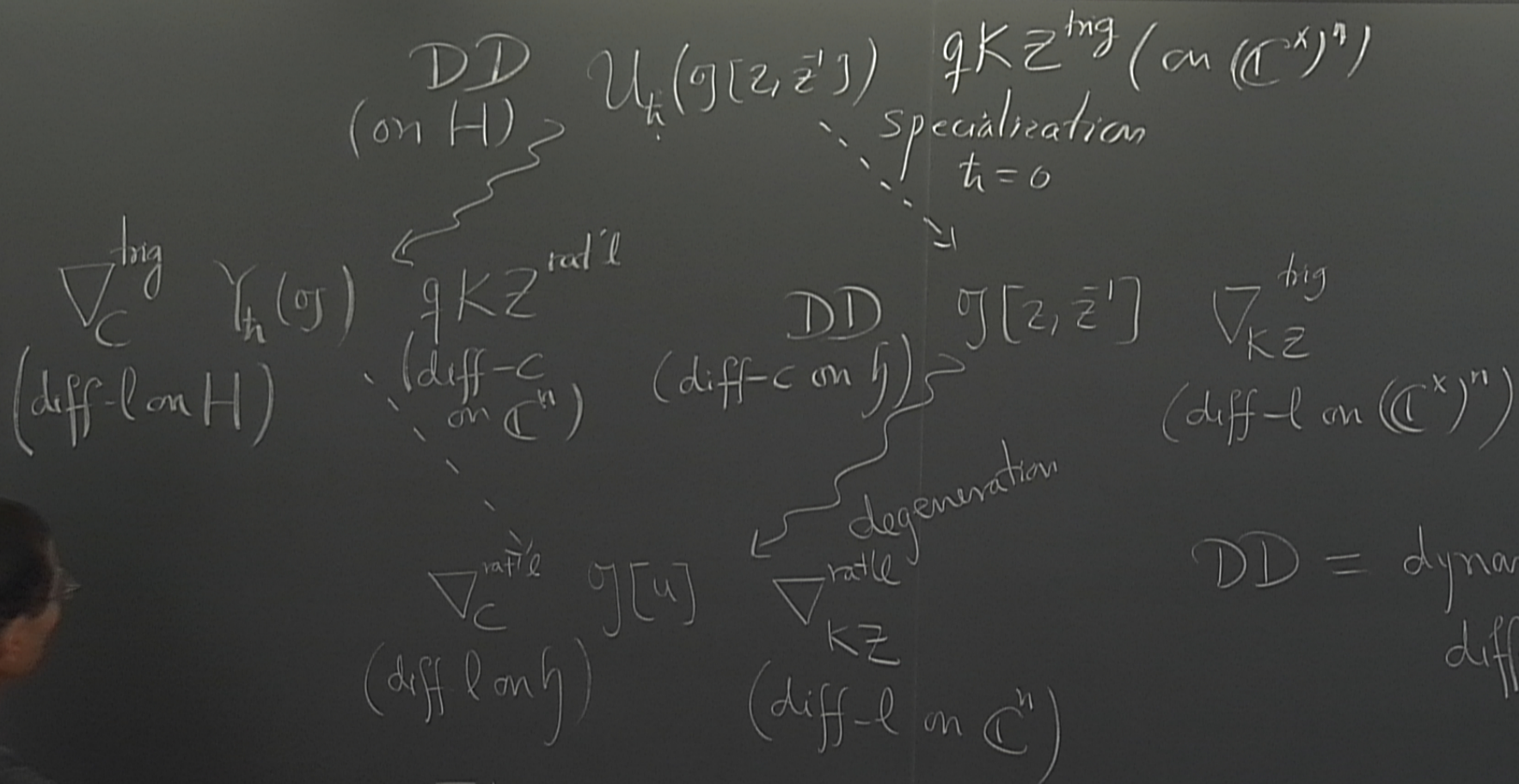


Title: Learning Seminar on Maulik-Okounkov

Date: Jun 23, 2016 02:00 PM

URL: <http://pirsa.org/16060125>

Abstract:



$DD = \text{dynamical difference}$

Etingof's diamond

(3)

$$\Delta \otimes I (R(u)) = R_{13}(u) R_{23}(u)$$

$$I \otimes \Delta (R(u)) = R_{13}(u) R_{12}(u)$$

$$Y_{\hbar}(\mathfrak{g})$$

$$q \in \mathbb{Z}^{\text{rat}} \ell$$

R-matrix

Stable Envelopes

Yangian Y graded Hopf alg

$$Y_{\hbar} = U(\mathfrak{g}[\hbar])$$

$$\mathbb{C}[\hbar] \quad \deg(\hbar) = 1$$

$$T_a \hookrightarrow Y \quad a \in \mathbb{C}$$

(Hopf alg auto)

$$\exists! R(u) \in Y \otimes Y[[\bar{u}]]$$

$$(1) \quad R(u) = 1 + \frac{\hbar \Omega}{u} + O(\bar{u}^2) \quad (\Omega \in \mathfrak{g} \otimes \mathfrak{g} \text{ Casimir})$$

$$(2) \quad (T_u \otimes 1) \Delta^{\text{op}}(x) = R(u) [(T_u \otimes 1) \Delta(x)] R(\bar{u})^{-1}$$

$$(3) \quad \Delta \otimes 1 (R(u)) = R$$

$$1 \otimes \Delta (R(u)) = R$$

$$\underline{\text{Cor.}} \quad R_{12}(u) R_{13}(u+v) R_{23}(v) \\ = R_{23}(v) R_{13}(u+v) R_{12}(u)$$

$$U(h[u]) \subset Y \\ U(g) = (Y)_0$$

Hopf alg

$$\mathbb{C}[\hbar]$$

$$\deg(\hbar) = 1$$

$\mathbb{C}$

$[\bar{u}']$

$$+ O(\bar{u}^2) \quad (\Omega \in \mathfrak{g} \otimes \mathfrak{g} \text{ Casimir})$$

$$(3) \quad \Delta \otimes 1 (R(u)) = R_{13}(u) R_{23}(u)$$

$$1 \otimes \Delta (R(u)) = R_{13}(u) R_{12}(u)$$

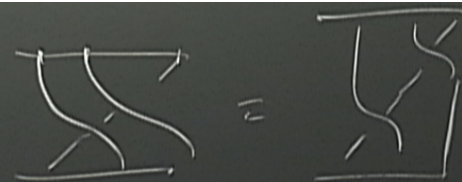
$$(4) \quad R(u)^{-1} = R_{21}(-u)$$

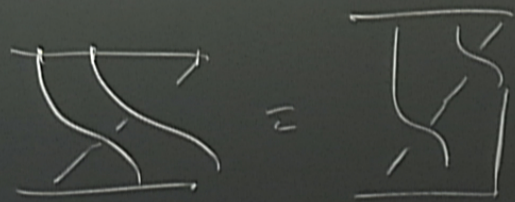
$$\text{Cor} \quad R_{12}(u) R_{13}(u+v) R_{23}(v)$$

$$= R_{23}(v) R_{13}(u+v) R_{12}(u)$$

$$\mathcal{U}(\hbar[u]) \subset Y$$

$$\mathcal{U}(\mathfrak{g}) = (Y)_0$$





$_{23}(u)$

$_{12}(u)$

$$(u) R_{13}(u+v) R_{23}(v) = R_{23}(v) R_{13}(u+v) R_{12}(u)$$

$$t_{1,1} = h_1 \otimes u$$

$V_1, \dots, V_n : (\text{f.d.}) \text{ repn. of } Y$

For a function

$$\Psi(\Lambda, \underline{a}) \in V_1 \otimes \dots \otimes V_n$$

$$\Lambda \in H$$

$$\underline{a} = (a_1, \dots, a_n) \in \mathbb{C}^n$$

(=rk(g))

$$\frac{\partial \Psi}{\partial \omega_i} = \left[\frac{1}{2\chi} \left(\sum_{d \geq 0} \frac{(d, \omega_i)}{e^d - 1} (e_d f_d + f_d e_d) \right) + \frac{1}{2\chi} \left(t_i(\underline{a}) \right) \right] \Psi$$

$n (\mathbb{C}^x)^n$

$\mathfrak{g} = \text{f.d.-simple}$
Lie alg.

$$\psi(\lambda, \underline{a} + \chi \epsilon_i) = B_i(\lambda, \underline{a}) \psi(\lambda, \underline{a})$$

∇_{kz}^{big}

(diff-l on $(\mathbb{C}^x)^n$)

DD = dynamical
difference

Yangian Y graded

$$Y / {}_{\hbar} Y = \mathcal{U}(\mathfrak{g}[u])$$

$$T_a \subset Y \quad a \in$$

(Hopf alg. auto)

$$\exists ! R(u) \in Y \otimes Y[[\hbar]]$$

$$(1) R(u) = 1 + \frac{\hbar \Omega}{u}$$

$$(2) (\overline{T}_u \otimes 1) \Delta^{\text{op}}(x)$$

$$(u+v) R_{23}(v)$$

$${}_{23}(v) R_{13}(u+v) R_{12}(u)$$

$$t_{u,1} = h_1 \otimes u$$

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$$\Psi(\Lambda, \underline{a}) \in \mathbb{C}$$

$$\Lambda \in H$$

$$\underline{a} = (a_1, \dots, a_n) \in \mathbb{C}^n$$

$$\frac{\partial \Psi}{\partial \omega_i} = \left[\frac{1}{2\lambda} \left(\sum_{d \geq 0} \frac{(d, \omega_i)}{e^d - 1} (e_a f_d + f_d e_a) \right) + \frac{1}{2\lambda} \left(t_i(\underline{a}) \right) \right] \Psi$$

$$\chi \in \mathbb{C}^{\text{step}}$$

$$B_i(\Lambda, \underline{a})$$

$$= R_{i-1,i}(a_{i-1} - a_i - \bar{x})^{-1} \cdot R_{i,i}(a_i - a_{i+1} - \bar{x})^{-1}$$

$$\Lambda^{(i)}$$

$$R_{i,n}(a_i - a_n) \cdots R_{i,i+1}(a_i - a_{i+1})$$

$x^n)$

dynamical
difference

$$B_i(\Lambda, \underline{a})$$

$$= R_{i-1,i}(a_{i-1} - a_i - \bar{x})^{-1} \cdot R_{i,i}(a_i - a_{i+1} - \bar{x})^{-1}$$

$$\Lambda^{(i)}$$

$$R_{i,n}(a_i - a_n) \cdots R_{i,i+1}(a_i - a_{i+1})$$

$$qKZ \text{ consistent} \Leftrightarrow \begin{cases} YBE \\ [\Lambda \otimes \Lambda, R] = 0 \end{cases}$$

$$\frac{1}{\hbar} \gamma$$

$$T_a \circ \gamma$$

(Hopf al)

$$\exists ! R(u)$$

$$(1) R(u)$$

$$(2) (T_u \otimes 1)$$

Stable Envelopes

$(X, \omega) \hookrightarrow$

A Hamiltonian

$\mathbb{C}_h^x$ rescales ω

$$(T = A \times \mathbb{C}_h^x)$$

$(X, \omega) \leftarrow$

A Hamiltonian

$\mathbb{C}_h^x$ rescales ω

$$(T = A \times \mathbb{C}_h^x)$$

$\mathcal{C} \subset \mathcal{O}_R$

Chamber

$$\text{Stab}_e: H_T(X^A) \rightarrow H_T(X)$$

unique $H_T(\text{pt})$ -mod hom

$$\text{s.t. } \gamma \in H_T(F)$$

$F \subset X^A$
connected
component

$$\Gamma = \text{Stab}_e(\gamma)$$

Satisfies

unique $H_T(pt)$ mod

s.t. $\gamma \in H_T(F)$

$F \subset X^A$
connected
component

$$\Gamma = \text{Stab}_e(\gamma)$$

Satisfies

$$H_{C^X}(F) \otimes H_A(pt)$$

$$(2) \quad \Gamma|_F = (\pm) E$$

$$(3) \quad \deg_A(\Gamma|_{F'}) < \dots$$

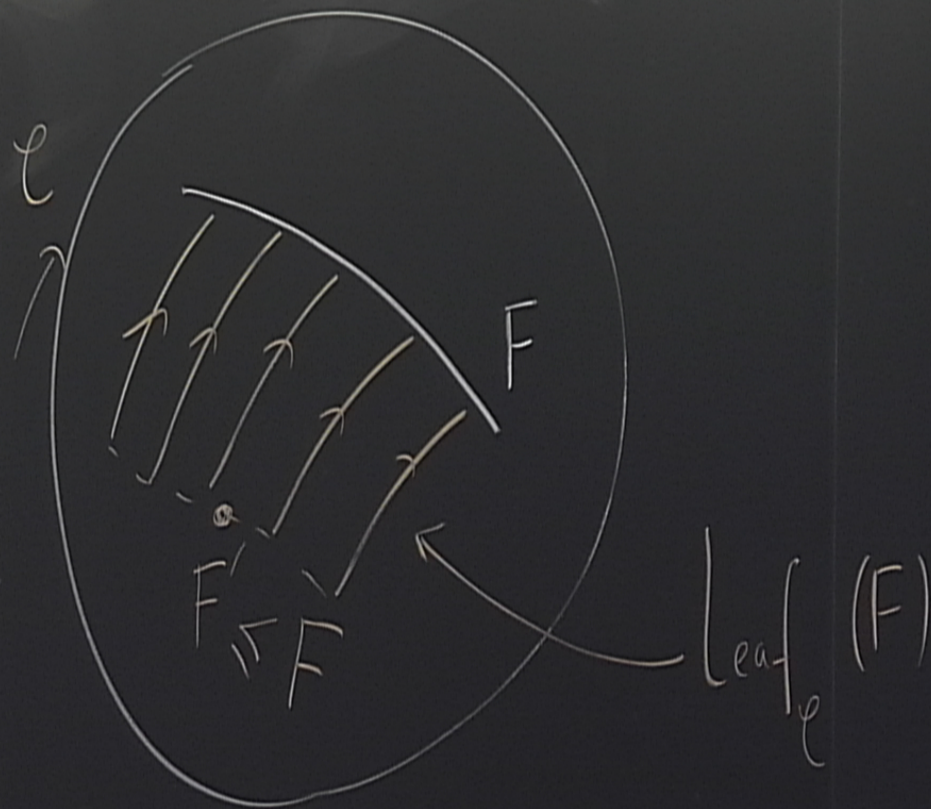
for $F' < F$

$\text{leaf}_e(F')$

$\leq F$

$\text{Euler}(N_F^+)$

$\frac{1}{2} \text{codim } F'$
 X



0")

ig

Σ

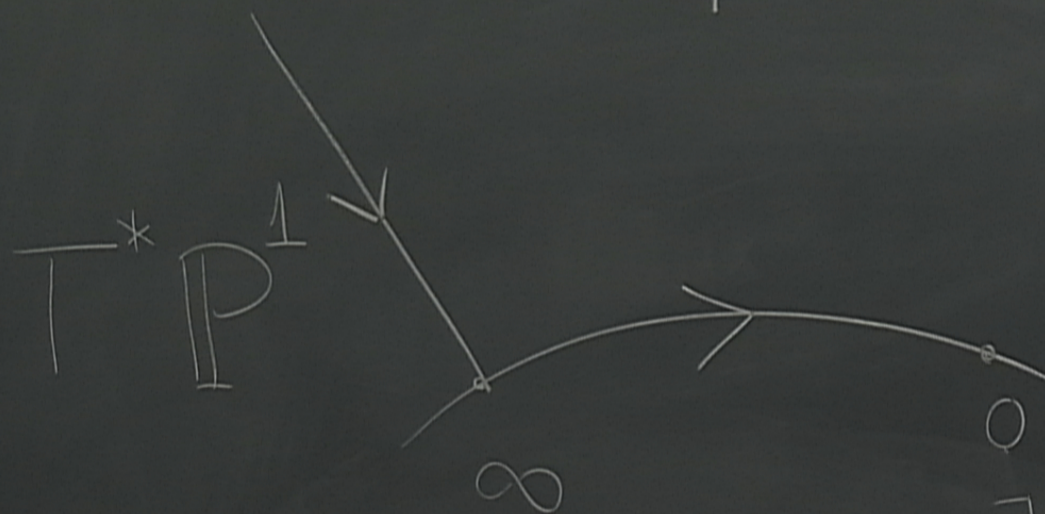
l on $(\mathbb{C}^x)^n$)

= dynamical
difference

$$R_{e',e} = \text{Stab}_{e'}^{-1} \circ \text{Stab}_e$$

$$\in \text{End} \left(H_T(X^A) \right) \otimes \mathbb{C}(\alpha)$$

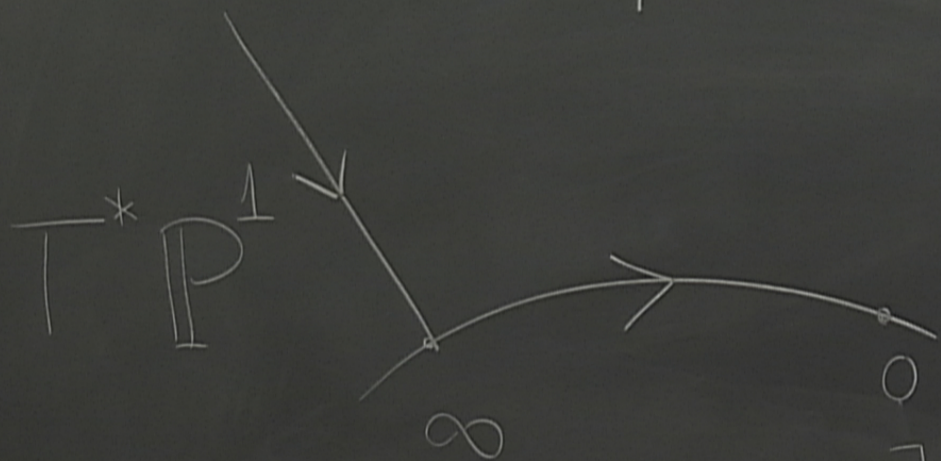
∇_c^{big}
Quantum
Cohomology



$$\text{Stab}_e = \begin{bmatrix} u+h & 0 \\ h & -u \end{bmatrix}$$

$$\text{Stab}_{-e} = \begin{bmatrix} u & h \\ 0 & -u+h \end{bmatrix}$$

h = weight of $\mathbb{C}^x$ scaling



$$\text{Stab}_e = \begin{bmatrix} u+h & 0 \\ h & -u \end{bmatrix}$$

$$\text{Stab}_{-e} = \begin{bmatrix} u & h \\ 0 & -u+h \end{bmatrix}$$

h = weight of $\mathbb{C}^x$ scaling

$$\leadsto R(u) = \frac{1}{u-h} \begin{bmatrix} u & -h \\ -h & u \end{bmatrix}$$

= weight of
 $\mathbb{D}^X$ scaling

$$R(u) = \frac{1}{u-h} \begin{bmatrix} u & -h \\ 1 & u \\ -h & u \end{bmatrix}$$

$$= \frac{u \text{Id} - h(\text{permutation})}{u-h} \mathcal{C} \mathcal{C}^T R$$

Cha

$$\text{Stab}_e H_T(X^A) \rightarrow H_T(X)$$

unique $H_T(\text{pt})$ -mod hom

$$\gamma \in H_T(F) \quad F \subset X^A$$

connected component

$$\Gamma = \text{Stab}_e(\gamma)$$

satisfies

$$H_{\text{cx}}(F) \otimes H_A(\text{pt})$$

$$\mathbb{C}[\pi]$$

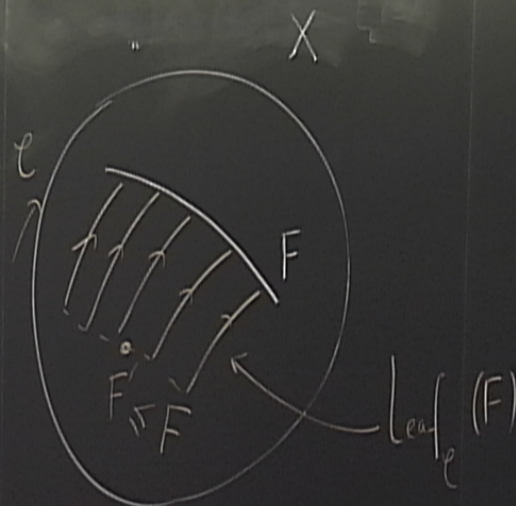
$$(1) \text{Supp}(\Gamma) \subset \bigcup_{F' \leq F} \text{leaf}_e(F')$$

$$(2) \Gamma|_F = (\pm) \text{Euler}(N_F^+)$$

$$(3) \deg_A(\Gamma|_{F'}) < \frac{1}{2} \text{codim}_X F'$$

$$\text{for } F' < F$$

divisible by π



$t_h = 0$

DD (diff-c on h) $\rightarrow \sigma[z, \bar{z}']$ $\nabla_{k\bar{z}}^{big}$ (diff-l on $(\mathbb{C}^x)^n$)

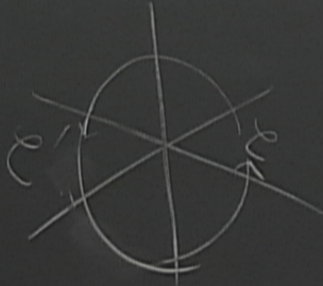
degeneration

$[u]$ $\nabla_{k\bar{z}}^{rate}$ (diff-l on $\mathbb{C}^n$)

's diamond

DD = dynamical difference

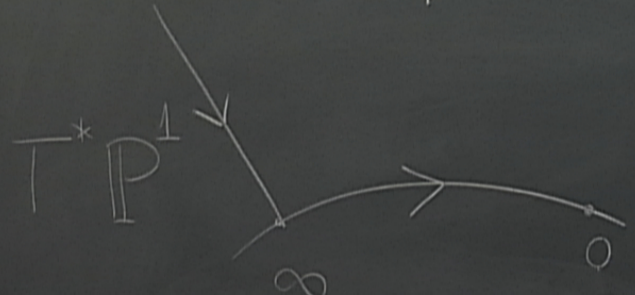
$e' \circ e = YBE$



$R_{e',e} = \text{Stab}_{e'} \circ \text{Stab}_e$

$\in \text{End} \left(H_T(X^A) \right) \otimes \mathbb{C}(\alpha)$

$T^* \mathbb{P}^1$



$\text{Stab}_e = \begin{bmatrix} u+t & 0 \\ t & -u \end{bmatrix}$

$\text{Stab}_{-e} = \begin{bmatrix} u & t \\ 0 & -u+t \end{bmatrix}$

$t_h = \text{weil}$
 $\mathbb{C}^x$ so

$\sim R$

$$\boxed{\begin{array}{c} \nabla_C^{\text{big}} \quad \gamma_h(\sigma) \quad gKZ^{\text{rat}} \\ \text{(diff-l on } H) \quad \text{(diff-c on } \mathbb{C}^n) \end{array}}$$

$$\text{DD} \quad \gamma[z, \bar{z}'] \quad \nabla_{KZ}^{\text{big}} \\ \text{(diff-c on } h)$$

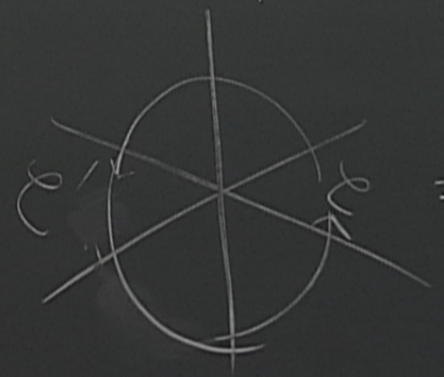
$$\nabla_{KZ}^{\text{big}} \\ \text{(diff-l on } (\mathbb{C}^x)^n)$$

$$\nabla_C^{\text{rat}} \quad g[u] \\ \text{(diff-l on } h)$$

$$\swarrow \text{degeneration} \\ \nabla_{KZ}^{\text{rat}} \\ \text{(diff-l on } \mathbb{C}^n)$$

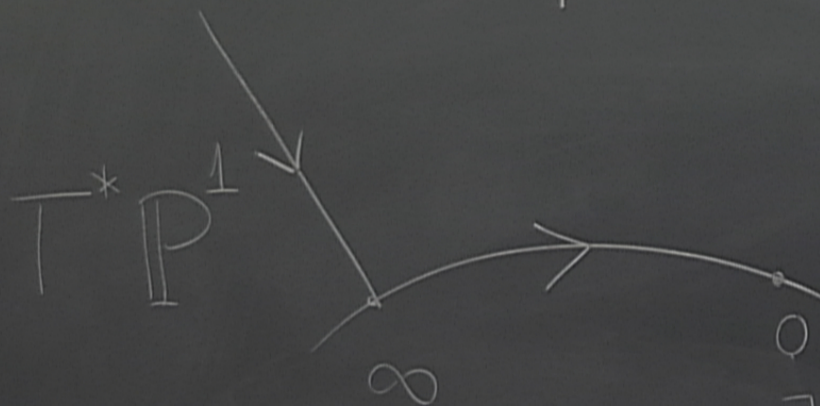
DD = dynamical difference

Etingof's diamond



$$R_{e',e} = \text{Stab}_{e'}^{-1} \circ \text{Stab}_e$$

$$\in \text{End} \left(H_{\overline{T}}(X^A) \right) \otimes \mathbb{C}(\alpha)$$



$$\text{Stab}_e = \begin{bmatrix} u+h & 0 \\ 0 & 1 \end{bmatrix}$$

Have to invert Eu

$$\text{Normal } F/X = N$$

$$\frac{x}{x+h} =$$

h = weight of $\mathbb{C}^x$ scaling

$$R(u) = \begin{bmatrix} u & -h \\ 1 & u \end{bmatrix}$$

$$A = (\mathbb{C})$$

$$A_1 = \mathbb{C}^\times$$

$$M(w)^{A_1} = M(w^{(1)}) \times M(w^{(2)} + w^{(3)})$$

$$\sigma_2 = \sigma_1 + \mathbb{C}(0, a_2 > a_3)$$

Hexagon identities

$$M(\underline{w})$$

$$\underline{w} = w^{(1)} + w^{(2)} + w^{(3)}$$

$$A = (\mathbb{C}^x)^3$$

$$A_1 = \mathbb{C}^x$$

$$M(w)^{A_1} = M(w^{(1)}) \times M(w^{(2)} + w^{(3)})$$

$$\mathcal{R}_{e'_1, e_1}$$

$$\left. \begin{array}{l} e_1 : a > 0 \\ e'_1 : a < 0 \end{array} \right\} \text{ in } A_1$$

Hexagon identities ✓

$$M(\underline{w})$$

$$\underline{w} = w^{(1)} + w^{(2)} + w^{(3)}$$

$$A = (\mathbb{C}^x)^3$$

$$A_1 = \mathbb{C}^x$$

$$M(w)^{A_1} = M(w^{(1)}) \times M(w^{(2)} + w^{(3)})$$

$$R_{e'_1, e_1} = R_{13}(a_1 - a_3) R_{12}(a_1 - a_2)$$

$$\left. \begin{array}{l} e_1 : a > 0 \\ e'_1 : a < 0 \end{array} \right\} \text{ in } A_1$$

