Title: Entanglement in QFT

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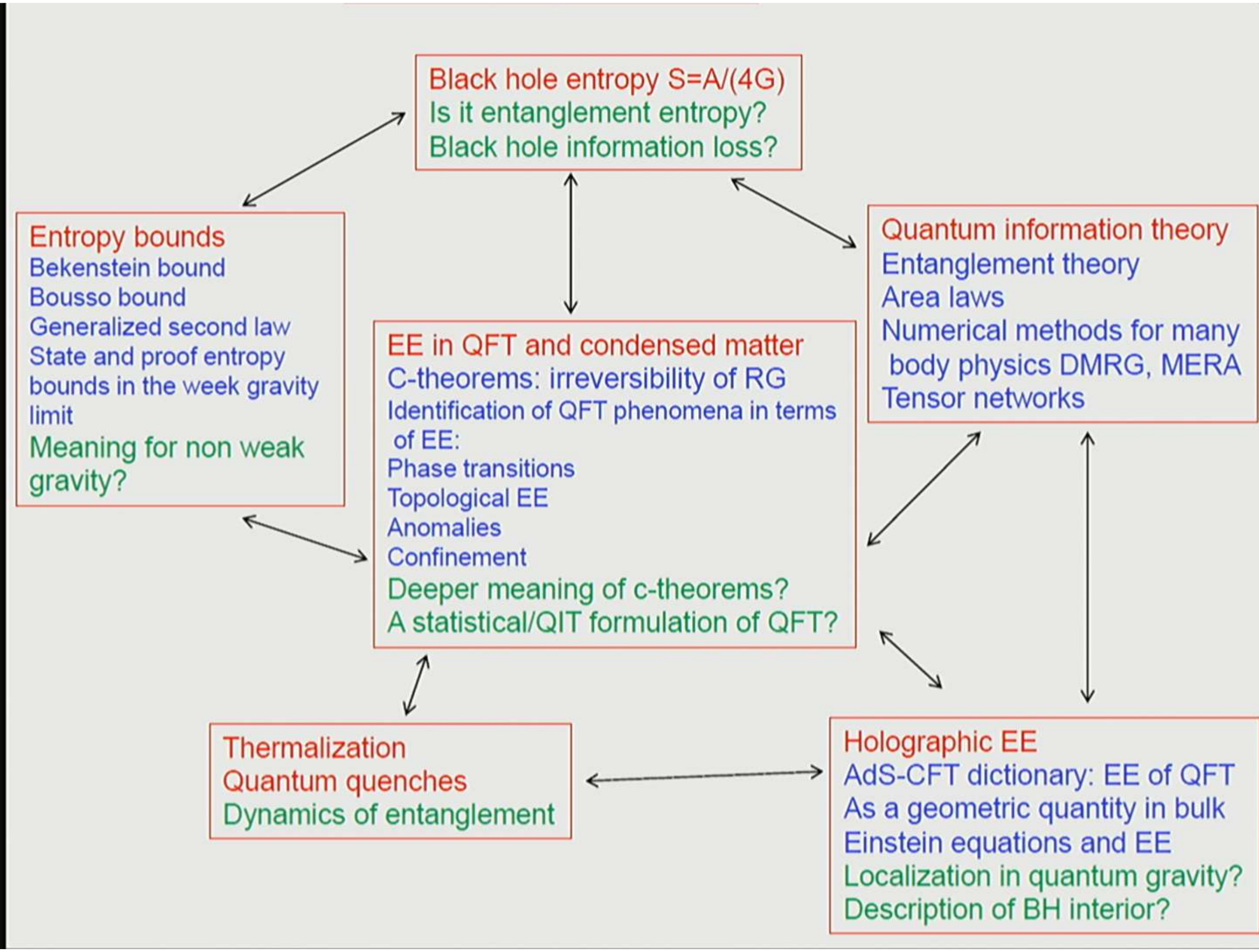
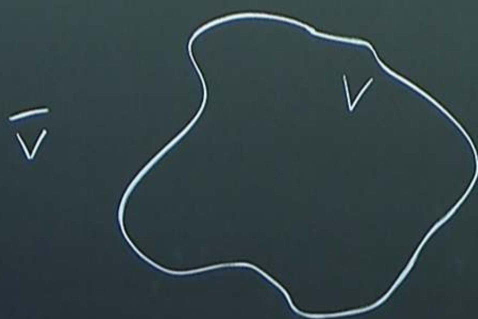
Abstract:


Black hole entropy S=A/(4G)
Is it entanglement entropy?
Black hole information loss?
Entropy bounds
Bekenstein bound
Bousso bound
Generalized second law
State and proof entropy bounds in the week gravity limit
Meaning for non weak gravity?
EE in QFT and condensed matter
C-theorems: irreversibility of RG
Identification of QFT phenomena in terms of EE:
Phase transitions
Topological EE
Anomalies
Confinement
Deeper meaning of c-theorems?
A statistical/QIT formulation of QFT?
Quantum information theory
Entanglement theory
Area laws
Numerical methods for many body physics DMRG, MERA
Tensor networks
Thermalization
Quantum quenches
Dynamics of entanglement
Holographic EE
AdS-CFT dictionary: EE of QFT
As a geometric quantity in bulk
Einstein equations and EE
Localization in quantum gravity?
Description of BH interior?

EE in QFT



$$\rho_V = \mathrm{tr}_{\bar{V}}\, \rho \qquad \mathrm{tr}(\rho_V \mathcal{O}_V) = \langle \mathcal{O}_V \rangle$$

$$S(V) = -\mathrm{tr}\, \rho_V \log \rho_V$$

N

$2^N \times 2^N$

$(\rho_V O_V) = \langle O_V \rangle$

$$H = \int d^{d-1}x \, \frac{1}{2}\left(\pi^2 + (\nabla\phi)^2 + m^2\phi^2\right)$$

$$[\phi_i, \pi_j] = i\delta_{ij}$$

$$\langle \phi_i \phi_j \rangle = X_{ij} \qquad \langle \pi_i \pi_j \rangle = P_{ij} \qquad \langle \phi_i \pi_j \rangle = \frac{i\delta_{ij}}{2} + \cancel{D_{ij}}$$

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \langle \phi_1 \phi_2 \rangle \langle \phi_3 \phi_4 \rangle + \langle \phi_1 \phi_3 \rangle \langle \phi_2 \phi_4 \rangle + \langle \phi_1 \phi_4 \rangle \langle \phi_2 \phi_3 \rangle$$

Peschel (2003)

$$\rho_V = N\, e^{-\sum \left(\phi_i X_{ij} \phi_j + \pi_i M_{ij} \pi_j\right)} = \prod_\ell e^{-\varepsilon_\ell a_\ell^+ a_\ell} \left(1 - e^{-\varepsilon_\ell}\right)$$

$\varepsilon_\ell \leftarrow E\beta$

$$\mathrm{tr}(\rho\, \phi_i \phi_j) = X_{ij} \qquad X =$$

$$\phi_i = \alpha_{ij} a_j^+ + \alpha_{ij} a_j$$

$$\pi_i = -i \beta_{ij} a_j^+ + i \beta_{ij} a_j$$

$+ \cancel{D}_{ij}$

$\rangle\langle \phi_2 \phi_3 \rangle$

$\pi_j)$

$$E\beta$$

$$= \prod_{\ell} e^{-\varepsilon_\ell a_\ell^+ a_\ell} \left(1 - e^{-\varepsilon_\ell}\right)$$

$$\mathrm{tr}(\rho\, \phi_i \phi_j) = X_{ij} \qquad X = \alpha\,(2n+1)\,\alpha^T$$

$$\phi_i = \alpha_{ij}\, a_j^+ + \alpha_{ij}\, a_j$$

$$n_{kk} = \langle a_k^+ a_k \rangle = \left(e^{\varepsilon_k} - 1\right)^{-1}$$

$$\pi_i = -i\beta_{ij}\, a_j^+ + i\beta_{ij}\, a_j$$

$$[a_k^\dagger, a_j] = \delta_{kj}$$

$$E\beta$$

$, \pi_j)$

$$= \prod_\ell e^{-\varepsilon_\ell a_\ell^\dagger a_\ell} \left(1 - e^{-\varepsilon_\ell}\right)$$

$$\mathrm{tr}(\rho \, \phi_i \phi_j) = X_{ij}$$

$$X = \alpha \, (2n+1) \, \alpha^T$$

$$\phi_i = \alpha_{ij} a_j^\dagger + \alpha_{ij} a_j$$

$$\pi_i = -i\beta_{ij} a_j^\dagger + i\beta_{ij} a_j$$

$$n_{kk} = \langle a_k^\dagger a_k \rangle = \left(e^{\varepsilon_k} - 1\right)^{-1}$$

$$P = \beta \, (2n+1) \, \beta^T$$

$$[\phi_i, \pi_j] = i\delta_{ij} \Rightarrow \boxed{\alpha \beta^T = -\frac{1}{2}}$$

$$\alpha \, \frac{1}{4} (2n+1)^2 \alpha^{-1} = X.P$$

$$C = \sqrt{XP} \rightarrow \nu_k$$

$$\boxed{\nu_k = \frac{1}{2} \coth\left(\varepsilon_k / 2\right)}$$

$-i\beta_{ij}\, a_j^{+} + i\beta_{ij}\, a_j$

$[\phi_i, \pi_j] = i\,\delta_{ij} \Rightarrow \boxed{\alpha\,\beta^T = -\frac{1}{2}}$

$\eta_{kk} = \langle a_k^{+} a_k \rangle = (e^{\ } - 1)$

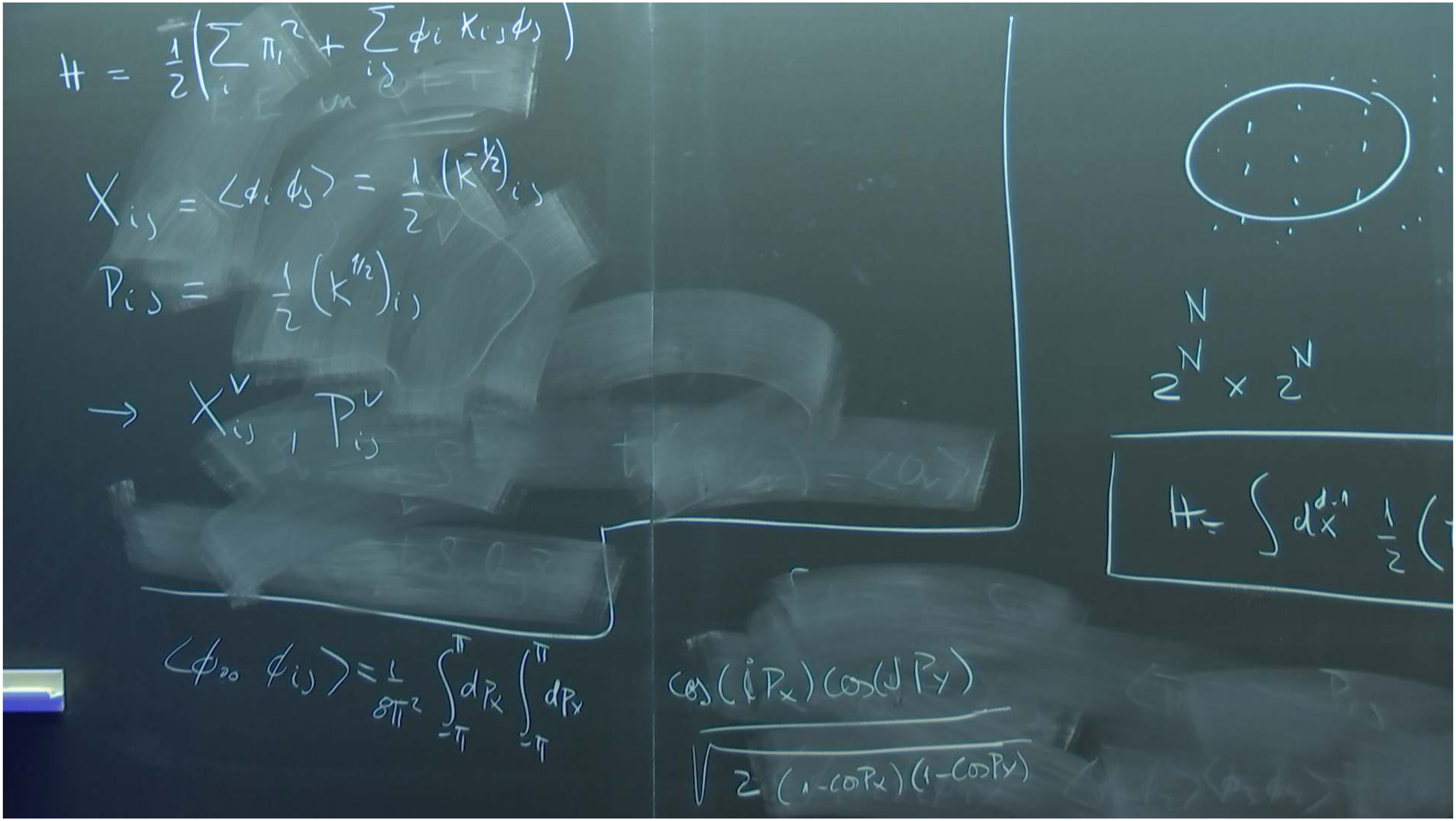
$P = \beta\,(2n+1)\,\beta^T$

$\alpha\ \frac{1}{4}(2n+1)^{2}\ \alpha^{-1} = X.P$

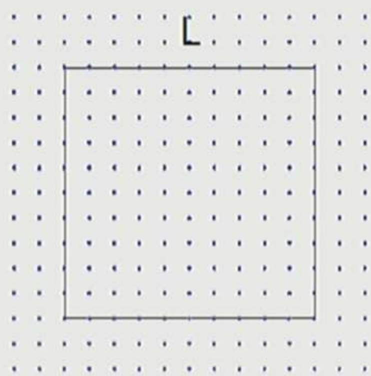
$C = \sqrt{XP} \rightarrow \nu_k$

$$\boxed{\nu_k = \frac{1}{2}\coth\left(\varepsilon_k/2\right)}$$

$$S = \sum_\ell \left(-\log\left(1 - e^{-\varepsilon_\ell}\right) + \frac{\varepsilon_\ell\, e^{-\varepsilon_\ell}}{1 - e^{\varepsilon_\ell}}\right) = \mathrm{tr}\left((C + 1/2)\log(C + 1/2) - (C - 1/2)\log(C - 1/2)\right)$$


$H = \frac{1}{2}\left(\sum_i \pi_i^2 + \sum_{ij} \phi_i K_{ij} \phi_j\right)$
$X_{ij} = \langle \phi_i \phi_j \rangle = \frac{1}{2}\left(K^{-1/2}\right)_{ij}$
$P_{ij} = \frac{1}{2}\left(K^{1/2}\right)_{ij}$
$\rightarrow X^V_{ij}, P^V_{ij}$
$\langle \phi_{00} \phi_{ij} \rangle = \frac{1}{8\pi^2} \int_{-\pi}^{\pi} dp_x \int_{-\pi}^{\pi} dp_y \frac{\cos(i p_x)\cos(j p_y)}{\sqrt{2(1-\cos p_x)(1-\cos p_y)}}$
N
$2^N \times 2^N$
$H = \int d^{d-1}x \; \frac{1}{2}($

Massless (gapless) scalar field model. Vacuum (fundamental) state in a square lattice
Similar to phonons in a solid

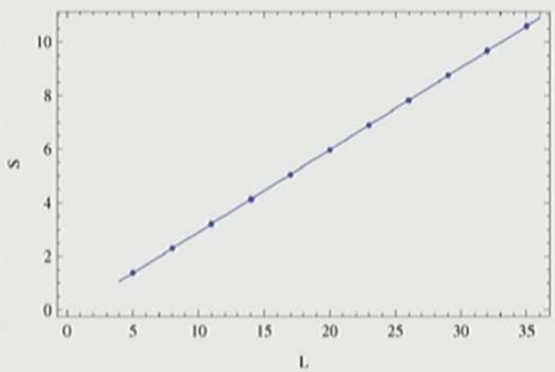


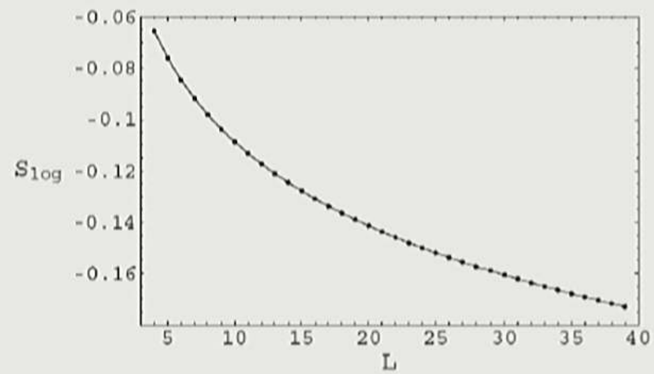
$$H = \frac{1}{2}\int d^2x \left(\dot{\phi}(x)^2 + (\nabla\phi(x))^2\right)$$

$$\rightarrow H = \frac{1}{2}\sum_i \epsilon^2 \left(\dot{\phi}_i^{\,2} + \sum_{j\sim i}\frac{(\phi_i - \phi_j)^2}{\epsilon^2}\right)$$

For interacting spin systems the Hilbert space dimension grows as 2^N

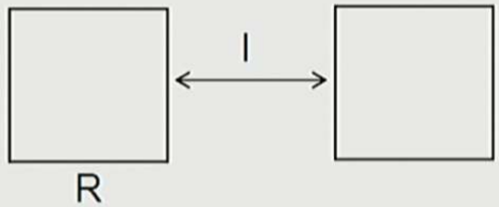
For coupled Harmonic oscillators we have only to diagonalize matrices of $N \times N$





S=.075 (4 L/ε)-0.047 Log[L/ε] +const=.075 (perimeter/ε)-0.047 Log[L/ ε]+const

We have an «area» term and a logarithmic correction. These are divergent as ε ->0



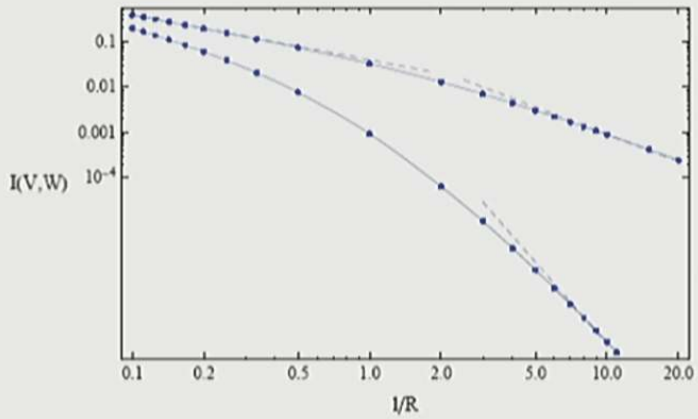
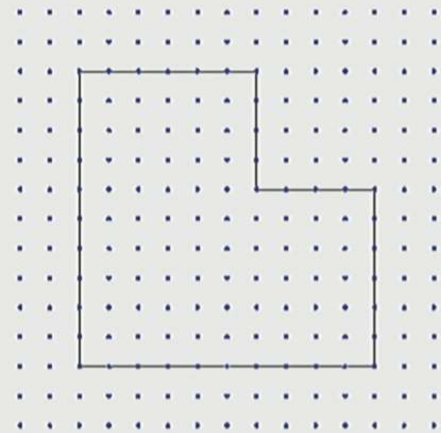


Figure 9: Log log plot of the mutual information for two squares of side R separated by a distance l, as a function of l/R. The curve at the top is the mutual information for the scalar and the lower one in the mutual information of the gauge model. The dashed lines are asymptotic behaviors. For small l/R we expect $I(V,W) \sim .0397R/l$ for both models, while for large distances we expect $I(V,W) \sim (l/R)^2$ for the scalar and $I(V,W) \sim (l/R)^6$ for the Maxwell field.

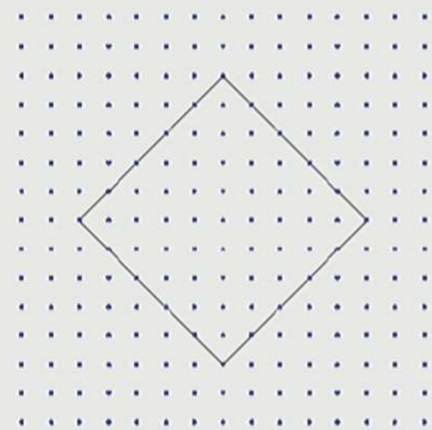
S=.075 (perimeter/ε)-(6/4) 0.047 Log[L/ε]+const



The same «area» term. A logarithmic coefficient growing with the number of vertices.
(All vertices have the same angle S(A)=S(-A) for a global pure state)

In general:

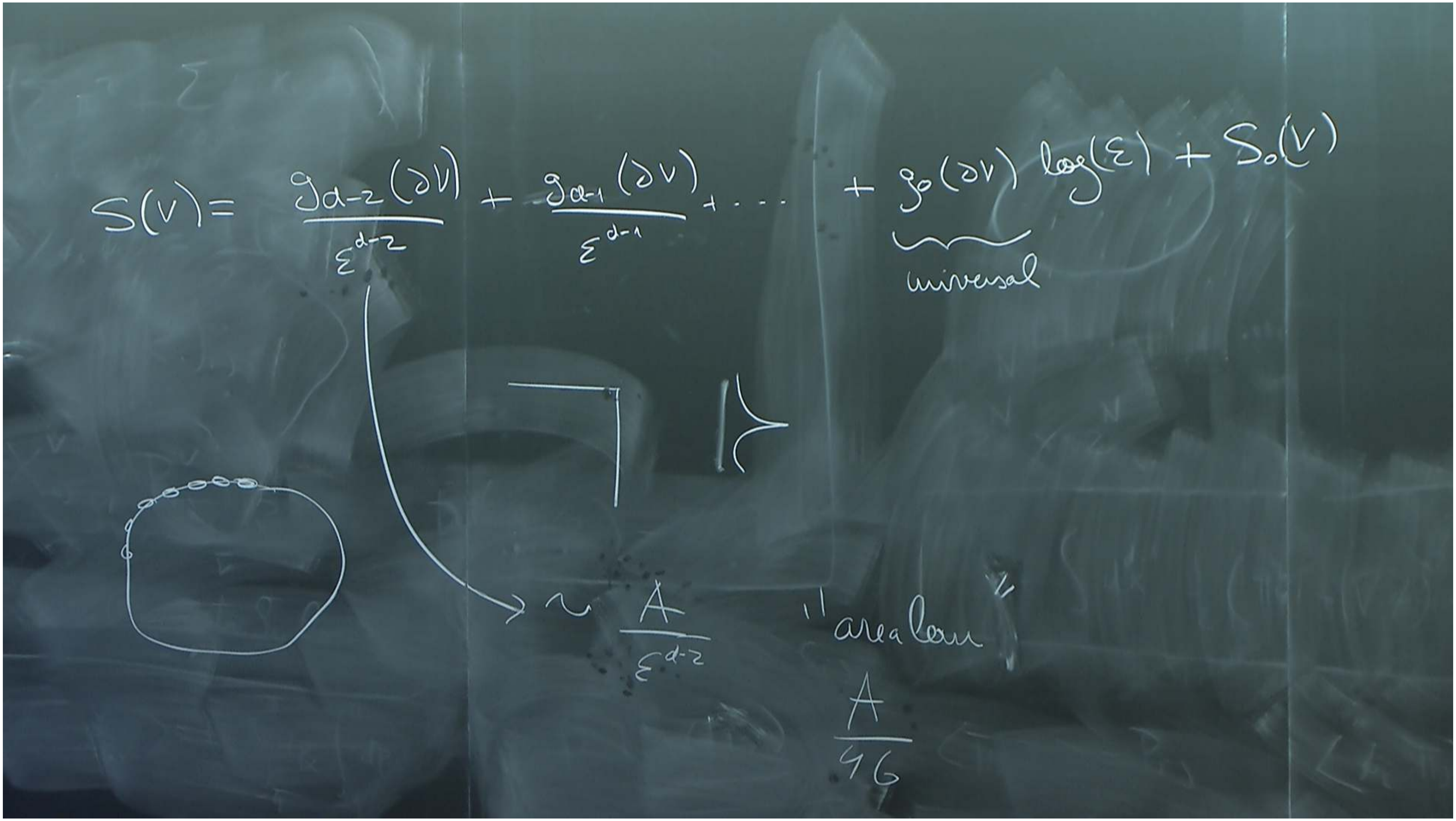
$$S(A) = c_1\,(\text{perimeter}/\epsilon) - \sum_{\text{vertices}} c_{\log}(\theta)\log(R/\epsilon) + \text{const}$$



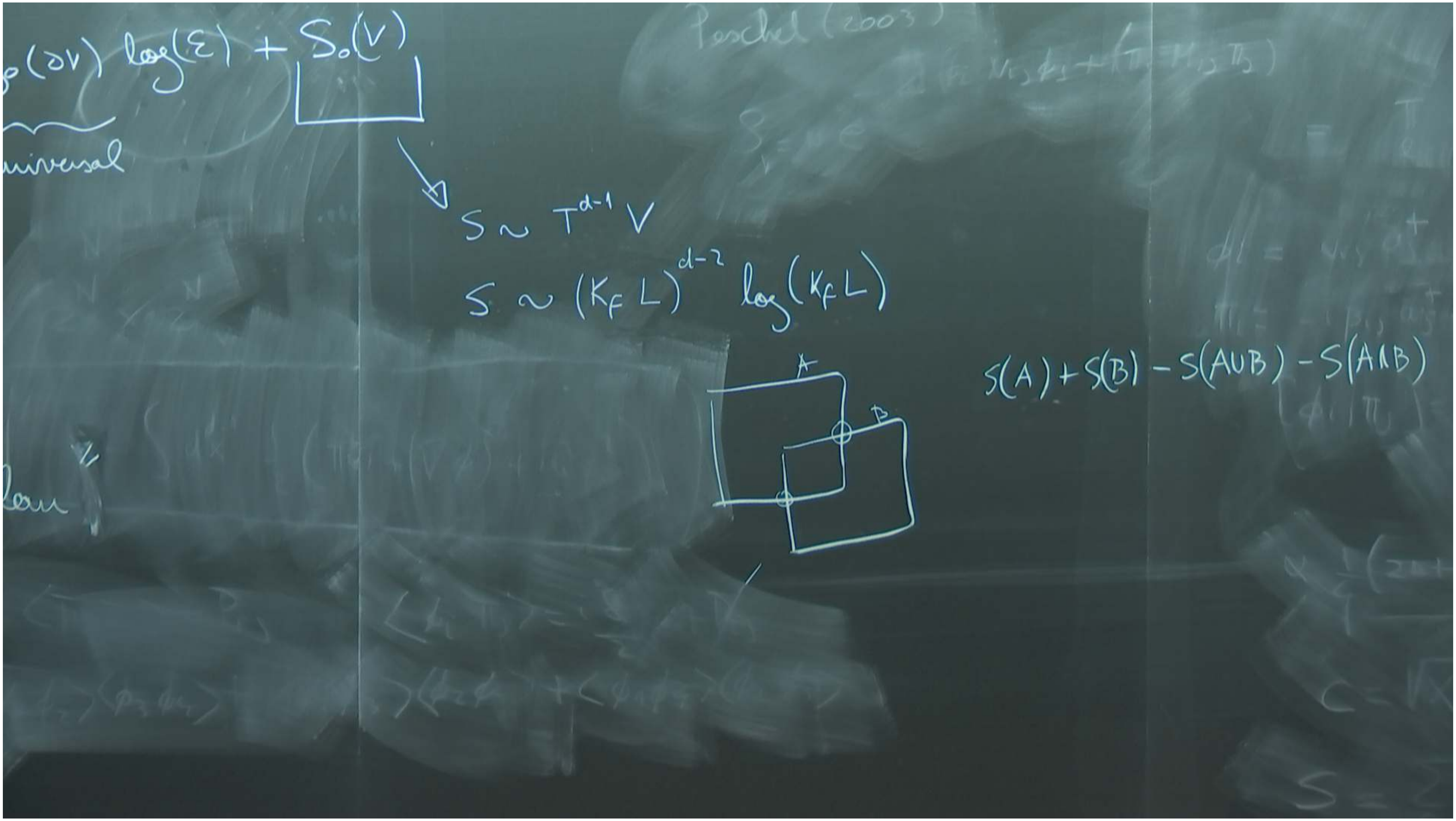
S=.085 (perimeter/ε)-0.047 Log[L/ε]+const

Bad: area term does not have the rotational symmetry of the theory in the continuum limit

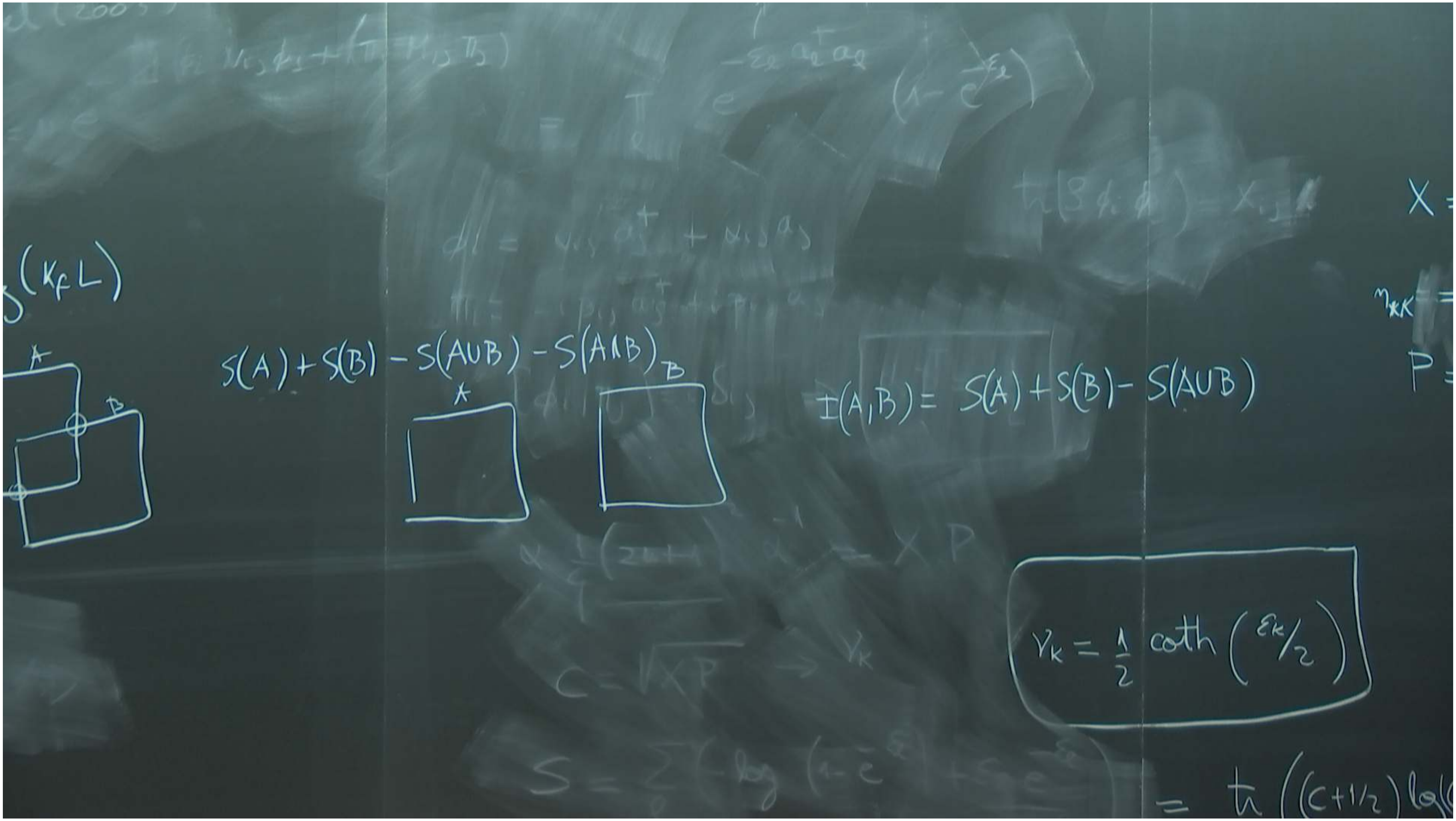
Good: the logarithmic term does not notice the lattice



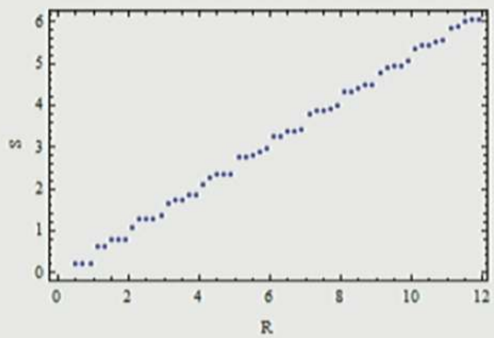
$S(V) = \frac{g_{d-2}(\partial V)}{\varepsilon^{d-2}} + \frac{g_{d-1}(\partial V)}{\varepsilon^{d-1}} + \cdots + \underbrace{g_0(\partial V)}_{\text{universal}} \log(\varepsilon) + S_0(V)$
$\sim \frac{A}{\varepsilon^{d-2}}$
"area law"
$\frac{A}{4G}$

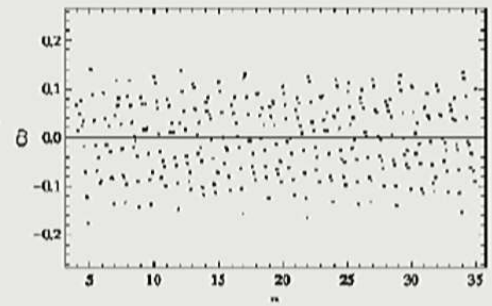


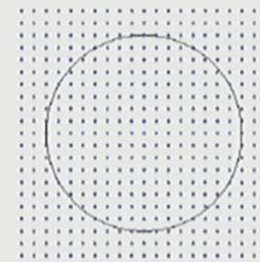
(∂V) log(ε) + S₀(V)
universal
Peschel (2003)
S ~ T^{d-1} V
S ~ (K_F L)^{d-2} log(K_F L)
A
B
S(A) + S(B) - S(A∪B) - S(A∩B)


S(A) + S(B) − S(A∪B) − S(A∩B)
A
B
I(A,B) = S(A) + S(B) − S(A∪B)
ν_k = 1/2 coth(ε_k/2)
= tr((C+1/2) log(

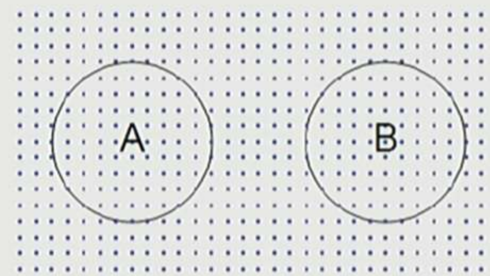
How to extract unambiguous information from the finite term?





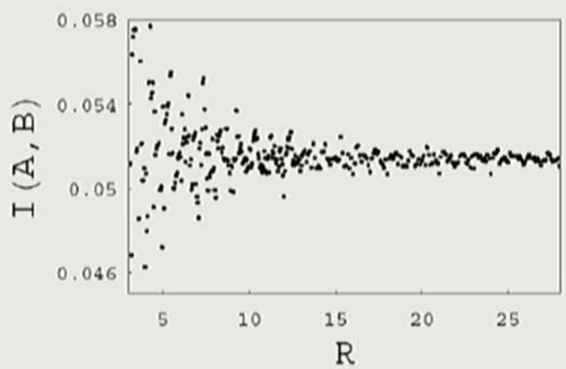


Circles in a square lattice (no log term): $S(R) = c_1 R + c_0$



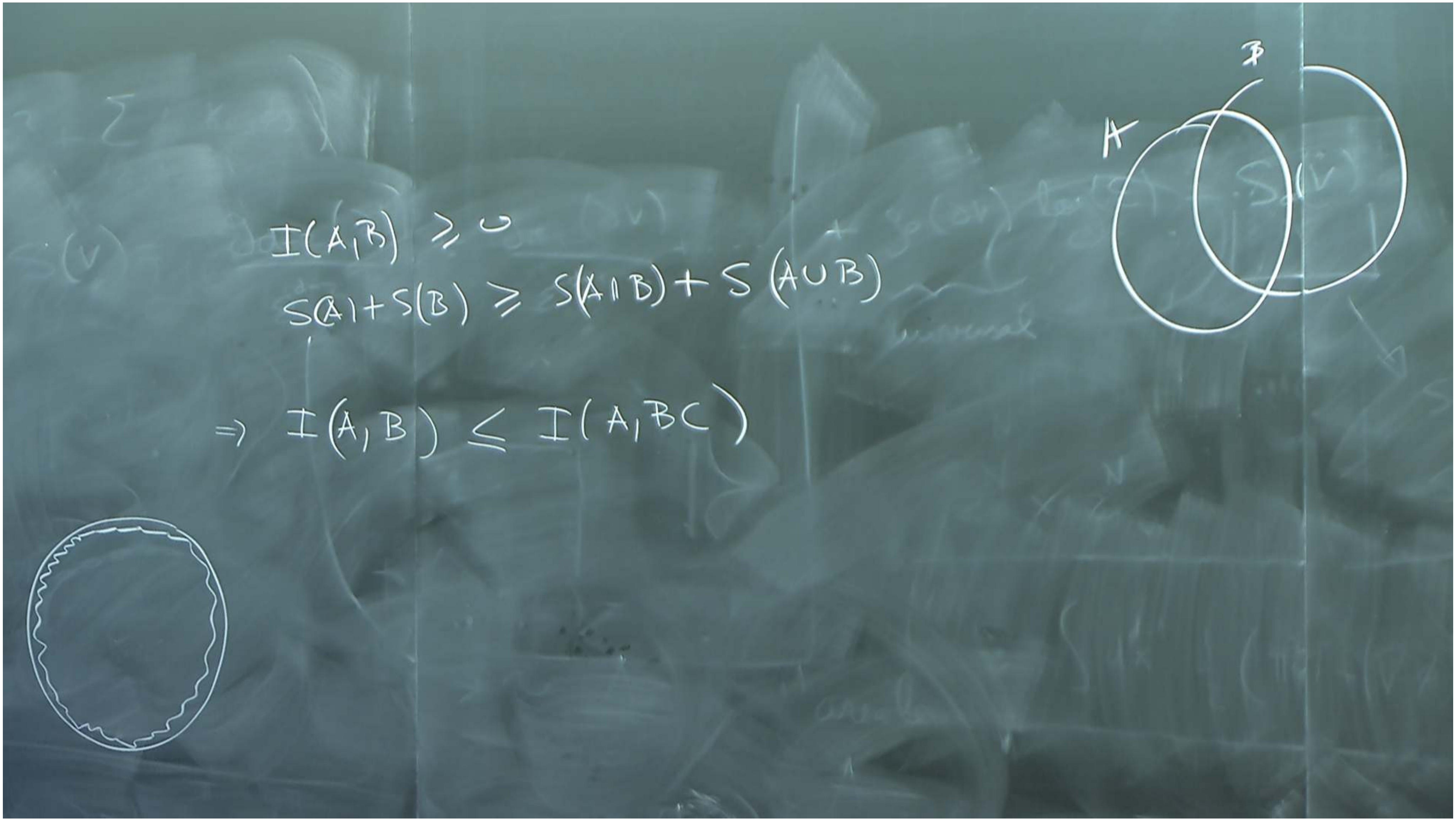
Mutual information $I(A,B) = S(A) + S(B) - S(A \cup B)$

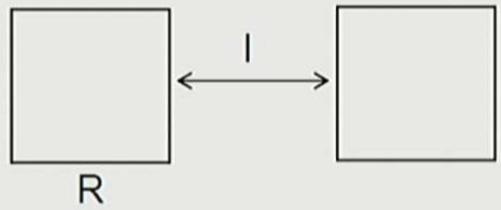
The boundary divergences cancel out in the combination.



Very little large distance entanglement
Compare I(A,B)=0.05 with log(2)=0.69
Less than 1/10 bit for infinitely many degree of freedom!

A lot of short distance entanglement:
I(A,B) diverges when A and B touch each other.
This reflects the locality of the theory


$I(A,B) \geq 0$
$S(A)+S(B) \geq S(A \cap B)+S(A \cup B)$
$\Rightarrow I(A,B) \leq I(A,BC)$
A
B



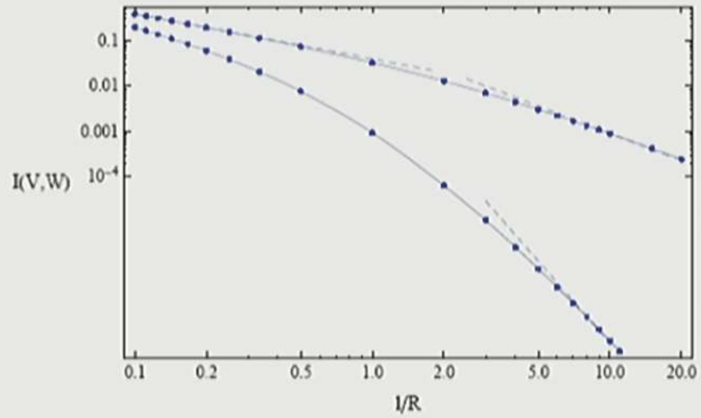
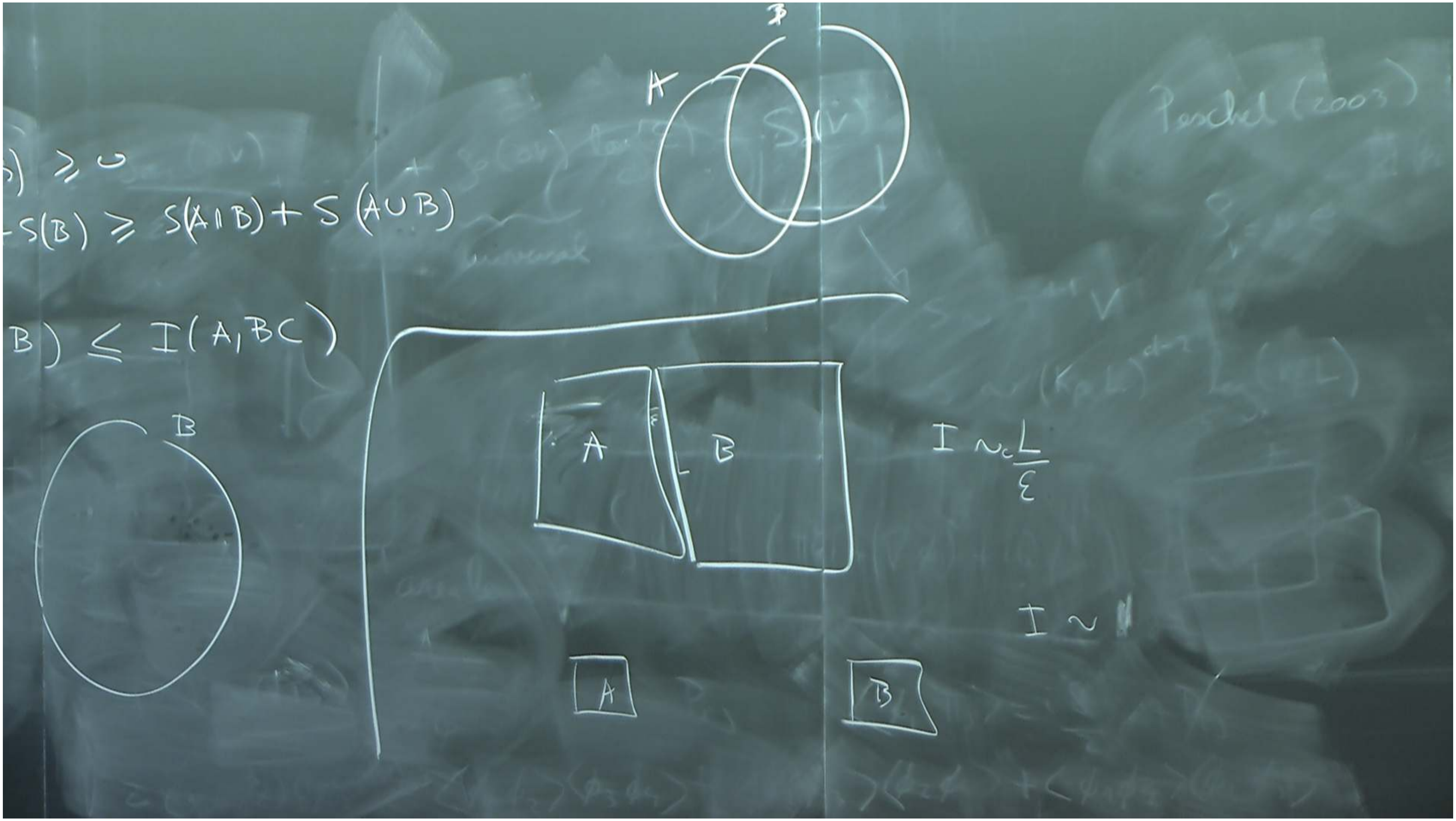
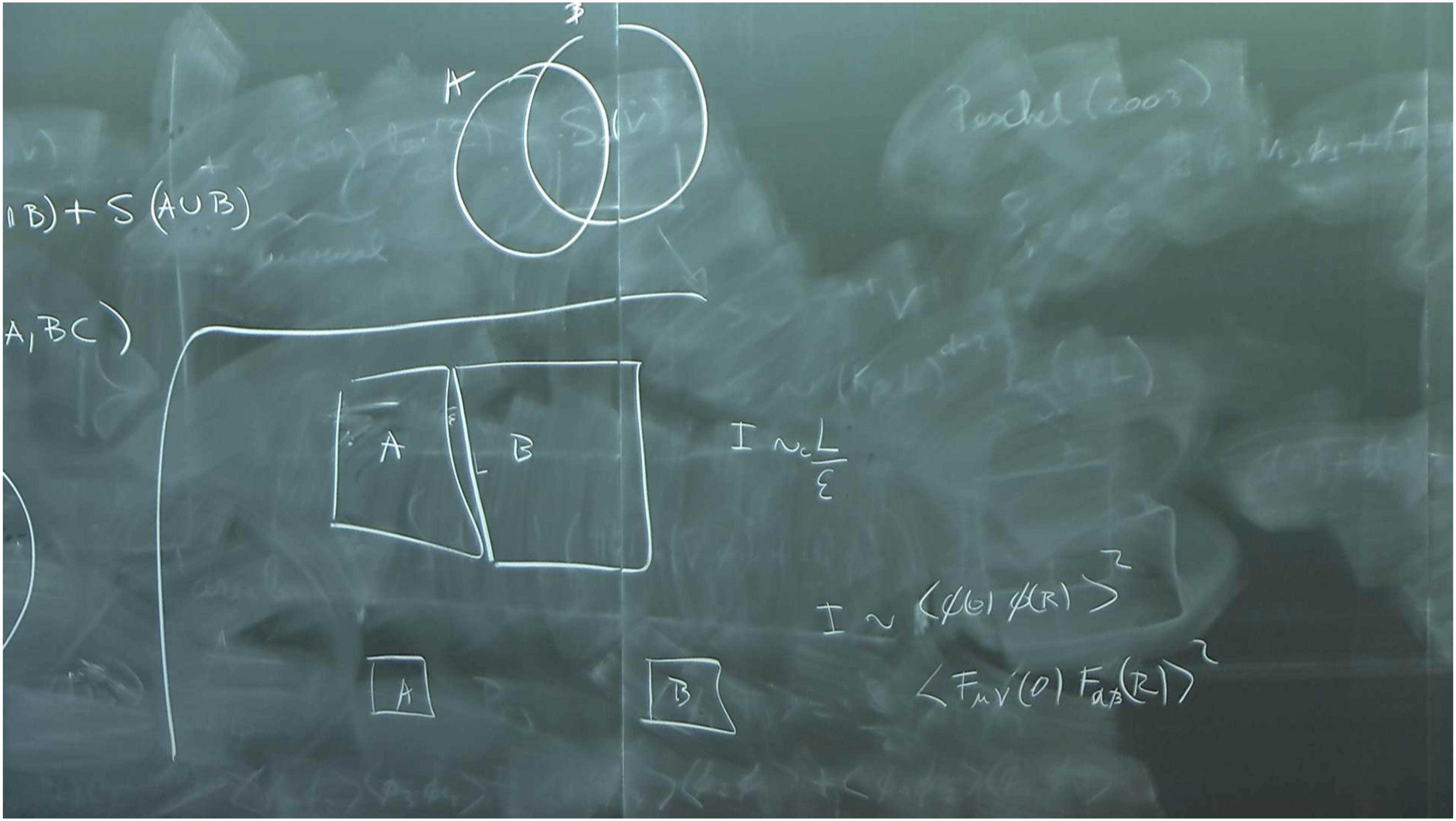
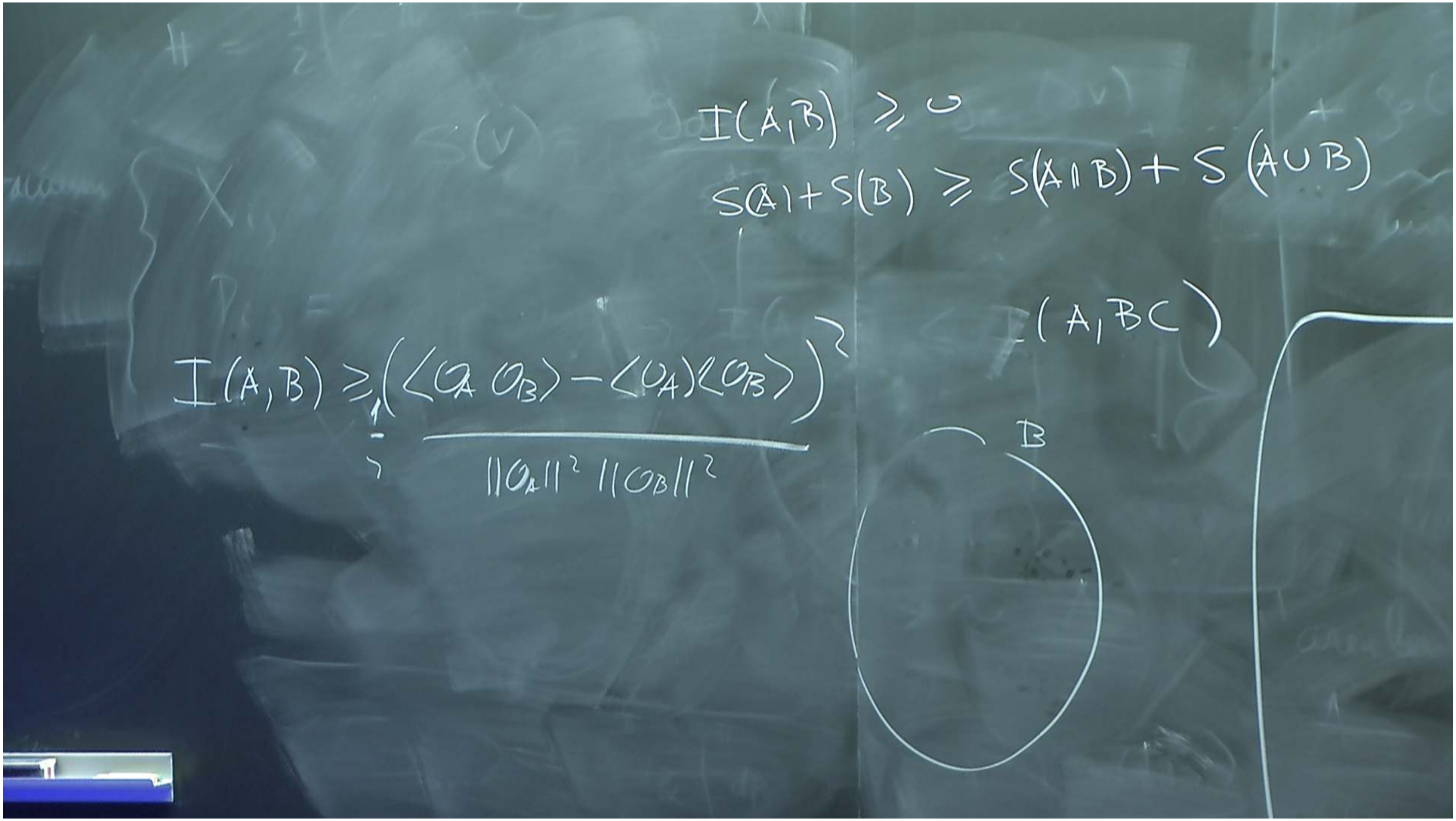


Figure 9: Log log plot of the mutual information for two squares of side R separated by a distance l, as a function of l/R. The curve at the top is the mutual information for the scalar and the lower one in the mutual information of the gauge model. The dashed lines are asymptotic behaviors. For small l/R we expect $I(V,W) \sim .0397R/l$ for both models, while for large distances we expect $I(V,W) \sim (l/R)^2$ for the scalar and $I(V,W) \sim (l/R)^6$ for the Maxwell field.


≥ 0
S(B) ≥ S(A∩B) + S(A∪B)
B) ≤ I(A,BC)
A
B
Peschel (2003)
A
B
I ~ L/ε
I ~
A
B


B) + S(A∪B)
A, BC)
A
B
Peschel (2003)
A
B
L
ε
I ∼ L/ε
I ∼ ⟨φ(0) φ(R)⟩²
⟨F_μν(0) F_αβ(R)⟩²
A
B



$I(A,B) \geq 0$
$S(A) + S(B) \geq S(A \cap B) + S(A \cup B)$
$I(A, BC)$
$I(A,B) \geq \frac{1}{2} \frac{(\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle)^2}{\|O_A\|^2 \|O_B\|^2}$
B

Replica method

$$S_n = \frac{1}{1-n}\log\left(\mathrm{tr}\,\rho^n\right) \qquad \text{Renyi entropies}$$

$$S_{n\to 1} = S$$

$$\langle x_0, t_0 | x, 0\rangle = \int_{x_0,t_0}^{x,0} \mathcal{D}x(t)\, e^{iS[x]} = \langle x_0 | e^{\overbrace{iHt_0}^{-H\tau}} | x\rangle \longrightarrow \langle x_0 | e^{-E_0\tau} | 0\rangle\langle 0 | x\rangle$$

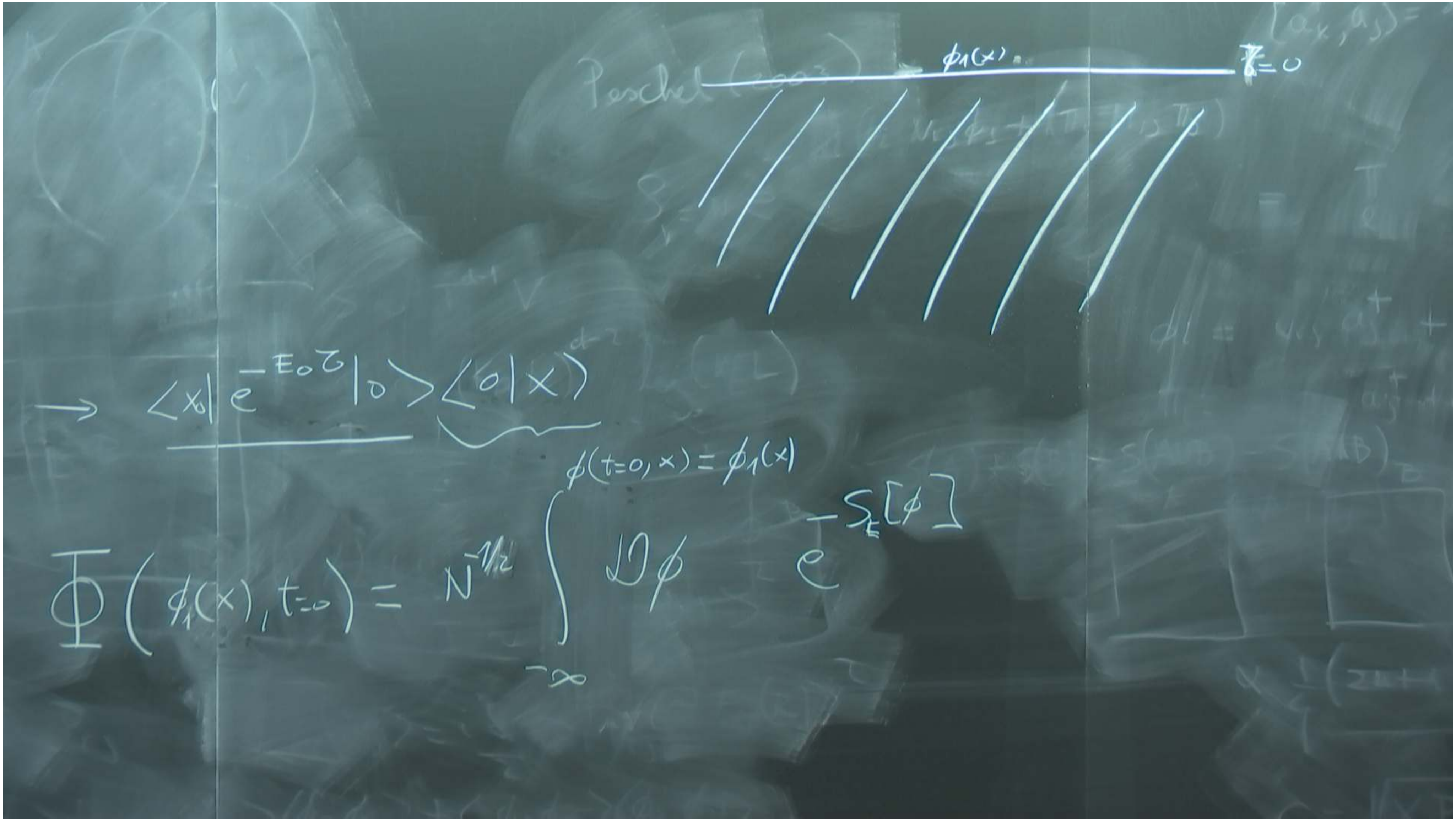
$$t_0 = i\tau \qquad \tau \to \infty$$

$$S_{n\to 1} = S$$

$$\langle x_0, t_0 | x, 0\rangle = \int_{x_0, t_0}^{x,0} \mathcal{D}x(t)\, e^{iS[x]} = \langle x_0 | e^{\overbrace{iHt_0}^{-H\tau}} | x\rangle \longrightarrow$$

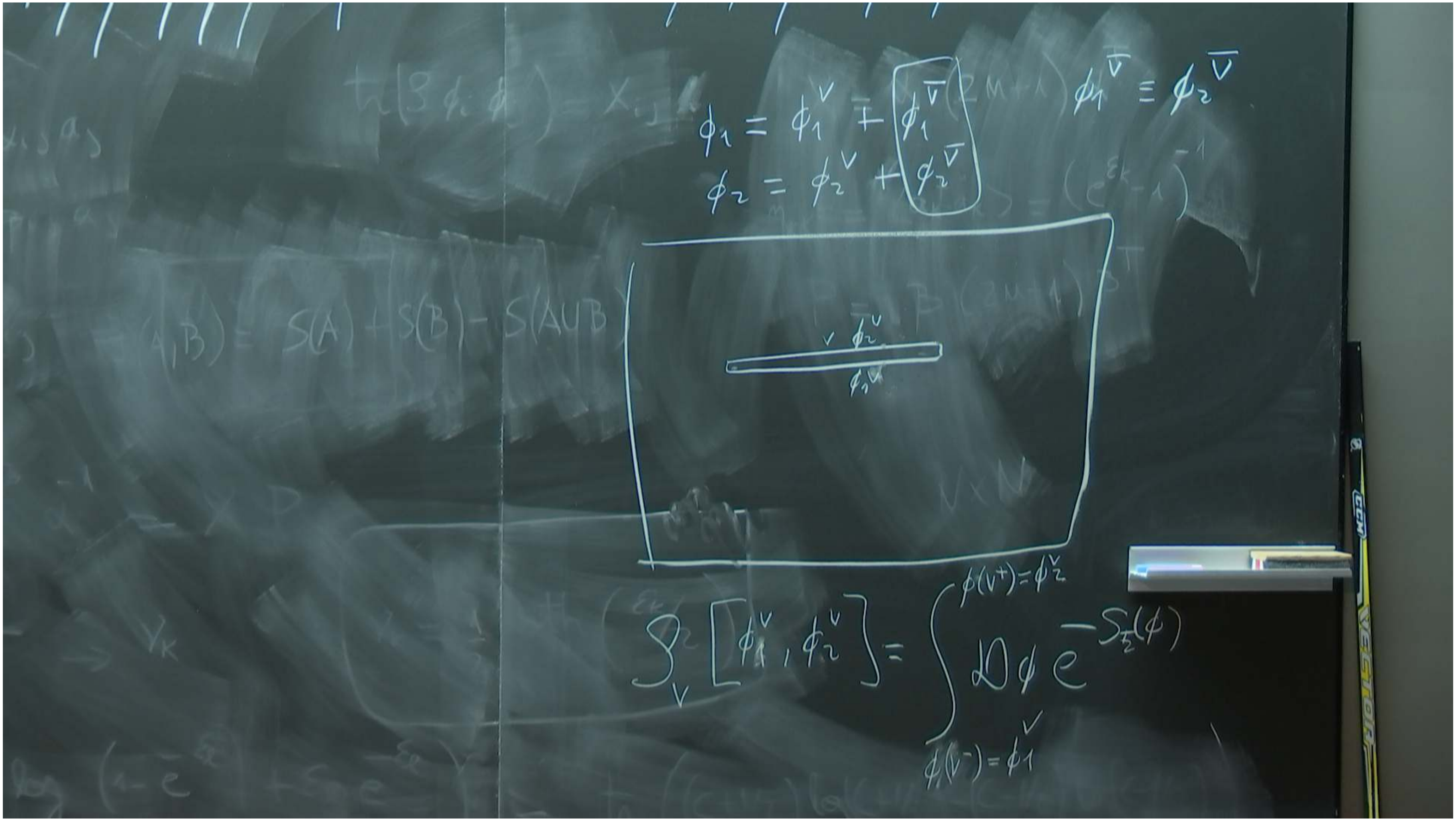
$$t_0 = i\tau \qquad \tau \to \infty$$

$$\Psi_0(x) = \int_{-\infty}^{x,0} \mathcal{D}x\, e^{-S_E[x]}$$


φ1(x)
τ = 0
→ ⟨x0| e^{-E0 τ} |0⟩⟨0|x⟩
φ(t=0, x) = φ1(x)
Φ(φ1(x), t=0) = N^{-1/2} ∫_{-∞} Dφ e^{-S_E[φ]}

$$\Psi_0(x) = \int_{-\infty}^{x,0} \mathcal{D}x \, e^{-S_E[x]}$$

$$\rho_0[\phi_1, \phi_2] = \Phi_0^*(\phi_1)\, \Phi_0(\phi_2)$$



$\phi_1 = \phi_1^V + \phi_1^{\bar{V}}$
$\phi_2 = \phi_2^V + \phi_2^{\bar{V}}$
$\phi_1^{\bar{V}} \equiv \phi_2^{\bar{V}}$
ϕ_2^V
ϕ_1^V
$\phi(V^+) = \phi_2^V$
$\mathcal{S}_V[\phi_1^V, \phi_2^V] = \int \mathcal{D}\phi \, e^{-S_E(\phi)}$
$\phi(V^-) = \phi_1^V$
$S(A) + S(B) - S(A \cup B)$