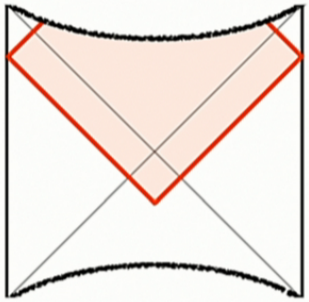
$$\text{Complexity} = \frac{\text{Action}(\mathcal{W})}{\pi\hbar}$$

(Another) conjecture



$$\frac{d\,\mathrm{Action}(\mathcal{W})}{dt} \leq 2M$$

(Neutral) black holes are fastest computers in nature.

$\frac{1}{G\ell_{\mathrm{AdS}}}$ **volume**  *vs.* $\frac{1}{\hbar}$ **action** 

prefer action because:

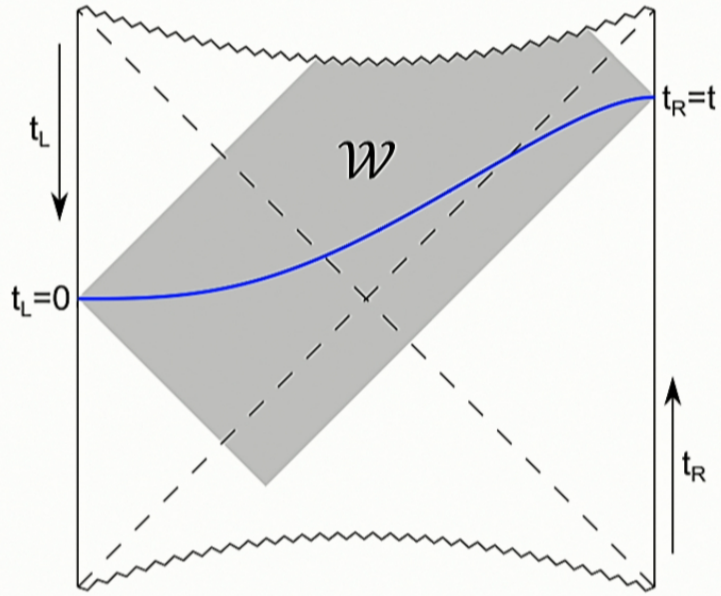
- naturally "dimensionless" (no arbitrary length scale)
- d(action)/dt = *2M* (independent of size and dimension)

connect to $\frac{d\mathrm{Complexity}}{dt} \leq \frac{2M}{\pi\hbar}$

black holes saturate limit implies

$$\mathrm{Complexity} = \frac{\mathrm{Action}}{\pi\hbar}$$

Wheeler-DeWitt patch



$$\text{Action}(\mathcal{W}) = \frac{1}{16\pi G_N}\int_{\mathcal{W}}\sqrt{-g}\,(\mathcal{R}-2\Lambda) + \frac{1}{8\pi G_N}\int_{\partial\mathcal{W}}\sqrt{|h|}K,$$

Complexity Equals Action ?

FURTHER WORK:

- **precise definition of complexity?**
- precise definition of action?
- relate imprecision in two definitions?
- reference state? ("complexity of formation")
- classical proof that black holes maximize action?
- more general black holes?
- higher-derivative theories and singularities?
- principle of least computation?
- complexity and horizon transparency?
- lots of puzzles!

Complexity lower bounds?

How many circuits can we make with g gates acting on n qubits with Δ steps?

There are $g\binom{n}{2}$ choices to make at each step, therefore we can make at most

$$\#\ \text{circuits} = (gn^2)^{\Delta}.$$

If we have a collection of different circuits we want to make $\mathcal{E}$, then if we want to ensure we can make all of them, we can determine the minimal number of steps

$$\Delta \geq \frac{\log|\mathcal{E}|}{\log(gn^2)}.$$

How random?

For an ensemble $\mathcal{E}$, we can characterize how generic it is by using the frame potential

$$F_{\mathcal{E}}^{(k)} = \frac{1}{|\mathcal{E}|^2} \sum_{U,V \in \mathcal{E}} |\mathrm{tr}\{V^\dagger U\}|^{2k}.$$

The frame potential is maximized by $\mathcal{E} = I$ and minimized by the Haar ensemble

$$F_{Haar}^{(k)} < F_{\mathcal{E}}^{(k)} < F_{I}^{(k)}.$$

If for an ensemble $F_{Haar}^{(k)} < F_{\mathcal{E}}^{(k)}$, then that ensemble is a k-design. It will reproduce k moments of the Haar ensemble.

Complexity and chaos

For any ensemble $\mathcal{E}$ of unitary operators, with $\tilde{B} = U^\dagger B U$

$$\frac{1}{2^{4kn}} \sum_{A_1,\cdots,B_1,\cdots} \left| \langle A_1 \tilde{B}_1 \cdots A_k \tilde{B}_k \rangle_{\mathcal{E}} \right|^2 = \frac{1}{2^{2(k+1)n}} \cdot F_{\mathcal{E}}^{(k)}$$

where summations are over all possible Pauli operators.

If we assume H is generic, then our ensemble could be of Hamiltonians. A disorder average of out-of-time-order correlators would give the frame potential. If the system self-averages, then we might use this to lower-bound the complexity for a particular system.