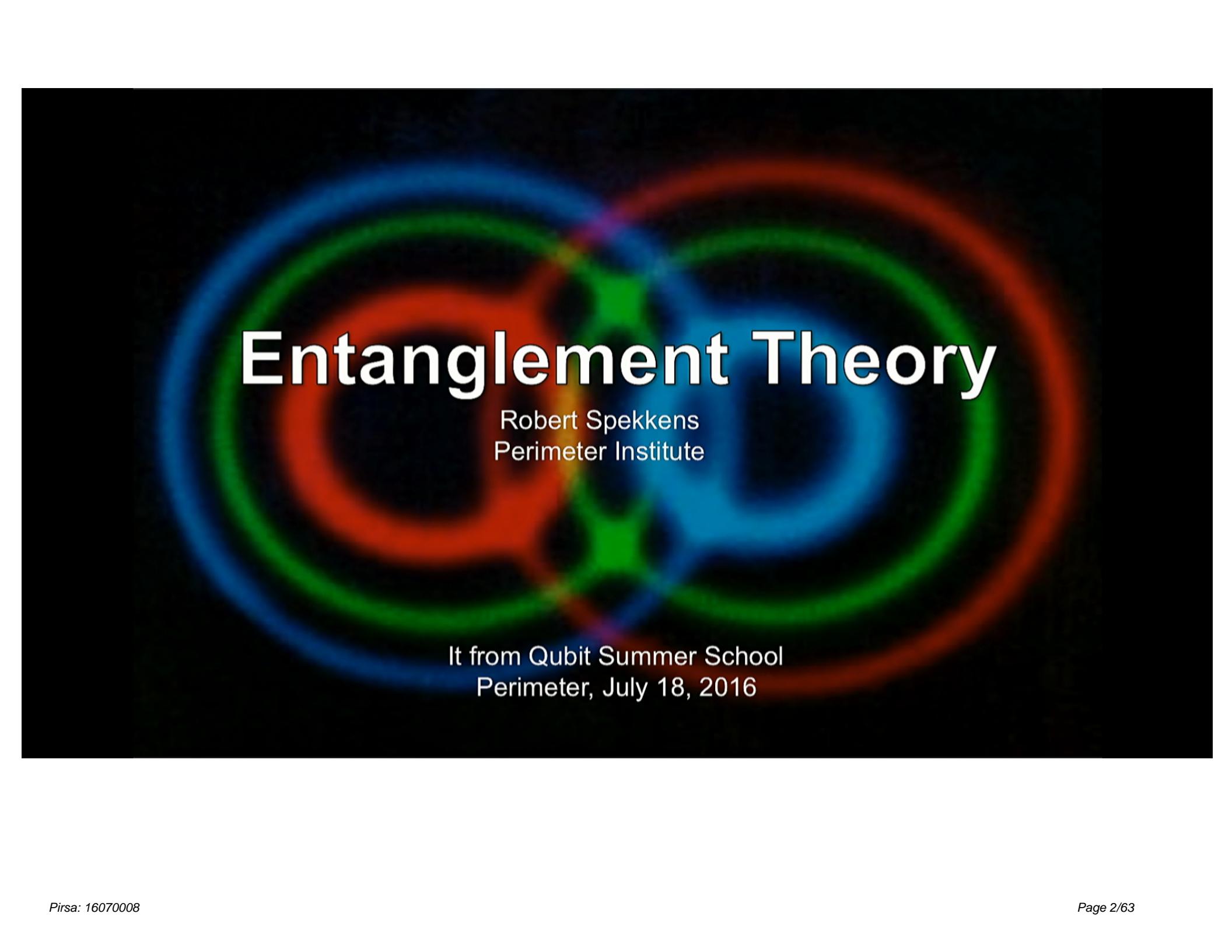


Title: Entanglement

Date: Jul 18, 2016 11:00 AM

URL: <http://pirsa.org/16070008>

Abstract:



Entanglement Theory

Robert Spekkens
Perimeter Institute

It from Qubit Summer School
Perimeter, July 18, 2016

Pure states of a bipartite system are vectors

$$|\Psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$$

Product states

$$|\Psi\rangle_{AB} = |\phi\rangle_A \otimes |\chi\rangle_B \quad \text{for some } \phi \text{ and } \chi$$

Entangled states

$$|\Psi\rangle_{AB} \neq |\phi\rangle_A \otimes |\chi\rangle_B \quad \text{for any } \phi \text{ and } \chi$$

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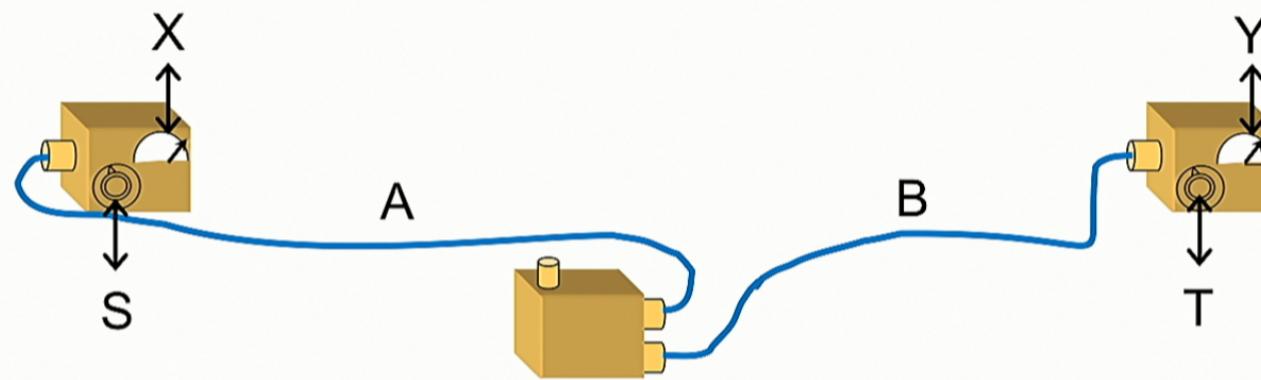
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Entangled states

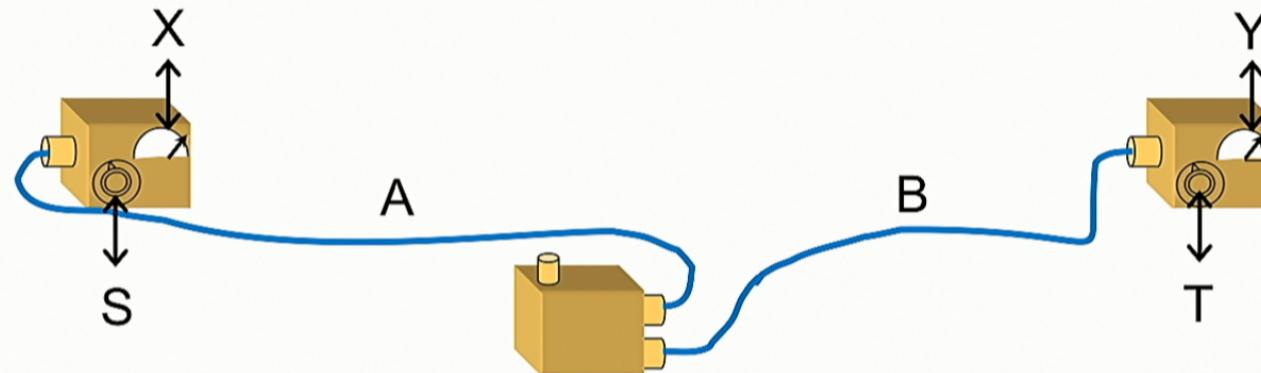
$$|\Psi\rangle_{AB} \neq |\phi\rangle_A \otimes |\chi\rangle_B \quad \text{for any } \phi \text{ and } \chi$$

Example: Two qubits $\mathcal{H}_A = \mathbb{C}_2 \quad \mathcal{H}_B = \mathbb{C}_2$

$$\begin{aligned} |\Phi^+\rangle_{AB} &= \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B) \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \end{aligned}$$



Initially
 $|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$



Set $S=0$

$$\begin{array}{c} \psi_{0,0} \\ \swarrow \\ \psi_{0,1} \end{array}$$

Set $S=1$

$$\begin{array}{c} \psi_{1,0} \\ \uparrow \\ \psi_{1,1} \end{array}$$

Initially

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\rho_B = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}I$$

Upon measuring S and learning outcome X

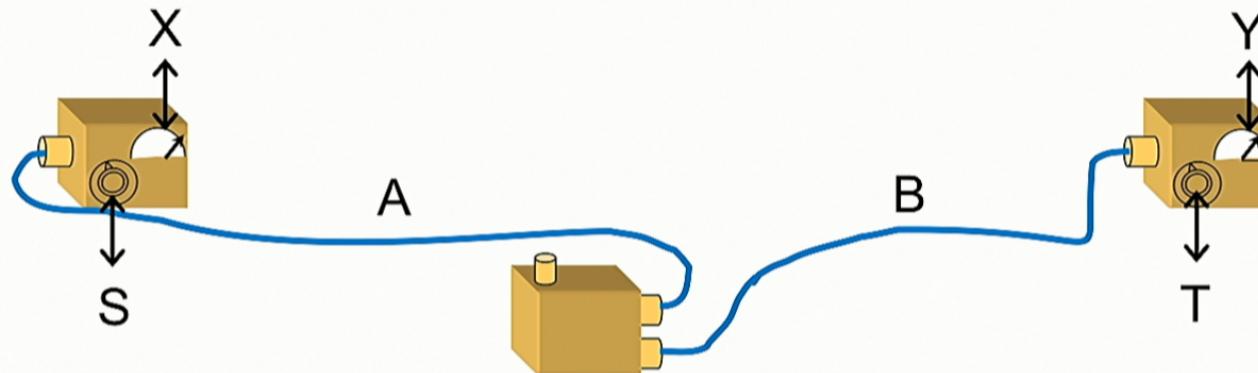
$$\rho_B = |\psi_{S,X}\rangle\langle\psi_{S,X}|$$

$$\alpha, \beta \in \mathbb{R}$$

$$(\alpha\langle 0|_A + \beta\langle 1|_A)|\Phi^+\rangle_{AB}$$

$$= (\alpha\langle 0|_A + \beta\langle 1|_A)(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$= \alpha|0\rangle_B + \beta|1\rangle_B$$



Set **S=0**

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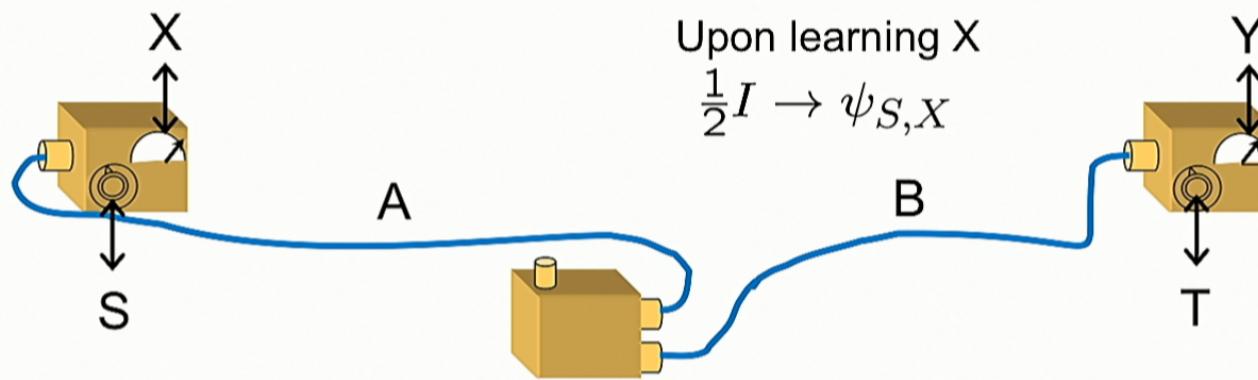
$$\rho_B = |\psi_{S,X}\rangle\langle\psi_{S,X}|$$

Upon measuring S and **not** learning outcome X

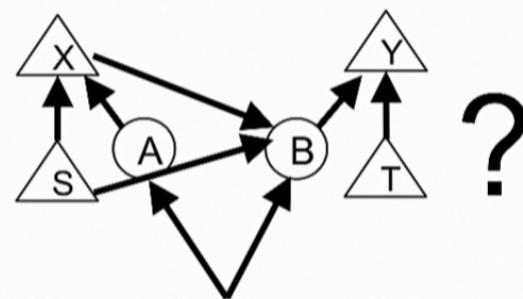
$$\rho_B = \frac{1}{2}|\psi_{S,0}\rangle\langle\psi_{S,0}| + \frac{1}{2}|\psi_{S,1}\rangle\langle\psi_{S,1}|$$

$$\frac{1}{2}|\psi_{0,0}\rangle\langle\psi_{0,0}| + \frac{1}{2}|\psi_{0,1}\rangle\langle\psi_{0,1}|$$

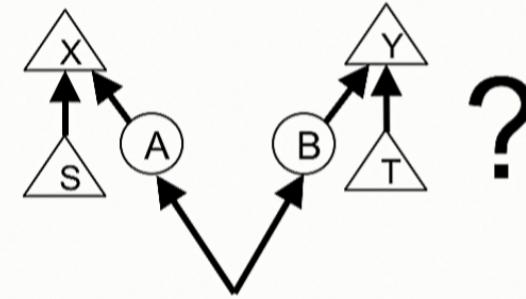
$$= \frac{1}{2}|\psi_{1,0}\rangle\langle\psi_{1,0}| + \frac{1}{2}|\psi_{1,1}\rangle\langle\psi_{1,1}| = \frac{1}{2}I$$



$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



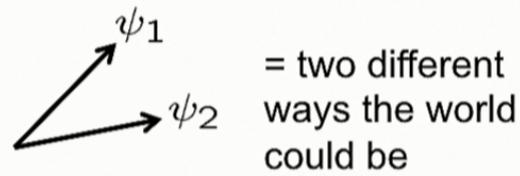
nonlocal action



local action

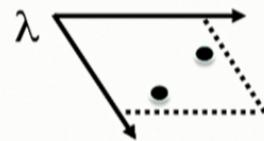
“... the change in the psi-function through observation then does not correspond essentially to the change in a real matter of fact but rather to the alteration in *our knowledge* of this matter of fact.” [emphasis in original]
---Einstein, 1948 letter to Heitler

ψ is ontic



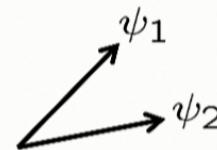
= two different ways the world could be

Example:



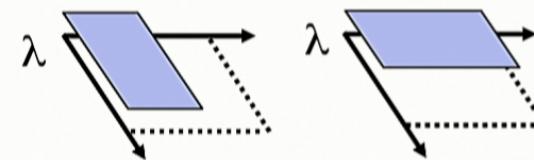
ψ is epistemic

VS.

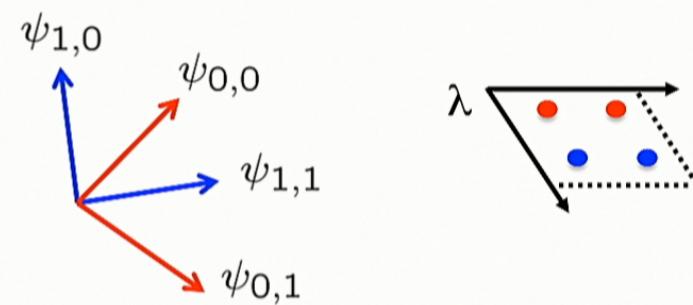


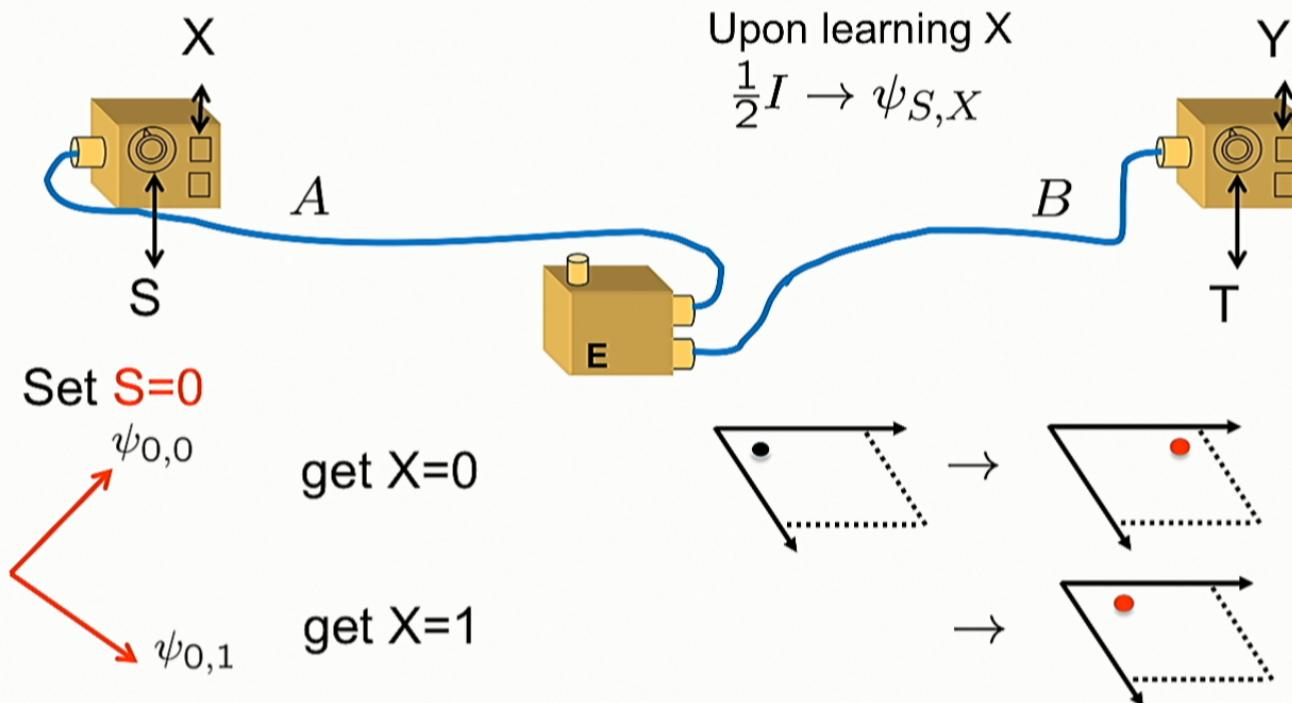
= two different ways of having incomplete information about the world

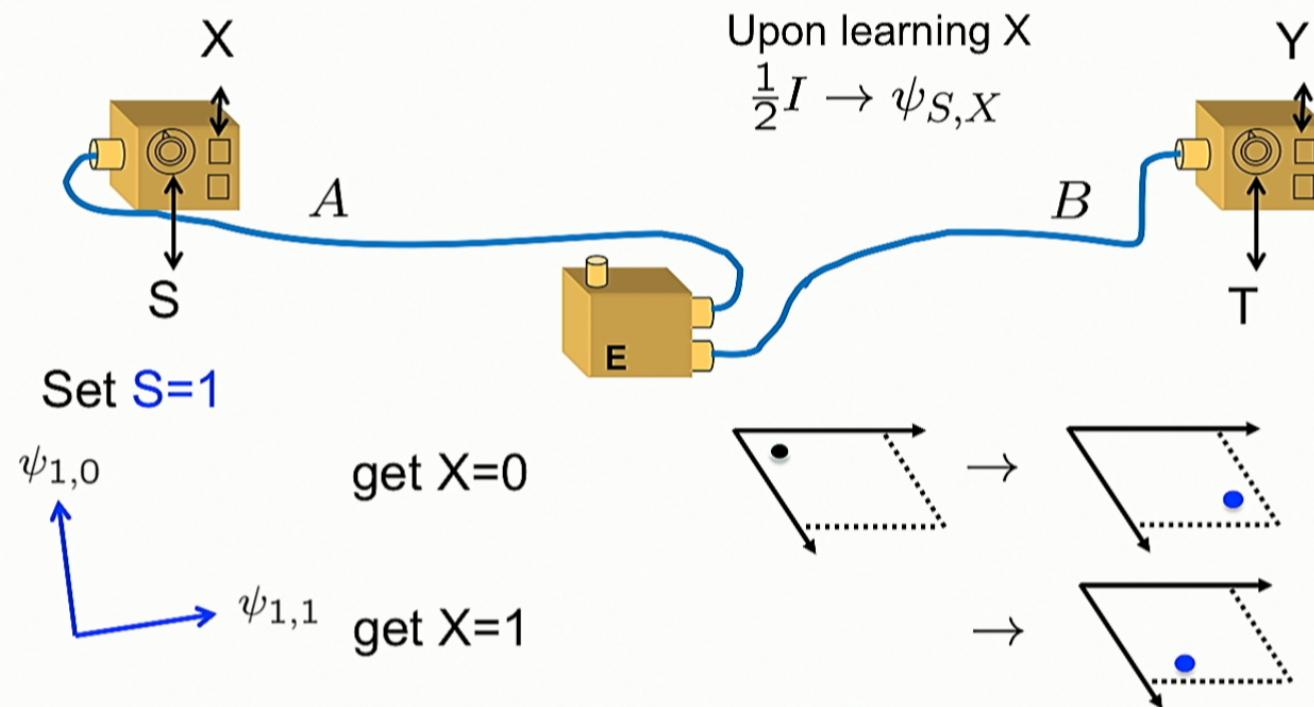
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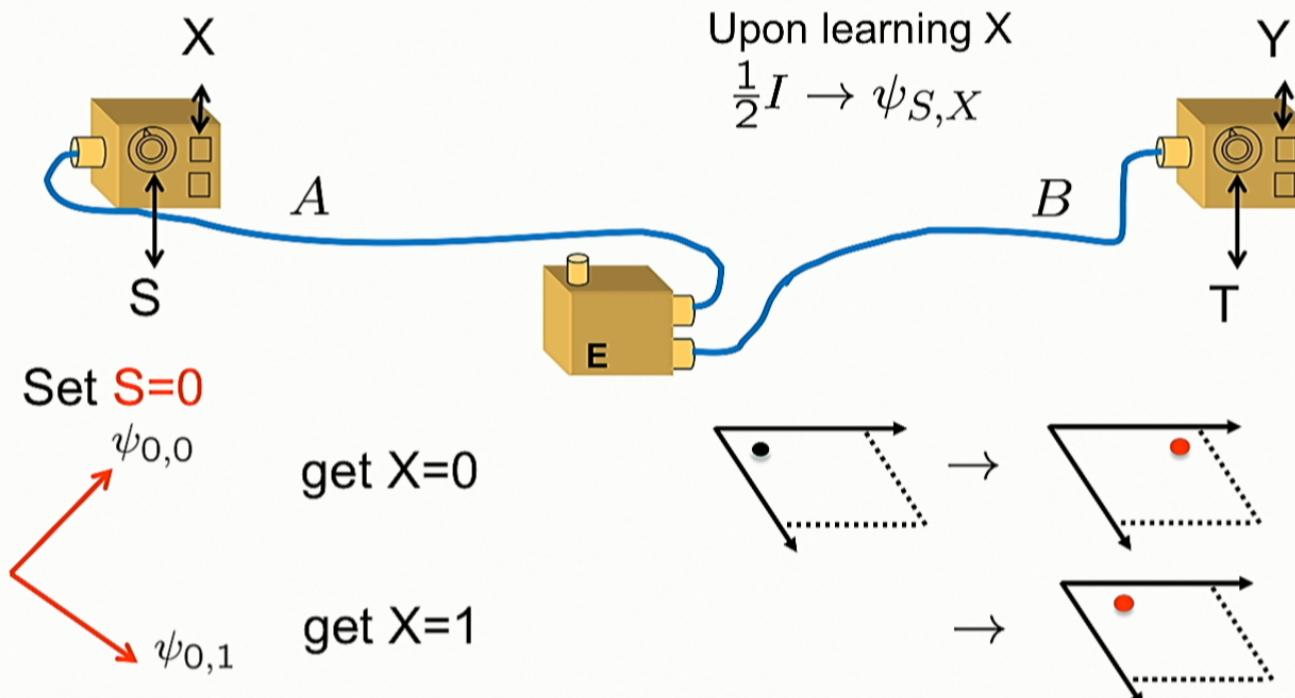


ψ is ontic

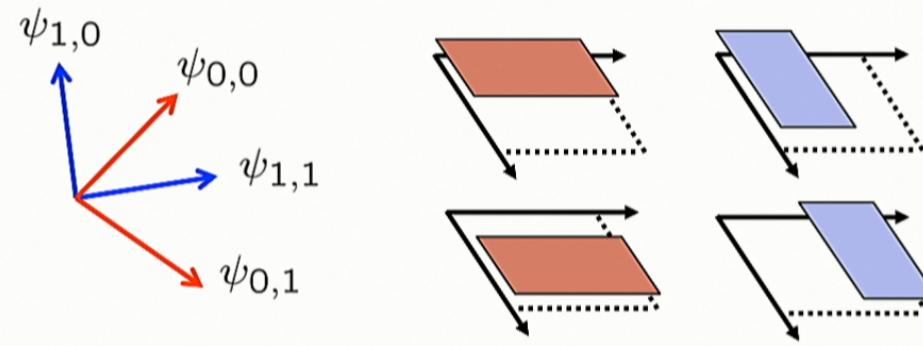


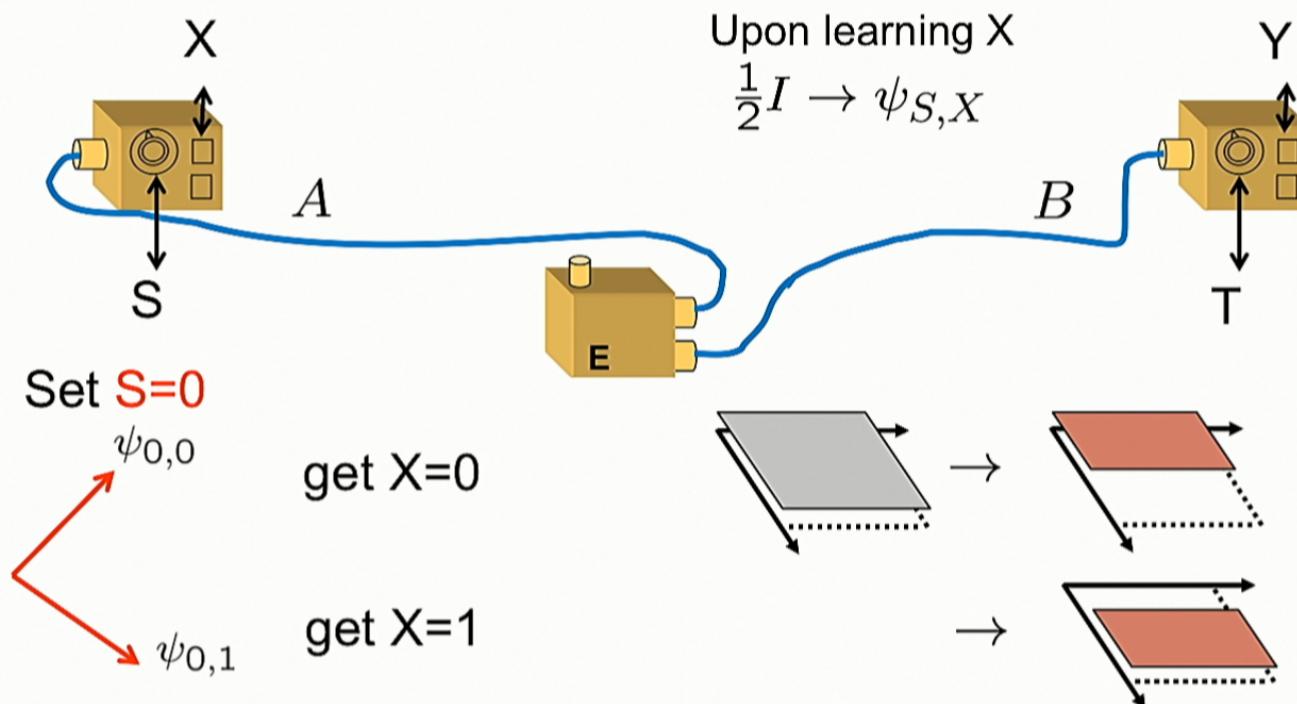


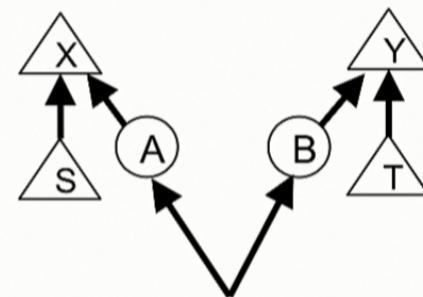
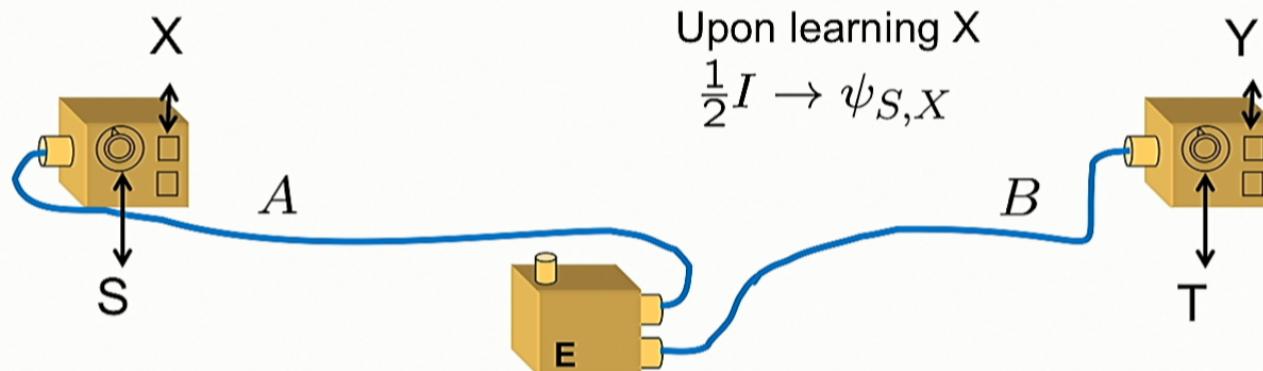




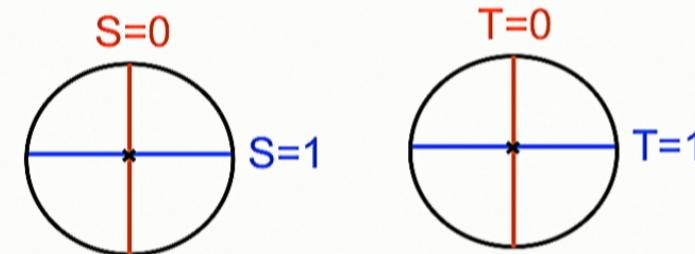
ψ is epistemic



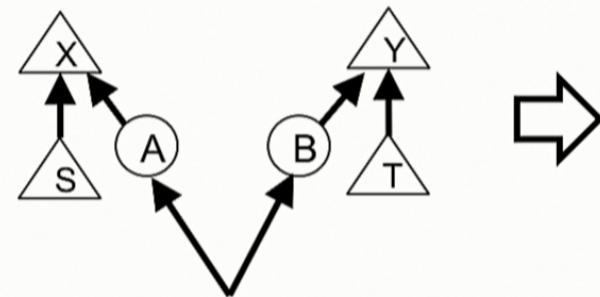
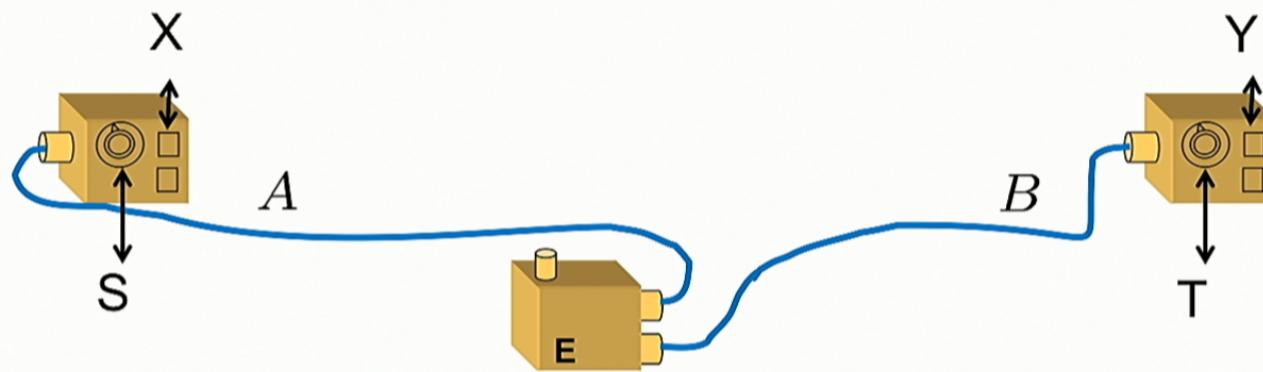




In particular, a local model exists for the sorts of measurements in EPR



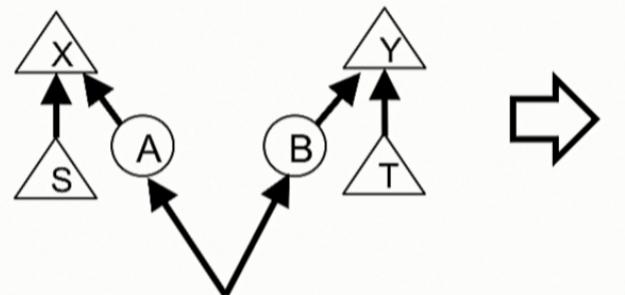
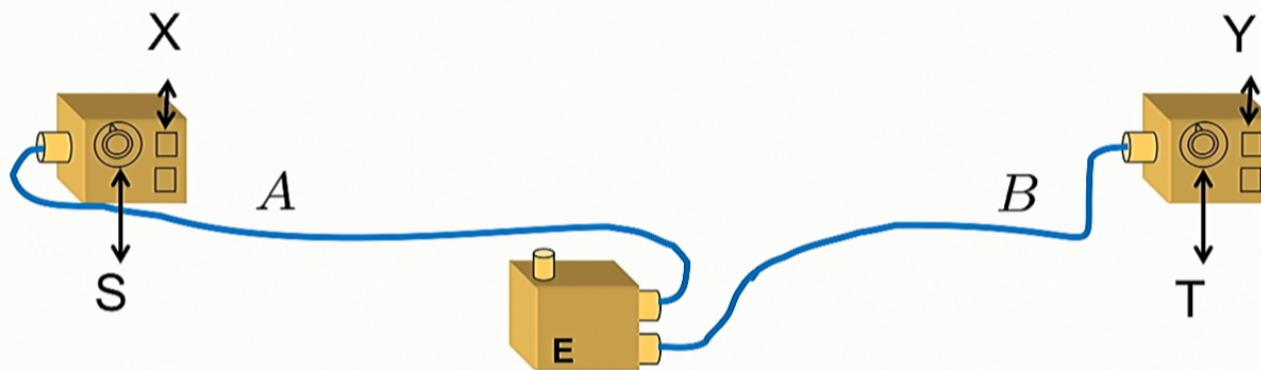
Bell's theorem



Clauser-Horne-Shimony-Holt inequality

$$\frac{1}{4}p(\text{agree}|00) + \frac{1}{4}p(\text{agree}|01)$$

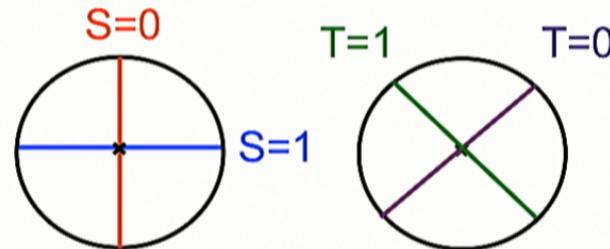
$$+ \frac{1}{4}p(\text{agree}|10) + \frac{1}{4}p(\text{disagree}|11) \leq \frac{3}{4}$$



Clauser-Horne-Shimony-Hole inequality

$$\frac{1}{4}p(\text{agree}|00) + \frac{1}{4}p(\text{agree}|01)$$

$$+ \frac{1}{4}p(\text{agree}|10) + \frac{1}{4}p(\text{disagree}|11) \leq \frac{3}{4}$$



$$p(\text{agree}|00) = p(\text{agree}|01)$$

$$= p(\text{agree}|10) = p(\text{disagree}|11)$$

$$= \cos^2(\pi/8) = \frac{1}{2} + \frac{1}{2\sqrt{2}} \quad \text{Violates CHSH}$$

Violation of a Bell inequality is one of our best notions of nonclassicality

Entanglement is *necessary* for a Bell inequality violation

Nonclassicality= Entanglement ?

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Problem: entanglement is *not sufficient* for Bell inequality violation

Violation of a Bell inequality is one of our best notions of nonclassicality

Entanglement is *necessary* for a Bell inequality violation

Nonclassicality= Entanglement ?

Problem: entanglement is *not sufficient* for Bell inequality violation

For many quantum-over-classical advantages in information processing, entanglement is also necessary but not sufficient

1935 Einstein Podolsky and Rosen argument

1964 Bell's theorem

1991 Ekert's protocol for key distribution based on Bell

1992 Bennett and Wiesner's superdense coding scheme

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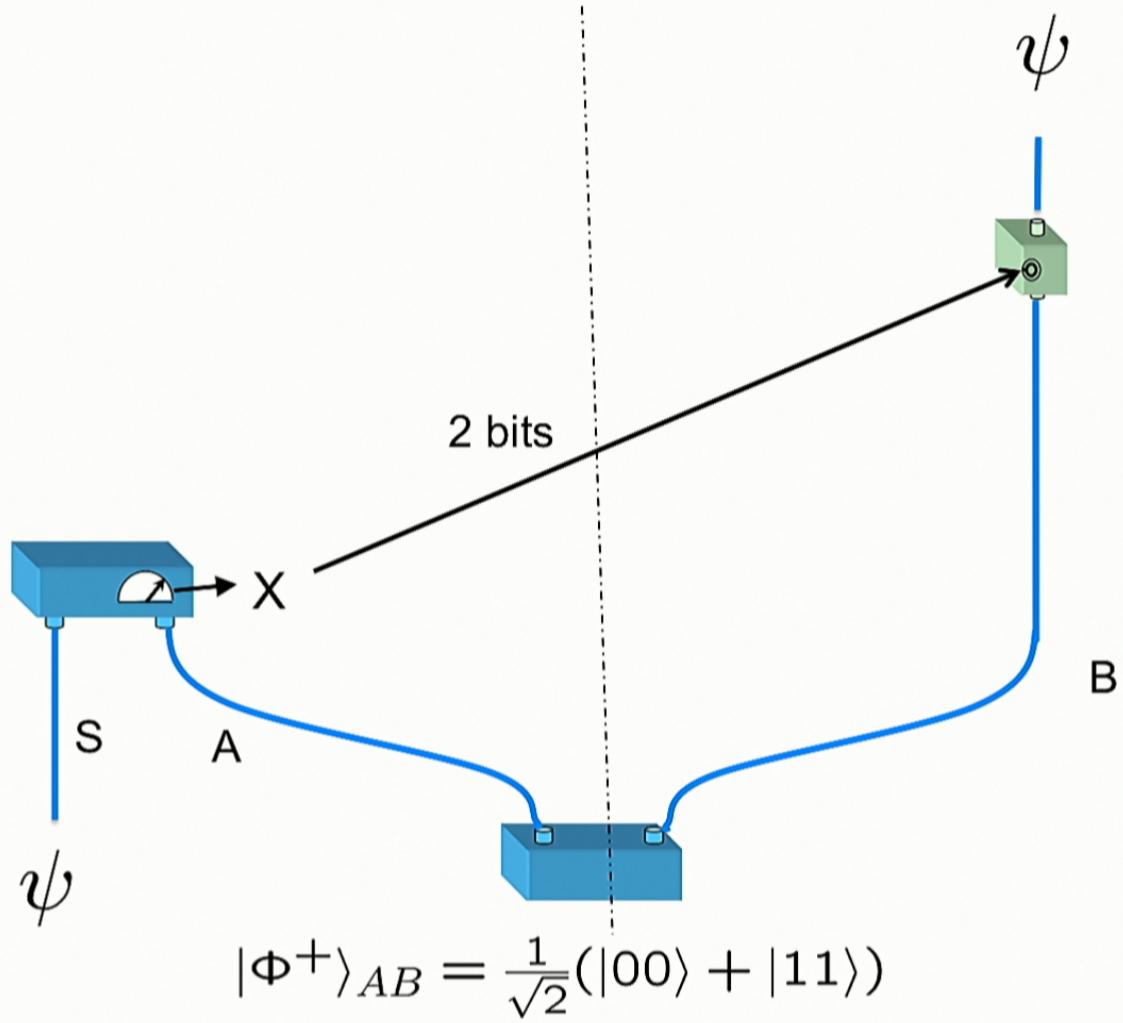
1964 Bell's theorem

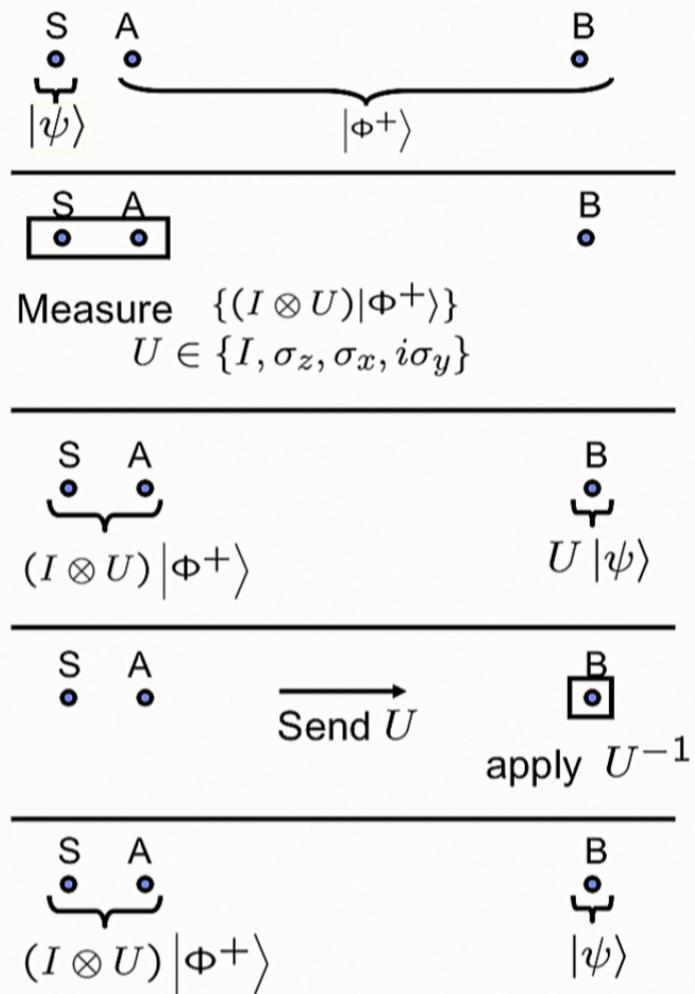
1991 Ekert's protocol for key distribution based on Bell

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Since: Entanglement is a resource for information processing!





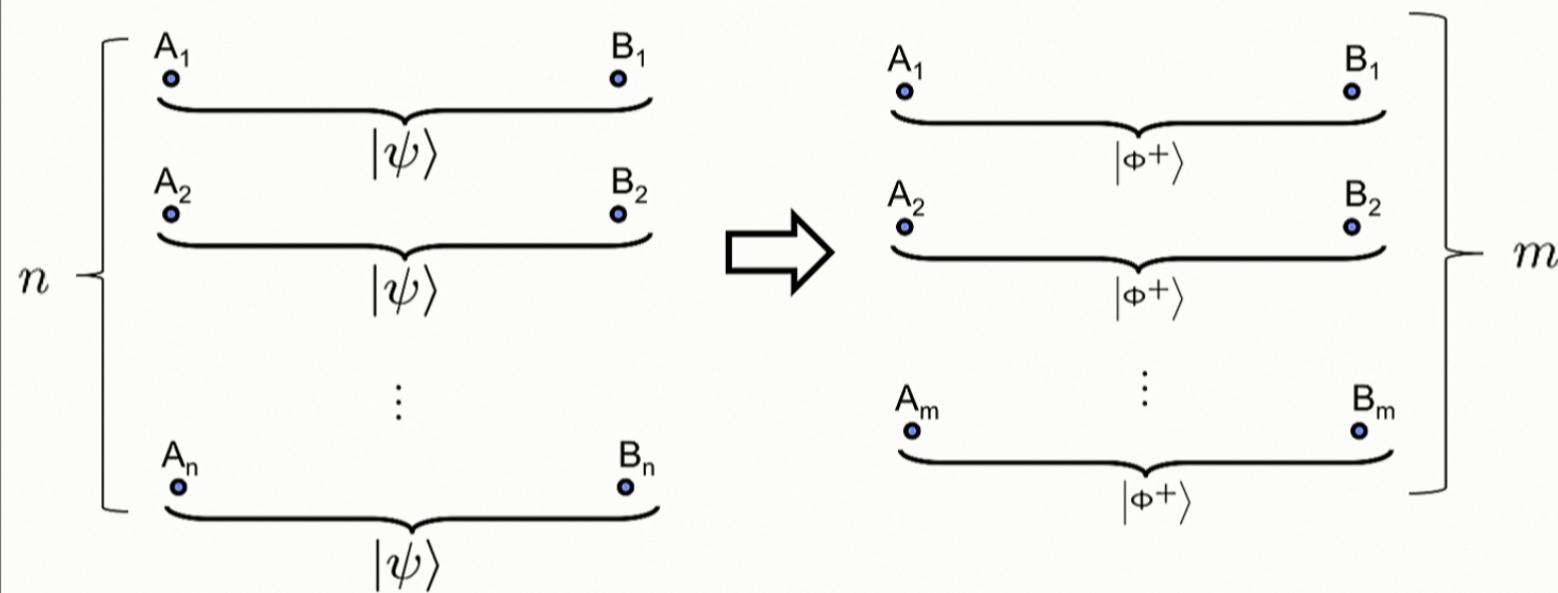
$$\begin{aligned}
 |\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \\
 |\Phi^+\rangle &= |0\rangle|0\rangle + |1\rangle|1\rangle \\
 &= (I \otimes I)|\Phi^+\rangle \\
 |\Phi^-\rangle &= |0\rangle|0\rangle - |1\rangle|1\rangle \\
 &= (I \otimes \sigma_z)|\Phi^+\rangle \\
 |\Psi^+\rangle &= |0\rangle|1\rangle + |1\rangle|0\rangle \\
 &= (I \otimes \sigma_x)|\Phi^+\rangle \\
 |\Psi^-\rangle &= |0\rangle|1\rangle - |1\rangle|0\rangle \\
 &= (I \otimes i\sigma_y)|\Phi^+\rangle
 \end{aligned}$$

The state after meas't is easily determined by noting that

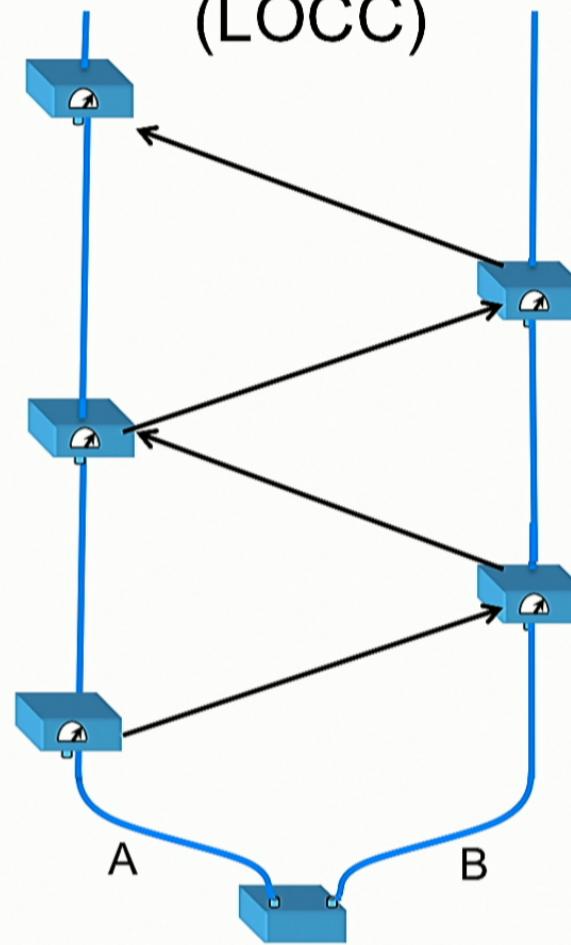
$$\begin{aligned}
 |\psi\rangle|\Phi^+\rangle &= |\Phi^+|\psi\rangle + |\Phi^-)(\sigma_z|\psi\rangle) \\
 &\quad + |\Psi^+)(\sigma_x|\psi\rangle) + |\Psi^-)(i\sigma_y|\psi\rangle)
 \end{aligned}$$

How many qubits can be teleported using a given bipartite pure state?

Rate for $|\psi\rangle_{AB}^{\otimes n} \rightarrow |\Phi^+\rangle_{AB}^{\otimes m}$



Local Operations and Classical Communicaiton (LOCC)



How many qubits can be teleported using a given bipartite pure state?

Rate for $|\psi\rangle_{AB}^{\otimes n} \xrightarrow{LOCC} |\Phi^+\rangle_{AB}^{\otimes m}$

$$R(\psi \rightarrow \Phi^+) = \sup_{\mathcal{E} \in \text{LOCC}} \left\{ \frac{m}{n} : \lim_{n \rightarrow \infty} \|\mathcal{E}(\psi^{\otimes n}) - (\Phi^+)^{\otimes m}\|_1 = 0 \right\}$$

$$\|A\|_1 \equiv \text{tr}(\sqrt{A^\dagger A})$$

Distillable entanglement

$$R(\psi \rightarrow \Phi^+) = -\text{Tr}(\rho_\psi \log \rho_\psi)$$

von Neumann entropy of
reduced density operator of ψ
= entanglement entropy of ψ

Teleportation is just one way in which the resource of entanglement can be used as a resource under the restriction of LOCC

Other measures of entanglement quantify the degree of success for other tasks.

Abstractly, a measure of entanglement can be **defined** as any function over quantum states which cannot be increased by LOCC

The states that are freely preparable under LOCC:

$$\rho_{AB} = \sum_i p_i \sigma_i^A \otimes \omega_i^B$$

“Separable states”

The states that are nontrivial resources under LOCC:

$$\rho_{AB} \neq \sum_i p_i \sigma_i^A \otimes \omega_i^B$$

“Entangled states”

Note: LOCC is a **practical**
restriction

---the universe doesn't care about
classical channels

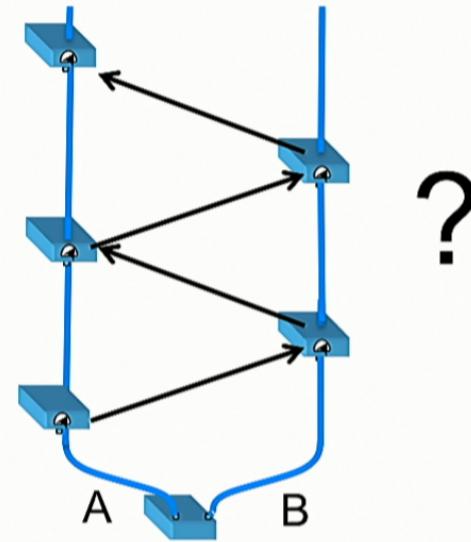
Deterministic state conversion

$$\rho \xrightarrow{LOCC} \sigma \quad \rightleftharpoons \quad \begin{matrix} \text{Conditions on} \\ \rho \text{ and } \sigma \end{matrix}$$

\uparrow

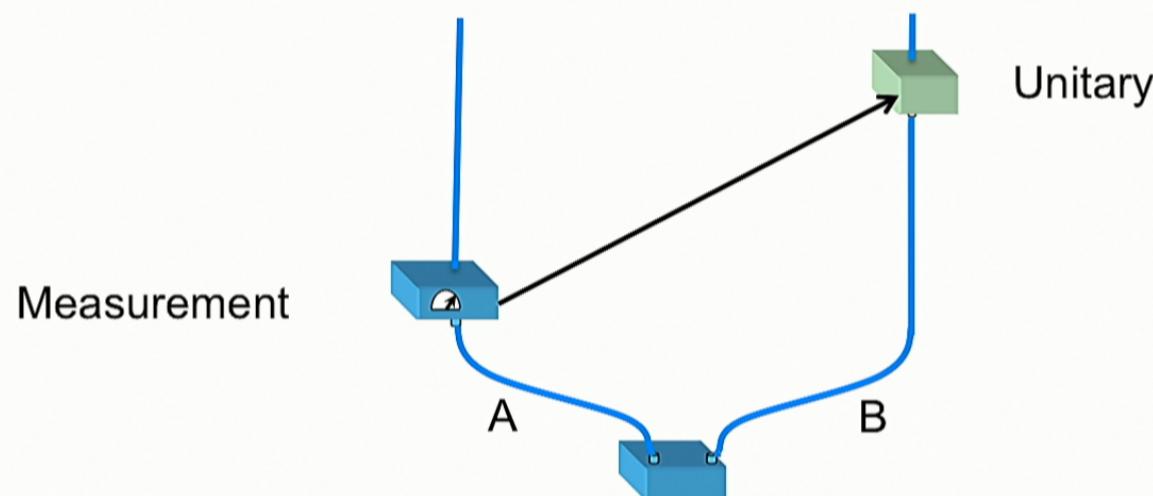
$$\exists \mathcal{E} \in \text{LOCC}$$
$$\mathcal{E}(\rho) = \sigma$$

$$|\chi\rangle_{AB} = \cos\theta|00\rangle + \sin\theta|11\rangle$$



$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\chi\rangle_{AB} = \cos\theta|00\rangle + \sin\theta|11\rangle$$



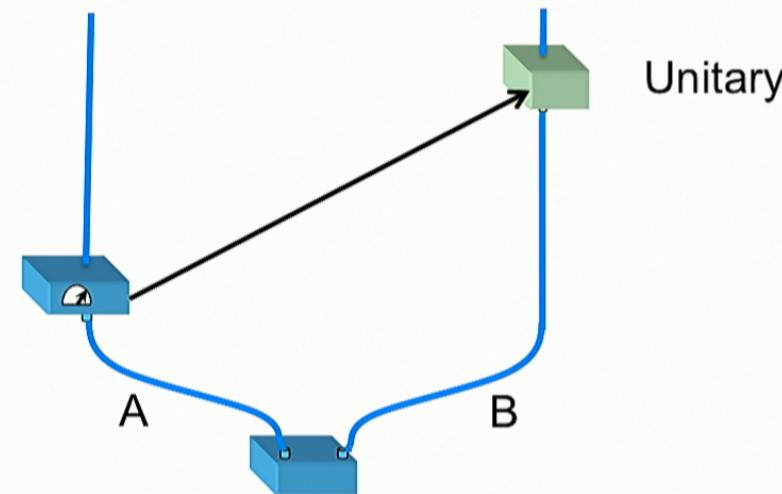
$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\chi\rangle_{AB} = \cos\theta|00\rangle + \sin\theta|11\rangle$$

Measure POVM $\{E_k^A\}$

$$|\psi_k\rangle_{AB} = M_k^A \otimes I^B |\psi\rangle_{AB}$$

where $(M_k^A)^\dagger M_k^A = E_k$



$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\{P_k\}$$

$$P_k P_{k'} = P_k S_{kk'}$$

$$\sum_k P_k = 1$$

$$\rho_k := \text{Tr}(\rho P_k)$$

$$\{E_k\}$$

$$E_k \geq 0$$

$$\sum_k E_k = 1$$

$$\rho_k = \text{Tr}(\rho E_k)$$

Deterministic state conversion

$$\psi \xrightarrow{LOCC} \phi \quad \Updownarrow \quad \text{Conditions on } \psi \text{ and } \phi$$

Deterministic state conversion

Nielsen's theorem

$$\psi \xrightarrow{LOCC} \phi \quad \Leftrightarrow \quad \lambda(\rho_\phi) \succ \lambda(\rho_\psi)$$

$\lambda(\rho)$ = vector of eigenvalues of ρ

where $x \succ y$ is the majorization relation
for $x := (x_1, x_2, \dots, x_d) \in \mathbb{R}_+^d$

define $x_1^\downarrow \geq x_2^\downarrow \cdots \geq x_d^\downarrow$

Similarly for y

$$\begin{aligned} x_1^\downarrow &\geq y_1^\downarrow \\ x_1^\downarrow + x_2^\downarrow &\geq y_1^\downarrow + y_2^\downarrow \end{aligned}$$

⋮

$$x_1^\downarrow + x_2^\downarrow + \cdots + x_{d-1}^\downarrow \geq y_1^\downarrow + y_2^\downarrow + \cdots + y_{d-1}^\downarrow$$

Deterministic state conversion

Nielsen's theorem

$$\psi \xrightarrow{LOCC} \phi \quad \Leftrightarrow \quad \lambda(\rho_\phi) > \lambda(\rho_\psi)$$

Example:

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{LOCC} |\chi\rangle_{AB} = \cos\theta|00\rangle + \sin\theta|11\rangle$$

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Example:

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{LOCC} |\chi\rangle_{AB} = \cos \theta |00\rangle + \sin \theta |11\rangle$$
$$\rho_{\Phi^+}^B = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \qquad \qquad \rho_\chi^B = \cos^2 \theta |0\rangle\langle 0| + \sin^2 \theta |1\rangle\langle 1|$$

Deterministic state conversion

Nielsen's theorem

$$\psi \xrightarrow{LOCC} \phi \quad \rightleftharpoons \quad \lambda(\rho_\phi) > \lambda(\rho_\psi)$$

Proof in three steps:

$$\psi \xrightarrow{LOCC} \phi$$

1

$$\rightleftharpoons \rho_\phi \xrightarrow{\text{mixture of unitaries}} \rho_\psi$$

Deterministic state conversion

Nielsen's theorem

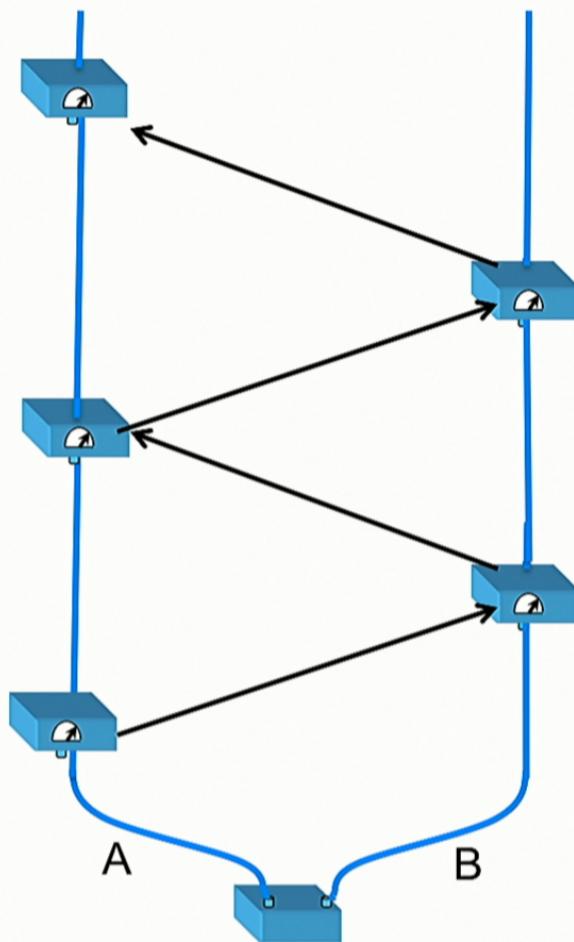
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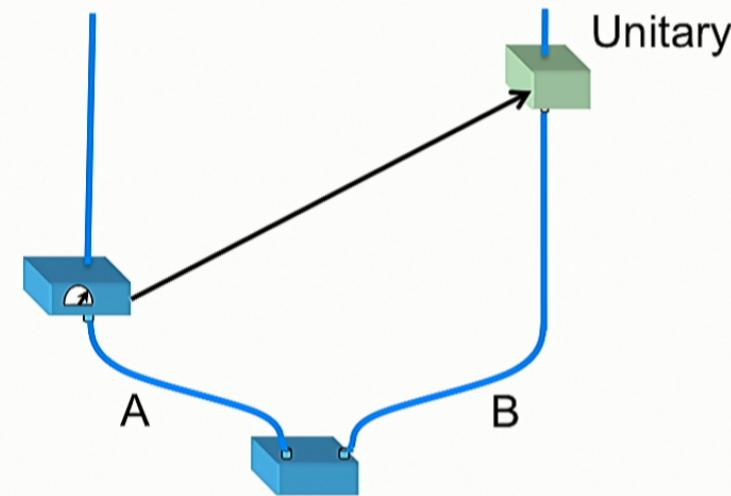
$$\begin{aligned} \psi &\xrightarrow{LOCC} \phi \\ &\xrightleftharpoons[1]{\qquad\qquad\qquad} \rho_\phi \xrightarrow{\text{mixture of unitaries}} \rho_\psi \\ &\xrightleftharpoons[2]{\qquad\qquad\qquad} \lambda(\rho_\phi) \xrightarrow{\text{mixture of permutations}} \lambda(\rho_\psi) \\ &\xrightleftharpoons[3]{\qquad\qquad\qquad} \lambda(\rho_\phi) \succ \lambda(\rho_\psi) \end{aligned}$$

1

For pure states



=



Result of Lo and Popescu

1

$$\psi \xrightarrow{LOCC} \phi$$

On Bob's side

$$\rho_{\psi}^B \xrightarrow{LOCC} \rho_{\phi}^B$$

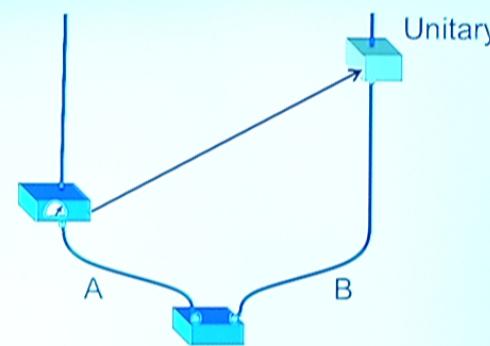
After Alice's mmt

$$\rho_{\psi}^B \longrightarrow \{p_k, \rho_k^B\}$$

$$\sum_k p_k \rho_k^B = \rho_{\psi}^B$$

After CC of k, Bob applies unitary V_k^B such that

$$\forall k : V_k^B \rho_k^B (V_k^B)^\dagger = \rho_{\phi}^B$$



2

$$\rho \xrightarrow{\text{mixture of unitaries}} \sigma \quad \rightleftharpoons \quad \lambda(\rho) \xrightarrow{\text{mixture of permutations}} \lambda(\sigma)$$

$$\sigma = \mathcal{E}(\rho) \quad \mathcal{E} \text{ mixture of unitaries}$$

$$\sum_k \lambda_k(\sigma) |s_k\rangle\langle s_k| = \mathcal{E}(\sum_j \lambda_j(\rho) |r_j\rangle\langle r_j|)$$

$$\lambda_k(\sigma) = \sum_j \underbrace{\langle s_k | \mathcal{E}(|r_j\rangle\langle r_j|) |s_k\rangle}_{D_{kj}} \lambda_j(\rho)$$

$$\lambda(\sigma) = D\lambda(\rho)$$

$$\mathcal{E} \text{ mixture of unitaries} \quad \rightleftharpoons \quad \begin{aligned} D &\text{ doubly stochastic} \\ \sum_k D_{kj} &= 1 \\ \sum_j D_{kj} &= 1 \end{aligned}$$

3

$$x \xrightarrow{\text{mixture of permutations}} y \quad \Leftrightarrow \quad x \succ y$$

Theorem of Hardy Littlewood Polya

2d example: $y = px + (1 - p)\text{SWAP}x$

$$y_1 = px_1 + (1 - p)x_2$$

$$y_2 = px_2 + (1 - p)x_1$$

$$y_1^\downarrow = px_1^\downarrow + (1 - p)x_2^\downarrow \quad \text{if } p \geq \frac{1}{2}$$

$$y_1^\downarrow \leq x_1^\downarrow$$

$$x \succ y$$

$$\begin{aligned}
 \psi &\xrightarrow{\textit{LOCC}} \phi \\
 \stackrel{1}{\Updownarrow} & \rho_\phi \xrightarrow{\text{mixture of unitaries}} \rho_\psi \\
 \stackrel{2}{\Updownarrow} & \lambda(\rho_\phi) \xrightarrow{\text{mixture of permutations}} \lambda(\rho_\psi) \\
 \stackrel{3}{\Updownarrow} & \lambda(\rho_\phi) \succ \lambda(\rho_\psi)
 \end{aligned}$$

$$\psi \xrightarrow{\textit{LOCC}} \phi \Updownarrow \lambda(\rho_\phi) \succ \lambda(\rho_\psi)$$

Measures of entanglement

Def'n: A function E from states to the reals is a **measure of entanglement** if

$$\psi \xrightarrow{LOCC} \phi \Rightarrow E(\psi) \geq E(\phi)$$

Measures of entanglement

Def'n: A function E from states to the reals is a **measure of entanglement** if

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$$\psi \xrightarrow{LOCC} \phi \quad \rightleftharpoons \quad \lambda(\rho_\phi) \succ \lambda(\rho_\psi)$$

Definition of Schur-concave function $G : \mathbb{R}_+^d \rightarrow \mathbb{R}$

$$x \succ y \Rightarrow G(x) \leq G(y).$$

Measures of entanglement

Def'n: A function E from states to the reals is a **measure of entanglement** if

$$\psi \xrightarrow{LOCC} \phi \Rightarrow E(\psi) \geq E(\phi)$$

$$\psi \xrightarrow{LOCC} \phi \quad \rightleftharpoons \quad \lambda(\rho_\phi) \succ \lambda(\rho_\psi)$$

Definition of Schur-concave function $G : \mathbb{R}_+^d \rightarrow \mathbb{R}$

$$x \succ y \Rightarrow G(x) \leq G(y).$$

$$\psi \xrightarrow{LOCC} \phi \Rightarrow G(\lambda(\rho_\phi)) \leq G(\lambda(\rho_\psi))$$

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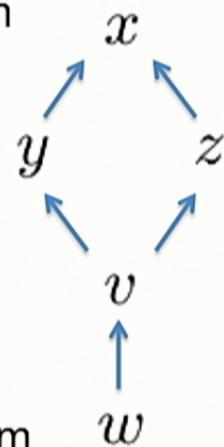
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$$\psi \xrightarrow{LOCC} \phi \Rightarrow G(\lambda(\rho_\phi)) \leq G(\lambda(\rho_\psi))$$

$E(\psi) = G(\lambda(\rho_\psi))$ is a measure of entanglement

Probability distributions
under the majorization order

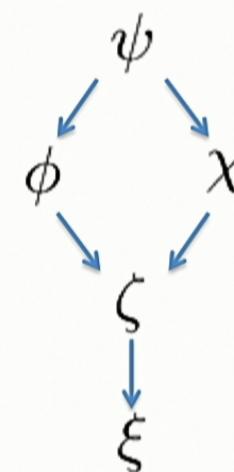
Most uniform



Least uniform

Bipartite pure states
under the LOCC order

Most entangled



least entangled

Definition of Schur-concave function $G : \mathbb{R}_+^d \rightarrow \mathbb{R}$

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Generating Schur-concave fn's:

For every concave function $g : \mathbb{R}_+ \rightarrow \mathbb{R}$

i.e., $\forall a \in \mathbb{R} : g''(a) \leq 0$

$$g(wa + (1 - w)a') \leq wg(a) + (1 - w)g(a')$$

the symmetric function

$$G(x) := \sum_{i=1}^d g(x_i)$$

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$$g(a) = -a^2 \quad \text{concave}$$

$$G(x) := - \sum_{i=1}^d x_i^2 \quad \text{Schur-concave}$$

$$G(\lambda(\rho_\psi)) := - \sum_{i=1}^d \lambda_i (\rho_\psi)^2 = -\text{Tr}(\rho_\psi^2)$$

negative of purity

$$g(a) = \pm a^\alpha \quad \alpha \in \mathbb{R} \setminus \{0, 1\} \quad (\text{sign chosen to make } g \text{ concave})$$

$$G(x) := \pm \sum_{i=1}^d x_i^\alpha \quad \text{Schur-concavity preserving}$$
$$\bar{G}(x) := (f \circ G)(x) \quad \text{where} \quad f(t) := \frac{\operatorname{sgn}\alpha}{\alpha-1} \log(t)$$

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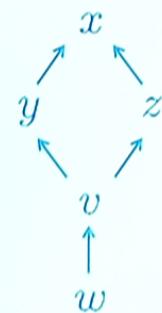
Order- α Renyi entropy

$$\bar{G}(\lambda(\rho_\psi)) := \pm \frac{1}{\alpha-1} \log \sum_{i=1}^d \lambda_i (\rho_\psi)^\alpha = \pm \frac{1}{\alpha-1} \log \text{Tr}(\rho_\psi^\alpha)$$

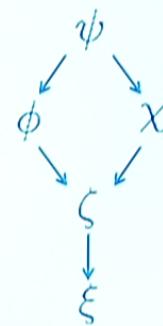
$$E(\psi) = \pm \frac{1}{\alpha-1} \log \text{Tr}(\rho_\psi^\alpha)$$

Order- α Renyi entanglement

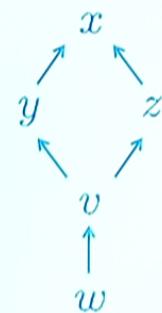
Probability distributions
under majorization form a
partial order



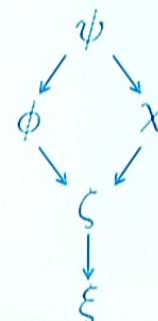
Bipartite pure states
under LOCC form a
partial order



Probability distributions
under majorization form a
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Bipartite pure states
under LOCC form a
partial order



Therefore, there cannot be
“one measure to rule them all”

