

Title: Information is the key!

Date: Jun 14, 2016 03:30 PM

URL: <http://pirsa.org/16060090>

Abstract: 

Most physicists take it for granted that the experimental violation of Bell's inequality provides evidence that it is not possible to completely describe the state of a physical system in terms of purely local information when this system is entangled with some other system. We disagree. Provided we redefine appropriately what is the information-theoretic state of a quantum system, it becomes possible to recover the whole from the description of its parts. This is in sharp contrast with the standard formalism of quantum mechanics in which the density matrix provides all there is to say about the state of a system. According to our formalism, there is no need to invoke supernatural nonlocality in order to explain everything standard quantum mechanics tells us that we can observe. We show, however, that this is inconsistent with the usual belief held among Everettians that the universal wavefunction can be taken as the complete representation of reality. Inspired by Plato and Kant, we introduce and contrast the notions of noumenal and phenomenal states of physical systems: the former corresponds to the complete but unknowable state of the system and the latter to what can be perceived about it with the help of arbitrary technology. We exhibit an explicit epimorphism from the former to the latter, which explains the relationship between all that there is and all that can be apprehended. Joint work with Paul Raymond-Robichaud

# Information is the Key!

Gilles Brassard      Paul Raymond-Robichaud  
Université de Montréal



# nature physics

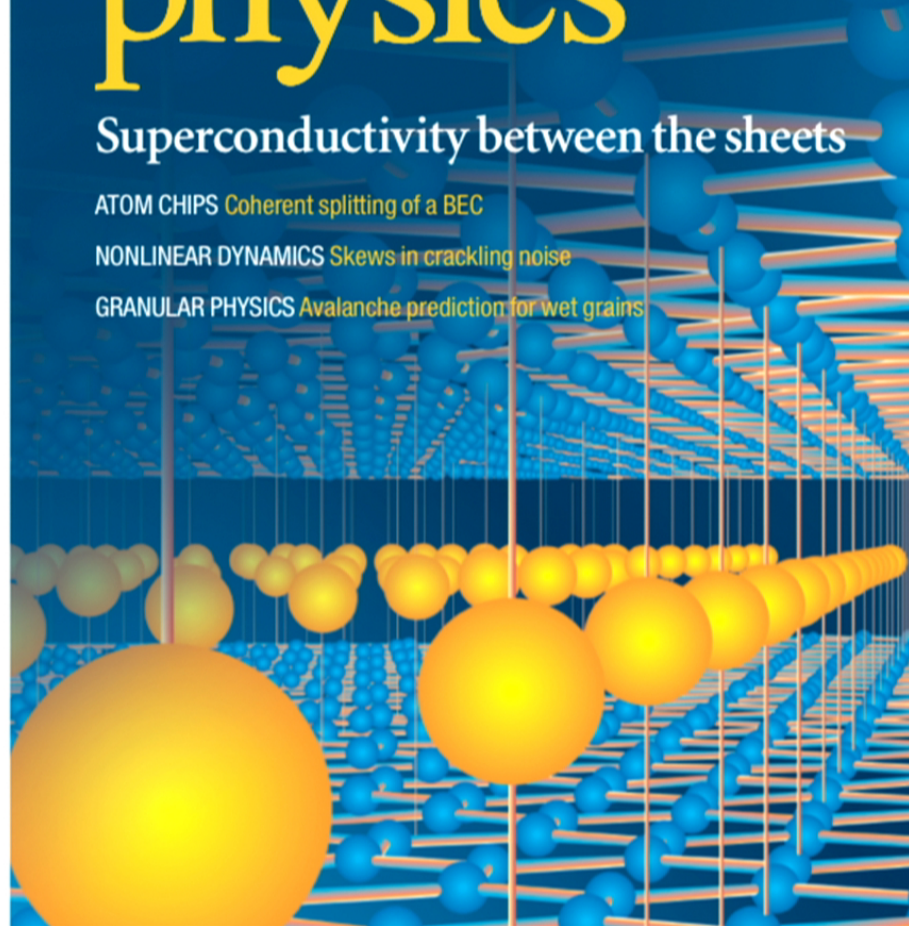
VOL. 1 NO. 1 October 2005  
[www.nature.com/naturephysics](http://www.nature.com/naturephysics)

## Superconductivity between the sheets

ATOM CHIPS Coherent splitting of a BEC

NONLINEAR DYNAMICS Skews in crackling noise

GRANULAR PHYSICS Avalanche prediction for wet grains



## COMMENTARY

# Is information the key?

GILLES BRASSARD

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Quantum information science has brought us novel means of calculation and communication. But could its theorems hold the key to understanding the quantum world at its most profound level? Do the truly fundamental laws of nature concern — not waves and particles — but information?

This year marks the centenary of quantum mechanics. Despite earlier work by Max Planck, it was Albert Einstein's Nobel prize-winning 1905 paper<sup>1</sup> on the photoelectric effect that gave us what is arguably the greatest scientific theory of all time. Subsequently, the stones that make up the exquisite structure of quantum mechanics were laid out, one by one, by a stream of legendary giants such as Niels Bohr, Erwin Schrödinger and Werner Heisenberg — sometimes to the horror of Einstein. An almost inevitable consequence of this collective foundational effort over so many years is that quantum mechanics, for all its elegance, is built upon a rather disjointed, *ad hoc* set of axioms.

Quantum mechanics has forced us to rethink the nature of the physical world, its teachings often running counter to our misleading macroscopic experience. It is time to pause and reflect on what we've learned in the course of these 100 years. Alongside Christopher Fuchs<sup>2</sup>, I contend that there is a fresh perspective to be taken on the axioms of quantum mechanics that could yield a more satisfactory foundation for the theory.

### NEW HORIZONS

Quantum mechanics has changed our outlook on the world. The transistor, the laser, superconductivity, the atomic bomb — these early applications of the theory are but a few among those that have reshaped the way we live. The transistor made possible a dramatic increase in computation speed. However, given enough time, cog-and-wheel devices such as Charles Babbage's analytical engine are, in principle, capable of the same calculations. In a very real sense, the modern electronic computer is essentially a classical

device. Could genuinely quantum-mechanical effects be harnessed for computing purposes?

In the early 1980s, it occurred to Richard Feynman<sup>3</sup> and David Deutsch<sup>4</sup> that a quantum computer could become so efficient that it would far outperform its classical counterpart. For example, an atom can be simultaneously in its ground and excited states. If we assign classical bit 0 to one state and bit 1 to the other (Fig. 1), this gives us a quantum bit, or qubit. If we string together ten qubits, they can be collectively in all 1024 classical states of ten bits, and we can compute using all those states in parallel. If we replace those ten qubits by one thousand, we obtain  $2^{1000}$  (roughly  $10^{301}$ ) simultaneous operations. This entails an amount of parallelism that could not be matched by a classical computer the size of the Universe, in which each elementary particle would be harnessed as a processing unit.

Quantum computing was at first regarded as a mere theoretical concept, but interest in it grew when Peter Shor discovered a way to use its capabilities to factorize large numbers efficiently<sup>5</sup>. Such a computer would threaten the public-key cryptographic schemes currently in use, in particular for the secure transmission of credit card numbers over the Internet. Electronic commerce in its current form is saved from a catastrophic collapse only because the construction of a full-size quantum computer is, for the moment, eluding our technological capabilities. And we can only shiver to think of the effect that such a collapse of classical cryptography could have on national security.

Even though the potential of quantum computers is mind-boggling, that does not change the theoretical notion of what is computable. The mathematical theory of computability is rooted in

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*nature physics* | VOL 1 | OCTOBER 2005 | www.nature.com/naturephysics

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## COMMENTARY

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# John Archibald Wheeler



By the late 1970s and onward, [...] to the best students who came asking for a research project, Wheeler would say,

"Derive quantum theory from an information theoretic principle".

Platonik / Kantian view

Noumenal  
world



# Platonic / Kantian View

Noumenal  
world

All that there is  
(*Ding an sich*)



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Kant: The noumenal world may exist, but it is completely unknowable through human sensation

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Phenomenal  
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world

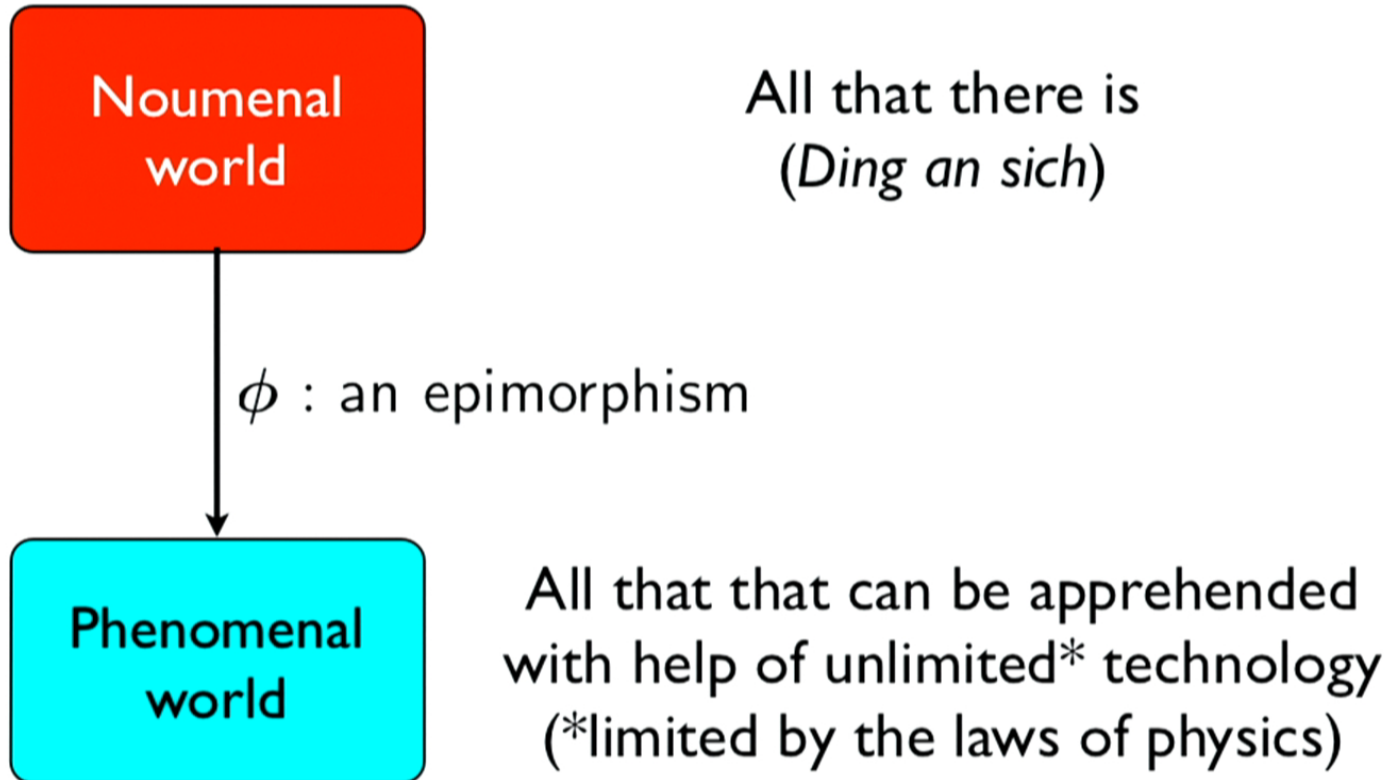
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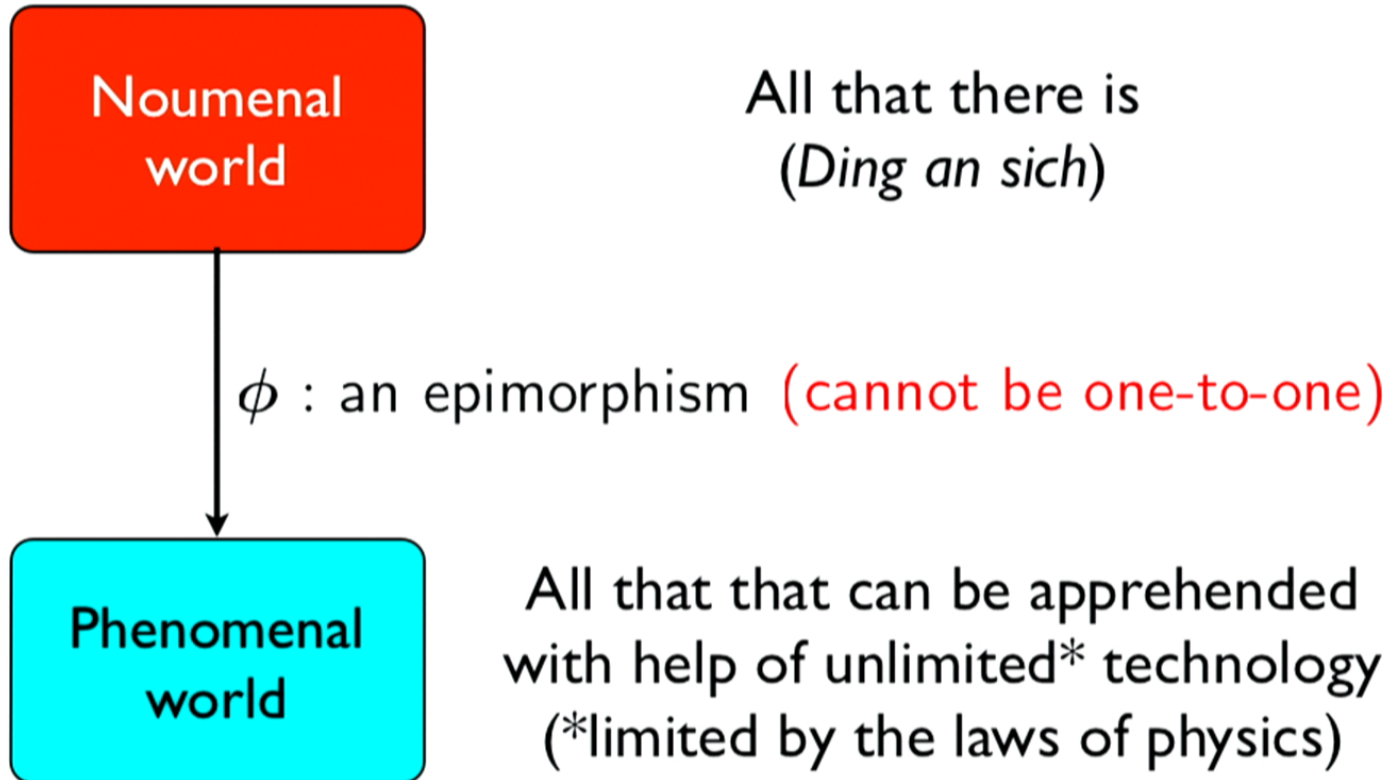
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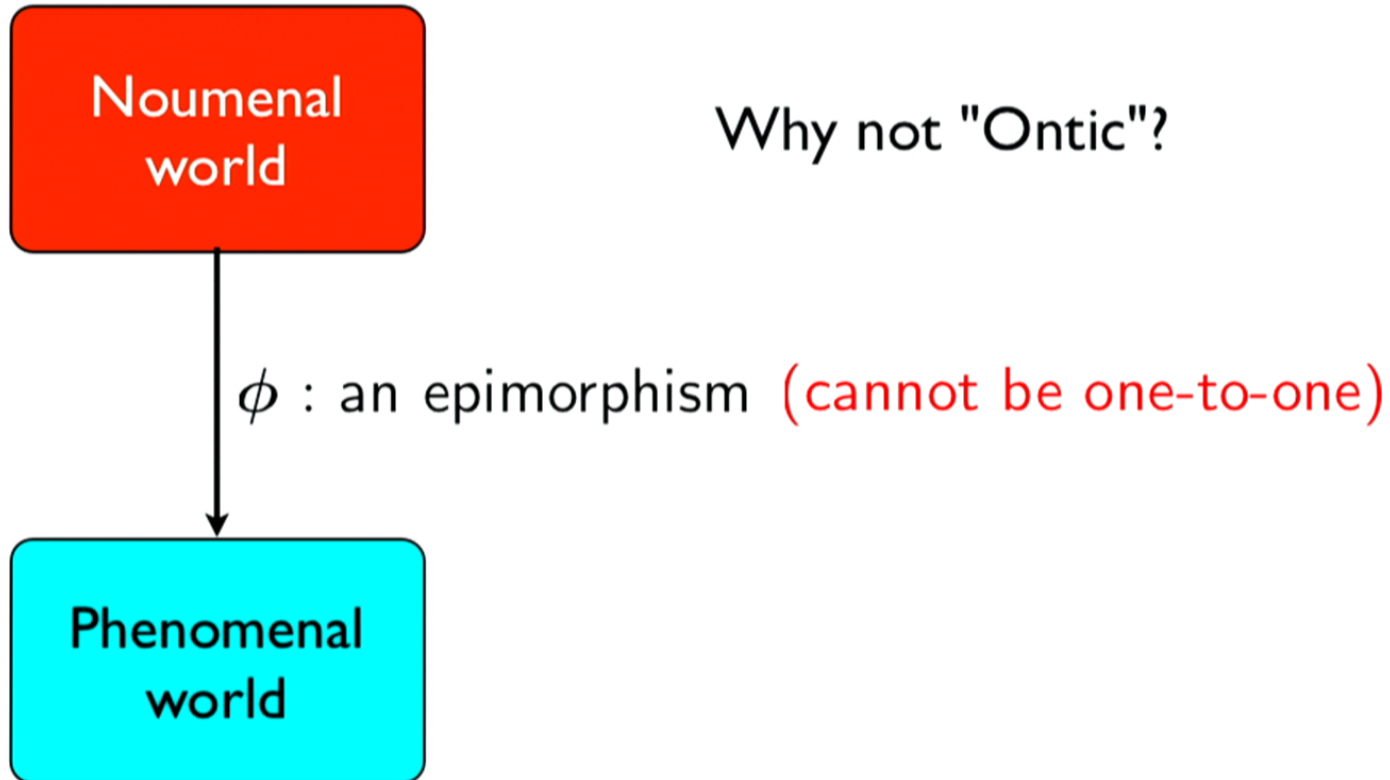
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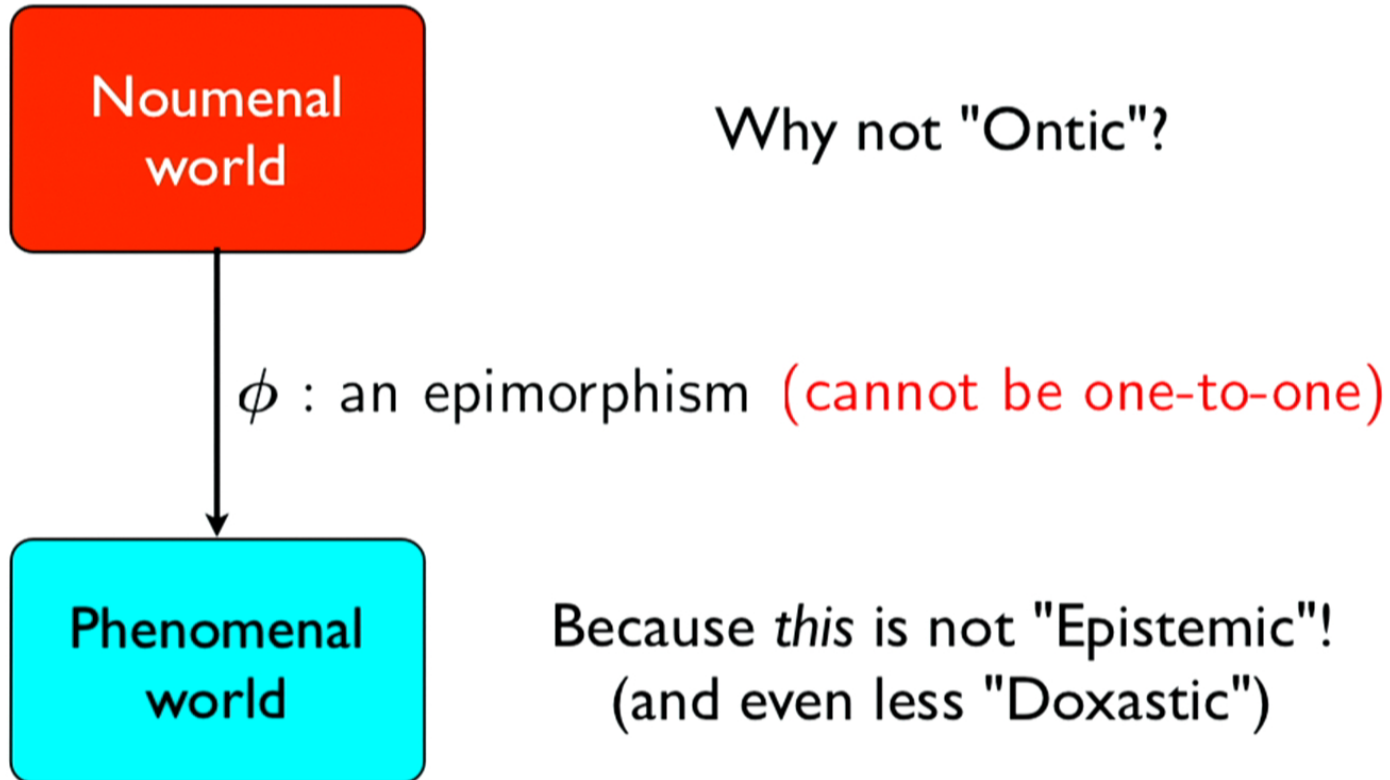
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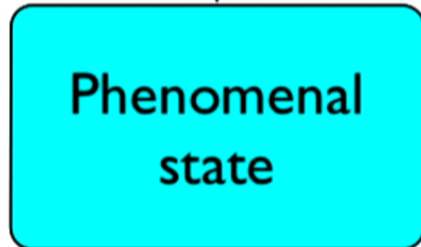


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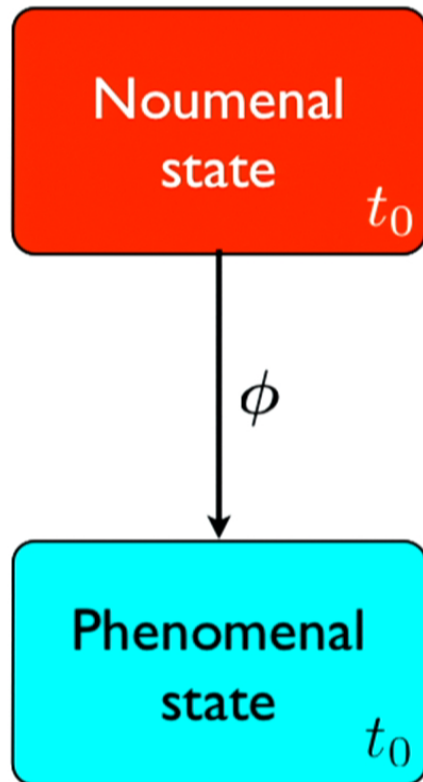
# State of a System



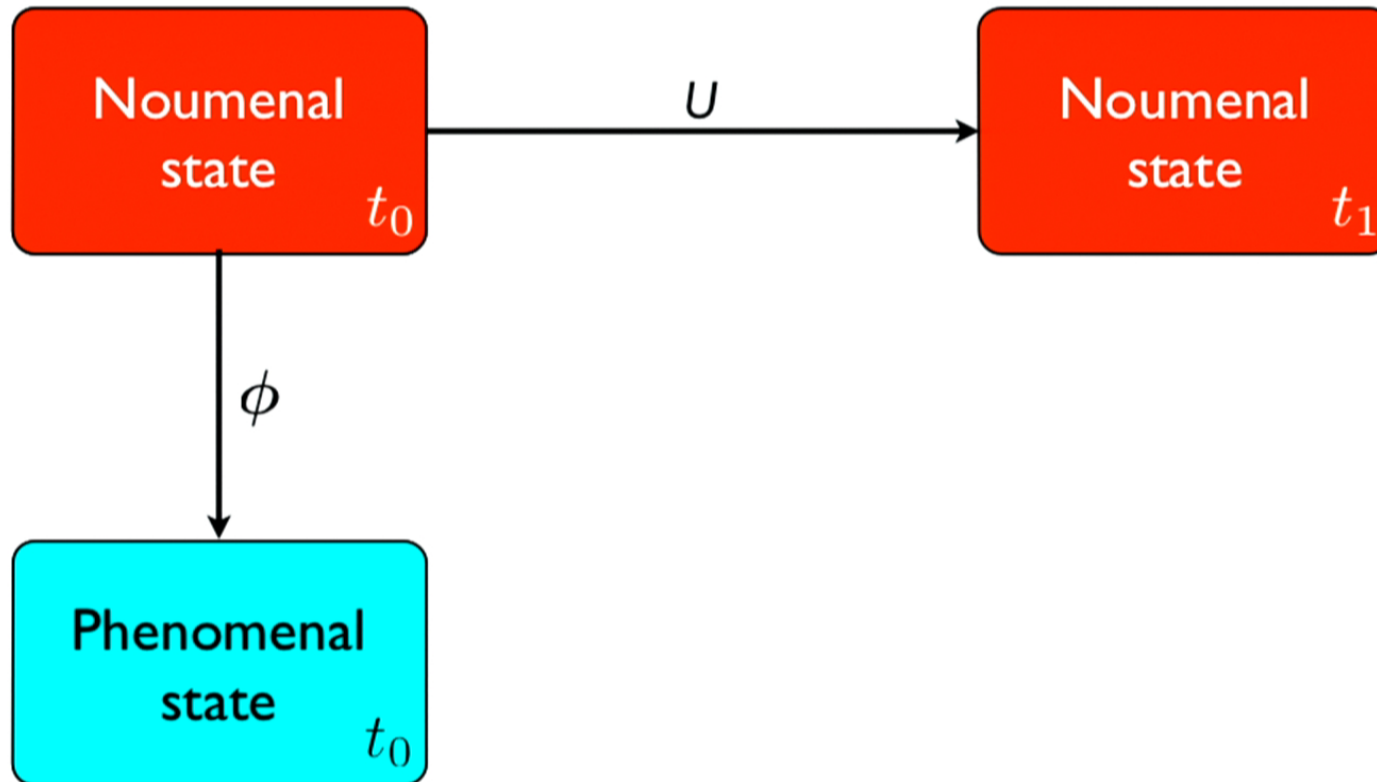
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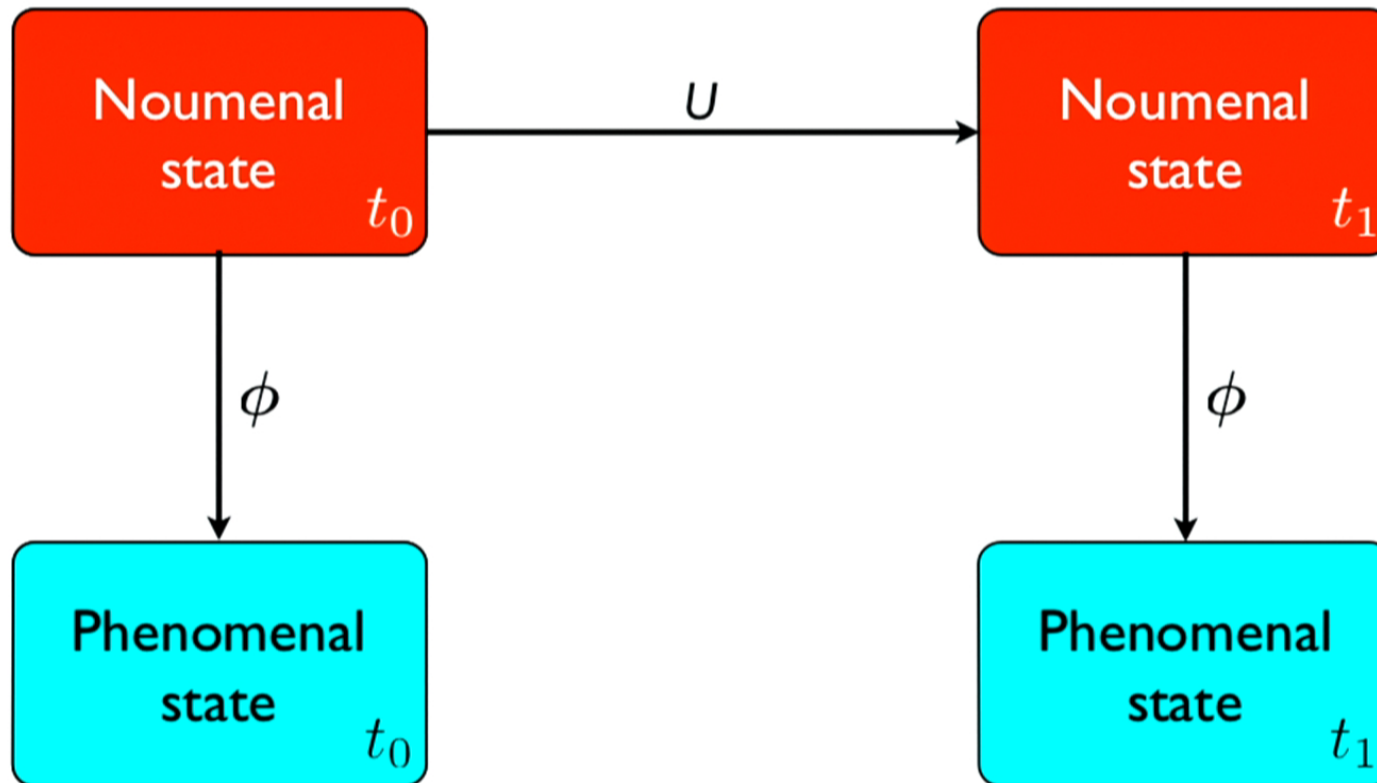
# Evolution of a System



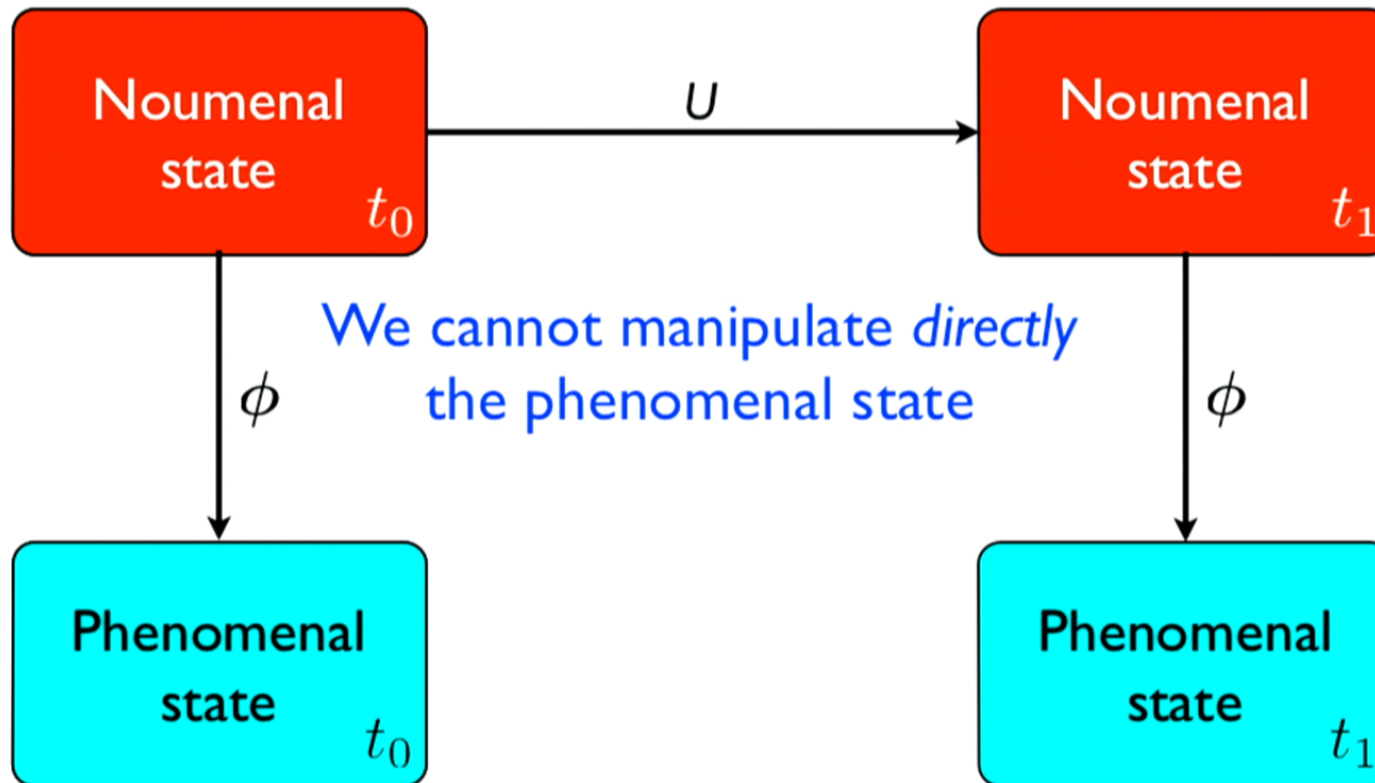
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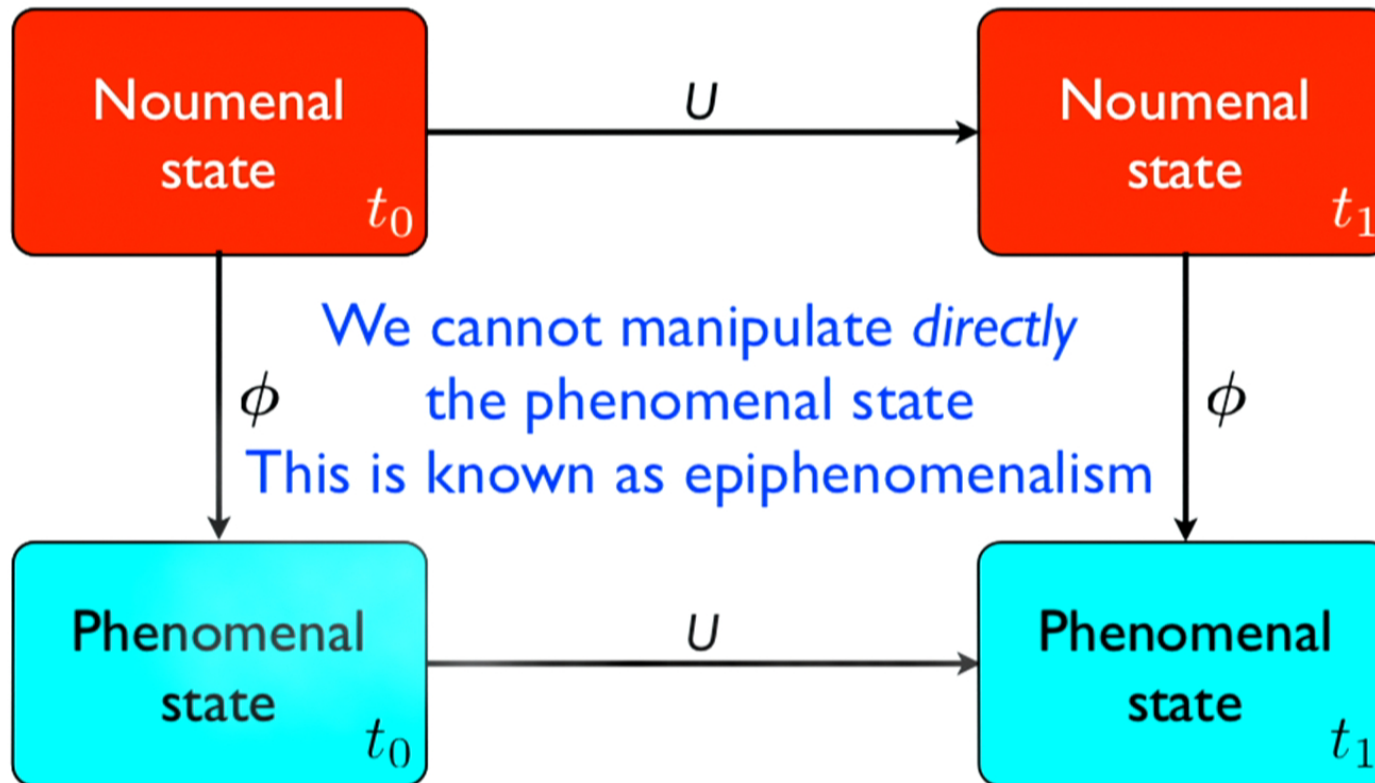
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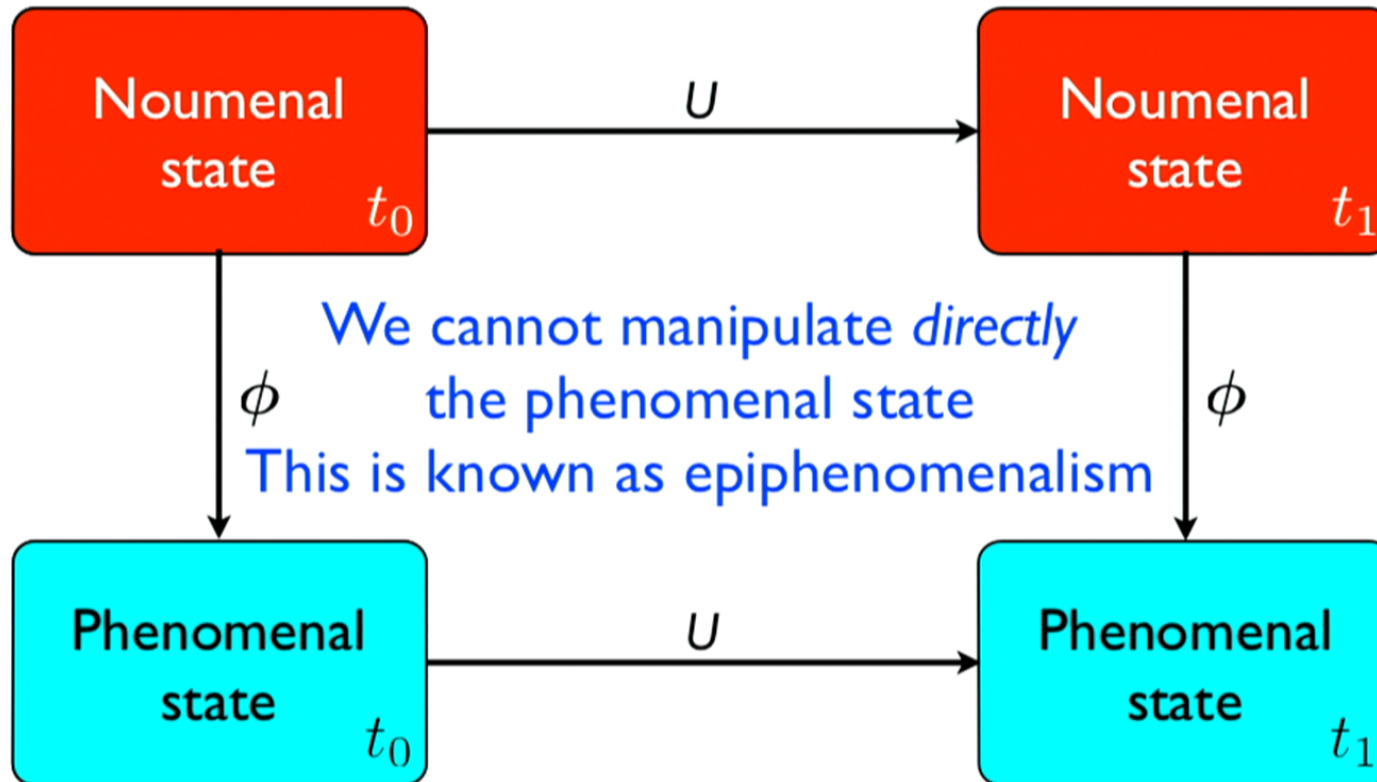
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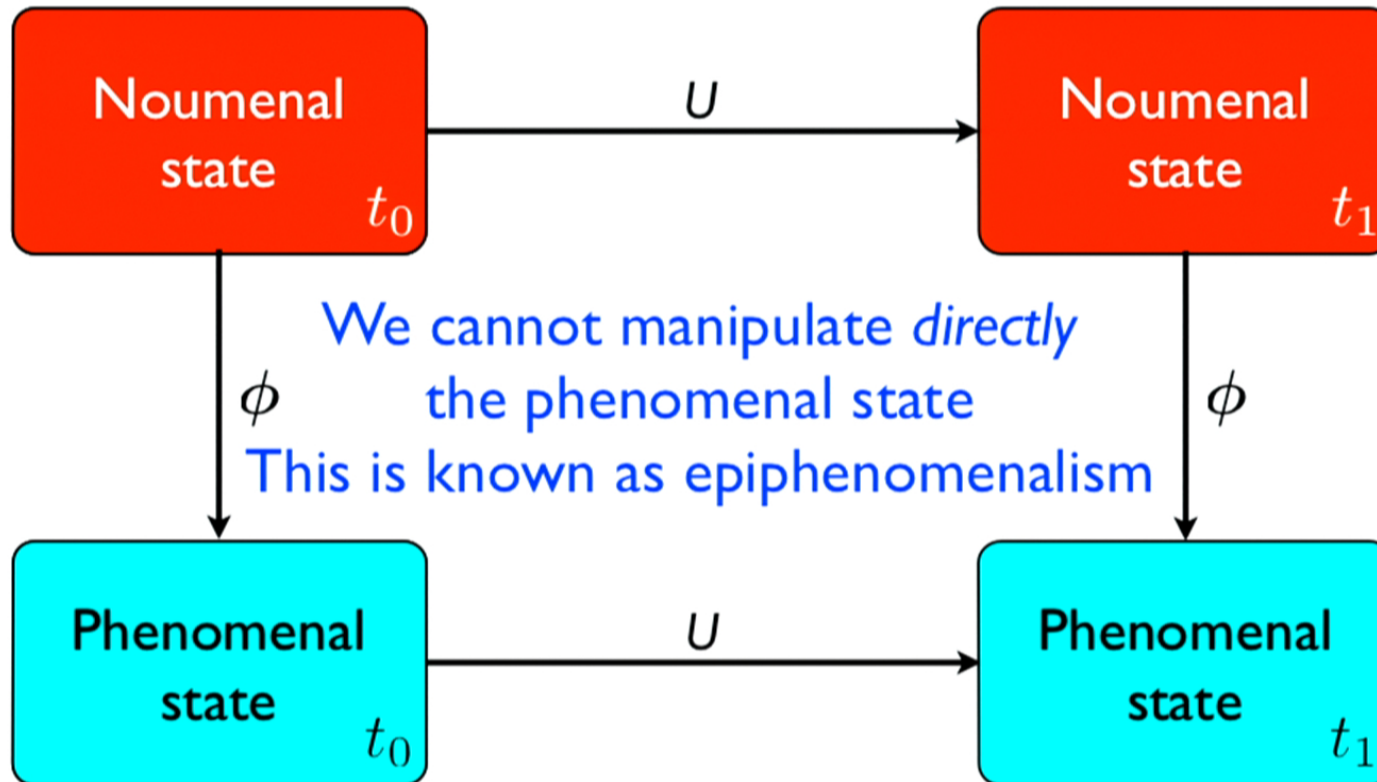
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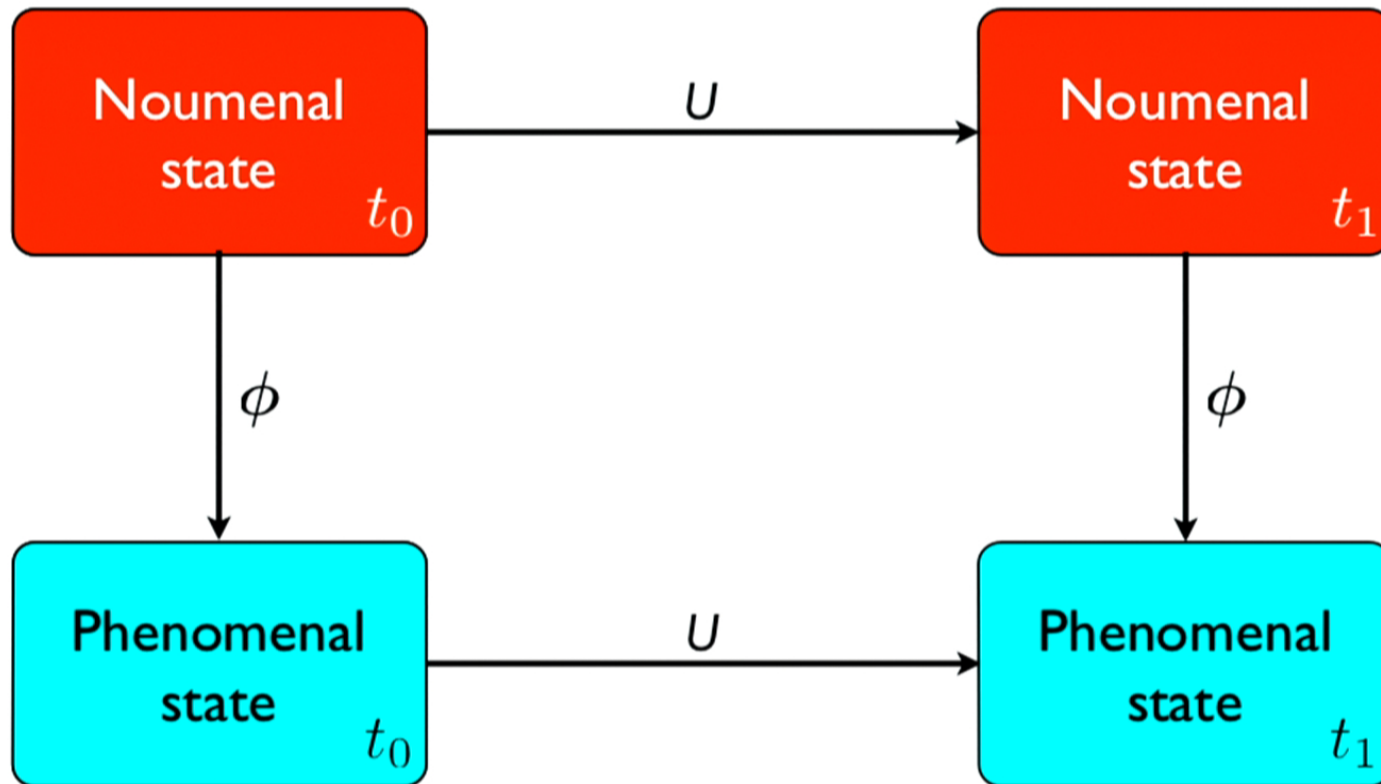


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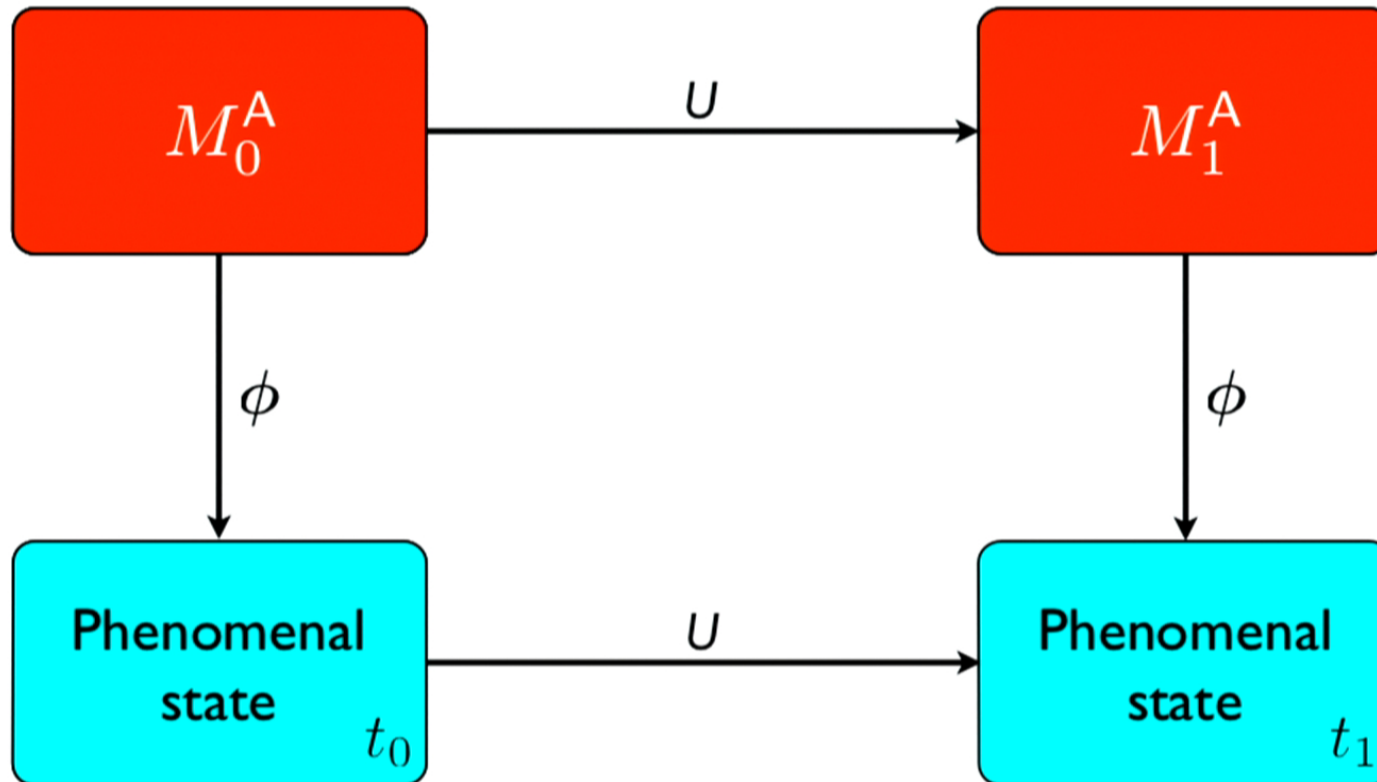


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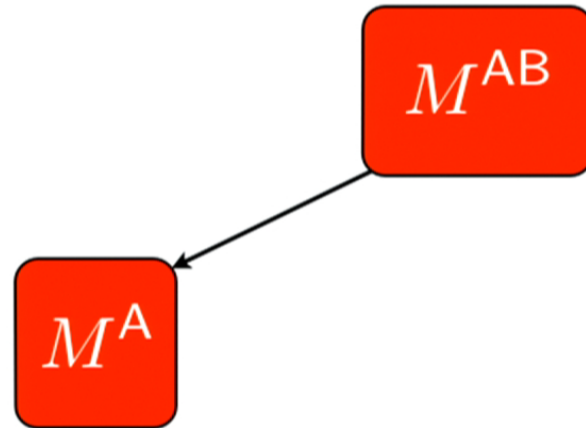


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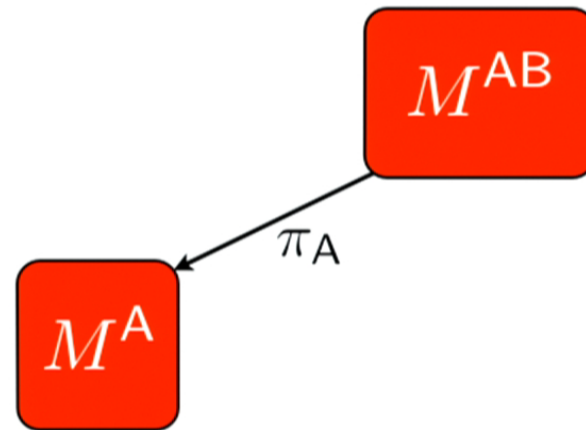
# Evolution of a System A



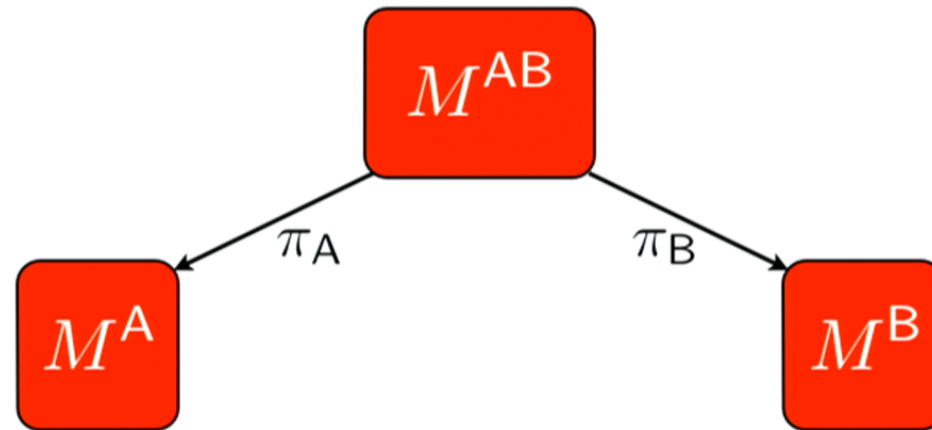
# Splitting of a System



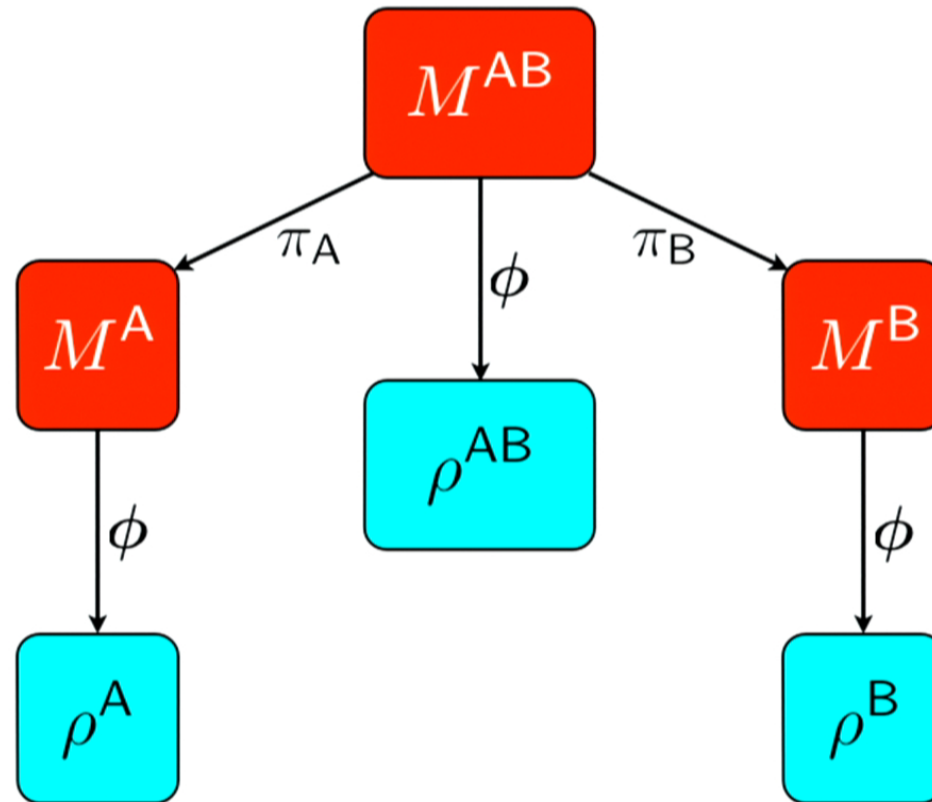
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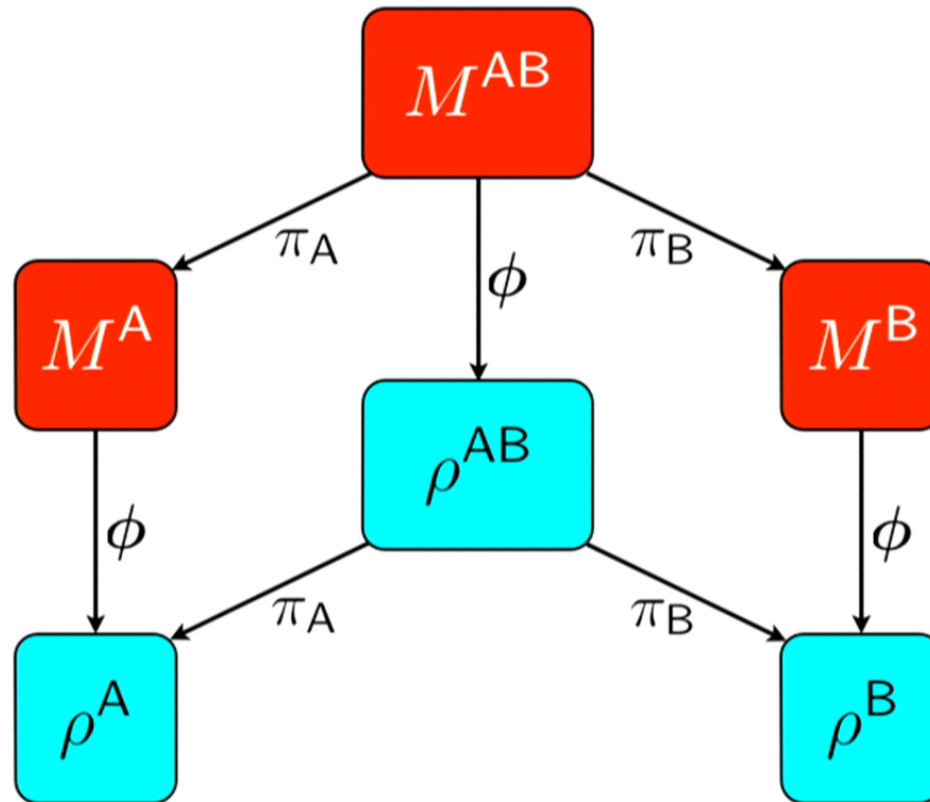
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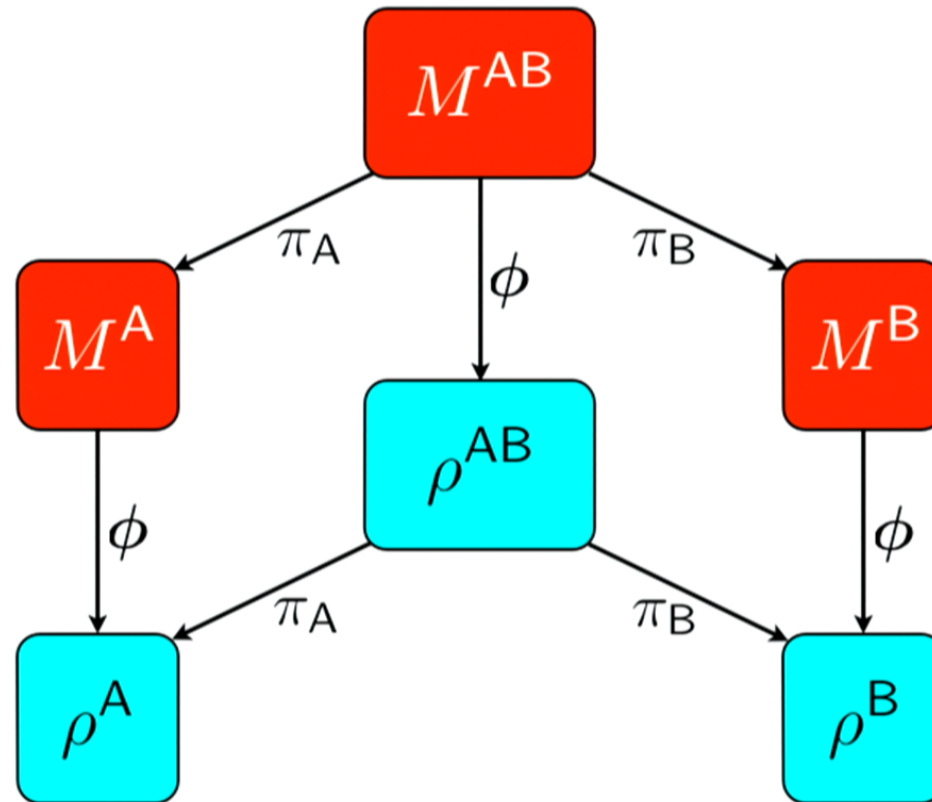
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# Two Theorems

- 1) Local-realistic theories are non-signalling
- 2) Non-signalling theories  
can be *made* local-realistic  
with the proper choice of noumenal states

This includes quantum mechanics

as well as Popescu-Rohrlich nonlocal boxes

## **Quantum Nonlocality as an Axiom**

**Sandu Popescu<sup>1</sup> and Daniel Rohrlich<sup>2</sup>**

*Received July 2, 1993; revised July 19, 1993*

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*In the conventional approach to quantum mechanics, indeterminism is an axiom and nonlocality is a theorem. We consider inverting the logical order, making nonlocality an axiom and indeterminism a theorem. Nonlocal "superquantum" correlations, preserving relativistic causality, can violate the CHSH inequality more strongly than any quantum correlations.*

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nature  
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## Nonlocality beyond quantum mechanics

Sandu Popescu

Nonlocality is the most characteristic feature of quantum mechanics, but recent research seems to suggest the possible existence of nonlocal correlations stronger than those predicted by theory. This raises the question of whether nature is in fact more nonlocal than expected from quantum theory or, alternatively, whether there could be an as yet undiscovered principle limiting the strength of nonlocal correlations. Here, I review some of the recent directions in the intensive theoretical effort to answer this question.

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\* There are technical caveats



Theorem: It is impossible for  
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According to *Nouvelle Cuisine* (1990)  
**not** his original 1964 paper



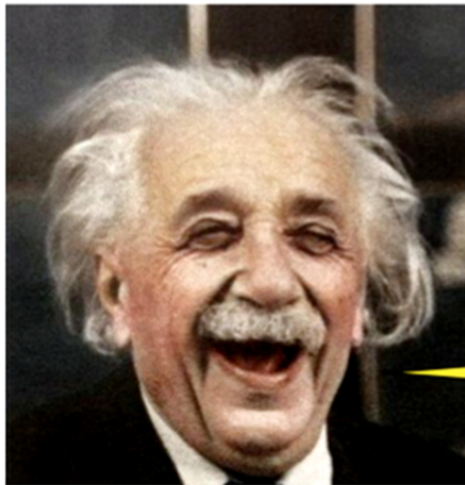
Theorem: It is impossible for Nature to be local-realistic (assuming quantum mechanics is phenomenally correct)

What is proved by impossibility proofs is lack of imagination



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I am enough of the artist to draw freely upon my imagination.

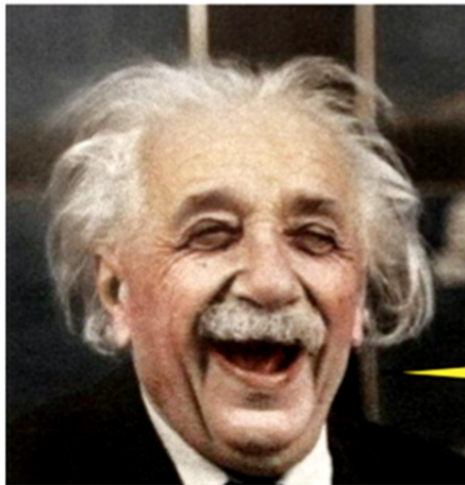
**Imagination is more important than knowledge.**

Knowledge is limited. Imagination encircles the world.



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Theorem: It is impossible to explain Quantum Mechanics with local hidden variables



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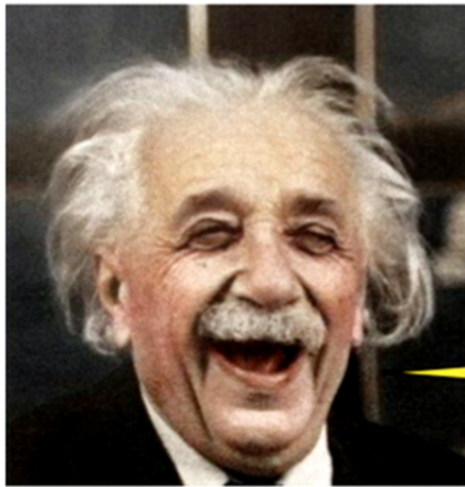
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~~Theorem: It is impossible for Nature to be local-realistic (assuming quantum mechanics is phenomenally correct)~~

Theorem: It is impossible to explain Quantum Mechanics with local hidden variables (as correctly stated in his 1964 paper)



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One that requires just a little more imagination!

Two keys towards a solution:

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- 2) Measurement outcomes never have to become fixed and definite:

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Two keys towards a solution:

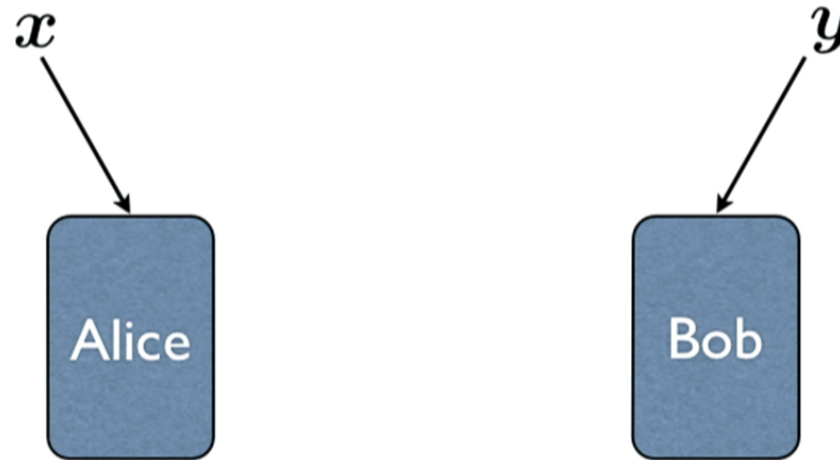
- 1) Tests of Bell inequality are not complete until Alice's and Bob's data are compared.  
**This cannot be faster than the speed of light!**
- 2) Measurement outcomes never have to become fixed and definite:  
**All possible results can occur simultaneously!**

# Popescu-Rohrlich Boxes

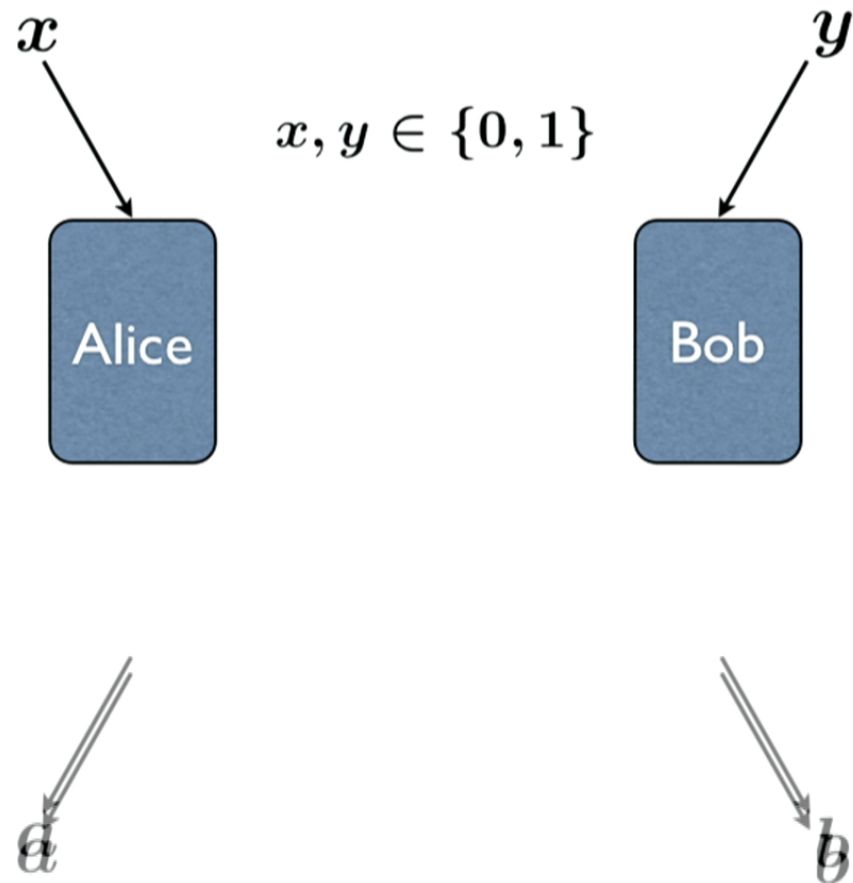
Alice

Bob

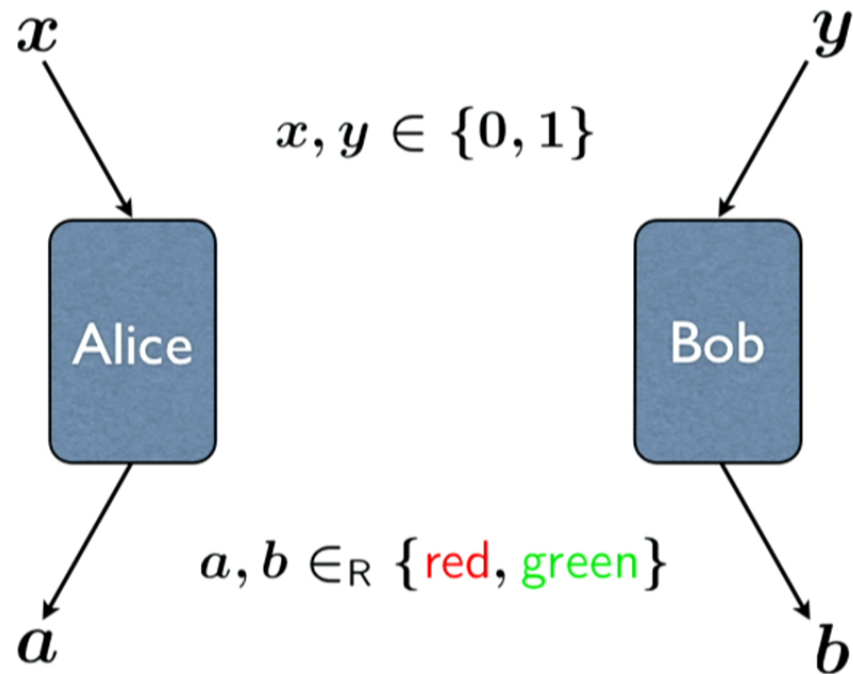
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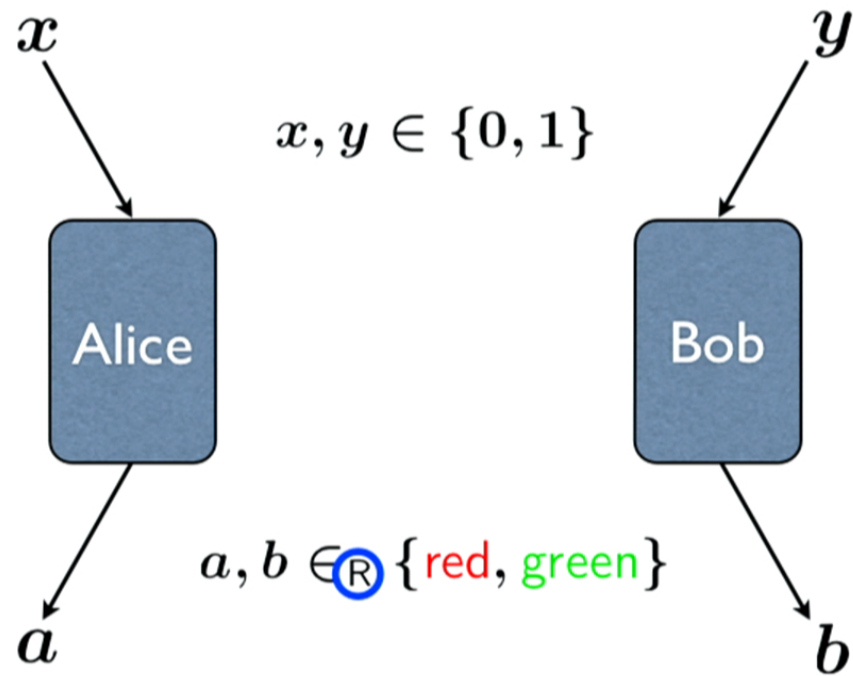
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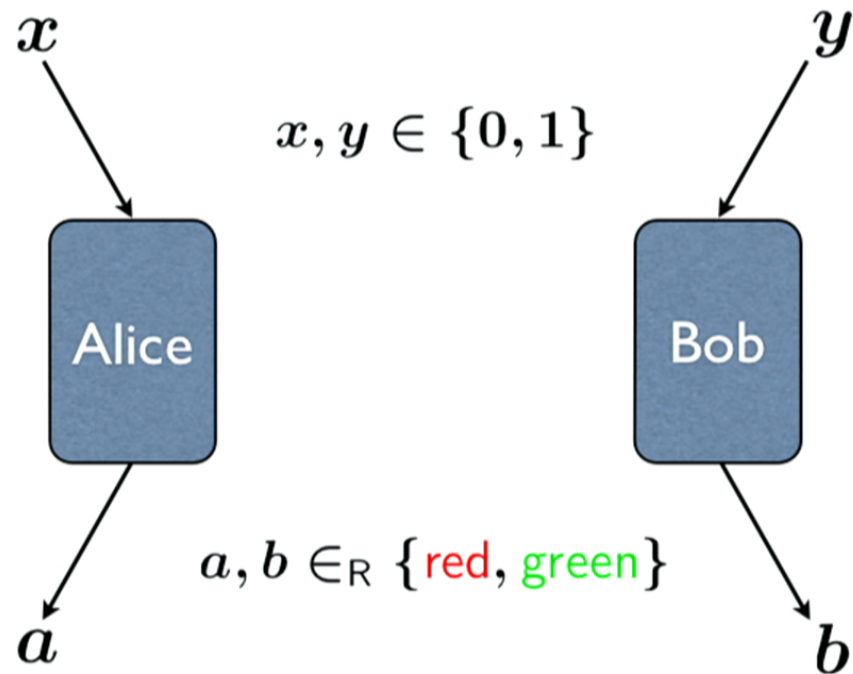
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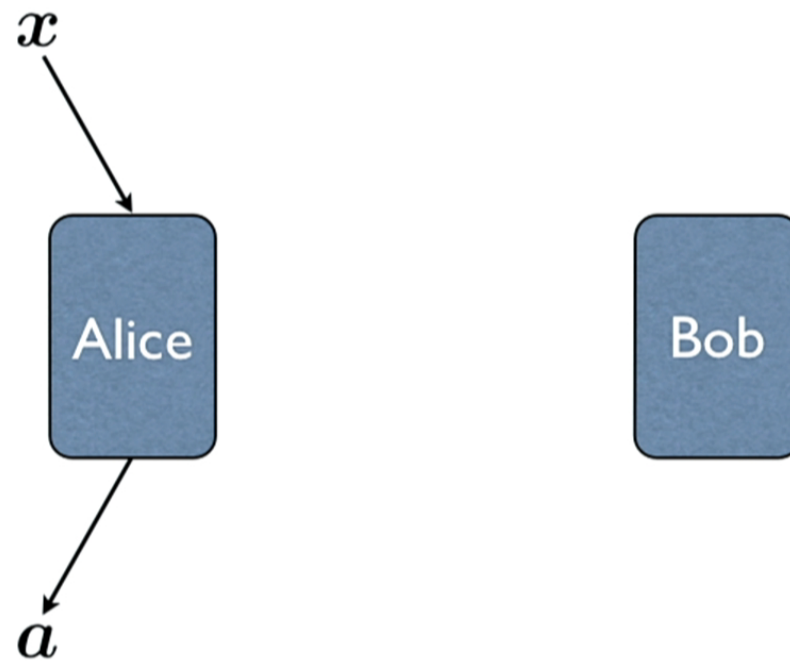
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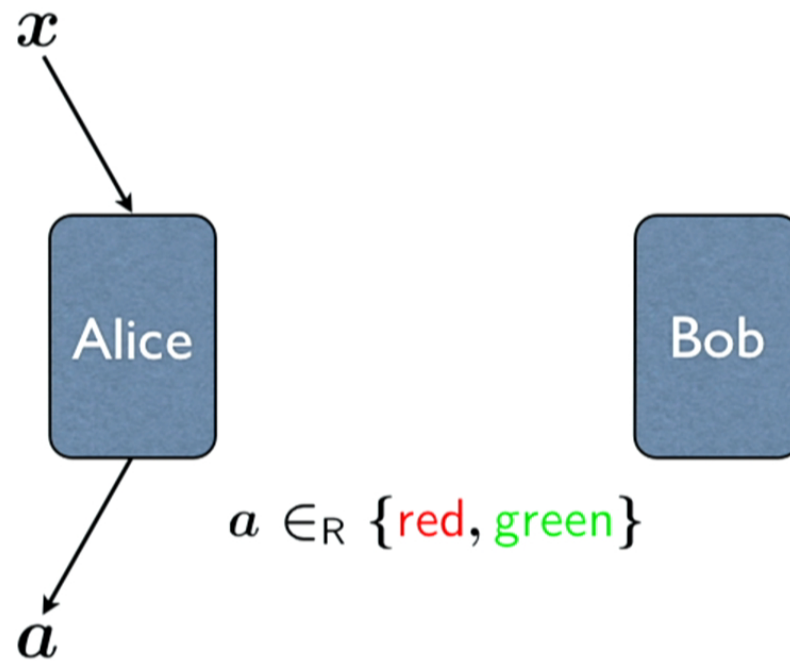
$$a \neq b \Leftrightarrow x = y = 1$$



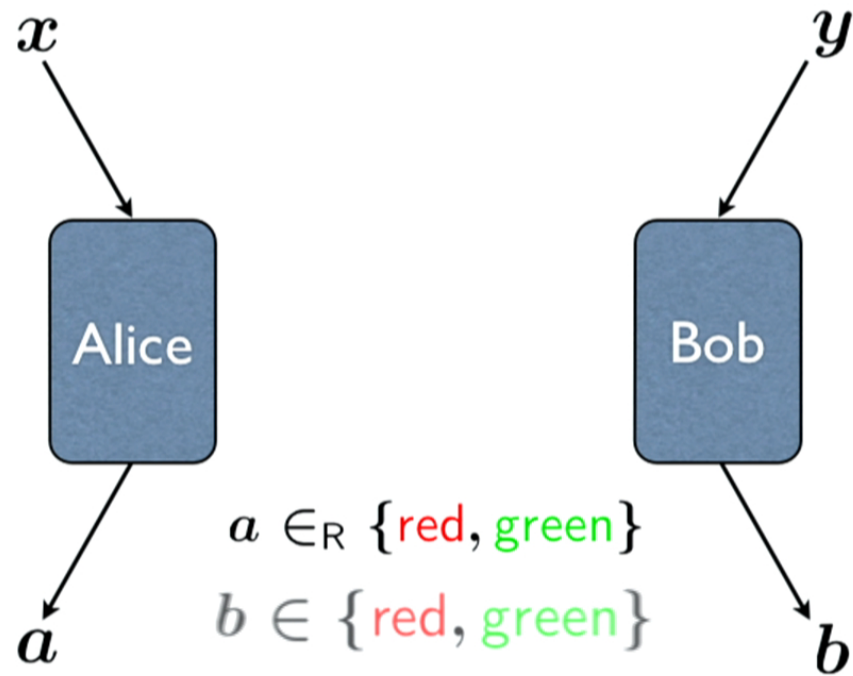
# PR Boxes are Atemporal



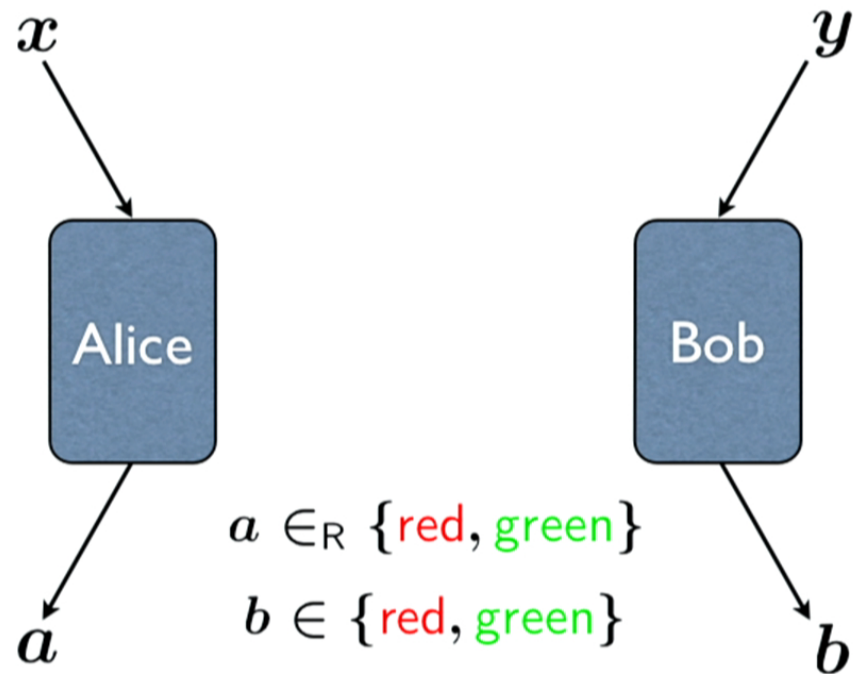
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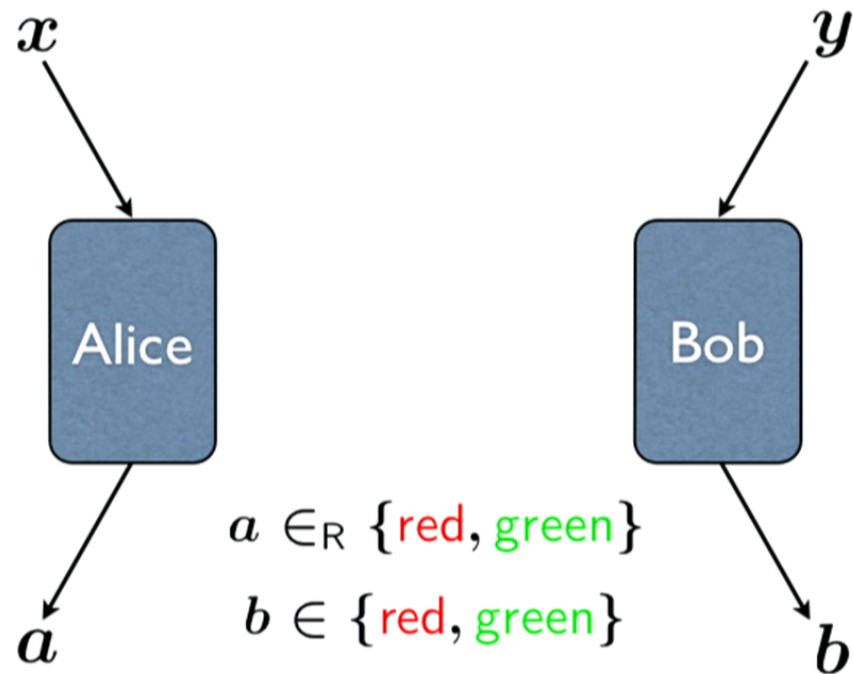
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# Popescu-Rohrlich Boxes

- \* Cannot be used to communicate:  
They are causal and atemporal
- \* Can be simulated classically with probability 75% at best
- \* Can be simulated quantumly with probability  $\cos^2 \frac{\pi}{8} \approx 85\%$  at best

# Popescu-Rohrlich Boxes

- \* Violate CHSH Bell inequality more than Quantum Mechanics

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- \* Yet, they can be "implemented" in a local realistic toy universe

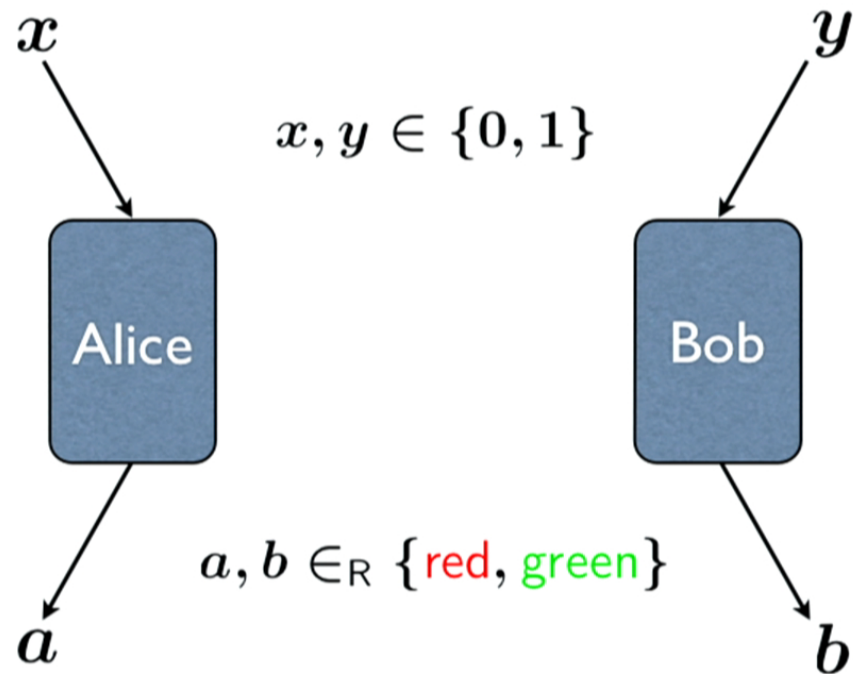
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- \* Yet, they can be "implemented" in a local realistic toy universe
- \* Hence a violation of Bell inequalities is NOT a proof of nonlocality!  
This applies to Quantum Mechanics!

# Popescu-Rohrlich Boxes



$$a \neq b \Leftrightarrow x = y = 1$$

# Parallel Lives: A local Realistic interpretation of nonlocal boxes

Gilles Brassard and Paul Raymond-Robichaud, Université de Montréal

**"What is proved by impossibility proofs is lack of imagination" - John Bell**  
**"The imagination is more important than knowledge" - Albert Einstein**


**Abstract:**  
 We show how local realism can be reconciled with bipartite correlations that are usually considered to be nonlocal. For this purpose we conduct a thought experiment in an Imaginary World.

**Imaginary World:**  
 Our imaginary world follows the principles of Locality and Realism.

**Principle of Locality:** No action taken at a point A can have any effect at a point B that is speed faster than light.

**Principle of Realism:** There is a real world and observations are determined by the state of the real world.


This world has two inhabitants, Alice and Bob, which are each carrying a PR box, introduced by Popescu and Brukner.



A PR box has a "0" and a "1" button. Whenever a button is pushed, it instantaneously flashes a red or green light with equal probability. If Alice and Bob both push a button, they will discover when they meet that they have seen different colours precisely when they both have pushed the "1" button.

(Note: That the PR box does not enable instantaneous communication between Alice and Bob.)

**Principle: Only we (and the boxes) know this protocol!**  
 They travel far apart to their operations. Alice and Bob flip coins and push the corresponding button simultaneously.



Alice's button is pushed, the box flashes either a green or red light.

The experiment is performed with sufficient randomness that Alice's cannot know the result of Bob's coin flip (which is crucial to Bob's best before it can be flushed to work together in their world).

After many experiments, they meet and realize that they have seen different results.

**Is the Locality-Realism Assumption?**  
 Alice's pushing of a button cannot have any instantaneous effect on Bob's system. The principle of Locality.

After Alice pushes her button, she can know with certainty what colour Bob will see depending on which button he pushes. (For example, if Alice pushes "1" and sees green, she knows that if Bob pushes "0" he will see green).

Since it is possible for Alice to predict with certainty what light Bob will see when he pushes a button, without influencing his system, it means that his observations were predetermined.

The observations of Bob should be described by local hidden variables  $\lambda_0$  and  $\lambda_1$ .

$\lambda_0 = 0$  if Bob will observe green after pushing "0"  
 $\lambda_0 = 1$  if Bob will observe red after pushing "0"

$\lambda_1 = 0$  if Bob will observe green after pushing "1"  
 $\lambda_1 = 1$  if Bob will observe red after pushing "1"

Likewise, Alice's system should be described by local hidden variables  $\mu_0$  and  $\mu_1$ .

$\mu_0 = 0$  if Alice will observe green after pushing "0"  
 $\mu_0 = 1$  if Alice will observe red after pushing "0"

$\mu_1 = 0$  if Alice will observe green after pushing "1"  
 $\mu_1 = 1$  if Alice will observe red after pushing "1"

A local hidden variable theory would give a local realistic interpretation of this experiment.

**Local hidden variable theories can only produce PR boxes that work 75% of the time.**  
 Proof: A hidden variable theory of these boxes must satisfy the following 4 equations:

Summing these equations on both sides and rearranging the terms:

$$A_0 + B_0 = \text{EVEN}$$

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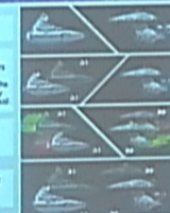
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Many people have concluded that any world that could produce PR boxes that work more than 75% of the time cannot be Local and Realistic. Remarkably, quantum mechanics enables PR boxes that work 85% of the time. Must we conclude that quantum mechanics cannot be Local and Realistic?

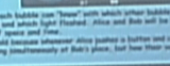
**Here is how the seemingly impossible is accomplished!**

Each spacetime lives inside a bubble.



When Alice pushes a button on her box there ("0"), her bubble splits into two bubbles. Each bubble contains a copy of its spacetime and its entanglement. Inside one bubble, Alice has seen the red light flash inside the other. She has seen the green light flash. From her view, the two bubbles are being parallel lives. They cannot interact in any way and will never meet again. Notice that this phenomenon is strictly local.

The same phenomenon takes place when Bob pushes his button (here "0") on the box. Let's see what happens when they travel toward each other.



Each of the two bubbles that contain Alice is allowed to interact with and see only a single bubble that contains Bob, namely the one that satisfies the equations described above.

Note that both a perfect matching is always possible. Furthermore, each bubble can "meet" with which other bubble to interact provided it keeps a local memory of which button was pushed and which light flashed. Alice and Bob will be under the illusion of correlations that seem to emerge from outside of space and time.

In our imaginary world, the Einstein-Podolsky-Rosen argument does not hold because whenever Alice pushes a button and can predict something about Bob, she is really predicting, not what is happening simultaneously at Bob's place, but how that event will need in the future.

**Conclusion:**  
 The virtues of our imaginary world is to demonstrate in an exceedingly simple way that local reality can produce correlations that are impossible in any classical theory based on local hidden variables.

In quantum mechanics, a theory analogous to this one can be traced back to at least Deutsch and Hayden.

Perhaps we live parallel lives?

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# Parallel Lives: A local realistic interpretation of "nonlocal" boxes

Gilles Brassard and Paul Raymond-Robichaud, Université de Montréal

"What is proved by impossibility proofs is lack of imagination" - John Bell  
 "Imagination is more important than knowledge" - Albert Einstein

## Abstract:

We show how local realism can be consistent with bipartite correlations that are usually considered to be nonlocal. For this purpose, we conduct a thought experiment in an imaginary world.

## Imaginary World:

Our imaginary world follows the principles of Locality and Realism.

**Principle of Locality:** No action taken at a point A can have any effect at a point B at a speed faster than light.

**Principle of Realism:** There is a real world and observations are determined by the state of the real world.

This world has two inhabitants, Alice and Bob, which are each carrying a PR box, introduced by Popescu and Rohrlich.



A PR box has a "0" and a "1" button. Whenever a button is pushed, it instantaneously flashes a red or green light with equal probability. If Alice and Bob both push a button, they will discover when they meet that they have seen different colours precisely when they both have pushed the "1" button.

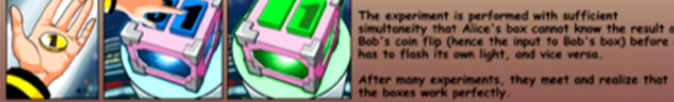
(Note that the PR box does not enable instantaneous communication between Alice and Bob)

## Alice and Bob will test the boxes with this protocol:

They travel far apart in their spaceships. Alice and Bob flip coins and push the corresponding button simultaneously.



Once a button is pushed, the box flashes either a green or red light.



The experiment is performed with sufficient simultaneity that Alice's box cannot know the result of Bob's coin flip (hence the input to Bob's box) before it has to flash its own light, and vice versa.

After many experiments, they meet and realize that the boxes work perfectly.

## The Einstein-Podolsky-Rosen Argument:

- Alice's pushing of a button cannot have any instantaneous effect on Bob's system by the principle of Locality.
- After Alice pushes her button, she can know with certainty what colour Bob will see depending on which button he pushes. (For example, if Alice pushes "1" and sees green, she knows that if Bob pushes "0" he will see green)
- Since it is possible for Alice to predict with certainty what light Bob will see when he pushes a button, without influencing his system, it means that his observations were predetermined.
- The observations of Bob should be described by local hidden variables  $B_0$  and  $B_1$ .
 

$B_0 = 0$ if Bob will observe green after pushing "0"	$B_1 = 0$ if Bob will observe green after pushing "1"
$B_0 = 1$ if Bob will observe red after pushing "0"	$B_1 = 1$ if Bob will observe red after pushing "1"
- Likewise, Alice's system should be described by local hidden variables  $A_0$  and  $A_1$ .
 

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- A local hidden variable theory would give a local realistic explanation of this experiment.

## Bell's Theorem: Local hidden variable theories can only produce PR boxes that work at most 75% of the time.

**Proof:** A hidden variable theory of these boxes must satisfy the following 4 equations:

$$A_0 + B_0 = \text{EVEN}$$

$$A_0 + B_1 = \text{EVEN}$$

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Summing these equations on both sides and rearranging the terms:

$$(A_0 + B_0) + (A_0 + B_1) + (A_1 + B_0) + (A_1 + B_1) = \text{EVEN} + \text{EVEN} + \text{EVEN} + \text{ODD}$$

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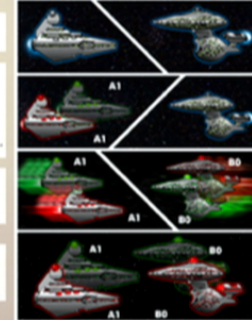
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## Here is how the seemingly impossible is accomplished:

Each spaceship lives inside a bubble.



When Alice pushes a button on her box (here "1"), her bubble splits into two bubbles. Each bubble contains a copy of its spaceship and its inhabitant. Inside one bubble, Alice has seen the red light flash; inside the other, she has seen the green light flash. From now on, the two bubbles are living parallel lives. They cannot interact in any way and will never meet again. Notice that this phenomenon is strictly local.

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## Conclusion:

The virtue of our imaginary world is to demonstrate in an exceedingly simple way that local reality can produce correlations that are impossible in any classical theory based on local hidden variables.

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## References:

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Poster art and design by Louis Fermat-Lacour (2012)



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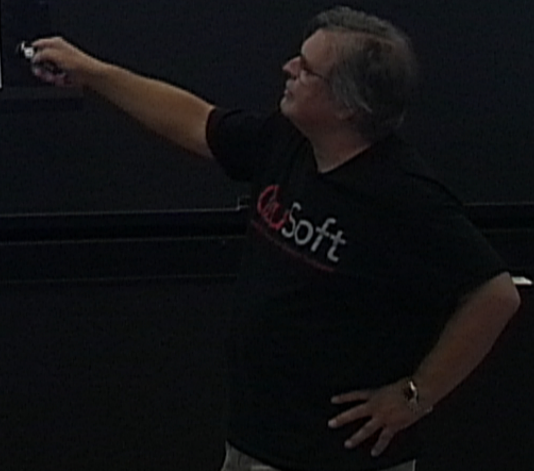
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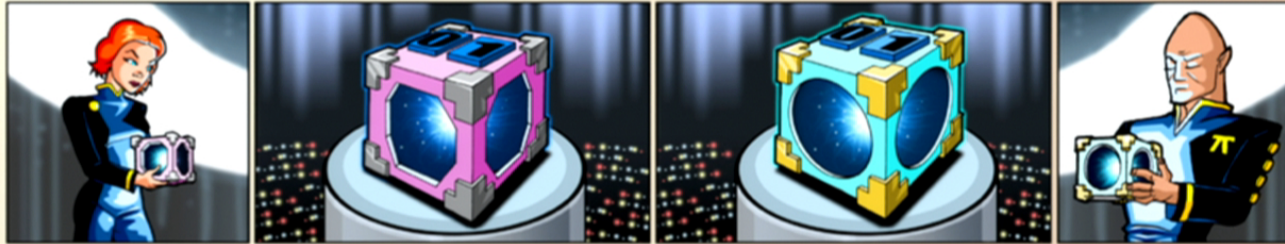
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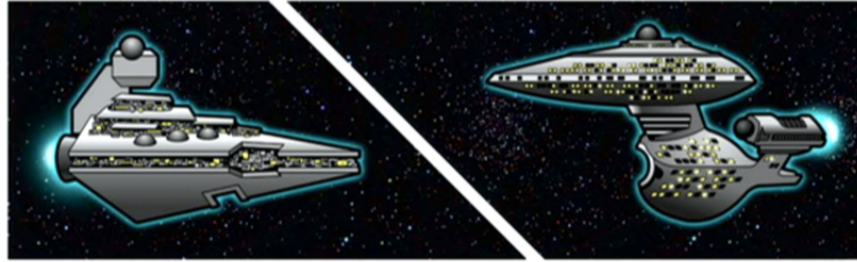
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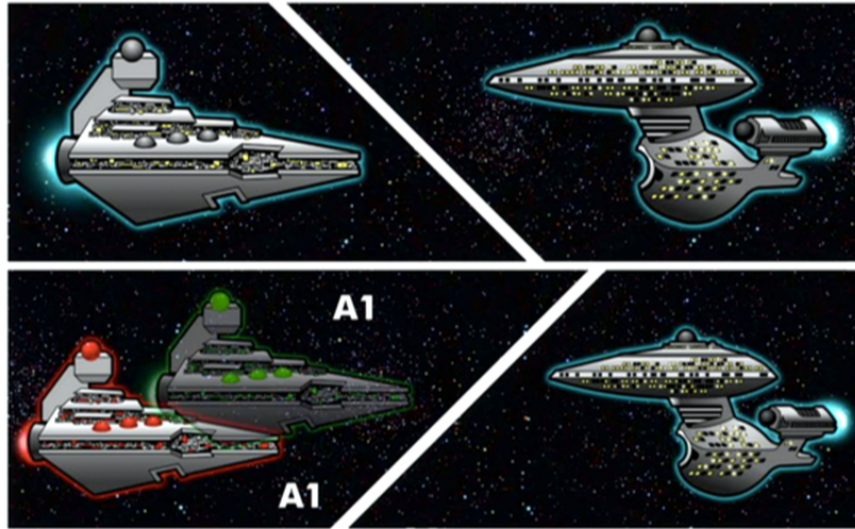


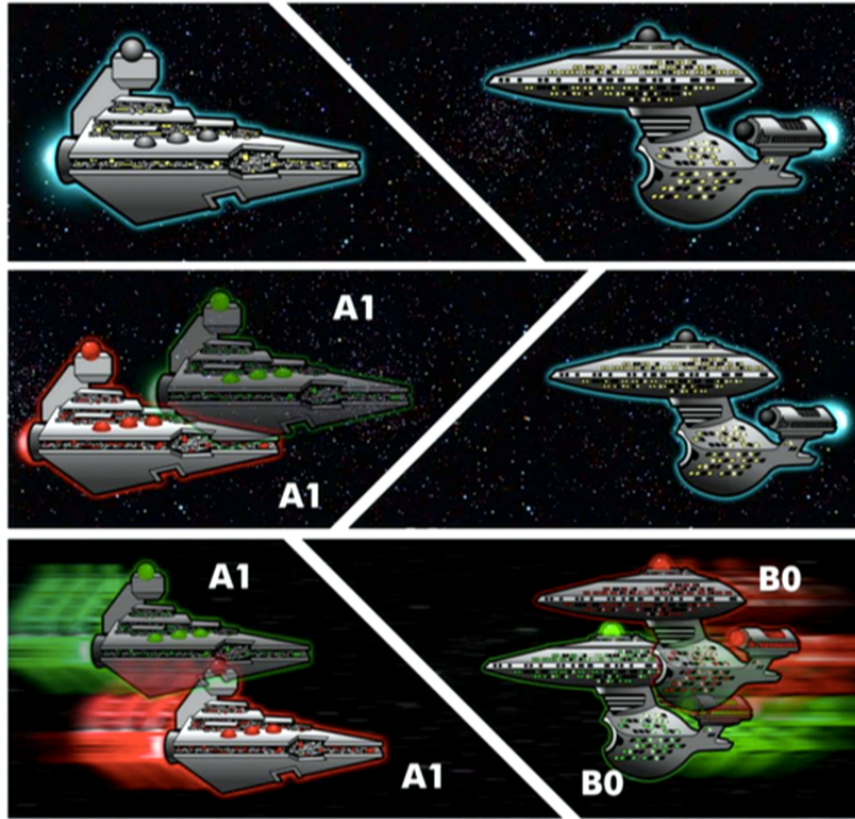
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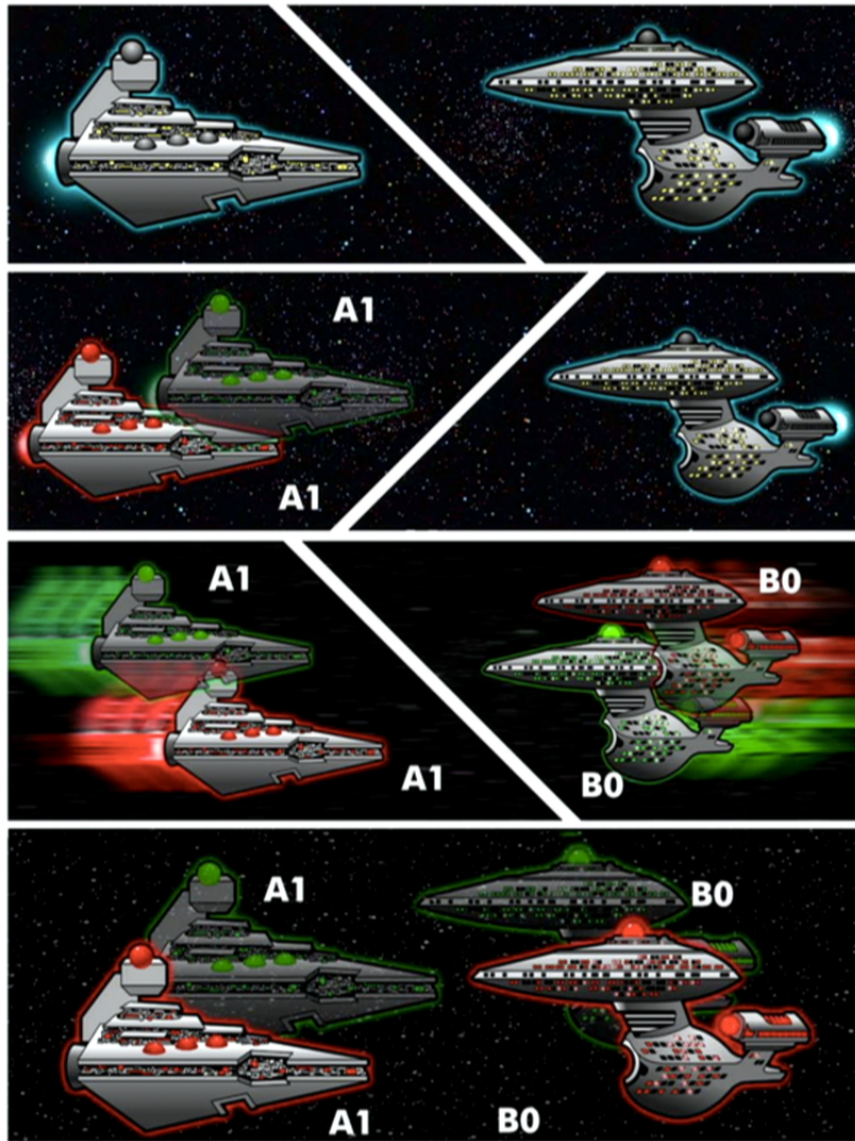
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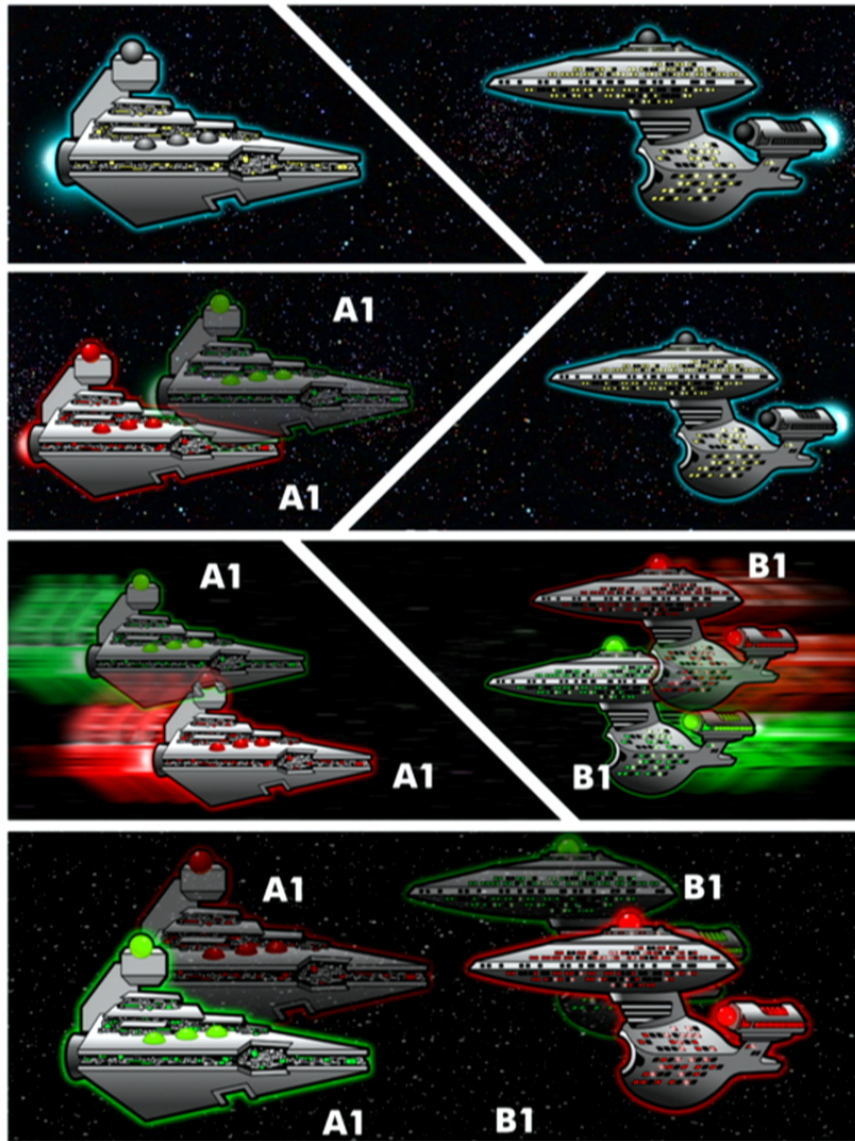
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In our imaginary world, the EPR argument does not hold because whenever Alice pushes a button and can predict something about Bob, she is really predicting not what is happening simultaneously at Bob's place but how their various lives will meet in the future.

This **proves** that  
it is **wrong** to claim that  
any world that violates  
Bell inequalities  
**has to be** nonlocal

**How about Quantum Mechanics?**

## States

For a system  $A$  associated with a Hilbert Space of dimension  $n$ , its noumenal state  $M^A$  is formally defined by an *evolution matrix*  $[W]^A$ , which is an  $n \times n$  matrix whose entries are matrices:

$$[W]_{i,j}^A \stackrel{\text{def}}{=} W^\dagger \left( |j\rangle\langle i| \otimes I^{\bar{A}} \right) W$$

for some unitary  $W$  on the global state, which corresponds to all that happened to the universe since the beginning of time.

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# Local Evolution





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**Theorem**

$$U[W]^A = \left[ (U \otimes V) W \right]^A$$

for any unitary  $V$  acting on  $\bar{A}$

## Local Evolution

### Proof

$$\begin{aligned} \left( U[W]^A \right)_{i,j} &= \sum_{m,n} U_{i,m} [W]_{m,n}^A U_{n,j}^\dagger \\ &= \sum_{m,n} \langle i|U|m\rangle \left( W^\dagger (|n\rangle\langle m| \otimes \bar{I}^A) W \right) \langle n|U^\dagger|j\rangle \\ &= \sum_{m,n} W^\dagger \left( (|n\rangle\langle n| \langle n|U^\dagger|j\rangle \langle i|U|m\rangle \langle m|) \otimes \bar{I}^A \right) W \\ &= W^\dagger \left( \left( \sum_{m,n} |n\rangle\langle n| U^\dagger|j\rangle\langle i|U|m\rangle\langle m| \right) \otimes \bar{I}^A \right) W \\ &= W^\dagger \left( (U^\dagger |j\rangle\langle i| U) \otimes \bar{I}^A \right) W \\ &= W^\dagger \left( (U^\dagger |j\rangle\langle i| U) \otimes (V^\dagger \bar{I}^A V) \right) W \\ &= W^\dagger (U^\dagger \otimes V^\dagger) (|j\rangle\langle i| \otimes \bar{I}^A) (U \otimes V) W \\ &= \left[ (U \otimes V) W \right]_{i,j}^A \end{aligned}$$



## Separation

The noumenal state of system  $A$  can be obtained from that of system  $AB$  by a projection operation  $\pi_A$  defined as

$$\left( \pi_A [W]^{AB} \right)_{i,j} \stackrel{\text{def}}{=} \sum_k [W]_{(i,k),(j,k)}^{AB}$$

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$$\begin{aligned} \left( \pi_A [W]^{AB} \right)_{i,j} &= \sum_k [W]_{(i,k),(j,k)}^{AB} \\ &= \sum_k W^\dagger \left( |j\rangle \langle i|^A \otimes |k\rangle \langle k|^B \otimes I^{\overline{AB}} \right) W \\ &= W^\dagger \left( |j\rangle \langle i|^A \otimes I^B \otimes I^{\overline{AB}} \right) W \\ &= [W]_{i,j}^A \end{aligned}$$

## Merging

The state of a joint system  $AB$  can be obtained from the evolution matrices of systems  $A$  and  $B$  by the joint product  $\odot$  defined as

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# Merging

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$$\begin{aligned} & \left( [W]^A \odot [W]^B \right)_{(i,k),(j,l)} \\ &= [W]_{i,j}^A [W]_{k,l}^B \\ &= \left( W^\dagger \left( |j\rangle\langle i|^A \otimes I^B \otimes I^{\overline{AB}} \right) W \right) \left( W^\dagger \left( I^A \otimes |l\rangle\langle k|^B \otimes I^{\overline{AB}} \right) W \right) \\ &= W^\dagger \left( |j\rangle\langle i|^A \otimes I^B \otimes I^{\overline{AB}} \right) \left( I^A \otimes |l\rangle\langle k|^B \otimes I^{\overline{AB}} \right) W \\ &= W^\dagger \left( |j\rangle\langle i|^A \otimes |l\rangle\langle k|^B \otimes I^{\overline{AB}} \right) W \\ &= [W]_{(i,k),(j,l)}^{AB} \end{aligned}$$



## Recovering the Phenomenal State

Let  $|\psi\rangle$  be a vector in the Hilbert space of the universe.

We define  $\rho^A \stackrel{\text{def}}{=} [W]^A |\psi\rangle$  by:

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$$[W]^A |\psi\rangle = [W |\psi\rangle]^A$$

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$$[W |\psi\rangle]^A \stackrel{\text{def}}{=} \text{tr}_{\bar{A}} \left( W |\psi\rangle \langle \psi| W^\dagger \right) .$$

## Recovering the Phenomenal State

### Proof

$$\begin{aligned} \left( [W]^A |\psi\rangle \right)_{ij} &= \langle \psi | [W]_{ij}^A | \psi \rangle \\ &= \langle \psi | \left( W^\dagger (|j\rangle\langle i| \otimes I^{\bar{A}}) W \right) | \psi \rangle \\ &= \langle \psi | \left( W^\dagger \left( |j\rangle\langle i| \otimes \sum_k |k\rangle\langle k| \right) W \right) | \psi \rangle \\ &= \sum_k \left( \langle \psi | W^\dagger |j\rangle\langle k| \right) \left( \langle i| \langle k| W | \psi \rangle \right) \\ &= \sum_k \langle i| \langle k| \left( W | \psi\rangle\langle \psi| W^\dagger \right) |j\rangle\langle k| \\ &= \left( \text{tr}_{\bar{A}} \left( W | \psi\rangle\langle \psi| W^\dagger \right) \right)_{ij} = [W | \psi]_{ij}^A \end{aligned}$$

## Proportion versus Probability

The *proportion* of a system  $A$ , with evolution Matrix  $[W]^A$  in state  $|i\rangle$  is given by

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## Separate evolution

### Theorem

$$\left( (U \otimes V) [W]^{AB} \right) = U[W]^A \odot V[W]^B$$



## Commuting Diagrams

And indeed we have

$$\begin{array}{ccc} M^A & \xrightarrow{U} & U(M^A) \\ \phi \downarrow & & \downarrow \phi \\ \rho^A & \xrightarrow{U} & U(\rho^A) \end{array}$$

# Commuting Diagrams

## Theorem

$$U([W]^A |\psi\rangle) = (U[W]^A) |\psi\rangle$$

## Proof

$$\begin{aligned} & U([W]^A |\psi\rangle) \\ &= U[W |\psi\rangle]^A \\ &= [(U \otimes I) W |\psi\rangle]^A \\ &= [(U \otimes I) W]^A |\psi\rangle \\ &= (U[W]^A) |\psi\rangle \end{aligned}$$

# Commuting Diagrams

As well as

$$\begin{array}{ccc} M^{AB} & \xrightarrow{\pi_A} & M^A \\ \phi \downarrow & & \downarrow \phi \\ \rho^{AB} & \xrightarrow{\pi_A = \text{tr}_B} & \rho^A \end{array}$$

## Commuting Diagrams

As well as

$$\begin{array}{ccc} M^{AB} & \xrightarrow{\pi_A} & M^A \\ \phi \downarrow & & \downarrow \phi \\ \rho^{AB} & \xrightarrow{\pi_A = \text{tr}_B} & \rho^A \end{array}$$

Which means:

$$\left( \pi_A [W]^{AB} \right) |\psi\rangle = \pi_A \left( [W]^{AB} |\psi\rangle \right)$$

# Commuting Diagrams

## Theorem

$$\left(\pi_A[W]^{AB}\right) |\psi\rangle = \pi_A\left([W]^{AB} |\psi\rangle\right)$$

## Proof

$$\begin{aligned} & \left(\pi_A[W]^{AB}\right) |\psi\rangle \\ &= [W]^A |\psi\rangle \\ &= [W |\psi\rangle]^A \\ &= \pi_A\left([W |\psi\rangle]^{AB}\right) \\ &= \pi_A\left([W]^{AB} |\psi\rangle\right) \end{aligned}$$

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Some precursors who had enough imagination!

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"A wild, thought-provoking ride through the Twilight Zone world of quantum theory . . . ."  
—Michio Kaku, author of *Hyperspace*

Some pr

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\* It was

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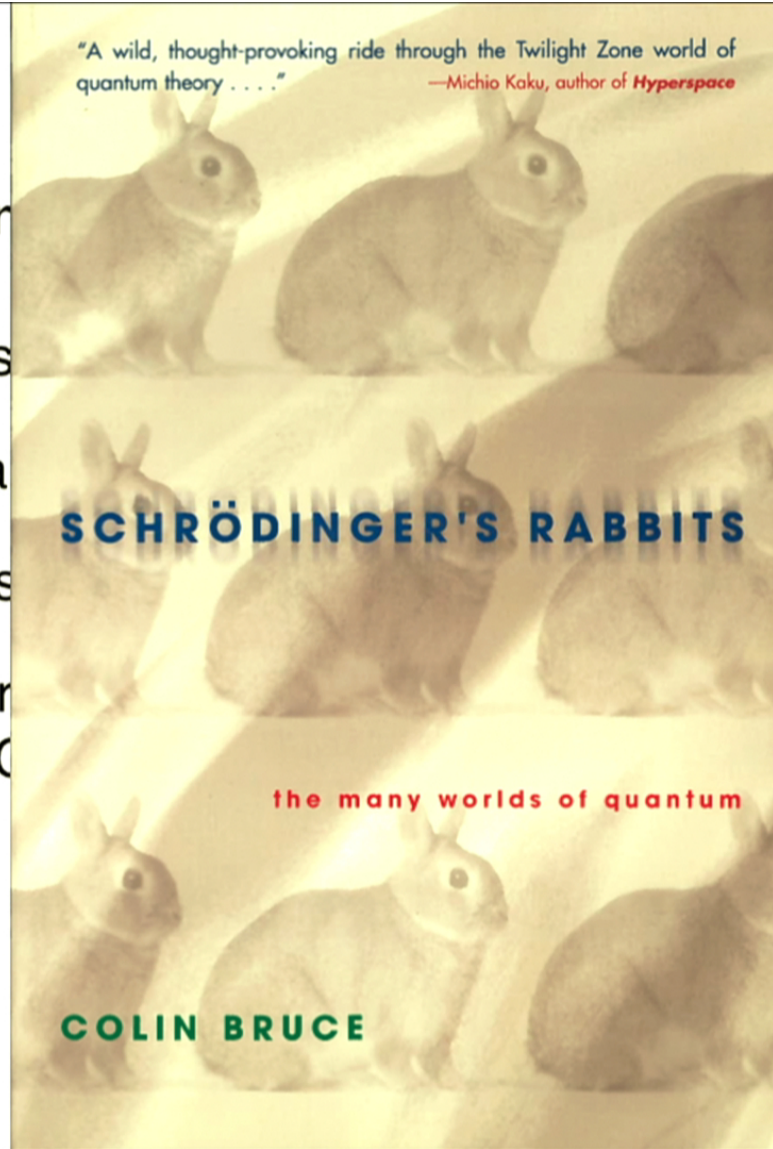
\* Descr  
(aka C

2004)

**SCHRÖDINGER'S RABBITS**

the many worlds of quantum

**COLIN BRUCE**



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- \* We made it simpler
- \* We proved that the universal wavefunction *cannot*  
be a complete description of physical reality

# Parallel Lives vs Many Worlds

According to the Desiderata, all states are separable:

$$|\psi^+\rangle = M^A \otimes M^B$$

1) For the total Bell state:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

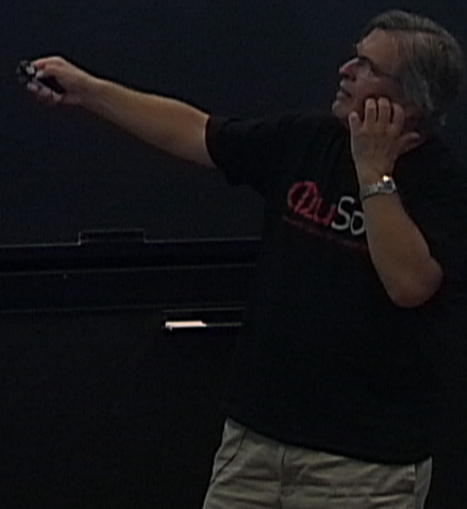
2) The negation gate:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

## Parallel Lives vs Many Worlds

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According to the Desiderata, all states are separable:

$$|\Psi^+\rangle = M^A \odot M^B$$

By separate evolution:

$$(N \otimes N) |\Psi^+\rangle = N(M^A) \odot N(M^B)$$

Since

$$|\Psi^+\rangle = (N \otimes N) |\Psi^+\rangle$$



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A contradiction!

