Title: Interference Energy Spectrum of the Infinite Square Well

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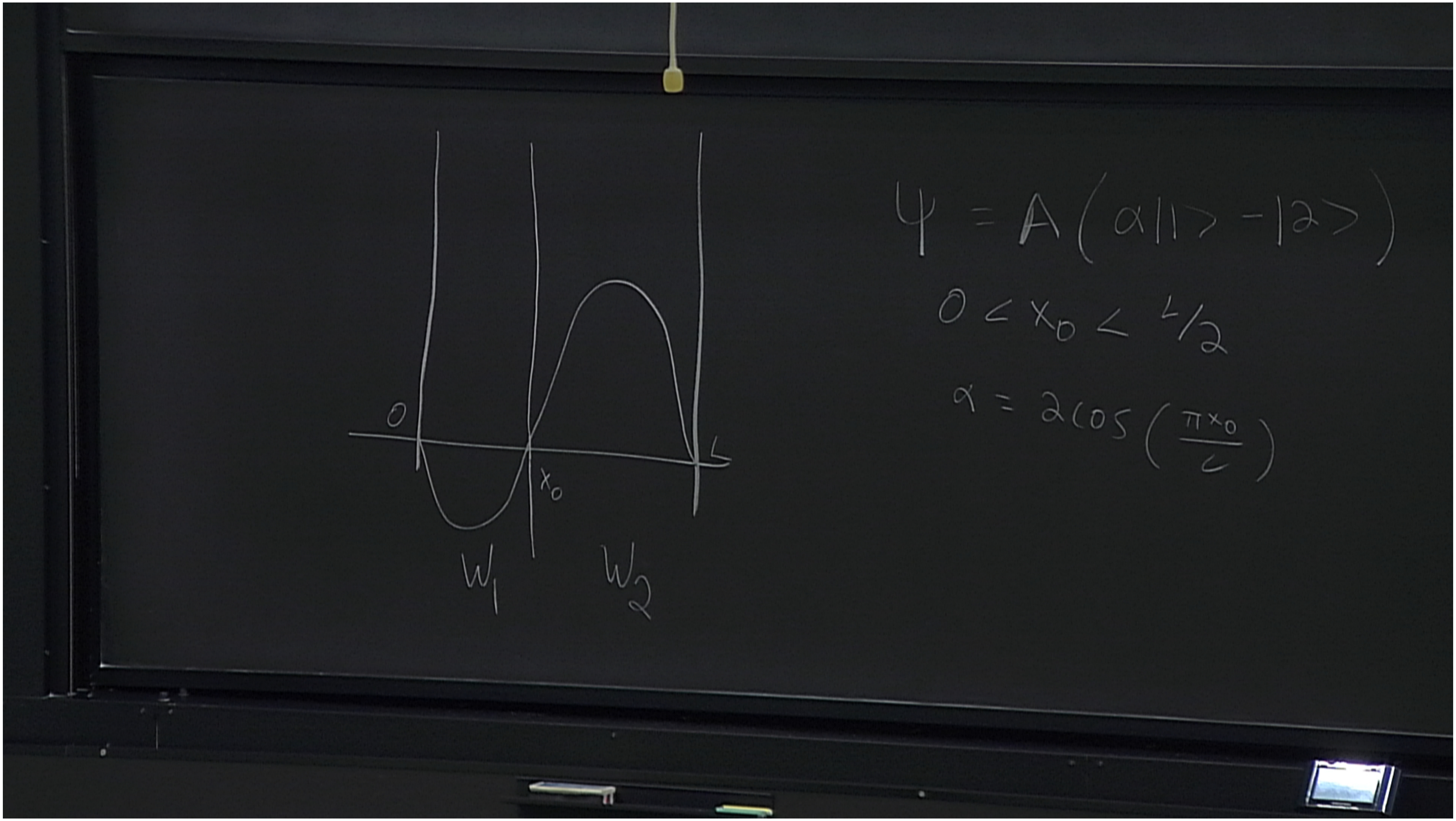
Abstract: <p>Certain superposition states of the 1-D infinite square well have transient zeros at locations other than the nodes of the eigenstates that comprise them. It is shown that if an infinite potential barrier is suddenly raised at some or all of these zeros, the well can be split into multiple adjacent infinite square wells without affecting the wavefunction. While the average energy of the state was unchanged, this splitting effects a change of the energy eigenbasis of the state to a basis that does not commute with the original, and a subsequent measurement of the energy now reveals a completely different spectrum, which we call the interference energy spectrum of the state. Numerical simulations were used to verify that a barrier can be rapidly raised at a zero of the wavefunction without significantly affecting it. The concept of state inherent energy spectra is further explored as it relates to quantum state time evolution, probability theory, and weak value measurements. Of particular interest, this procedure can result in measurable energies that are greater than the energy of the highest mode in the original superposition, raising questions about the conservation of energy akin to those that have been raised in the study of superoscillations.</p>


0
x_0
L
W_1
W_2
$\psi = A(\alpha|1\rangle - |2\rangle)$
$0 < x_0 < L/2$
$\alpha = 2\cos\left(\frac{\pi x_0}{L}\right)$

$$\rho^0 = 1 = \int_0^L \bar{\psi}\psi\,dx = \underbrace{\int_0^{x_0} \bar{\psi}\psi\,dx}_{\rho_1^F} + \underbrace{\int_{x_0}^L \bar{\psi}\psi\,dx}_{\rho_2^F}$$

$$\langle E\rangle_0 = \int_0^L \bar{\psi}\hat{H}\psi\,dx \stackrel{?}{=} \int_0^{x_0} \bar{\psi}\hat{H}\psi\,dx + \int_{x_0}^L \bar{\psi}\hat{H}\psi\,dx = \langle E\rangle_F$$

$$E_n^{w_1} = \frac{\pi^2 \hbar^2 n^2}{2M x_0^2}$$

$$E_m^{w_2} = \frac{\pi^2 \hbar^2 m^2}{2M (L - x_0)^2}$$

$$a_n = \int_0^{x_0} \psi \sqrt{\frac{2}{x_0}} \sin\left(\frac{\pi n x}{x_0}\right) dx$$

$$b_m = \ldots\ldots$$

$$P(E_n^{w_1}) = |a_n|^2$$

$$P(E_m^{w_2}) = |b_m|^2$$

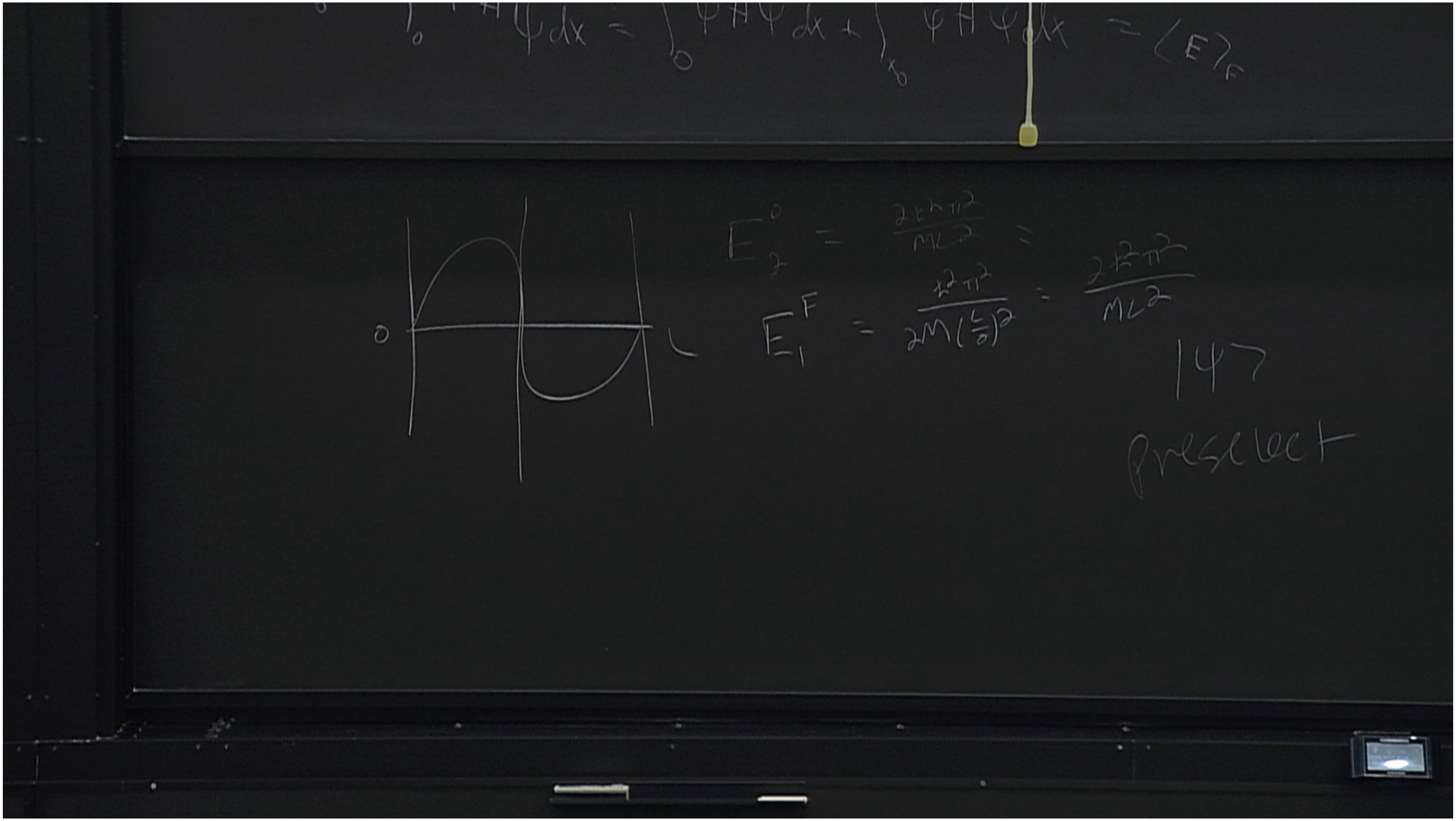
$$\frac{\pi n x}{x_0}\Big) dx$$

$$\sum_n P(E_n^{w_1}) + \sum_m P(E_m^{w}) = 1$$

$$|a_n|^2$$

$$\langle E \rangle_F = \sum_n P(E_n^{w_1}) E_n^{w_1} + \sum_m P(E_m^{w_2}) E_m^{w_2} = \langle E \rangle_0$$

$$|a_m|^2$$


$E_2^0 = \frac{2\hbar^2\pi^2}{mL^2} =$
$E_1^F = \frac{\hbar^2\pi^2}{2m(\frac{L}{2})^2} = \frac{2\hbar^2\pi^2}{mL^2}$
$|\psi\rangle$
preselect