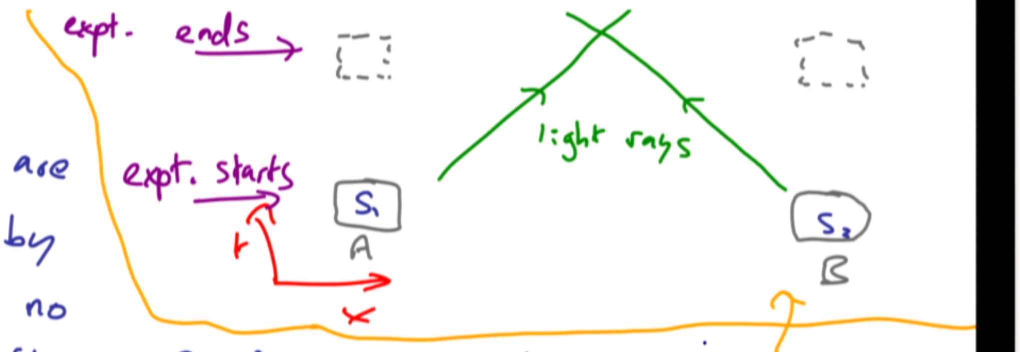
Title: Bellâ€™s Theorem

Date: Jul 18, 2016 05:00 PM

URL: http://pirsa.org/16070010

Abstract:

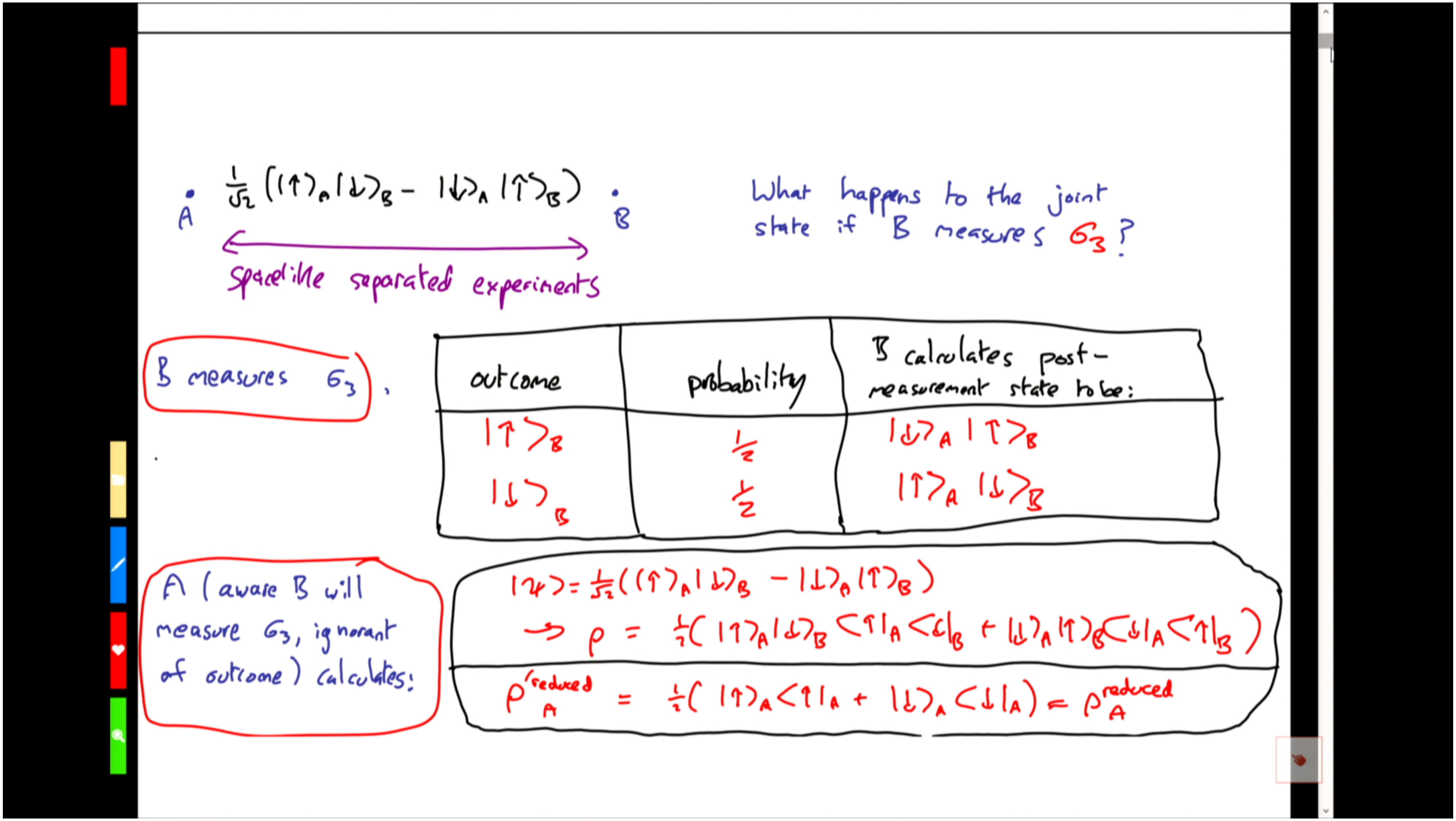
Spacelike separated systems



One way of ensuring $S_1 + S_2$ are isolated (from one another) is by space like separation: there is no time for a state of S_2 to influence S_1 (or vice versa) in this setup.

Nor is there time for measurement outcomes on S_2 to be communicated to an observer A at S_1, even if A and B are trying to collaborate.

$\rho_1(t)$ encodes ALL the information available to A throughout the experiment. No actions by B on S_2 – neither applying Hamiltonians nor measurements – affect $\rho_1(t)$. Operations on entangled subsystems don't give any means of superluminal signalling. "Peaceful coexistence" of QM and SR!



A • $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\right)$ • B

spacelike separated experiments

What happens to the joint state if B measures σ_3?

B measures σ_3.

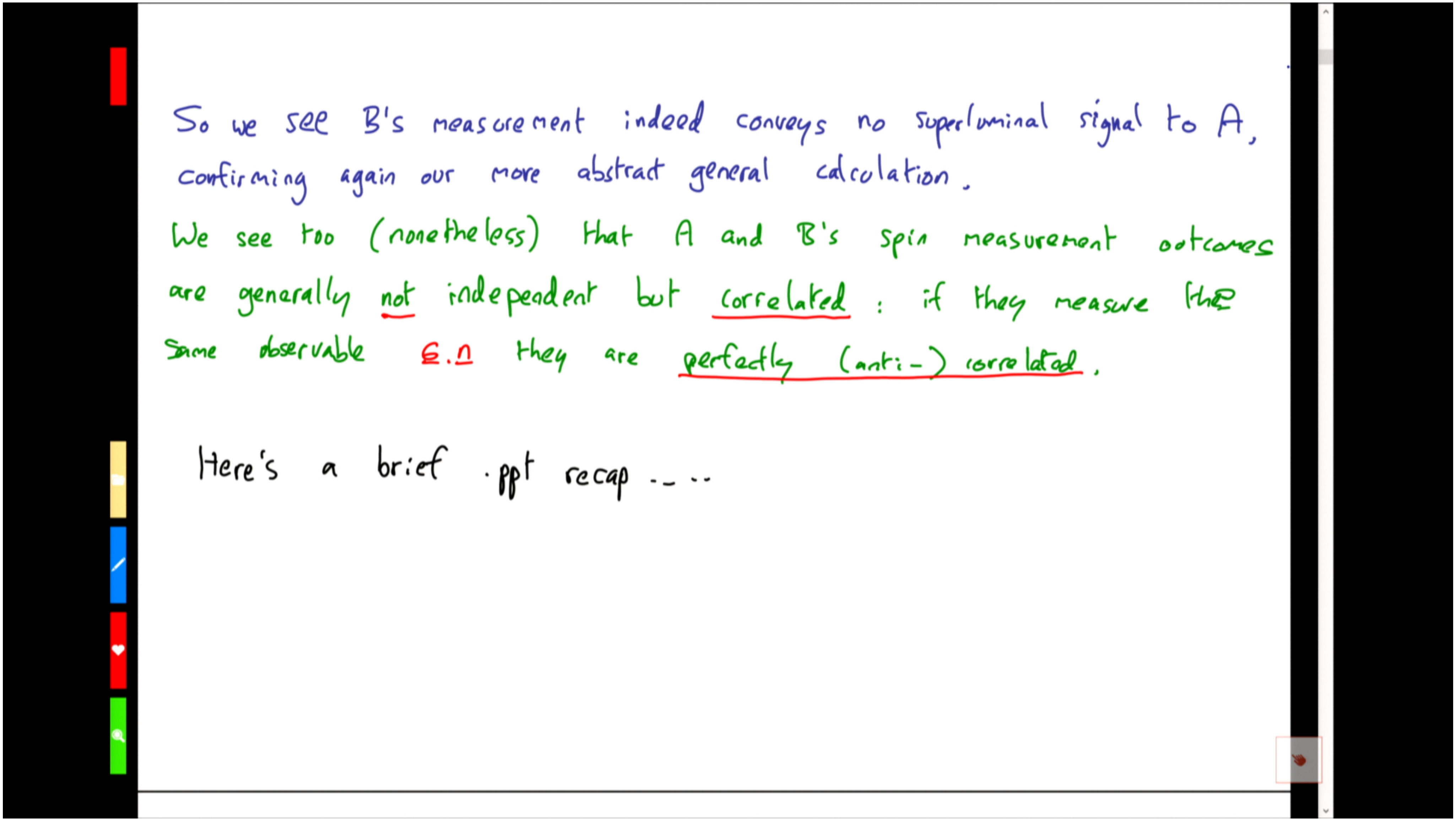
outcome	probability	B calculates post-measurement state to be:
$|\uparrow\rangle_B$	$\frac{1}{2}$	$|\downarrow\rangle_A|\uparrow\rangle_B$
$|\downarrow\rangle_B$	$\frac{1}{2}$	$|\uparrow\rangle_A|\downarrow\rangle_B$

A (aware B will measure σ_3, ignorant of outcome) calculates:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\right)$$

$$\rightarrow \rho = \frac{1}{2}\left(|\uparrow\rangle_A|\downarrow\rangle_B\langle\uparrow|_A\langle\downarrow|_B + |\downarrow\rangle_A|\uparrow\rangle_B\langle\downarrow|_A\langle\uparrow|_B\right)$$

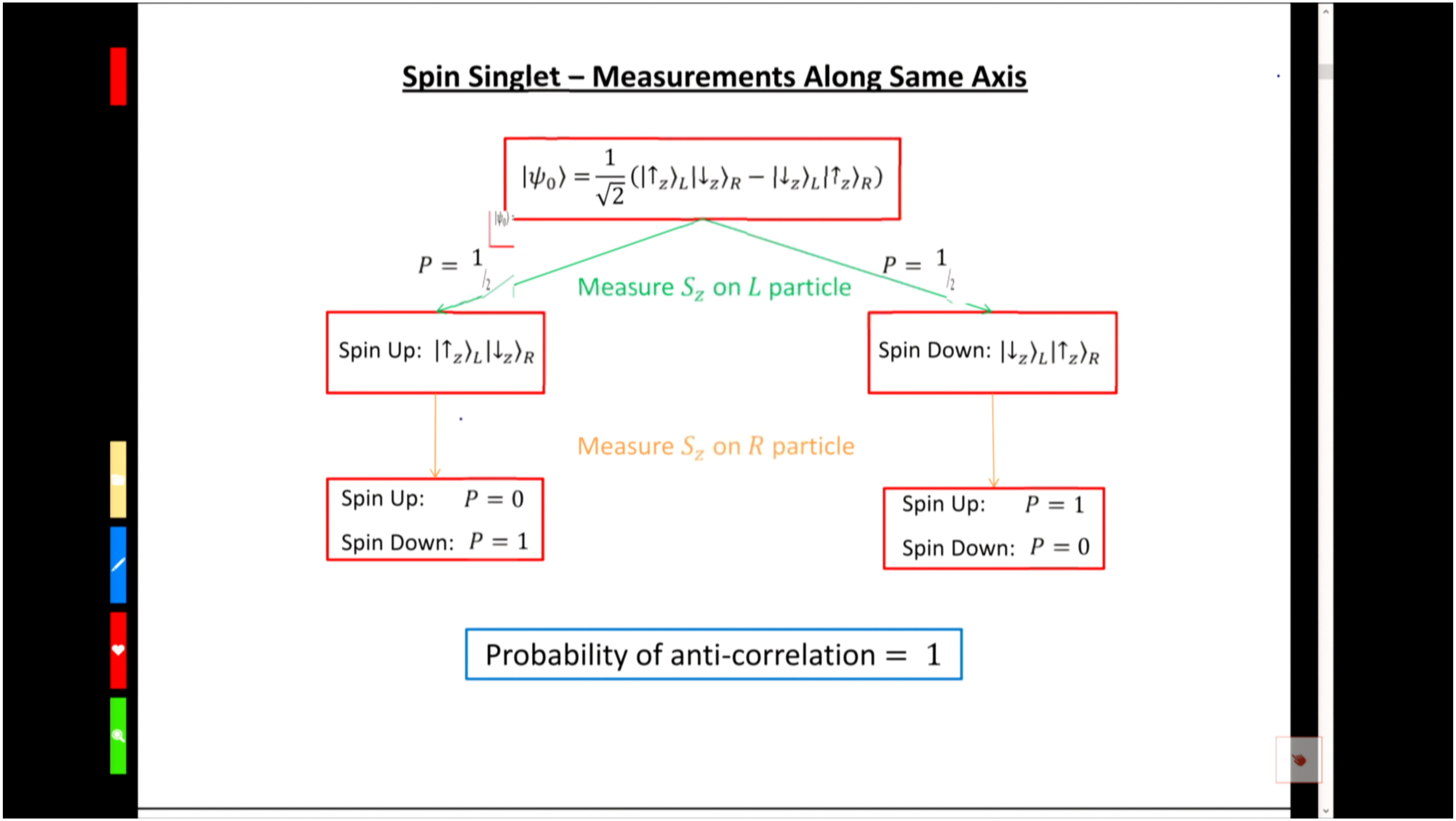
$$\rho'^{\text{reduced}}_A = \frac{1}{2}\left(|\uparrow\rangle_A\langle\uparrow|_A + |\downarrow\rangle_A\langle\downarrow|_A\right) = \rho^{\text{reduced}}_A$$



So we see B's measurement indeed conveys no superluminal signal to A, confirming again our more abstract general calculation.

We see too (nonetheless) that A and B's spin measurement outcomes are generally not independent but correlated: if they measure the same observable $\underline{\sigma}.\underline{n}$ they are perfectly (anti-) correlated.

Here's a brief .ppt recap ....



Spin Singlet – Measurements Along Same Axis

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle_L|\downarrow_z\rangle_R - |\downarrow_z\rangle_L|\uparrow_z\rangle_R\right)$$

$|\psi_0\rangle =$

$P = 1/2$

Measure S_z on L particle

$P = 1/2$

Spin Up: $|\uparrow_z\rangle_L|\downarrow_z\rangle_R$

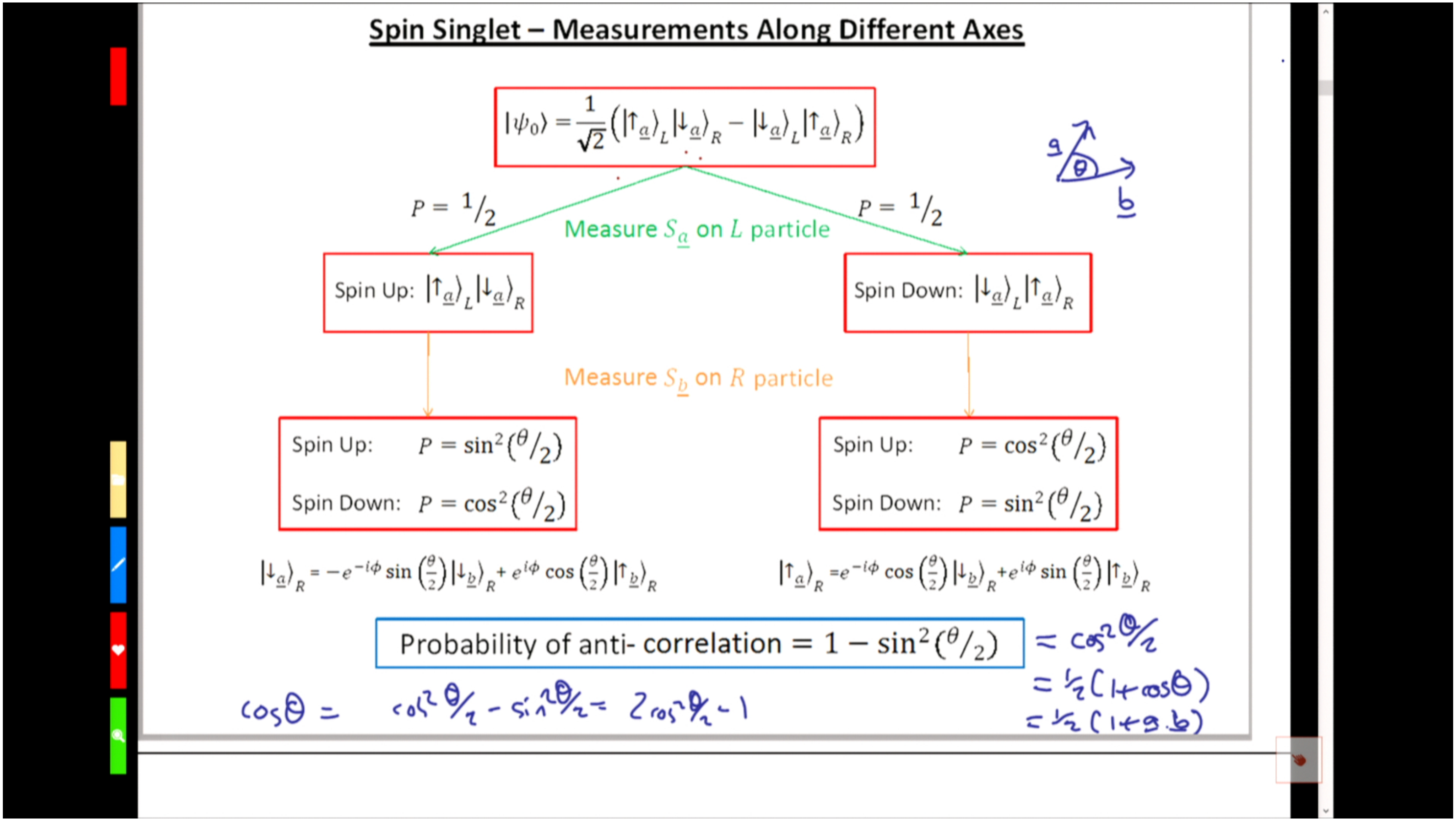
Spin Down: $|\downarrow_z\rangle_L|\uparrow_z\rangle_R$

Measure S_z on R particle

Spin Up: $P = 0$
Spin Down: $P = 1$

Spin Up: $P = 1$
Spin Down: $P = 0$

Probability of anti-correlation $= 1$



Spin Singlet – Measurements Along Different Axes

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R - |\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R\right)$$

$P = {}^1/_2$

Measure $S_{\underline{a}}$ on L particle

$P = {}^1/_2$

Spin Up: $|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R$

Spin Down: $|\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R$

Measure $S_{\underline{b}}$ on R particle

Spin Up: $P = \sin^2({}^\theta/_2)$

Spin Down: $P = \cos^2({}^\theta/_2)$

Spin Up: $P = \cos^2({}^\theta/_2)$

Spin Down: $P = \sin^2({}^\theta/_2)$

$$|\downarrow_{\underline{a}}\rangle_R = -e^{-i\phi}\sin\left(\frac{\theta}{2}\right)|\downarrow_{\underline{b}}\rangle_R + e^{i\phi}\cos\left(\frac{\theta}{2}\right)|\uparrow_{\underline{b}}\rangle_R$$

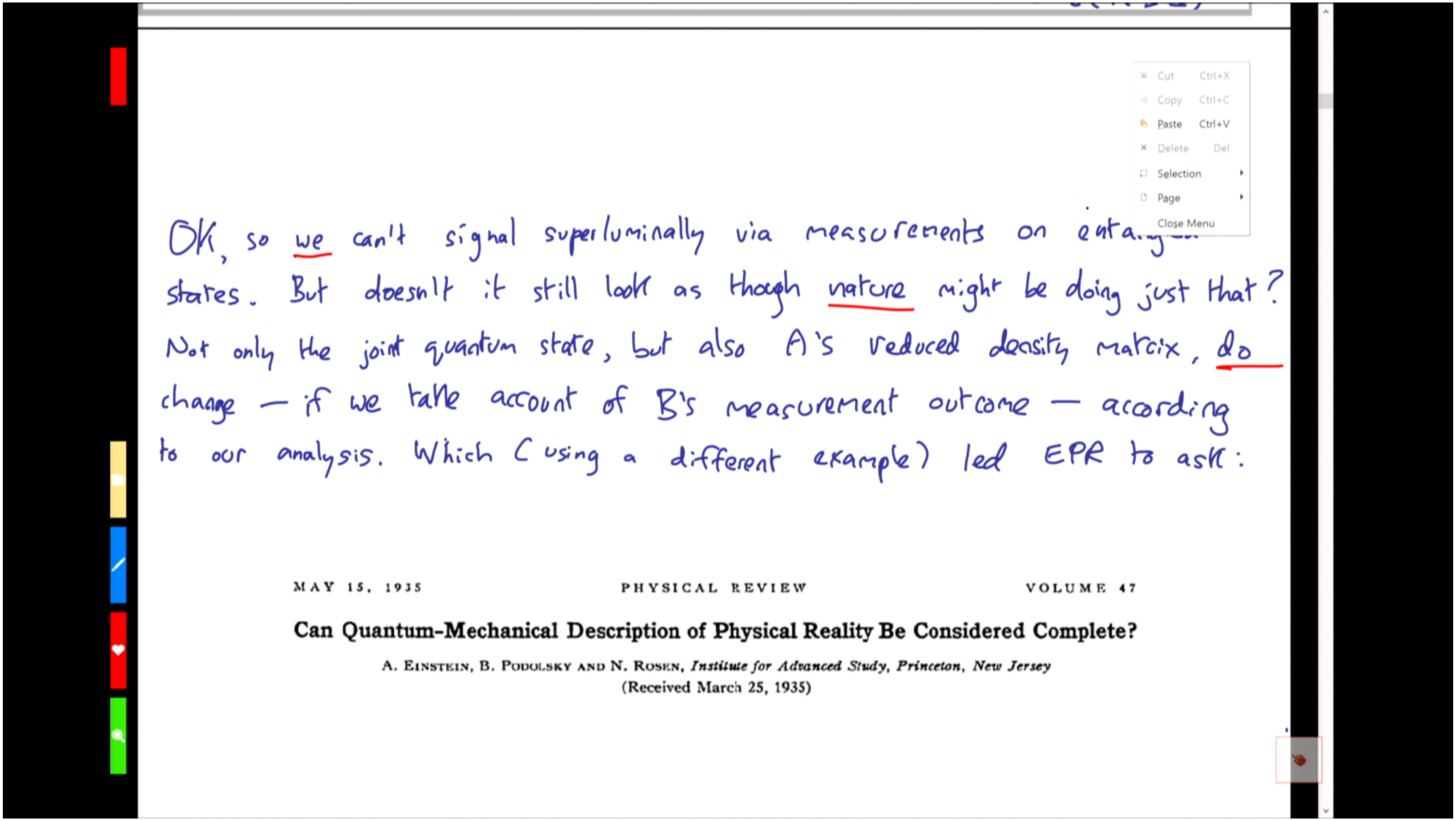
$$|\uparrow_{\underline{a}}\rangle_R = e^{-i\phi}\cos\left(\frac{\theta}{2}\right)|\downarrow_{\underline{b}}\rangle_R + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|\uparrow_{\underline{b}}\rangle_R$$

$$\boxed{\text{Probability of anti- correlation} = 1 - \sin^2({}^\theta/_2)} = \cos^2 \theta/2$$

$$= \tfrac{1}{2}(1+\cos\theta)$$

$$= \tfrac{1}{2}(1+\underline{a}\cdot\underline{b})$$

$$\cos\theta = \cos^2\theta/2 - \sin^2\theta/2 = 2\cos^2\theta/2 - 1$$



OK, so we can't signal superluminally via measurements on entangled states. But doesn't it still look as though nature might be doing just that? Not only the joint quantum state, but also A's reduced density matrix, do change — if we take account of B's measurement outcome — according to our analysis. Which (using a different example) led EPR to ask:

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Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

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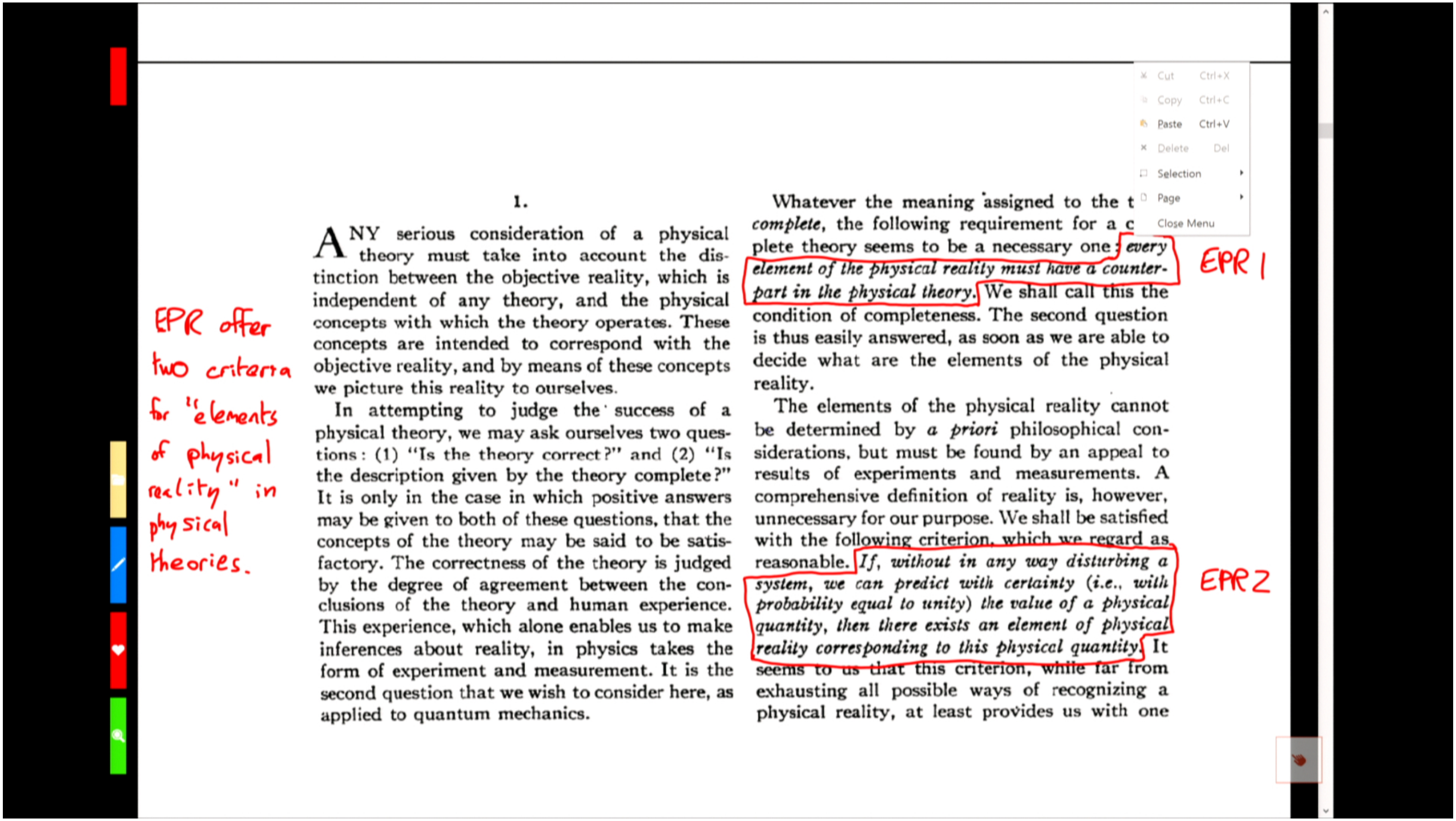
(Received March 25, 1935)



EPR offer two criteria for "elements of physical reality" in physical theories.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

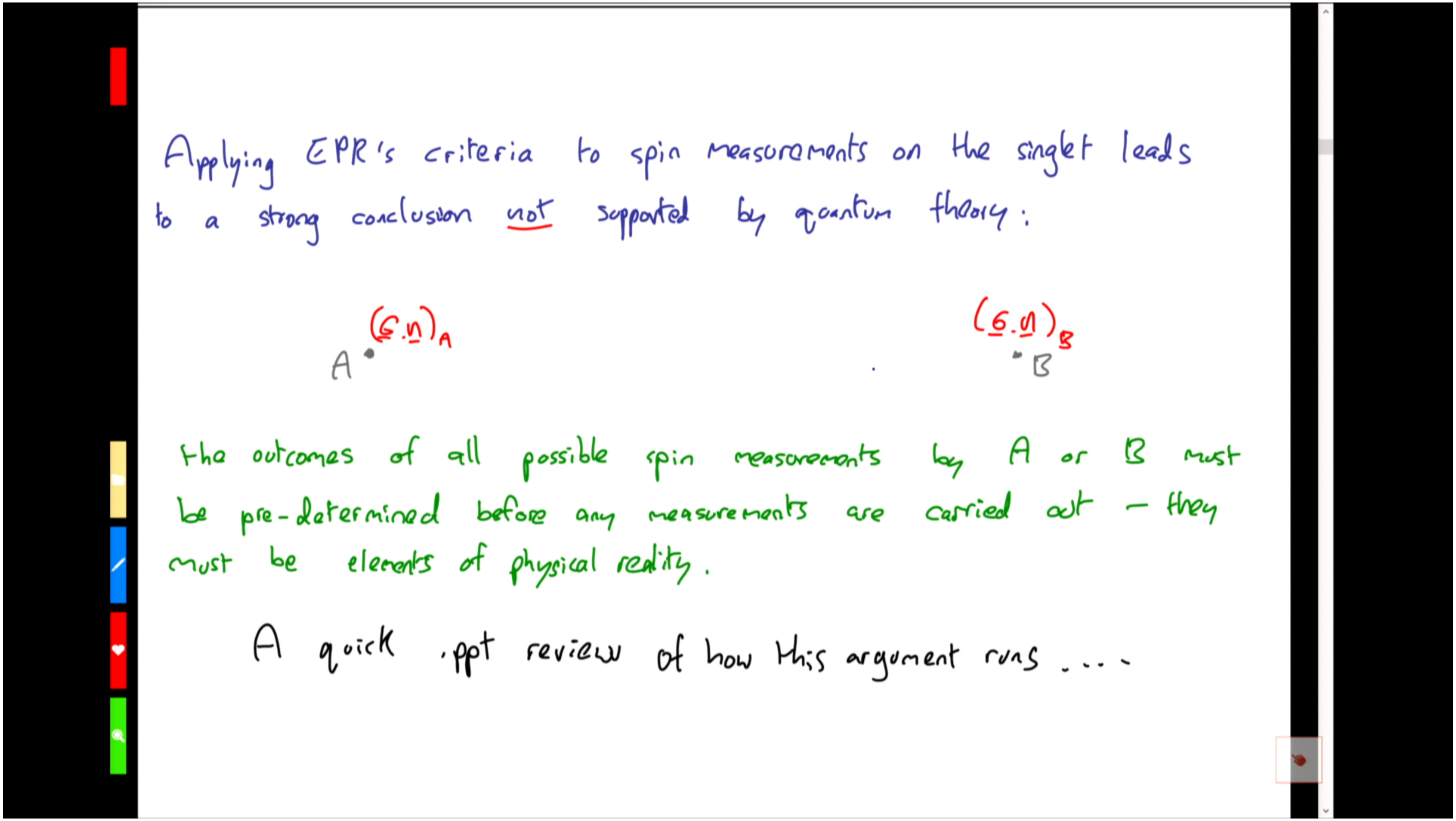
In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the t *complete*, the following requirement for a c plete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

EPR 1

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

EPR 2



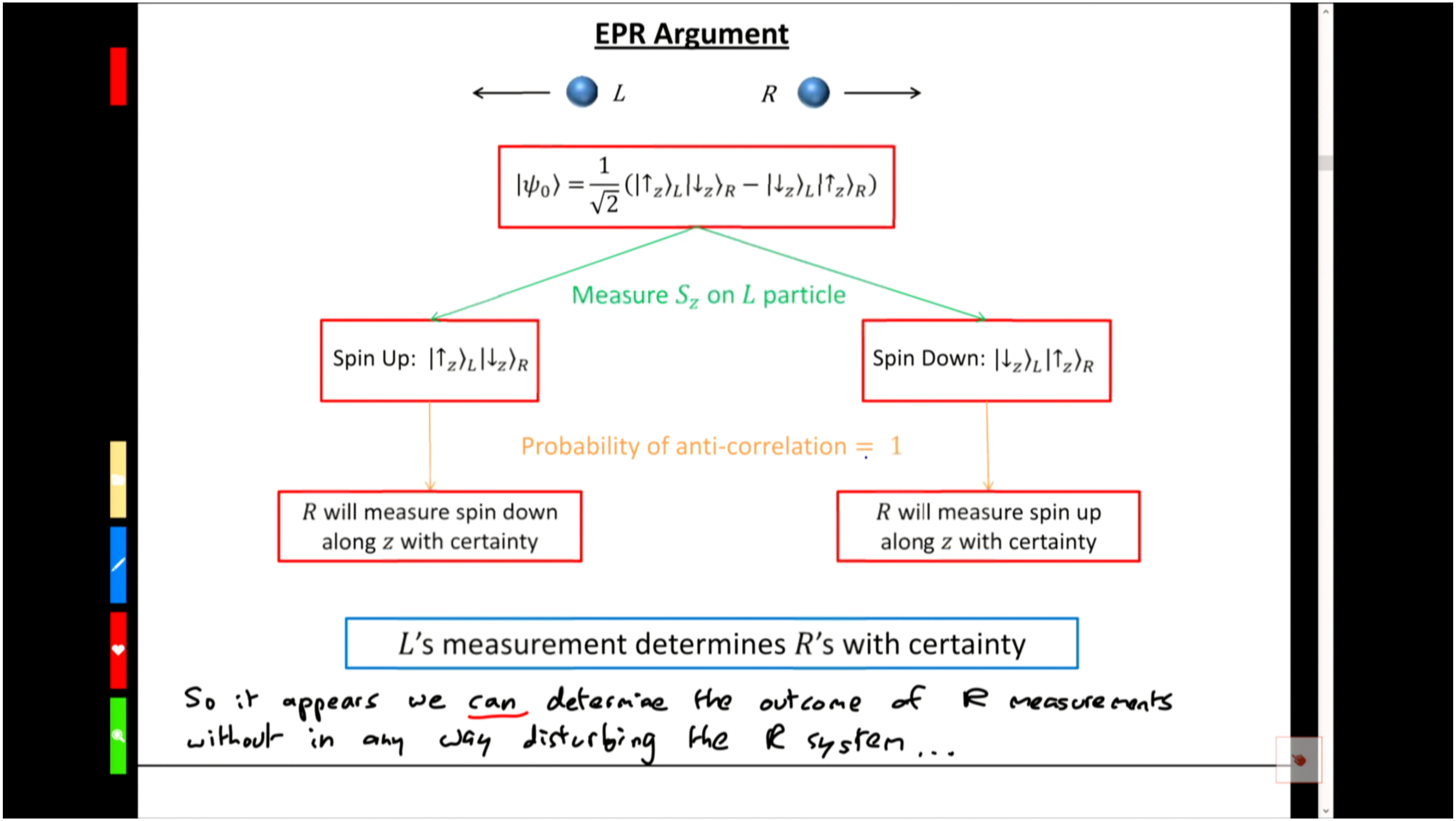
Applying EPR's criteria to spin measurements on the singlet leads to a strong conclusion not supported by quantum theory:

A $(\underline{\sigma}\cdot\underline{n})_A$

$(\underline{\sigma}\cdot\underline{n})_B$ B

the outcomes of all possible spin measurements by A or B must be pre-determined before any measurements are carried out – they must be elements of physical reality.

A quick .ppt review of how this argument runs ...



EPR Argument

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_L|\downarrow_z\rangle_R - |\downarrow_z\rangle_L|\uparrow_z\rangle_R)$$

Measure S_z on L particle

Spin Up: $|\uparrow_z\rangle_L|\downarrow_z\rangle_R$

Spin Down: $|\downarrow_z\rangle_L|\uparrow_z\rangle_R$

Probability of anti-correlation = 1

R will measure spin down along z with certainty

R will measure spin up along z with certainty

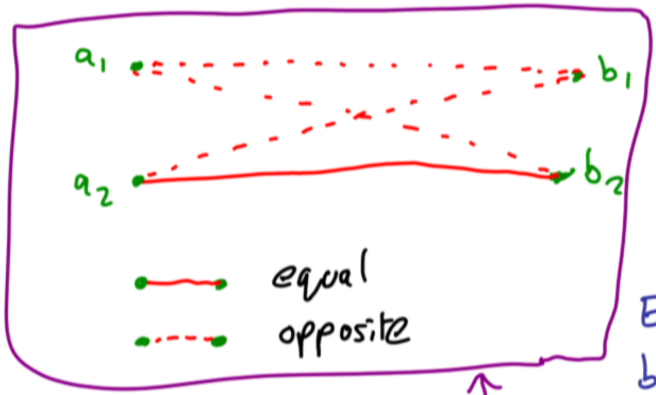
L's measurement determines R's with certainty

So it appears we can determine the outcome of R measurements without in any way disturbing the R system...

(Hardy, Phys. Lett. A 161 (1991) 21-25)

Experimental Implications of the EPR argument: Bell and CHSH inequalities

Suppose EPR are correct: the outcomes of all L and R spin measurements on the singlet are pre-determined. Let A_1, A_2 be two possible measurements on L, and $a_1, a_2 \in \{\pm 1\}$ their pre-determined outcomes; similarly B_1, B_2 on R.



We can ask: is $a_1 \neq b_1$?

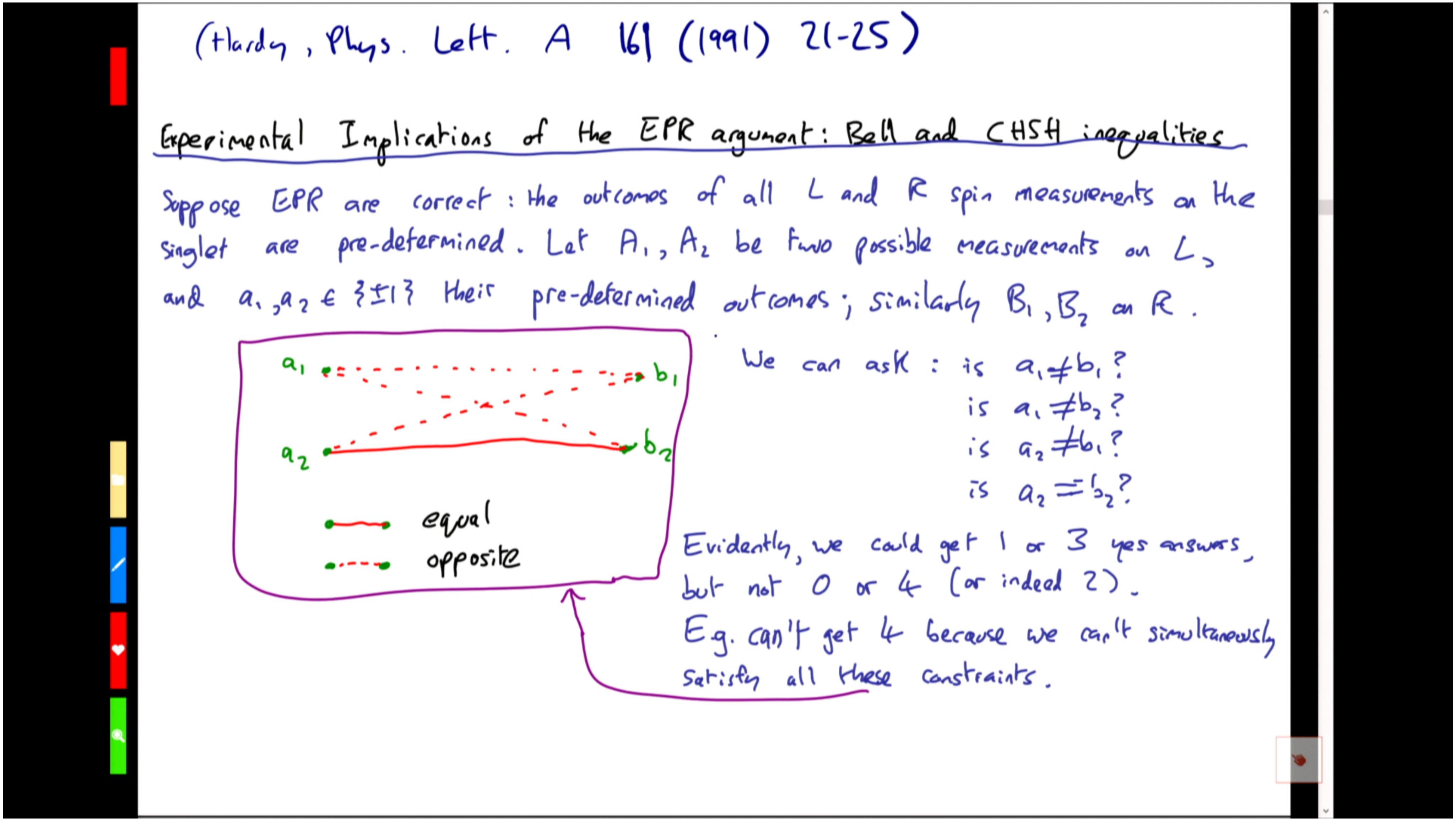
is $a_1 \neq b_2$?

is $a_2 \neq b_1$?

is $a_2 = b_2$?

Evidently, we could get 1 or 3 yes answers, but not 0 or 4 (or indeed 2).

E.g. can't get 4 because we can't simultaneously satisfy all these constraints.



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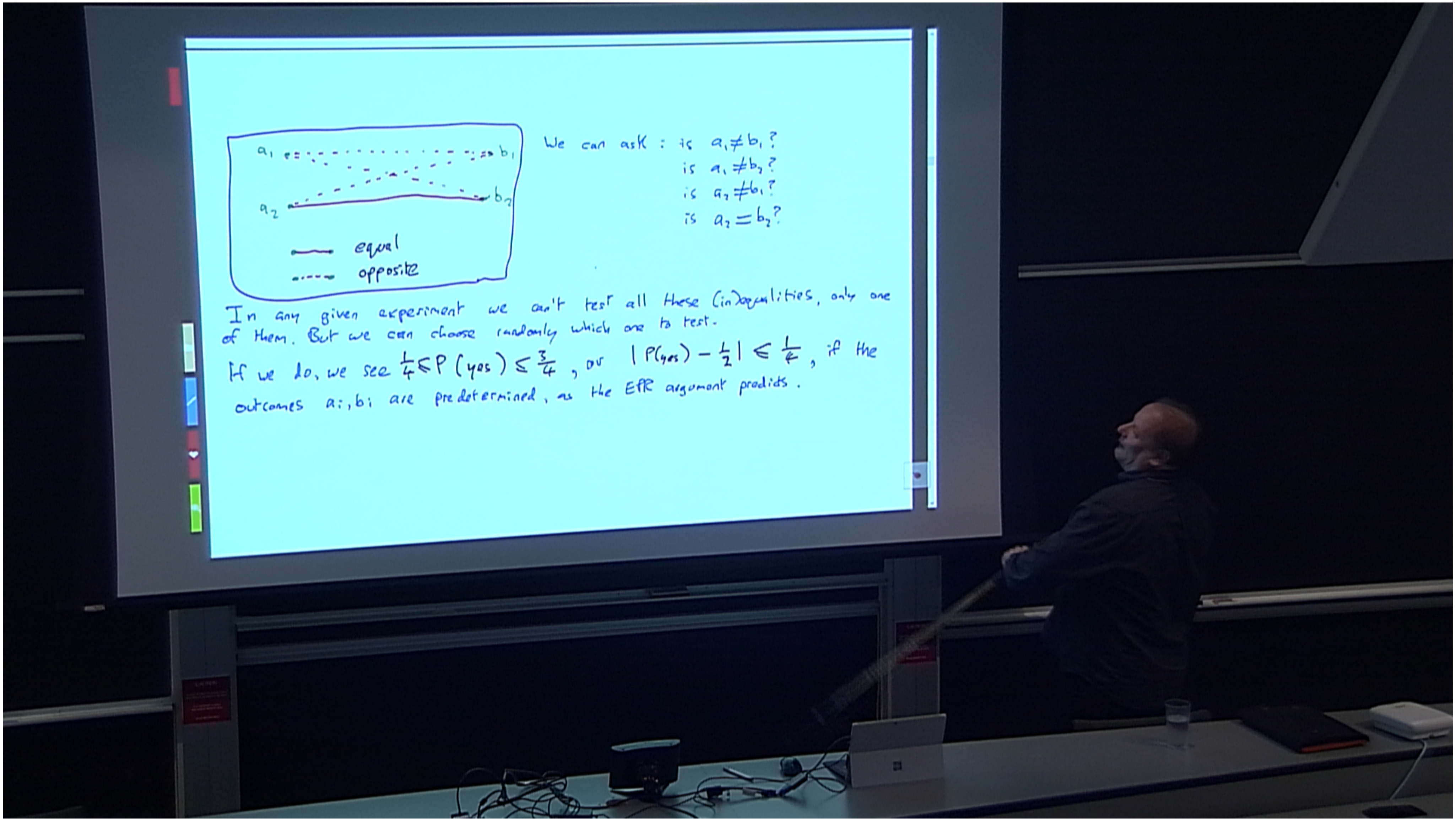


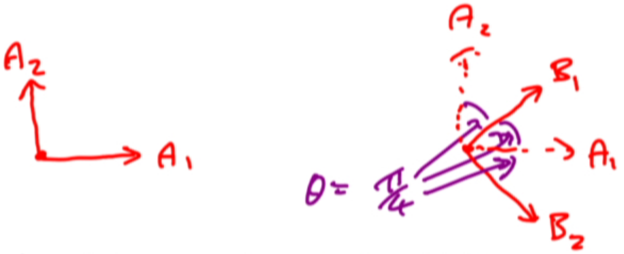
We can ask : is $a_1 \neq b_1$?
is $a_1 \neq b_2$?
is $a_2 \neq b_1$?
is $a_2 = b_2$?

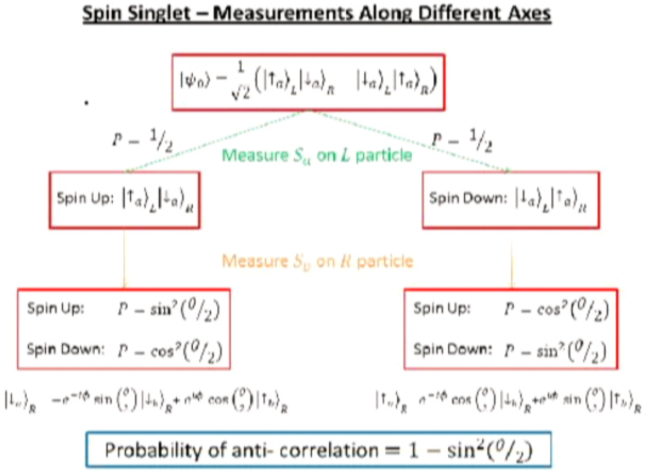
In any given experiment we can't test all these (in)equalities, only one of them. But we can choose randomly which one to test.

If we do, we see $\frac{1}{4} \leq P(\text{yes}) \leq \frac{3}{4}$, or $|P(\text{yes}) - \frac{1}{2}| \leq \frac{1}{4}$, if the outcomes a_i, b_i are predetermined, as the EPR argument predicts.

What does quantum mechanics say?

Depends on the angles between the axes defining A_1, A_2, B_1, B_2.





Spin Singlet – Measurements Along Different Axes

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Spin Down: $|\downarrow_a\rangle_L|\uparrow_a\rangle_R$

Measure S_b on R particle

Spin Up: $P = \sin^2(\theta/2)$
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$|\uparrow_a\rangle_R = e^{-i\phi}\cos\left(\frac{\theta}{2}\right)|\downarrow_b\rangle_R + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|\uparrow_b\rangle_R$

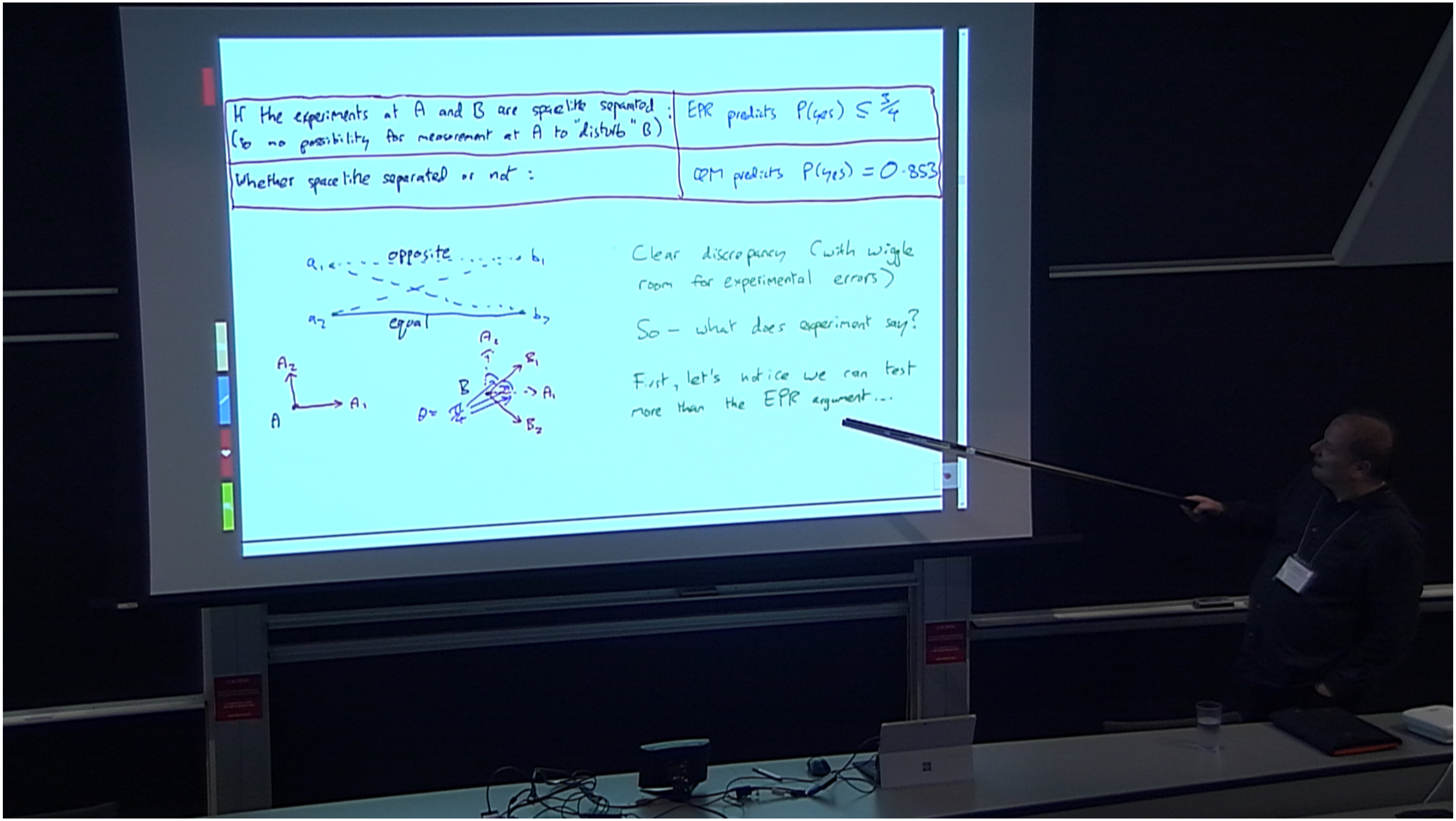
Probability of anti- correlation $= 1 - \sin^2(\theta/2)$

$$P(a_1 \neq b_1) = P(a_1 \neq b_2) = P(a_2 \neq b_1)$$
$$= \cos^2\left(\tfrac{\pi}{8}\right) = \tfrac{1}{2}\left(1 + \tfrac{1}{\sqrt{2}}\right)$$

$$P(a_2 = b_2) = \cos^2\left(\tfrac{\pi}{8}\right) = \tfrac{1}{2}\left(1 + \tfrac{1}{\sqrt{2}}\right)$$

$\therefore$ P(yes) for any random choice of these questions is $\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = 0.853.. > \frac{3}{4}$

And this is the largest possible quantum value for 2-input 2-output tests (Tsirelson)

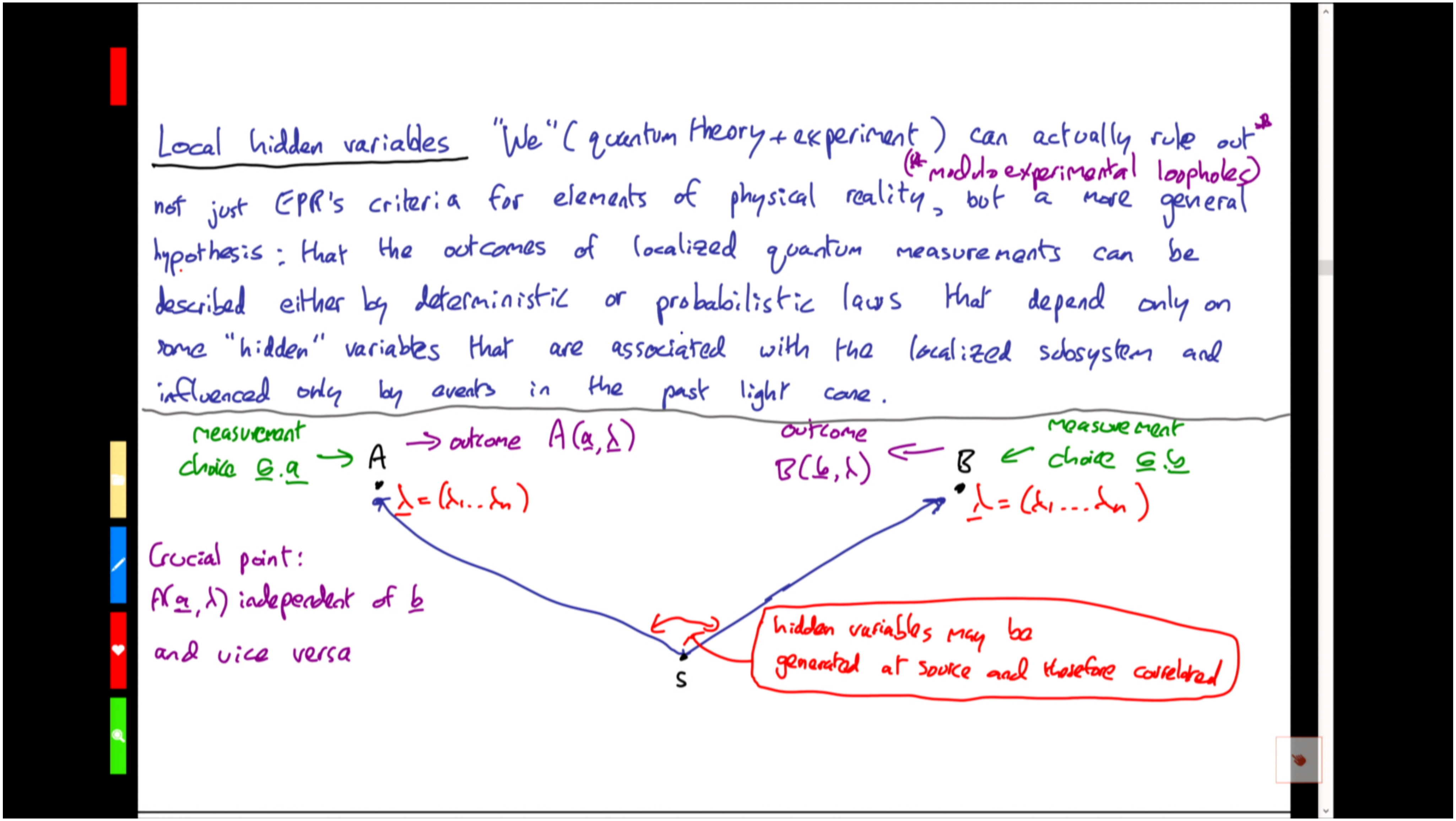


If the experiments at A and B are spacelike separated (so no possibility for measurement at A to "disturb" B) :	EPR predicts $P(\text{yes}) \leq \frac{3}{4}$
Whether spacelike separated or not :	QM predicts $P(\text{yes}) = 0.853$

Clear discrepancy (with wiggle room for experimental errors)

So – what does experiment say?

First, let's notice we can test more than the EPR argument...



Local hidden variables

"We" (quantum theory + experiment) can actually rule out* (* modulo experimental loopholes) not just EPR's criteria for elements of physical reality, but a more general hypothesis: that the outcomes of localized quantum measurements can be described either by deterministic or probabilistic laws that depend only on some "hidden" variables that are associated with the localized subsystem and influenced only by events in the past light cone.

measurement choice $\in \underline{a}$ → A → outcome $A(\underline{a}, \underline{\lambda})$

$\underline{\lambda} = (\lambda_1 .. \lambda_n)$

outcome $B(\underline{b}, \lambda)$ ← B ← measurement choice $\in \underline{b}$

$\underline{\lambda} = (\lambda_1 ... \lambda_n)$

S

hidden variables may be generated at source and therefore correlated

Crucial point:
$A(\underline{a}, \lambda)$ independent of $\underline{b}$
and vice versa

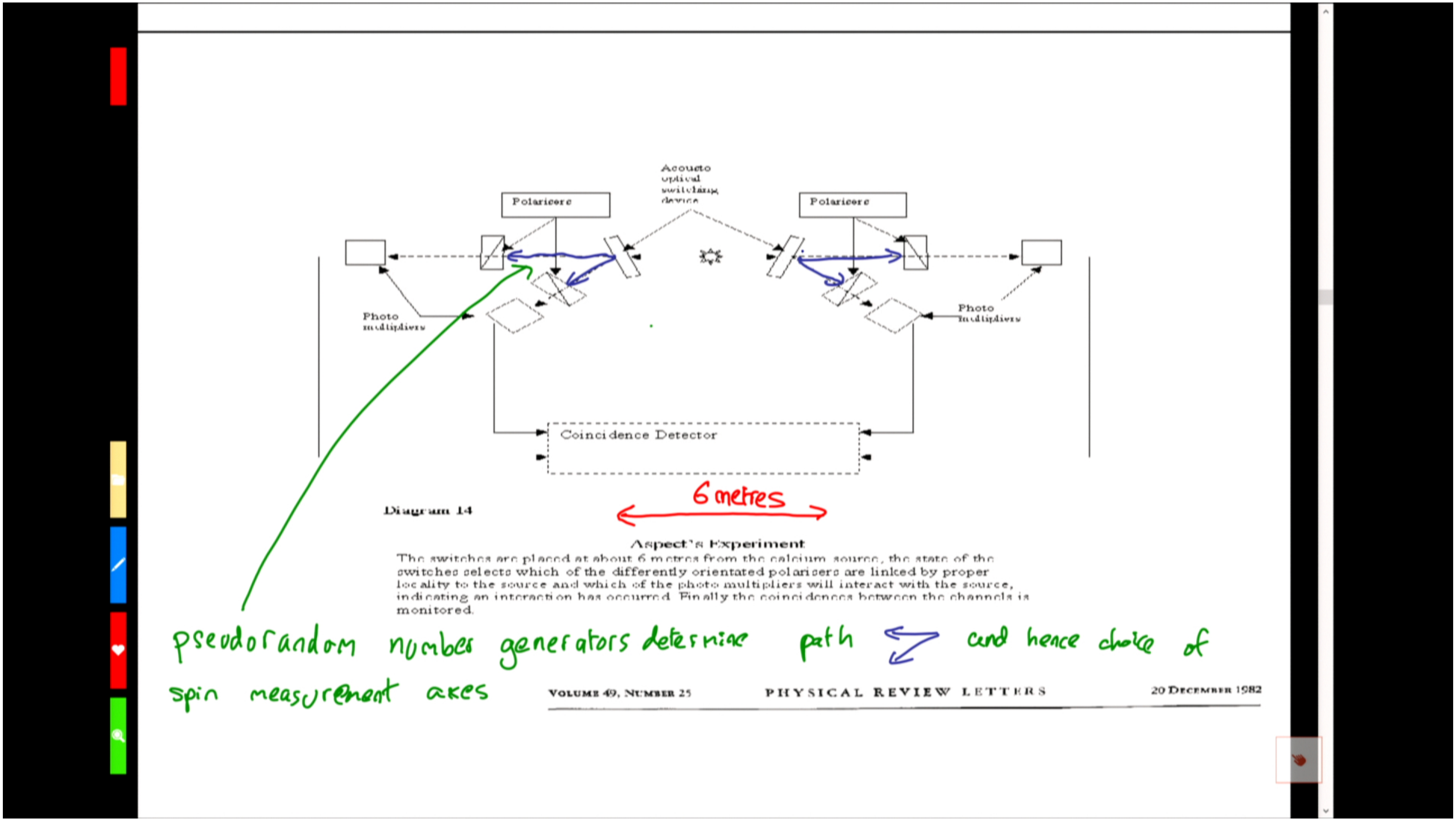


Diagram 14

Aspect's Experiment

The switches are placed at about 6 metres from the calcium source, the state of the switches selects which of the differently orientated polarisers are linked by proper locality to the source and which of the photo multipliers will interact with the source, indicating an interaction has occurred. Finally the coincidences between the channels is monitored.

pseudorandom number generators determine path and hence choice of spin measurement axes

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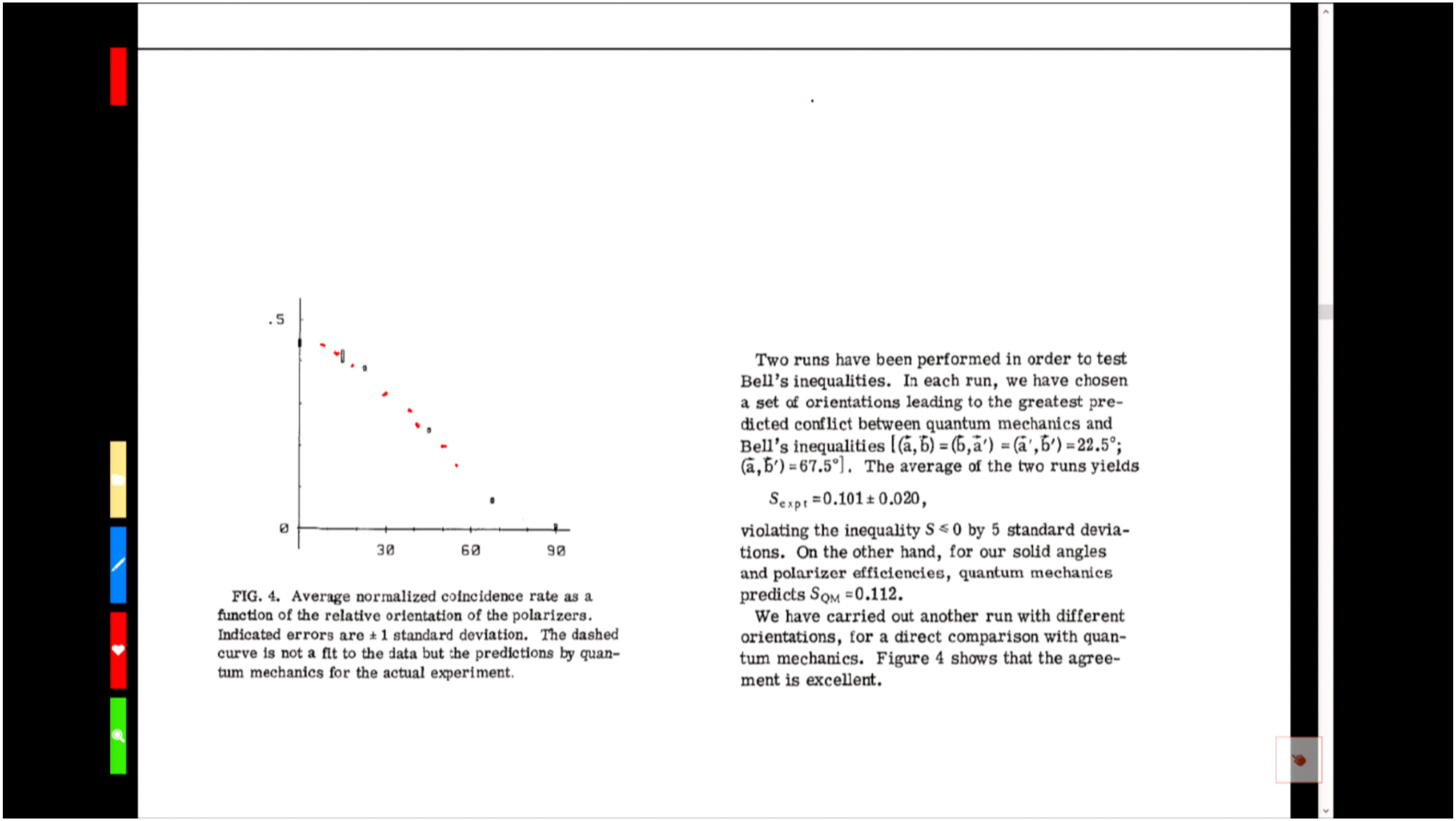


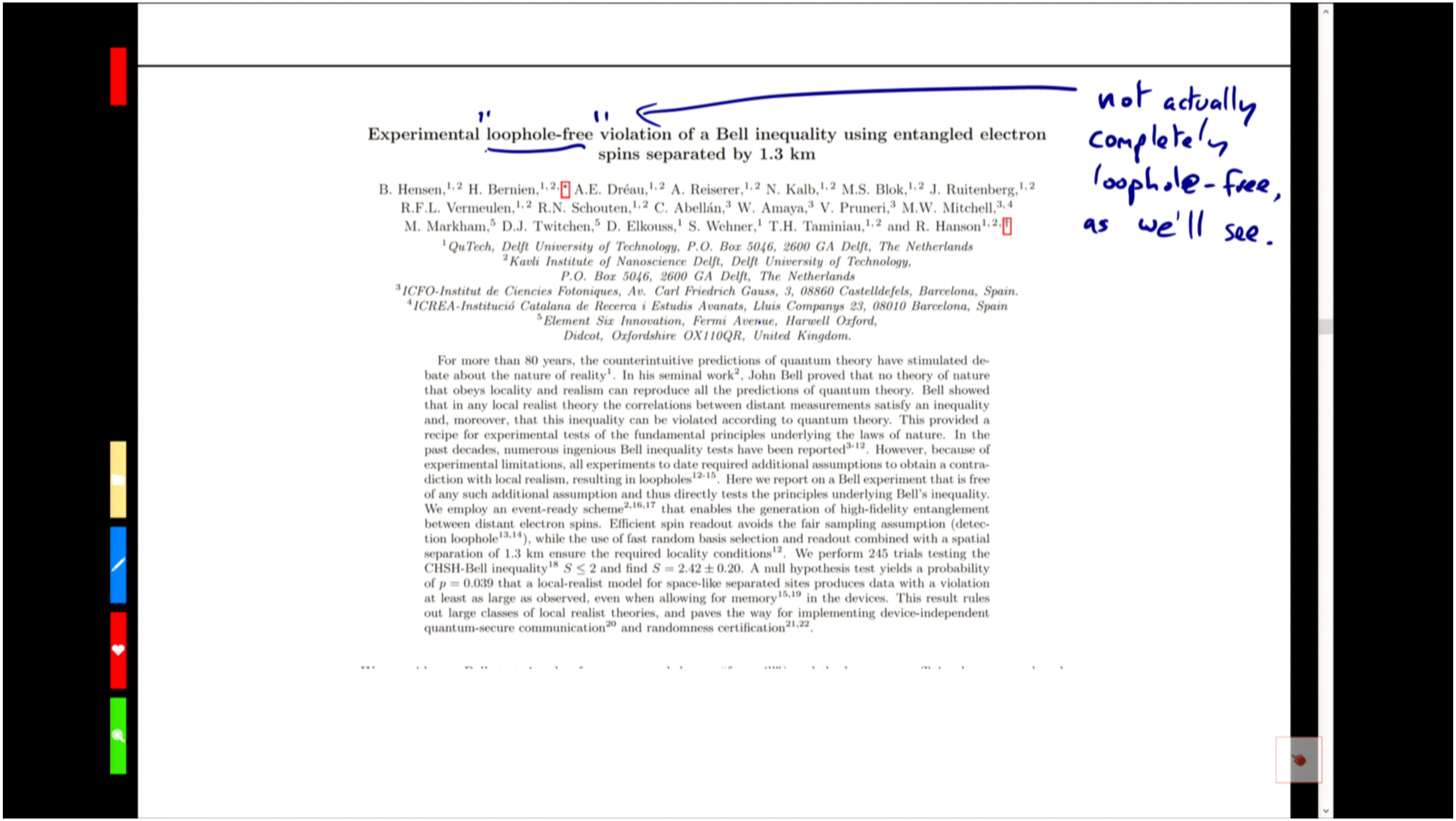
FIG. 4. Average normalized coincidence rate as a function of the relative orientation of the polarizers. Indicated errors are ± 1 standard deviation. The dashed curve is not a fit to the data but the predictions by quantum mechanics for the actual experiment.

Two runs have been performed in order to test Bell's inequalities. In each run, we have chosen a set of orientations leading to the greatest predicted conflict between quantum mechanics and Bell's inequalities $[(\vec{a},\vec{b})=(\vec{b},\vec{a}')=(\vec{a}',\vec{b}')=22.5°; (\vec{a},\vec{b}')=67.5°]$. The average of the two runs yields

$$S_{\text{expt}}=0.101\pm 0.020,$$

violating the inequality $S\leq 0$ by 5 standard deviations. On the other hand, for our solid angles and polarizer efficiencies, quantum mechanics predicts $S_{\text{QM}}=0.112$.

We have carried out another run with different orientations, for a direct comparison with quantum mechanics. Figure 4 shows that the agreement is excellent.



Experimental "loophole-free" violation of a Bell inequality using entangled electron spins separated by 1.3 km

not actually completely loophole-free, as we'll see.

B. Hensen,[1,2] H. Bernien,[1,2,*] A.E. Dréau,[1,2] A. Reiserer,[1,2] N. Kalb,[1,2] M.S. Blok,[1,2] J. Ruitenberg,[1,2]
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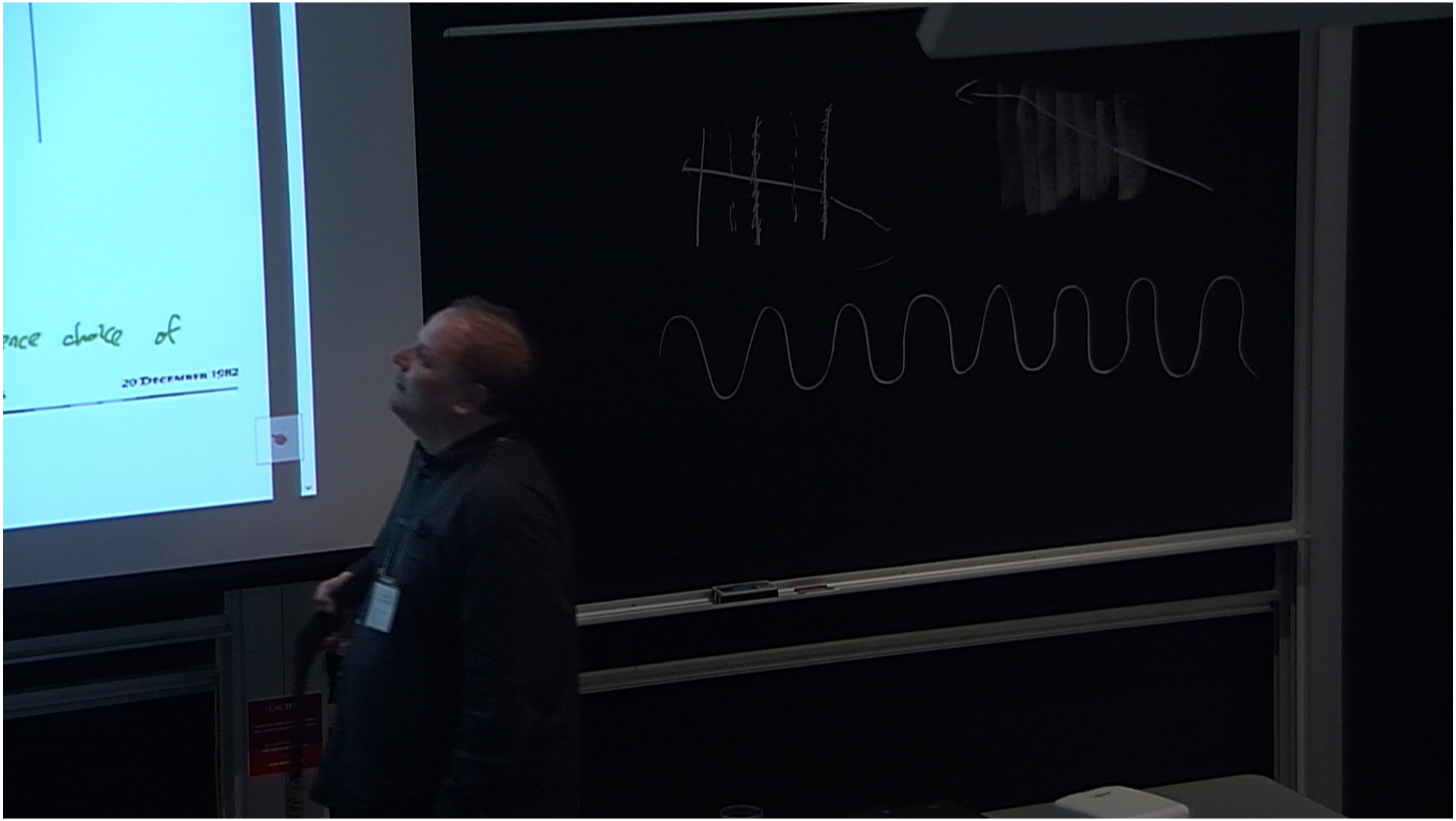
For more than 80 years, the counterintuitive predictions of quantum theory have stimulated debate about the nature of reality[1]. In his seminal work[2], John Bell proved that no theory of nature that obeys locality and realism can reproduce all the predictions of quantum theory. Bell showed that in any local realist theory the correlations between distant measurements satisfy an inequality and, moreover, that this inequality can be violated according to quantum theory. This provided a recipe for experimental tests of the fundamental principles underlying the laws of nature. In the past decades, numerous ingenious Bell inequality tests have been reported[3-12]. However, because of experimental limitations, all experiments to date required additional assumptions to obtain a contradiction with local realism, resulting in loopholes[12-15]. Here we report on a Bell experiment that is free of any such additional assumption and thus directly tests the principles underlying Bell's inequality. We employ an event-ready scheme[2,16,17] that enables the generation of high-fidelity entanglement between distant electron spins. Efficient spin readout avoids the fair sampling assumption (detection loophole[13,14]), while the use of fast random basis selection and readout combined with a spatial separation of 1.3 km ensure the required locality conditions[12]. We perform 245 trials testing the CHSH-Bell inequality[18] $S \leq 2$ and find $S = 2.42 \pm 0.20$. A null hypothesis test yields a probability of $p = 0.039$ that a local-realist model for space-like separated sites produces data with a violation at least as large as observed, even when allowing for memory[15,19] in the devices. This result rules out large classes of local realist theories, and paves the way for implementing device-independent quantum-secure communication[20] and randomness certification[21,22].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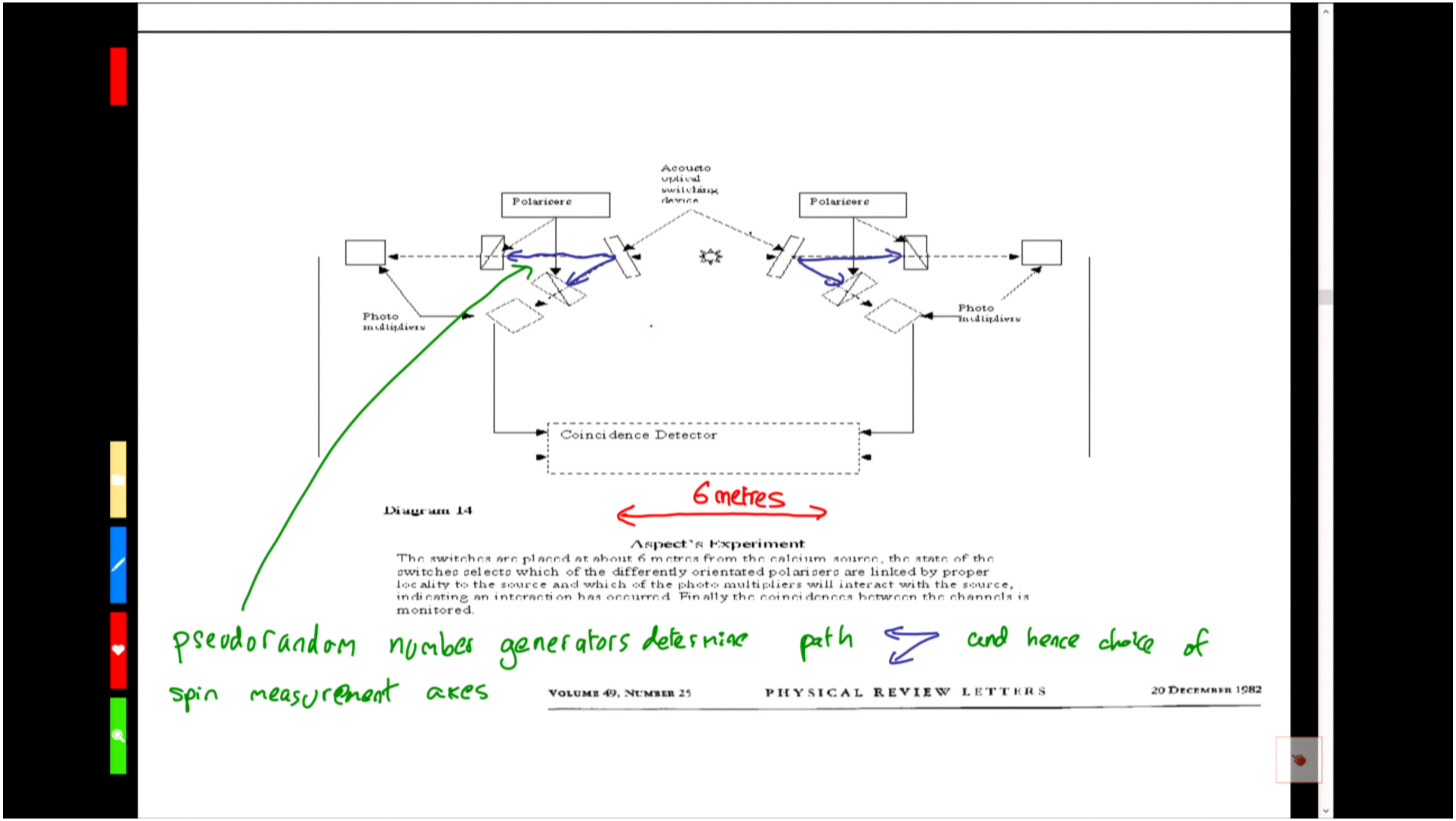

6 metres

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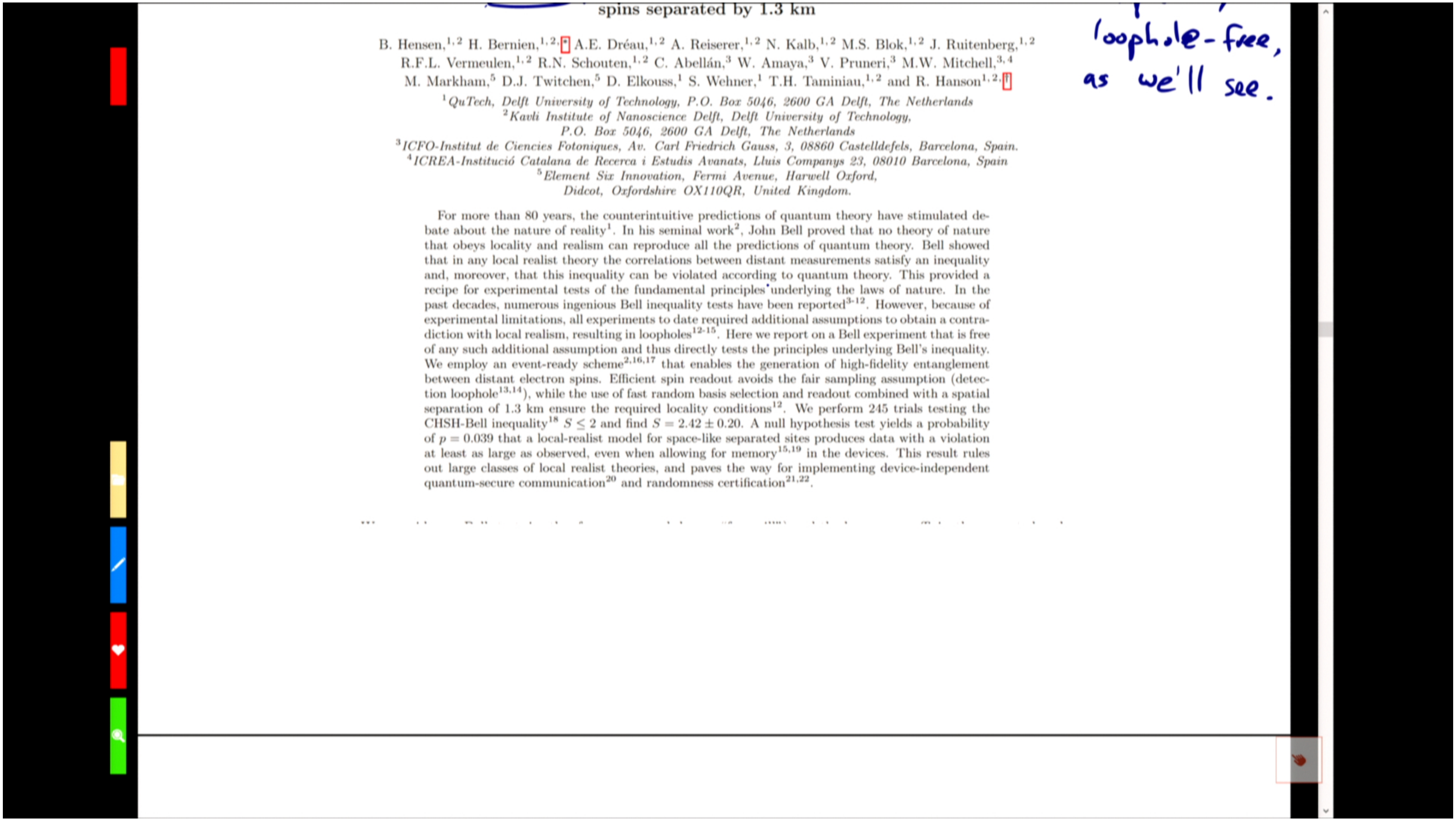

spins separated by 1.3 km

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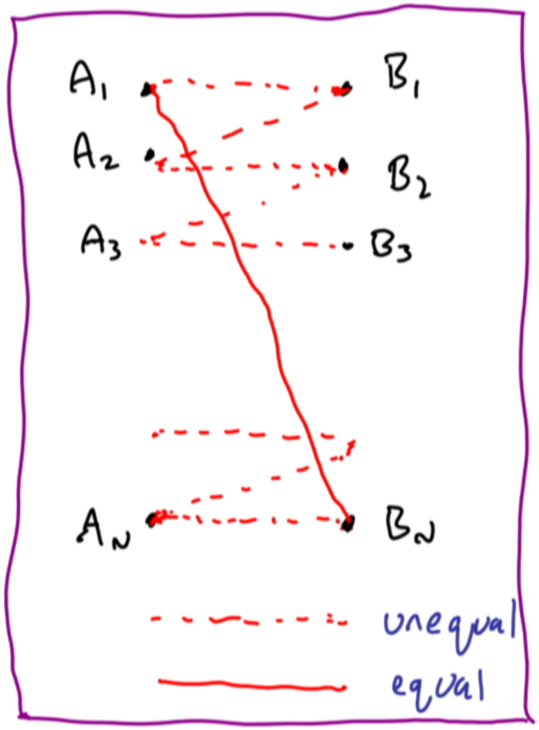
For more than 80 years, the counterintuitive predictions of quantum theory have stimulated debate about the nature of reality[1]. In his seminal work[2], John Bell proved that no theory of nature that obeys locality and realism can reproduce all the predictions of quantum theory. Bell showed that in any local realist theory the correlations between distant measurements satisfy an inequality and, moreover, that this inequality can be violated according to quantum theory. This provided a recipe for experimental tests of the fundamental principles underlying the laws of nature. In the past decades, numerous ingenious Bell inequality tests have been reported[3-12]. However, because of experimental limitations, all experiments to date required additional assumptions to obtain a contradiction with local realism, resulting in loopholes[12-15]. Here we report on a Bell experiment that is free of any such additional assumption and thus directly tests the principles underlying Bell's inequality. We employ an event-ready scheme[2,16,17] that enables the generation of high-fidelity entanglement between distant electron spins. Efficient spin readout avoids the fair sampling assumption (detection loophole[13,14]), while the use of fast random basis selection and readout combined with a spatial separation of 1.3 km ensure the required locality conditions[12]. We perform 245 trials testing the CHSH-Bell inequality[18] $S \leq 2$ and find $S = 2.42 \pm 0.20$. A null hypothesis test yields a probability of $p = 0.039$ that a local-realist model for space-like separated sites produces data with a violation at least as large as observed, even when allowing for memory[15,19] in the devices. This result rules out large classes of local realist theories, and paves the way for implementing device-independent quantum-secure communication[20] and randomness certification[21,22].

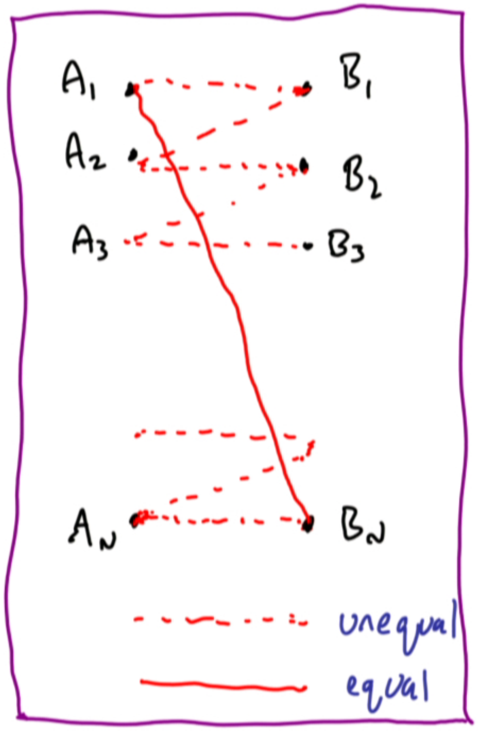
Extensions of Bell-CHSH inequality I: the Braunstein-Caves inequalities



Instead of 2 measurement choices on each side, we can consider N.

Using the same argument, at least one of the relations $a_1 \neq b_1$, $b_1 \neq a_2$, $a_2 \neq b_2$, $\ldots$, $a_N \neq b_N$, $b_N = a_1$ must fail to hold.

Braunstein-Caves inequality (cont.)

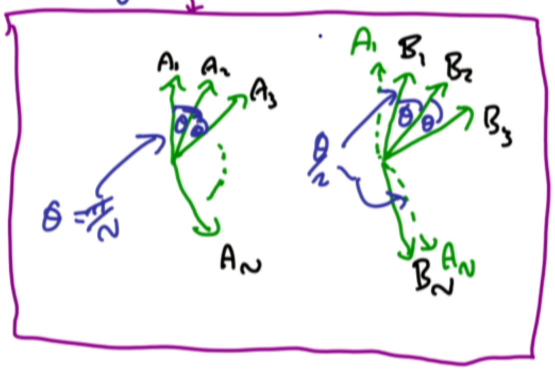


Not all these relations can simultaneously hold.

If we do 2N random tests we expect at least 1 violation, according to EPR/LHV.

I.e. $P(\text{violation}) \geq \frac{1}{2N}$

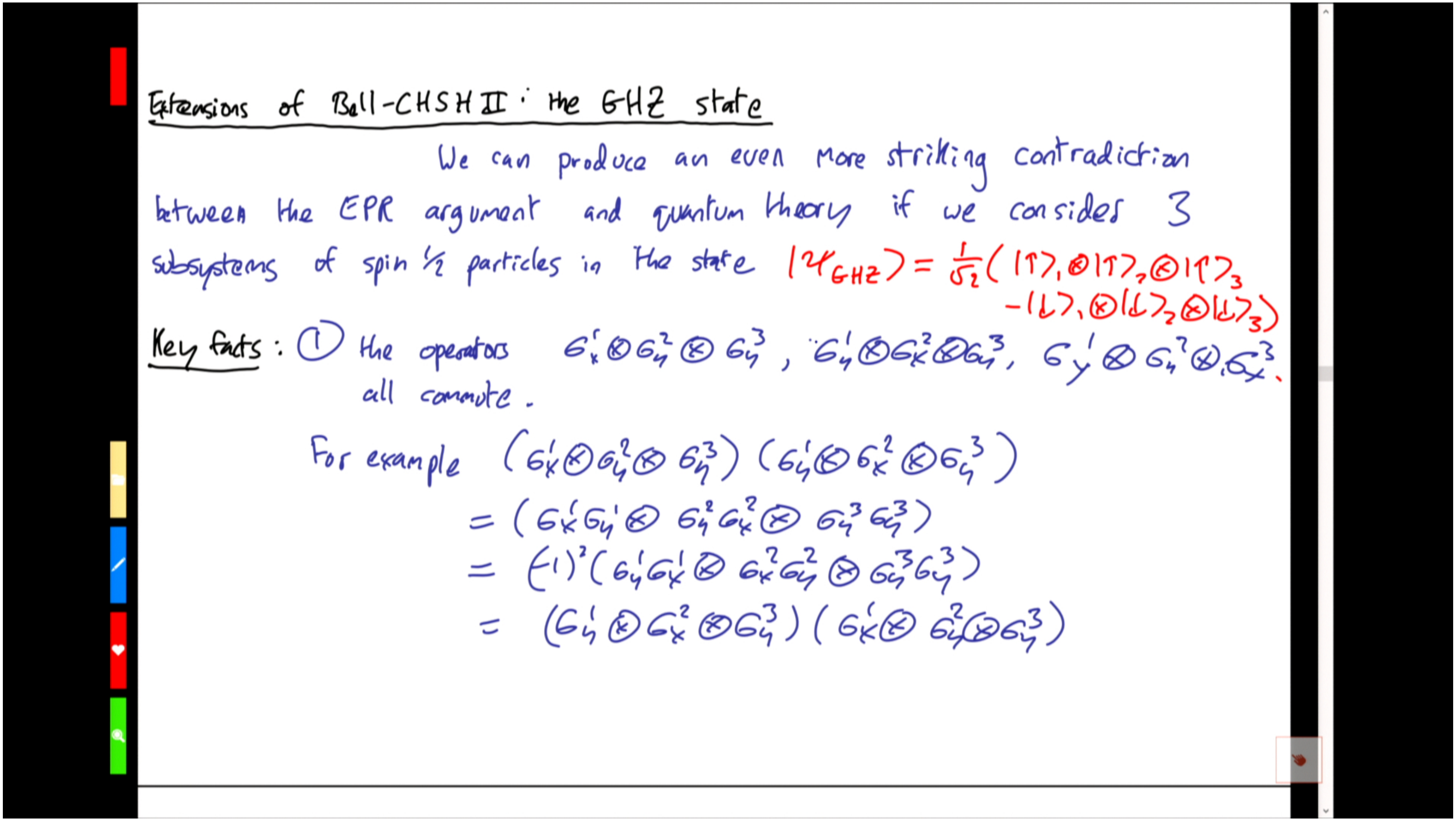
But QM predicts prob. of violation $\sim 1/N^2$ for eg. this choice of measurements on the singlet



$P(A_1 \neq B_1) = \cos^2\left(\frac{\pi}{4N}\right)$

$P(\text{violation}) \doteq \frac{\pi^2}{32N^2}$

and same holds true for each (in)equality.



Extensions of Bell-CHSH II: the GHZ state

We can produce an even more striking contradiction between the EPR argument and quantum theory if we consider 3 subsystems of spin ½ particles in the state $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \otimes |\uparrow\rangle_3 - |\downarrow\rangle_1 \otimes |\downarrow\rangle_2 \otimes |\downarrow\rangle_3\right)$

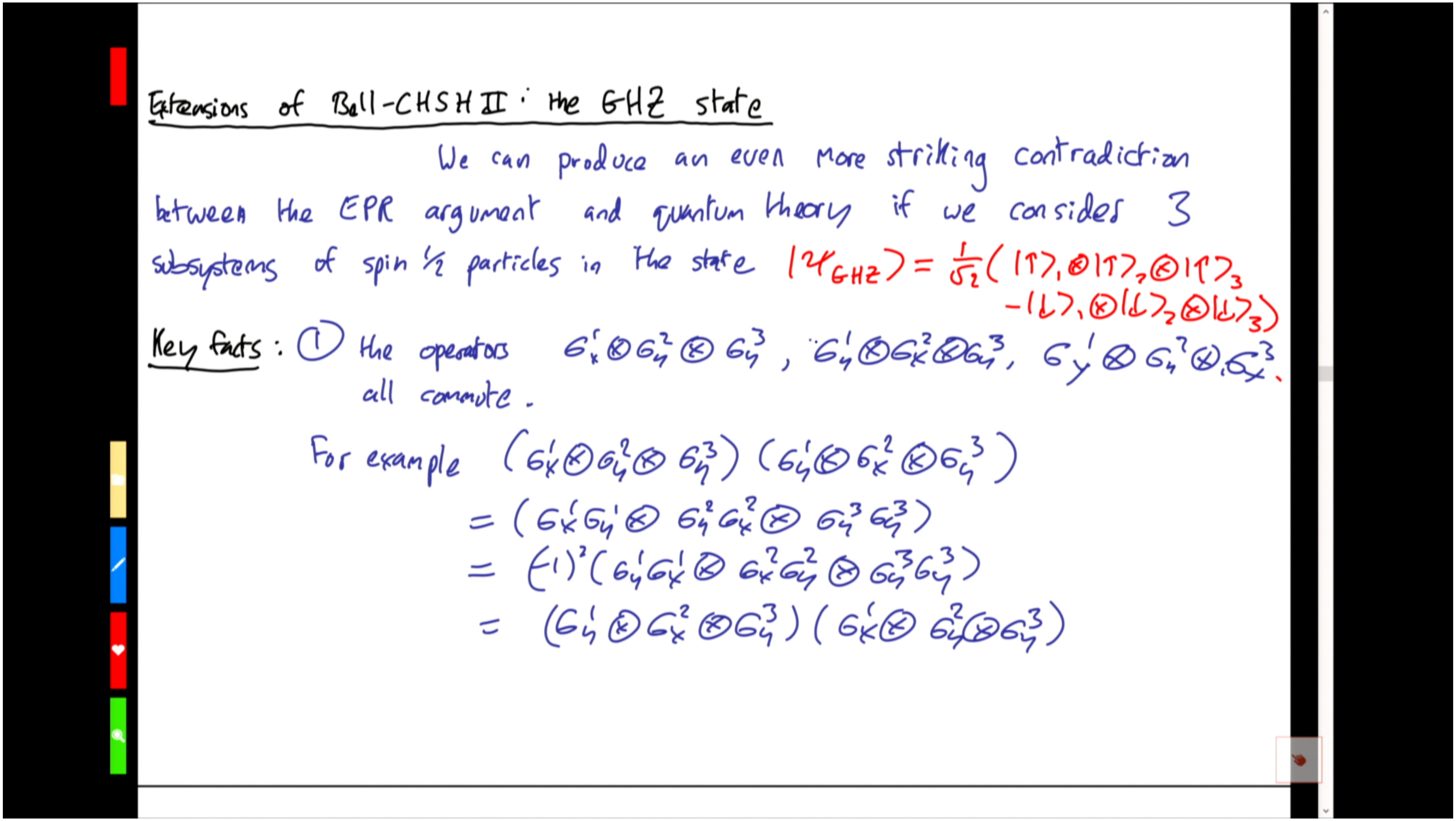
Key facts: ① The operators $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$ all commute.

For example $(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)$

$$= (\sigma_x^1\sigma_y^1 \otimes \sigma_y^2\sigma_x^2 \otimes \sigma_y^3\sigma_y^3)$$

$$= (-1)^2(\sigma_y^1\sigma_x^1 \otimes \sigma_x^2\sigma_y^2 \otimes \sigma_y^3\sigma_y^3)$$

$$= (\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$$



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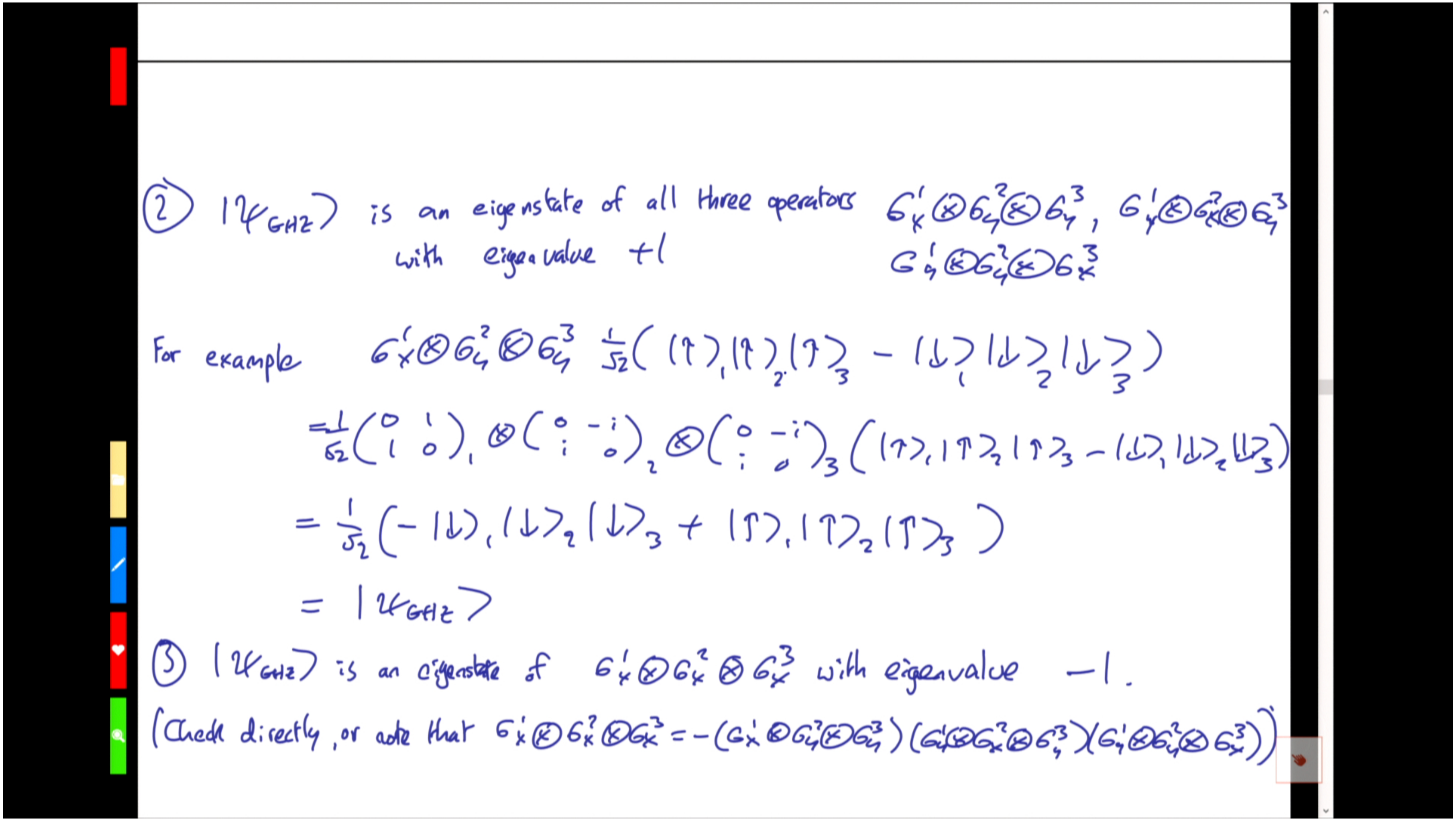


② $|\psi_{GHZ}\rangle$ is an eigenstate of all three operators $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$ with eigenvalue $+1$

For example $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3 \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 - |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3 \right)$

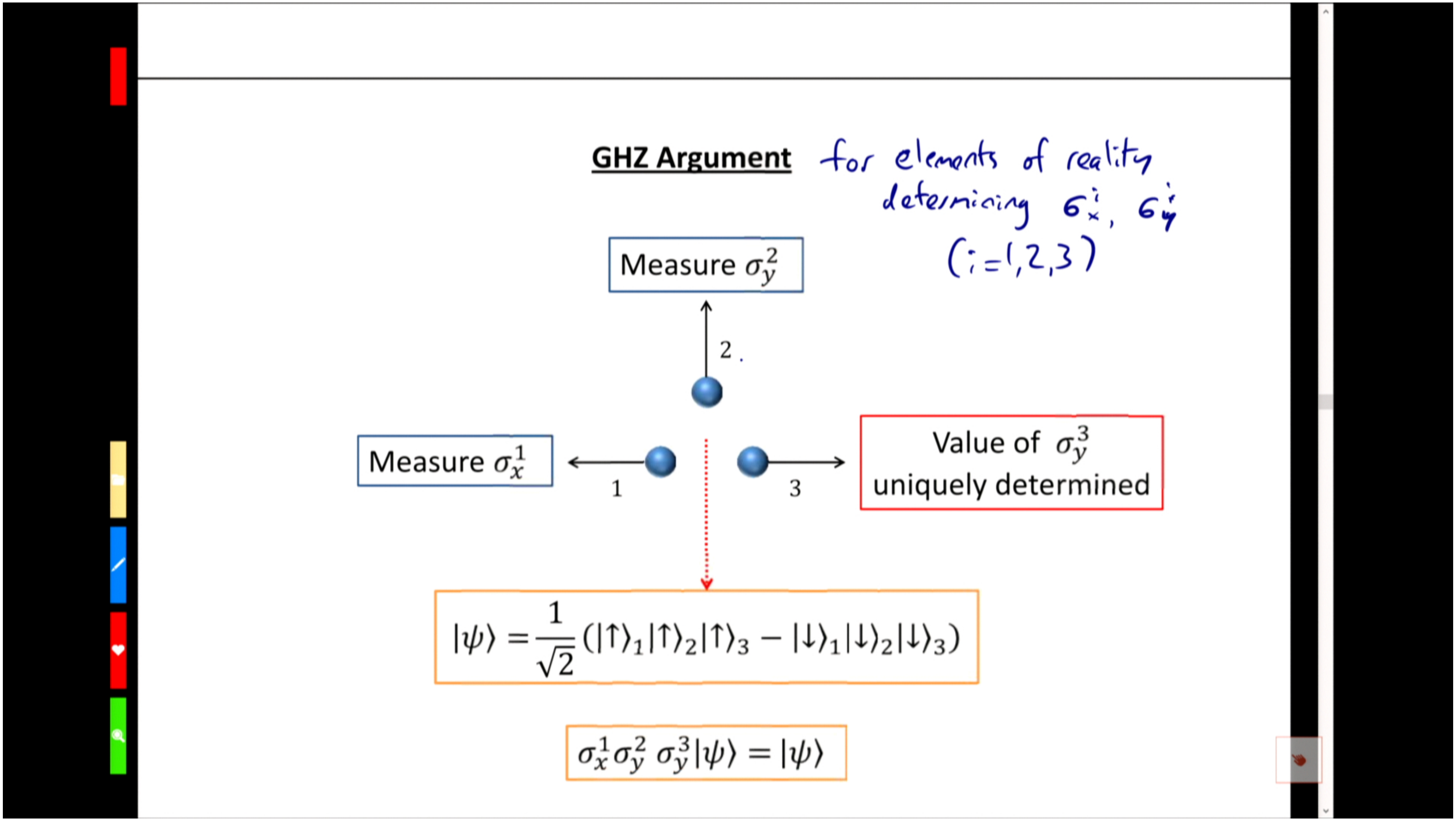
$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_1 \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_2 \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_3 \left(|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 - |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3 \right)$$

$$= \frac{1}{\sqrt{2}} \left(- |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3 + |\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 \right)$$

$$= |\psi_{GHZ}\rangle$$

③ $|\psi_{GHZ}\rangle$ is an eigenstate of $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$ with eigenvalue -1.

(Check directly, or note that $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 = -(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)$)



GHZ Argument

for elements of reality determining σ_x^i, σ_y^i ($i=1,2,3$)

Measure σ_y^2

2

Measure σ_x^1

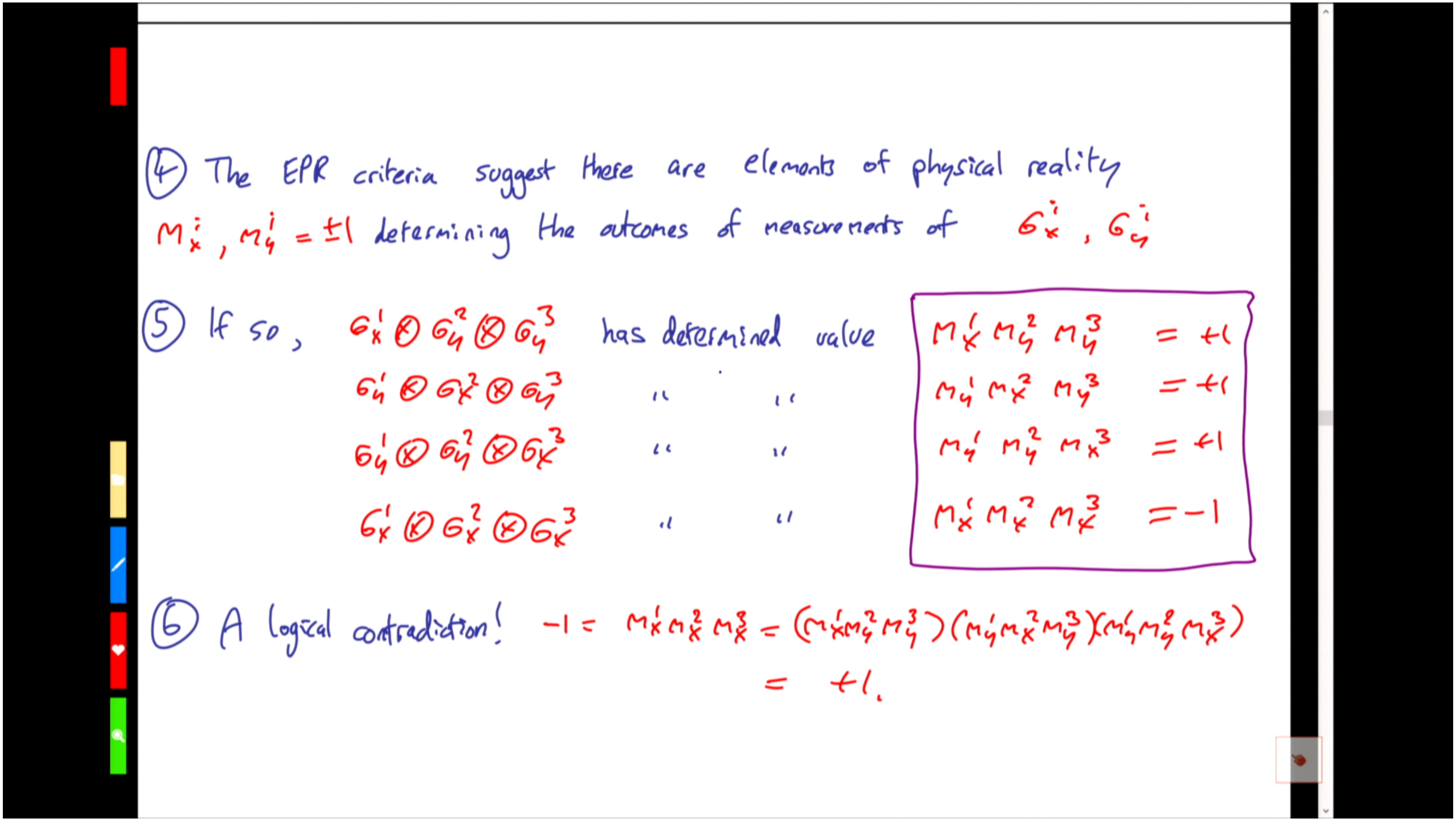
1

3

Value of σ_y^3 uniquely determined

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3)$$

$$\sigma_x^1\sigma_y^2\,\sigma_y^3|\psi\rangle = |\psi\rangle$$



④ The EPR criteria suggest there are elements of physical reality $M_x^i, M_y^i = \pm 1$ determining the outcomes of measurements of σ_x^i, σ_y^i

⑤ If so, $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$ has determined value

$\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$ " "

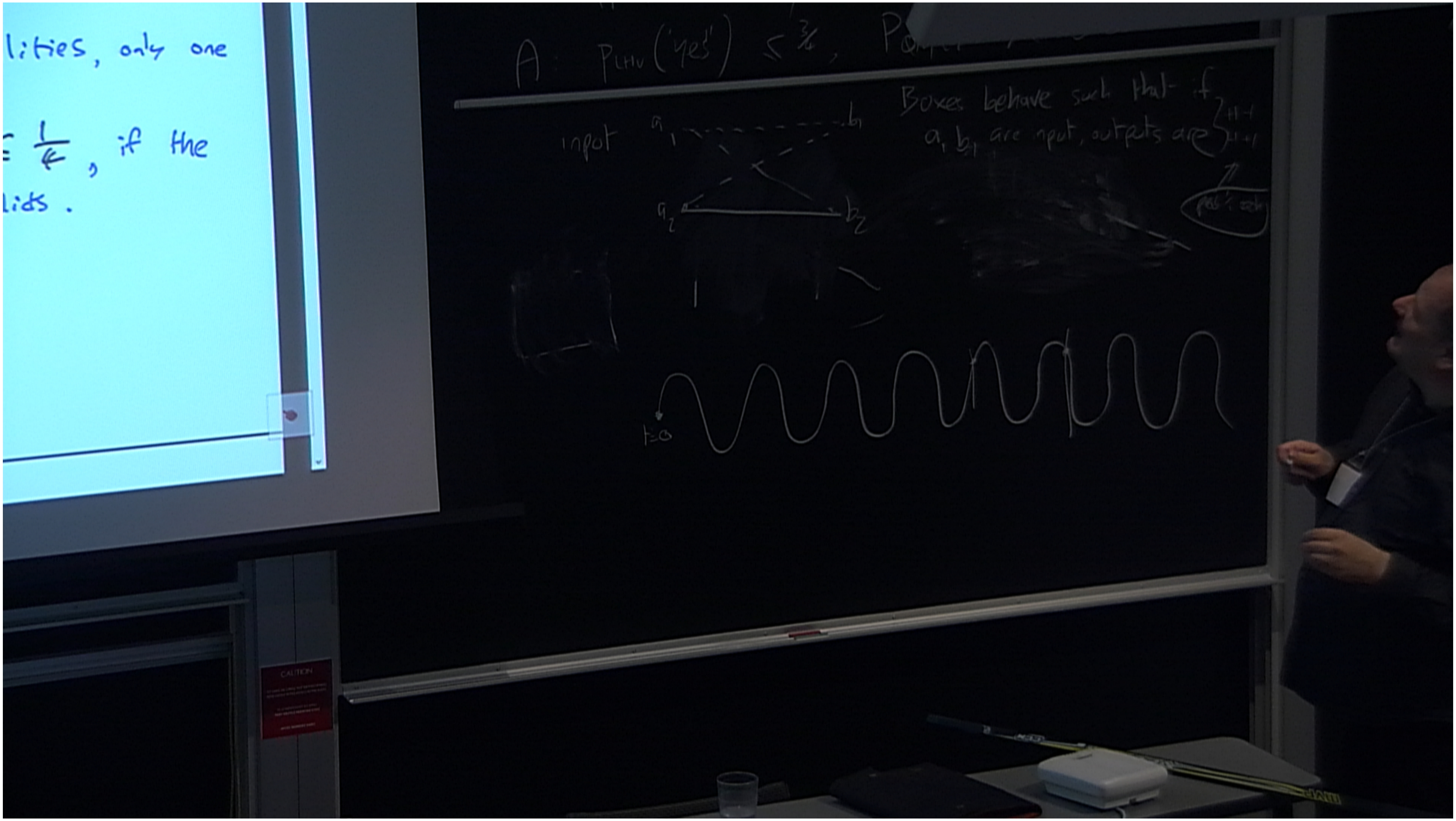
$\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$ " "

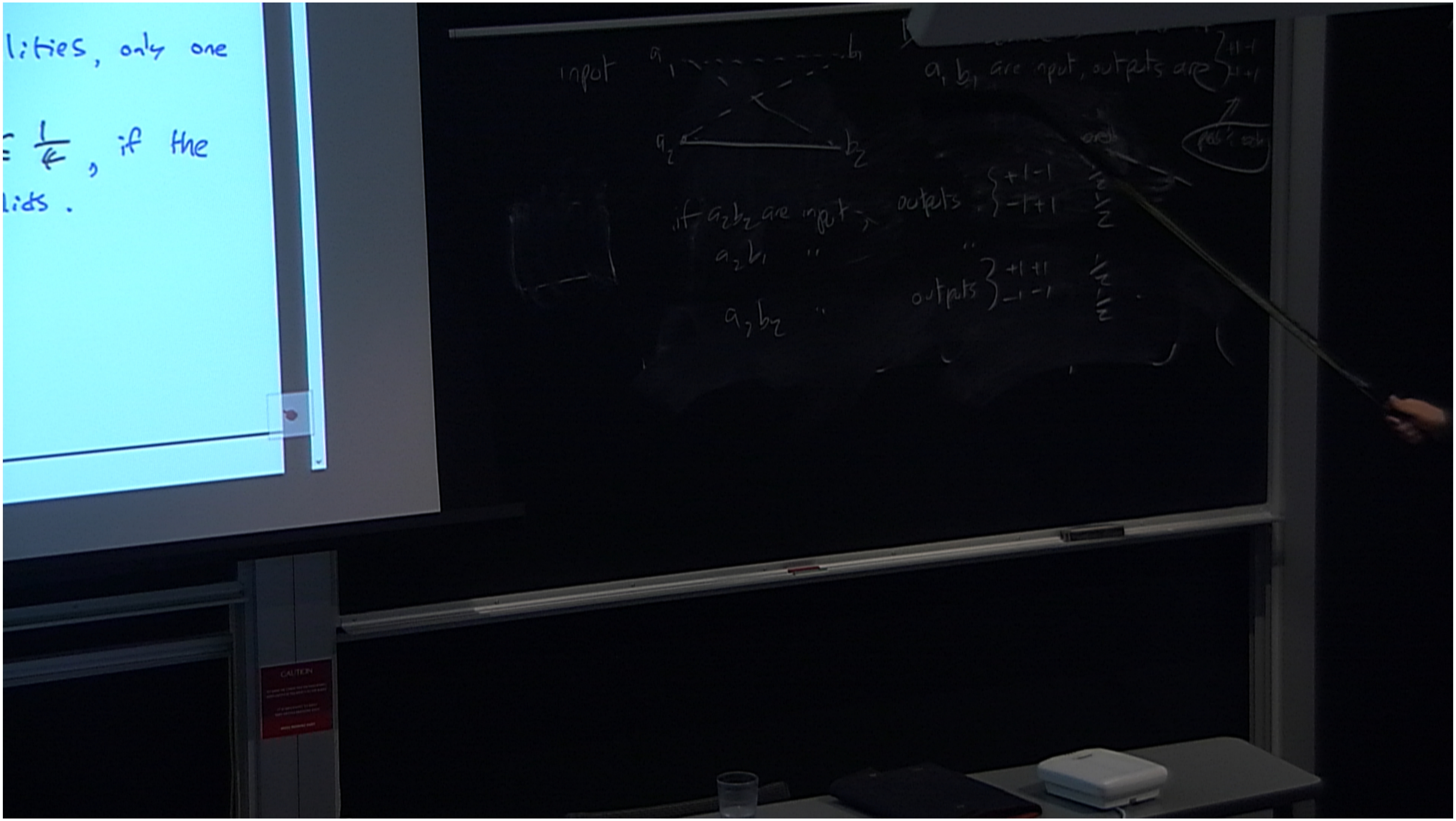
$\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$ " "

$$\boxed{\begin{aligned} M_x^1 M_y^2 M_y^3 &= +1 \\ M_y^1 M_x^2 M_y^3 &= +1 \\ M_y^1 M_y^2 M_x^3 &= +1 \\ M_x^1 M_x^2 M_x^3 &= -1 \end{aligned}}$$

⑥ A logical contradiction! $-1 = M_x^1 M_x^2 M_x^3 = (M_x^1 M_y^2 M_y^3)(M_y^1 M_x^2 M_y^3)(M_y^1 M_y^2 M_x^3)$

$= +1.$

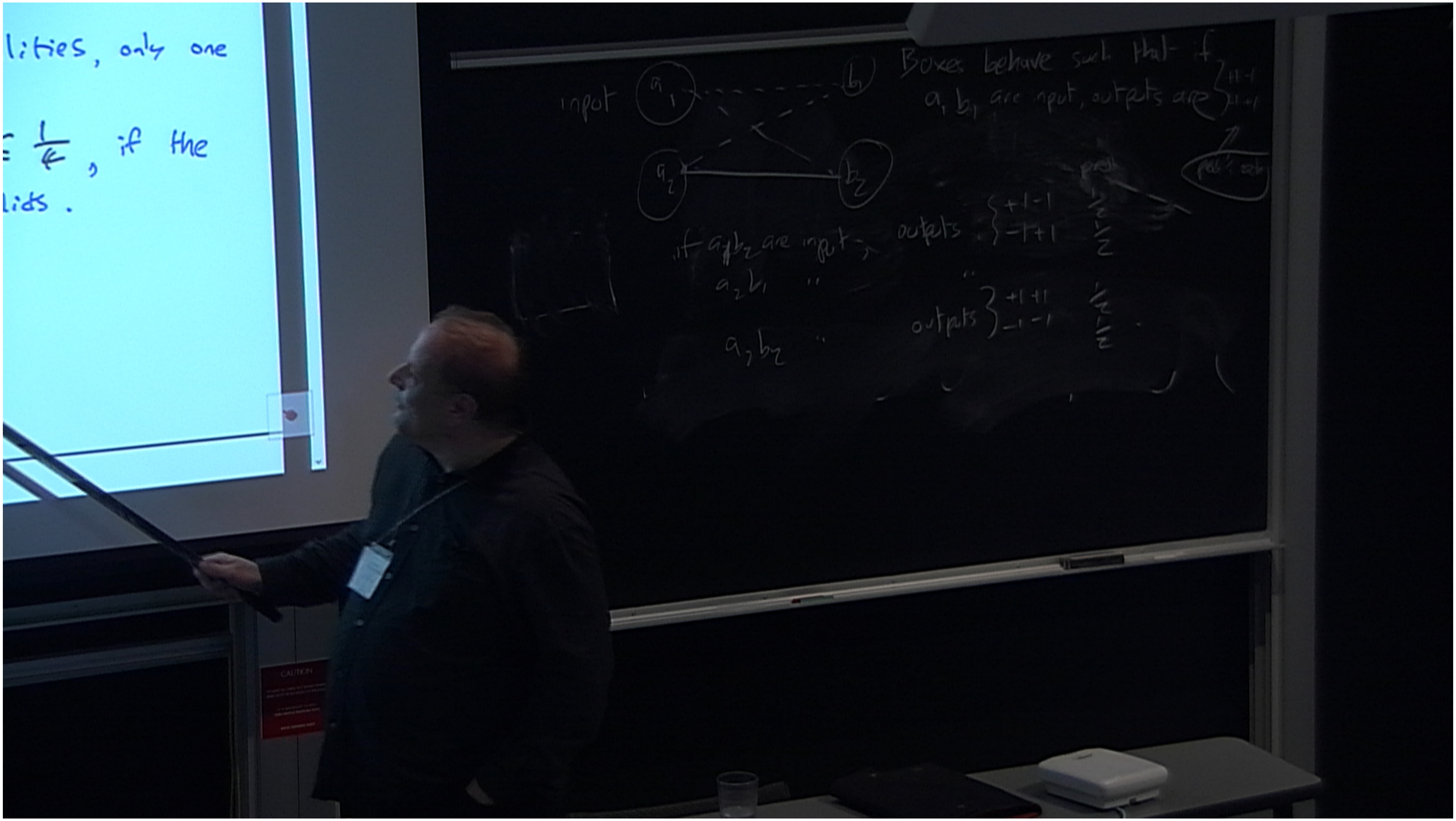

lities, only one
≤ 1/4, if the
lids.
A: P_LHV('yes') ≤ 3/4, P
Boxes behave such that if
a₁ b₁ are input, outputs are
+1 -1
-1 +1
input
a₁
b₁
a₂
b₂
t=0
CAUTION

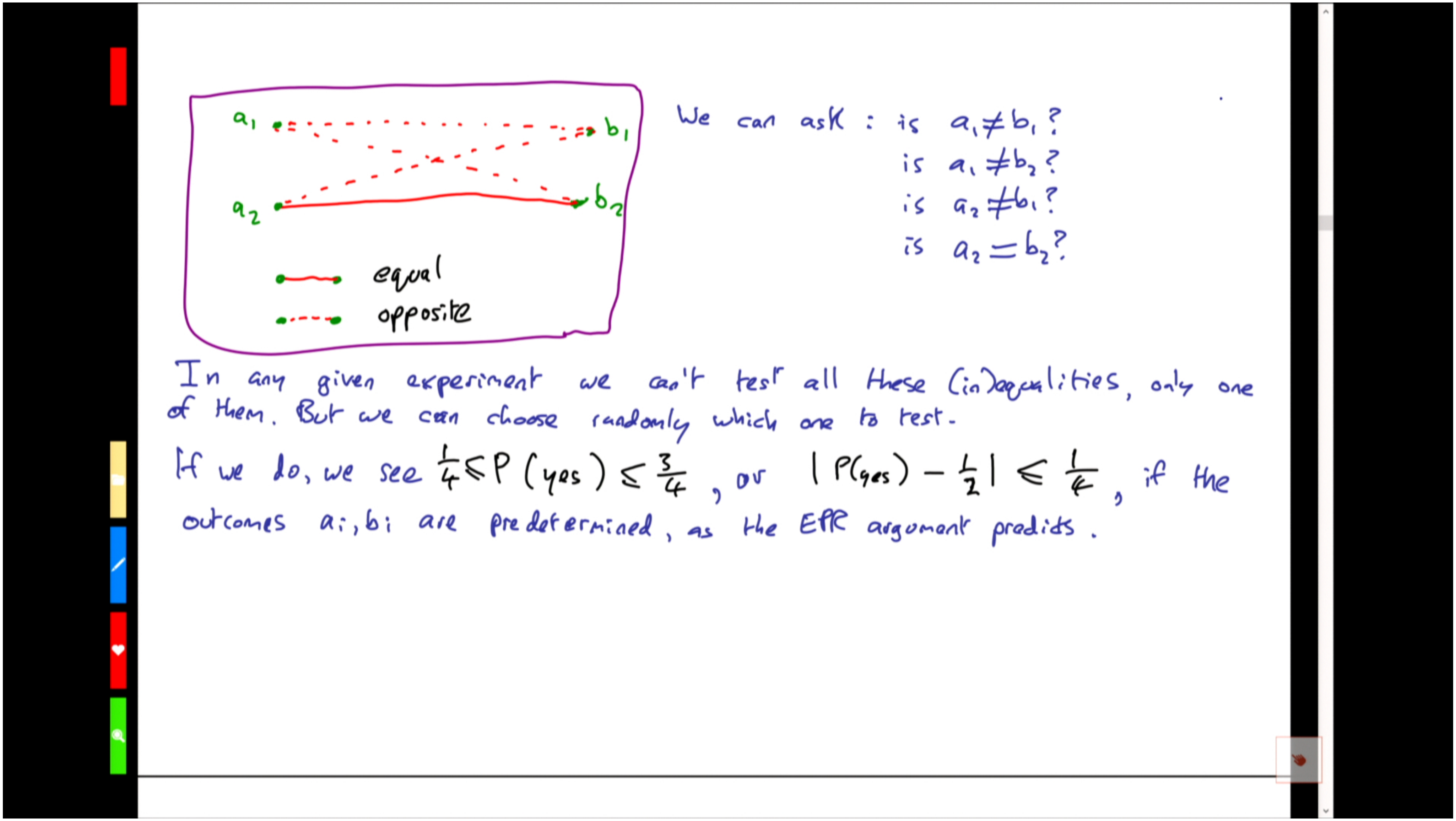


lities, only one

$\leq \frac{1}{4}$, if the

lids.


lities, only one
≤ 1/4, if the
lds.
input
a_1
b_1
a_2
b_2
Boxes behave such that if
$a_1 b_1$ are input, outputs are
+1 -1
+1 +1
prob ½ each
if $a_1 b_2$ are input,
$a_2 b_1$ "
$a_2 b_2$ "
outputs +1 -1, -1 +1
prob
½
½
outputs +1 +1, -1 -1
½
½



We can ask: is $a_1 \neq b_1$?

is $a_1 \neq b_2$?

is $a_2 \neq b_1$?

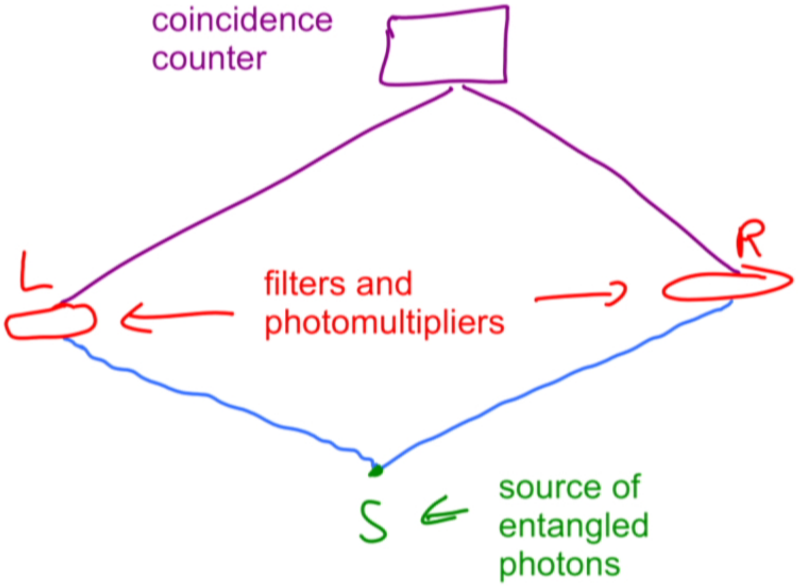
is $a_2 = b_2$?

In any given experiment we can't test all these (in)equalities, only one of them. But we can choose randomly which one to test.

If we do, we see $\frac{1}{4} \leqslant P(\text{yes}) \leq \frac{3}{4}$, or $\left| P(\text{yes}) - \frac{1}{2} \right| \leqslant \frac{1}{4}$, if the outcomes a_i, b_i are predetermined, as the EPR argument predicts.

ANOTHER LOOPHOLE: NOT YET (FULLY) CLOSED

Schematic standard Bell experiment



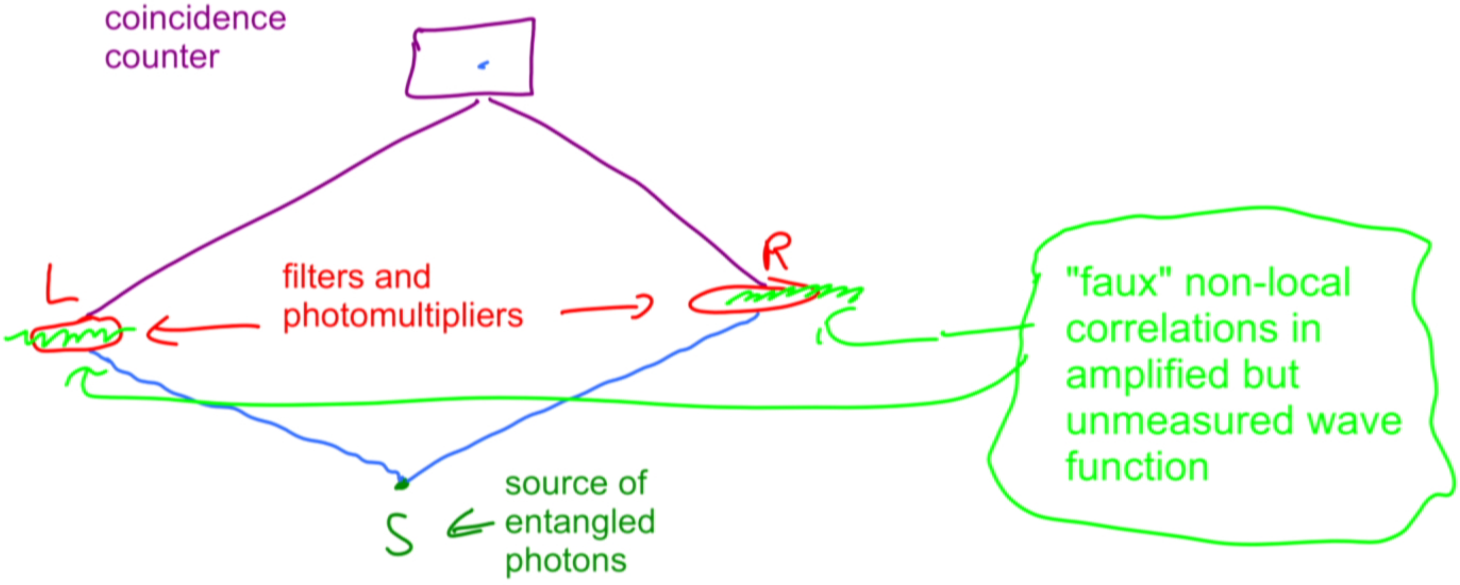
coincidence counter

filters and photomultipliers

source of entangled photons

If we take the view that quantum theory involves a transition from potential (wave function) to actual (measurements) at some uncertain scale, standard Bell experiments don't **necessarily** establish quantum nonlocality, because of ....

The collapse locality loophole

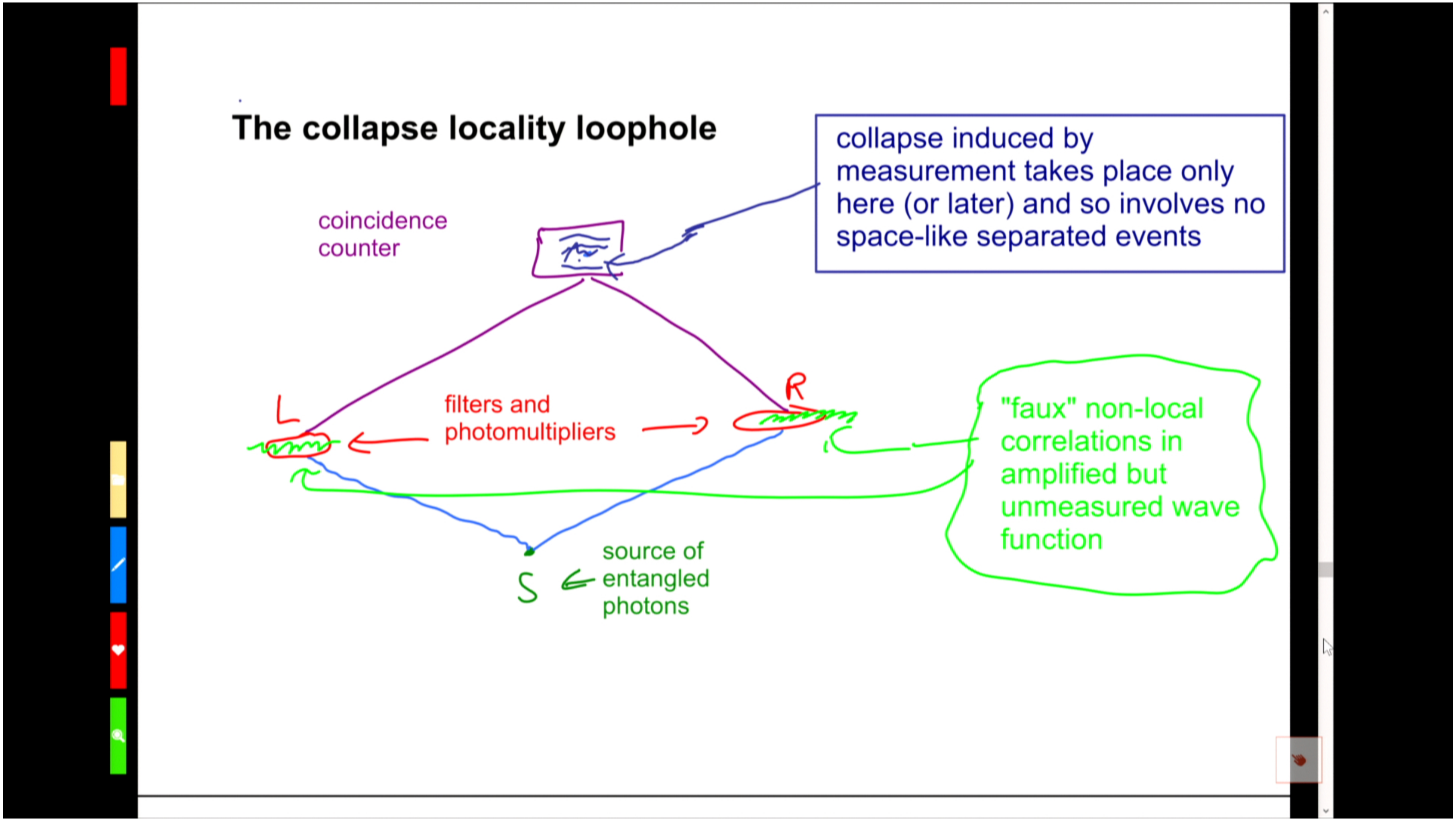


coincidence
counter

filters and
photomultipliers

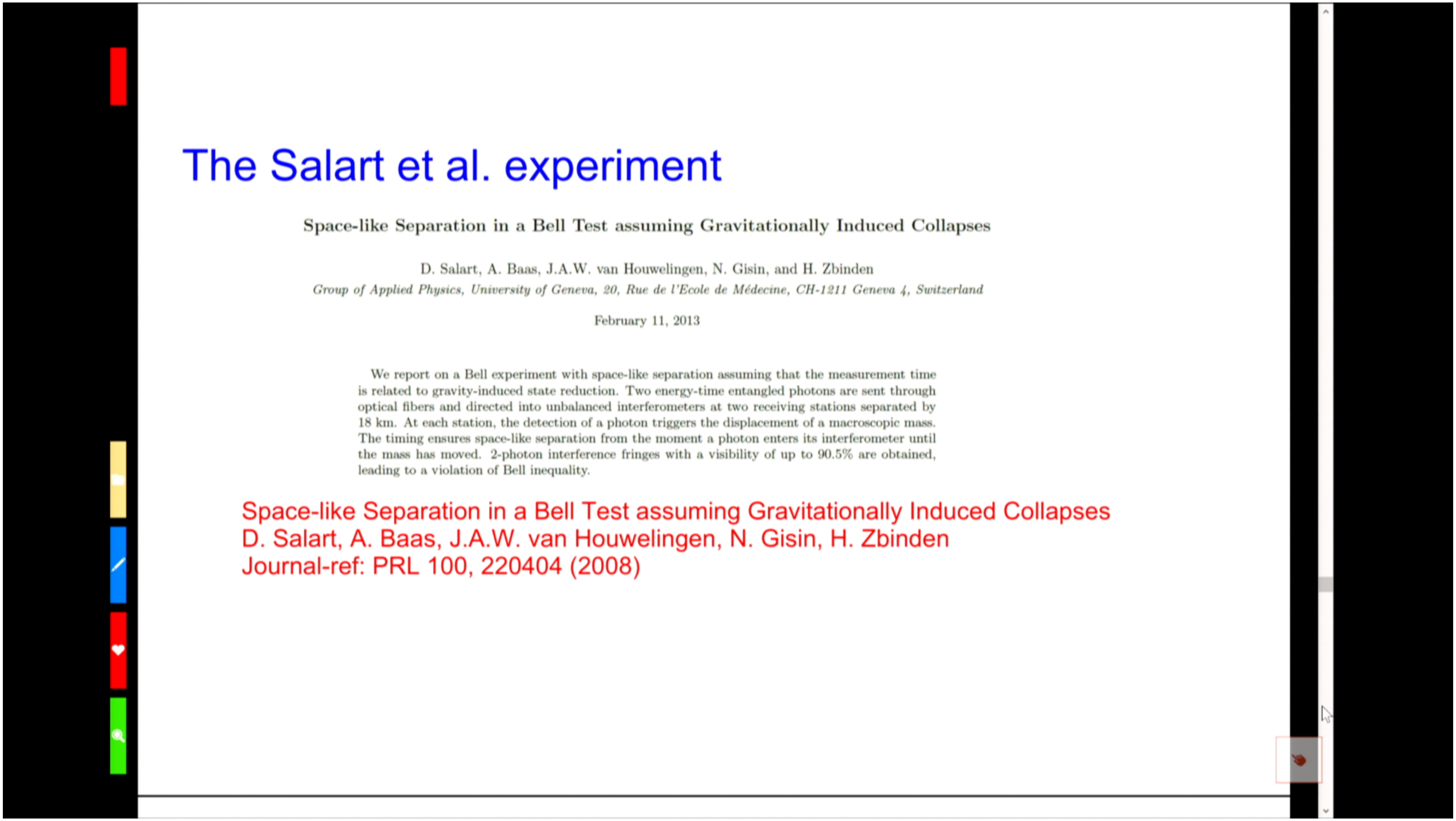
source of
entangled
photons

"faux" non-local
correlations in
amplified but
unmeasured wave
function



The collapse locality loophole

collapse induced by measurement takes place only here (or later) and so involves no space-like separated events



The Salart et al. experiment

Space-like Separation in a Bell Test assuming Gravitationally Induced Collapses

D. Salart, A. Baas, J.A.W. van Houwelingen, N. Gisin, and H. Zbinden
Group of Applied Physics, University of Geneva, 20, Rue de l'Ecole de Médecine, CH-1211 Geneva 4, Switzerland

February 11, 2013

We report on a Bell experiment with space-like separation assuming that the measurement time is related to gravity-induced state reduction. Two energy-time entangled photons are sent through optical fibers and directed into unbalanced interferometers at two receiving stations separated by 18 km. At each station, the detection of a photon triggers the displacement of a macroscopic mass. The timing ensures space-like separation from the moment a photon enters its interferometer until the mass has moved. 2-photon interference fringes with a visibility of up to 90.5% are obtained, leading to a violation of Bell inequality.

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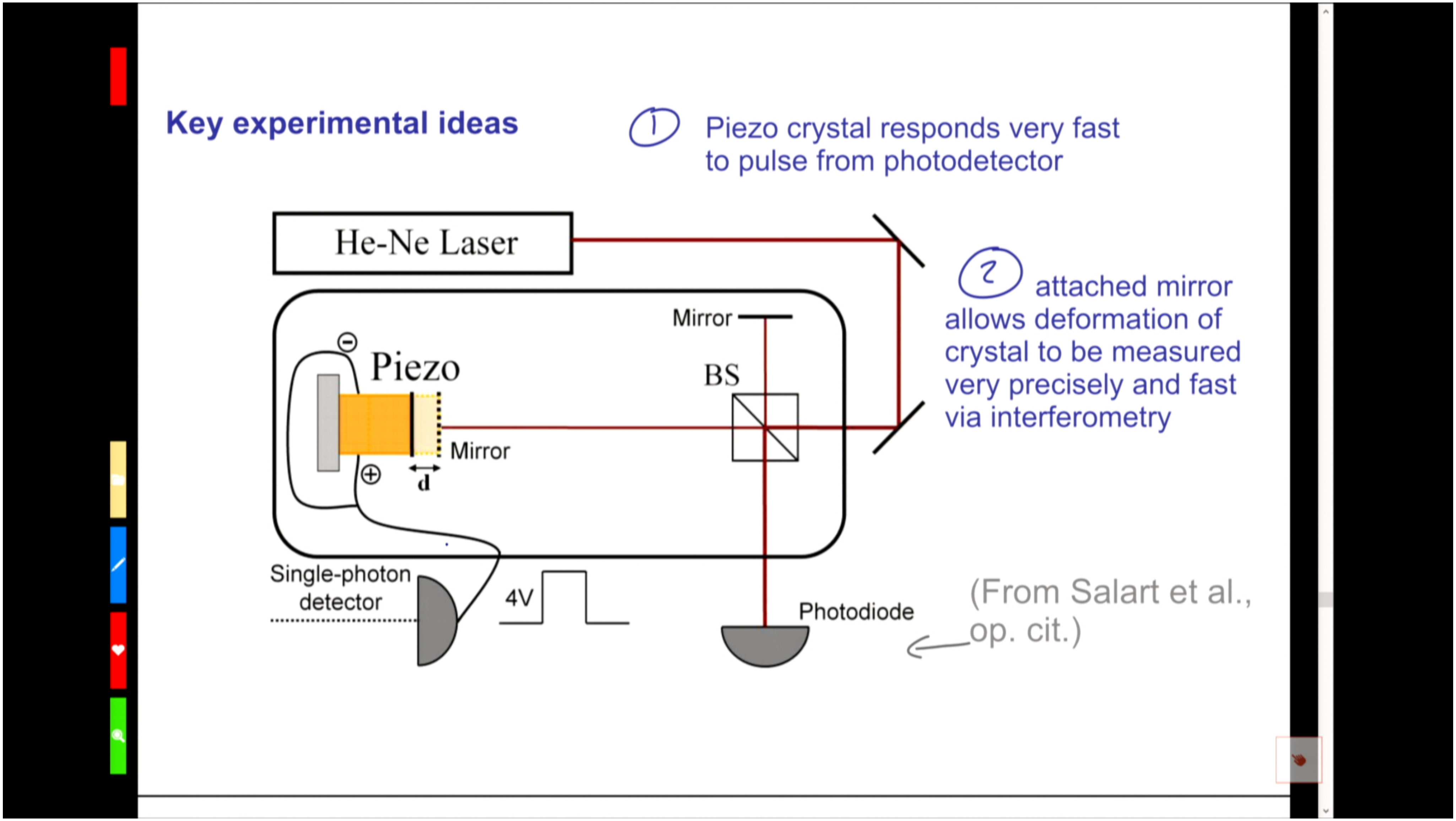
Journal-ref: PRL 100, 220404 (2008)

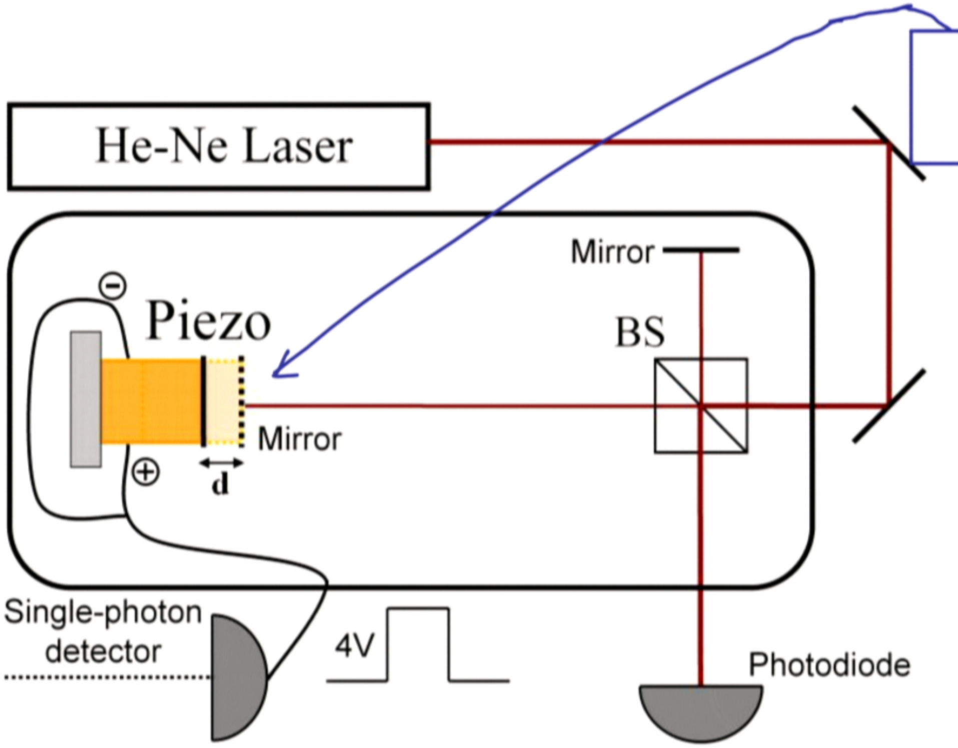


Key experimental ideas

① Piezo crystal responds very fast to pulse from photodetector

② attached mirror allows deformation of crystal to be measured very precisely and fast via interferometry

(From Salart et al., op. cit.)



Time from photons entering detector to

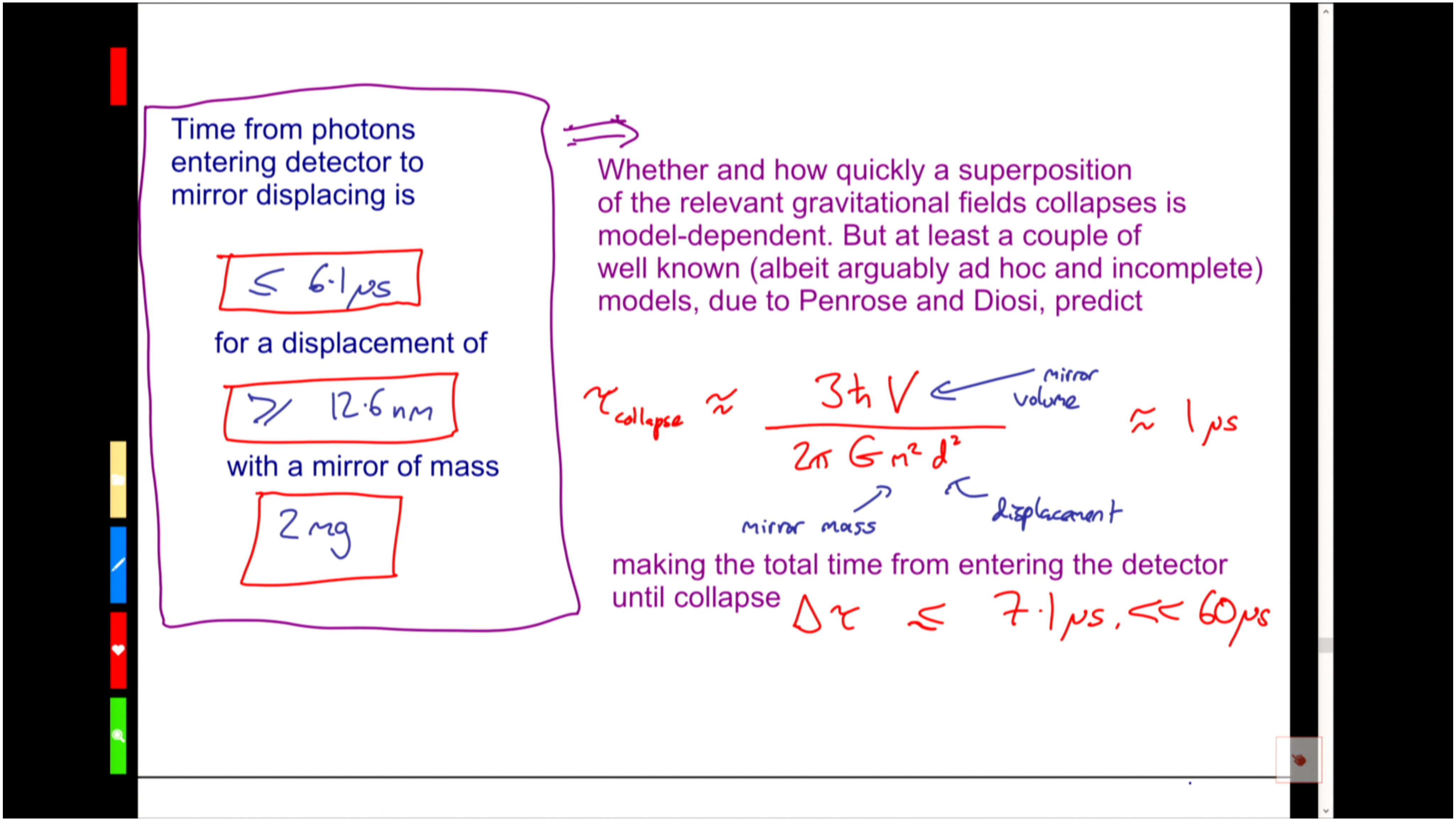
$\boxed{\leq 6.1\,\mu s}$

for a displacement of

$\boxed{\geq 12.6\,nm}$

with mirror of mass

$\boxed{2\,mg}$



Time from photons entering detector to mirror displacing is

$$\boxed{\lesssim 6.1\,\mu s}$$

for a displacement of

$$\boxed{\gtrsim 12.6\,nm}$$

with a mirror of mass

$$\boxed{2\,mg}$$

$\Longrightarrow$

Whether and how quickly a superposition of the relevant gravitational fields collapses is model-dependent. But at least a couple of well known (albeit arguably ad hoc and incomplete) models, due to Penrose and Diosi, predict

$$\tau_{collapse} \approx \frac{3\hbar V}{2\pi G m^2 d^2} \approx 1\,\mu s$$

mirror volume

mirror mass

displacement

making the total time from entering the detector until collapse $\Delta\tau \lesssim 7.1\,\mu s, \ll 60\,\mu s$

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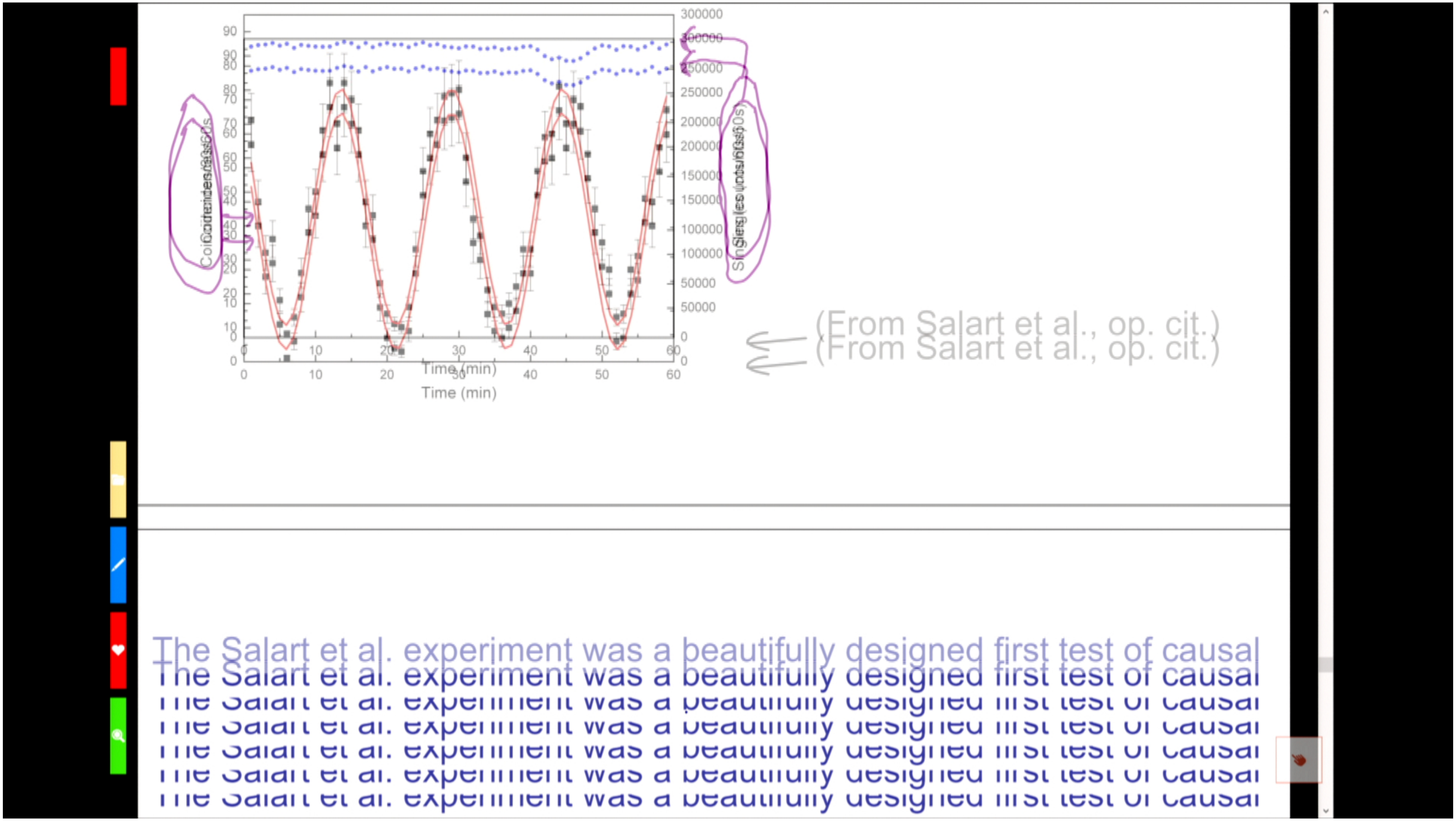
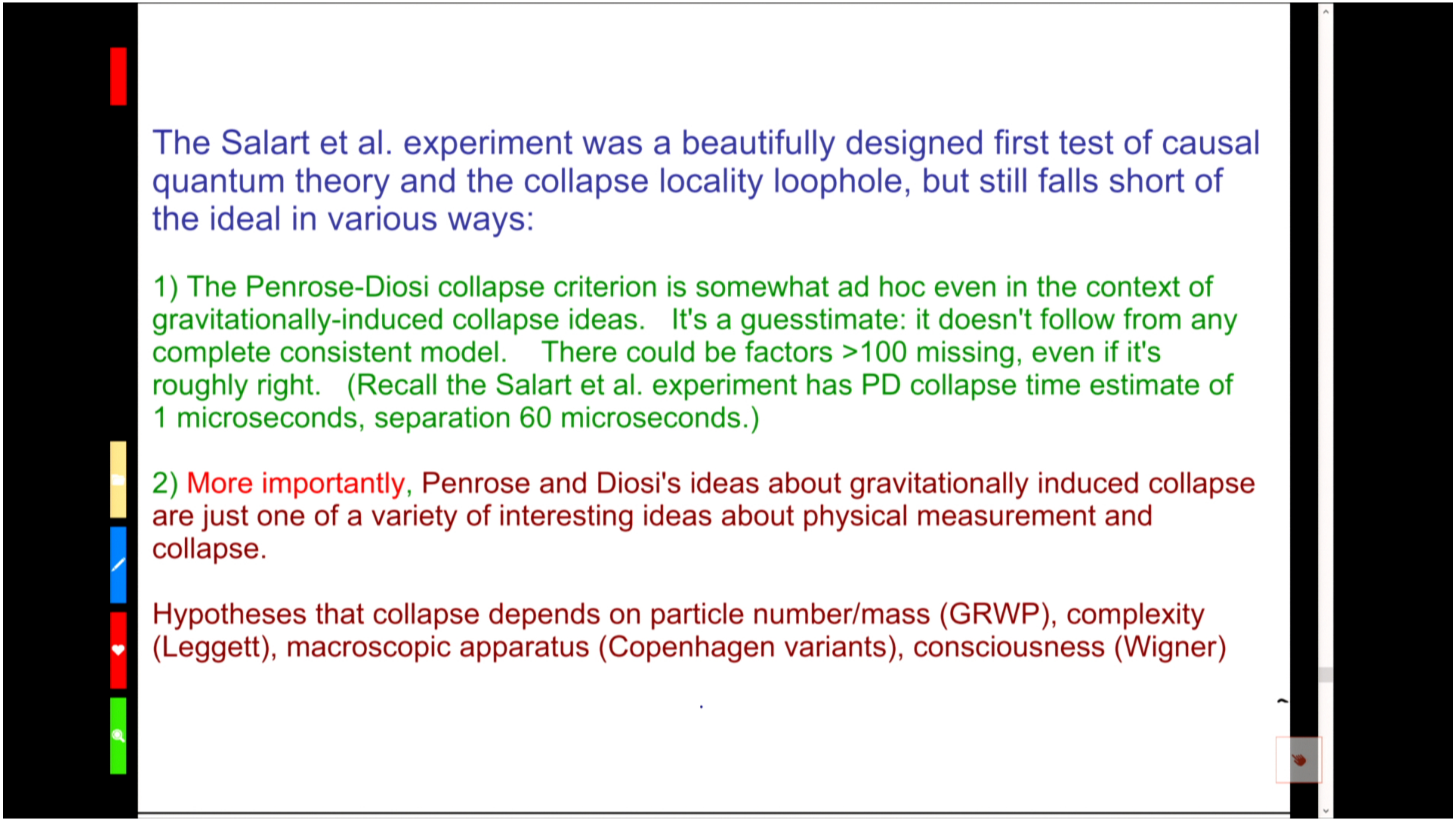


(From Salart et al., op. cit.)

The Salart et al. experiment was a beautifully designed first test of causal

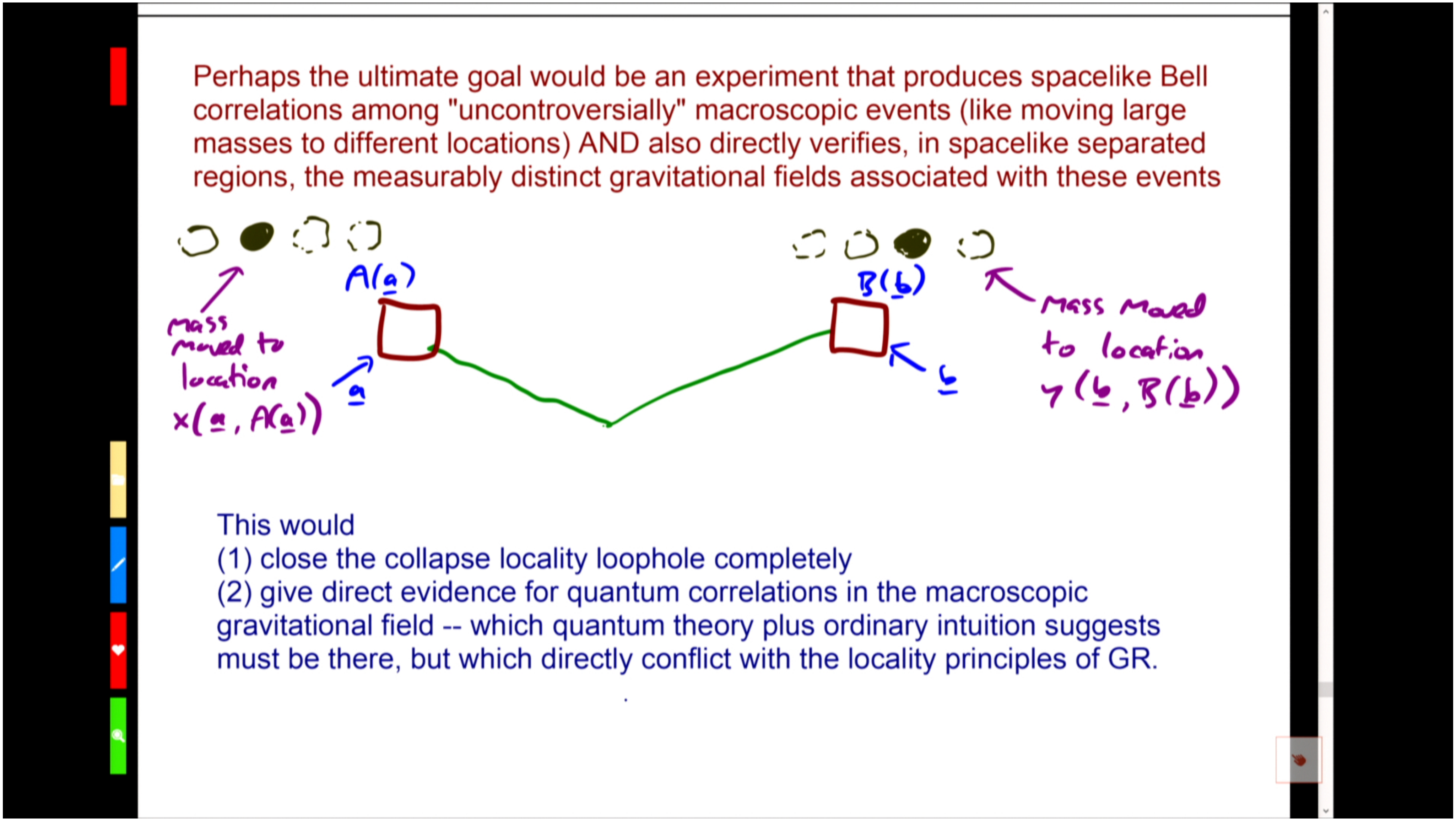


The Salart et al. experiment was a beautifully designed first test of causal quantum theory and the collapse locality loophole, but still falls short of the ideal in various ways:

1) The Penrose-Diosi collapse criterion is somewhat ad hoc even in the context of gravitationally-induced collapse ideas. It's a guesstimate: it doesn't follow from any complete consistent model. There could be factors >100 missing, even if it's roughly right. (Recall the Salart et al. experiment has PD collapse time estimate of 1 microseconds, separation 60 microseconds.)

2) More importantly, Penrose and Diosi's ideas about gravitationally induced collapse are just one of a variety of interesting ideas about physical measurement and collapse.

Hypotheses that collapse depends on particle number/mass (GRWP), complexity (Leggett), macroscopic apparatus (Copenhagen variants), consciousness (Wigner)



Perhaps the ultimate goal would be an experiment that produces spacelike Bell correlations among "uncontroversially" macroscopic events (like moving large masses to different locations) AND also directly verifies, in spacelike separated regions, the measurably distinct gravitational fields associated with these events

This would
(1) close the collapse locality loophole completely
(2) give direct evidence for quantum correlations in the macroscopic gravitational field -- which quantum theory plus ordinary intuition suggests must be there, but which directly conflict with the locality principles of GR.