

Title: Focus Lecture

Date: Jul 20, 2016 05:00 PM

URL: <http://pirsa.org/16070021>

Abstract:

# BLACK HOLE THERMODYNAMICS

a) MOTIVATION: SCHWARZSCHILD BH.

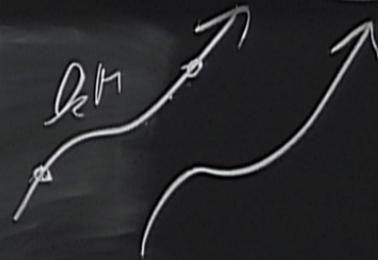
$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$f = 1 - \frac{2m}{r}$$
$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

ASYMPTOTIC MASS (TOTAL ENERGY)

NOETHER: SYMMETRY  $\leftrightarrow$  CONSERVED Q.  
 $\leftarrow$  KILLING FIELDS

$$\nabla_{\mu} k_{\nu} + \nabla_{\nu} k_{\mu} = 0$$

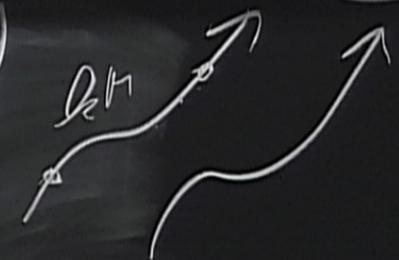
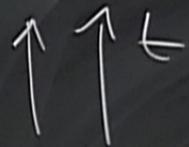


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SPEC.  $k = \partial_t$



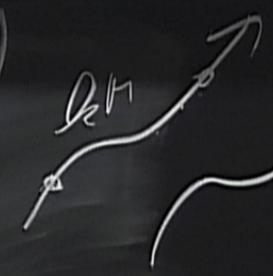
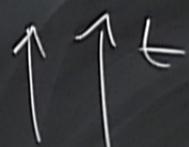
SPACETIME IS STATIC  $\Rightarrow$  ENERGY IS CONSERVED

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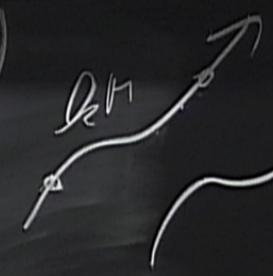
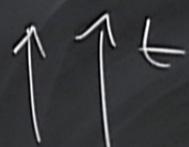
$$M = -\frac{1}{8\pi} \int_{S_{\infty}^2} *dk = M.$$

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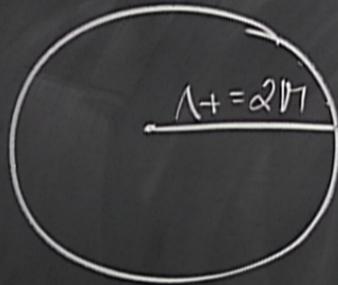


SPACETIME IS STATIC  $\Rightarrow$  ENERGY IS CONSERVED

$$M = -\frac{1}{8\pi} \int_{S_{\infty}^2} *dk = M.$$

• BLACK HOLE HORIZON  
(BOUNDARY OF BH)

$$\boxed{f=0} \Leftrightarrow \boxed{r=r_+=2M}$$



$$r_+ = 2M$$

CAUTION

DO NOT STAND ON THE BOARD. PLEASE STAY ON THE FLOOR OF THE ROOM.

IT IS PROHIBITED TO SMOKE OR DRINK IN THE ROOM.

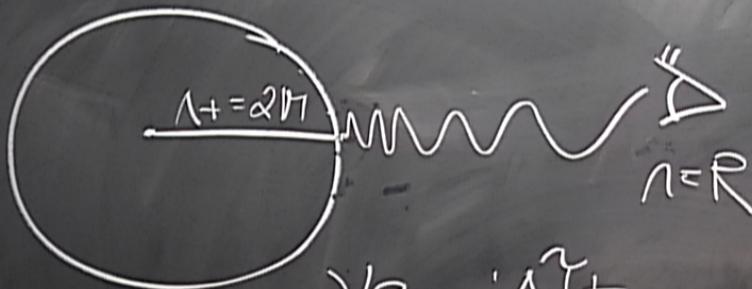
PLEASE KEEP THE BOARD CLEAN.

• BLACK HOLE HORIZON  
(BOUNDARY OF BH)

$$f=0$$

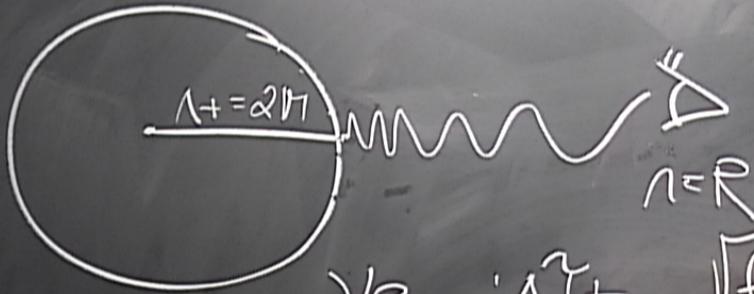
$\Leftrightarrow$

$$r=r_+=2M$$



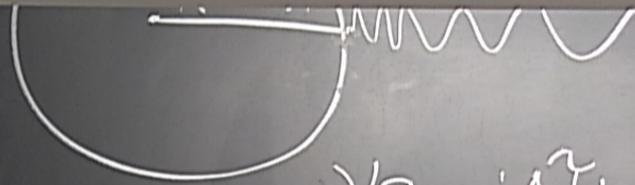
$$\frac{v_R}{v_+} = \frac{\Delta r_+}{\Delta r_R}$$

(BOUNDARY OF BH')



$$\frac{\nu_R}{\nu_+} = \frac{\Delta t_+}{\Delta t_R} = \frac{\sqrt{1 + \frac{\Delta t}{r_+}}}{\sqrt{1 - \frac{2M}{R}}} \rightarrow 0$$

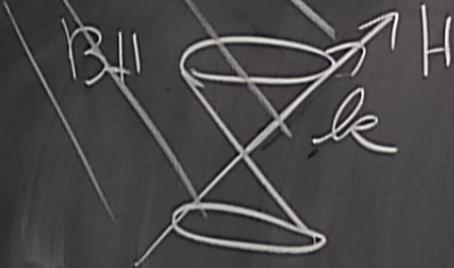
THINGS LOOK INFINITELY REDSHIFTED  $\Rightarrow$  BLACK



$$\frac{\nu_R}{\nu_+} = \frac{\Delta\tau_+}{\Delta\tau_R} = \frac{\sqrt{f_+ \Delta t}}{\sqrt{f_R \Delta t}} = \frac{\sqrt{1 - \frac{2m}{r_+}}}{\sqrt{1 - \frac{2m}{R}}} \rightarrow 0$$

THINGS LOOK INFINITELY REDSHIFTED  $\Rightarrow$  BLACK

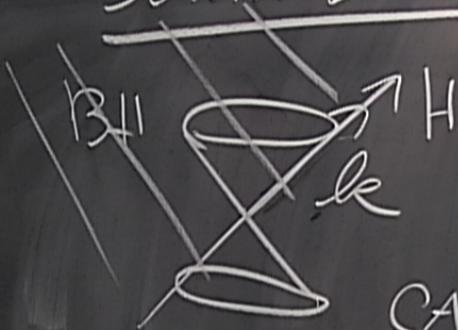
SURFACE GRAVITY: KILLING HORIZON: NULL SURFACE  
 BY KILLING FIELD  $k = \partial_t$



$$\boxed{k^\mu \nabla_\mu k^\nu = \underset{\uparrow}{\kappa} k^\nu}$$

CAUTION  
 DO NOT TOUCH THE BOARD  
 AT AN UNDESIRABLE TIME  
 YOUR INSTRUCTOR WILL  
 BE VERY ANGRY

SURFACE GRAVITY: KILLING HORIZON: NULL SURFACE  
 BY KILLING FIELD  $k = \partial_t$



$$k^\mu \nabla_\mu k^\nu = \underbrace{\partial_r k^\nu}_{\uparrow}$$

CAN SHOW

$$\partial_r = \frac{f'(r_+)}{2} = \frac{1}{2} \frac{2M}{r_+^2} = \frac{1}{4M} = \frac{1}{2r_+}$$

CAUTION  
 DO NOT TOUCH THE BOARD  
 IF AN EMERGENCY CALL  
 911 OR 911-1111  
 PLEASE PRESS 1111

# NEWTONIAN ACCELERATION

$$\mathcal{R} = \frac{M}{r^2} = \frac{1}{4M}$$

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$$\mathcal{R} = \frac{M}{r^2} = \frac{1}{4M}$$

• HORIZON AREA  $dt=0$

$$\mathcal{R} = \frac{1}{r^2} = \frac{1}{4M^2}$$

• HORIZON AREA

$$dt=0=dr$$

$$d\sigma^2 = r^2 d\Omega^2$$

$$A = \int \sqrt{d\sigma^2} d\theta d\phi = \int r^2 \sin\theta d\theta d\phi = \underline{4\pi r^2}$$

$\mathcal{K} = \frac{2}{r^2} \quad \frac{4\pi}{r^2}$   
 • HORIZON AREA  $dt=0=dr$   $d\eta^2 = r^2 d\Omega^2$   
 $A = \int \sqrt{d\eta^2} d\theta d\varphi = \int r^2 \sin\theta d\theta d\varphi = \underline{4\pi r^2}$   
 • OBSERVATION:  $dM = \frac{1}{2\pi} \frac{dA}{4}$   
 $\Rightarrow \boxed{dM = \frac{\mathcal{K}}{2\pi} \frac{dA}{4}}$

$\mathcal{L} = \frac{1}{2} \dot{r}^2 - \frac{GM}{r}$

• HORIZON AREA  $dt=0=dr$   $d\eta^2 = r^2 d\Omega^2$

$A = \int \sqrt{dA} d\theta d\varphi = \int r^2 \sin\theta d\theta d\varphi = \underline{4\pi r^2}$

• OBSERVATION:  $dM = \frac{1}{4} dA$

$\Rightarrow \boxed{dM = \frac{\mathcal{L}}{2\pi} \frac{dA}{4}}$

$$\mathcal{R} = \frac{1}{r^2} = \frac{1}{4M^2}$$

• HORIZON AREA

$$dt=0=dr$$

$$d\gamma^2 = r^2 d\Omega^2$$

$$A = \int \sqrt{d\gamma^2} d\theta d\phi = \int r^2 \sin\theta d\theta d\phi = 4\pi r^2$$

• OBSERVATION:

$$dM = \frac{r}{2} \quad dA = 8\pi r dr$$

$$\Rightarrow \boxed{dM = \frac{\mathcal{R}}{2\pi} \frac{dA}{4}}$$

ROTATING ( $J$ ) CHARGED ( $Q$ )

0.  $\omega = \text{CONST}$

1.  $dM = \frac{\omega}{2\pi} \frac{dA}{4} + \text{WORK TERMS}$   
 $\omega d\phi = \Phi dQ$   $\Leftrightarrow dE = T dS + \text{work}$   
 $T \sim \omega$

2.  $dA \geq 0$

3. CANNOT REDUCE  $\omega \rightarrow 0$  IN A FINITE # OF STEPS

NEWTONIAN ACCELERATION

c) BLACK HOLE THERMODYNAMICS

WHEELER'S CUP OF TEA. WHERE DID THE

C) BLACK HOLE THERMODYNAMICS

WHEELER'S CUP OF TEA. WHERE DID THE ENTROPY GO?

BEKENSTEIN:

$$\boxed{S \sim A} \neq \boxed{\alpha \sim T}$$

WHEELER'S COPY

BEKENSTEIN:

$$S \sim A \quad \nabla \quad \partial \sim T$$

• HAWKING 74. WHEN QUANTUM EFFECTS TAKEN INTO ACCOUNT, BH RADIATES AWAY AS BLACK BODY.

$$T = \frac{\hbar \alpha}{2\pi k_B}, \quad S = \frac{A}{4\hbar G_H}$$

$$2^{-2} \pi^2 / + = \frac{1}{4M} = \frac{1}{2A+}$$



BEKENSTEIN:

$$S \sim A \quad \neq \quad |\alpha \sim |$$

- HAWKING 79. WHEN QUANTUM EFFECTS TAKEN INTO ACCOUNT, BH RADIATES AWAY AS BLACK BODY.

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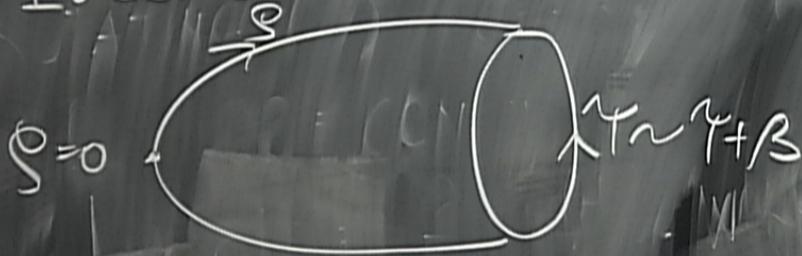
$$\alpha = \frac{r(1+)}{2} = \frac{1}{2} \frac{2M}{r^2} \Big|_+ = \frac{1}{4M} = \frac{1}{2r_+}$$

$$T = \frac{\hbar \mathcal{H}}{2\pi k_B}, \quad S = \frac{A}{4\hbar G_N}$$

DER.  
• EUCLIDEAN PI,  
TUNNELLING,  
LQC, STRING T.

d) EUCLIDEAN TRICK

EUCLIDEAN SEHK: - GIGAR



• PARTITION FUNCTION

$$Z = \int Dg e^{-S_E(g)} \approx \int \mathcal{L}^{-S_E(g_c)}$$
$$S_E = \int \frac{d^4x \sqrt{g} R}{16\pi G_N}$$

CAUTION

$$S_E = S_E(\text{CIGAR}) - S_E(\text{FLAT}) = \frac{\beta M}{2}$$

• FREE ENERGY  $F = -\frac{1}{\beta} \log Z = M/2$

ENTROPY  $S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right| = \frac{1}{16\pi T^2} = \pi A^2 = A/4$

# KILLING FIELDS

## 2) FINAL REMARKS

• KINEMATIC EFFECT ... DOES NOT RELY ON E.E.

• GREYBODY  $\langle N \rangle = \frac{\sum \omega}{e^{\frac{\omega}{T}} - 1}$  GREY BODY FACTOR

KILLING FIELDS

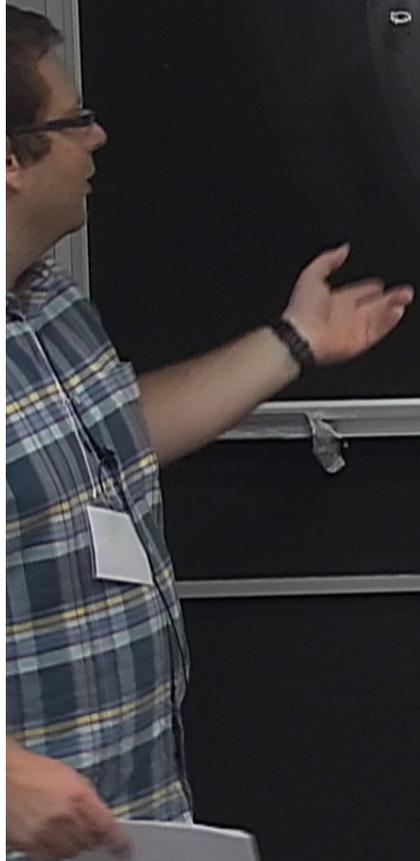
- GREY BODY

GREY BODY FACTOR

$$\langle N \rangle = \frac{\omega}{e^{\frac{\omega}{T}} - 1}$$

- BH LOOSES MASS

$$\frac{dM}{dt} \sim 6 \pi^4 \frac{1}{M^4} A \sim \frac{1}{M^2}$$



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OR THE SURFACE OF THE BOARD  
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(110) (110) (110)

$$T = \frac{1}{8\pi M}$$

$$C = T \frac{\partial S}{\partial T} = -\frac{1}{8\pi T^2}$$

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• PHASE TRANSITIONS

SCHW

$$f = 1 - \frac{2M}{r}$$

$$F = M/2$$

BORING

