Title: Foliated Quantum Codes, with a chance of Anyonic Time

Date: Jun 29, 2016 04:00 PM

URL: http://pirsa.org/16060124

Abstract: <p>Raussendorf introduced a powerful model of fault tolerant measurement based quantum computation, which can be understood as a layering (or â€œfoliationâ€•) of a multiplicity of Kitaevâ€™s toric code. I will discuss our generalisation of Raussendorfâ€™s construction to an arbitrary CSS code. We call this a Foliated Quantum Code. Decoding this foliated construction is not necessarily straightforward, so I will discuss an example in which we foliate a family of finite-rate quantum turbo codes, and present the results of numerical simulations of the decoder performance.</p>

<p>If I have time, I will discuss some ongoing work (with Gavin Brennan) on relational time applied to topological quantum field theories, in particular, how anyonic systems with essentially trivial dynamics can still exhibit correlations that track â€œtime".</p>

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

Foliated Quantum Codes

Tom Stace
University of Queensland, Australia

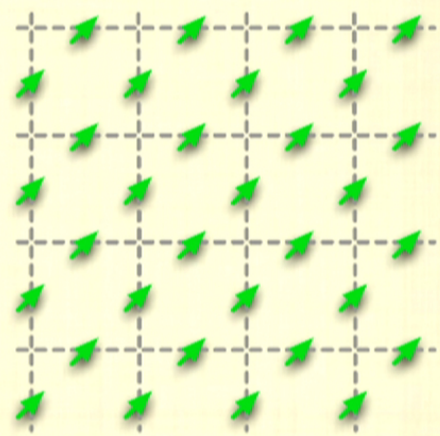
with: Andrew Bolt, David Poulin, Guillaume Duclos-Cianci

eQus

ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

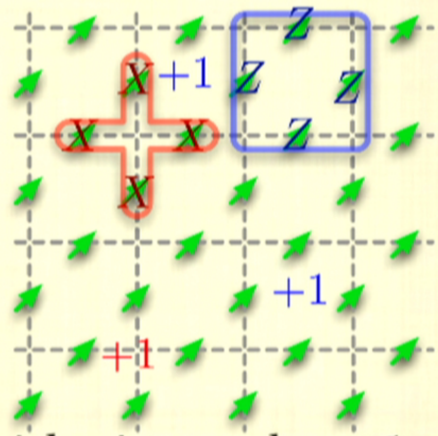
Toric codes defined

- A toric code is an encoding of a *logical* quantum state into a set of *physical* "spins".
- Physical spins live on the links of an $L \times L$ lattice:
- Periodic boundary conditions (i.e. embeddable on a torus)
- Kitaev, quant-ph/9707021



Toric codes defined

- A toric code is defined by its stabiliser operators:
  stars and **plaquettes**
- Plaquettes commute with other plaquettes.
- Stars commute with other stars.
- Plaquettes commute with stars



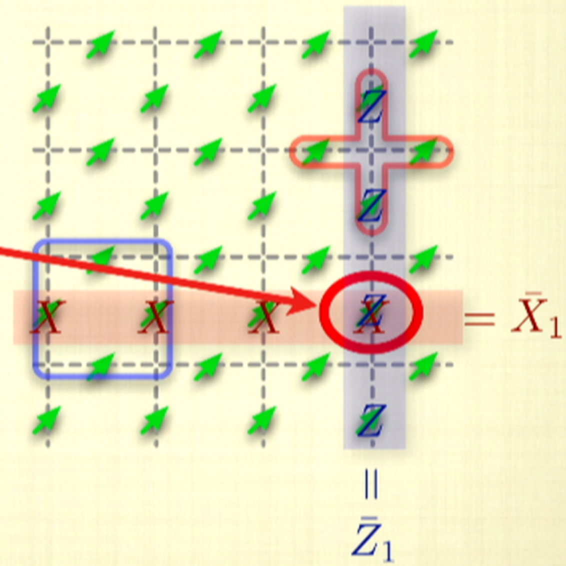
$$[X_i X_j, Z_i Z_j] = 0$$

- The code is a simultaneous eigenstate of all the stabilisers, with eigenvalue +1

Logical qubit operators

- What are the logical qubit Pauli operators?
- Need to find anticommuting operators that commute with stabilisers, and are independent of them.
- For logical qubit

$$[\bar{Z}_1, \bar{X}_1]_+ = [Z, X]_+ = 0$$



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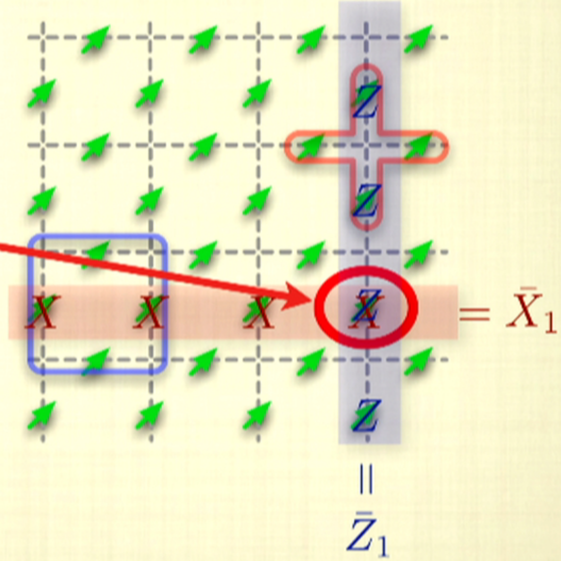
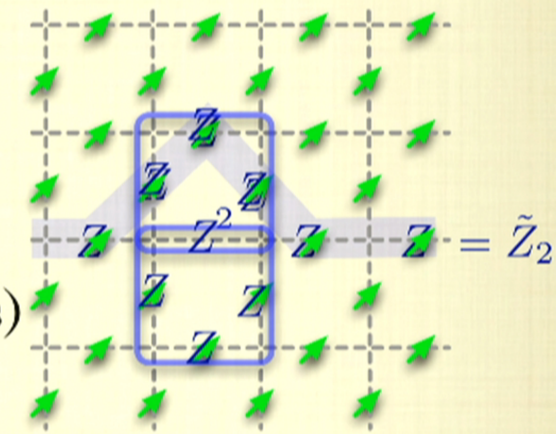


Losing Memory

- Can **deform logical operators** around the loss

$$\bar{Z}_2 \rightarrow P\bar{Z}_2 = \tilde{Z}_2$$



- Can find products of stabilizers that are independent of lost qubit.
- **Super-plaquettes** (and **super-stars**) detect ends of error chains
- Losses are located errors: we know where they are.
- i.e. redundancy in stabilizers and logical operators.

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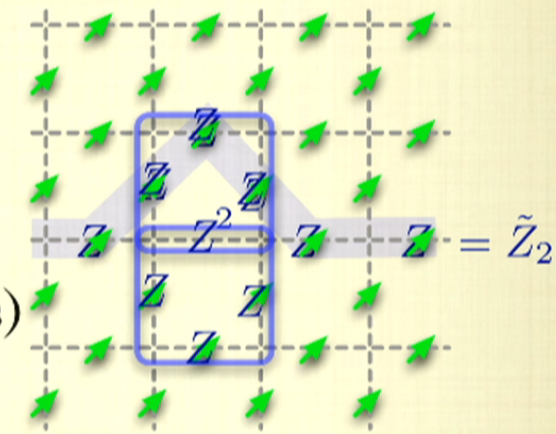

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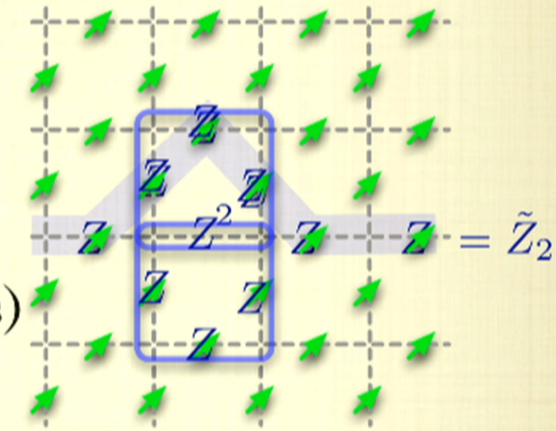

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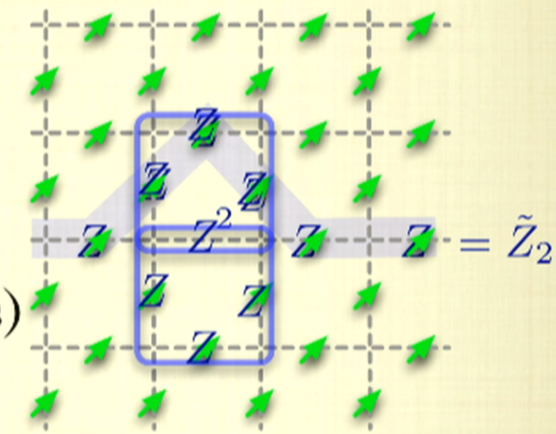
$$\bar{Z}_2 \to P\bar{Z}_2 = \tilde{Z}_2$$



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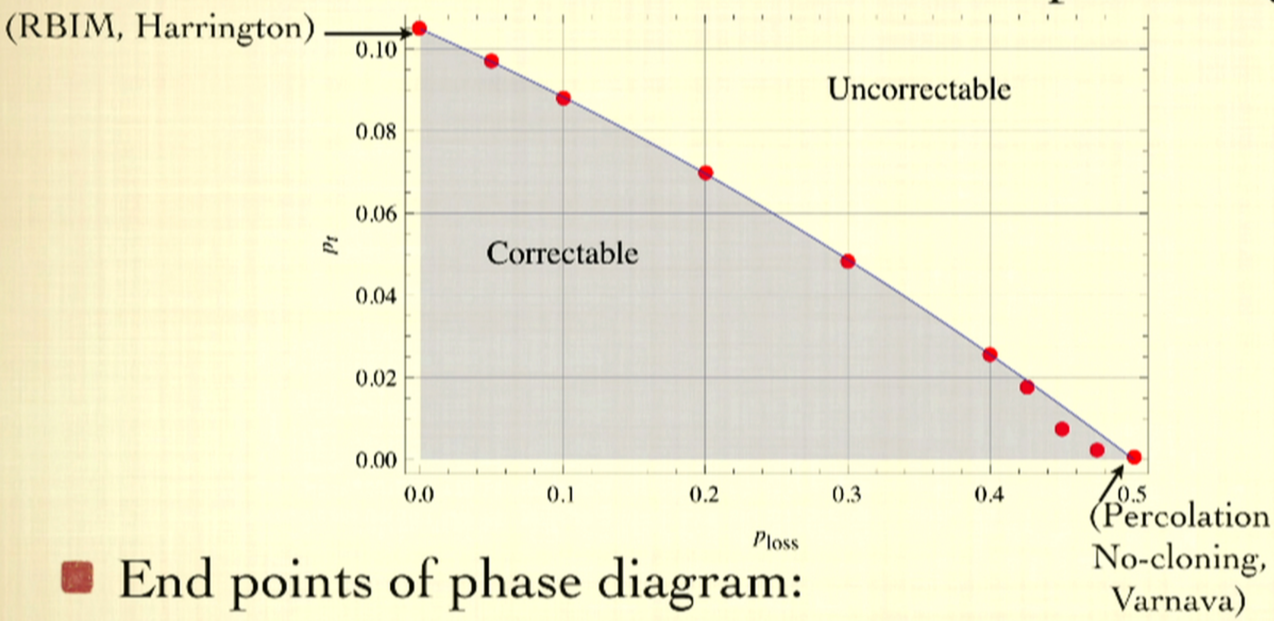
Losing Memory gracefully

- Numerical error threshold vs. loss rate 'phase diagram'



- End points of phase diagram:

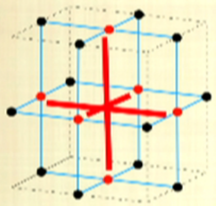
$$(p_{\text{loss}}, p_{\text{err}}) = (0, 0.104) \text{ and } (0.5, 0)$$

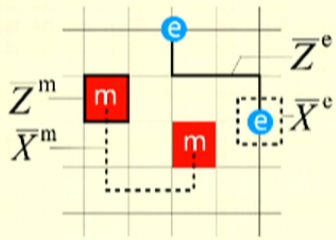
Stace, Barrett, Doherty PRL 102, 200501 (2009)

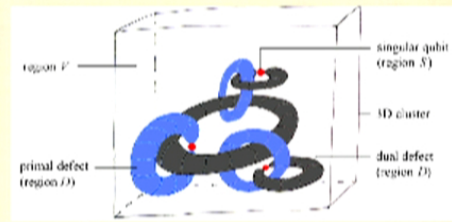
FTQC?

- How about fault tolerance and quantum computing?
- A 3D lattice does the job...

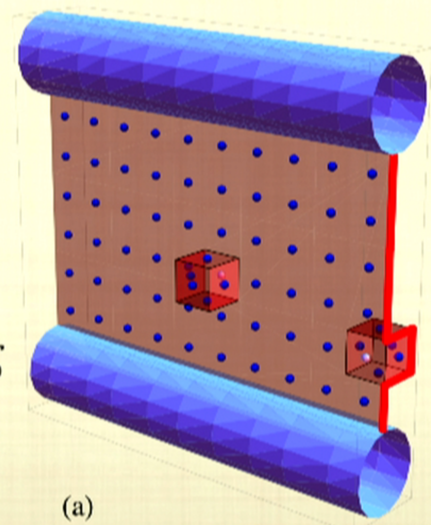
Raussendorf, Harrington, Goyal NJP 9, 199 (2007)







- Same tricks work (deforming logical operators and making super-stars)



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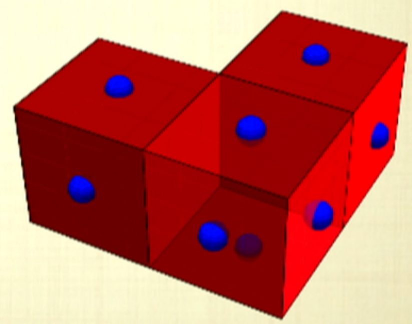
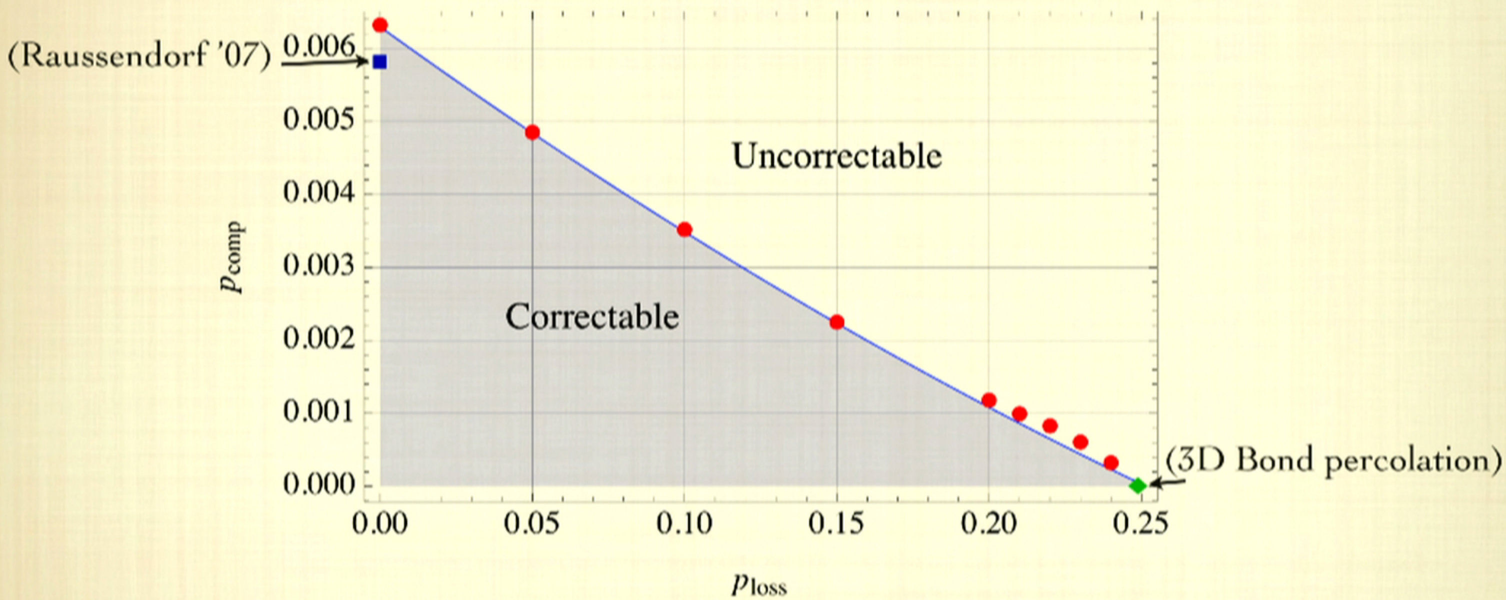


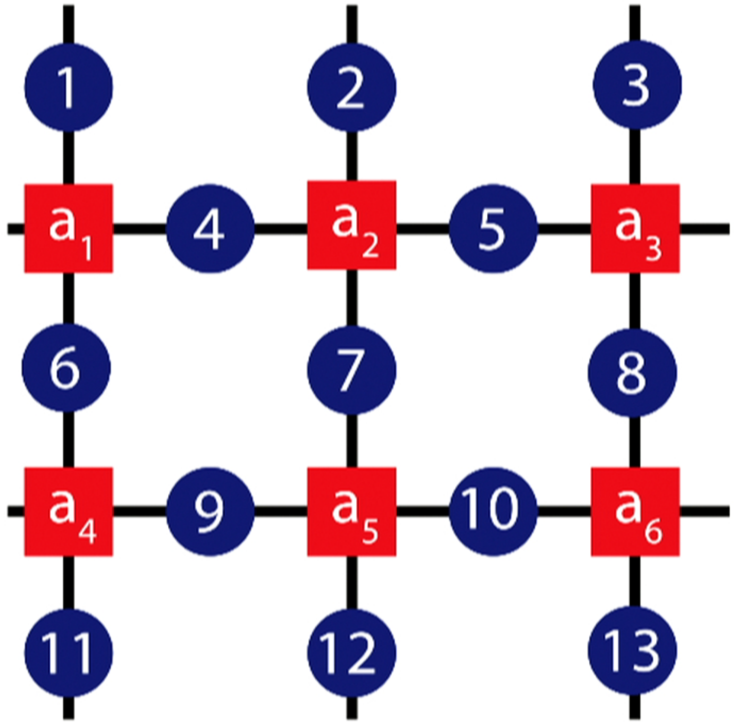
FTQC threshold

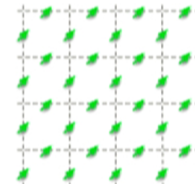


Barrett, Stace PRL 105, 200502 (2010)

Cluster State Progenitors

Toric code Cluster State Progenitors:





Cluster State Progenitors

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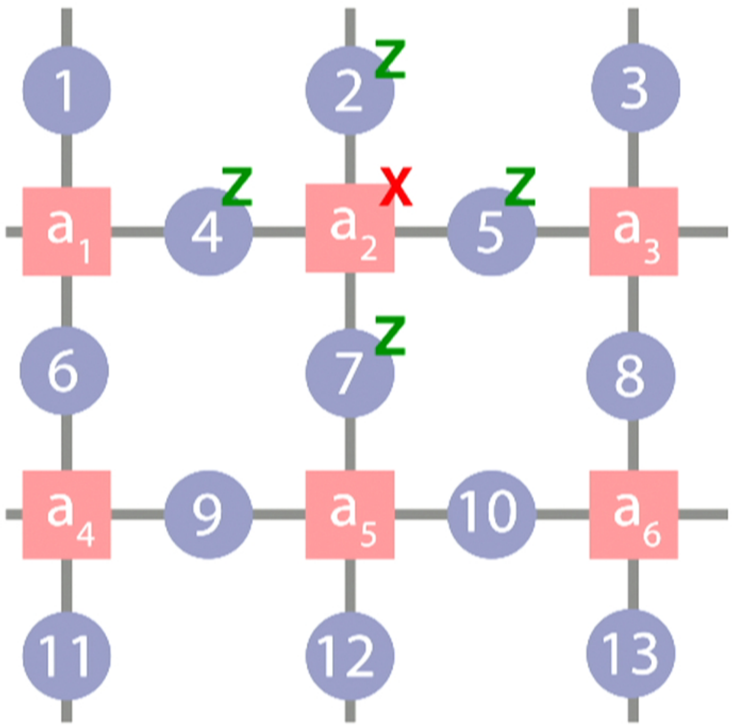

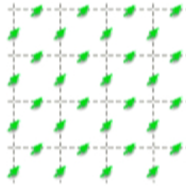

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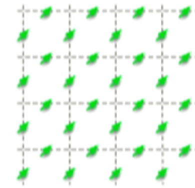

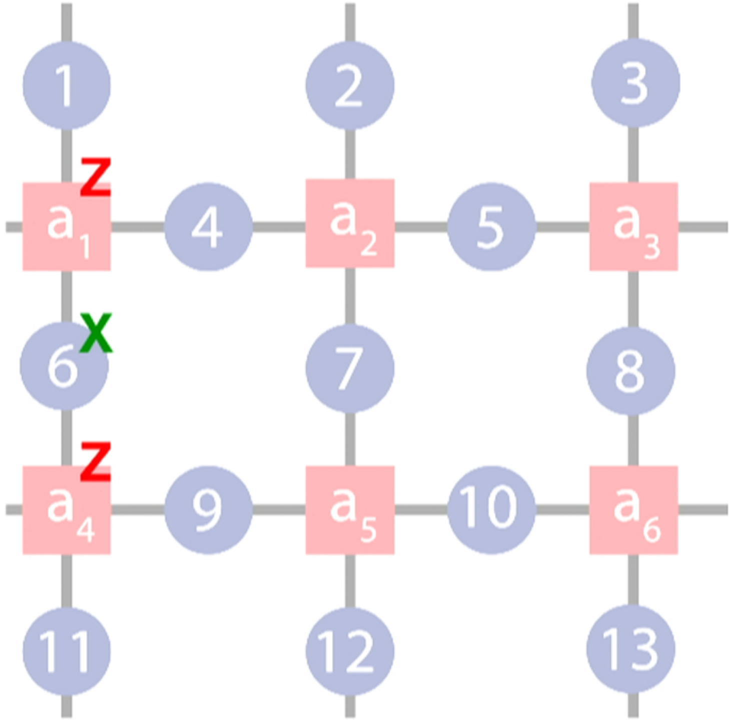

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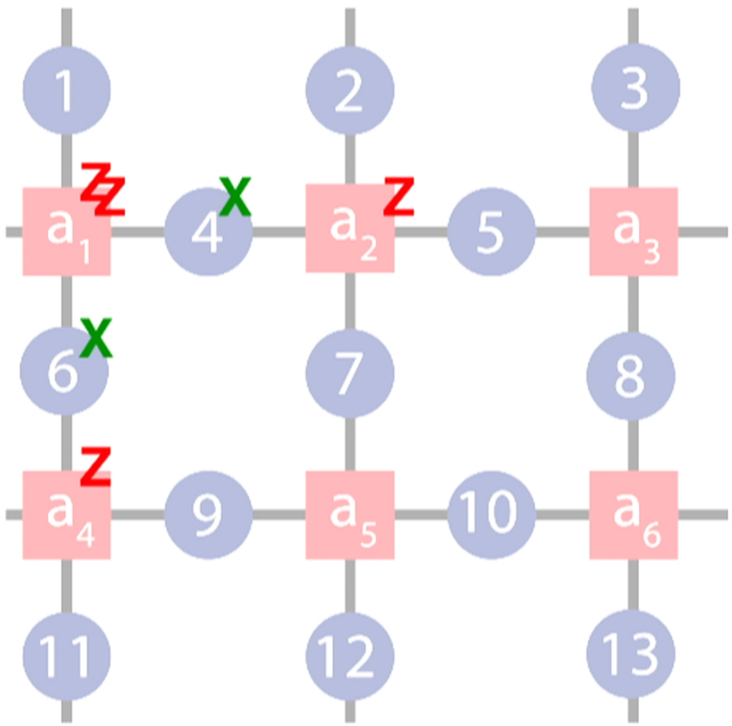
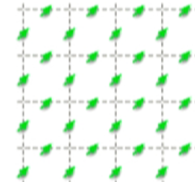
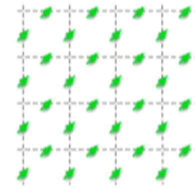
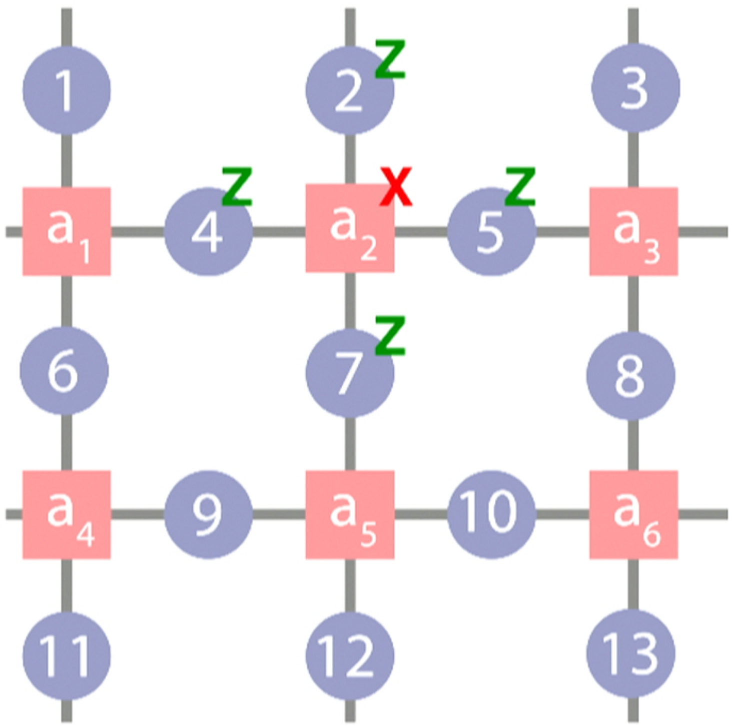




Cluster State Progenitors



Toric code Cluster State Progenitors:



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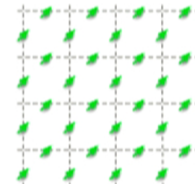

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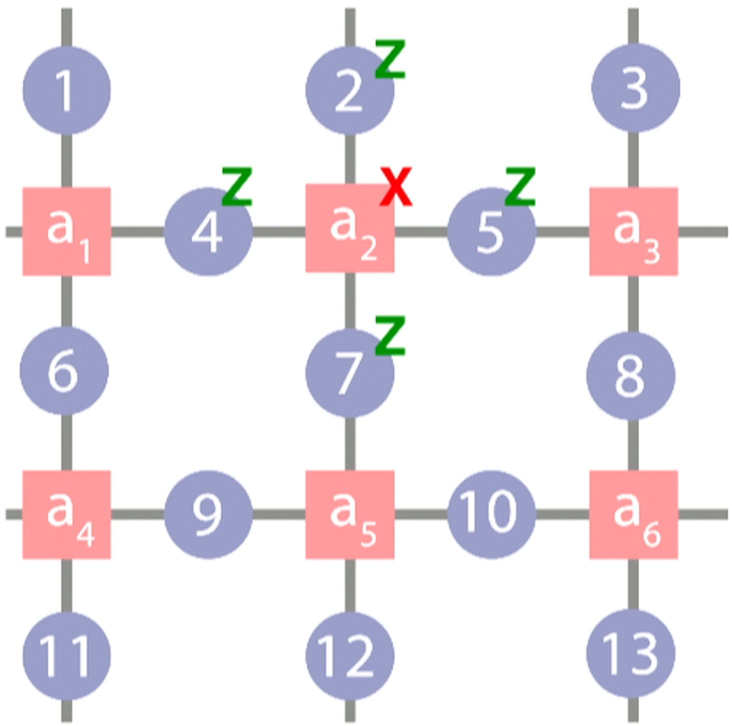

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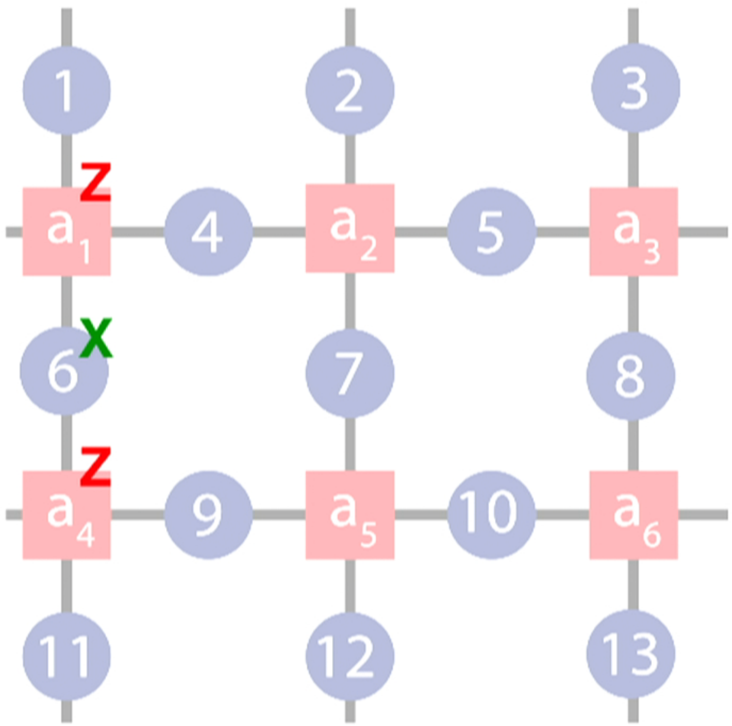

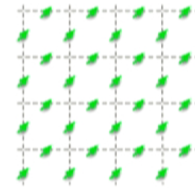

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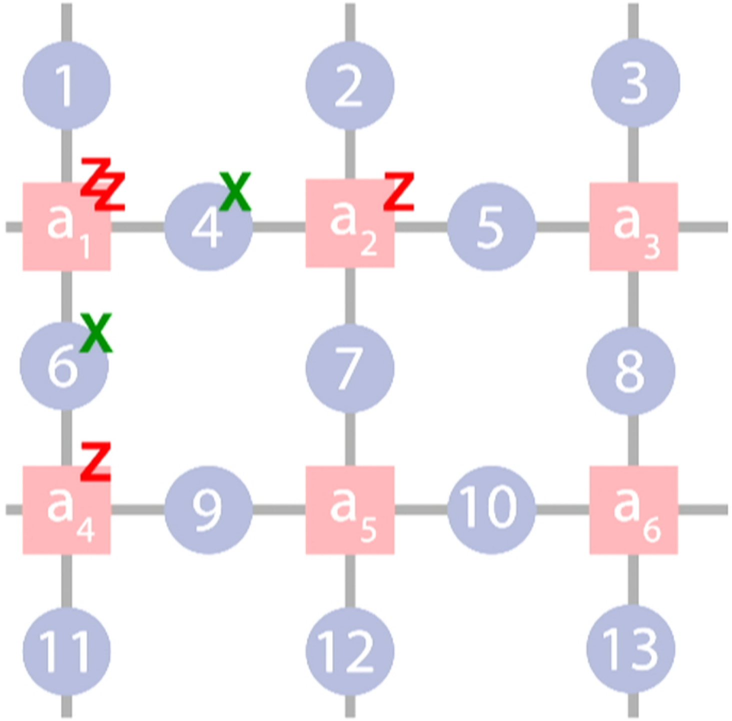

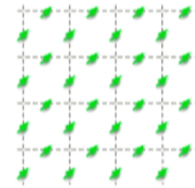

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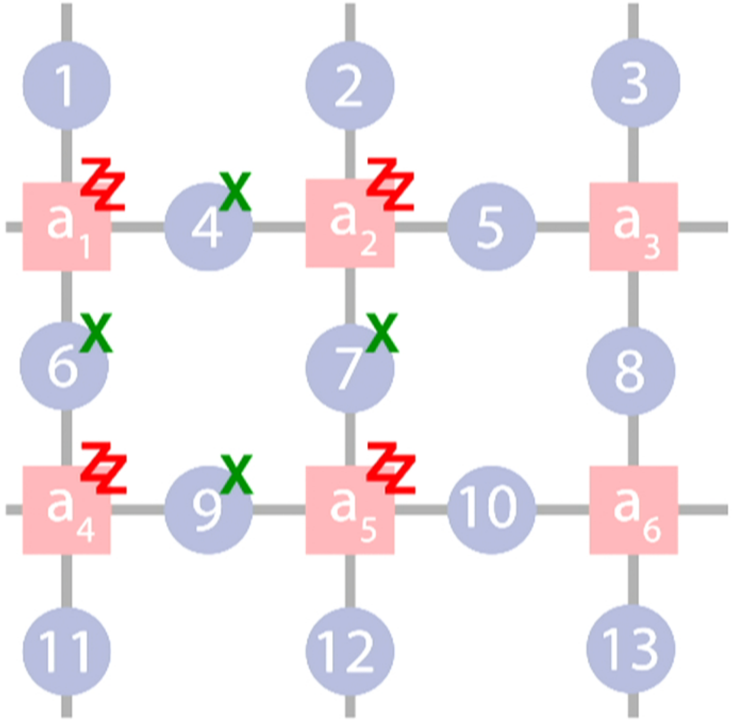
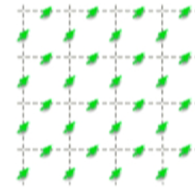
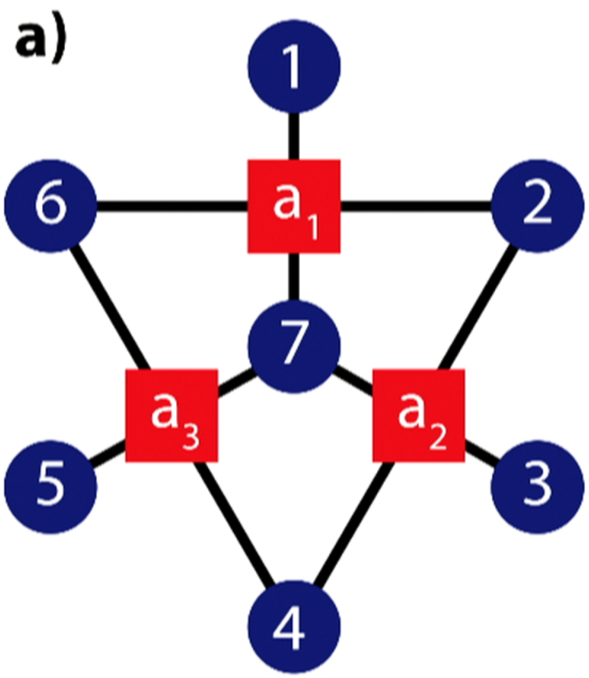


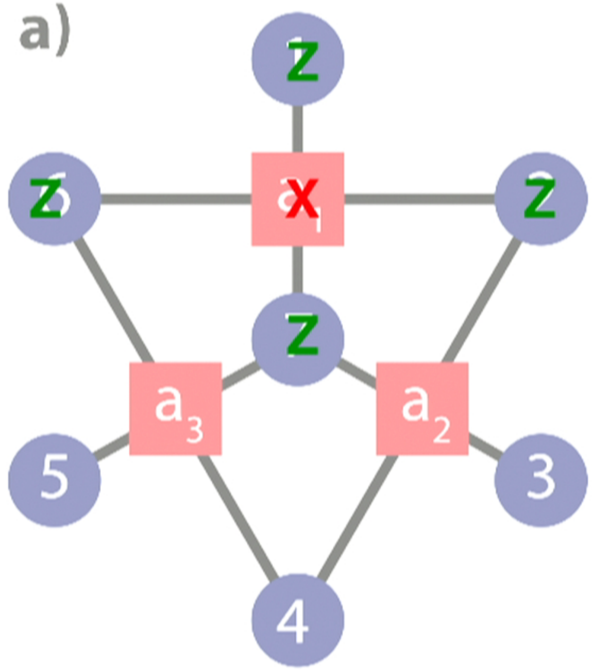


Steane code Cluster State Progenitors:

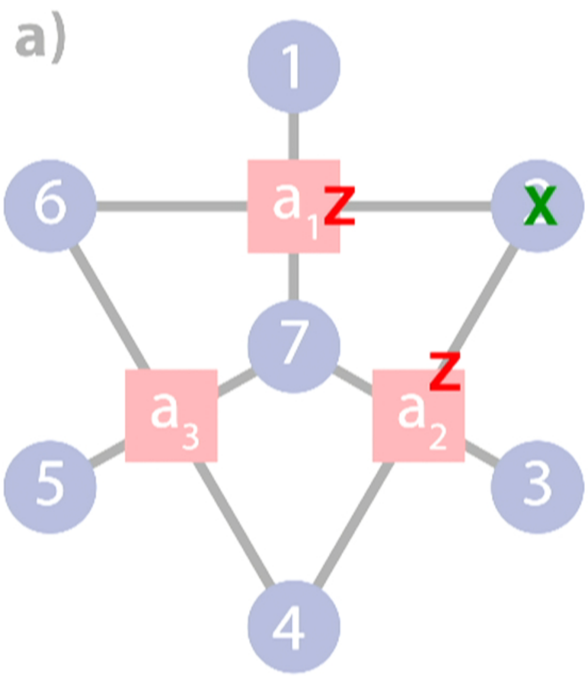


Steane code Cluster State Progenitors:

a)



Steane code Cluster State Progenitors:



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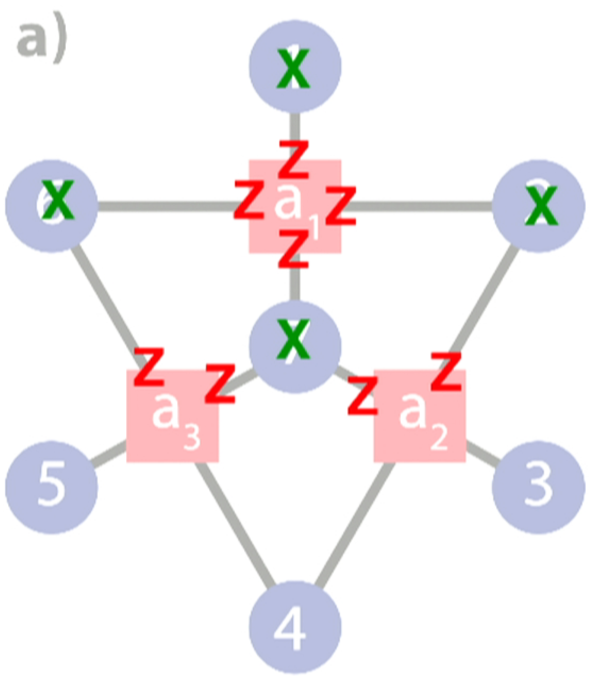
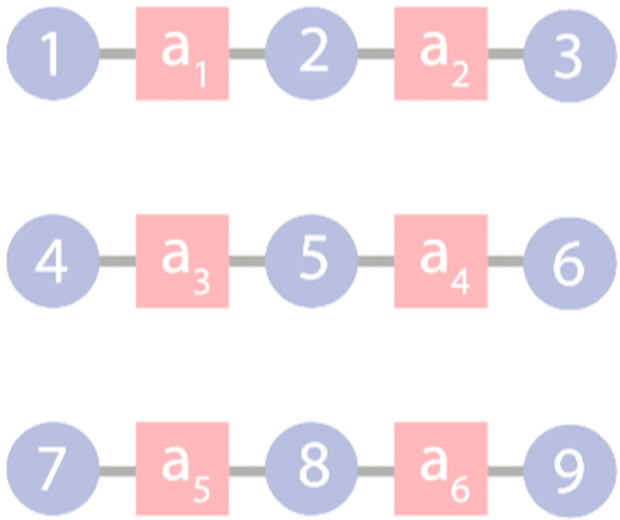
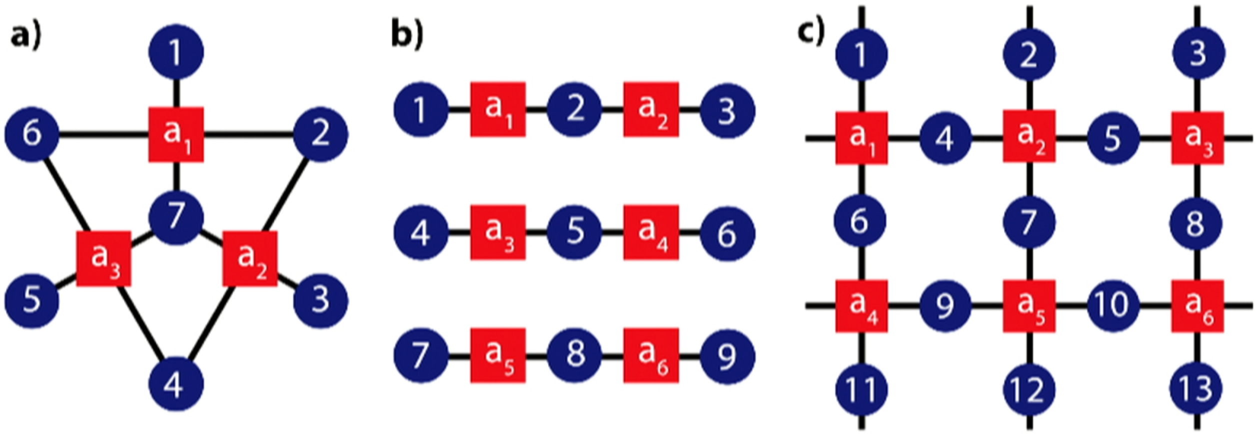


Shor code Cluster State Progenitors:



Assertion: can construct CSS code states from cluster states (defined on code qubits plus ancilla), by single-qubit X- measurement



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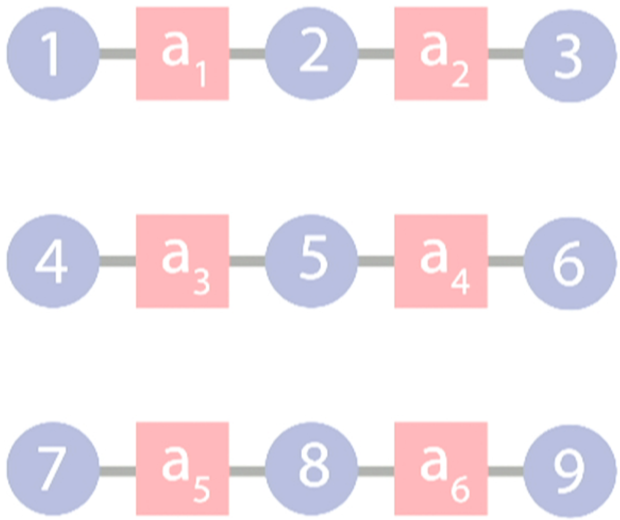
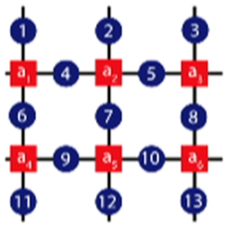
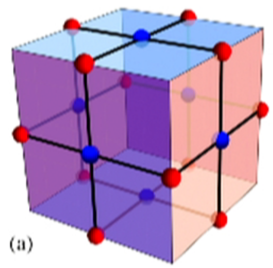


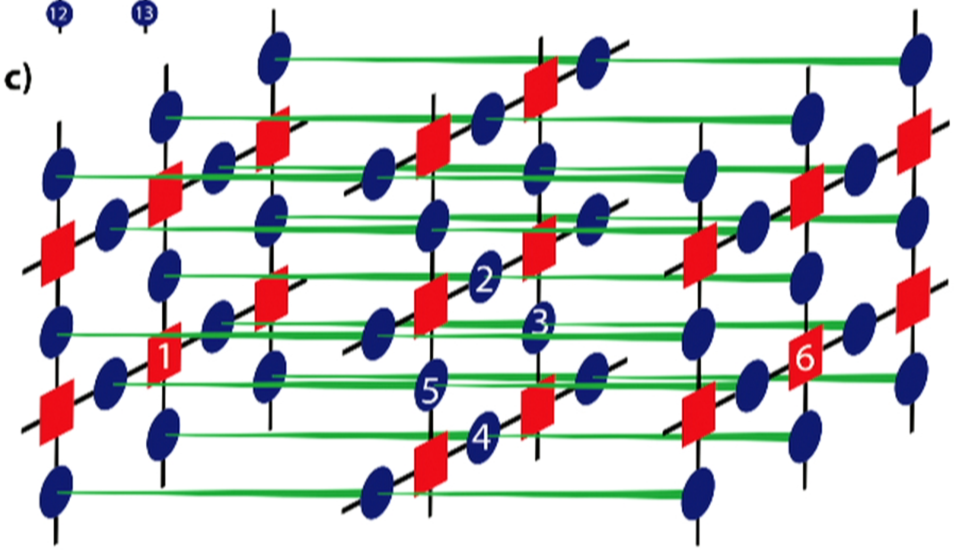
Raussendorf=Foliated Surface Code





(a)

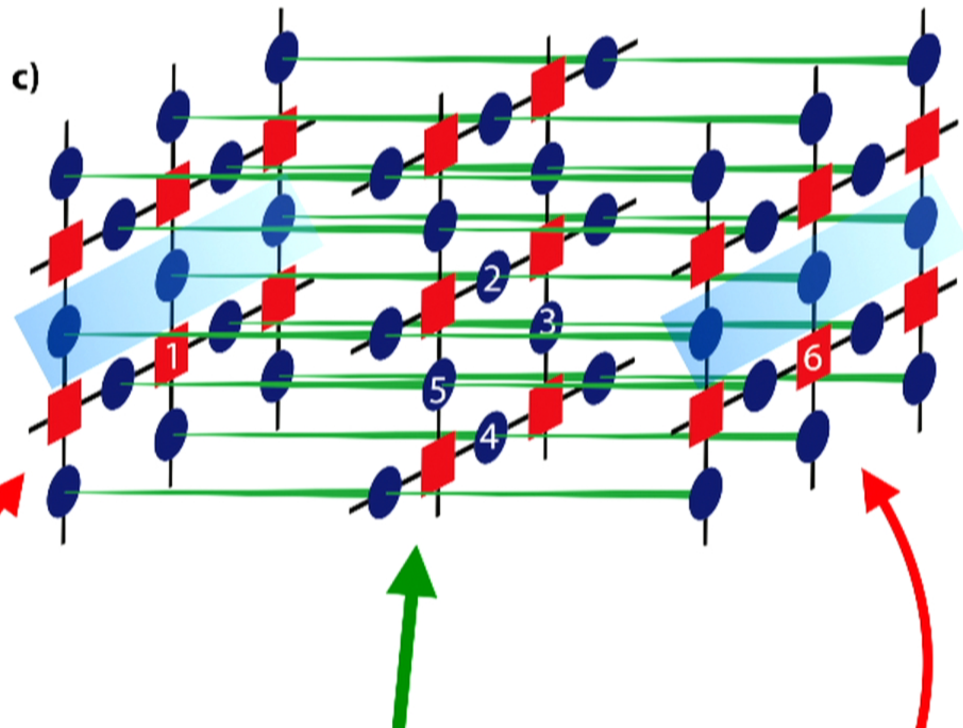
c)



Long-range quantum entanglement in noisy cluster states

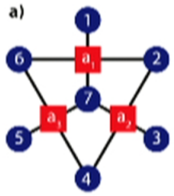
Robert Raussendorf, Sergey Bravyi, and Jim Harrington

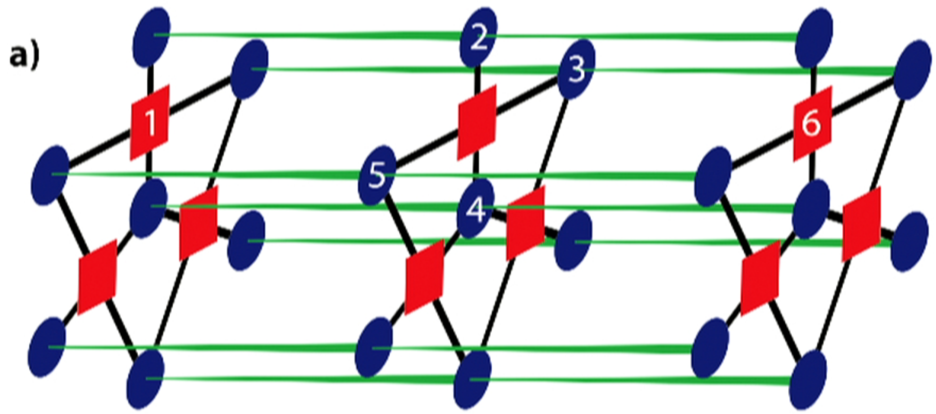
Phys. Rev. A **71**, 062313 – Published 14 June 2005



Measure “bulk” qubits in X basis leaves end surface code faces entangled: $X_L X_R$ & $Z_L Z_R$

Foliated Steane Code

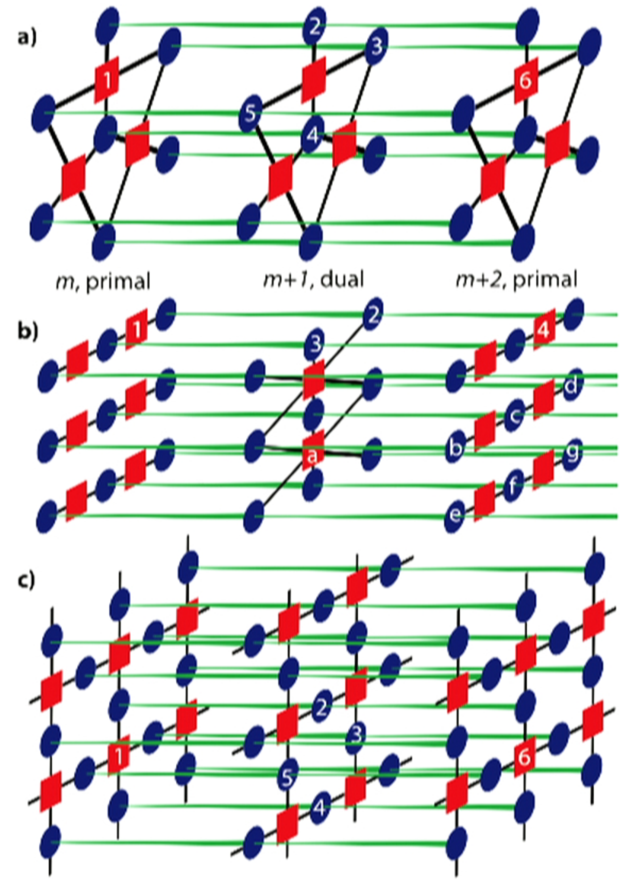


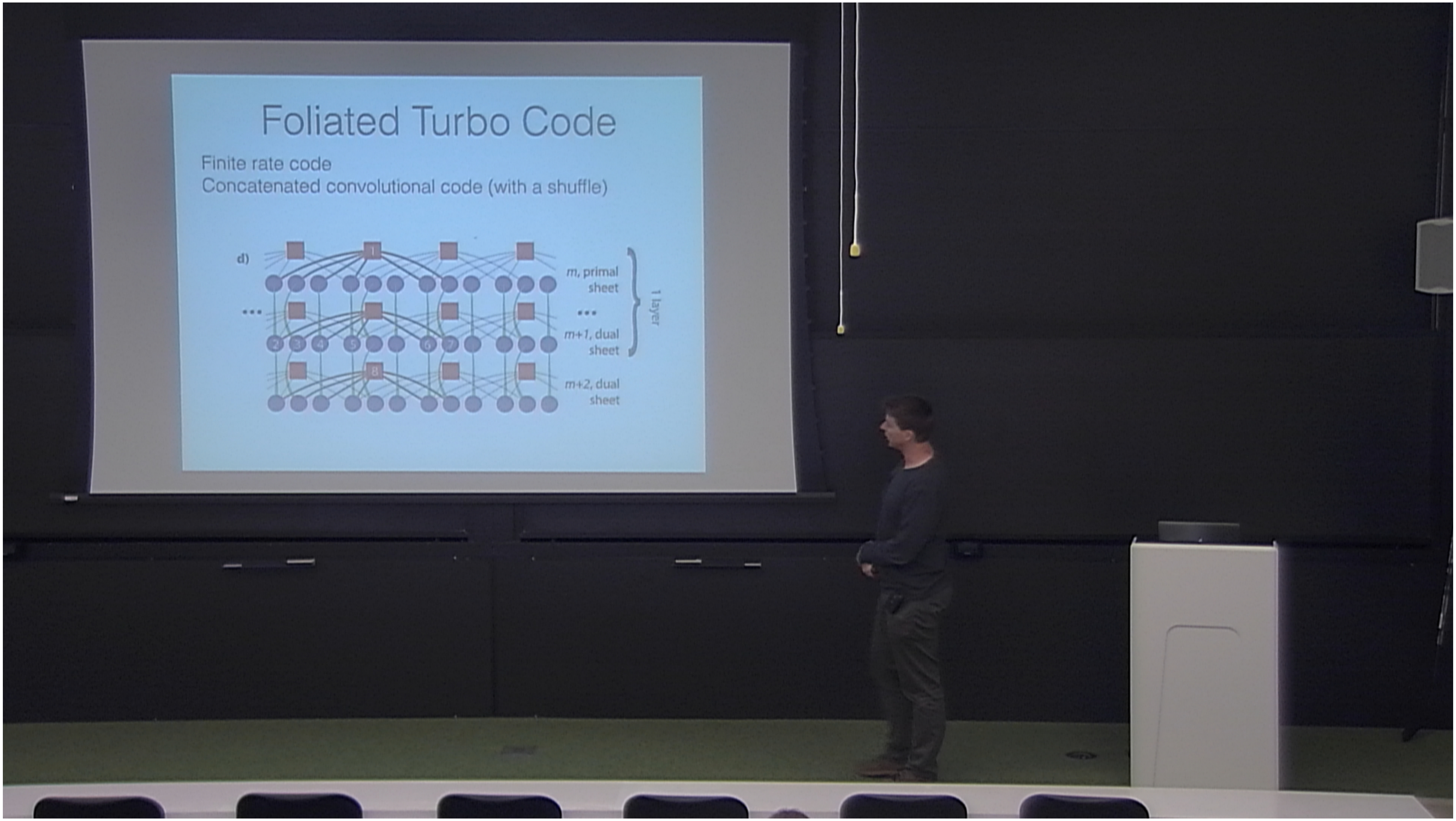


Assertion: can foliate code cluster states together.

Bulk measurements leave end faces entangled

Errors are detected by parity check operators

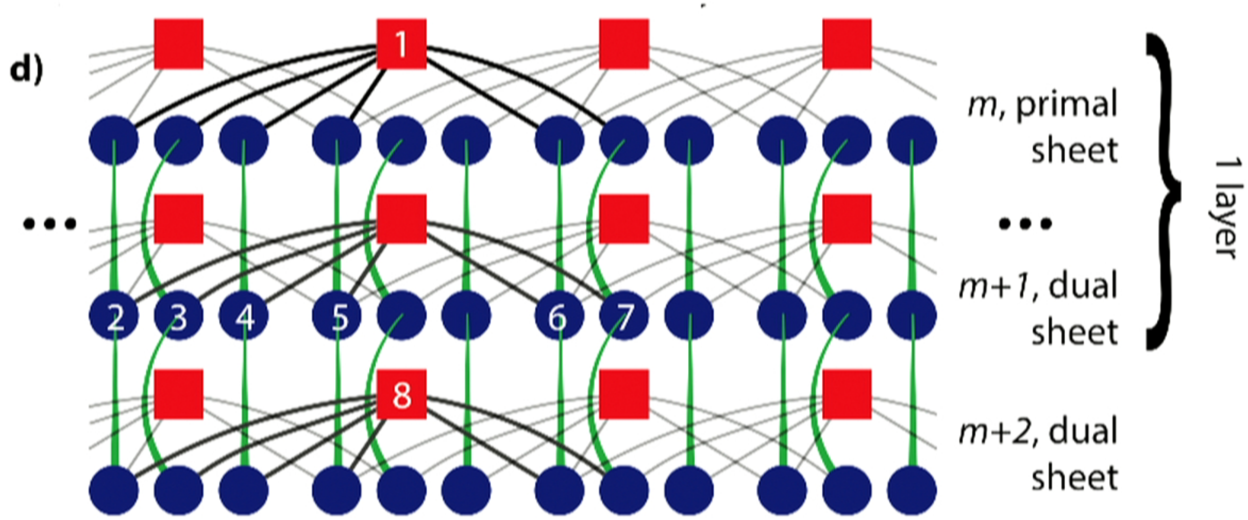



Foliated Turbo Code
Finite rate code
Concatenated convolutional code (with a shuffle)
d)
m, primal
sheet
m+1, dual
sheet
m+2, dual
sheet
1 layer

Foliated Turbo Code

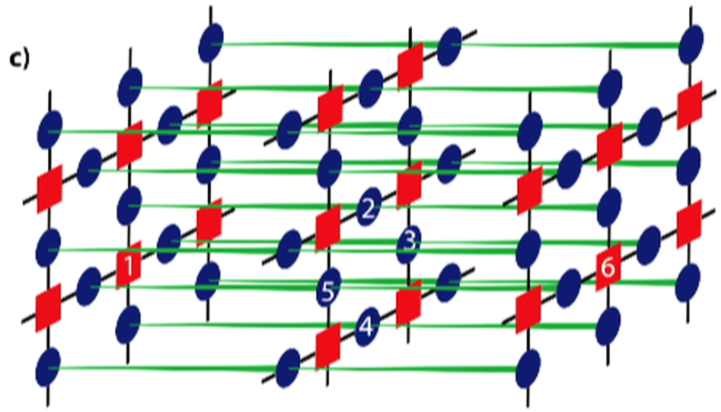
Finite rate code

Concatenated convolutional code (with a shuffle)



Error Decoding

- Assume CSS code has a “good” decoder
- Can we build a decoder for foliated codes using CSS decoder as a subroutine?
- n.b. not generally done for surface code



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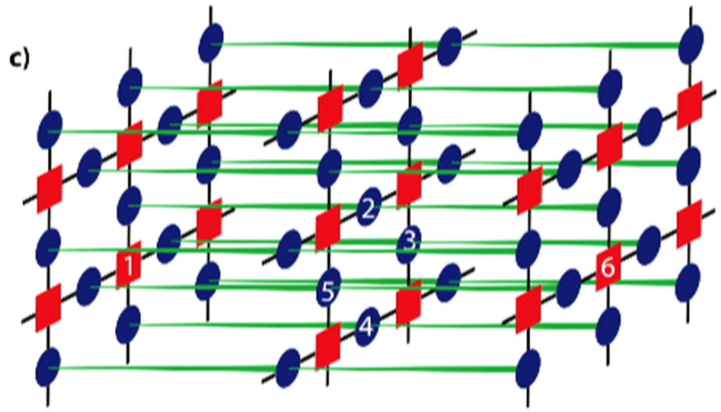
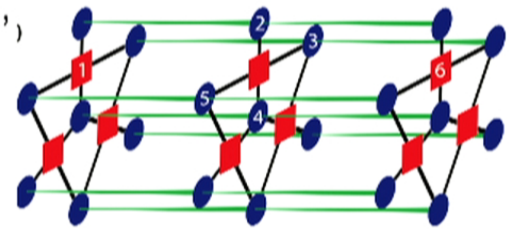


Heuristic Decoder

1. For each code qubit, *j*, in sheet *m*, CSS decoder calculates an in-sheet error model probability distribution,



$$P_m(\sigma_j \mid S_m \cup P_{m\pm1}(a_k))$$

subject to the measured code syndrome S_m, and an assumed error distribution, $P_{m\pm1}(a_k)$, for errors on ancilla qubits, a_k, in adjacent dual sheets.

2. From 1 fix code qubit error model, $P_m(\sigma_j)$, and calculate error model on the dual-sheet ancilla qubits, $P_{m\pm1}(a_k \mid S_{m\pm1} \cup P_m(\sigma_j))$

3. Iterate to step 1, using the result of Step 2 for $P_{m\pm1}(a_k)$, repeating until each error model converges.

Turbo Code CSS Decoder

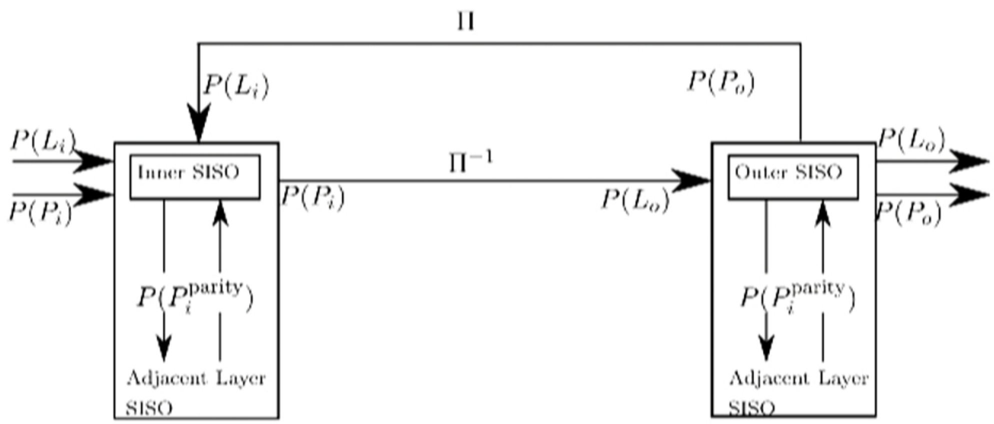
$$P = \begin{bmatrix} 1+D+D^2 & 1+D+D^3 & 1+D+D^2+D^3 & 1 \end{bmatrix}$$

$$H_{ft} = \begin{bmatrix} 1+D+D^2+D^3+D^4 & 1+D^2+D^3+D^5 & 1+D^2+D^3+D^4+D^5 & 1 \end{bmatrix}$$

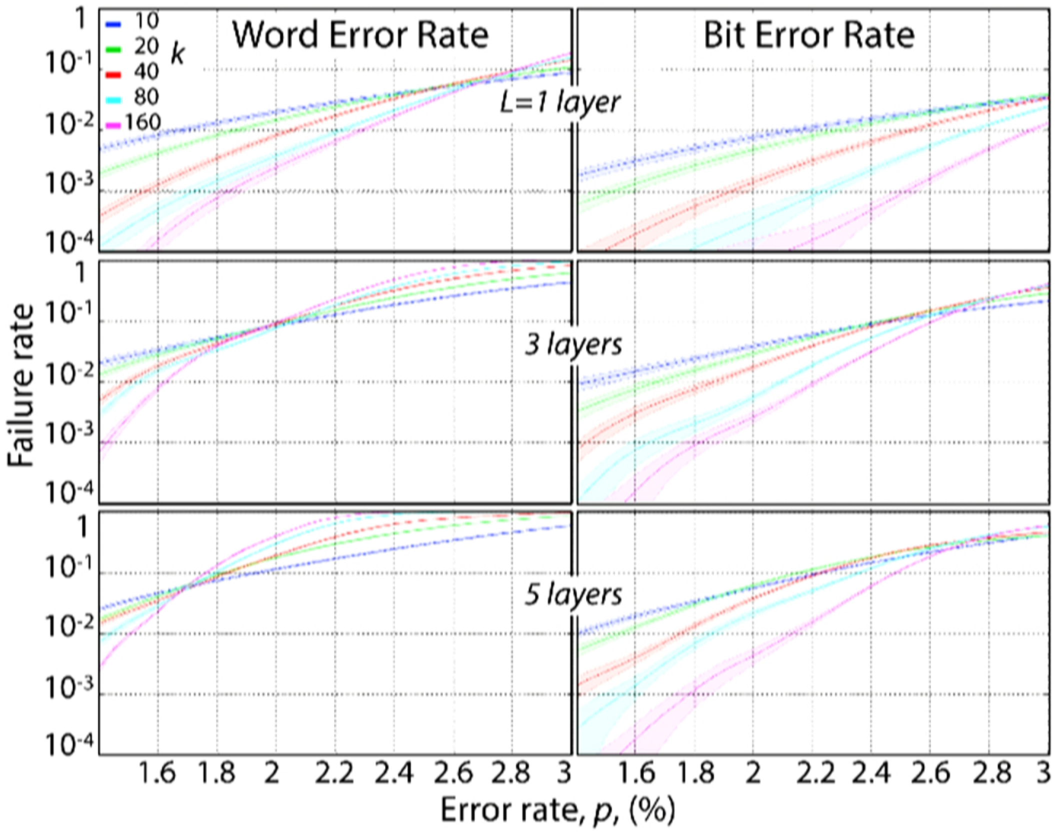
$$G_{ft}^T = \begin{bmatrix} D^2 & 1+D^2+D^3 & 1+D+D^3 & 0 \end{bmatrix}$$

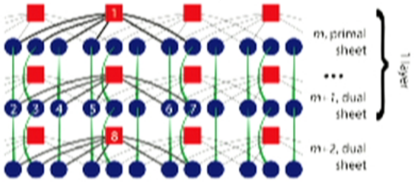
$$ISF_{ft} = \begin{bmatrix} 1+D & 1 & 1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} h_1 & h_2 & h_3 & & & \\ & h_1 & h_2 & h_3 & & \\ & & h_1 & h_2 & h_3 \ldots \end{bmatrix}$$



- Results for turbo code [[n, k=n/25, d=25]]





Conclusions

- Can clusterize CSS codes
- Can foliate clusterized CSS codes
- Heuristic decoder for foliated code using soft CSS decoder
- Promising performance from numerical studies
- Potential for quantum repeater networks with finite rate codes
- Code deformation, quasi-transversal codes etc for FTQC?