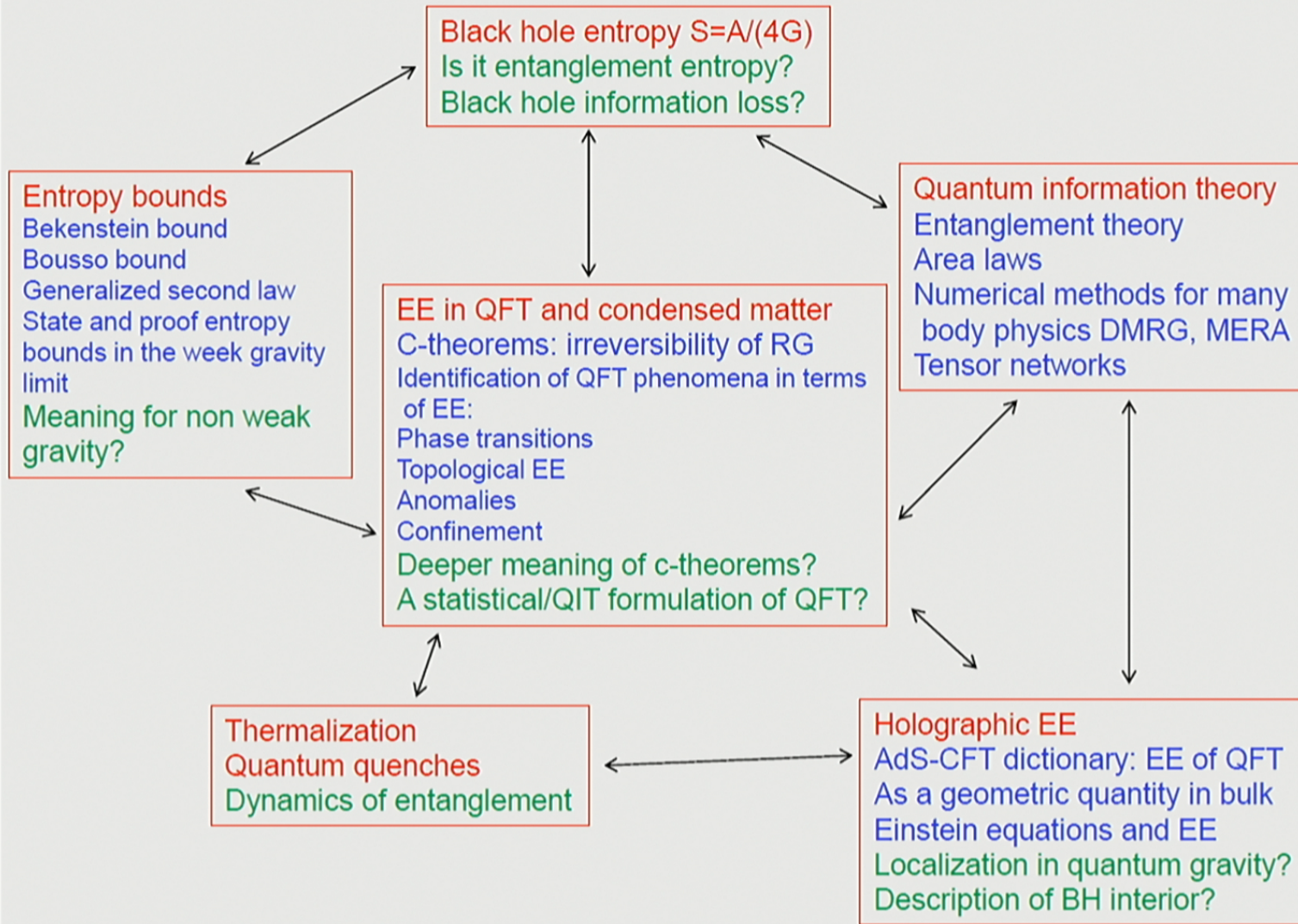


Title: Entanglement in QFT

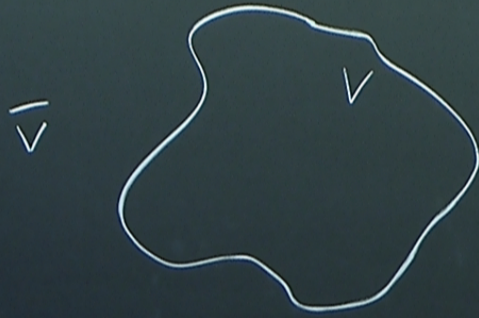
Date: Jul 20, 2016 11:00 AM

URL: <http://pirsa.org/16070020>

Abstract:



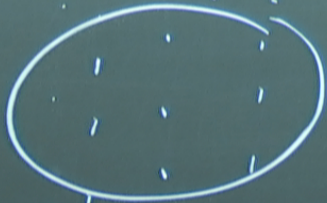
$EE \sim \underline{\underline{Q \neq T}}$



$$\rho_v = \tau_v S$$

$$S(v) = - \tau_v \rho_v \log \rho_v$$

$$\tau_v(\rho_v, \langle \rho_v \rangle) = \langle \rho_v \rangle$$



$$\mathbb{Z}^N \times \mathbb{Z}^N$$

$$\langle \mathcal{O}_v \rangle = \langle \mathcal{O}_v \rangle$$

$$H = \int d^d x \frac{1}{2} \left(\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 \right)$$

$$[\phi_i, \pi_j] = i \delta_{ij}$$

$$\langle \phi_i \phi_j \rangle = X_{ij}$$

$$\langle \pi_i \pi_j \rangle = P_{ij}$$

$$\langle \phi_i \pi_j \rangle = \frac{i \delta_{ij}}{2} + \cancel{D_{ij}}$$

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \langle \phi_1 \phi_2 \rangle \langle \phi_3 \phi_4 \rangle + \langle \phi_1 \phi_3 \rangle \langle \phi_2 \phi_4 \rangle + \langle \phi_1 \phi_4 \rangle \langle \phi_2 \phi_3 \rangle$$

Peschel (2003)

$$S_V = N E - \sum (\phi_i \alpha_{ij} \phi_j + \pi_i M_{ij} \pi_j) = \prod e^{-\epsilon_{ij} a_i^\dagger a_j} (1 - e^{-\epsilon_{ij}})$$

$$\phi_i = \alpha_{ij} a_j^\dagger + \alpha_{ij} a_j$$

$$\pi_i = -i \beta_{ij} a_j^\dagger + i \beta_{ij} a_j$$

$$\ln(\mathcal{B} \phi_i \phi_j) = X_{ij} \quad X =$$

+ β_{ij}
 $\rightarrow (\phi_2 \phi_3)$

$$\langle \pi_0 \rangle = \prod_l e^{-\epsilon_l a_l^\dagger a_l} (1 - e^{-\epsilon_l})^{-1}$$

\uparrow $E\beta$
 $-\epsilon_l a_l^\dagger a_l$

$$\phi_i = \alpha_{ij} a_j^\dagger + \alpha_{ij} a_j$$

$$\pi_i = -i\beta_{ij} a_j^\dagger + i\beta_{ij} a_j$$

$$\text{tr}(\beta \phi_i \phi_j) = X_{ij} \quad X = \alpha (2M+1) \alpha^T$$

$$\eta_{kk} = \langle \alpha_k^\dagger \alpha_k \rangle = (e^{\epsilon_{k-1}})^{-1}$$

$$[a_k^+, a_j] = \delta_{kj} \in \beta$$

$$\pi_j) = \prod e^{-\epsilon_k a_k^+ a_k} (1 - e^{-\epsilon_k})$$

$$\phi_i = \alpha_{ij} a_j^+ + \alpha_{ij} a_j$$

$$\pi_i = -i\beta_{ij} a_j^+ + i\beta_{ij} a_j$$

$$[\phi_i, \pi_j] = i\delta_{ij} \Rightarrow \alpha \beta^T = -\frac{1}{2}$$

$$\alpha \frac{1}{c} (2m+1)^2 \alpha^{-1} = X \cdot P$$

$$c = \sqrt{X \cdot P} \rightarrow \gamma_k$$

$$\text{tr}(\beta \phi_i \phi_j) = X_{ij}$$

$$X = \alpha (2m+1) \alpha^T$$

$$\eta_{kk} = \langle a_k^+ a_k \rangle = (e^{\epsilon_k - 1})^{-1}$$

$$P = \beta (2m+1) \beta^T$$

$$\gamma_k = \frac{1}{2} \coth\left(\frac{\epsilon_k}{2}\right)$$

$$-i\beta_{ij} a_j^\dagger + i\beta_{ij} a_j$$

$$[\phi_i, \pi_j] = i\delta_{ij} \Rightarrow \boxed{\alpha \beta^T = -\frac{1}{2}}$$

$$\alpha \frac{1}{4} (2m+1)^2 \alpha^{-1} = X \cdot P$$

$$C = \sqrt{XP} \rightarrow \gamma_k$$

$$\boxed{\gamma_k = \frac{1}{2} \coth(\epsilon_k/2)}$$

$$S = \sum_{\ell} \left(-\log(1 - e^{-\epsilon_{\ell}}) + \frac{\epsilon_{\ell} e^{-\epsilon_{\ell}}}{1 - e^{-\epsilon_{\ell}}} \right) = \ln \left((C+1/2) \log(C+1/2) - (C-1/2) \log(C-1/2) \right)$$

$$\eta_{kk} = \langle \alpha_k^\dagger \alpha_k \rangle = (e^{-1})$$

$$P = \beta (2m+1) \beta^T$$

$$H = \frac{1}{2} \left(\sum_i \pi_i^2 + \sum_{ij} \phi_i K_{ij} \phi_j \right)$$

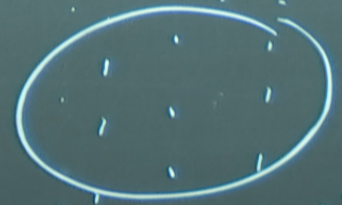
$$X_{ij} = \langle \phi_i \phi_j \rangle = \frac{1}{2} (K^{-1/2})_{ij}$$

$$P_{ij} = \frac{1}{2} (K^{1/2})_{ij}$$

$$\rightarrow X_{ij}^v, P_{ij}^v$$

$$\langle \phi_{20} \phi_{10} \rangle = \frac{L}{8\pi^2} \int_{-\pi}^{\pi} dP_x \int_{-\pi}^{\pi} dP_y$$

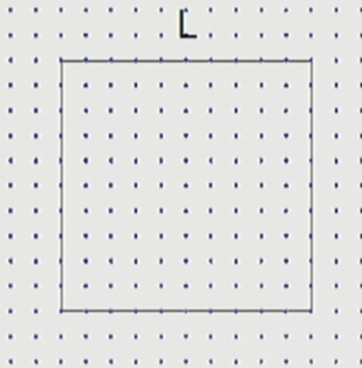
$$\frac{\cos(iP_x) \cos(jP_y)}{\sqrt{2(1-\cos P_x)(1-\cos P_y)}}$$



$$2^N \times 2^N$$

$$H = \int d^d x \frac{1}{2} \left(\dots \right)$$

Massless (gapless) scalar field model. Vacuum (fundamental) state in a square lattice
 Similar to phonons in a solid

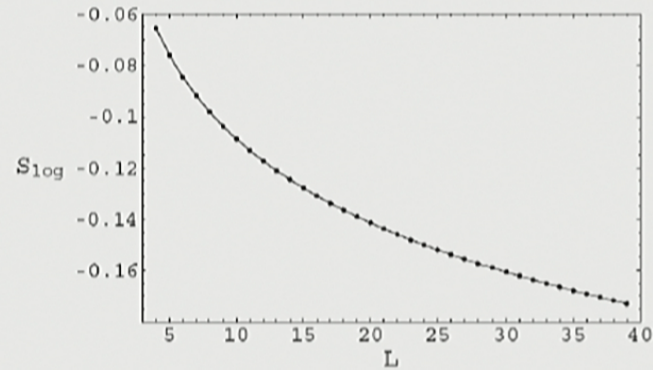
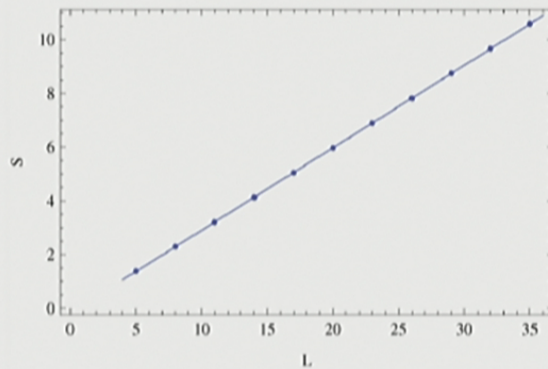


$$H = \frac{1}{2} \int d^2x \left(\dot{\phi}(x)^2 + (\nabla\phi(x))^2 \right)$$

$$\rightarrow H = \frac{1}{2} \sum_i \epsilon^2 \left(\dot{\phi}_i^2 + \sum_{j \sim i} \frac{(\phi_i - \phi_j)^2}{\epsilon^2} \right)$$

For interacting spin systems the Hilbert space dimension grows as 2^N

For coupled Harmonic oscillators we have only to diagonalize matrices of $N \times N$



$$S = .075 (4 L/\epsilon) - 0.047 \text{Log}[L/\epsilon] + \text{const} = .075 (\text{perimeter}/\epsilon) - 0.047 \text{Log}[L/\epsilon] + \text{const}$$

We have an «area» term and a logarithmic correction. These are divergent as $\epsilon \rightarrow 0$

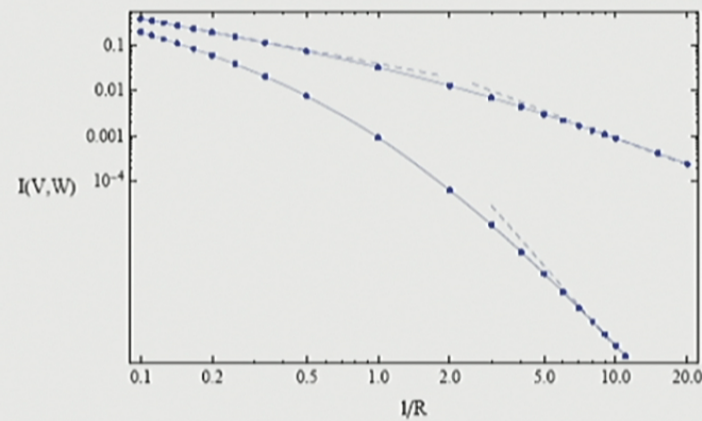
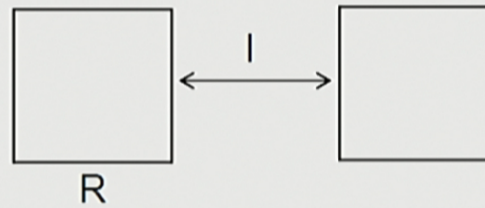


Figure 9: Log log plot of the mutual information for two squares of side R separated by a distance l , as a function of l/R . The curve at the top is the mutual information for the scalar and the lower one in the mutual information of the gauge model. The dashed lines are asymptotic behaviors. For small l/R we expect $I(V, W) \sim .0397R/l$ for both models, while for large distances we expect $I(V, W) \sim (l/R)^2$ for the scalar and $I(V, W) \sim (l/R)^6$ for the Maxwell field.

$$S = .075 (\text{perimeter}/\epsilon) - (6/4) 0.047 \text{Log}[L/\epsilon] + \text{const}$$

The same «area» term. A logarithmic coefficient growing with the number of vertices.
 (All vertices have the same angle $S(A) = S(-A)$ for a global pure state)

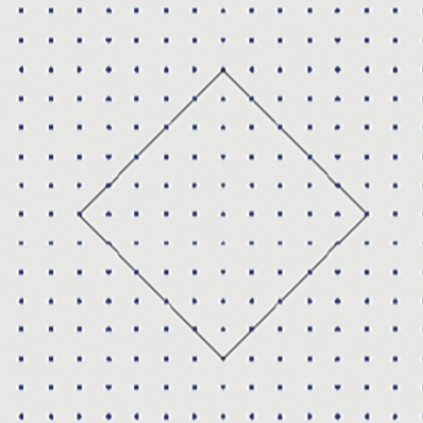
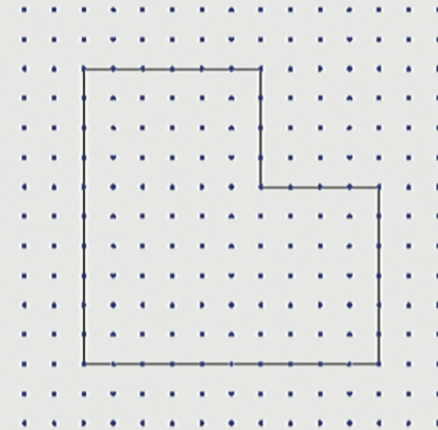
In general:

$$S(A) = c_1 (\text{perimeter}/\epsilon) - \sum_{\text{vertices}} c_{\log}(\theta) \log(R/\epsilon) + \text{const}$$

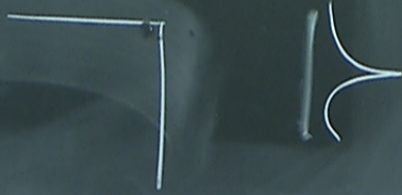
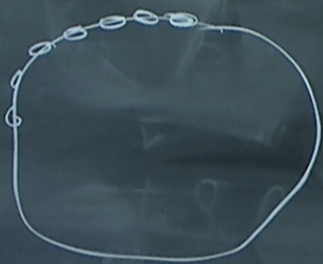
$$S = .085 (\text{perimeter}/\epsilon) - 0.047 \text{Log}[L/\epsilon] + \text{const}$$

Bad: area term does not have the rotational symmetry of the theory in the continuum limit

Good: the logarithmic term does not notice the lattice



$$S(V) = \frac{g_{d-2}(\partial V)}{\varepsilon^{d-2}} + \frac{g_{d-1}(\partial V)}{\varepsilon^{d-1}} + \dots + \underbrace{g_0(\partial V)}_{\text{universal}} \log(\varepsilon) + S_0(V)$$



$$\sim \frac{A}{\varepsilon^{d-2}}$$

"area law"

$$\frac{A}{4G}$$

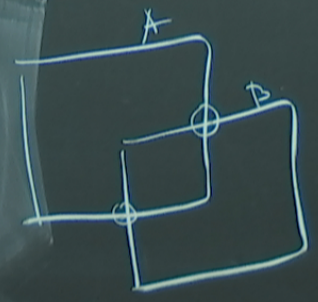
$$p(\partial V) \log(\epsilon) + S_0(V)$$

universal



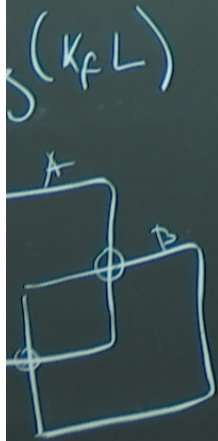
$$S \sim T^{d-1} V$$

$$S \sim (K_F L)^{d-2} \log(K_F L)$$

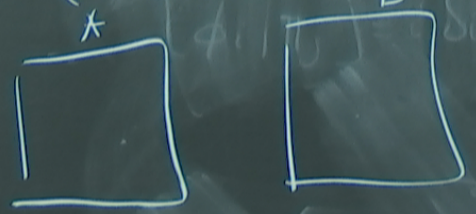


$$S(A) + S(B) - S(A \cup B) - S(A \cap B)$$

Peschel (2003)



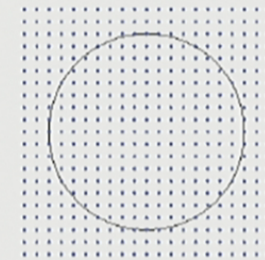
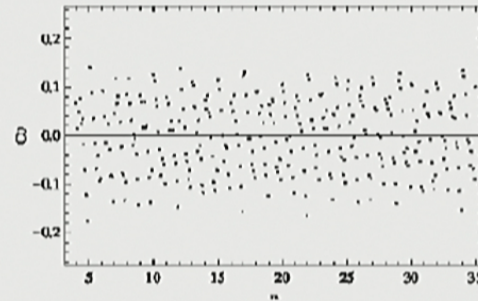
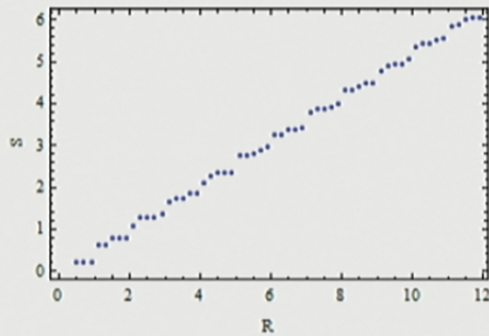
$$S(A) + S(B) - S(A \cup B) - S(A \cap B)$$



$$I(A, B) = S(A) + S(B) - S(A \cup B)$$

$$\gamma_k = \frac{1}{2} \operatorname{coth} \left(\frac{\epsilon_k}{2} \right)$$

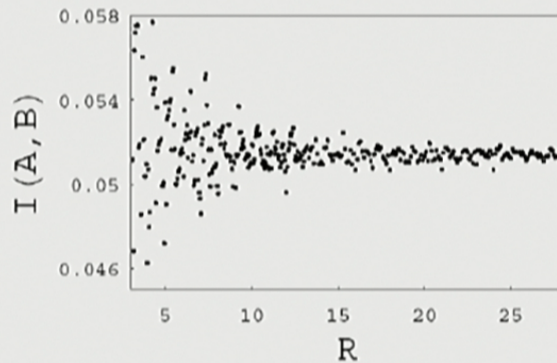
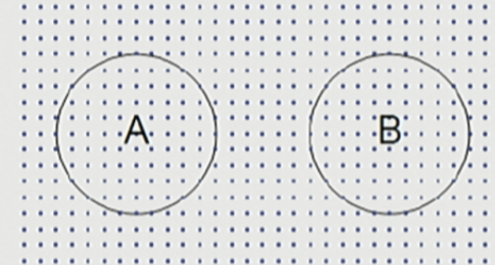
How to extract unambiguous information from the finite term?



Circles in a square lattice (no log term): $S(R) = c_1 R + c_0$

Mutual information $I(A, B) = S(A) + S(B) - S(A \cup B)$

The boundary divergences cancel out in the combination.



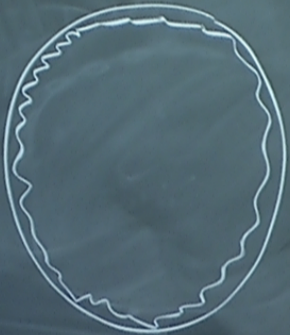
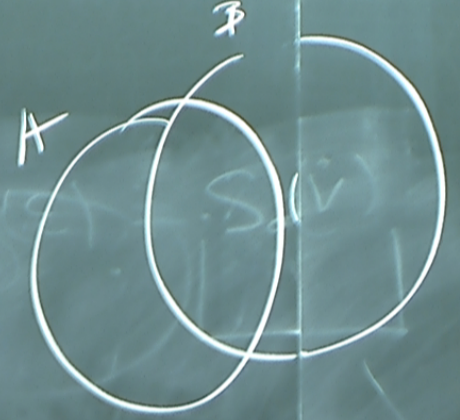
Very little large distance entanglement
 Compare $I(A, B) = 0.05$ with $\log(2) = 0.69$
 Less than 1/10 bit for infinitely many degree of freedom!

A lot of short distance entanglement:
 $I(A, B)$ diverges when A and B touch each other.
 This reflects the locality of the theory

$$I(A, B) \geq 0$$

$$S(A) + S(B) \geq S(A|B) + S(A \cup B)$$

$$\Rightarrow I(A, B) \leq I(A, BC)$$



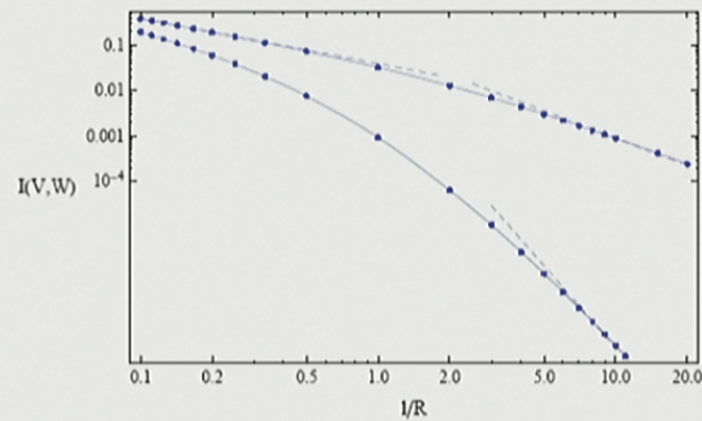
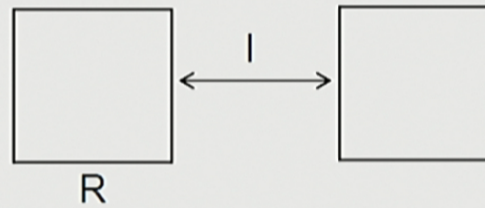
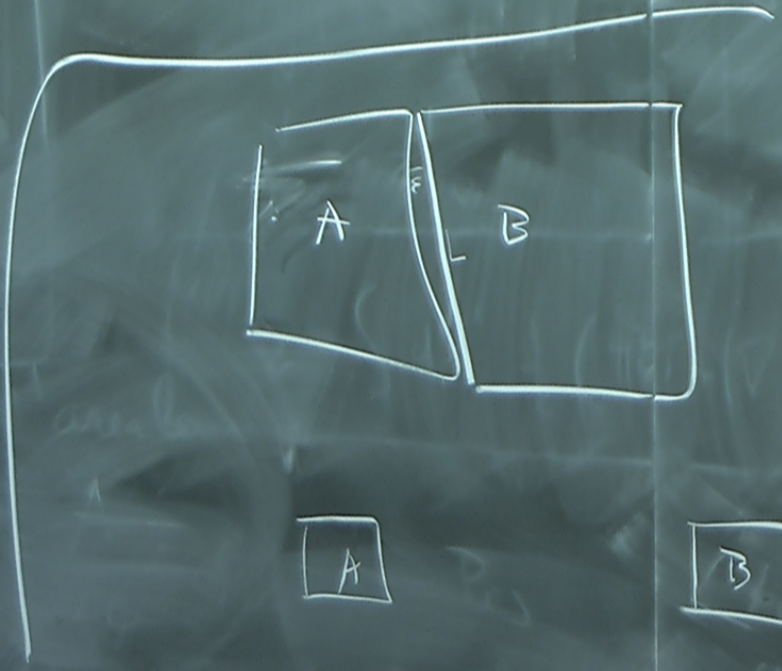
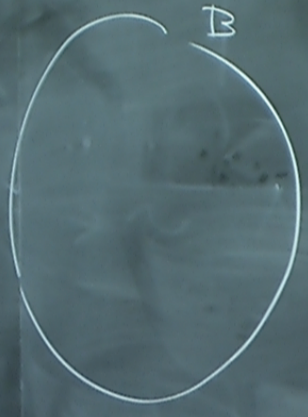
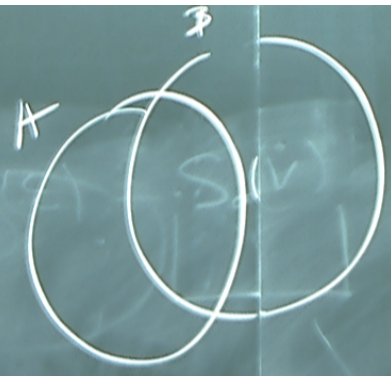


Figure 9: Log log plot of the mutual information for two squares of side R separated by a distance l , as a function of l/R . The curve at the top is the mutual information for the scalar and the lower one in the mutual information of the gauge model. The dashed lines are asymptotic behaviors. For small l/R we expect $I(V, W) \sim .0397R/l$ for both models, while for large distances we expect $I(V, W) \sim (l/R)^2$ for the scalar and $I(V, W) \sim (l/R)^6$ for the Maxwell field.

$$a) \geq 0$$

$$-S(B) \geq S(A|B) + S(A \cup B)$$

$$B) \leq I(A, B, C)$$



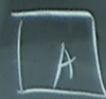
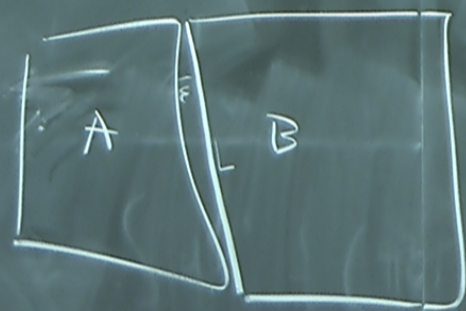
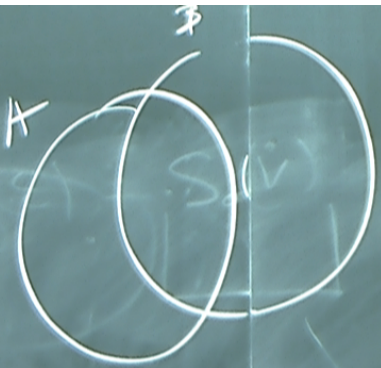
$$I \sim \frac{L}{\epsilon}$$

$$I \sim \epsilon$$

Paschel (2003)

$$|B) + S(A \cup B)$$

A, B, C)



$$I \sim \frac{L}{\epsilon}$$

$$I \sim \langle \psi(0) | \psi(R) \rangle$$

$$\langle F_{\mu\nu}(0) | F_{\alpha\beta}(R) \rangle$$

Peschel (2003)

$$I(A, B) \geq 0$$

$$S(A) + S(B) \geq S(A|B) + S(A \cup B)$$

$$I(A, B) \geq \frac{(\langle \rho_A \rho_B \rangle - \langle \rho_A \rangle \langle \rho_B \rangle)^2}{\|\rho_A\|^2 \|\rho_B\|^2}$$

(A, BC)

B

Replica method

$$S_m = \frac{1}{1-m} \log \left(\frac{1}{Z^m} \right)$$

Rényi entropies

$$S_{m \rightarrow 1} = S$$

$$\langle x_0, t_0 | x_0, 0 \rangle = \int_{x_0, t_0}^{x_0, 0} \mathcal{D}x(t) e^{-S[x]} = \langle x_0 | e^{-\int_0^{t_0} H dt} | x_0 \rangle \rightarrow \langle x_0 | e^{-E_0 \tau} | 0 \rangle \langle 0 | x_0 \rangle$$

$$t_0 = i\tau \quad \tau \rightarrow \infty$$

$$S_{m \rightarrow 1} = S$$

$$\langle x_0, t_0 | x_1, 0 \rangle = \int_{x_0, t_0}^{x_1, 0} \mathcal{D}x(t) e^{iS[x]} = \langle x_0 | e^{-iH\tau} | x \rangle \rightarrow \langle x_0 | e^{-S_E[x]} | x \rangle$$

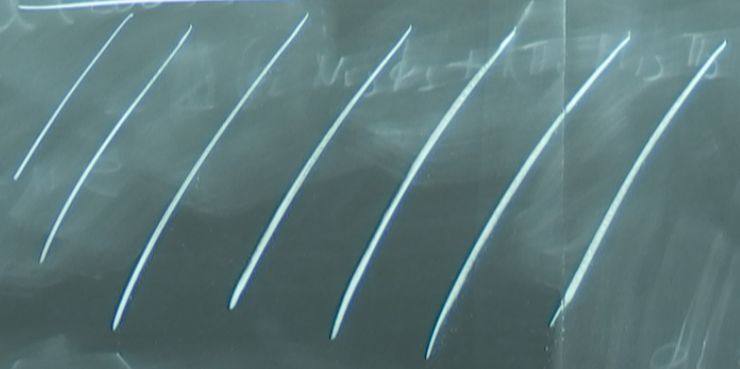
$$t_0 = i\tau$$

$$\tau \rightarrow \infty$$

$$\psi_0(x) =$$

$$\int_{-\infty}^{x_0} \mathcal{D}x e^{-S_E[x]}$$

Pöschel (1993) $\phi_1(x)$ $\mathbb{E} = 0$



$$\rightarrow \langle x_0 | e^{-E_0 \tau} | 0 \rangle \langle 0 | x \rangle$$

$$\phi(t=0, x) = \phi_1(x)$$

$$\Phi(\phi_1(x), t=0) = N^{-1/2} \int_{-\infty}^{\infty} \mathcal{D}\phi e^{-S_E[\phi]}$$

$$\Psi_0(x) =$$

$$\int_{-\infty}^{x,0} dx e^{-S_E[x]}$$

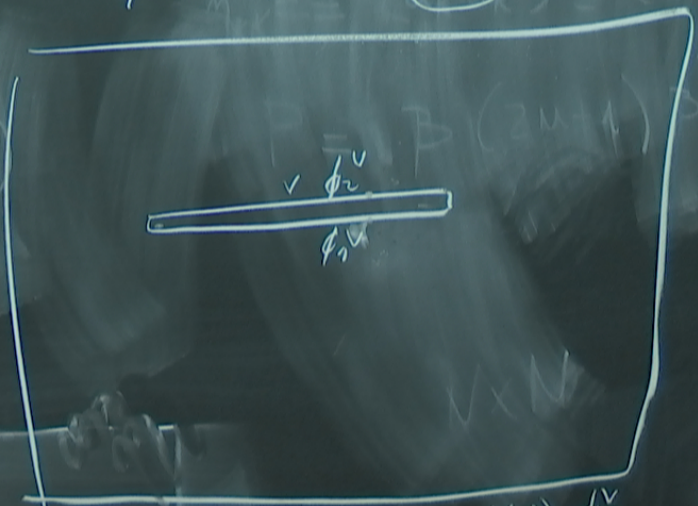
$$S_0[\phi_1, \phi_2] = \bar{\Phi}_0^*(\phi_1) \bar{\Phi}_0(\phi_2)$$

$$\ln(B \phi_1 \phi_2) = x_{12}$$

$$\phi_1 = \phi_1^{\vee} + \boxed{\phi_1^{\nabla}} \quad \phi_1^{\nabla} = \phi_2^{\nabla}$$

$$\phi_2 = \phi_2^{\vee} + \boxed{\phi_2^{\nabla}}$$

$$S(A, B) = S(A) + S(B) - S(A \cup B)$$



$$\int_{\vee} [\phi_1^{\vee}, \phi_2^{\vee}] = \int_{\phi_1^{\vee}(\nu^-)}^{\phi_2^{\vee}(\nu^+)} d\phi e^{-S_{\frac{1}{2}}(\phi)}$$