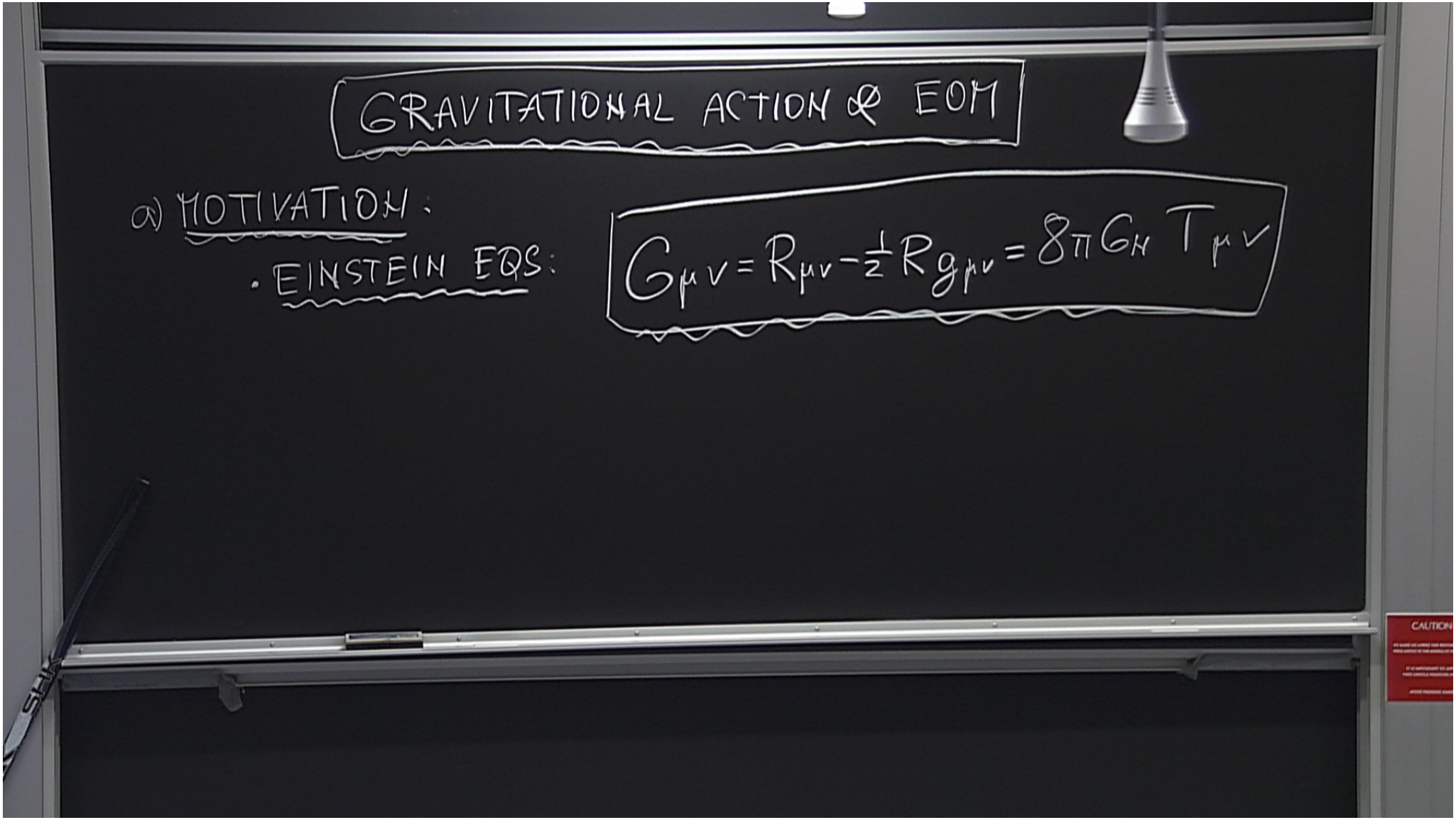
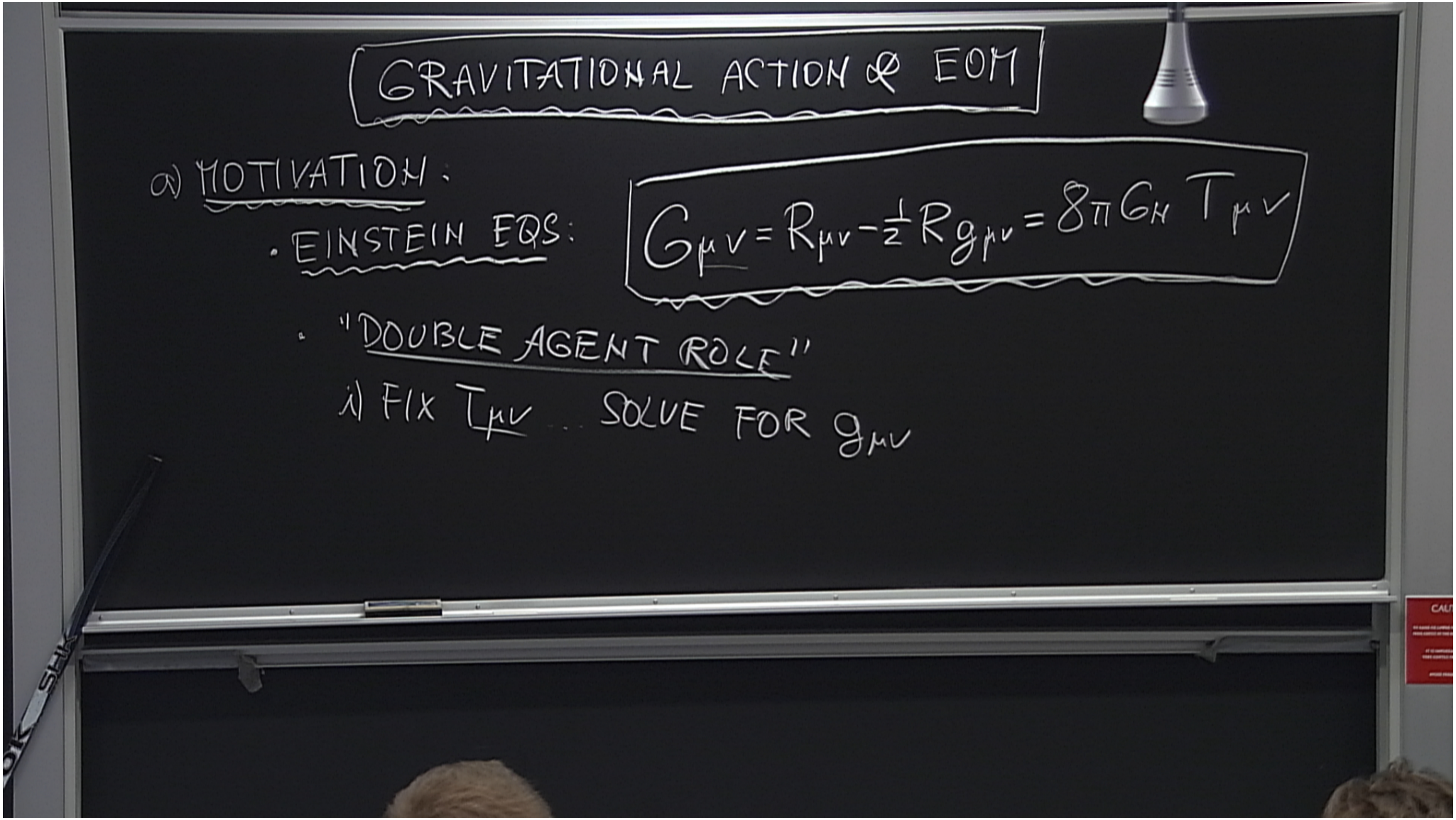
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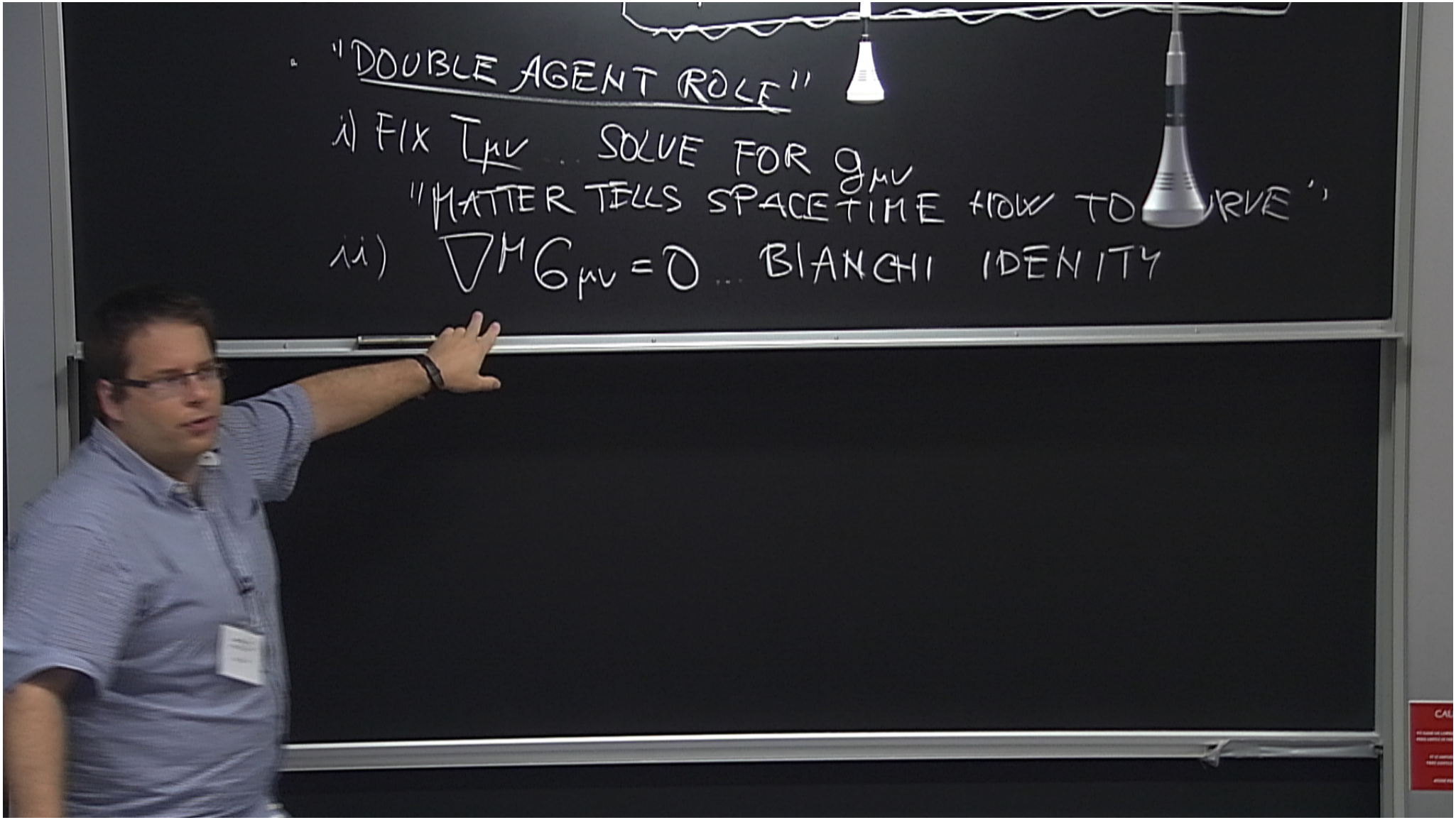
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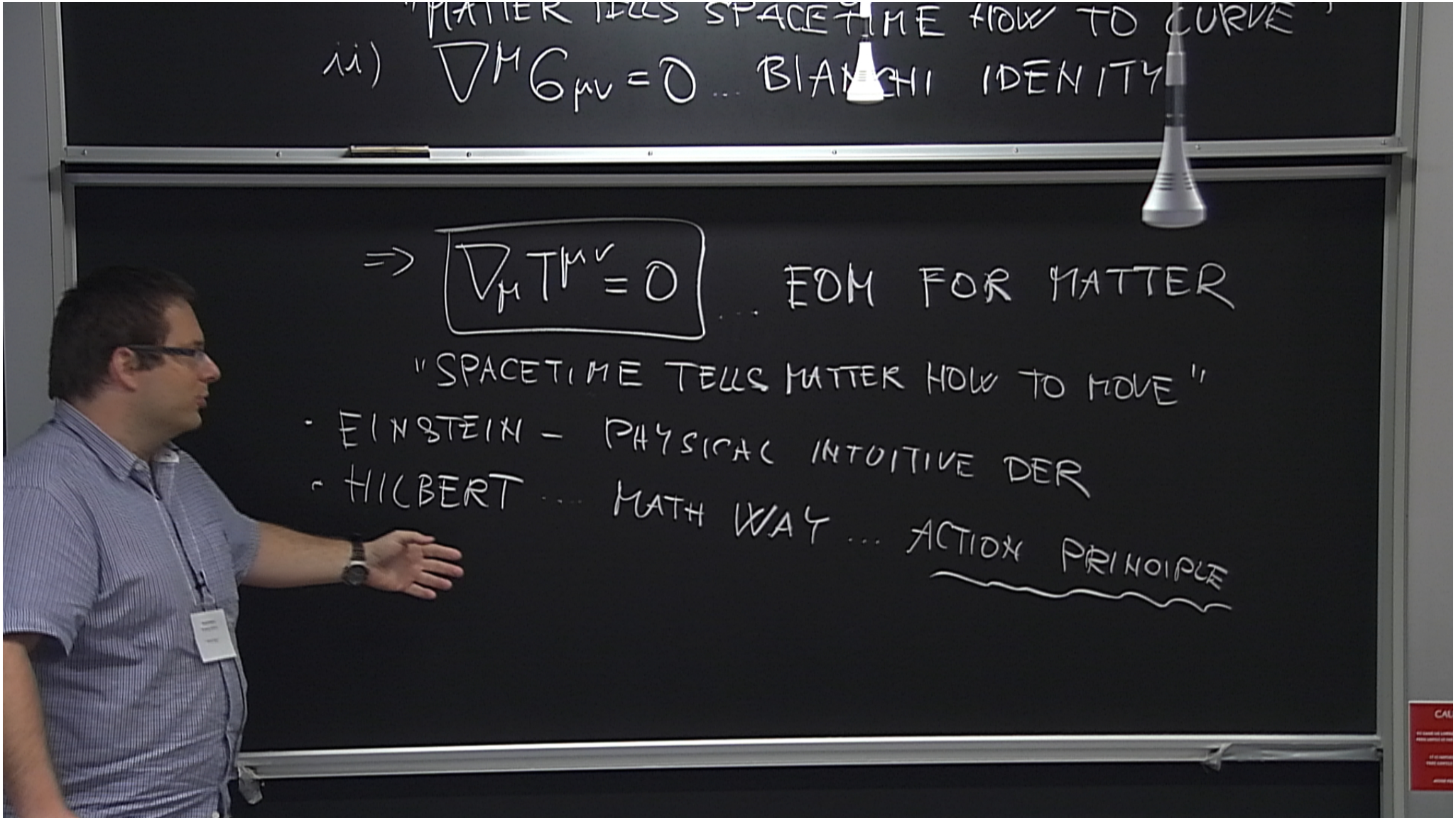
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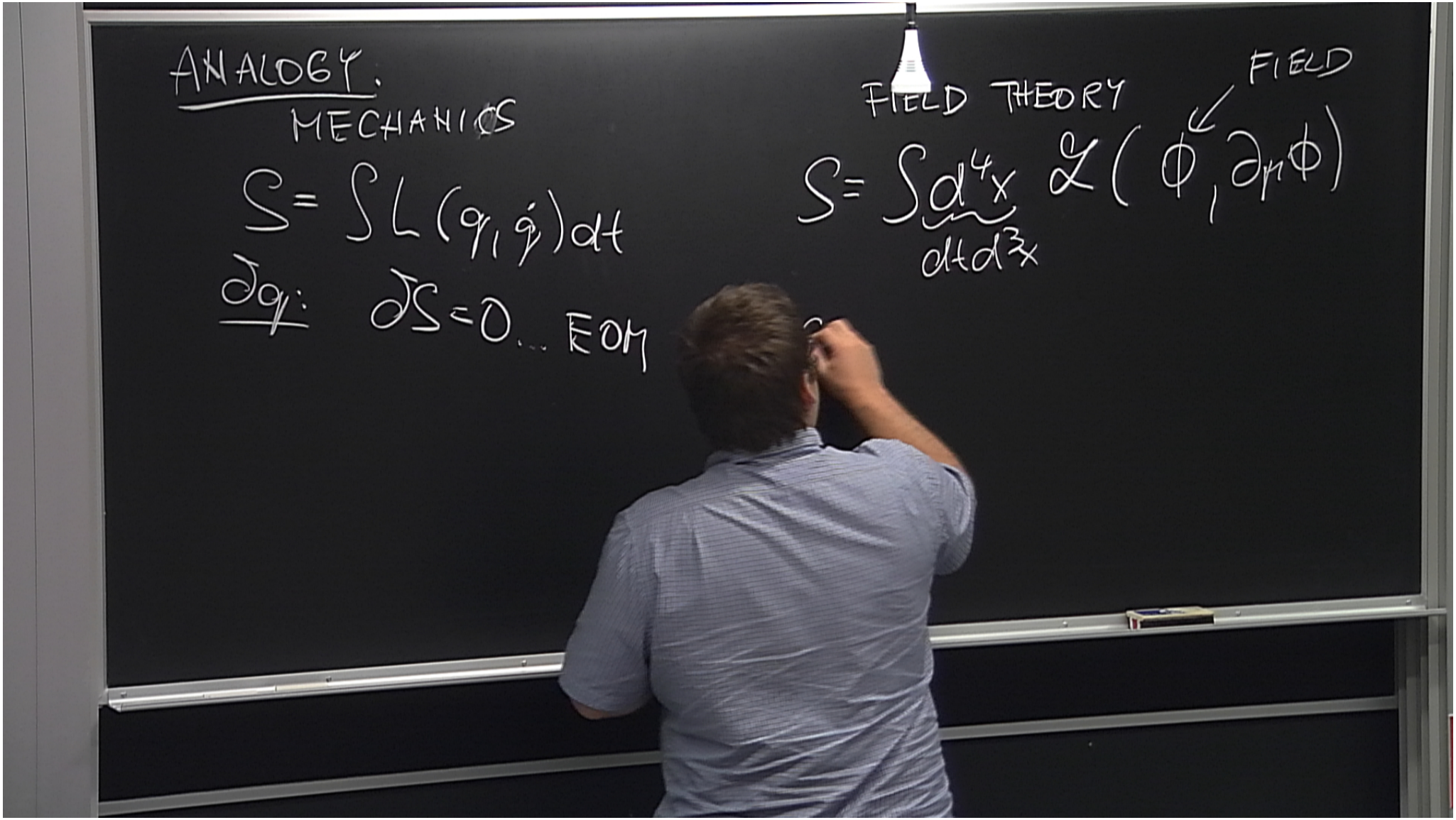
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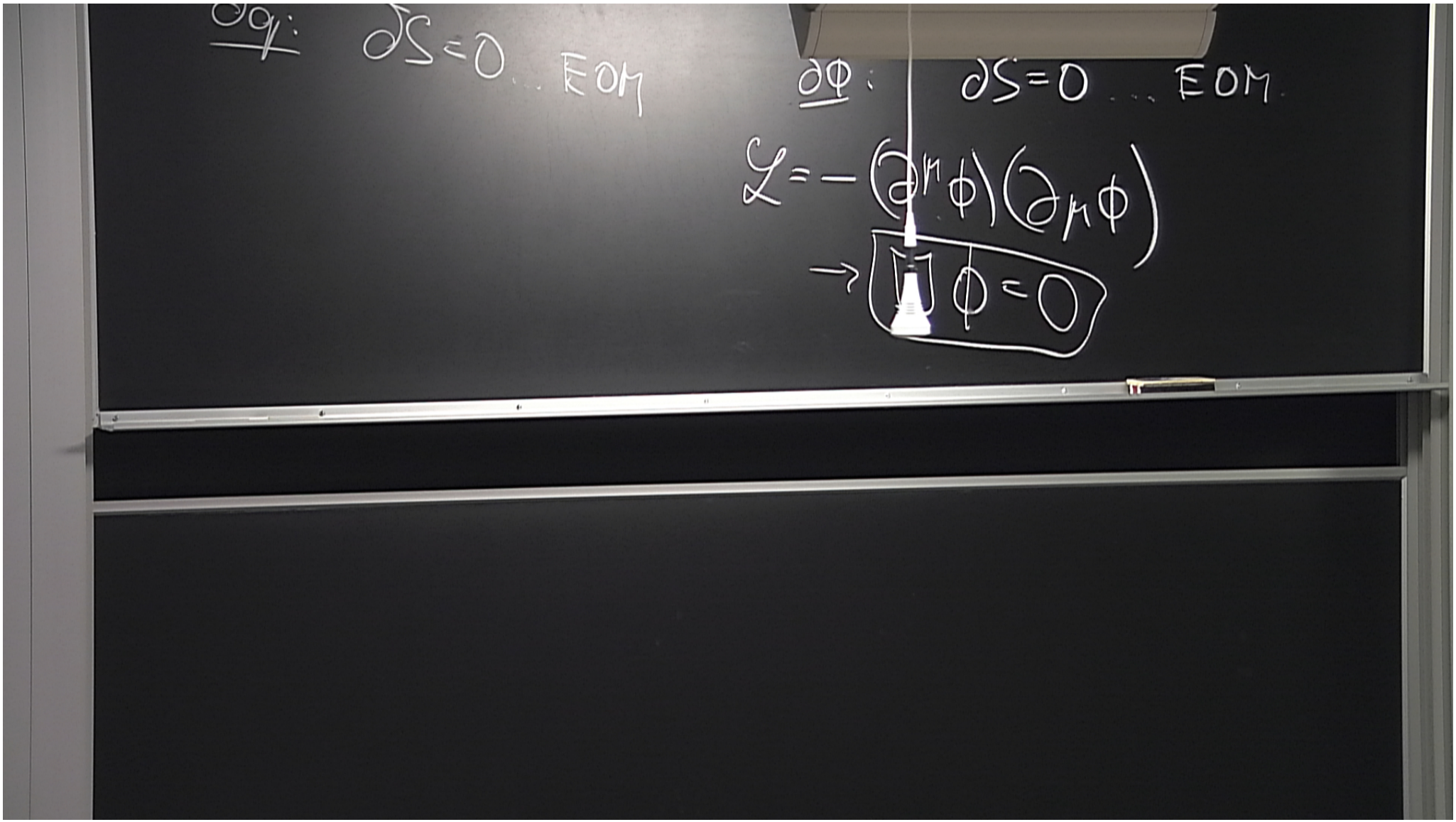

GRAVITATIONAL ACTION & EOM
a) MOTIVATION:
• EINSTEIN EQS:
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
CAUTION

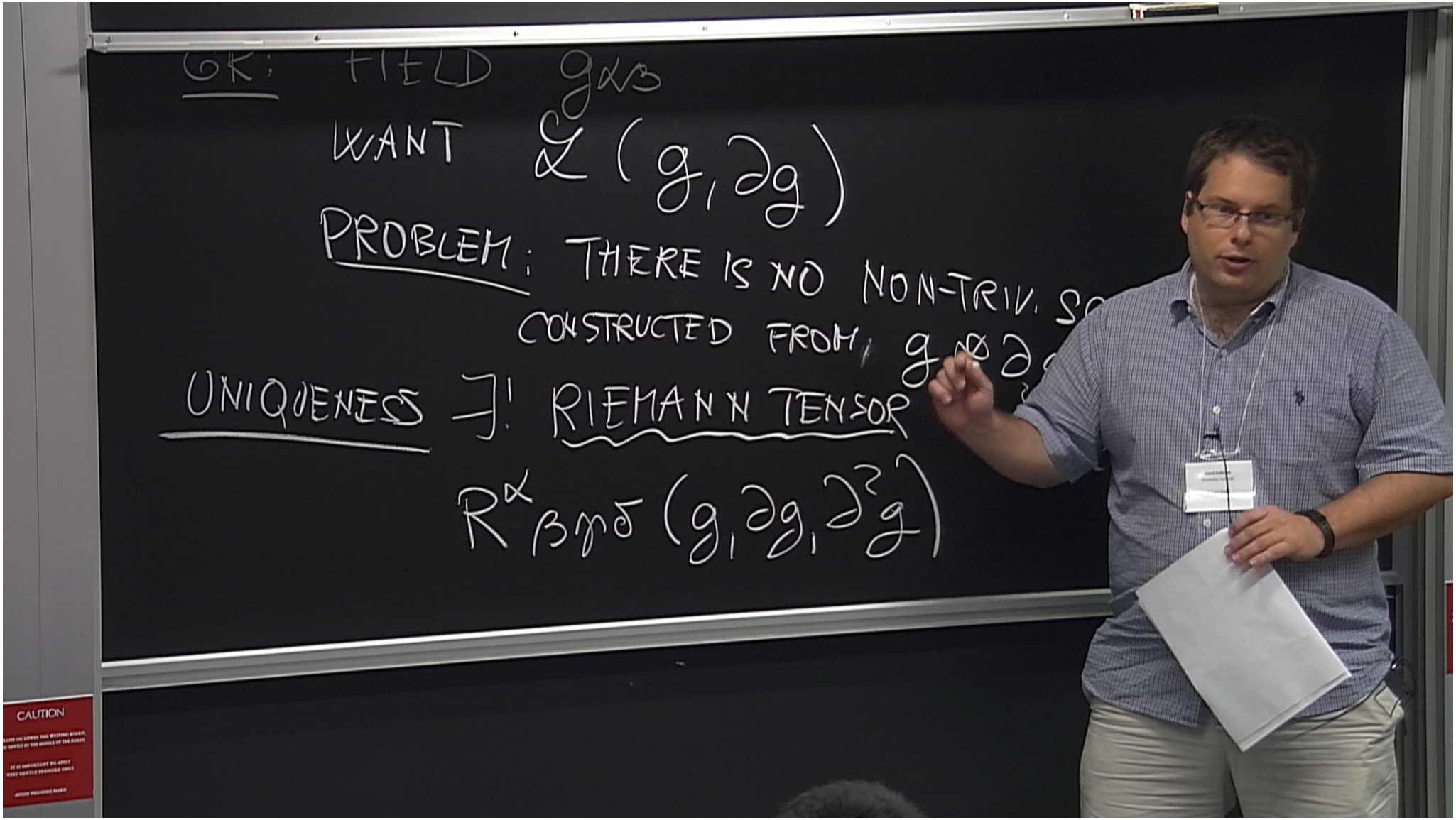

GRAVITATIONAL ACTION & EOM
a) MOTIVATION:
• EINSTEIN EQS: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
• "DOUBLE AGENT ROLE"
i) FIX $T_{\mu\nu}$ ... SOLVE FOR $g_{\mu\nu}$

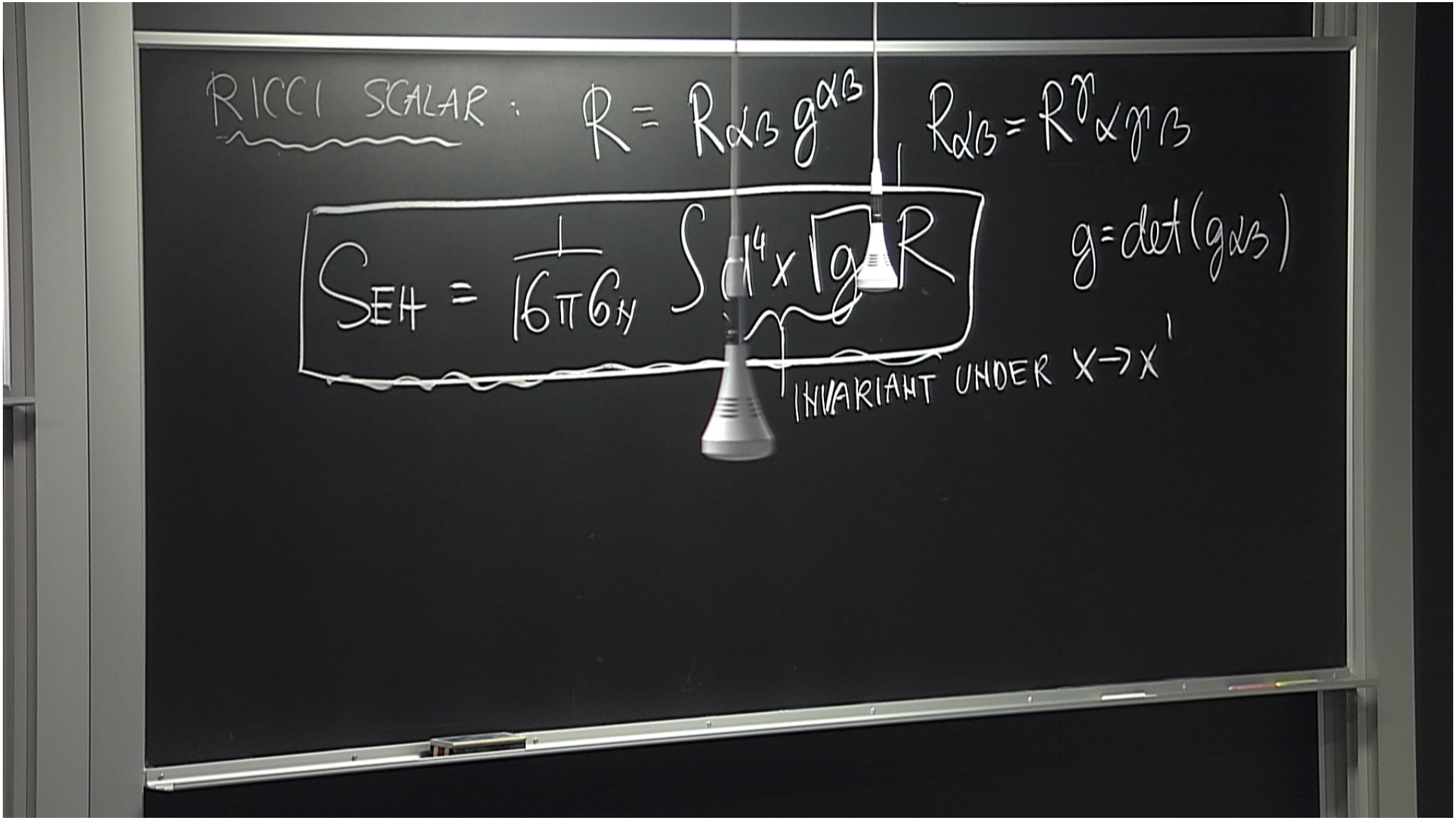

"DOUBLE AGENT ROLE"
i) FIX $T_{\mu\nu}$ .. SOLVE FOR $g_{\mu\nu}$
"MATTER TELLS SPACETIME HOW TO
ii) $\nabla^\mu G_{\mu\nu} = 0$ .. BIANCHI IDENITY

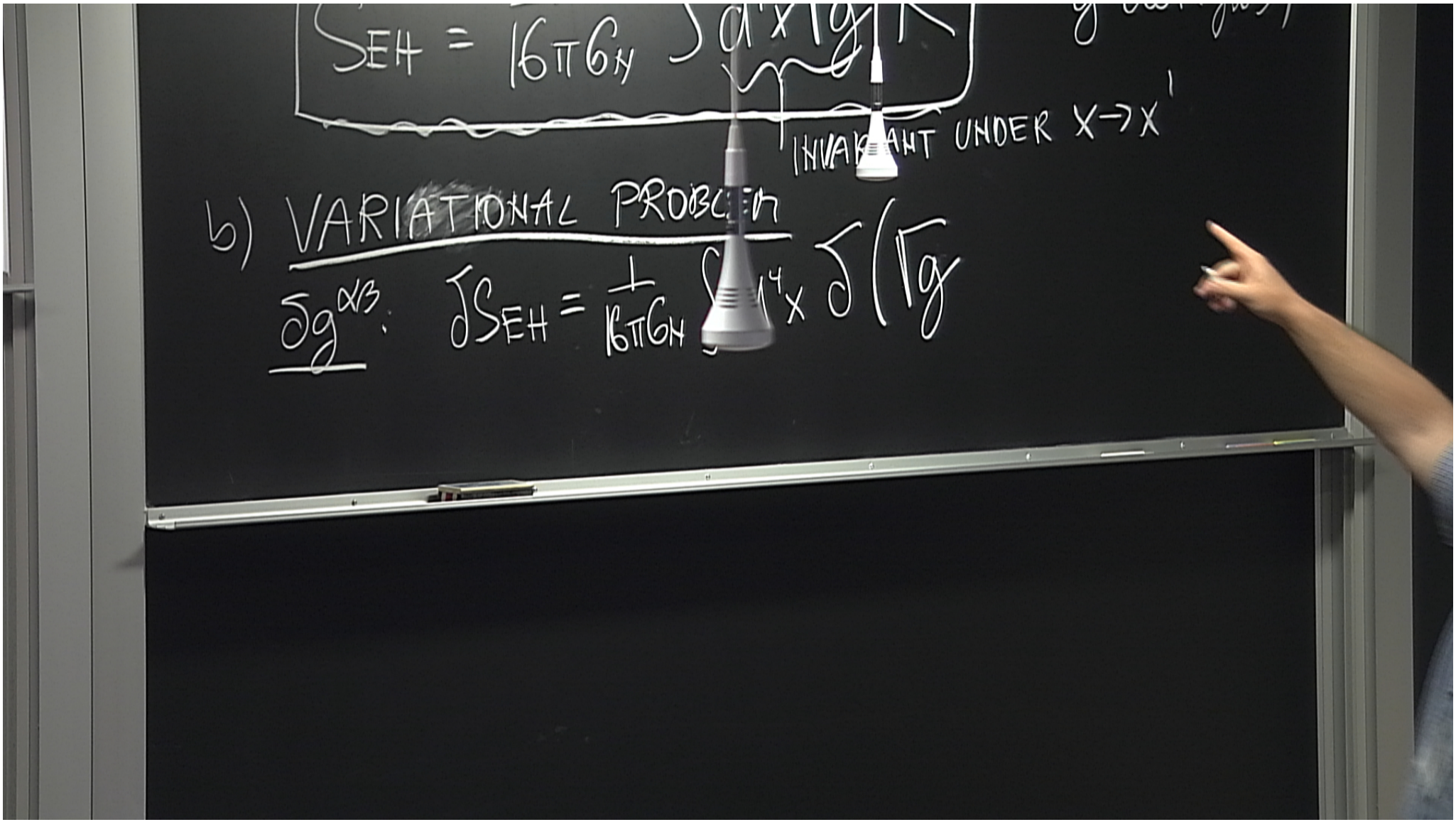

"MATTER TELLS SPACETIME HOW TO CURVE"
ii) $\nabla^\mu G_{\mu\nu} = 0$ ... BIANCHI IDENITY
$\Rightarrow \boxed{\nabla_\mu T^{\mu\nu} = 0}$ ... EOM FOR MATTER
"SPACETIME TELLS MATTER HOW TO MOVE"
- EINSTEIN - PHYSICAL INTOITIVE DER
- HILBERT ... MATH WAY ... ACTION PRINCIPLE

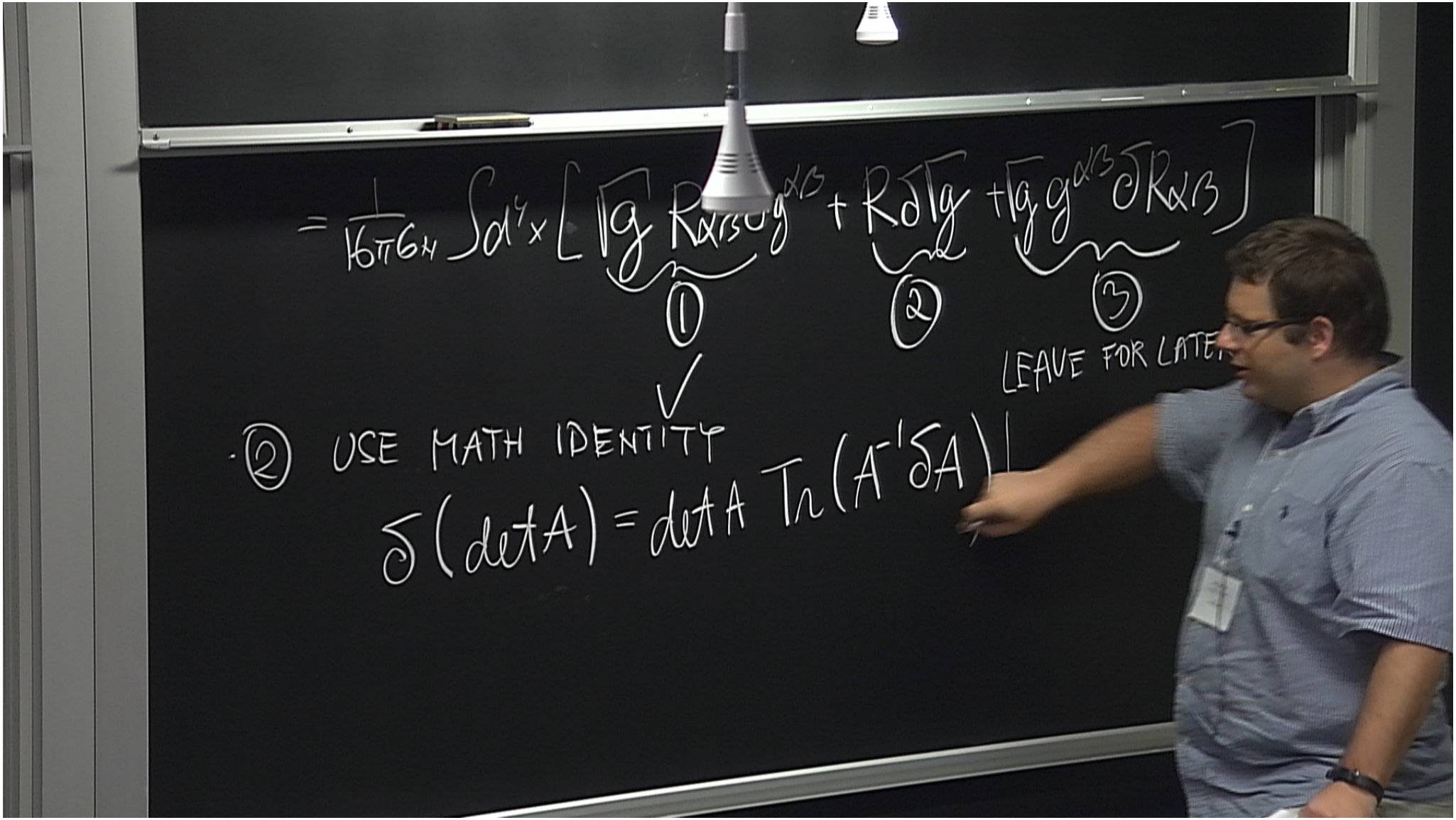

ANALOGY:
MECHANICS
$S = \int L(q, \dot{q})\, dt$
∂_q: $\partial S = 0$ ... EOM
FIELD THEORY
FIELD
$S = \int \underbrace{d^4x}_{dt\,d^3x}\, \mathcal{L}(\phi, \partial_\mu \phi)$

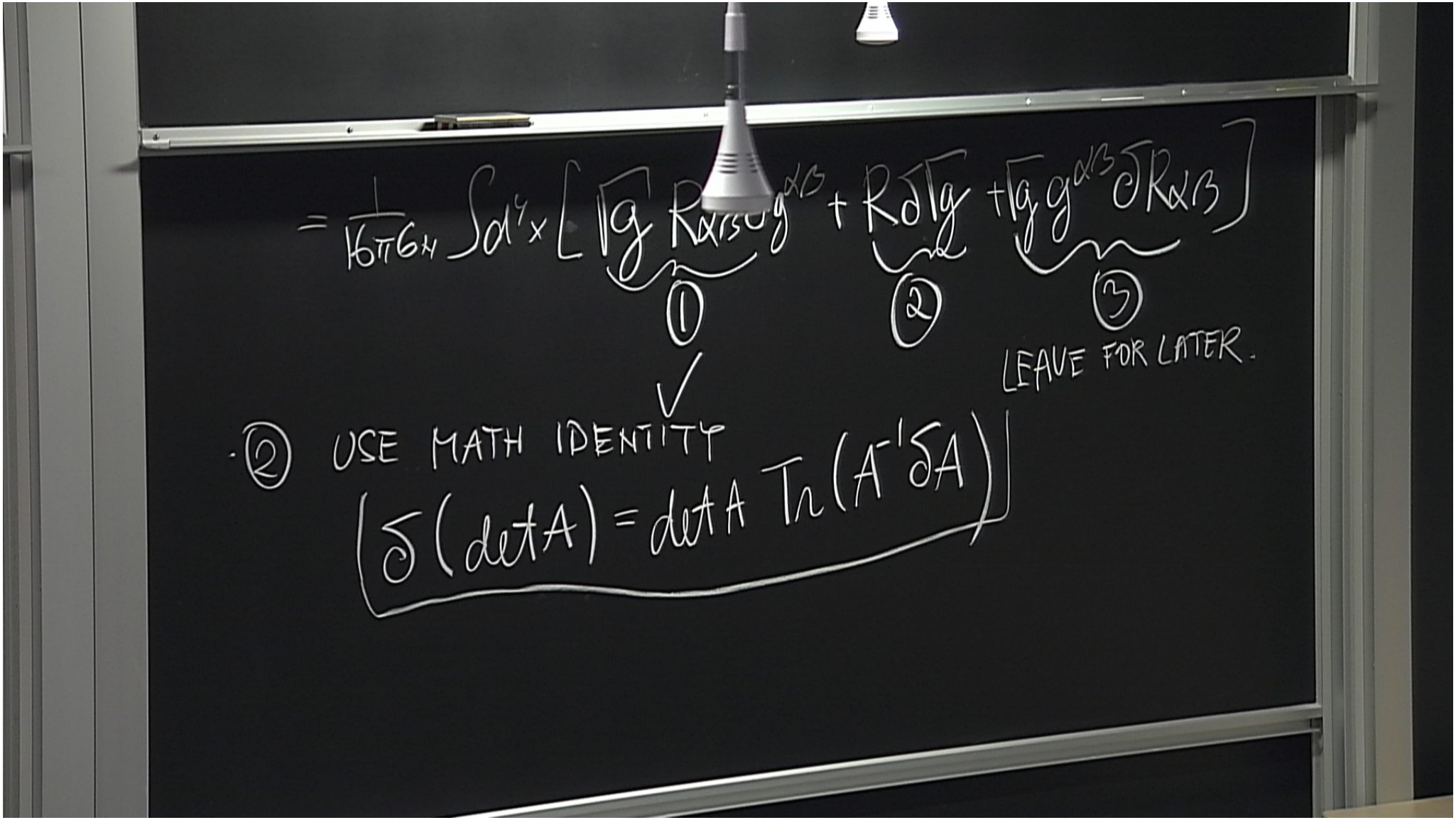

∂q: $\delta S = 0$ ... EOM
$\partial \Phi$: $\delta S = 0$ ... EOM
$\mathcal{L} = -(\partial^\mu \Phi)(\partial_\mu \Phi)$
$\rightarrow \Box \Phi = 0$

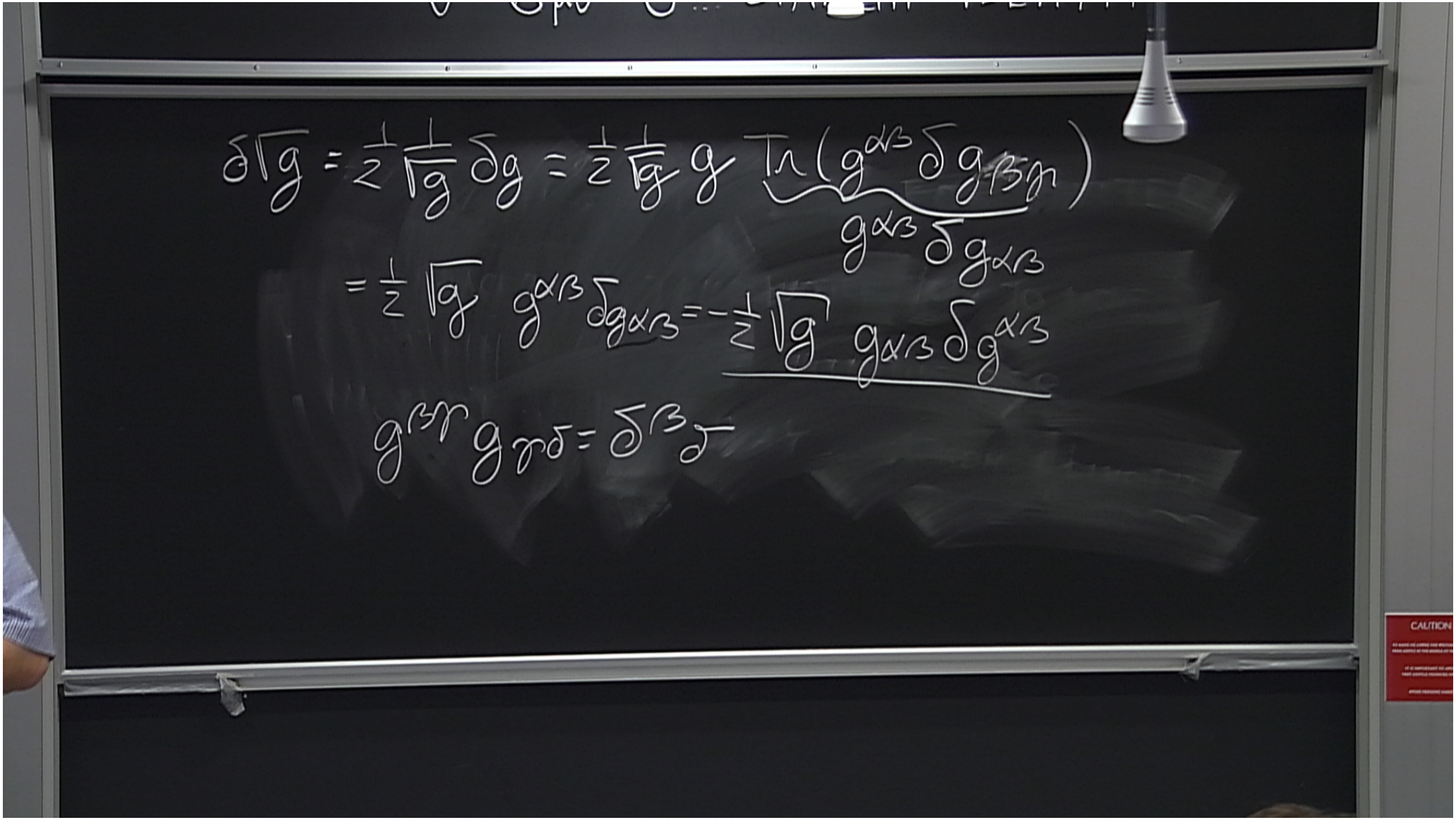

GR: FIELD $g_{\alpha\beta}$
WANT $\mathcal{L}(g, \partial g)$
PROBLEM: THERE IS NO NON-TRIV. SC
CONSTRUCTED FROM g, ∂
UNIQUENESS $\exists!$ RIEMANN TENSOR
$R^{\alpha}{}_{\beta\gamma\sigma}(g, \partial g, \partial^2 g)$
CAUTION

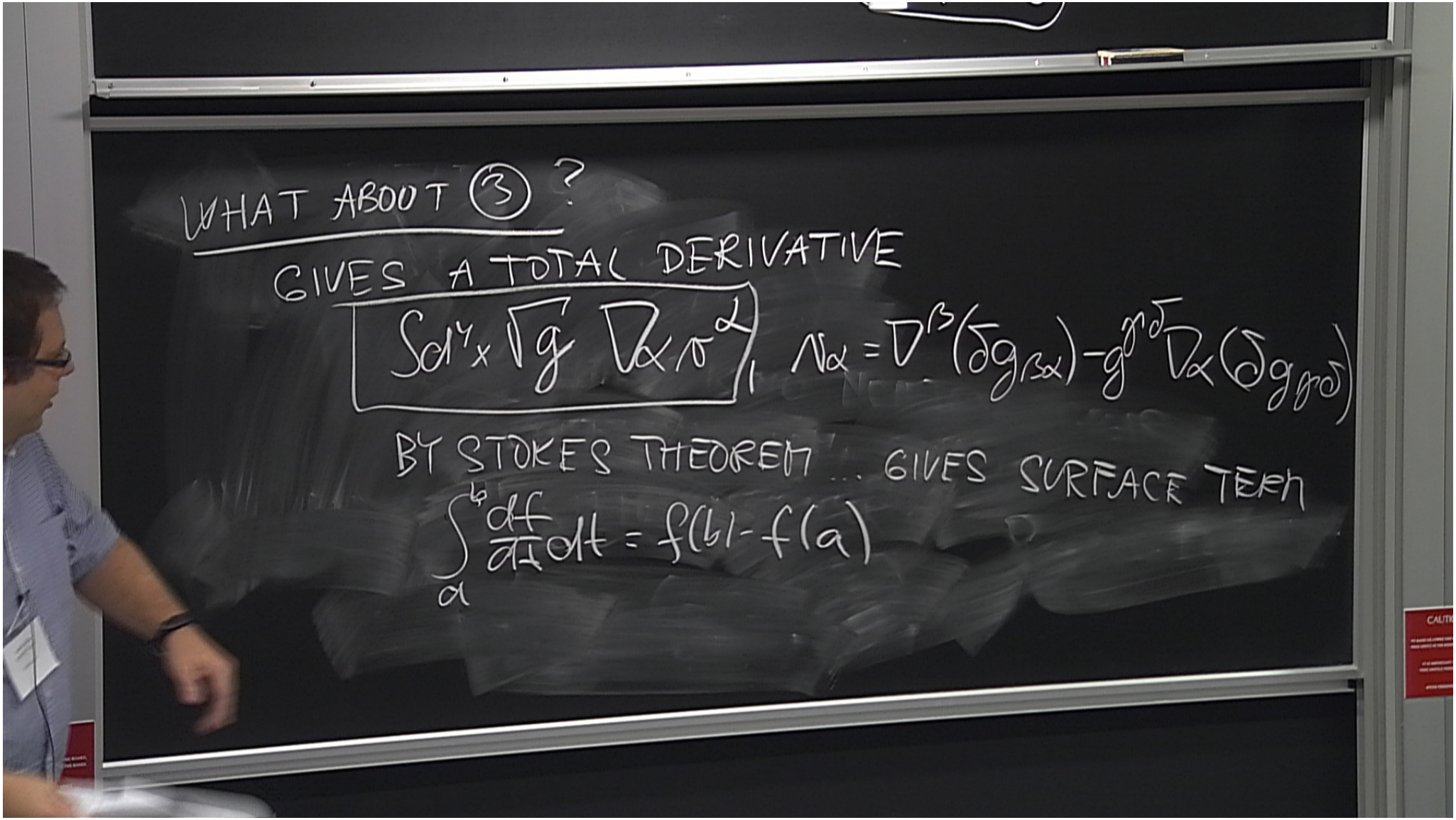

RICCI SCALAR : $R = R_{\alpha\beta} g^{\alpha\beta}$
$R_{\alpha\beta} = R^{\gamma}{}_{\alpha\gamma\beta}$
$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} R$
$g = \det(g_{\alpha\beta})$
INVARIANT UNDER $x \to x'$


$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R$
INVARIANT UNDER $x \to x'$
b) VARIATIONAL PROBLEM
$\delta g^{\alpha\beta}$: $\delta S_{EH} = \frac{1}{16\pi G_N} \int d^4x \, \delta(\sqrt{-g}$


$= \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{g} R_{\alpha\beta} \delta g^{\alpha\beta} + R \delta \sqrt{g} + \sqrt{g} g^{\alpha\beta} \delta R_{\alpha\beta} \right]$
① ✓
②
③
LEAVE FOR LATE
② USE MATH IDENTITY
$\delta(\det A) = \det A \, \mathrm{Tr}(A^{-1} \delta A)$


$= \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{g} R_{\alpha\beta} \delta g^{\alpha\beta} + R \delta \sqrt{g} + \sqrt{g} g^{\alpha\beta} \delta R_{\alpha\beta} \right]$
①
②
③
LEAVE FOR LATER.
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$\delta(\det A) = \det A \, \mathrm{Tr}(A^{-1} \delta A)$

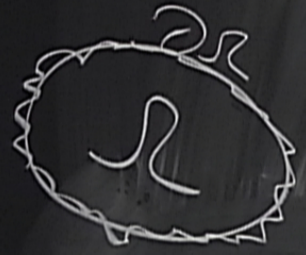

$\delta\sqrt{g} = \frac{1}{2}\frac{1}{\sqrt{g}}\delta g = \frac{1}{2}\frac{1}{g}g\,\mathrm{Tr}(g^{\alpha\beta}\delta g_{\beta\gamma})$
$g^{\alpha\beta}\delta g_{\alpha\beta}$
$= \frac{1}{2}\sqrt{g}\; g^{\alpha\beta}\delta g_{\alpha\beta} = -\frac{1}{2}\sqrt{g}\; g_{\alpha\beta}\delta g^{\alpha\beta}$
$g^{\beta\gamma}g_{\gamma\sigma} = \delta^{\beta}{}_{\sigma}$
CAUTION


WHAT ABOUT (3) ?
GIVES A TOTAL DERIVATIVE
$\int d^4x \sqrt{g}\, \nabla_\alpha \Lambda^2$, $\Lambda_\alpha = D^\beta(\delta g_{\beta\alpha}) - g^{\rho\sigma}\nabla_\alpha(\delta g_{\rho\sigma})$
BY STOKES THEOREM ... GIVES SURFACE TERM
$\int_a^b \frac{df}{dx} dt = f(b) - f(a)$

WHAT ABOUT (3)?

GIVES A TOTAL DERIVATIVE

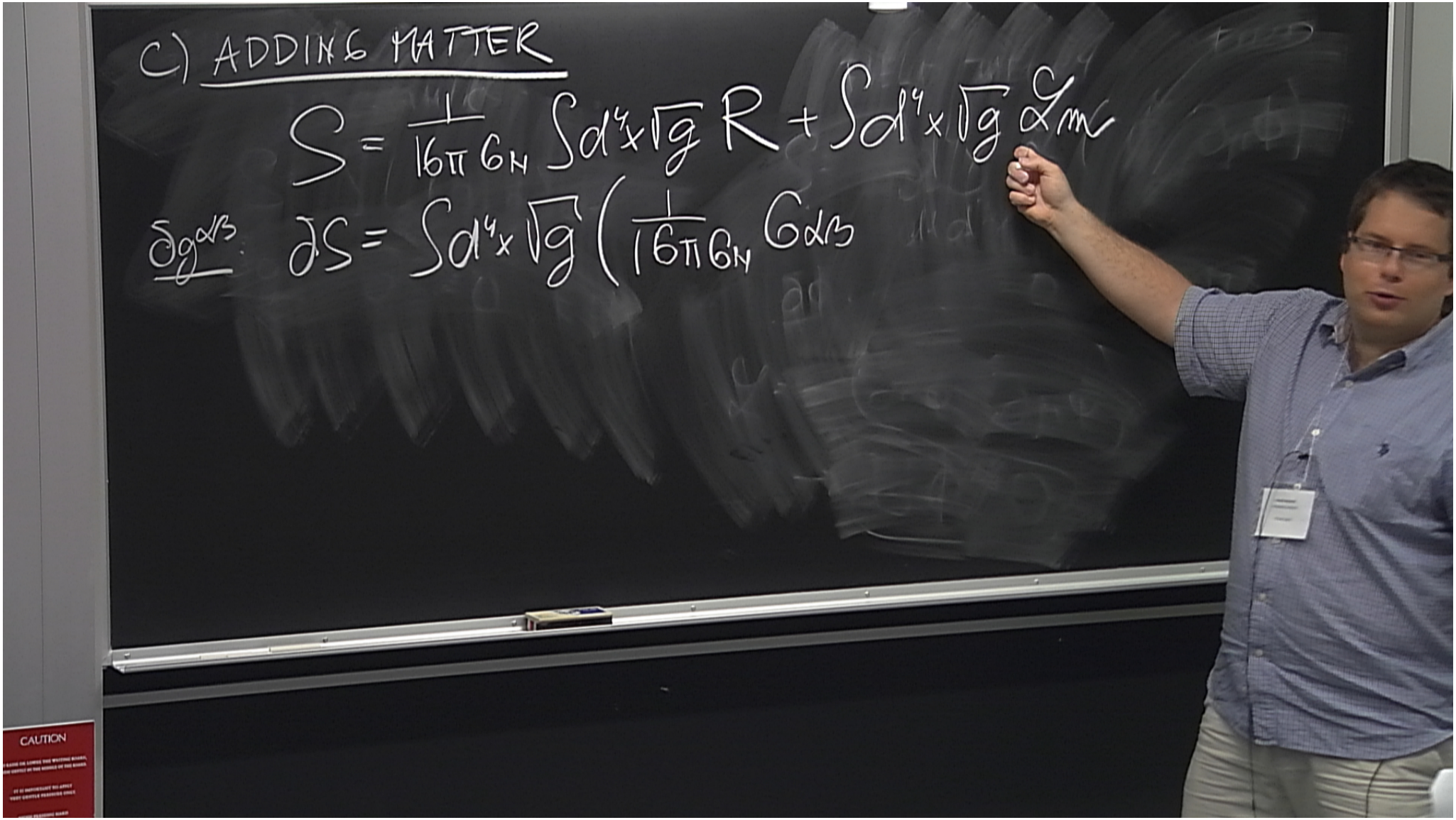
$$\int d^4x \sqrt{g}\, \nabla_\alpha \Lambda^\alpha, \quad \Lambda_\alpha = \nabla^\beta(\delta g_{\beta\alpha}) - g^{\rho\sigma}\nabla_\alpha(\delta g_{\rho\sigma})$$

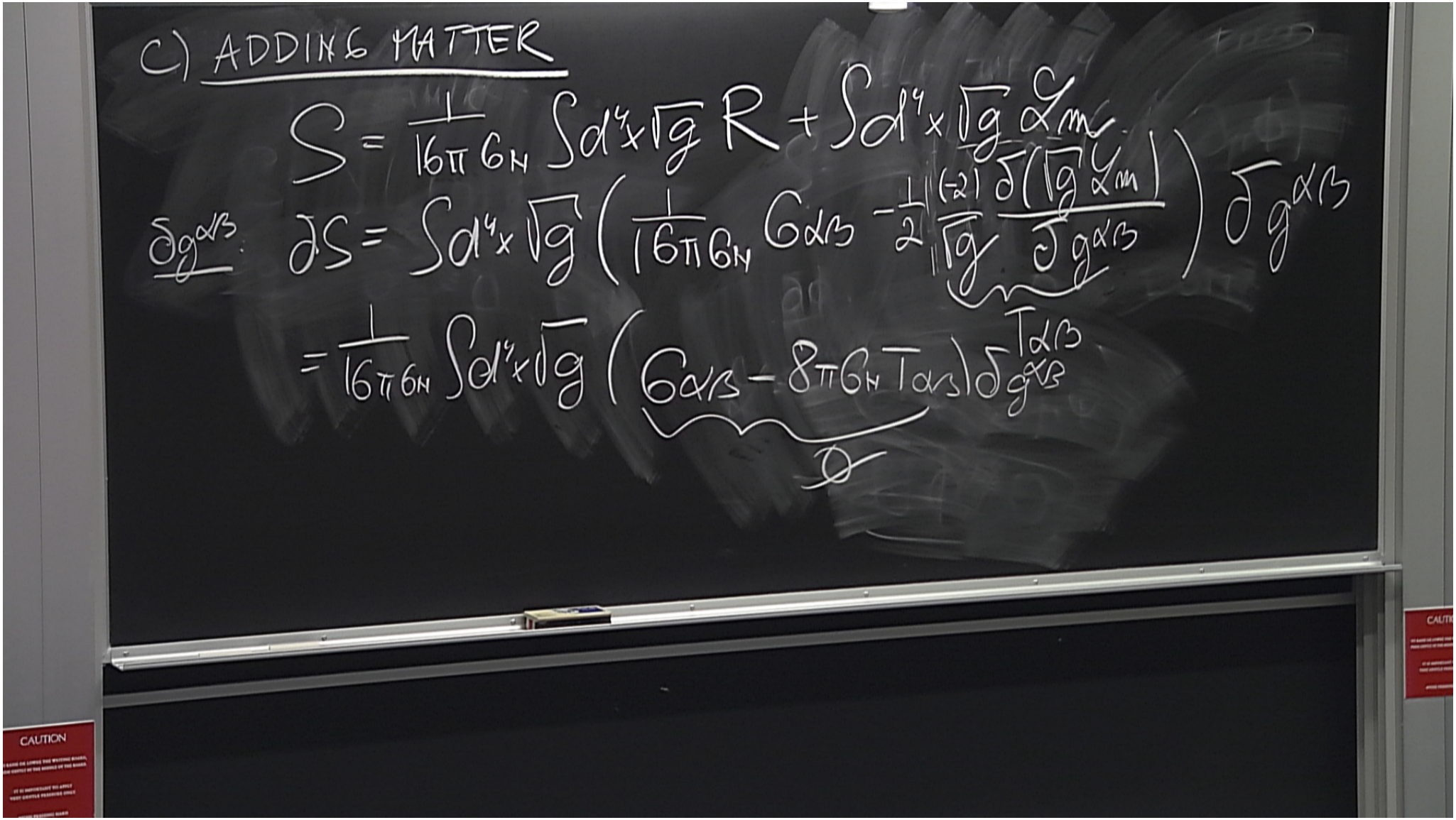


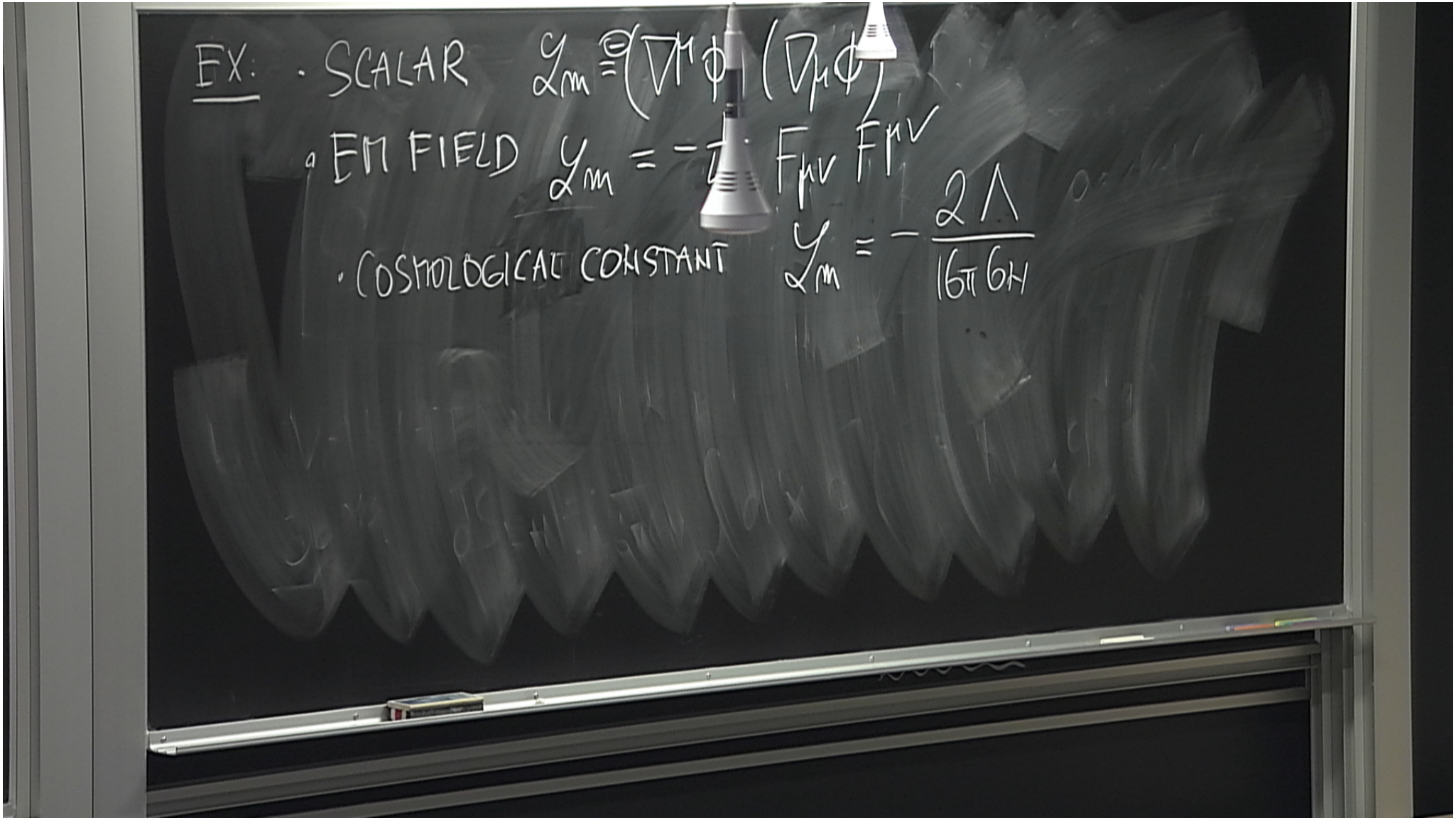
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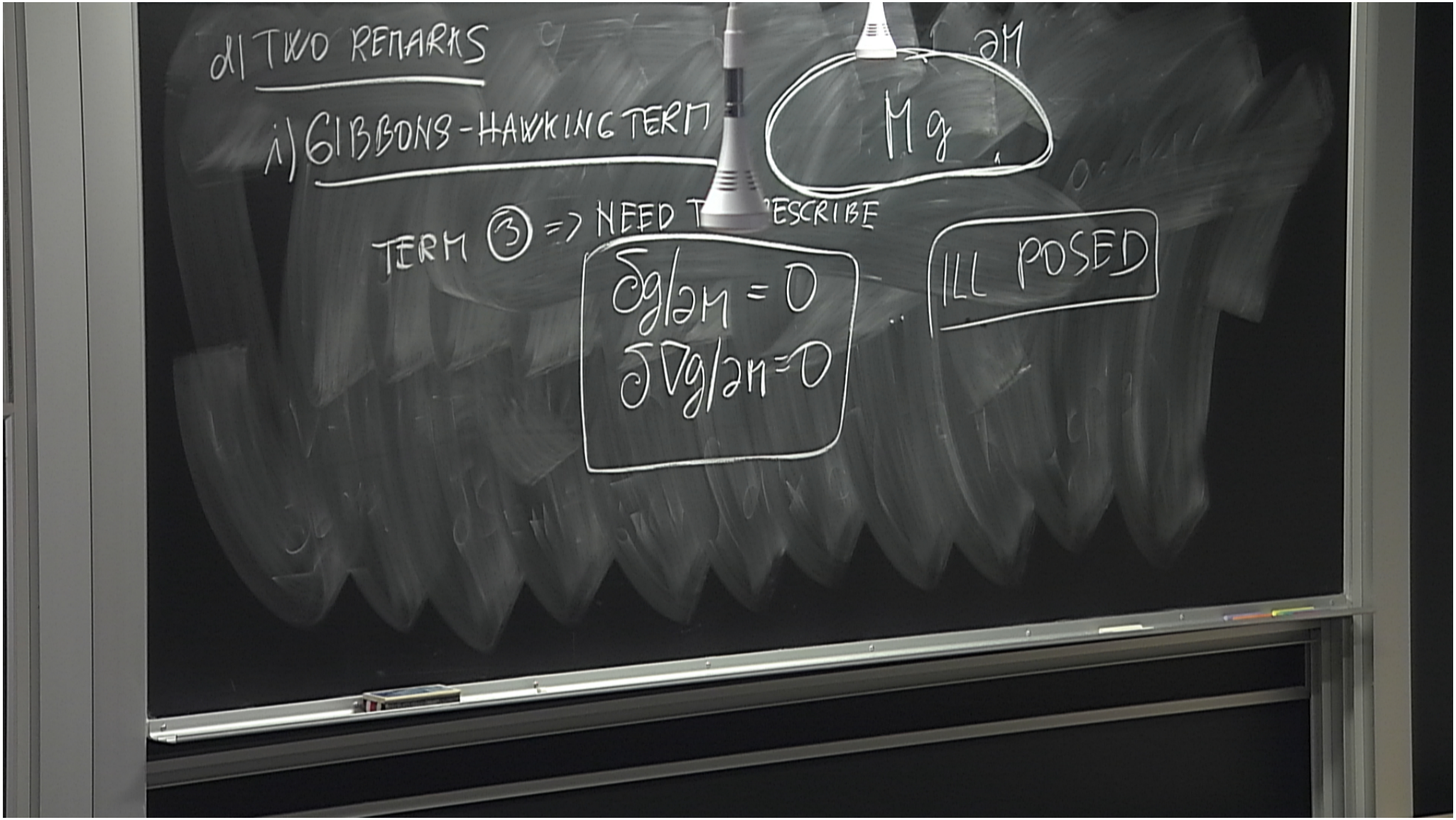
$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

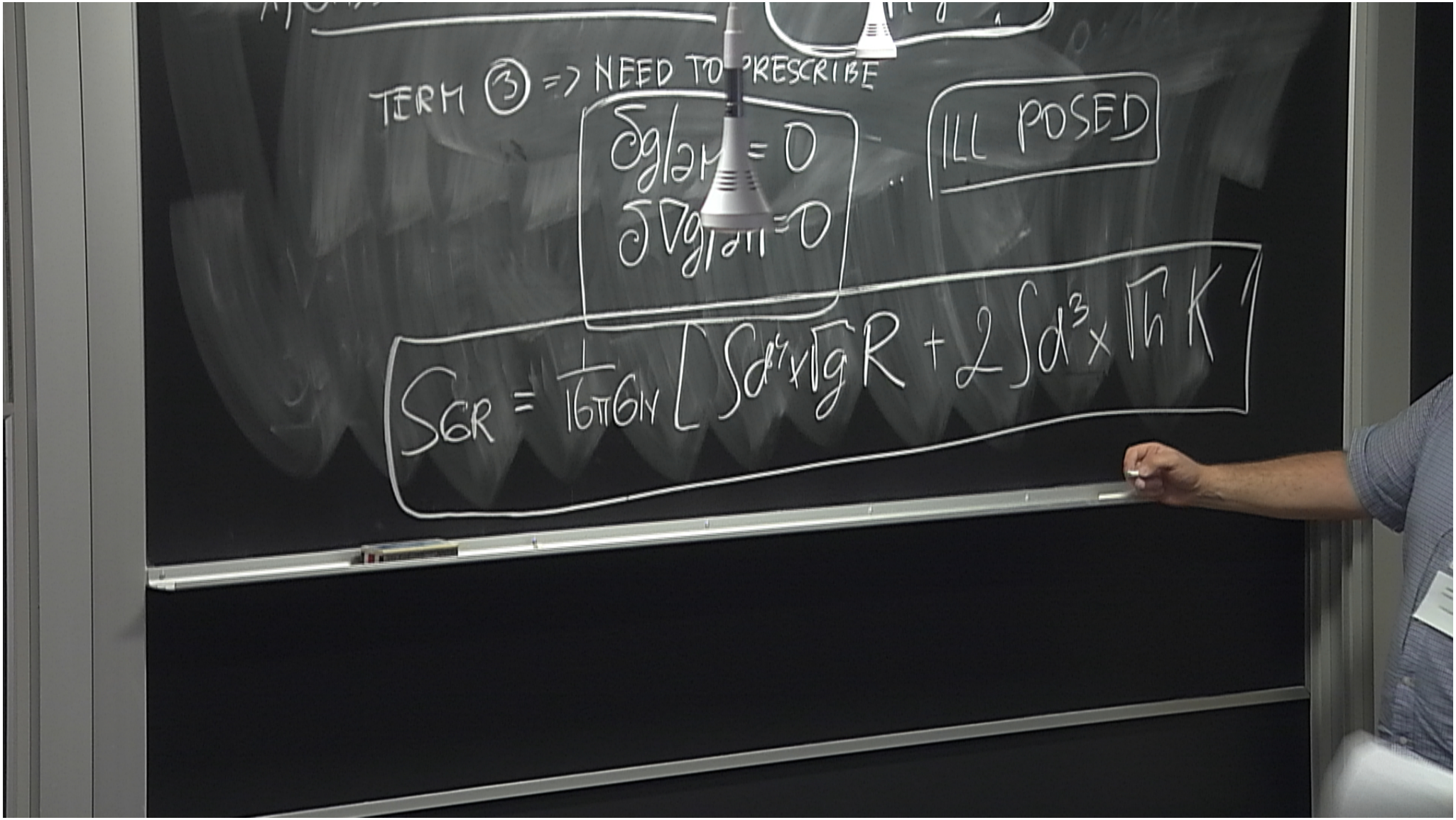
CAUTION

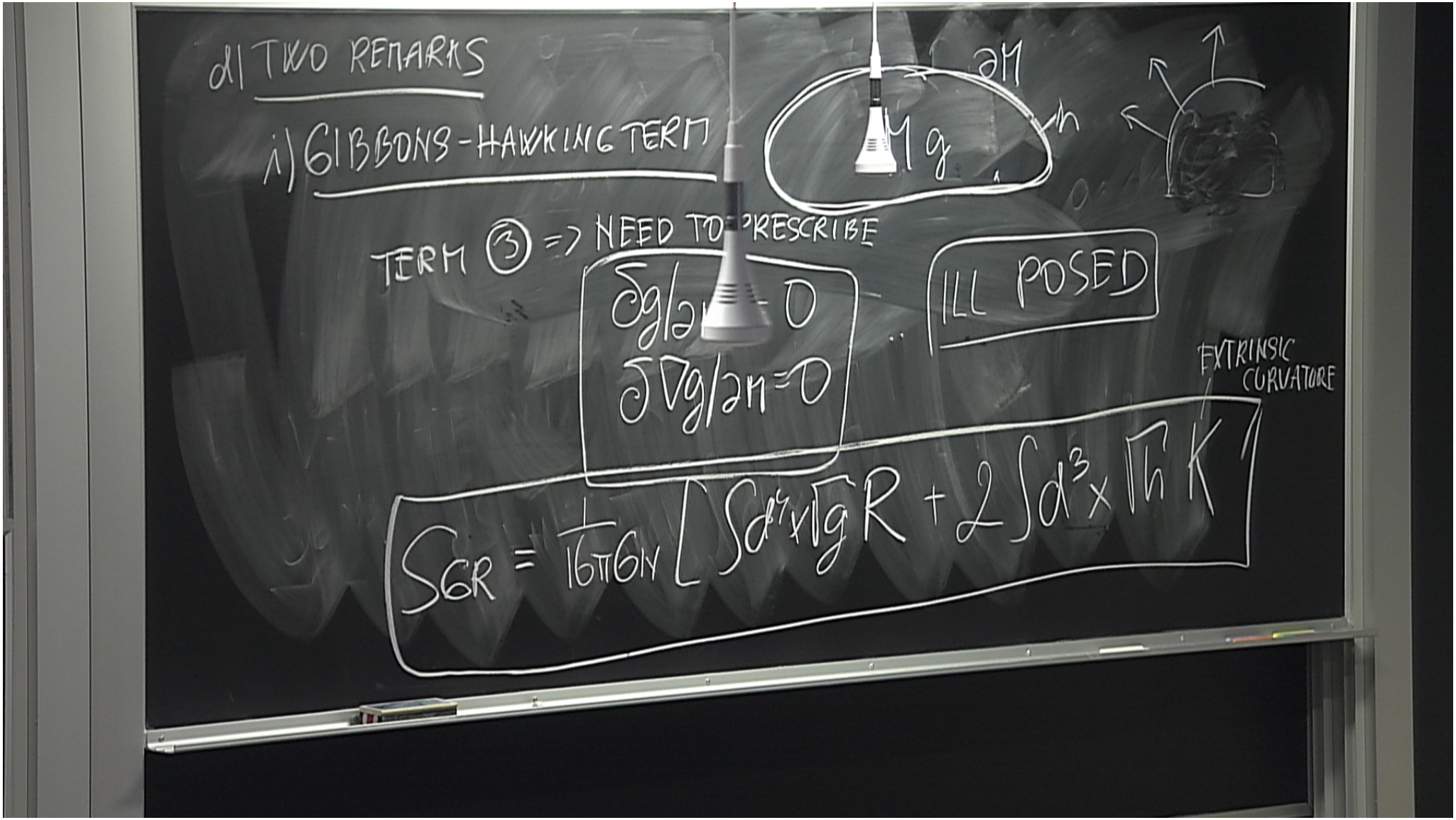

C) ADDING MATTER
$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,R + \int d^4x\sqrt{g}\,\mathcal{L}_m$
$\delta g^{\alpha\beta}$: $\delta S = \int d^4x\sqrt{g}\left(\frac{1}{16\pi G_N}G_{\alpha\beta}\right.$
CAUTION

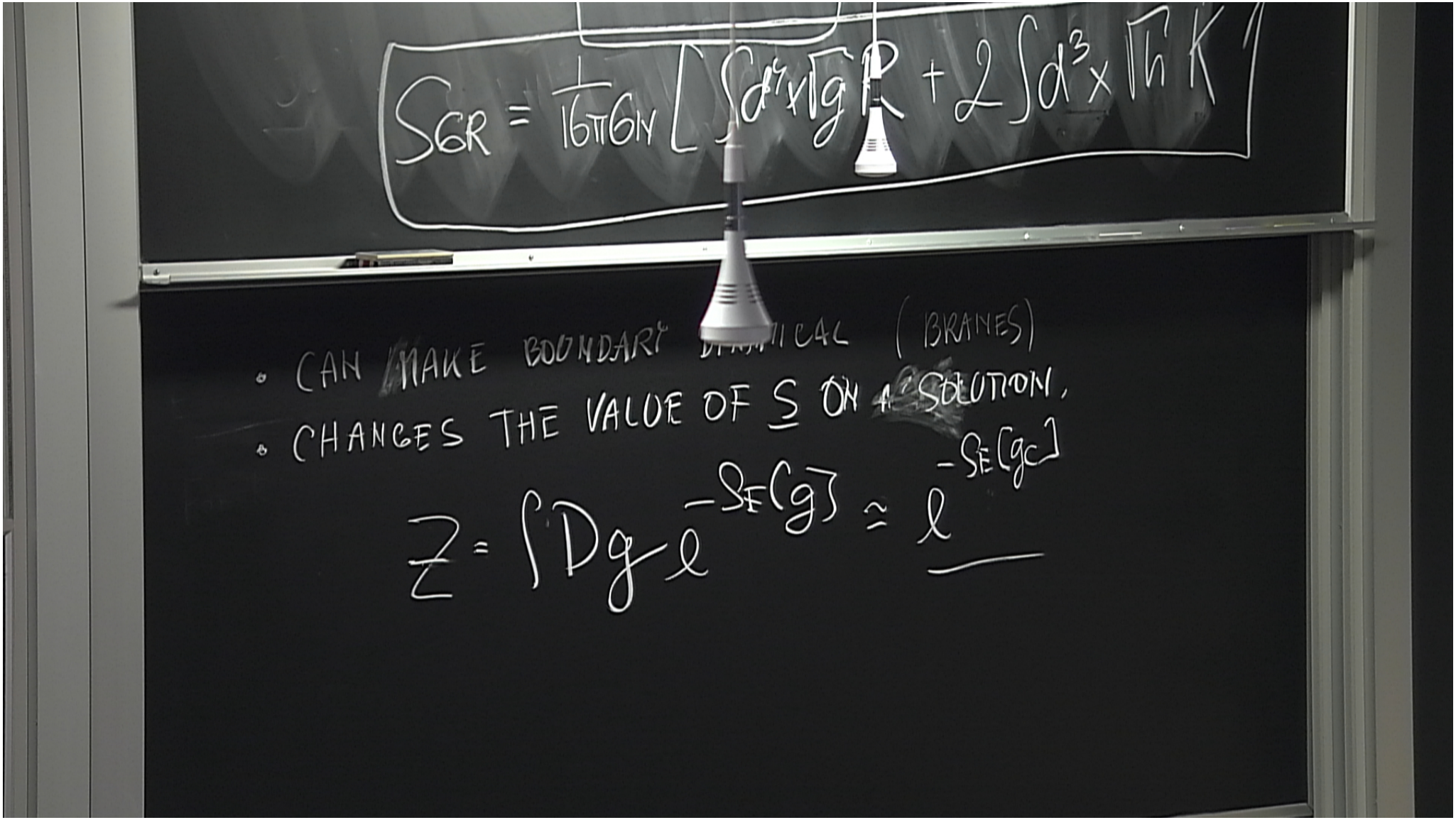

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$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,R + \int d^4x\sqrt{g}\,\mathcal{L}_m$
$\delta g^{\alpha\beta}$: $\delta S = \int d^4x\sqrt{g}\left(\frac{1}{16\pi G_N}G_{\alpha\beta} - \frac{1}{2}\frac{1}{\sqrt{g}}\frac{\delta(\sqrt{g}\mathcal{L}_m)}{\delta g^{\alpha\beta}}\right)\delta g^{\alpha\beta}$
$= \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\left(G_{\alpha\beta} - 8\pi G_N T_{\alpha\beta}\right)\delta g^{\alpha\beta}$
$T_{\alpha\beta}$
0
CAUTION

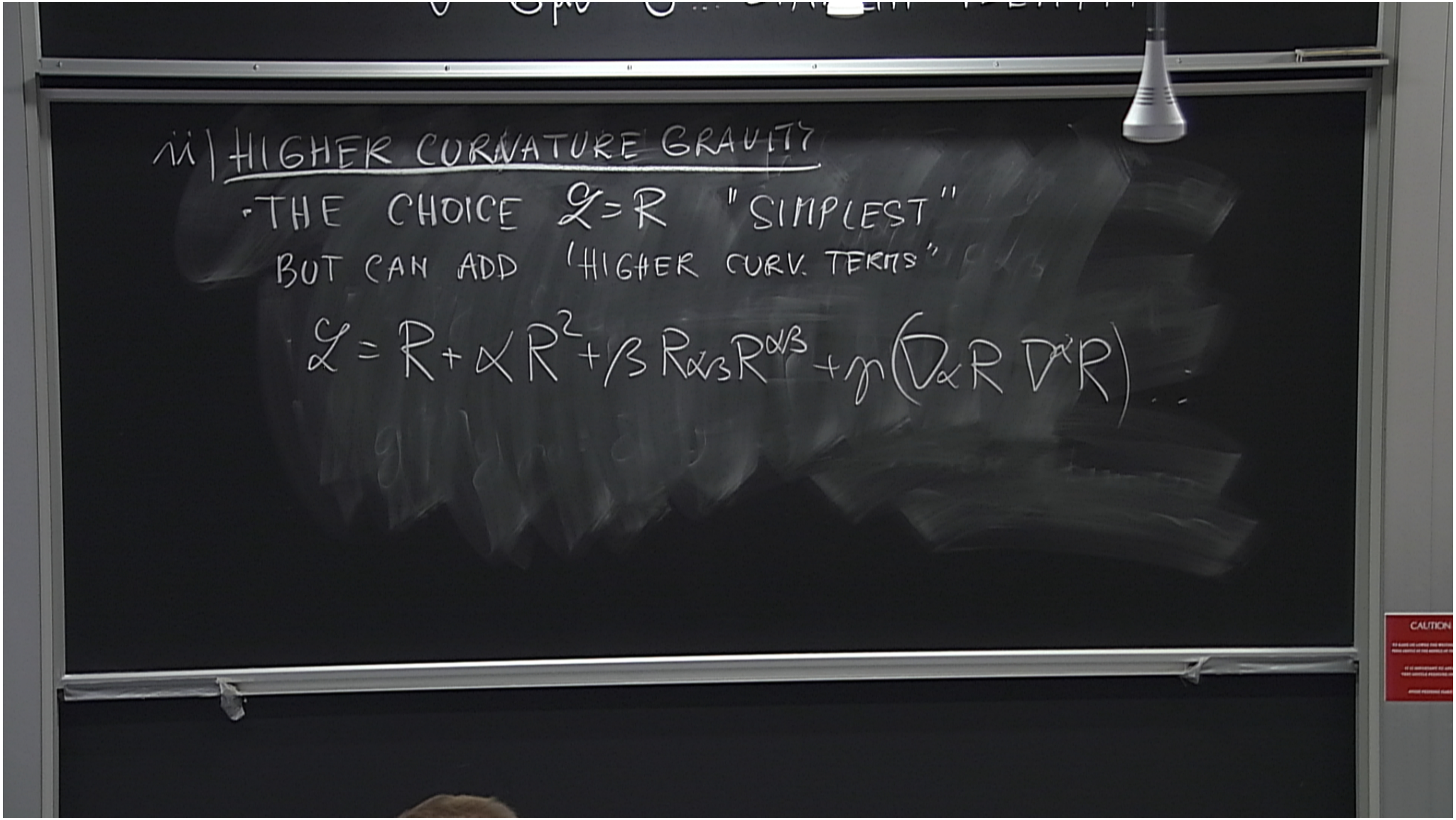

EX: · SCALAR $\mathcal{L}_m = (\nabla^\mu \phi)(D_\mu \phi)$
· EM FIELD $\mathcal{L}_m = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
· COSMOLOGICAL CONSTANT $\mathcal{L}_m = -\frac{2\Lambda}{16\pi G_N}$

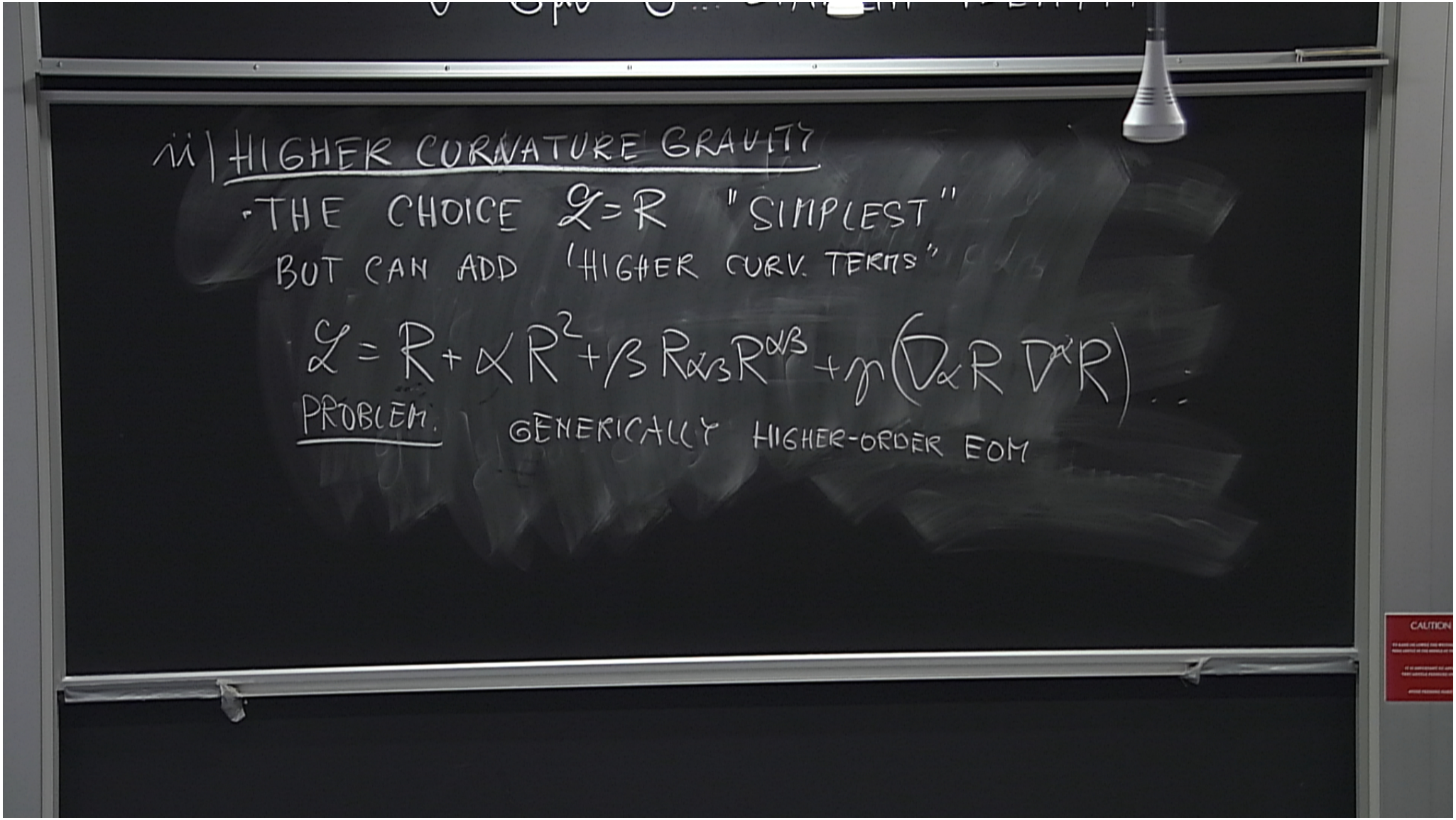

d) TWO REMARKS
i) GIBBONS-HAWKING TERM
TERM (3) => NEED T ESCRIBE
∂g/∂M = 0
∂∇g/∂M = 0
ILL POSED
∂M
Mg

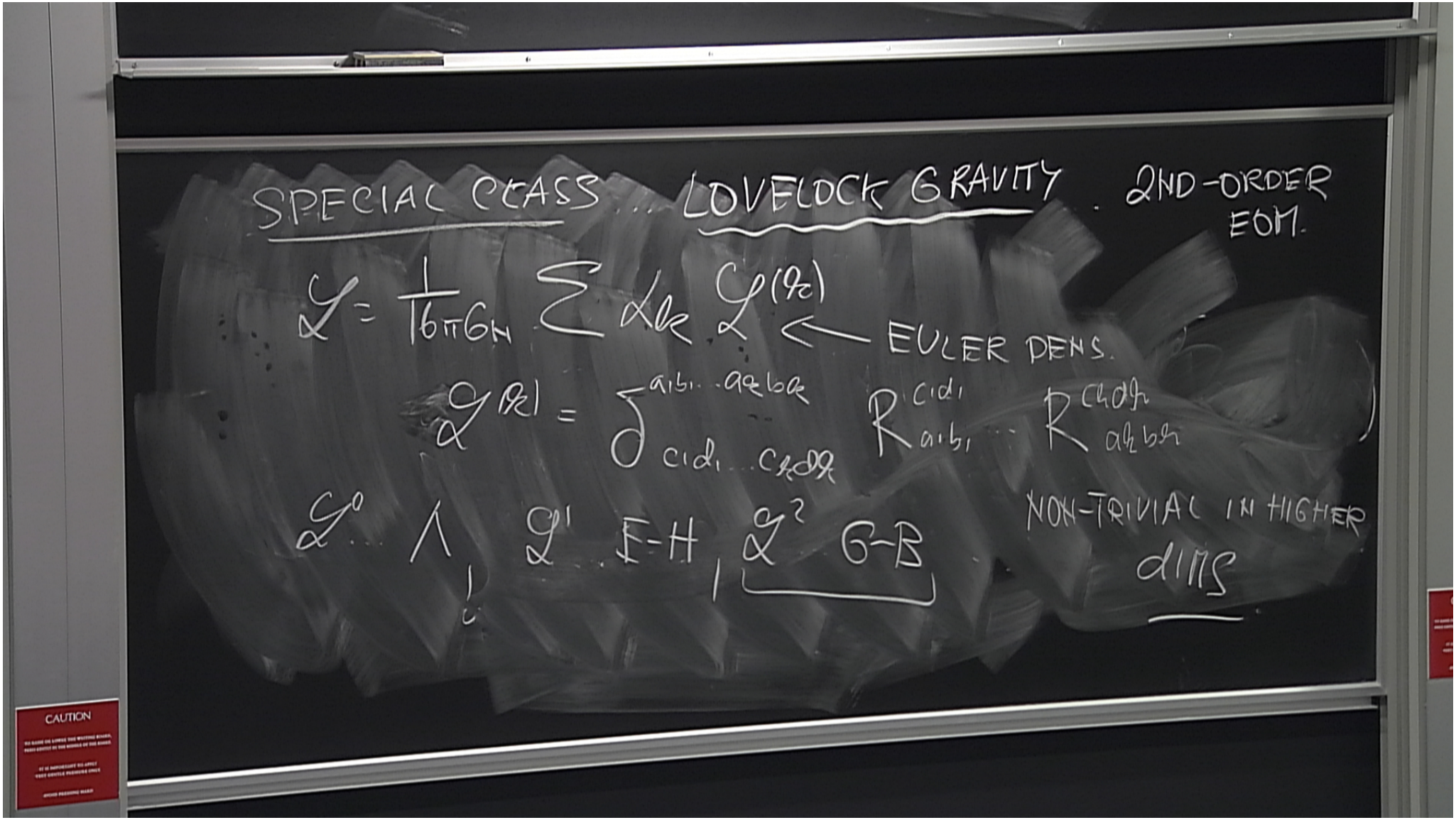

TERM (3) => NEED TO PRESCRIBE
ILL POSED
$S_{GR} = \frac{1}{16\pi G_N}\left[\int d^4x\sqrt{g}\,R + 2\int d^3x\sqrt{h}\,K\right]$


d) TWO REMARKS
i) GIBBONS-HAWKING TERM
TERM (3) => NEED TO PRESCRIBE
ILL POSED
EXTRINSIC CURVATURE


$S_{GR} = \frac{1}{16\pi G_N}\left[\int d^3x\sqrt{g}R + 2\int d^3x\sqrt{h}K\right]$
• CAN MAKE BOUNDARY DYNAMICAL (BRANES)
• CHANGES THE VALUE OF S ON A SOLUTION.
$Z = \int Dg\, e^{-S_E[g]} \simeq e^{-S_E[g_c]}$


iii) HIGHER CURVATURE GRAVITY
• THE CHOICE $\mathcal{L}=R$ "SIMPLEST"
BUT CAN ADD 'HIGHER CURV. TERMS"
$\mathcal{L}=R+\alpha R^2+\beta R_{\alpha\beta}R^{\alpha\beta}+\gamma(\nabla_\alpha R\,\nabla^\alpha R)$
CAUTION


iii) HIGHER CURVATURE GRAVITY
- THE CHOICE $\mathcal{L}=R$ "SIMPLEST"
BUT CAN ADD "HIGHER CURV. TERMS"
$\mathcal{L}=R+\alpha R^2+\beta R_{\alpha\beta}R^{\alpha\beta}+\gamma(\nabla_\alpha R\,\nabla^\alpha R)\ldots$
PROBLEM. GENERICALLY HIGHER-ORDER EOM
CAUTION


SPECIAL CLASS ... LOVELOCK GRAVITY
2ND-ORDER EOM
$\mathcal{L} = \frac{1}{16\pi G_N} \sum \alpha_k \mathcal{L}^{(k)}$
EULER DENS.
$\mathcal{L}^{(k)} = \delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_k d_k} R^{c_1 d_1}_{a_1 b_1} \ldots R^{c_k d_k}_{a_k b_k}$
$\mathcal{L}^0 \ldots \Lambda$
$\mathcal{L}^1 \ldots$ E-H
$\mathcal{L}^2$ G-B
NON-TRIVIAL IN HIGHER dIMS
CAUTION