Title: Why interactions matter: How the laws of dynamics determine the shape of physical reality

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Abstract: Measurements performed at variable strengths show that non-commuting physical properties are related by complex-valued statistics, where the complex phase expresses the action of transformations along orbits represented by the eigenstates. In strong measurements, the dynamics along the orbits is completely randomized, which means that the pure states prepared by such a measurement actually represent ergodic statistics where the coherence between components originates from quantum dynamics. The complex algebra of Hilbert space inner products describes the intersection of two ergodically randomized orbits, where the complex phase describes the action of propagation along the orbits. Since the same action also appears in classical descriptions of the dynamics it is possible to derive quantum states and their time evolution directly from the classical equations of motion, without the abstractions of operator algebra.

A representative example of this fundamental relation between classical dynamics and quantum coherence is the multi-photon interference in two-path interferometers, where the multi-photon interference fringes can be explained by the action enclosed by two classical orbits corresponding to the input and output photon number states. This example shows how the non-classical features of quantum statistics emerge from the effects of enclosed actions on the causality relations between the initial orbit prepared by ergodic randomization and the final orbit along which the system was sampled during the measurement. Since action relations take the same form in quantum mechanics and in the classical limit, any attempt to explain quantum mechanics should start with an analysis of the dynamics.

The conventional sense of reality only emerges from the consistency of causality relations,not from any abstract ``knowledge of reality". Our concepts of particles and trajectories only have an approximate validity which breaks down in the limit of small action. Reality always requires the dynamics of interaction, and hbar is an absolute limitation of physical reality. In this presentation, I hope to clarify that this absence of a microscopic material reality can be understood quite naturally in terms of the well known physics of dynamics and interactions, removing the need for any untestable platonic assumptions about a hypothetical ``reality out there".

Why interactions matter: How the laws of dynamics determine the shape of physical reality

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- Why are weak and strong measurements so different?
- Deterministic causality and the definition of action
- Emergence of "classical" realities from "quantum" dynamics


Why are weak and strong measurements
Deterministic causality and the definition
Emergence of "classical" realities


University
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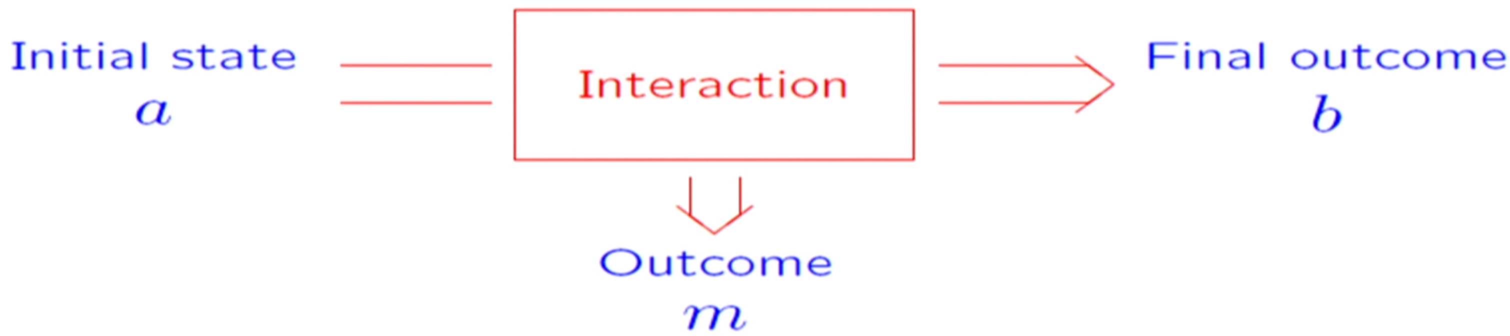
Measurements are physical:
Why the strength of interactions makes a difference

Physics describes relations between physical properties (a, m, b) -
But how can we find out what the correct relations are?

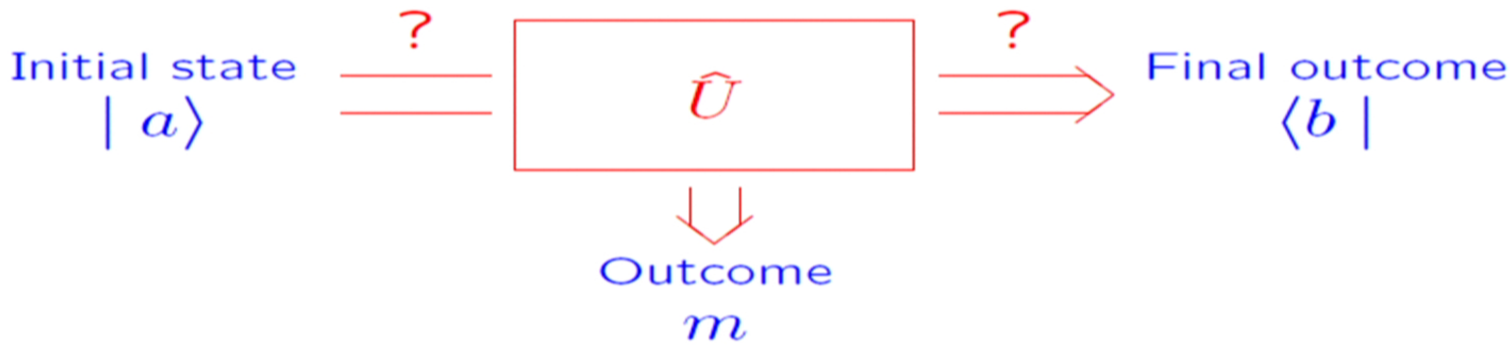


Nature is known by "touch and sight" -

Platonic realities are fictitious additions to science.

Uncertainties of measurement dynamics

Variable strength sequential measurements are a powerful method of exploring the relations between physical properties.



If $\hat{U}$ changes b, there is *no joint reality* of (a, m, b) before or after the interaction.

Deterministic relations in weak measurements

Weak values describe the objective relation between the eigen-states of three physical properties,

Initial state $| \, a \rangle$ $\Longrightarrow$ $\boxed{\hat{U}}$ $\Longrightarrow$ Final outcome $\langle b \, |$

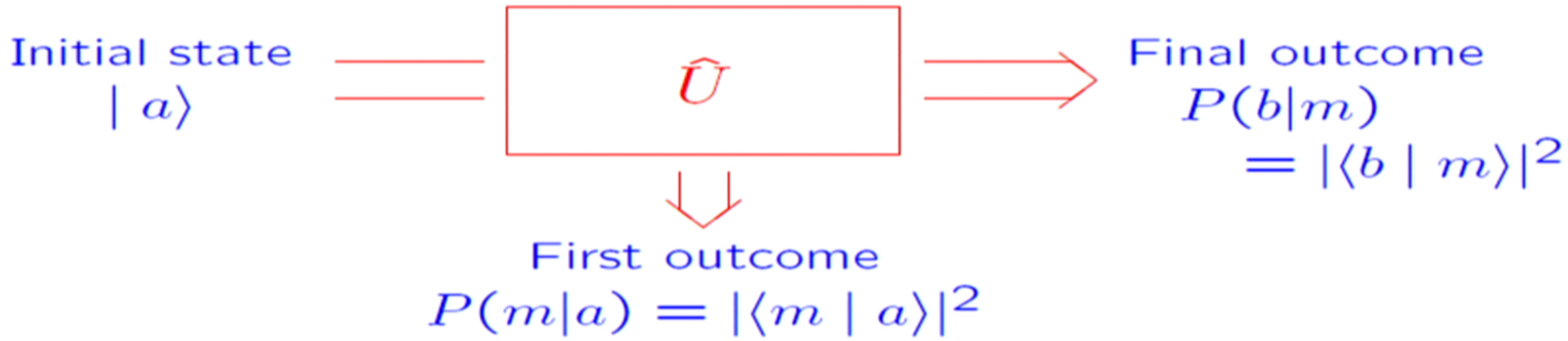
$\Downarrow$

$$P_{\text{weak}}(m|a,b) = \frac{\langle b \mid m \rangle \langle m \mid a \rangle}{\langle b \mid a \rangle}$$ weak statistical signature

Weak values describe the objective relations between three strong measurements (a, m, b) that have *no joint reality*.

Dynamic randomization in strong measurements

In strong measurements, b originates from the *dynamics* generated by the *ergodic orbit* $| m\rangle\langle m |$.



> Quantum states and projective measurements represent the ergodic average of a complete cycle along the orbit $| m\rangle\langle m |$!

How does ergodic randomization work?

The ergodic probability is defined as the time average of $P(x(t))$ over one or more periods T,

$$P_{\text{erg.}}(b) = \frac{1}{T} \int P(b(t))\, dt$$

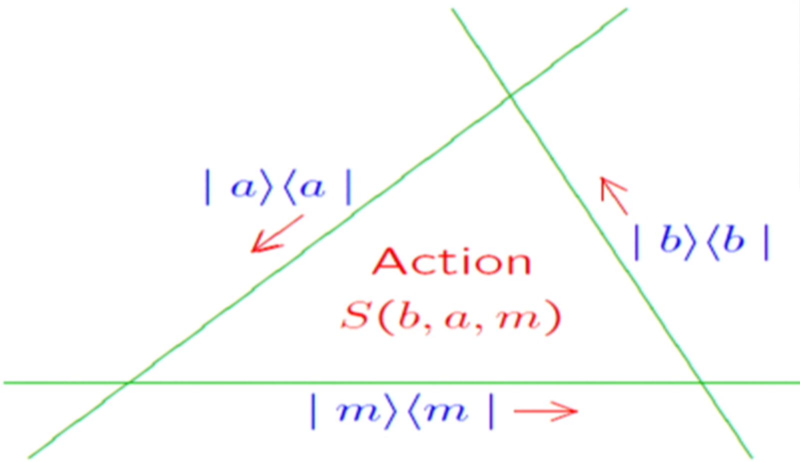
For dynamics generated by $\widehat{U}(t)$ with eigenstates $\mid m\rangle$,

$$\begin{aligned} P_{\text{erg.}}(b) &= \frac{1}{T} \int \langle b \mid \widehat{U}(t) \mid a\rangle\langle a \mid \widehat{U}^{\dagger}(t) \mid b\rangle\, dt \\ &= \sum_{m} \langle b \mid m\rangle\langle m \mid a\rangle\langle a \mid m\rangle\langle m \mid b\rangle \end{aligned}$$

In quantum dynamics, ergodic averaging results in *dephasing*.

Relations between different orbits: the origin and meaning of complex probabilities

Quantum states describe ergodic orbits. Complex inner products describe the *dynamics* of intersecting orbits.



$$\boxed{S(b,a,m) = \hbar \,\text{Arg}\left(\frac{\langle b \mid m\rangle\langle m \mid a\rangle}{\langle b \mid a\rangle}\right)}$$

Action S optimizes transformations:

$\text{Max}(|\langle b \mid \hat{U}_M \mid a\rangle|^2)$ for

$\hat{U}_M = \sum_m \exp(-\frac{i}{\hbar}S(b,a,m)) \mid m\rangle\langle m \mid$

Action physics

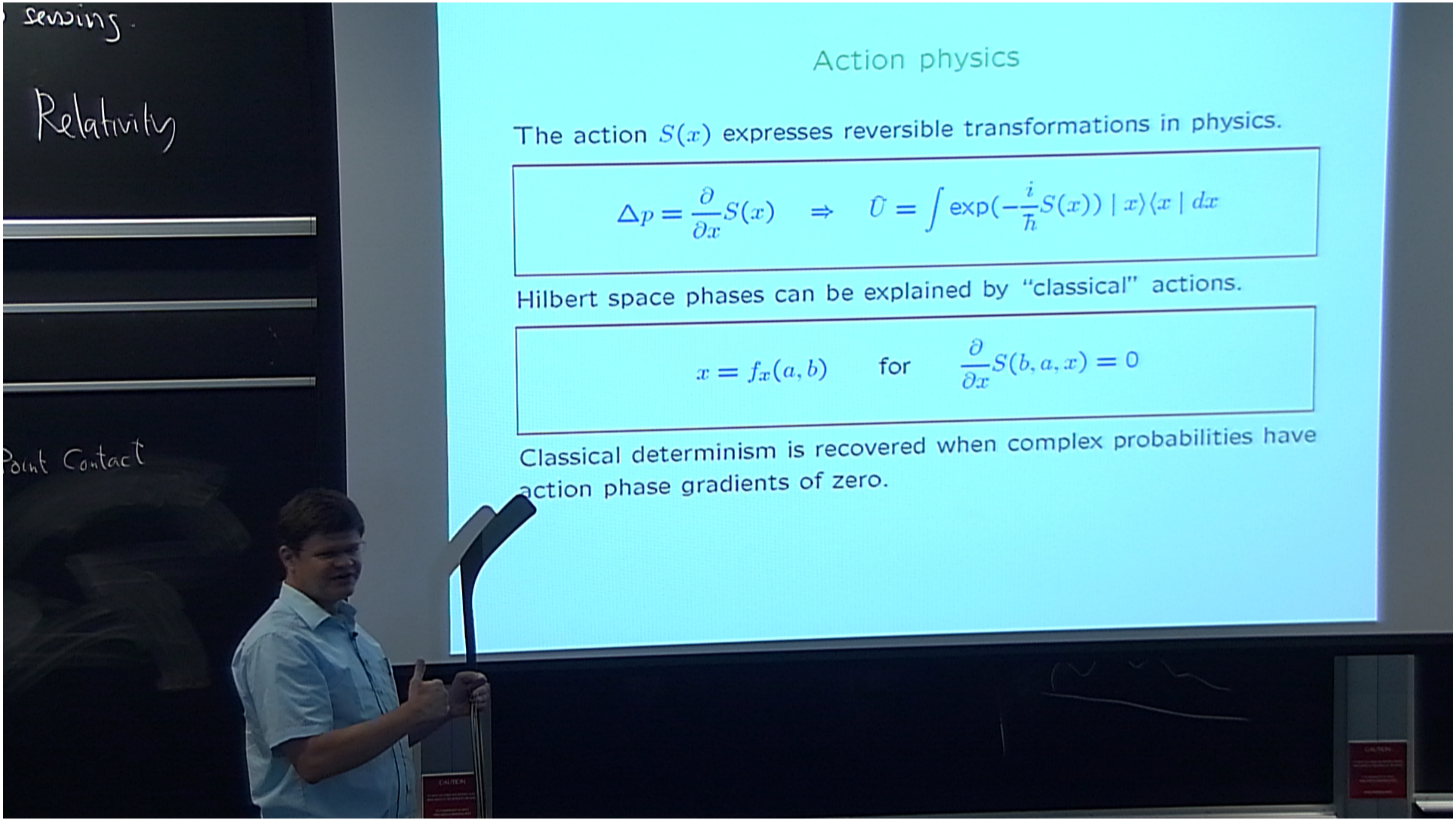
The action $S(x)$ expresses reversible transformations in physics.

$$\Delta p = \frac{\partial}{\partial x} S(x) \quad \Rightarrow \quad \hat{U} = \int \exp(-\frac{i}{\hbar} S(x)) \mid x \rangle \langle x \mid dx$$

Hilbert space phases can be explained by "classical" actions.

$$x = f_x(a, b) \qquad \text{for} \qquad \frac{\partial}{\partial x} S(b, a, x) = 0$$

Classical determinism is recovered when complex probabilities have action phase gradients of zero.


sensing.
Relativity
Point Contact
Action physics
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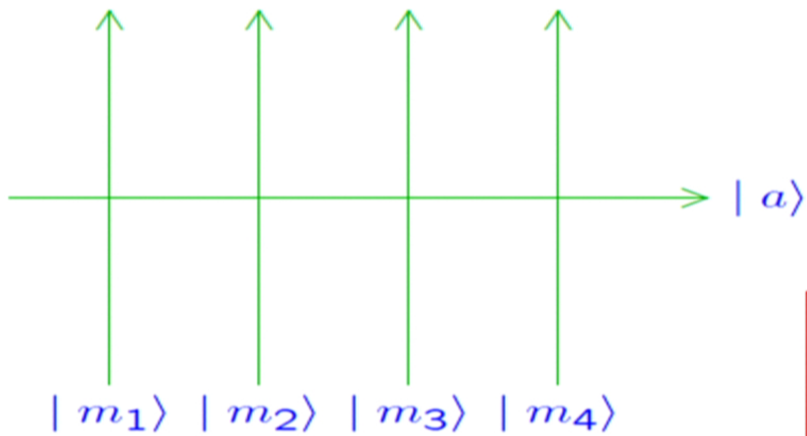
Quantum coherence as transformation distance

"Superpositions" describe intersections between orbits, where the phase localizes the intersection along the orbits.



Intersections:

$$\langle m \mid a\rangle$$

Enclosed action of three orbits:

$$\begin{aligned} S(b, a, m) = \hbar \mathrm{Arg}(\langle m \mid a\rangle) \\ + \hbar \mathrm{Arg}(\langle b \mid m\rangle) \\ + \hbar \mathrm{Arg}(\langle a \mid b\rangle) \end{aligned}$$

Quantum ergodicity of energy eigenstates

A classical action $S(B, A, E)$ can be derived from the time it takes to get from A to B at an energy of E,

$$\boxed{\frac{\partial}{\partial E} S(B, A, E) = t(B, A, E)}$$

The eigenstate $\mid m\rangle$ with $E = E_m$ can be expressed in the eigenstate basis $\mid b\rangle$ with $B = B_b$.

$$\boxed{\langle b \mid m\rangle \approx \sqrt{\frac{\Delta E \Delta B}{2\pi\hbar}} \sum \sqrt{\frac{\partial^2 S(B, A, E)}{\partial B\, \partial E}} \exp\left(i \frac{S(B, A, E)}{\hbar}\right)\Bigg|_{B=B_b; E=E_m}}$$

$\Delta E, \Delta B$ give the intervals between eigenvalues;
The sum runs over multiple solutions for t.

Getting the classical limit right

Action phases also describe the dynamics of states in Hilbert space (Eur. Phys. J. D 70, 118 (2016)).

$$\boxed{S(b,a,t) = \hbar \, \mathrm{Arg}\left(\langle b \mid \hat{U}(t) \mid a\rangle\right)}$$

The dynamics of the action is given by the energy,

$$\boxed{\frac{\partial}{\partial t} S(b,a,t) = -\mathrm{Re}\left(\frac{\langle b \mid \hat{H} \mid a(t)\rangle}{\langle b \mid a(t)\rangle}\right) = -E(b,a,t)}$$

The weak value replaces the classical energy in this generalized Hamilton-Jacobi equation.

Time evolution and weak values

The time evolution of the "intersection" $\langle b \mid a(t) \rangle$ is given by the weak values of energy:

$$\frac{\partial}{\partial t}\langle b \mid a(t)\rangle = -\frac{i}{\hbar}\,\frac{\langle b \mid \hat{H} \mid a(t)\rangle}{\langle b \mid a(t)\rangle}\,\langle b \mid a(t)\rangle$$

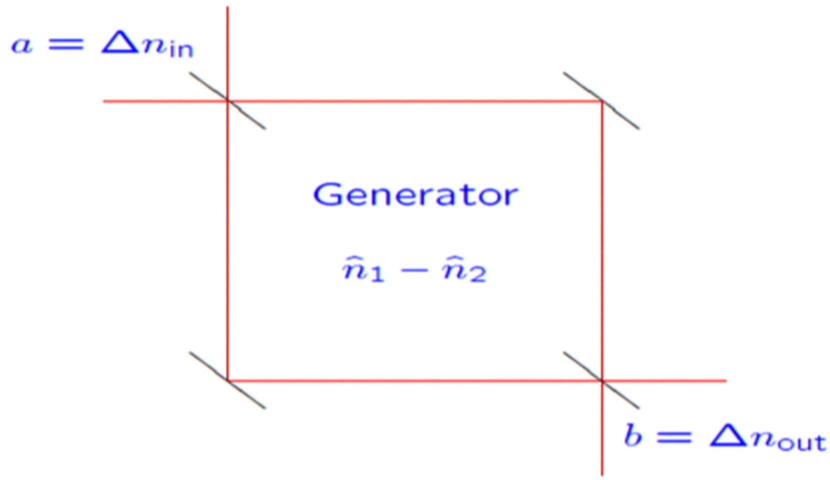
Higher order derivatives are determined by higher order weak values:

$$\frac{\partial^2}{\partial t^2}\langle b \mid a(t)\rangle = -\frac{1}{\hbar^2}\,\frac{\langle b \mid \hat{H}^2 \mid a(t)\rangle}{\langle b \mid a(t)\rangle}\,\langle b \mid a(t)\rangle$$

Equivalent to time dependent Schrödinger equations, these relations provide a better explanation of the physics of energy and time.

Observation of the action in multi-photon interferences

In N-photon interference, the phase-dependent action appears as interference between positive and negative generator values.



Phase shift: $\hat{U}(\phi) = \exp(-\frac{i}{2}(\hat{n}_1 - \hat{n}_2)\phi)$

$$\boxed{\frac{\partial^2}{\partial \phi^2}\langle b \mid a\rangle = -\frac{1}{\hbar^2} J_3^2(b,a,\phi)\langle b \mid a\rangle}$$

$$J_3 = \frac{\hbar}{2}\sqrt{\frac{\langle b \mid (\hat{n}_1 - \hat{n}_2)^2 \mid a\rangle}{\langle b \mid a\rangle}}$$

Action of interference fringes:

$$\langle b \mid a\rangle \approx A\cos(S(b,a,\phi)/\hbar)$$

$$\frac{\partial}{\partial \phi}S(b,a,\phi) = -J_3(b,a,\phi)$$

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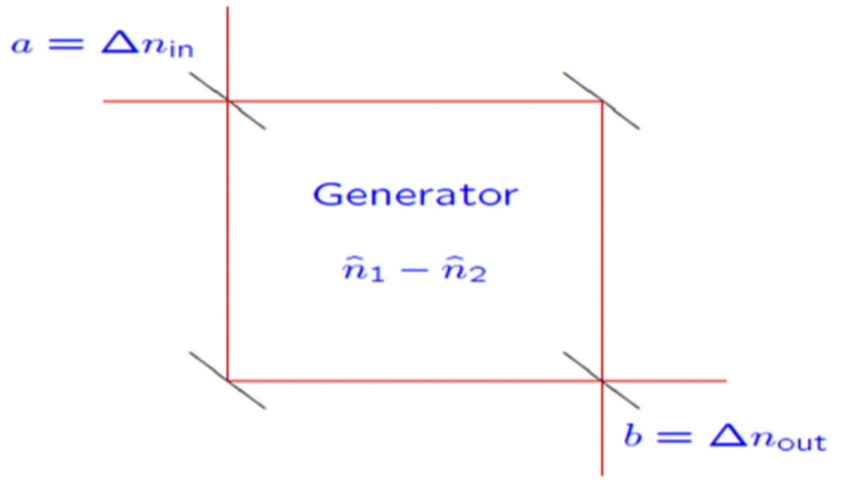

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Weak value (classical causality)

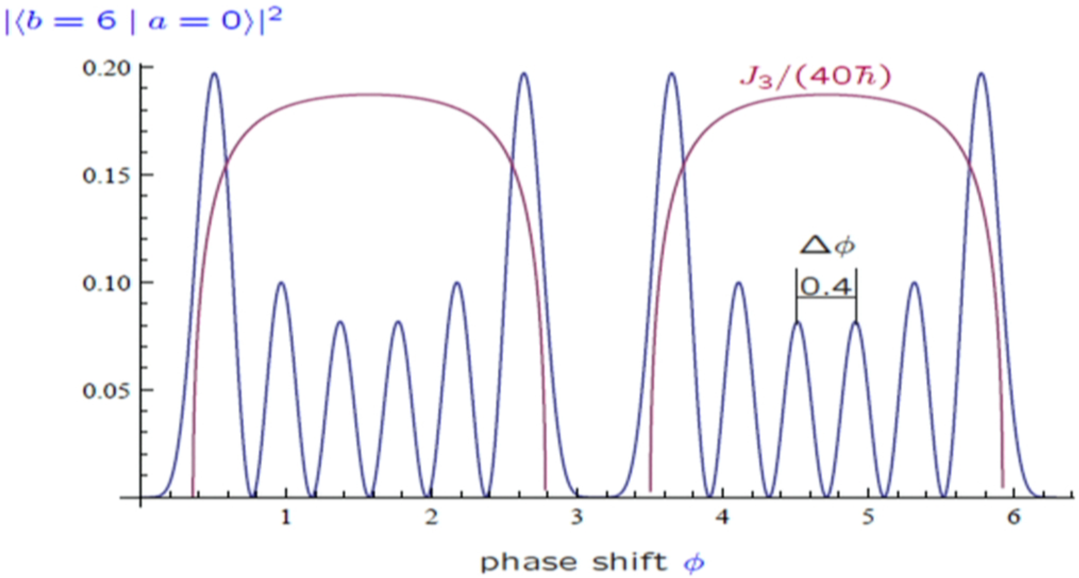
$$\Delta n_{\text{out}} = \cos\phi\,\Delta n_{\text{in}} + \sin\phi\sqrt{4 n_1 n_2 - \Delta n_{\text{in}}^2}$$

$$\Downarrow$$

$$J_3 = \frac{\hbar}{2}\sqrt{N^2 - \frac{a^2 - 2ab\cos\phi + b^2}{(\sin\phi)^2}}$$

Explanation of multi-photon interference fringes

$N = 16$ photons, input $(8,8)$, output $(11,5)$



Fringe width:

$$\Delta\phi = \frac{\pi\hbar}{J_3}$$

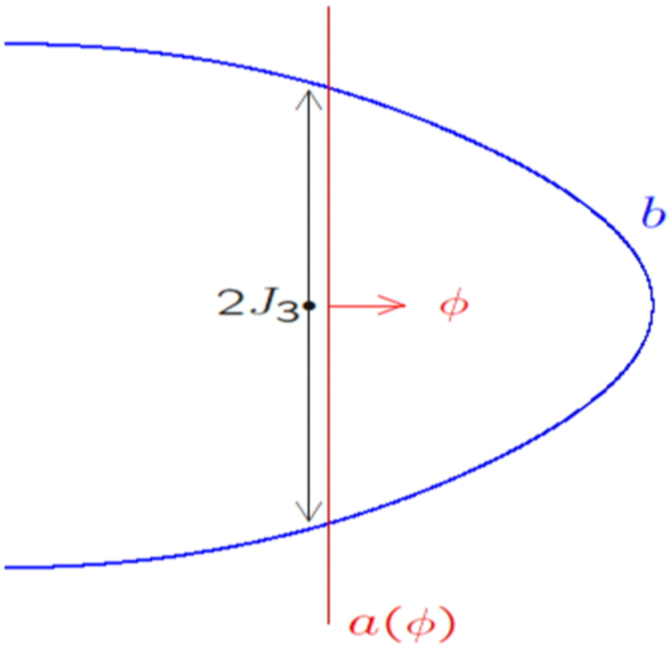
Path intensities:

$$J_3 = \frac{\hbar}{2}\sqrt{16^2 - \frac{6^2}{(\sin\phi)^2}}$$

Fringe estimate ($\phi = 0$):

$$\Delta\Phi_{\text{min.}} = \frac{\pi}{7.42} = 0.424$$

Interference is an effect of the enclosed action



The orbits a and b have two intersections with $\pm J_3$.

Phase shifts ϕ sweep the enclosed action $2S$,

$$\frac{\partial}{\partial \phi}(2S) = -2J_3$$

Interference fringes:

$$\langle b \mid a(\phi)\rangle = A \cos\left(\frac{1}{\hbar} S(\phi)\right)$$

Dynamic orbits versus static realities

Why is it impossible to break down the orbit $| a\rangle\langle a |$ into separate realities of (a, b) or(m, a) ?

$$\boxed{| a\rangle\langle a | = \sum_b | b\rangle\langle b | a\rangle\langle a | = \sum_m | m\rangle\langle m | a\rangle\langle a |}$$

The problem is that there is no static relation between (a, b) and (m, a). Instead, m connects a and a by dynamics,

$$| b\rangle\langle b | a\rangle\langle a | = \sum_m | b\rangle\langle b | m\rangle\langle m | a\rangle\langle a |$$

$$\langle b | \hat{U}_M | a\rangle = \sum_m \langle b | m\rangle\langle m | a\rangle \exp(-\frac{i}{\hbar}S(m))$$

Re-connecting the physics

Quantum state preparation results in ergodic randomization of a complete orbitenclosing a minimal action of $2\pi\hbar$.

Measurement samples a complete orbit before it can produce an outcome.

Quantum statistics are modulated by the action-phases enclosed by the two orbits.

The reality of physical systems is shaped by their dynamics.
Quantum phases describe this dynamics in the form of an action.

"And yet it moves ..."

Why do we stubbornly insist on fantasies of realities that nobody can see?

Quantum mechanics makes intuitive sense, because perfect control is an artificial asumption. We simply do not notice the details of the dynamics described by actions of $2\pi\hbar$.

Observations take time - there is no instantaneous snapshot of reality. Motion can only be observed by interactions that change the motion. Mathematical trajectories do not exist in the real world.

Action phases and complex correlations are consistent with our experience - mathematical trajectories and point particles are not!

Further reading

The story of the action phase - a trilogy in five parts:

1. "On the role of complex phases in the quantum statistics of weak measurements," H. F. Hofmann, New J. Phys. **13**, 103009 (2011).
2. "Complex joint probabilities as expressions of reversible transformations in quantum mechanics," H. F. Hofmann, New J. Phys. **14**, 043031 (2012).
3. "Derivation of quantum mechanics from a single fundamental modification of the relations between physical properties," H. F. Hofmann, Phys. Rev. A **89**, 042115 (2014).
4. "Quantum paradoxes originating from the nonclassical statistics of physical properties related to each other by half-periodic transformations," H. F. Hofmann, Phys. Rev. A **91**, 062123 (2015).
5. "On the fundamental role of dynamics in quantum physics," H. F. Hofmann, Eur. Phys. J. D **70**, 118 (2016).

Action and causality in quantum dynamics

$$a \xrightarrow[\text{Energy } E]{\text{Time } t} b$$

Action phases

$$E(t) = -\hbar \, \frac{\partial}{\partial t} \mathrm{Arg}(\langle b \mid \widehat{U}(t) \mid a \rangle)$$

$$t(E) = \hbar \, \frac{\partial}{\partial E} \mathrm{Arg}(\langle b \mid E \rangle \langle E \mid a \rangle)$$