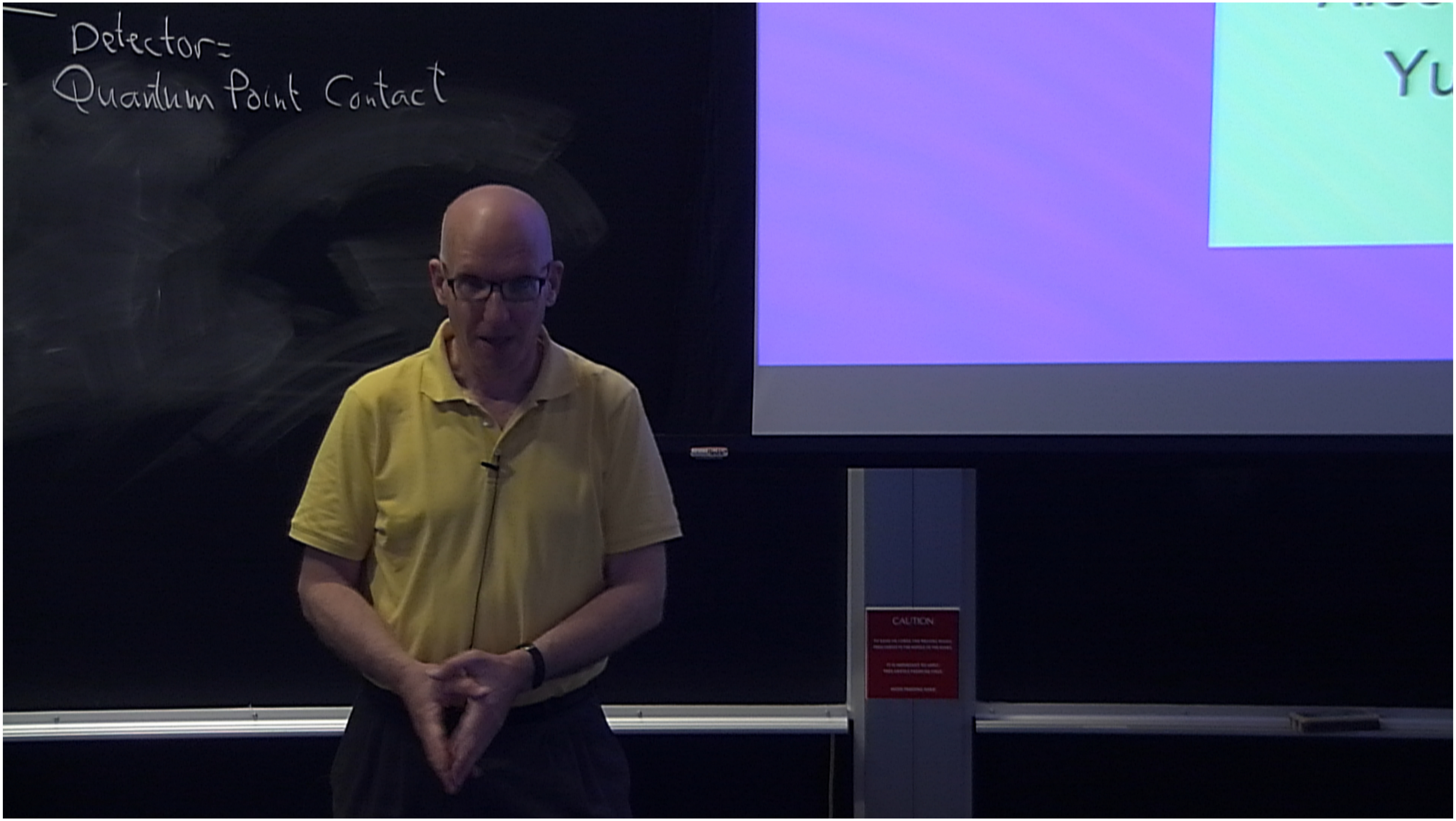
Title: TBA

Date: Jun 24, 2016 11:00 AM

URL: http://pirsa.org/16060072

Abstract:


Detector=
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CAUTION

What are topological excitations?

Measuring entanglement and non-locality of topological excitations

Weakly measuring entanglement and non-locality of topological excitations

THREE PILLARS OF QUANTUM MECHANICS

- ***WAVE EQUATION***

- ***NON—LOCALITY single particle (Einstein's box paradox, Elitzur-Vaidman bomb...)***

THREE PILLARS OF QUANTUM MECHANICS

- **WAVE EQUATION**
- **NON—LOCALITY single particle (Einstein's box paradox, Elitzur-Vaidman bomb...)**
- **NON-LOCALITY**
  entanglement
  topological excitations
  quantum statistics (fermions, bosons, anyons...)

topological excitations are usually associated with …

- ***topological protection against detrimental mechanisms of decoherence***
- ***quantum information processes***
- ***entanglement and non-locality***


Dirac fermions
TOPO-
ZOO
Majoranas
Dirac fermions
non-abelian anyons
Weil II
anyons
Weil

A Majorana fermion is a fermion
which is its own anti-particle

$$\Gamma = \Gamma^\dagger$$

Dirac equation : $(i\gamma^\mu \partial_\mu - m)\Psi = 0$

Complex solution Ψ

Dirac (1928)

Ψ particle $\neq \Psi^*$ antiparticle

Real solution Ψ

Majorana (1937)

Ψ particle $= \Psi^*$ antiparticle

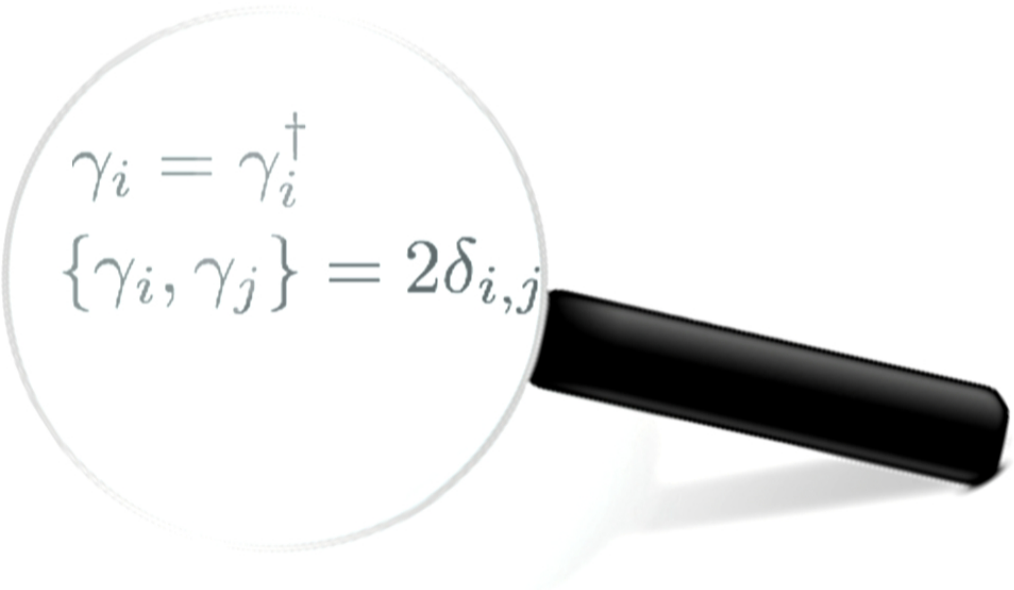
E. Majorana, Nuovo Cimento **14**, 171 (1937)

There are several categories of scientists in the world; those of second or third rank do their best but never get very far. Then there is the first rank, those who make important discoveries, fundamental to scientific progress. But then there are the geniuses, like Galilei and Newton. Majorana was one of these.

— (Enrico Fermi about Majorana, Rome 1938)

Majorana anyons

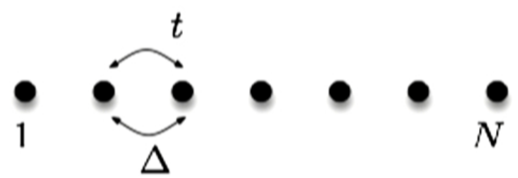
Zero energy excitations which
are their own antiparticles



The basics: Kitaev chain

p-wave spinless superconductor in 1D

$$H = -\mu \sum_{i=1}^{N} (c_i^\dagger c_i - 1/2) - \sum_{i=1}^{N-1} (t c_{i+1}^\dagger c_i - \Delta c_{i+1}^\dagger c_i^\dagger) + h.c.$$



[A. Yu. Kitaev (2001)]

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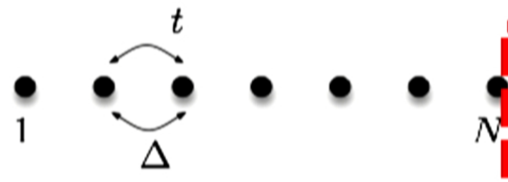

S-wave Superconductors

$$\Delta c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + h.c.$$

[A. Yu. Kitaev (2001)]

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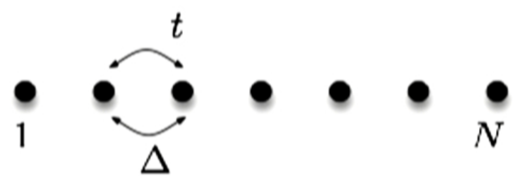

$$c_j = 1/2(\gamma_{b,j} + i\gamma_{a,j})$$

[A. Yu. Kitaev (2001)]

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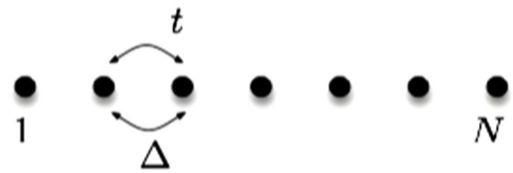
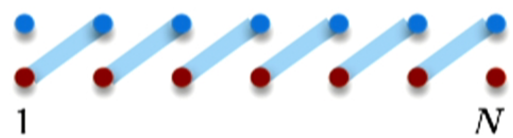


$$c_j = 1/2(\gamma_{b,j} + i\gamma_{a,j})$$

in the limit $t = \Delta\,,\ \ \mu = 0$



$$d_j = 1/2(\gamma_{a,j+1} + i\gamma_{b,j})$$

$$d_0 = 1/2(\gamma_{a,1} + i\gamma_{b,N})$$

$$H = 2\Delta \sum_{i=1}^{N-1} (d_i^\dagger d_i - 1/2)$$

[A. Yu. Kitaev (2001)]

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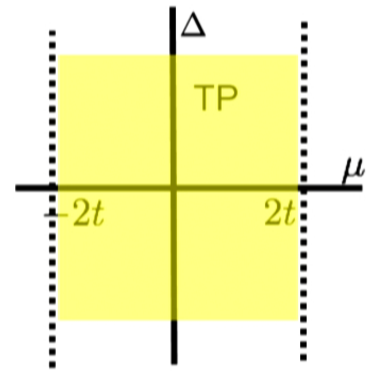
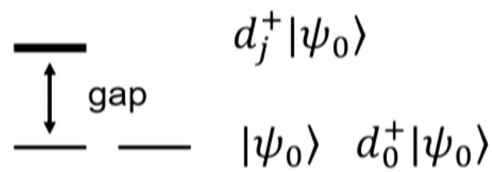
p-wave spinless superconductor in 1D

$$H = -\mu \sum_{i=1}^{N} (c_i^\dagger c_i - 1/2) - \sum_{i=1}^{N-1} (t c_{i+1}^\dagger c_i - \Delta c_{i+1}^\dagger c_i^\dagger) + h.c.$$





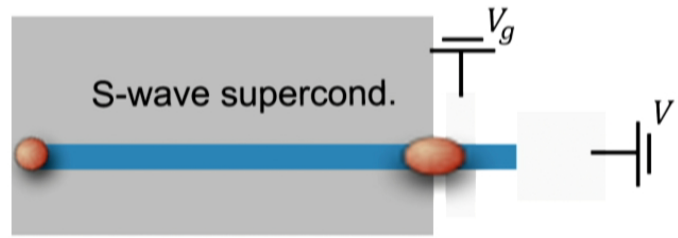
Searching for Majorana bound states

Engineering topological phases in nanostructures

[L. Fu and C.L. Kane (2008)]

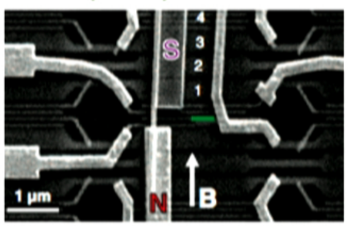
Semiconductor wires with spin-orbit coupling → experiments

[Y. Oreg et al. (2010), J. D. Sau et al. (2010)]

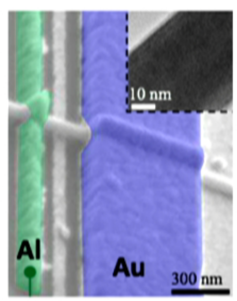


Experiments in nanostructures

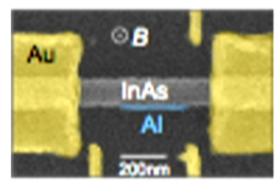
InSb wires
Delft (2012)



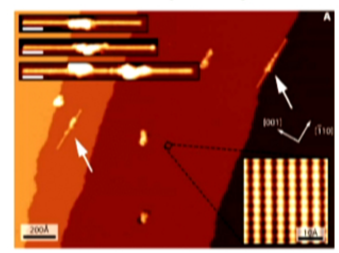
InAs wires
Weizmann (2012)



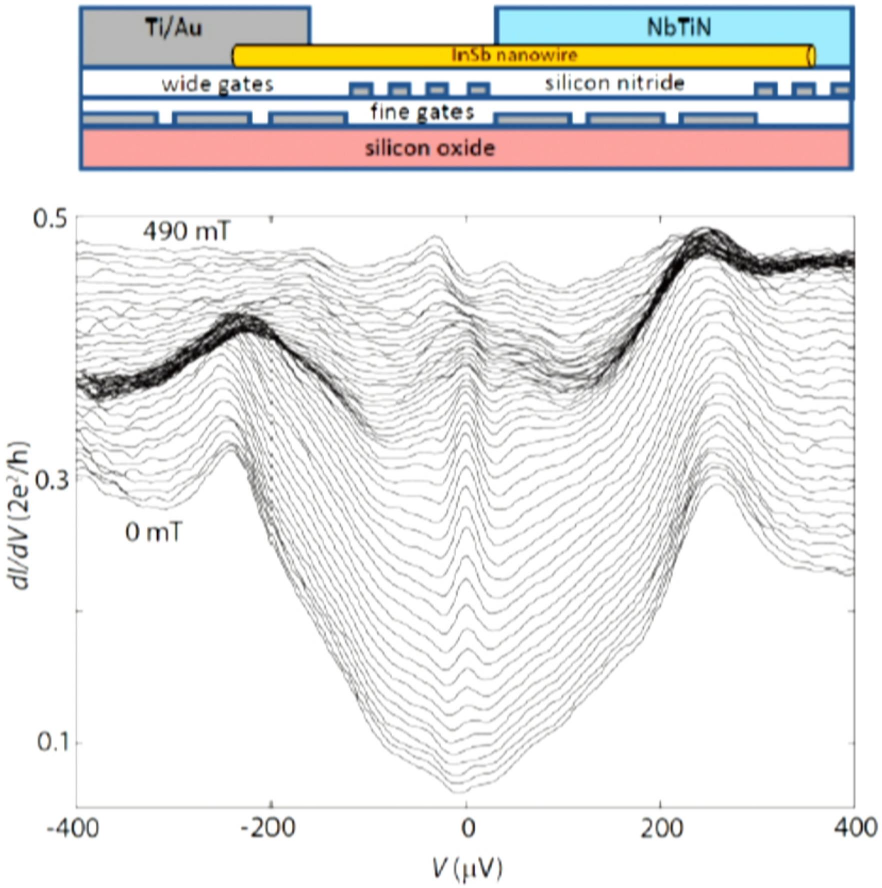
InAs wires
Copenhagen (2014)



Fe atomic chains
Princeton (2014)

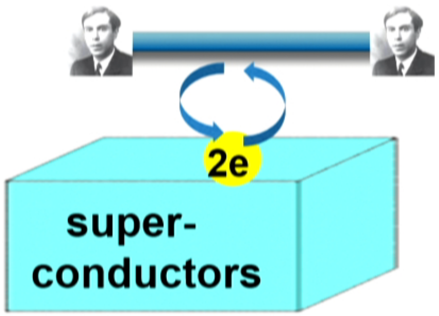


Experimental signatures of Majorana physics



Mourik et al., Science **336**, 1003 (2012)

More on Majorana bound states



$$d_0 = 1/2(\gamma_{a,1} + i\gamma_{b,N})$$

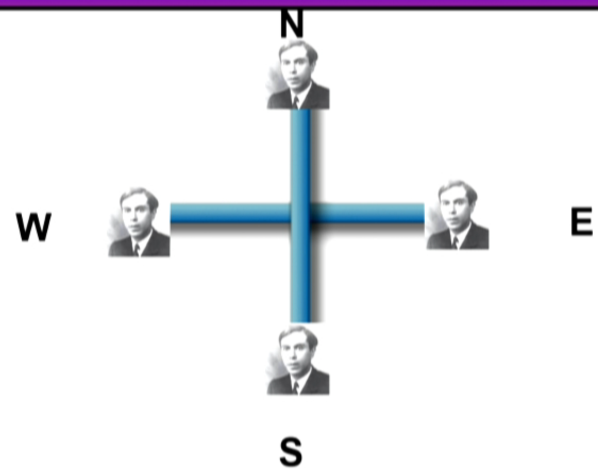
Two Majorana end-states $\Rightarrow$ One Dirac Fermion

$|0\rangle$ or $|1\rangle$

No Interference between and

Dirac's fermion fractionalizes !!

Coherent superposition of Majorana states



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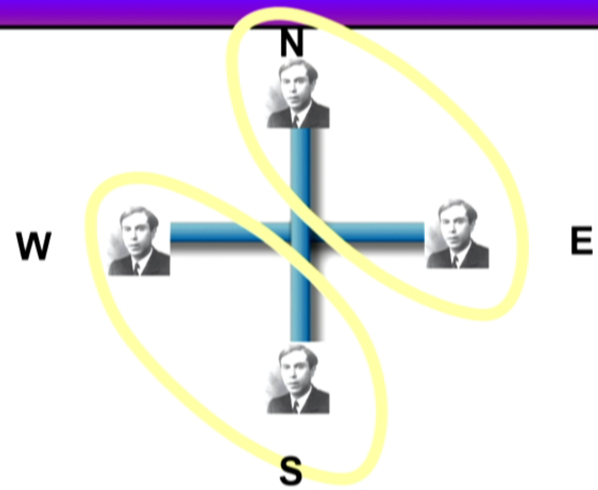


possible states:

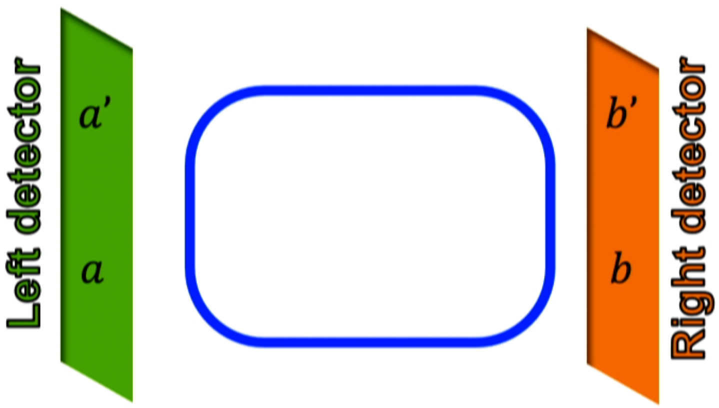
$$\left|NE,SW\right\rangle = \underbrace{\left|00\right\rangle,\left|11\right\rangle}_{\text{even}},\underbrace{\left|01\right\rangle,\left|10\right\rangle}_{\text{odd}}$$

Non-local signatures

Can we detect genuine quantum correlations, e.g. entanglement, in these systems?

Detecting quantum non-locality

Non-local quantum correlations (entanglement) can be operatively identified via correlation measurements



CHSH inequality

$$\mathcal{C} = |\langle ab\rangle - \langle ab'\rangle| + |\langle a'b\rangle + \langle a'b'\rangle|$$

Local (hidden variables) $\mathcal{C} \leq 2$

Quantum entangled states $2 < \mathcal{C} \leq 2\sqrt{2}$

measurements optimized for the given state

[J. Bell (1964), J.F. Clauser et al. (1969)]

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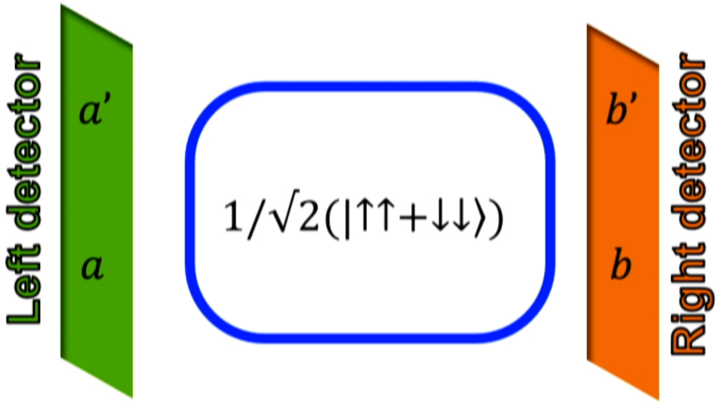

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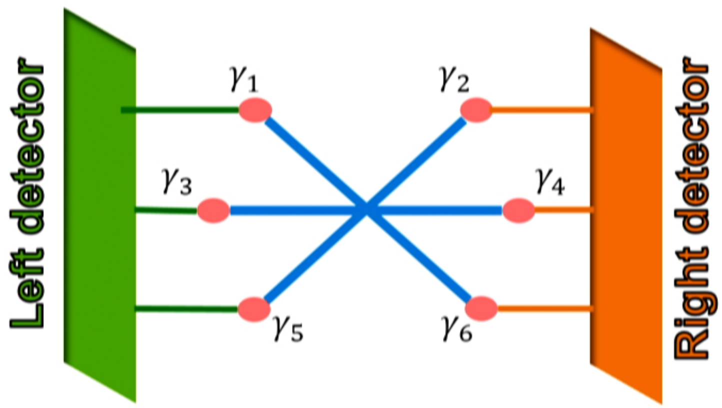
$$a = z, a' = x, b = \frac{1}{\sqrt{2}}(x+z), b' = \frac{1}{\sqrt{2}}(x-z)$$

measurements optimized for the given state

$$\mathcal{C} = 2\sqrt{2}$$

[J. Bell (1964), J.F. Clauser et al. (1969)]

Local operators



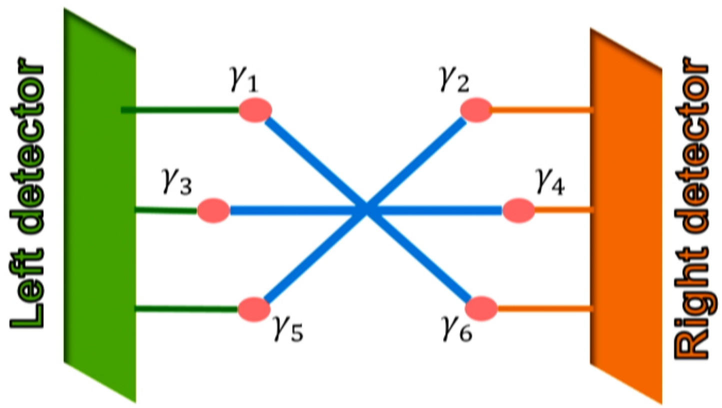
6 Majoranas:
Minimal viable setup

$L_{135} \ni \{-i\gamma_1\gamma_3 = \sigma_z, \text{-}i\gamma_3\gamma_5 = \sigma_x, -i\gamma_5\gamma_1 = \sigma_y\}$

$R_{426} \ni \{-i\gamma_4\gamma_2 = \sigma_z, \text{-}i\gamma_2\gamma_6 = -\sigma_y, -i\gamma_6\gamma_4 = \sigma_x\}$

Spatially separable algebras
of measurable operators

Local operators



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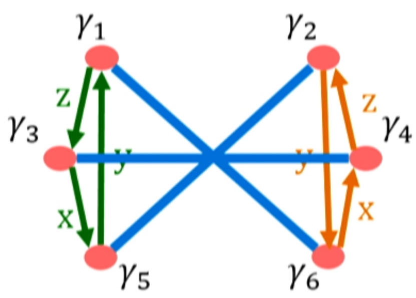

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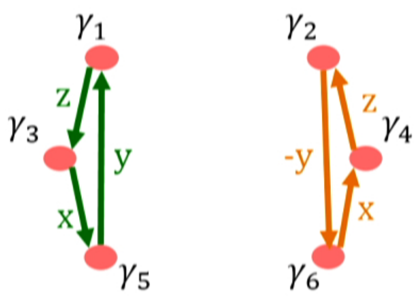
Spatially separable operators



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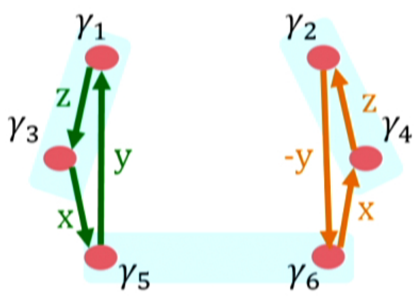
Spatially separable operators

$d_{13}|\varnothing\rangle = 0,\ d_{56}|\varnothing\rangle = 0,\ d_{42}|\varnothing\rangle = 0 \qquad |\Psi\rangle = (Ad^{+}_{13}d^{+}_{42}d^{+}_{56} + Bd^{+}_{13} + Cd^{+}_{42} - Dd^{+}_{56})|\varnothing\rangle$



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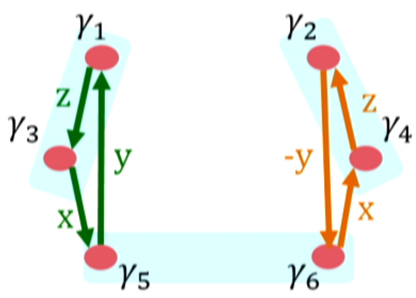

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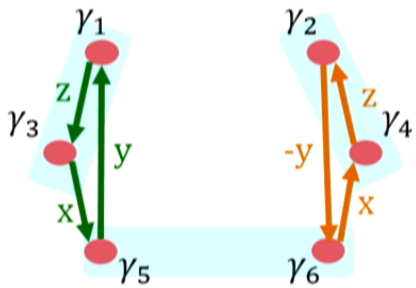

$\mathcal{C} \leq 2$ if $AD - BC$=0

$|\Psi\rangle = A|\uparrow\uparrow\rangle + B|\uparrow\downarrow\rangle + C|\downarrow\uparrow\rangle + D|\downarrow\downarrow\rangle$

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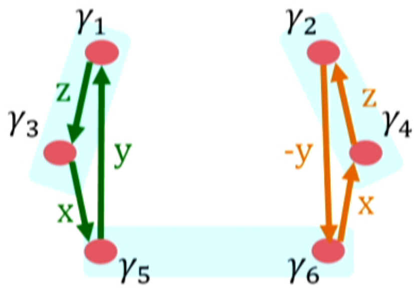
Detecting entanglement in different basis

$d_{13}|\varnothing\rangle = 0,\ d_{56}|\varnothing\rangle = 0,\ d_{42}|\varnothing\rangle = 0$ $\quad |\Psi\rangle = (Ad_{13}^{+}d_{42}^{+}d_{56}^{+} + Bd_{13}^{+} + Cd_{42}^{+} - Dd_{56}^{+})|\varnothing\rangle$



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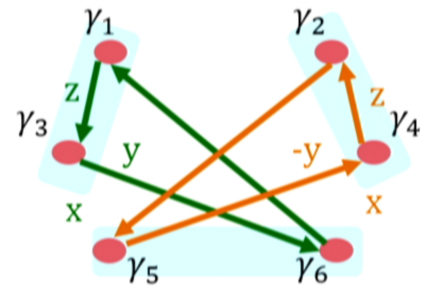


$\mathcal{C} \le 2$ if $AD + BC$=0

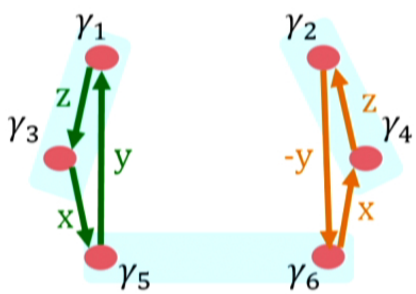
$|\Psi\rangle = A|\uparrow\uparrow\rangle + B|\uparrow\downarrow\rangle - C|\downarrow\uparrow\rangle + D|\downarrow\downarrow\rangle$

Non-local signatures

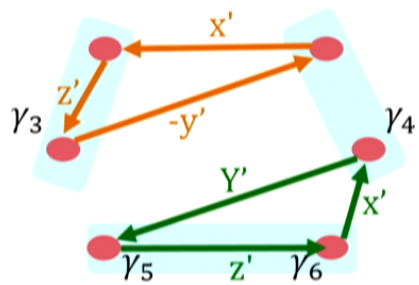
Any state of the system will show $\mathcal{C} > 2$ in at least one of the four measurement sets.

Any state will look non-local for some choice of measurement sets

Is it just algebra?



Formally the algebra holds for
2 spin ½ particles
→ generalized entanglement



The new operators cannot be measured by two spatially separable detectors in the spin system

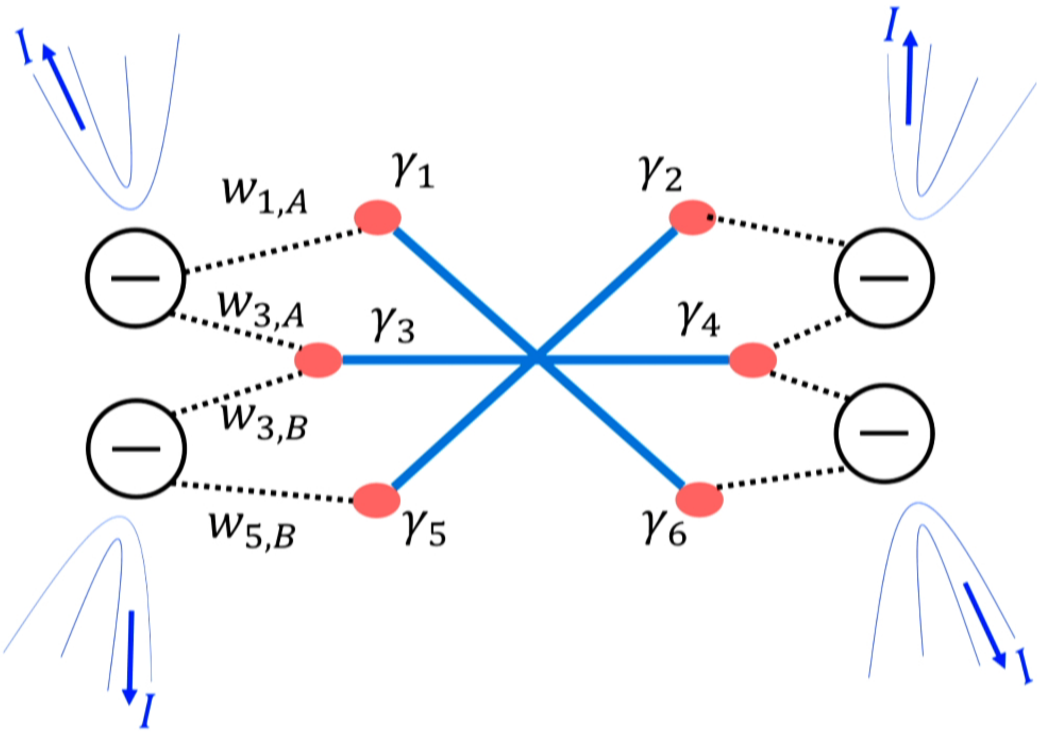
Majorana fractionalize fermions to make it possible

[Barnum et al. (2002),…]

Outlook

- The edge states of topological phases of matter naturally have non-local features.
- Any low-energy state of a Majorana system is entangled in some measurement configuration!
- The required measurements are accessible in current-technology nanostructures
- What for non-Ising anyons? What beyond the minimal geometry? Non-pure states?

What to measure? (cannot measure "charge of Majorana")



$$H_L = (w_{3,A}\gamma_3 + w_{1,A}\gamma_1)(c_{A,L} - c^{+}_{A,L}) + (w_{5,B}\gamma_5 + w_{3,B}\gamma_3)(c_{B,L} - c^{+}_{B,L})$$

Sensing the dots' charge configuration

$$(n_{A,L}, n_{B,L}) = \{(0,1), (1,0)\}$$

For weak measurements (short times, weak tunneling)