Title: Gravitational Positive Energy Theorems from Information Inequalities

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Abstract:

Gravitational Positive Energy Theorems from Information Inequalities

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It from Qubit Summer School

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Swampland Question

Given an effective theory of gravity, how can one judge whether it is realized as a low energy appropximation to a consistent quantum theory with **ultra-violet completion**, such as string theory?

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Constraints on Symmetry

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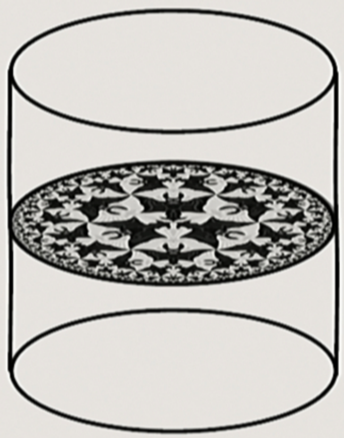
Conjectures:

- ☆ There are no global symmetry.
- ☆ All continuous gauge symmetries are compact.
- ☆ The spectrum of electric and magnetic charges forms a complete set consistent with the Dirac quantization condition.



Holographic understanding:

Harlow, arXiv: 1510.07911
Harlow + H.O., to appear

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More Conjectures:

Gravity is the weakest force in Nature.

Arkani-Hamed, Motl, Nicolis + Vafa, hep-th/0601001

Every symmetry is gauged.

With a gauge field, there is always a particle
whose **mass is smaller than its charge** in the Planck unit.

Black holes at the Reissner-Nordstrom bound
are unstable unless protected by supersymmetry.

Conjectures:

☆ The moduli space is **non-compact, complete, and has finite volume**.

☆ If we move a large distance T from a reference point, **a tower of light particles emerges** with mass of the order exp(- a T) for some a. The number of such light particles becomes infinite at T tends to the infinity.

☆ There is **no non-trivial one-cycle with minimal length** within a given homotopy class in the moduli space.

as formulated by Vafa + H.O., arXiv:0605264

These moduli space constraints have been proven for theories with N=3 or higher supersymmetry.

Cecotti, "Supersymmetric Field Theories," section 4.9.1

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Constraints on Calabi-Yau Topology

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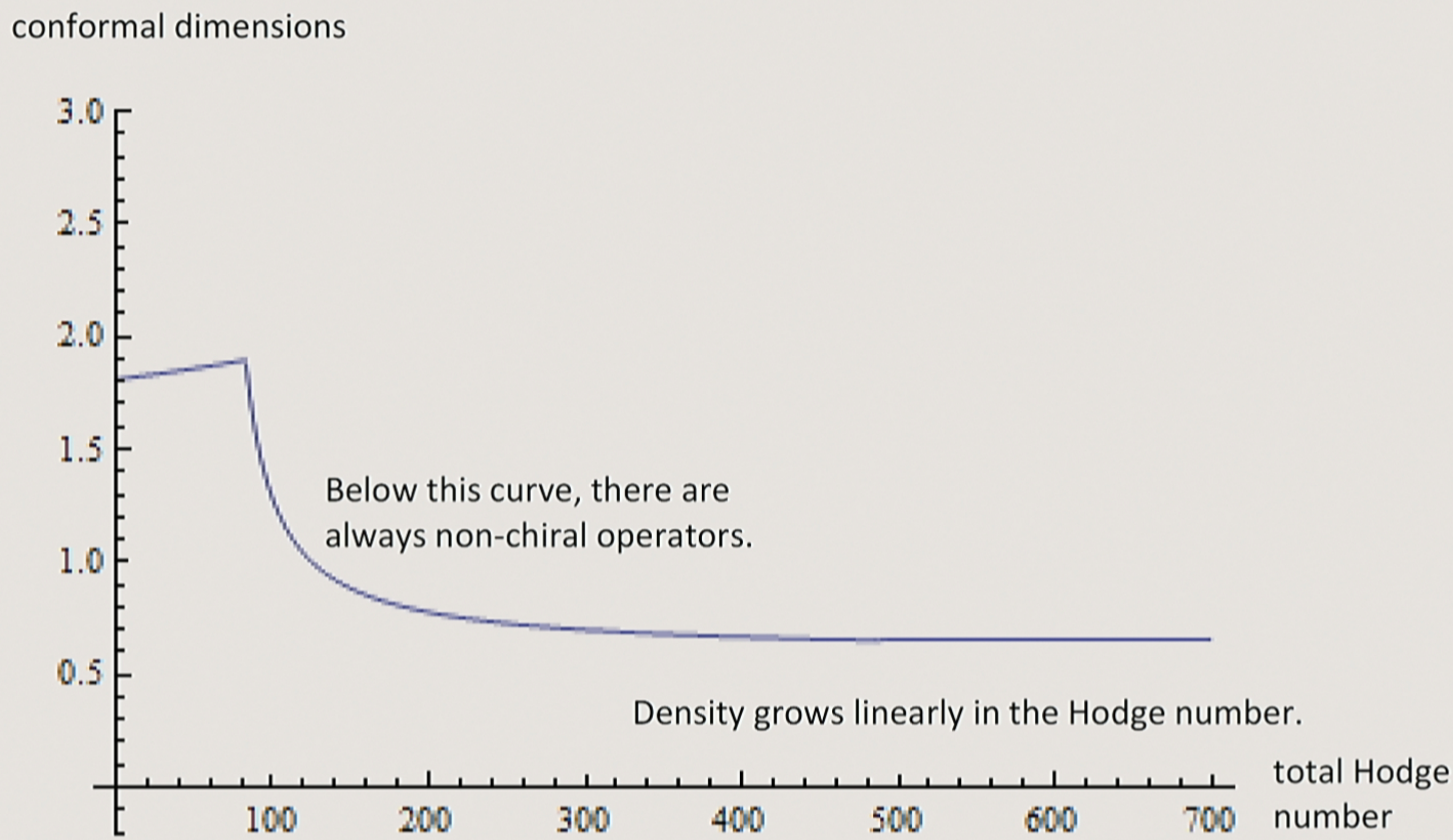
Modular invariance constraints

Keller + H.O., arXiv: 1209.4649



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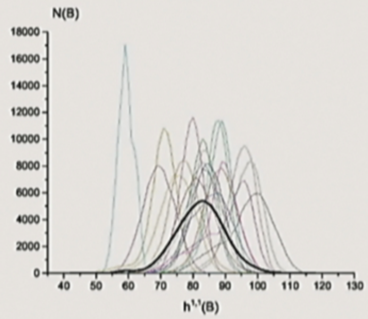
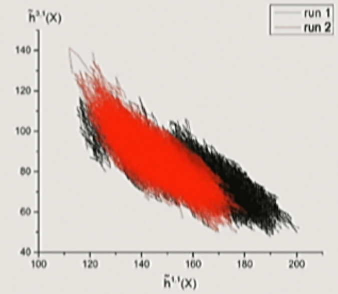
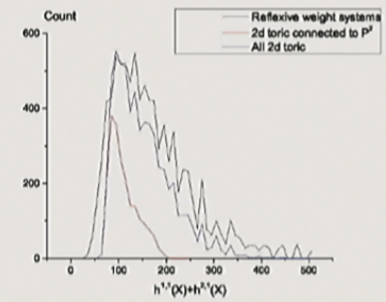
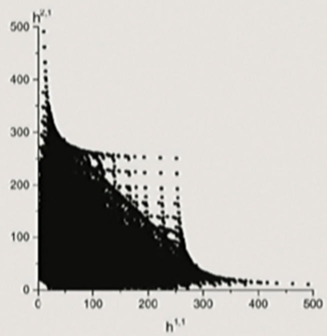
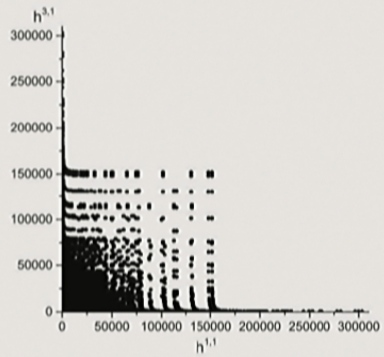
Recent experimental data on Calabi-Yau 3 and 4 folds











Taylor + Wang,
arXiv: 1510.04978, 1511.03209

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Holographic Constraints

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Suppose there is a low energy effective field theory whose gravity solutions asymptote to the anti-de Sitter space at the infinity.

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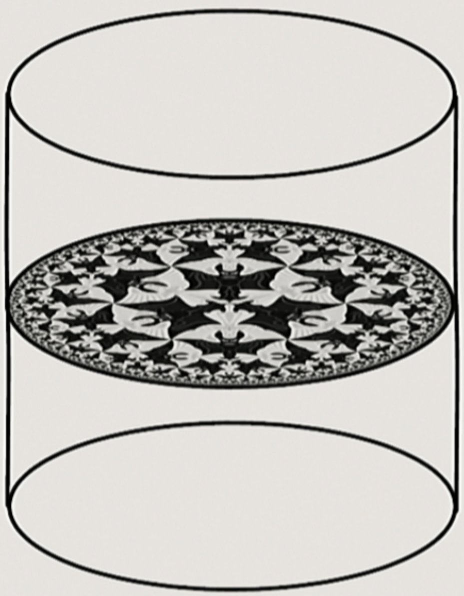
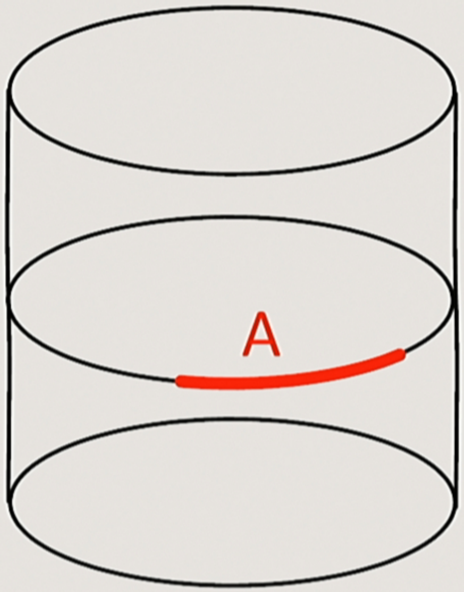


Holography of Quantum Gravity:

Consistent quantum gravity in AdS is equivalent to a conformal field theory on the boundary.

AdS/CFT Correspondence



Gravity theory in (d+1)-dim AdS is equivalent to d-dim CFT.

Entanglement Density Matrix ρ

For any state $|\psi\rangle$ in CFT, choose a spacelike region A.

$$\rho = tr_{\bar{A}} \, |\psi\rangle\langle\psi|$$

- ☆ The trace is on the Hilbert space over the complement of A.
- ☆ It is an operator acting on the Hilbert space over A.

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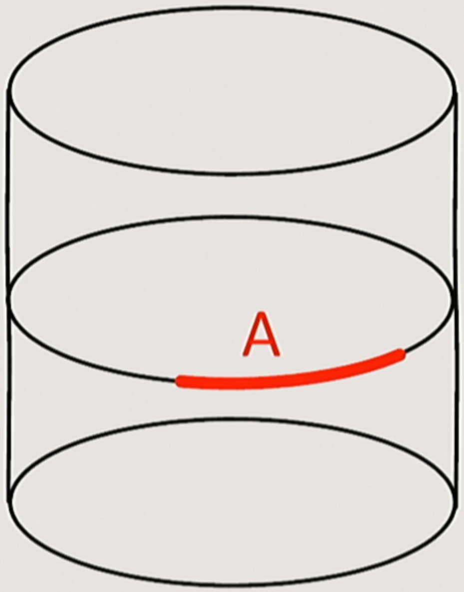


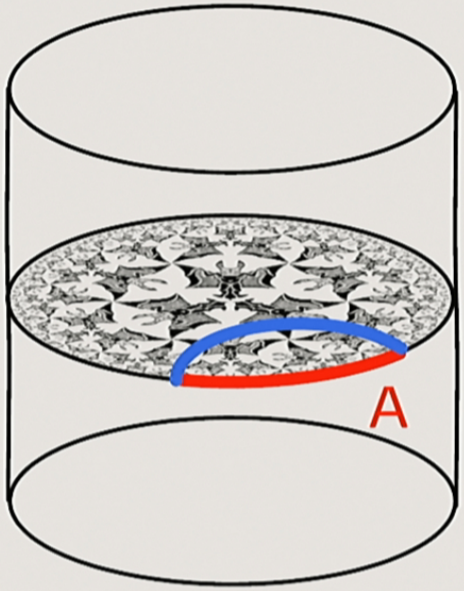
Entanglement Density Matrix ρ

$$\rho = tr_{\bar{A}}\, |\psi\rangle\langle\psi|$$

Entanglement Entropy *S*

$$S = -\mathrm{tr}\,\rho \log \rho$$

S measures the amount of entanglement between the region A and its complement.



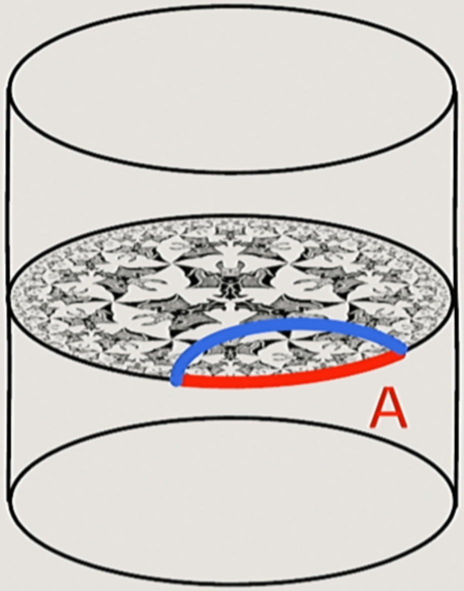
Entanglement Entropy S

$$S = -\operatorname{tr} \rho \log \rho$$

When the bulk gravity theory is described with smooth geometry, the entanglement entropy ***S*** is proportional to the area of the minimum surface ending of the boundary of A.

$$\mathrm{S} = \frac{1}{4G_N} Area(\Sigma)$$

Ryu-Takayanagi (2006)



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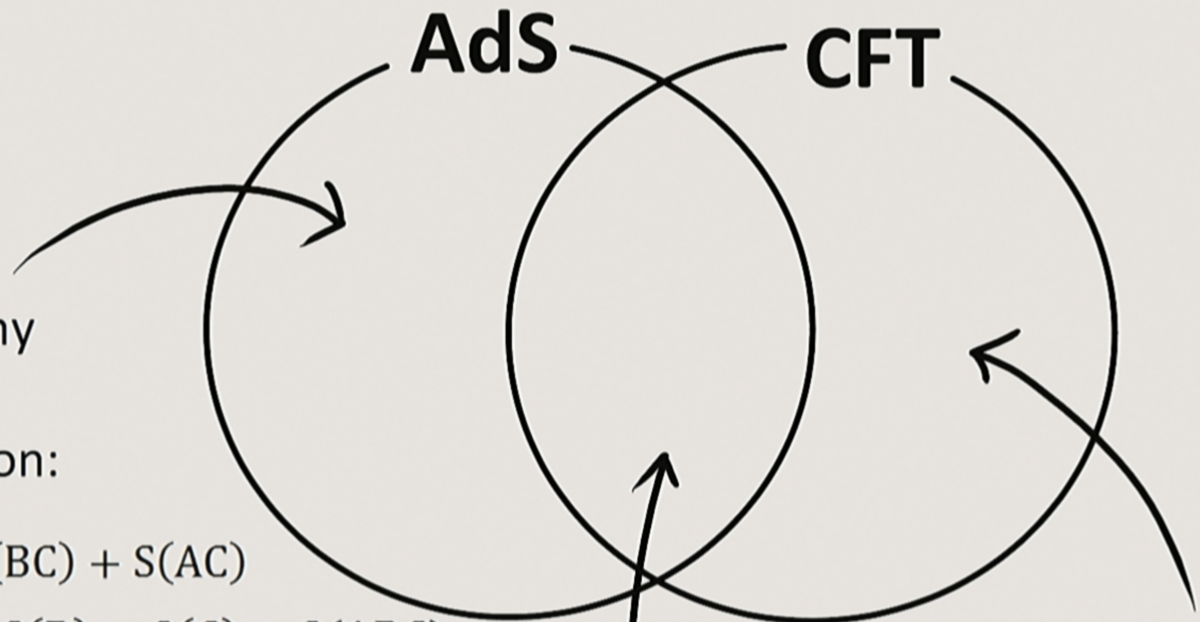

Monogamy of mutual information:

$$S(AB) + S(BC) + S(AC) \geq S(A) + S(B) + S(C) + S(ABC)$$

Positivity/monotonicity of relative entropy

Strong subadditivity:

$$S(AB) + S(BC) \geq S(B) + S(ABC)$$

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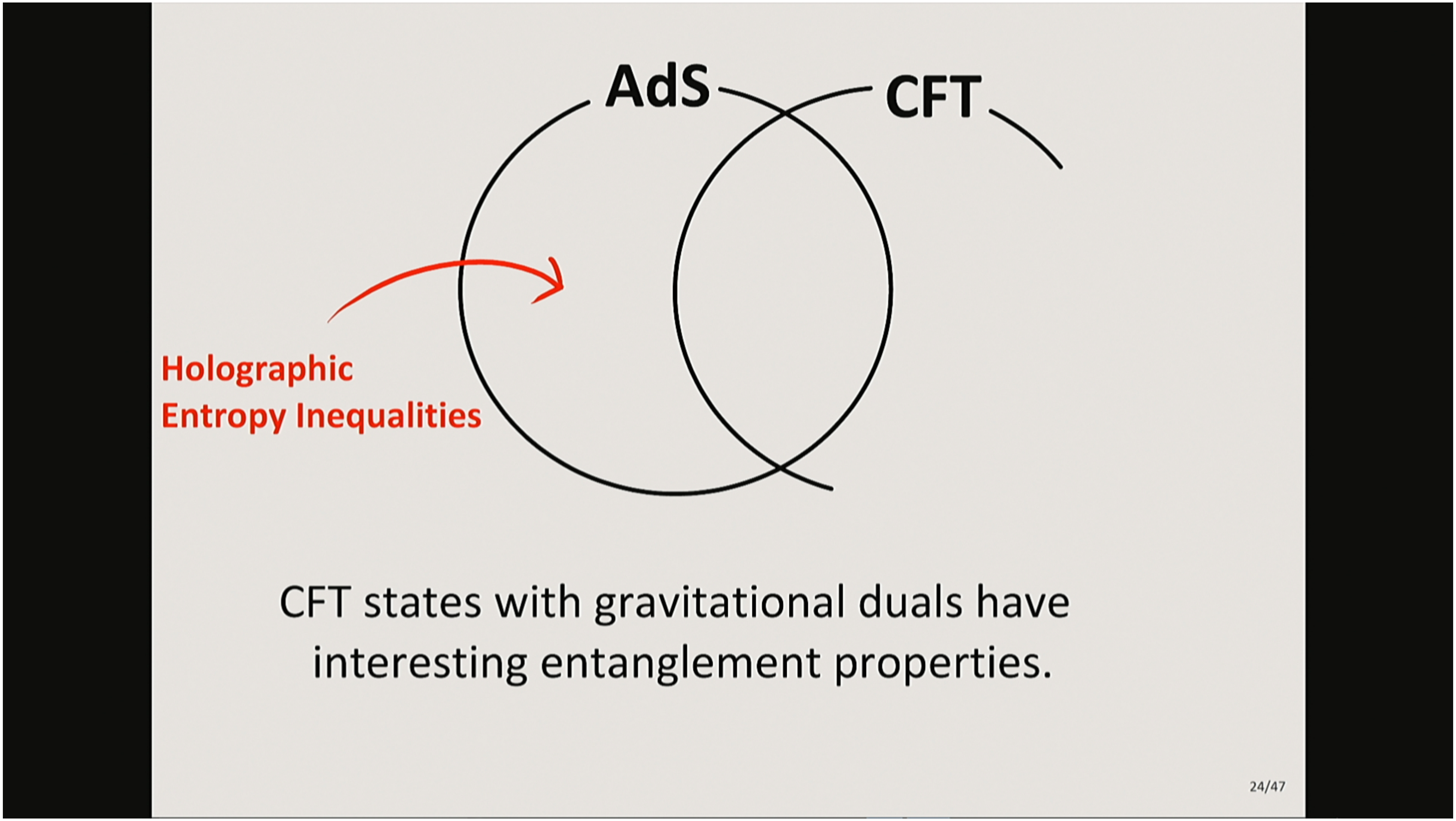
AdS
CFT
Holographic
Entropy Inequalities
CFT states with gravitational duals have
interesting entanglement properties.
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Entropy Inequalities

(Classical) Shannon Entropy:

There are ***infinite number*** of independent entropy inequalities for more than 3 regions.

$\Rightarrow$ Asymptotic performance for information processing tasks

Matus (2007)

(Quantum) von Neumann Entropy:

For more than 3 regions, the complete set of independent inequalities is ***not known***.

$\Rightarrow$ Numerical evidences that the number is infinite.

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For holographic states:

- ☆ **Finite algorithm** to classify all inequalities.
- ☆ There are **finitely many independent inequalities** for a fixed number of regions.
- ☆ Complete classification for 2, 3, 4 regions.
- ☆ A new family of inequalities for 5 and more regions.

Bao, Nezami, Stoica, Sully, Walter + H.O., arXiv:1505.07839

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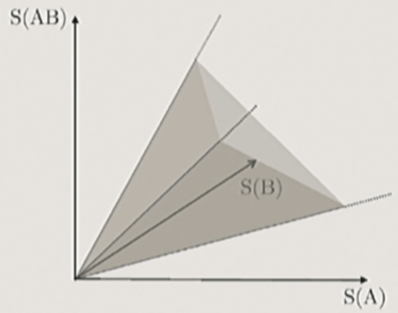
Holographic Entropy Cone



Entanglement entropies for n regions make a vector in $(2^n - 1)$ dimensions.

Entropy vectors of holographic states populate inside of a **convex rational polyhedral cone**.

The number of independent inequalities is finite for each n.

Are these implied by the gap conditions on CFT?

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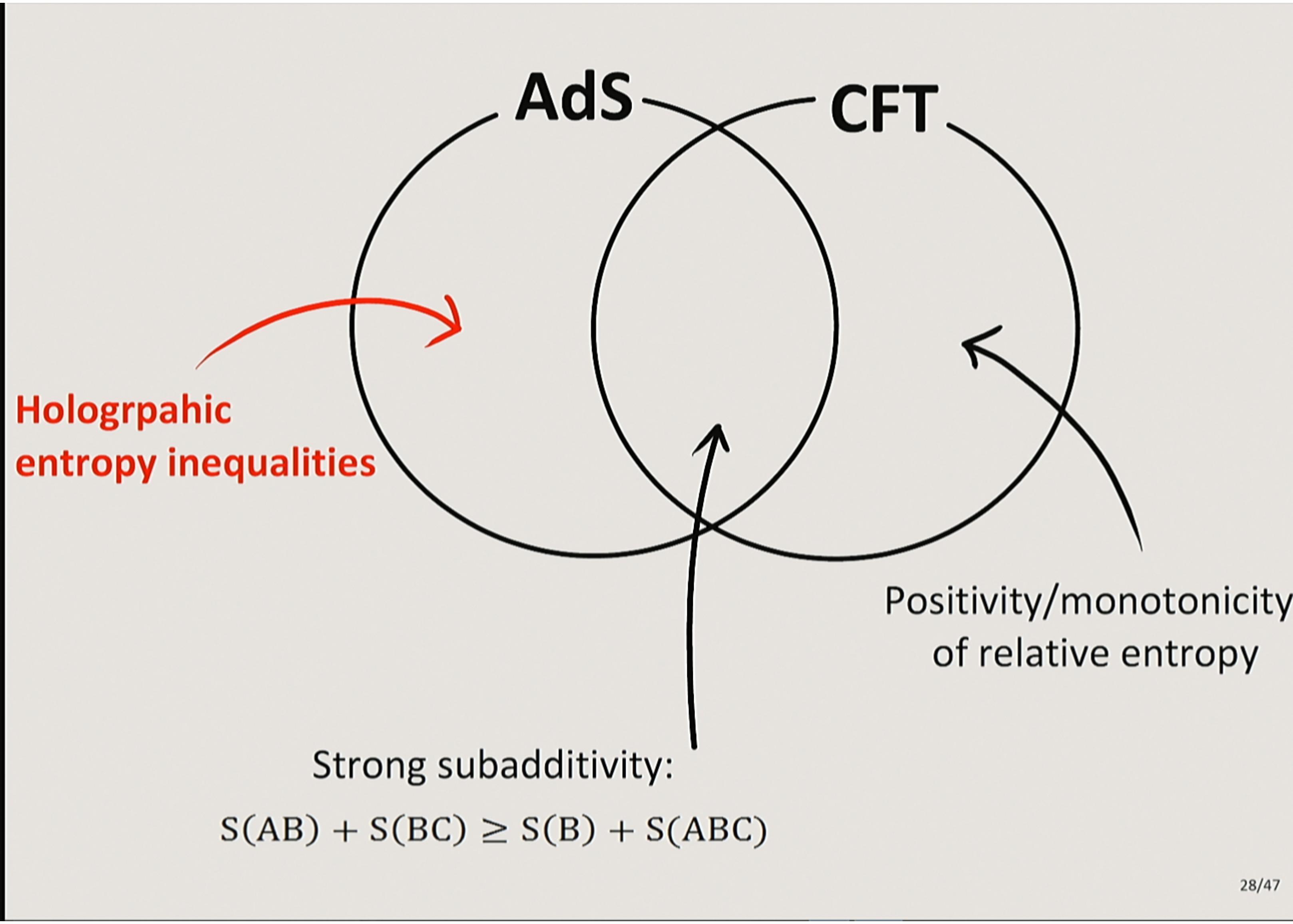
AdS
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entropy inequalities
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of relative entropy
Strong subadditivity:
S(AB) + S(BC) ≥ S(B) + S(ABC)
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Energy and Entropy

based on formalism
developed by Wald & collaborators

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Σ : subspace of a Cauchy surface

We will choose Σ to be part of an asymptotically AdS geometry bounded by the Ryu-Takayanagi surface and the AdS boundary.

$\Sigma \subset$ Cauchy surface , g : metric + matter on Σ.

$L(g)$: Lagrangian density

$$\delta L(g) = d\,\theta(\delta g) + \text{e.o.m.}$$

$$\Downarrow$$

Symplectic form

$$\Omega(\delta_1 g, \delta_2 g) = \int_\Sigma \omega(\delta_1 g, \delta_2 g) \equiv \int_\Sigma \delta_1 \theta(\delta_2 g) - \delta_2 \theta(\delta_1 g)$$

Analogy :

$$L(Q) = \frac{1}{2}\left(\frac{dQ}{dt}\right)^2 - V(Q)$$

$$\delta L(Q) = \frac{d}{dt}\left(\frac{dQ}{dt}\delta Q\right) + \text{e.o.m.}$$

$$= \frac{d}{dt}\theta(\delta Q) + \text{e.o.m.}$$

$$\theta(\delta Q) = P\,\delta Q$$

$$\delta\theta = \delta P \wedge \delta Q$$

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Hamiltonian H_ξ for a vector field ξ on Σ to generate $\mathcal{L}_\xi g$

$$\delta H_\xi = \Omega(\delta g, \mathcal{L}_\xi g)$$

$$= \int_\Sigma \delta\theta(\mathcal{L}_\xi g) - \mathcal{L}_\xi \theta(\delta g)$$

$$\left(\mathcal{L}_\xi \theta = \xi \cdot \underbrace{d\theta}_{\substack{\shortparallel \\ \delta L \,+\, \text{e.o.m.}}} + d(\xi\cdot\theta) \right)$$

$$= \int_\Sigma \delta\left(\theta(\mathcal{L}_\xi g) - \xi\cdot L\right)$$

$$- \oint_{\partial\Sigma} \xi\cdot\theta(\delta g)$$

Analogy :

$$\delta H = \delta p \frac{dQ}{dt} - \delta Q \frac{dP}{dt}$$

$$= \delta\left(p\frac{dQ}{dt}\right)$$

$$\underbrace{- \frac{d}{dt}(p\,\delta Q)}_{\substack{\shortparallel \\ \delta L \,+\, \text{e.o.m.}}}$$

$$= \delta\left(p\frac{dQ}{dt} - L\right)$$

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For a vector field ξ on Σ,

$$\delta H_\xi = \int_\Sigma \delta\left(\theta(\mathcal{L}_\xi g) - \xi \cdot L \right) - \oint_{\partial\Sigma} \xi \cdot \theta(\delta g) .$$

If $^\exists B$ on $\partial\Sigma$ such that $\xi \cdot \theta(\delta g) = \delta(\xi \cdot B)$,

$$H_\xi = \int_\Sigma J_\xi - \oint_{\partial\Sigma} \xi \cdot B$$

where

$$J_\xi = \theta(\mathcal{L}_\xi g) - \xi \cdot L .$$

e.g. pure Einstein gravity,

$L = \frac{1}{2}(R - \Lambda)\, e$, $\quad e$: spacetime volume form

$\theta(\delta g) = \frac{1}{2}(g^{\mu\nu} D^\rho - g^{\nu\rho} D^\mu)\, \delta g_{\nu\rho}\, e_\mu$, $\quad e_\mu$: volume form on Σ

$B \propto$ extrinsic curvature (Gibbons-Hawking term)

Relative Entropy

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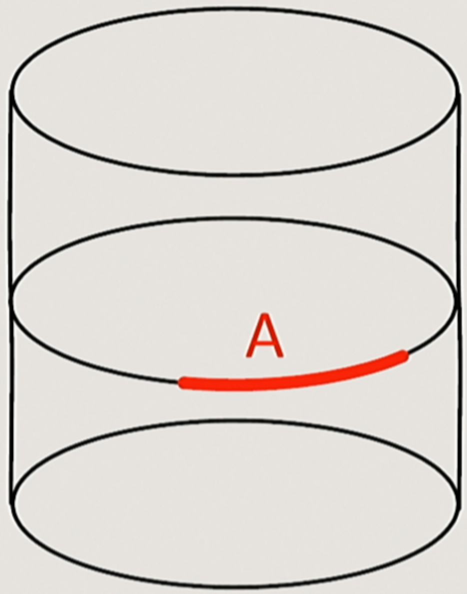


$|\psi_0\rangle$: vacuum in CFT

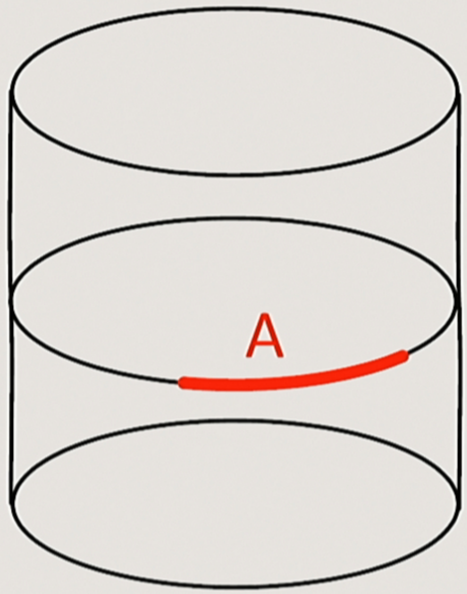
$\Leftrightarrow$ pure AdS geometry

$|\psi\rangle$: any CFT state

$\Leftrightarrow$ gravity solution

$$\rho_0 = \mathrm{tr}_{\bar{A}} \, |\psi_0\rangle\langle\psi_0|$$

$$\rho = \mathrm{tr}_{\bar{A}} \, |\psi\rangle\langle\psi|$$



Relative entropy :

$$S(\rho | \rho_0) = -\mathrm{tr}[\rho \log \rho_0] + \mathrm{tr}[\rho \log \rho]$$

measures the distance between

$$\rho_0 = \mathrm{tr}_{\bar{A}} |\Psi_0\rangle\langle\Psi_0|$$

$$\rho = \mathrm{tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

When A is a ball,

the modular Hamiltonian $= -\log \rho_0$ is simplified,

and $S(\rho | \rho_0)$ has a holographic expression.

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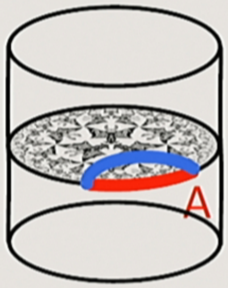
Relative Entropy

$$S(\rho \| \rho_0) = \underbrace{-\mathrm{tr}[\rho \log \rho_0]}_{=\ \langle \text{modular Hamiltonian} \rangle_\rho} + \underbrace{\mathrm{tr}[\rho \log \rho]}_{=\ -(\text{Entanglement Entropy})}$$



⟨modular Hamiltonian⟩$_\rho$ — Metric asymptotics on A

− (Entanglement Entropy) — Minimum surface area

Hamiltonian $H_\xi = \int_\Sigma J_\xi - \oint_{\partial\Sigma} \xi \cdot B$

$dJ_\xi = 0$ by e.o.m. $\Rightarrow \exists\, Q_\xi,\ J_\xi = dQ_\xi$.

$$H_\xi = \oint_{\partial\Sigma} (Q_\xi - \xi \cdot B)$$

$\exists\, \xi$, such that

$$S(\rho \| \rho_0) = H_\xi(\rho) - H_\xi(\rho_0).$$

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Relative Entropy = Canonical quasi-local energy

$$S(\rho | \rho_0) = H_\xi(\rho) - H_\xi(\rho_0)$$

Since $S(\rho | \rho_0) \geq 0$,

$H_\xi(\rho)$ is bounded below by the vacuum energy.

Lashkari, Lin, Stoica, van Raamsdonk + H.O. arXiv:1605.01075

For linear variation, $\rho = \rho_0 + \delta\rho$

$$S(\rho_0 + \delta\rho, \rho_0) = 0$$

implies the linearized Einstein equation in the bulk.

Faulkner, Guica, Hartman, Myers + Van Raamsdonk, arXiv: 1312.7856

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In the next leading order with backreaction from matters,

$$S(\rho|\rho_0) \geq 0, \quad \frac{d}{dR} S(\rho|\rho_0) \geq 0$$

↖ Radius of A

imply integrated positivity conditions on $T_{\mu\nu}$ of matters,

such as
$$\int_\Sigma \xi^0 T_{00} \sqrt{g_\Sigma} \geq 0$$

Lin, Marcolli, Stoica + H. O. arXiv: 1412.1879

Lashkari, Rabideau, Sabella-Garnier, Van Raamsdonk arXiv: 1412.3514

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Relative entropy = Canonical quasi-local energy

Positivity and monotonicity of the relative entropy

⇒ • Linearlized Einstein equations. arXiv:1312.7856

• Integrated positivity of T_{00} arXiv:1412.1879
1412.3514

• Positivity of quasi-local energy arXiv:1605.01075

Any low energy effective theory of a consistent ultraviolet complete quantum theory of gravity must satisfy these positive energy conditions.

How strong are these positive energy conditions?

Which low energy theories are ruled out by them?

Note: $S(\rho|\sigma)_{CFT} = S(\tilde{\rho}|\tilde{\sigma})_{bulk}$

Jafferis, Lewkowycz, Maldacena, Suh: 1512.06431

Dong, Harlow, Wall: 1601.05416

Harlow: 1607.03901

Or, can we prove a new type of positivity theorems for quasi-local energies?

c.f. Bekenstein bound Casini: 0804.2182

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Radon transform

For a perturbation near AdS,

$$\frac{d}{dR}\left(\frac{d}{dR}+\frac{1}{R}\right) S(\rho \mid \rho_0) = 16\pi^2 \int_{RT} T_{00}$$

For AdS, this is invertible.

$\Rightarrow$ T_{00} is reconstructible from $S(\rho \mid \rho_0)$.

For a general solution, for some vectors v and τ,

$$\frac{d}{dR}\left(\frac{d}{dR}+\frac{1}{R}\right) S(\rho \mid \rho_0) = 2\pi \int_{RT} v \cdot \left(J_\tau - d(\tau \cdot B)\right)$$

Can we invert this?

Swampland Question:

How to characterize an effective gravity theory that can emerge in a low energy approximation to a consistent quantum theory, such as string theory.

Constraints on Symmetry

Constraints on Moduli Space

Constraints on Calabi-Yau Topology

Positive Energy Theorems

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