Title: Everything you wanted to know about the reality of the quantum state, but were afraid to ask Matt Pusey

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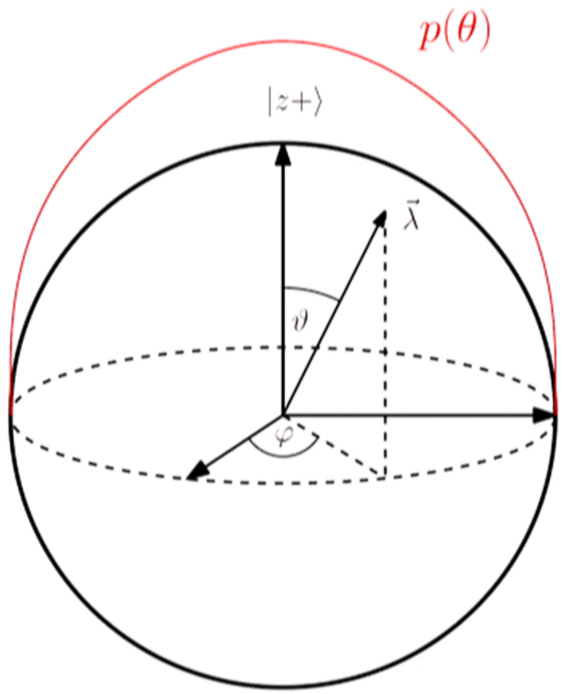
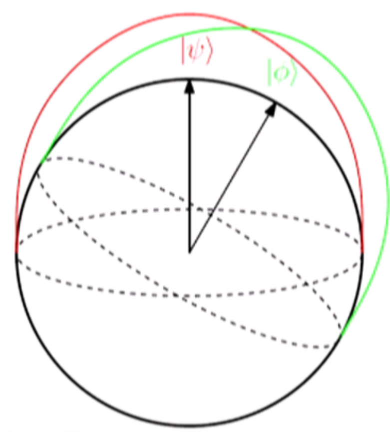
Abstract: <p>In this talk, I will outline the current state of the art in the study of the reality of the quantum state. The main theme will be that, although you cannot derive the reality of the quantum state in an ontological model without additional assumptions, you can place constraints on the amount of overlap between probability measures that begin to make psi-epistemic theories look implausible. These overlap bounds come from noncontextuality inequalities, and there are two types in the literature: those based on Cabello-Severini-Winter type inequalities and those based on Yu-Oh type inequalities. The latter type of overlap bound was not originally derived from noncontextuality, but thinking of them this way yields a new proof of the Yu-Oh inequality and gives rise to family of related inequalities. I will also explain why I think that most papers on ovelap bounds (including my own) have adopted sub-optimal measures, introduce better ones, and explain how this affects the choice of the best experimental protocol for demonstrating overlap bounds. </p>

The Kochen-Specker model for a qubit



$$\mu_{z+}(\Omega) = \int_\Omega p(\vartheta) \sin\vartheta d\vartheta d\varphi$$

$$p(\vartheta) = \begin{cases} \frac{1}{\pi}\cos\vartheta, & 0 \leq \vartheta \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \vartheta \leq \pi \end{cases}$$



S. Kochen and E. Specker, *J. Math. Mech.*, 17:59–87 (1967)

Models for arbitrary finite dimension

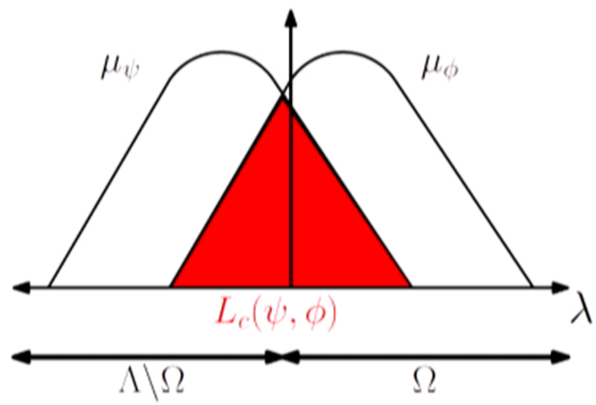
- Lewis et. al. provided a ψ-epistemic model for all finite d.
  - P. G. Lewis et. al., *Phys. Rev. Lett.* 109:150404 (2012) arXiv:1201.6554
- Aaronson et. al. provided a similar model in which every pair of nonorthogonal states is ontologically indistinct.
  - S. Aaronson et. al., *Phys. Rev. A* 88:032111 (2013) arXiv:1303.2834
- These models have the feature that, for a fixed inner product, the amount of overlap decreases with d.

Overlap bounds

Classical overlap

- *Classical overlap*:

$$L_c(\psi,\phi) := \inf_{\Omega\in\Sigma} \left[\mu_\psi(\Omega) + \mu_\phi(\Lambda\backslash\Omega)\right]$$



- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ if you know λ:

$$p_c(\psi,\phi) = \frac{1}{2}\left(2 - L_c(\psi,\phi)\right)$$

Quantum Symmetric overlap

- *Classical overlap*:

$$L_c(\psi,\phi) := \inf_{\Omega\in\Sigma}\left[\mu_\psi(\Omega) + \mu_\phi(\Lambda\backslash\Omega)\right]$$

- *Quantum overlap*:

$$L_q(\psi,\phi) := \inf_{0\leq E\leq I}\left[\langle\psi|\,E\,|\psi\rangle + \langle\phi|\,(I-E)\,|\phi\rangle\right]$$
$$= 1 - \sqrt{1 - |\langle\phi|\psi\rangle|^2}$$

- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ based on a quantum measurement:

$$p_q(\psi,\phi) = \frac{1}{2}\left(2 - L_q(\psi,\phi)\right)$$

ψ-ontology measures

- Given a set V of states, and another state $|\psi\rangle$, we can upper bound the average overlap

$$\langle L_c \rangle = \frac{1}{|V|} \sum_{|a\rangle \in V} L_c(\psi, a).$$

- Most works use this to bound the ratio:

$$k = \frac{\langle L_c \rangle}{\langle L_q \rangle}.$$

- Better to use the difference:
  - *Overlap deficit*: $\Delta L = \langle L_q \rangle - \langle L_c \rangle$

Previous results

	Dimension	$\|V\|$	$\langle L_c \rangle$	$\langle L_q \rangle$
Barrett et. al.[1]	Prime power $d \geq 4$	d^2	$1/d^2$	$1 - \sqrt{1 - 1/d}$
Leifer[2]	$d \geq 3$	2^{d-1}	$1/2^{d-1}$	$1 - \sqrt{1 - 1/d}$
Branciard[3]	$d \geq 4$	$n \geq 2$	$1/n$	$1 - \sqrt{1 - \frac{1}{4} n^{-1/(d-2)}}$
Amaral et. al.[4]	$d \geq n_j$	$n_j \geq ?$	$n_j^{\delta - 1}$	$1 - \sqrt{\frac{1}{2} + \epsilon}$

[1] J. Barrrett et. al., Phys. Rev. Lett. 112, 250403 (2014)
[2] ML, Phys. Rev. Lett. 112, 160404 (2014)
[3] C. Branciard, Phys. Rev. Lett. 113, 020409 (2014)
[4] B. Amaral et. al., Phys. Rev. A 92, 062125 (2015)

Optimizing for distinguishability deficit

	Optimal dimension	Optimal $\|V\|$	ΔL
Barrett et. al.	4	16	0.0715
Leifer	7	64	0.0586
Branciard	4	$n \to \infty$	0.134
Amaral et. al.	$d \to \infty$	$n_j \to \infty$	0.293

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Experiment

- Ringbauer et. al.[5] experiment (based on Branciard's construction) obtained:

$$k \leq 0.690 \pm 0.001$$

$$\Delta L \geq 0.047 \pm 0.010$$

- My analysis suggests larger ΔL should be obtainable from the Barrett et. al. construction.

[5] M. Ringbauer et. al. Nature Physics 11, 249–254 (2015).

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Overlap bounds from contextuality

Kochen-Specker noncontextuality

- Let $\mathcal{M}$ be a set of orthonormal bases in $\mathbb{C}^d$.
- An ontological model for $\mathcal{M}$ is *Kochen Specker noncontextual* if it is
  - *Outcome deterministic*: $\text{Pr}(a|M, \lambda) \in \{0, 1\}$
  - *Measurement noncontextual*: If there exist $M, N \in \mathcal{M}$ and $|a\rangle$ such that $|a\rangle \in M$ and $|a\rangle \in N$ then

$$\text{Pr}(a|M, \cdot) = \text{Pr}(a|N, \cdot).$$

- Define:

$$\Gamma_a^M = \{\lambda \in \Lambda | \text{Pr}(a|M, \lambda) = 1\} \qquad \Gamma_a = \bigcap_{\{M \in \mathcal{M} | |a\rangle \in M\}} \Gamma_a^M$$

Theorem: There exists a KS noncontextual model for $\mathcal{M}$ iff there exists a model where, for all $|\psi\rangle$, $M \in \mathcal{M}$, $|a\rangle \in M$,

$$\int_\Lambda \text{Pr}(a|M, \lambda) d\mu_\psi(\lambda) = \mu_\psi(\Gamma_a).$$

Deriving overlap bounds

- For a (finite) set V of states, a noncontextuality inequality is a bound of the form

$$\sum_{|a\rangle \in V} \mu_\psi(\Gamma_a) \leq \gamma.$$

- Let $\mathcal{M}$ be a covering set of bases for V. We have

$$\int_\Lambda \text{Pr}(a|M,\lambda) d\mu_a(\lambda) = |\langle a|a\rangle|^2 = 1$$

and since $\text{Pr}(a|M,\lambda) \leq 1$ this implies that $\mu_a(\Gamma_a^M) = 1$.

- Since $\Gamma_a = \cap_{M\in\mathcal{M}||a\rangle\in M} \Gamma_a^M$ is a finite intersection of measure one sets, we also have

$$\mu_a(\Gamma_a) = 1.$$

Deriving overlap bounds

- Now,

$$L_c(\psi, a) = \inf_{\Omega \in \Sigma} [\mu_\psi(\Omega) + \mu_a(\Lambda \backslash \Omega)]$$
$$\leq \mu_\psi(\Gamma_a) + \mu_a(\Lambda \backslash \Gamma_a)$$

- We just showed that $\mu_a(\Gamma_a) = 1$, so $\mu_a(\Lambda \backslash \Gamma_a) = 0$, and hence

$$L_c(\psi, a) \leq \mu_\psi(\Gamma_a).$$

- Hence,

$$\sum_{|a\rangle \in V} L_c(\psi, a) \leq \sum_{|a\rangle \in V} \mu_\psi(\Gamma_a) \leq \gamma.$$

General results

- Using Cabello, Severini and Winter's results[6], for a set of states V, we can derive

$$\frac{1}{|V|}\sum_{|a\rangle\in V} L_c(\psi, a) \leq \frac{\alpha(G)}{|V|},$$

where $\alpha(G)$ is the *independence number* of the *orthogonality graph* of V.

- Better bounds come from a different technique, introduced by Barrett et. al.[7], that was not based on contextuality.

- It turns out that their method is contextuality in disguise though.

[6] A. Cabello, S. Severini, A. Winter, Phys. Rev. Lett. 112:040401 (2014).
[7] J. Barrrett et. al., Phys. Rev. Lett. 112, 250403 (2014)

Antidistinguishability-based noncontextuality inequalities

Antidistinguishability

- Definition: A set $V = \{|a_j\rangle\}_{j=1}^{d}$ of states in $\mathbb{C}^d$ is *antidistinguishable* if there exists an orthonormal basis $\{\left|a_j^{\perp}\right\rangle\}_{j=1}^{d}$ such that, for all j,

$$\left|\left\langle a_j^{\perp}\middle|a_j\right\rangle\right|^2 = 0.$$

- Example:

$$|a_1\rangle = (1,0,0) \qquad \left|a_1^{\perp}\right\rangle = (0,1,0)$$

$$|a_2\rangle = (1,1,1) \qquad \left|a_2^{\perp}\right\rangle = (1,0,-1)$$

$$|a_3\rangle = (-1,1,1) \qquad \left|a_3^{\perp}\right\rangle = (1,0,1)$$

Implication for ontological models

- Theorem: If V is antidistinguishable then

$$\cap_{j=1}^{d} \Gamma_{a_j} = \emptyset.$$

- Proof: Because ontic states in $\cap_{j=1}^{d} \Gamma_{a_j}$ would have to assign probability 0 to all of the measurement outcomes $\left| a_j^{\perp} \right\rangle$.

Bonferroni inequalities

- On any measure space, the inclusion-exclusion principle states:

$$\mu(\cup_j X_j) = \sum_j \mu(X_j) - \sum_{j<k} \mu(X_j \cap X_k) + \sum_{j<k<m} \mu(X_j \cap X_k \cap X_m) - \ldots$$

- Bonferroni: Terminating this sequence gives an alternating sequence of upper and lower bounds, e.g.

$$\mu(\cup_j X_j) \leq \sum_j \mu(X_j)$$

$$\mu(\cup_j X_j) \geq \sum_j \mu(X_j) - \sum_{j<k} \mu(X_j \cap X_k).$$

- Set $X_j = \Gamma_\psi \cap \Gamma_{a_j}$ and note that $\mu_\psi(\Gamma_\psi) = 1$. Second inequality gives

$$1 \geq \sum_j \mu_\psi(\Gamma_{a_j}) - \sum_{j<k} \mu(\Gamma_\psi \cap \Gamma_{a_j} \cap \Gamma_{a_k})$$

Noncontextuality inequalities

- From previous slide:

$$1 \geq \sum_j \mu_\psi(\Gamma_{a_j}) - \sum_{j<k} \mu(\Gamma_\psi \cap \Gamma_{a_j} \cap \Gamma_{a_k})$$

- So, if $\{|\psi\rangle, |a_j\rangle, |a_k\rangle\}$ are antidistinguishable for all $j \neq k$, we get

$$\sum_j \mu_\psi(\Gamma_{a_j}) \leq 1.$$

- Example: Yu-Oh inequality[8]

$$|\psi\rangle = (1,0,0)^T$$

$$|a_0\rangle = (1,1,1) \qquad |a_1\rangle = (-1,1,1)$$

$$|a_2\rangle = (1,-1,1) \qquad |a_3\rangle = (1,1,-1)$$

[8] S. Yu, C. Oh, Phys. Rev. Lett. 108, 030402 (2012)

Generalization of Yu-Oh

- Let,

$$|\psi\rangle = (1, 0, 0, \cdots, 0).$$

- For $\boldsymbol{x} \in \{0,1\}^d$, let

$$|a_{\boldsymbol{x}}\rangle = (-1^{x_1}, -1^{x_2}, \ldots, -1^{x_n}).$$

- Then, $\{|\psi\rangle, |a_{\boldsymbol{x}}\rangle, |a_{\boldsymbol{x}'}\rangle\}$ is antidistinguishable for $\boldsymbol{x} \neq \boldsymbol{x}'$, so

$$\sum_{\boldsymbol{x}} \mu_\psi(\Gamma_{a_{\boldsymbol{x}}}) \leq 1$$

- In contrast, using CSW method on this set only gives

$$\sum_{\boldsymbol{x}} \mu_\psi(\Gamma_{a_{\boldsymbol{x}}}) \leq (2-\epsilon)^d$$

for some $\epsilon > 0$.

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Summary and Open questions

- Summary:
  - Several bounds exist showing $k \to 0$. Harder to get $\Delta L \approx 1$. Best current bound is $\Delta L \approx 0.293$.
  - Any noncontextuality inequality is an overlap bound.
  - Methods developed to bound overlaps yield new contextuality inequalities, sometimes with much tighter bounds.
- Open questions:
  - Error analysis for arbitrary noncontextuality-based overlap bounds.
  - What is the best possible bound on ΔL?
  - Applications in quantum information.

- Become neo-Copenhagen.
- Adopt a more exotic ontology:
  - Nonstandard logics and probability theories.
  - Ironic many-worlds.
  - Retrocausality.
  - Relationalism.

References

- Review articles:
  - ML, “Is the quantum state real? An extended review of ψ-ontology theorems”, *Quanta* 3:67–155 (2014), arXiv:1409.1570.
  - D. Jennings and ML, “No Return to Classical Reality”, *Contemp. Phys.* 56 (2015). arXiv:1501.03202.
- Overlap bounds and contextuality:
  - ML and O. Maroney, “Maximally epistemic interpretations of the quantum state and contextuality”, *Phys. Rev. Lett.* 110:120401 (2013) arXiv:1208.5132.
  - ML, “ψ-epistemic models are exponentially bad at explaining the distinguishability of quantum states ”*Phys. Rev. Lett.* 112:160404 (2014) arXiv:1401.7996.
  - ML, “Bounds on the epistemic interpretation of the quantum state from contextuality inequalities” in preparation.