Title: Gravity Basics

Date: Jul 20, 2016  09:00 AM

URL: http://pirsa.org/16070019

Abstract:

# L2: Black holes & AdS

Recap of L1:

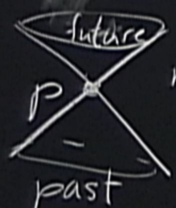
gravity = ST curvature

- ST tells matter how to move (e.g. geod. eq.: $\{x^\mu(\lambda)\}$ $\ddot{x}^\mu + \Gamma^\mu{}_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta = 0$)
- matter tells ST how to curve ($\leftarrow$ E. eq.: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$)

for $\{g_{\mu\nu}(x),\ T_{\mu\nu}(x)\}$

$$\begin{pmatrix} g_{00}(x^0, x^1, \dots) & g_{01}(\cdot\ -\ ) \\ & \ddots \end{pmatrix}$$

- local Lorentzian structure




no signal from p

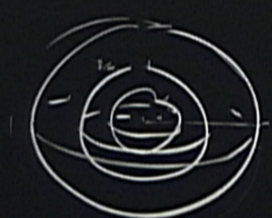
$\rightsquigarrow$ causal structure

A) Schwarzschild Soln = static, spher. sym. soln to vacuum ($T_{\mu\nu}=0$)
$\Lambda=0$, E-eq.

describes a BH, ST outside "star"

Choose coords based on symmetries:

- static: $t$ st. $g_{\mu\nu}$ indep. of $t$ ; $g_{\mu t}=0$
- spher. sym $\theta,\varphi$ s.t. only dep. on $\theta,\varphi$ comes through $d\Omega^2 \equiv d\theta^2+\sin^2\theta\, d\varphi^2$
- set $r$ to measure size of $S^2$: area of $S^2 = 4\pi r^2$



$$ds^2 = -f(r)\,dt^2 + h(r)\,dr^2 + r^2\,d\Omega^2$$

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soln: $f(r) = \frac{1}{h(r)} = 1 - \frac{2M}{r}\left(\frac{G_N}{c^2}\right)$ ;

$M =$ "mass" of BH

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

- $M = 0 \rightarrow$ Minkowski ST
- $M > 0$
  - as $r \to \infty$ $\quad ds^2 \to -dt^2 + dr^2 + r^2 d\Omega^2$ : Schw. is asymptotically flat
  - as $r = 2M$, $\quad g_{tt} = 0$, $g_{rr} = \infty$ ; coordinate singularity (remove via change of coords)
  - as $r = 0$. $\quad g_{rr} = 0$, $g_{tt} = \infty$ ; curvature singularity.

$$R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = \frac{48 M^2}{r^6} \xrightarrow[r \to 0]{} \infty$$

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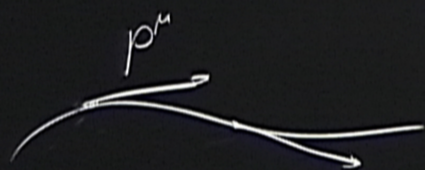
- $M<0$ : unphysical (naked singularity) symmetries $\rightsquigarrow$ CoM

$\partial_t \longrightarrow \quad E = \left(1-\frac{2M}{r(\lambda)}\right)\dot{t}(\lambda) \quad \rightarrow$ energy / unit mass

$\partial_\varphi \longrightarrow \quad L = r^2 \underset{1}{\sin^2\theta}\, \dot{\varphi} \quad \rightarrow$ ang. mom. — " —

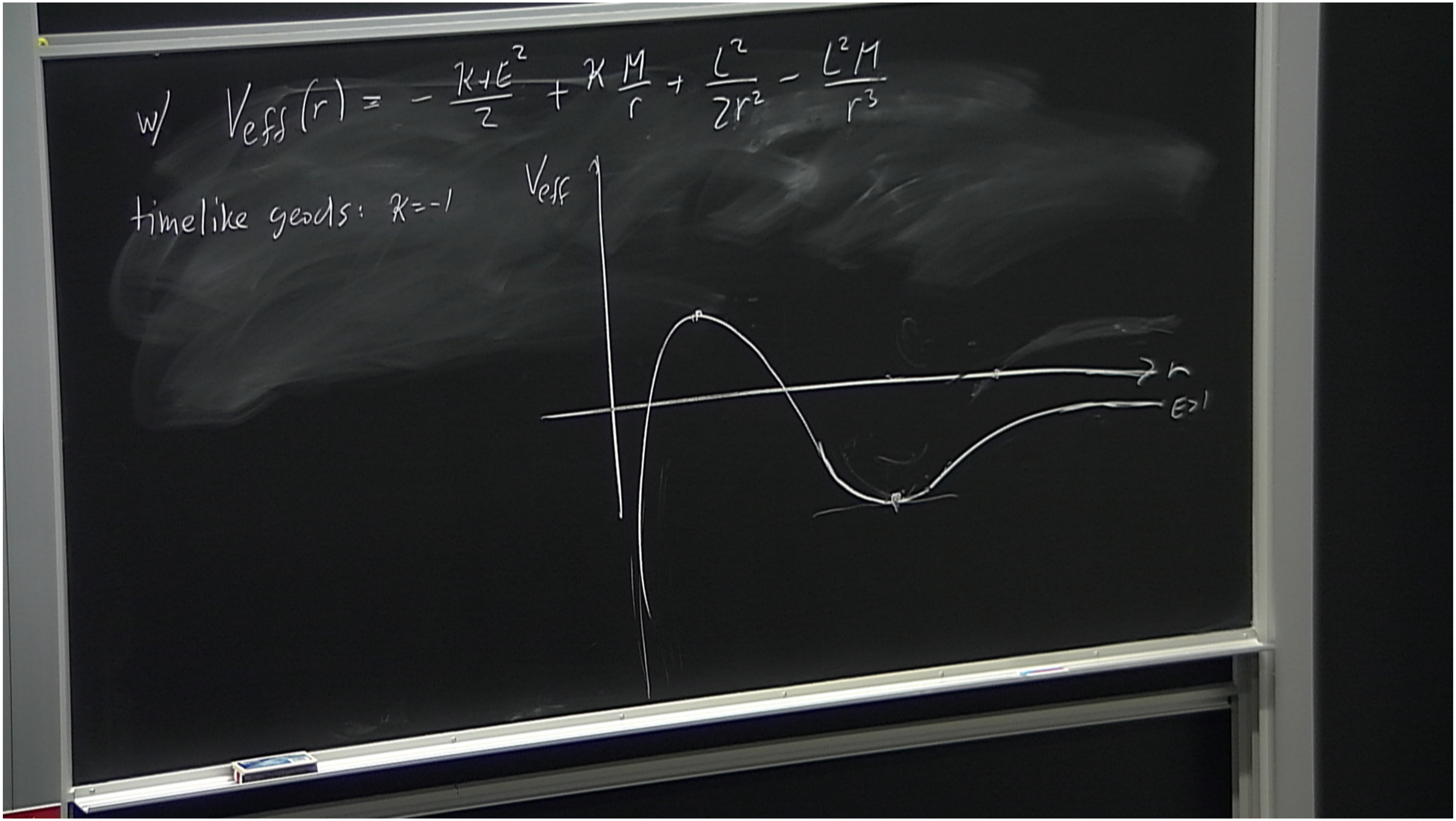
spher. sym $\Rightarrow$ WLOG set $\theta = \frac{\pi}{2}$

norm of tangent vect. $p^\mu$ $\quad \kappa = p_\mu p^\mu = \begin{cases} +1 & \text{spacelike} \\ 0 & \text{null} \\ -1 & \text{timelike} \end{cases}$ geods




$$= -\left(1-\frac{2M}{r}\right)\underset{E^2/(\;)^2}{\dot{t}^2} + \frac{\dot{r}^2}{\left(1-\frac{2M}{r}\right)} + r^2\left(\underset{0}{\dot{\theta}^2} + \underset{1}{\sin^2\theta}\,\underset{(L^2/r^2)^2}{\dot{\varphi}^2}\right)$$

$\rightsquigarrow$ effective 1-d motion: $\frac{1}{2}\dot{r}^2 + V_{eff}(r) = 0$


w/ $V_{eff}(r) = -\frac{\kappa+E^2}{2} + \kappa\frac{M}{r} + \frac{L^2}{2r^2} - \frac{L^2 M}{r^3}$
timelike geods: $\kappa=-1$
$V_{eff}$
$r$
$E>1$

Causal structure

Cons. null radial geods:

$$\left(1-\frac{2M}{r}\right)dt^2 = \frac{dr^2}{\left(1-\frac{2M}{r}\right)} \rightarrow \frac{dt}{dr} = \frac{\pm 1}{1-\frac{2M}{r}}$$

let $\underline{v = t + r_*}$ ↳ tortoise coord. $\qquad dr_* \equiv \frac{dr}{1-\frac{2M}{r}}$

Ingoing Eddington coords: $ds^2 = -\left(1-\frac{2M}{r}\right)dv^2 + 2\,dv\,dr + r^2 d\Omega^2$

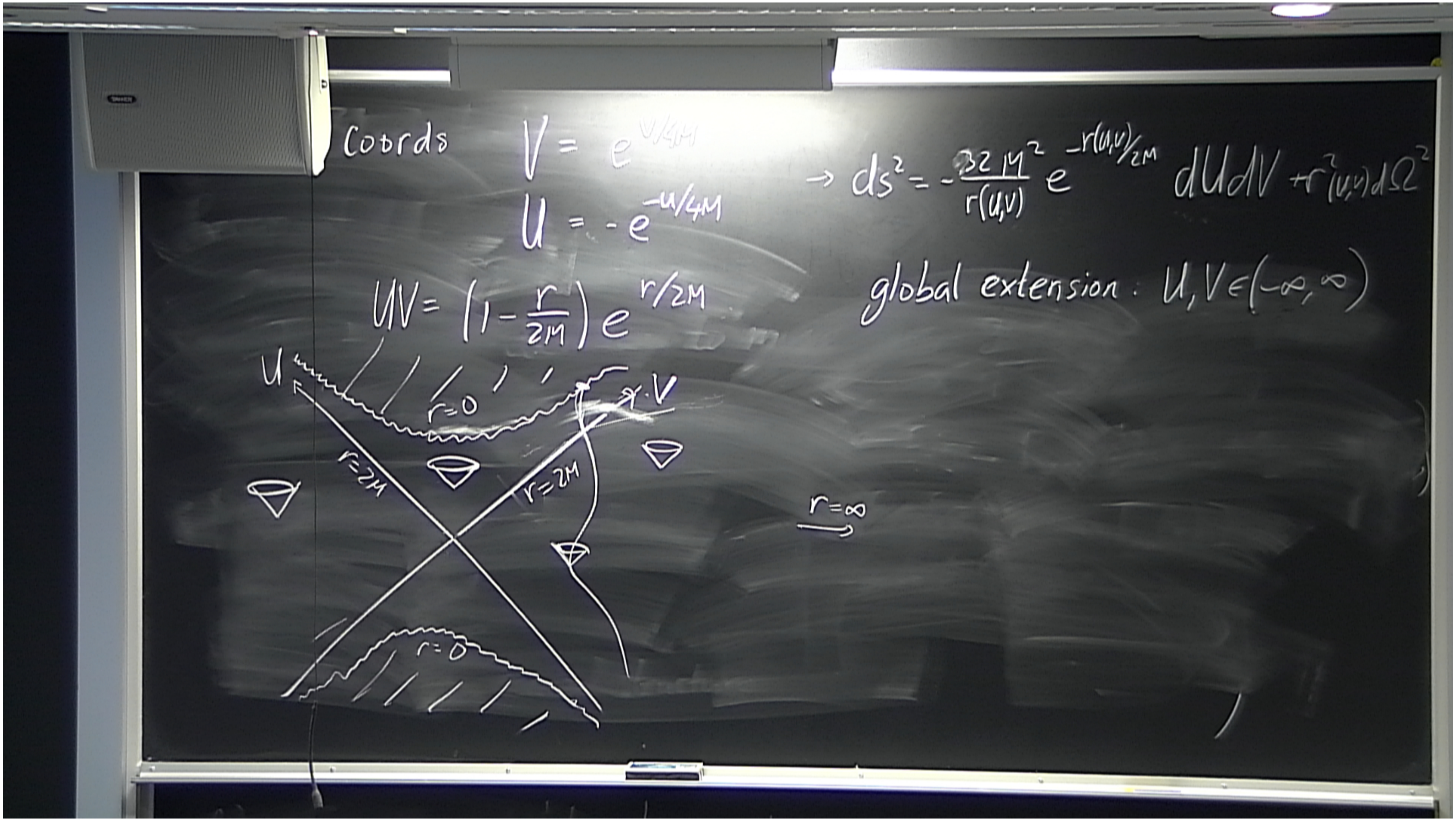
$(v, r, \theta, \varphi)$

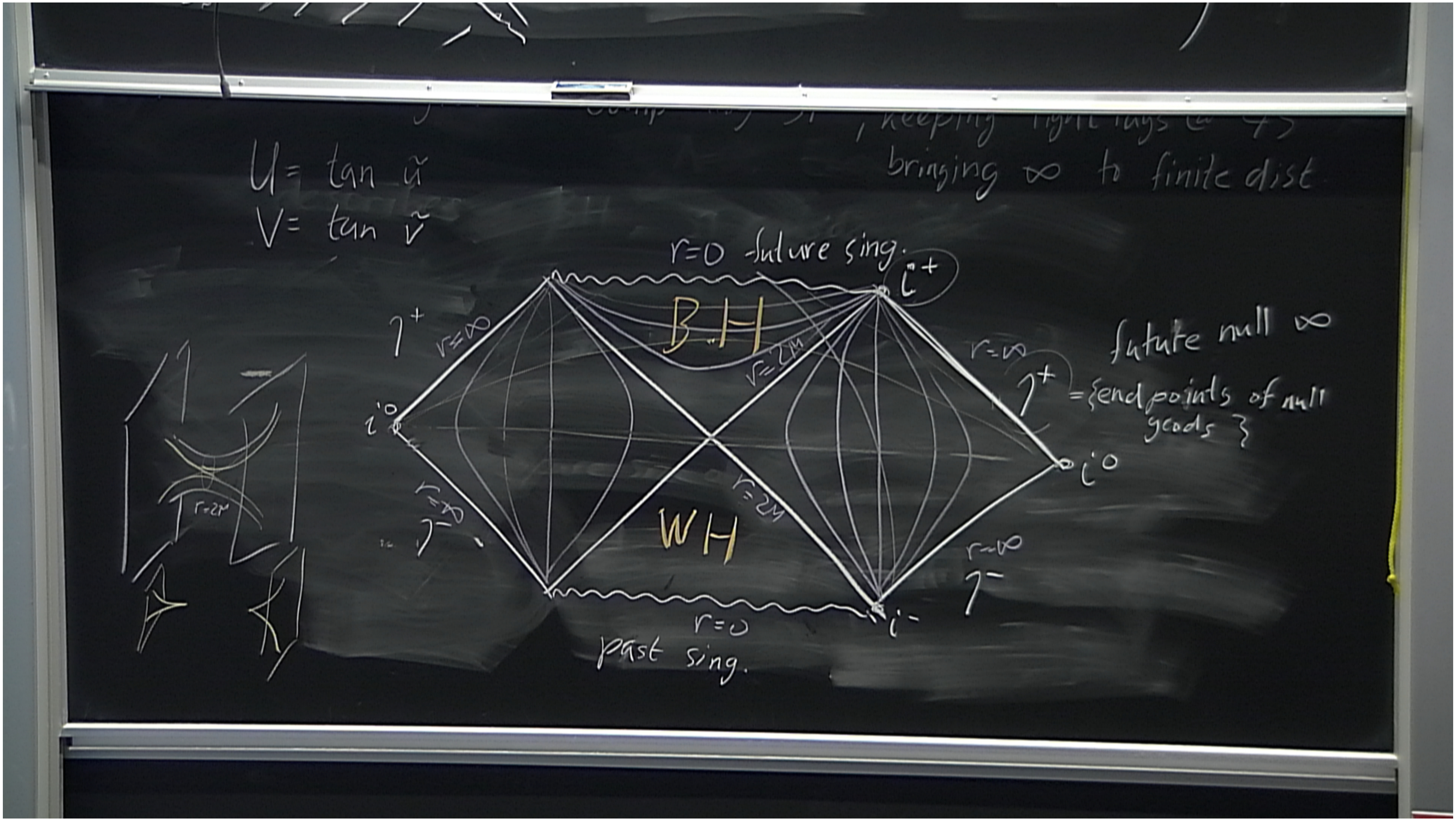
regular @ $r=2M$

but not time reversal sym.

& $v \to -\infty$ is reached in finite affine par.

$\underline{u = t - r_*} \rightarrow (v, u, \theta, \varphi)$


Coords $V = e^{v/4M}$
$U = -e^{-u/4M}$
$UV = \left(1-\frac{r}{2M}\right)e^{r/2M}$
$\to ds^2 = -\frac{32M^2}{r(U,V)}e^{-r(U,V)/2M}\,dU\,dV + r^2(U,V)\,d\Omega^2$
global extension: $U, V \in (-\infty, \infty)$
U
V
$r=0$
$r=2M$
$r=2M$
$r=0$
$r=\infty$


U = tan ũ
V = tan ṽ
bringing ∞ to finite dist
r=0 future sing.
i+
B.H
r=∞
r=2M
future null ∞
I+ = {end points of null geods}
i0
W H
r=0
past sing.
I−
i−

$$BH = M - I^-[\mathcal{J}^+]$$

ST — past of — future null $\infty$

$$\text{event horizon} \equiv \partial I^-[\mathcal{J}^+]$$

boundary of

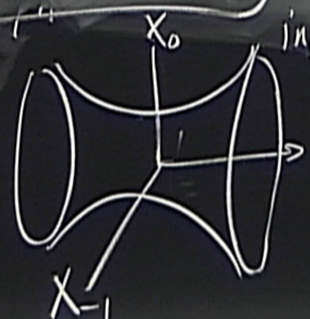
## AdS

= maximally symmetric soln. for $T_{\mu\nu} = 0$, $\Lambda < 0$

→ negatively curved

"Lorentzian hyperboloid"

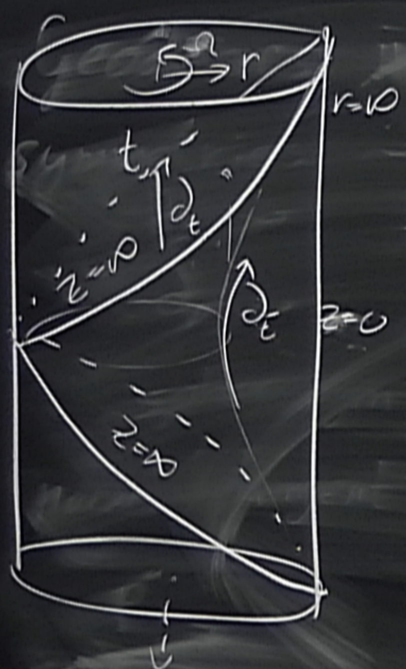
$$-X_{-1}^2 - X_0^2 + X_1^2 + \dots + X_d^2 = -\ell^2$$




into $ds^2 = -dX_{-1}^2 - dX_0^2 + dX_1^2 + \dots + dX_d^2$

$$\sim ds^2 = -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega_{d-1}^2$$

$\rightarrow AdS_{d+1}$




global AdS : static, spher. sym.

Poincare AdS : transl. sym

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + \sum_i (dx^i)^2 + dz^2\right)$$