

Title: Quantum paradoxes emerging in matter-wave interferometer experiments

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Abstract: Peculiarities of quantum mechanical predictions on a fundamental level are investigated intensively in matter-wave optical setups; in particular, neutron interferometric strategy has been providing almost ideal experimental circumstances for experimental demonstrations of quantum effects. In this device quantum interference between beams spatially separated on a macroscopic scale is put on explicit view.

Recently, a new counter-intuitive phenomenon, called quantum Cheshire-cat, is observed in a neutron interferometer experiment. Weak measurement and weak values justify the access of the neutrons' dynamics in the interferometer. Moreover, another experiment reported full determination of weak-values of neutrons' \hat{S}_z -spin; this experiment is further applied to demonstrate quantum Pigeonhole effect and quantum contextual. In my talk, I am going to give an overview of neutron interferometry for investigation of quantum paradoxes.

Quantum paradoxes emerging in matter-wave interferometer experiments

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Chapman University

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Quantum paradoxes emerging in matter-wave interferometer experiments

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I. Introduction:

neutron interferometer

entanglement between degrees of freedom

II. Quantum Cheshire-Cat

III. 1/2-spin weak-values:

quantum Pigeonhole effect

quantum contextuality

IV. Summary

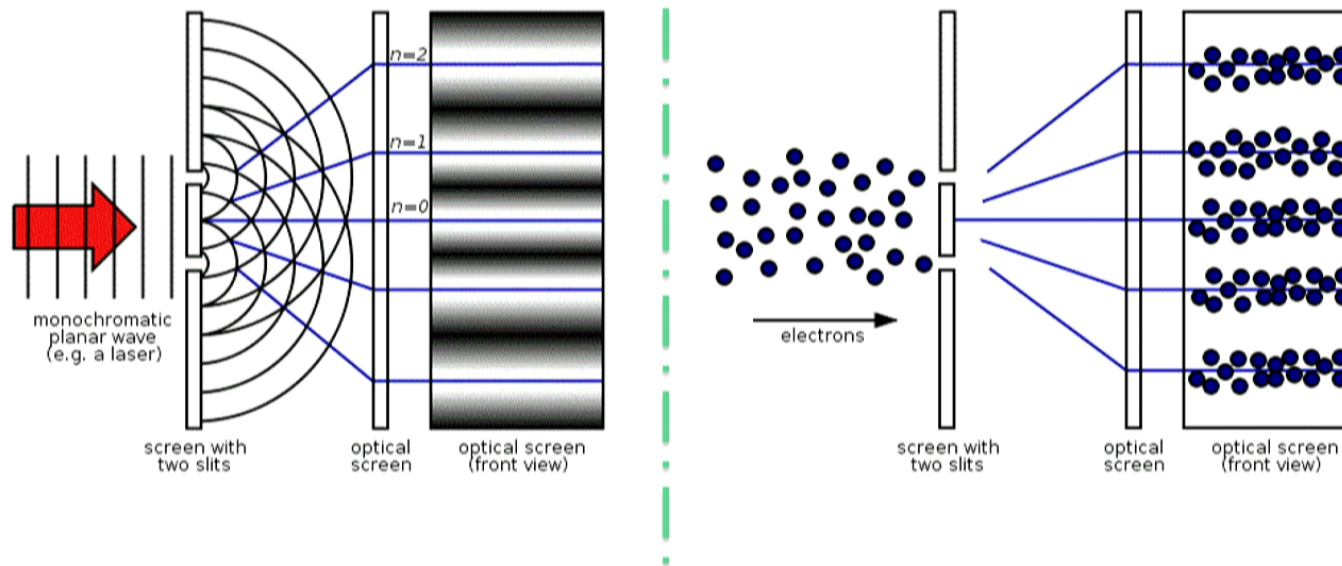
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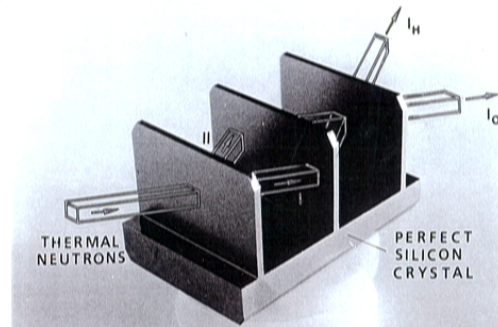
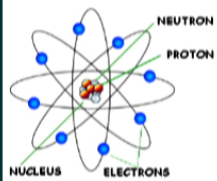
FWF



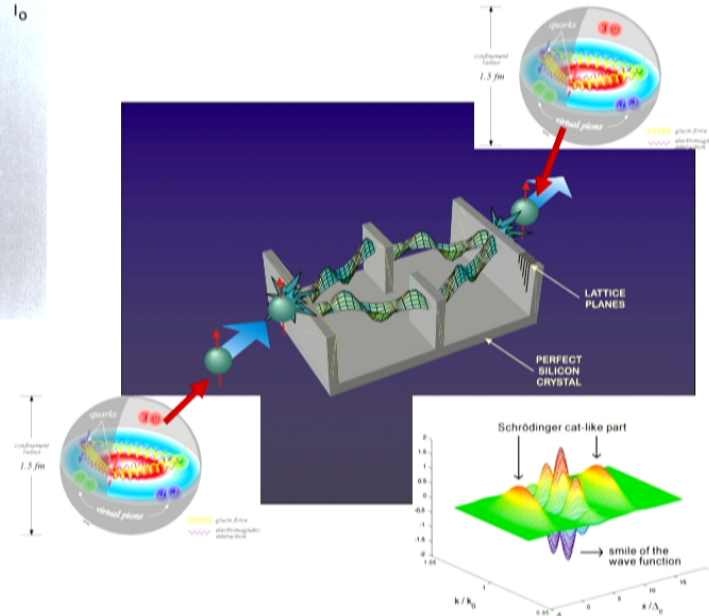
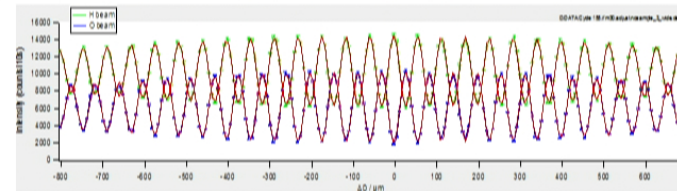
Waves in classical- and quantum-mechanics



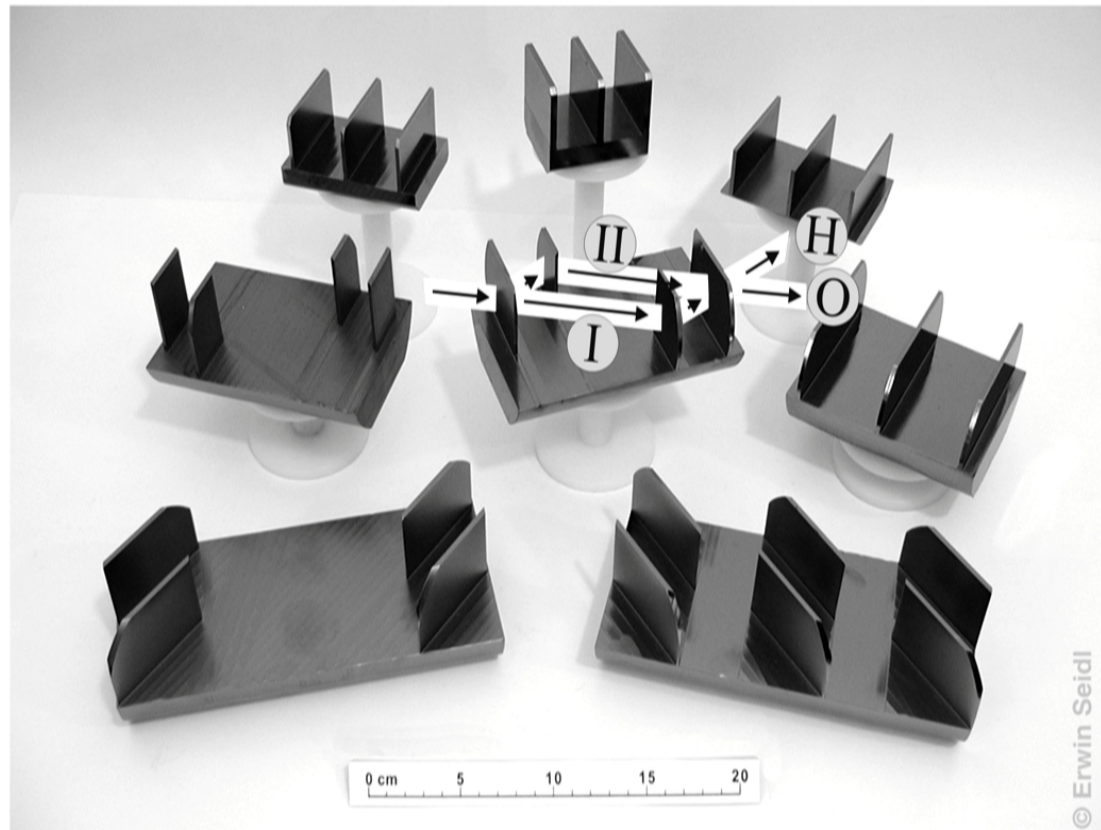
Neutronen interferometry: quantum skier



The New Yorker Collection, Ch.Adams 1940



Neutron interferometer family



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The neutron

Particle

$$m = 1.674928(1) \times 10^{-27} \text{ kg}$$

$$s = \frac{1}{2} \hbar$$

$$\mu = -9.6491783(18) \times 10^{-27} \text{ J/T}$$

$$\tau = 887(2) \text{ s}$$

$$R = 0.7 \text{ fm}$$

$$\alpha = 12.0(2.5) \times 10^{-4} \text{ fm}^3$$

u - d - d - quark structure

m ... mass, s ... spin, μ ... magnetic moment,
 τ ... β -decay lifetime, R ... (magnetic) confine-
 ment radius, α ... electric polarizability; all other
 measured quantities like electric charge, magnetic
 monopole and electric dipole moment are com-
 patible with zero

Feels four-forces

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m \cdot v}$$

Schrödinger

$$H\psi(\vec{r}, t) = i\hbar \frac{\delta\psi(\vec{r}, t)}{\delta t}$$

&

boundary conditions

Wave

$$\lambda_c = \frac{h}{m \cdot c} = 1.319695(20) \times 10^{-15} \text{ m}$$

For thermal neutrons

$$\lambda = 2\text{\AA}, v = 2\text{km/s}, E_{\text{kin}} = 20\text{meV}$$

$$\lambda_B = \frac{h}{m \cdot v} = 1.8 \times 10^{-10} \text{ m}$$

$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8} \text{ m}$$

$$\Delta_p = v \cdot \Delta t \cong 10^{-2} \text{ m}$$

$$\Delta_d = v \cdot \tau = 1.942(5) \times 10^6 \text{ m}$$

$$0 \leq \chi \leq 2\pi (4\pi)$$

λ_c ... Compton wavelength, λ_B ...
 deBroglie wavelength, Δ_c ...
 coherence length, Δ_p ... packet
 length, Δ_d ... decay length, δk ...
 momentum width, Δt ... chopper
 opening time, v ... group velocity, χ
 ... phase.



Neutrons in quantum mechanics

Particle and wave properties

$$p = mv = h/\lambda$$

(L. De Broglie)

Schroedinger equation

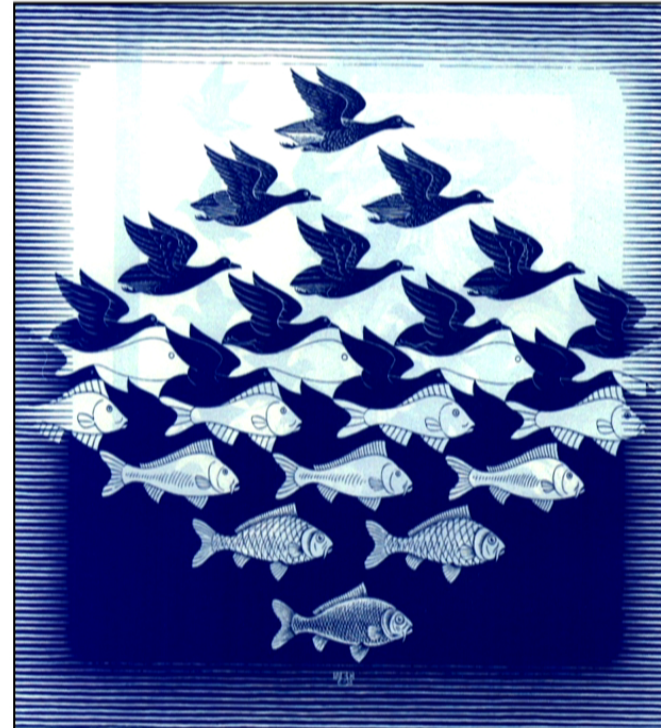
$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = H\Psi(\vec{r}, t)$$

(E. Schrödinger)

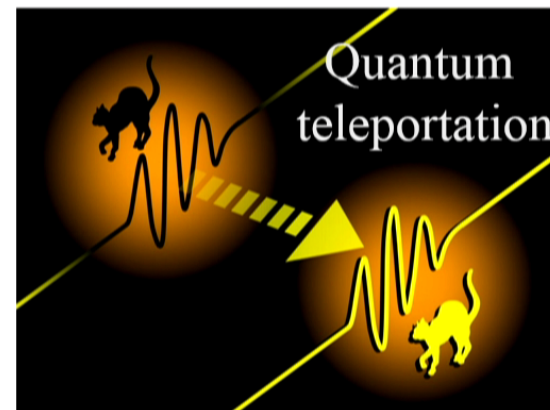
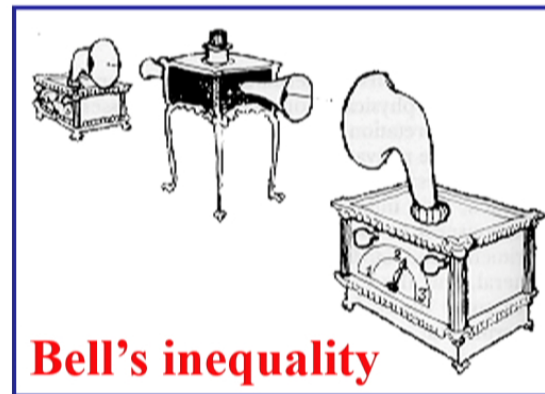
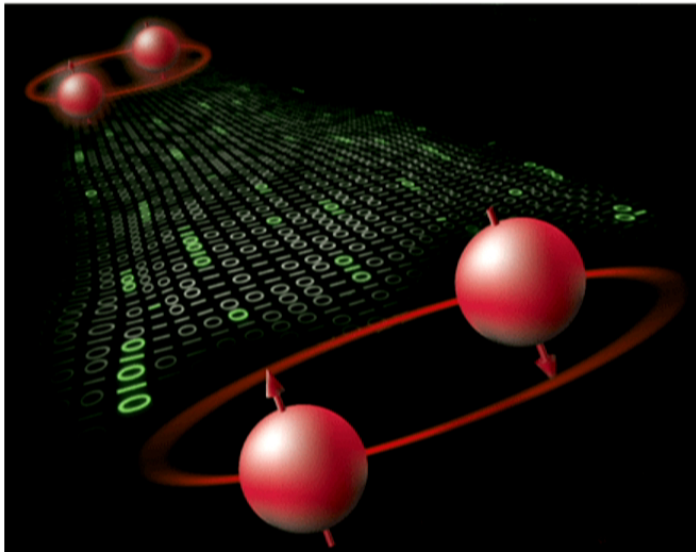
Uncertainty

$$\Delta x \Delta p \geq \hbar/2$$

(W. Heisenberg)



Quantum information technology: entanglement



Bi-partite and tri-partite entanglements

2-Particle Bell-State

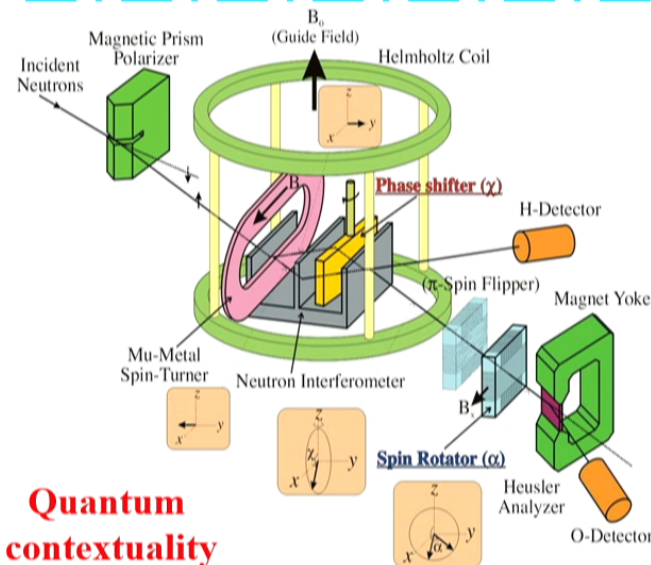
$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II} \}$$

I, II represent **2-Particles**

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_s \otimes |I\rangle_p + |\downarrow\rangle_s \otimes |II\rangle_p \}$$

s, p represent **2-Spaces**, e.g., spin & path



Quantum contextuality

Violation of Bell-like inequality

$$S' \equiv E'(\alpha_1, \chi_1) + E'(\alpha_1, \chi_2) - E'(\alpha_2, \chi_1) + E'(\alpha_2, \chi_2) = 2.051 \pm 0.019 > 2$$

Nature2003, NJP2011

Kochen-Specker-like contradiction

$$E_x \cdot E_y = 0.407 \xrightarrow{63\%} E' = \langle \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2 \rangle = -0.861$$

PRL2006/2009

Tri-partite entanglement (GHZ-state)

$$|\Psi_{\text{Neutron}}\rangle = \{ |\Psi_I\rangle \otimes |\uparrow\rangle \otimes |\Psi(E_0)\rangle + (e^{i\chi} |\Psi_{II}\rangle) \otimes (e^{i\alpha} |\downarrow\rangle) \otimes (e^{i\gamma} |\Psi(E_0 + \hbar\omega_r)\rangle) \}$$

$$M_{\text{Measured}} = 2.558 \pm 0.004 > 2$$

PRA2010/NJP2013

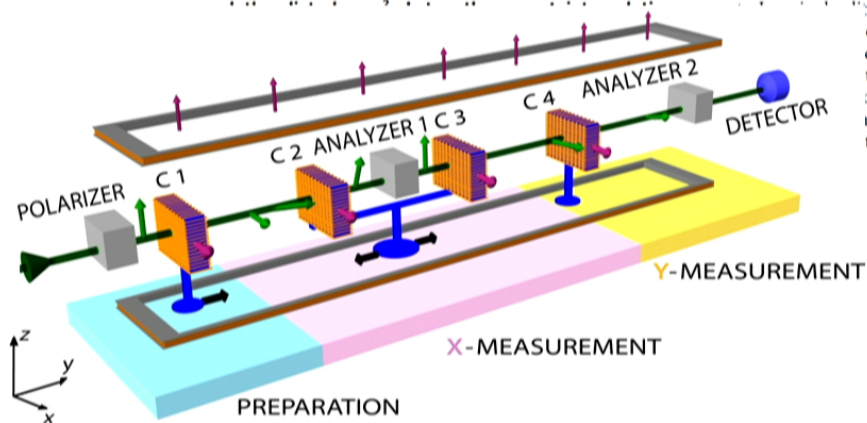
Experimental demonstration of a universally valid error-disturbance uncertainty relation in spin measurements

Jacqueline Erhart¹, Stephan Sponar¹, Georg Sulyok¹, Gerald Badurek¹, Masanao Ozawa² and Yuji Hasegawa^{1*}

The uncertainty principle generally prohibits simultaneous measurements of certain pairs of observables and forms the basis of indeterminacy in quantum mechanics¹. Heisenberg's original formulation, illustrated by the famous γ -ray microscope, sets a lower bound for the product of the measurement

as $\sigma(A)^2 = \langle \psi | A^2 | \psi \rangle - \langle \psi | A | \psi \rangle^2$. Note that a positive definite covariance term can be added to the right-hand side of equation (2), if squared, as discussed by Schrödinger⁵. For our experimental setting, this term vanishes. Robertson's relation (equation (2)) for standard deviations has been confirmed by many different experi-

ments: diffraction experiment¹⁵ the uncertainty relation (2), has been confirmed. A trade-off between squeezing coherent states of radiation fields¹⁶, and quantum state tomography demonstrations have been carried out¹⁷. Robertson's relation (equation (2)) has a mathematical basis, but applications for limitations on measurements, generally understood as limitations on state



Cheshire-cat



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Cheshire Cat

From Wikipedia, the free encyclopedia

*This article is about a character in *Alice's Adventures in Wonderland*. For other uses, see *Cheshire Cat (disambiguation)*.*

The **Cheshire Cat** (/ˈtʃəsɪrə/ or /tʃəˈsiriə/) is a fictional cat popularised by Lewis Carroll in *Alice's Adventures in Wonderland* and known for its distinctive mischievous grin.

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- Origins
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 - Church carvings
 - Heraldic lion
- Alice's Adventures in Wonderland
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 - Adaptations of the *Alice* books in film and other media
 - Disney film
 - 1999 TV film
 - 2010 film
 - Other major appearances and cultural references
 - Television
 - Anime and manga
 - Art
 - Businesses
 - Comics
 - Film
 - Games

Cheshire Cat

Alice character




The Cheshire cat as illustrator John Tenniel envisioned it in the 1866 publication

First appearance *Alice's Adventures in Wonderland*

Created by Lewis Carroll

Cheshire-cat



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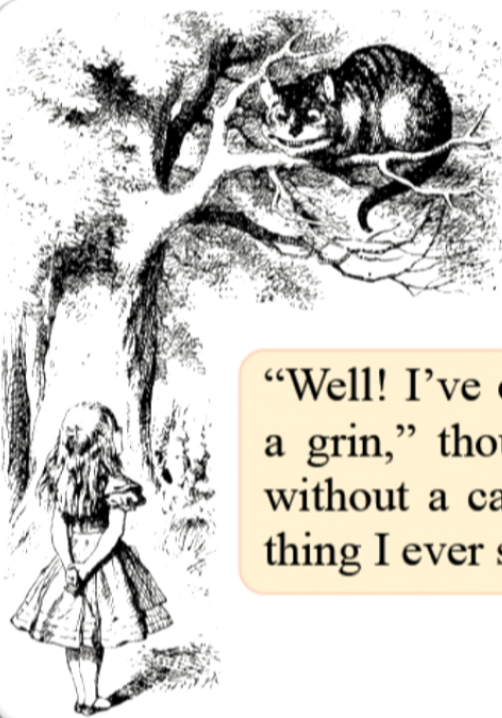
Article Talk

From Wikipedia, the free encyclopedia

This article is about the Cheshire Cat from Lewis Carroll's Alice's Adventures in Wonderland. For other uses, see Cheshire (disambiguation).

Cheshire Cat

The Cheshire Cat (/ˈtʃɛʃaɪr/), also known as the Grinning Cat, is a distinctive mischievous character that appears in Lewis Carroll's 1865 novel *Alice's Adventures in Wonderland*. The cat is known for its ability to disappear and reappear at will, leaving behind only its grin.



"Well! I've often seen a cat without a grin," thought Alice; "but a grin without a cat! It's the most curious thing I ever saw in all my life!"

1 Origins

- 1.1 Dairy farming
- 1.2 Cheese mould
- 1.3 Church carving
- 1.4 Heraldic lion

2 Alice's Adventures in Wonderland

3 Cultural uses

- 3.1 Adaptations of the Cheshire Cat
- 3.1.1 Disney
- 3.1.2 1999 TV series
- 3.1.3 2010 film
- 3.2 Other major adaptations
- 3.3 Television
- 3.4 Anime and manga
- 3.5 Art
- 3.6 Businesses
- 3.7 Comics
- 3.8 Film
- 3.9 Games

First appearance *Alice's Adventures in Wonderland*

Created by Lewis Carroll

Quantum Cheshire-cat

PHYSICS TEXTBOOK

Yakir Aharonov
Daniel Rohrlich

Quantum Pa

Quantum Theory for the Perplexed



New Journal of Physics

The open access journal for physics

Quantum Cheshire Cats

Yakir Aharonov^{1,2}, Sandu Popescu^{3,6}, Daniel Rohrlich^{4,6}
and Paul Skrzypczyk⁵

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Abstract. In this paper we present a quantum Cheshire Cat. In a pre- and post-selected experiment we find the Cat in one place, and its grin in another. The Cat is a photon, while the grin is its circular polarization.

Quantum Cheshire-cat

PHYSICS TEXTBOOK

Yakir Aharonov
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Quantum Paradoxes

Quantum Theory for the Perplexed



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Quantum Cheshire Cats

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¹ Tel Aviv University, School of Physics and Astronomy, Tel Aviv 6100, Israel
² Schmid College of Science and Technology, California State University, Fullerton, CA 92866, USA

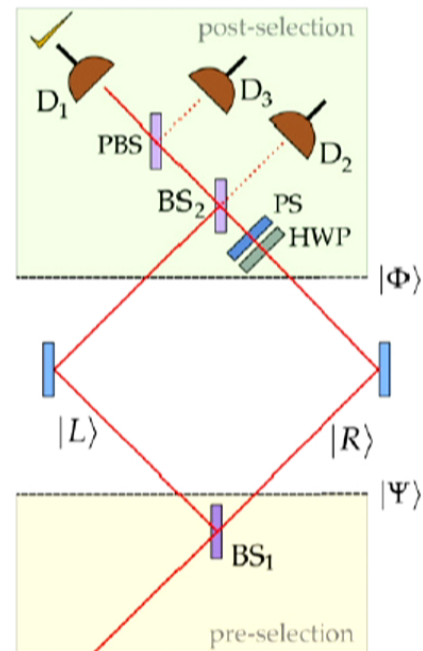
³ H. H. Wills Physics Laboratory, University of Bristol, BS8 1TL, United Kingdom

⁴ Physics Department, Ben-Gurion University, Beer-Sheva 84105, Israel

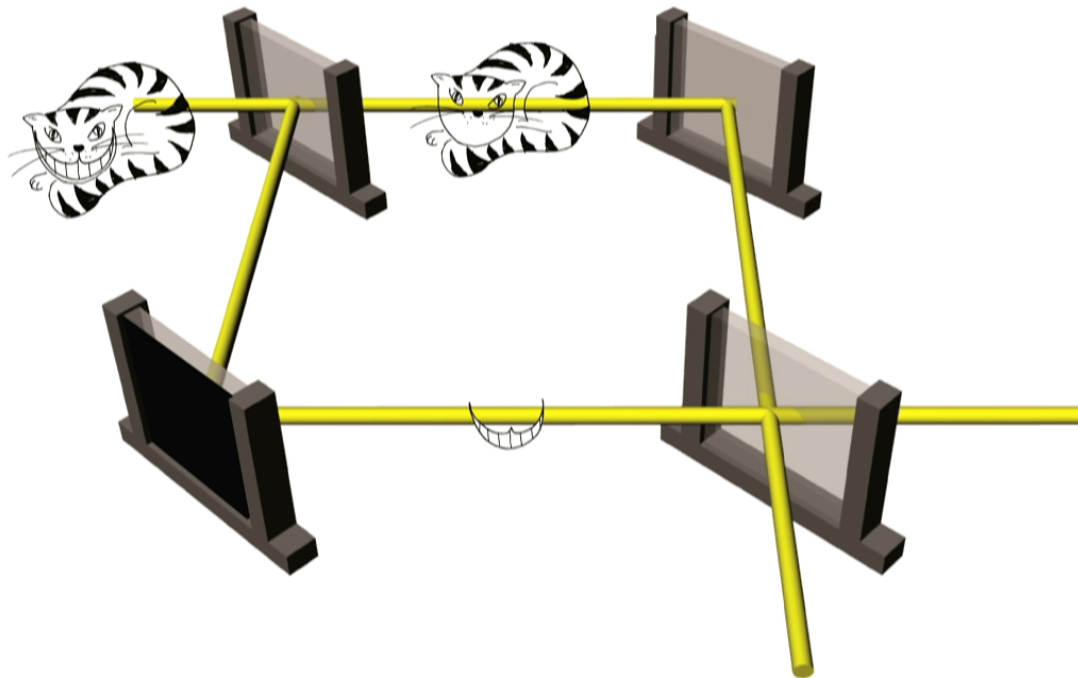
⁵ Department of Applied Mathematics, University of Cambridge, Centre for Mathematical Sciences, Cambridge CB3 0WA, United Kingdom
E-mail: s.popescu@gmail.com

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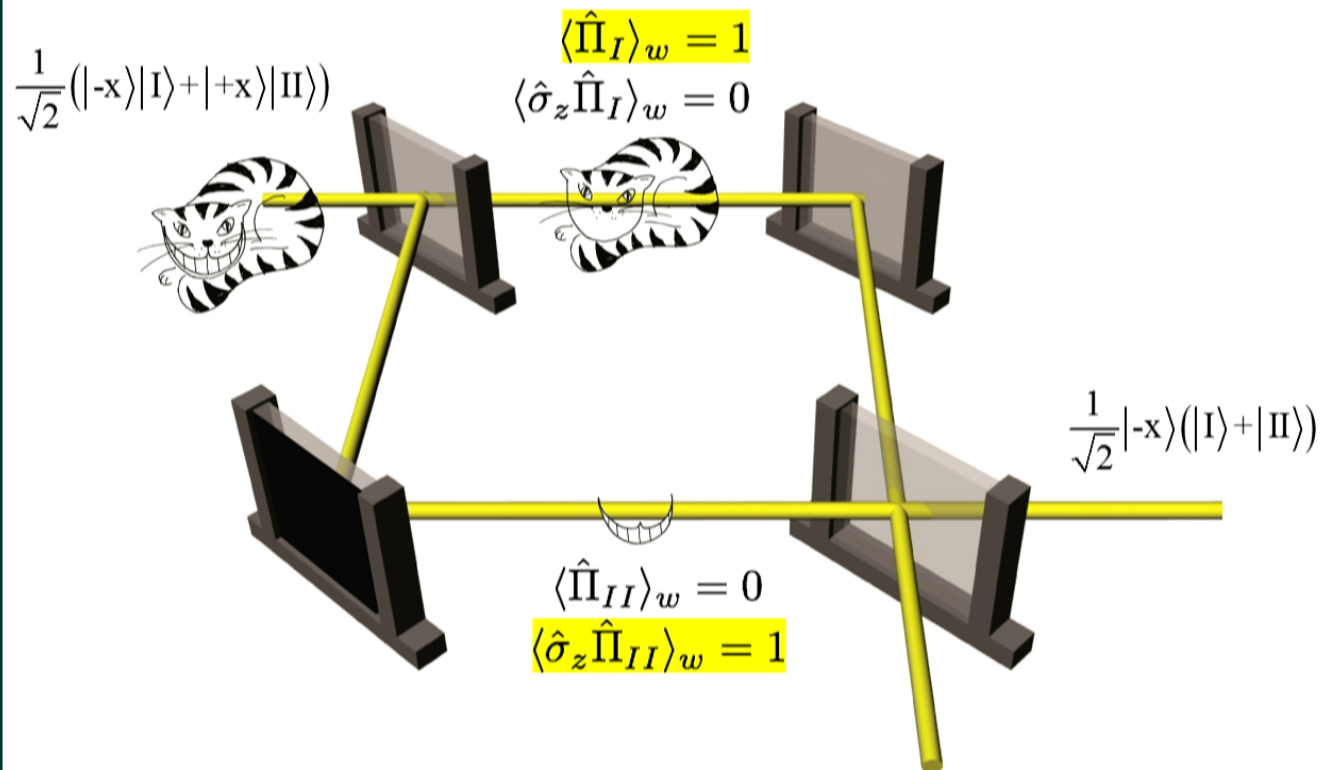


Quantum Cheshire-cat in neutron interferometer



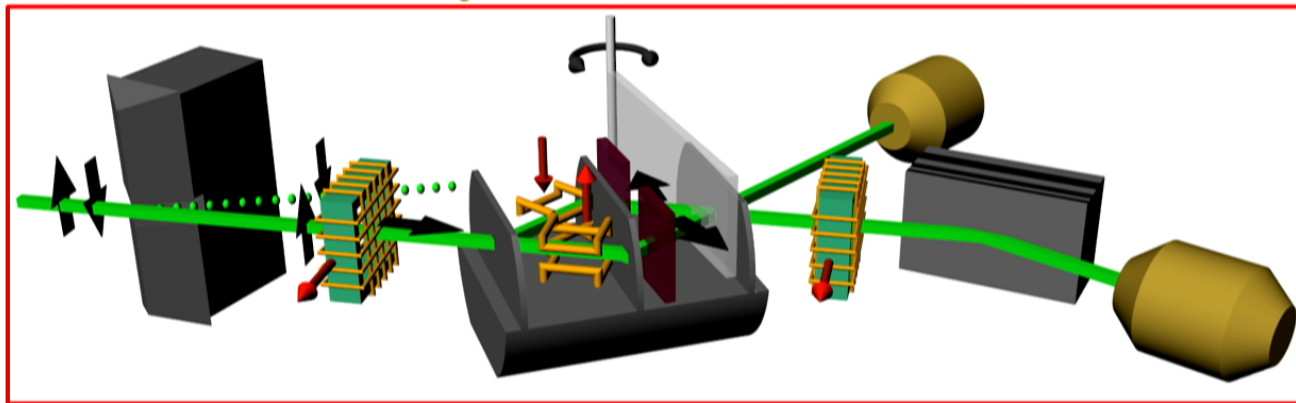
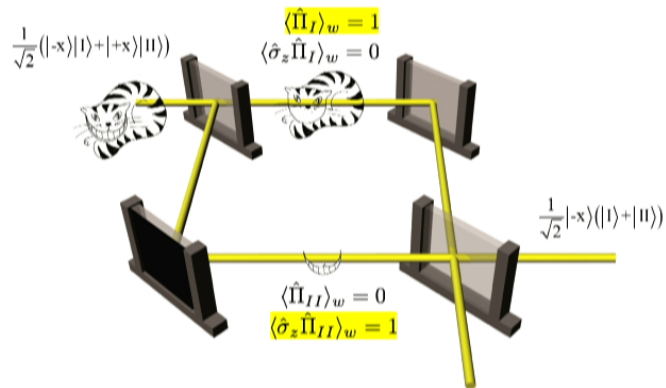
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Quantum Cheshire-cat in neutron interferometer



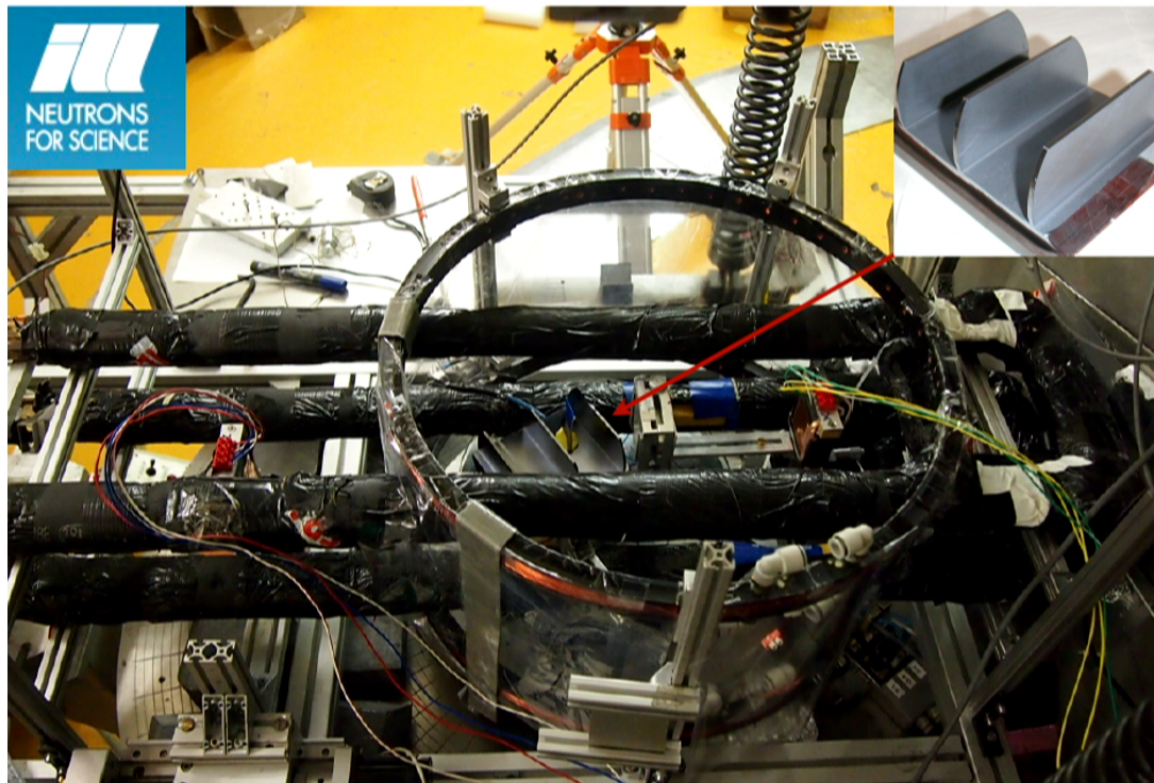
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Quantum Cheshire-cat in neutron interferometer



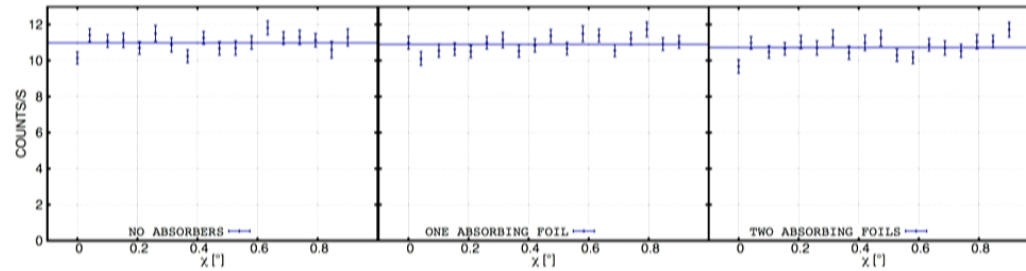
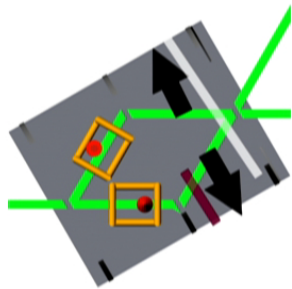
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Quantum Cheshire-cat: experiment



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Quantum Cheshire-cat: neutron(cat) in upper path

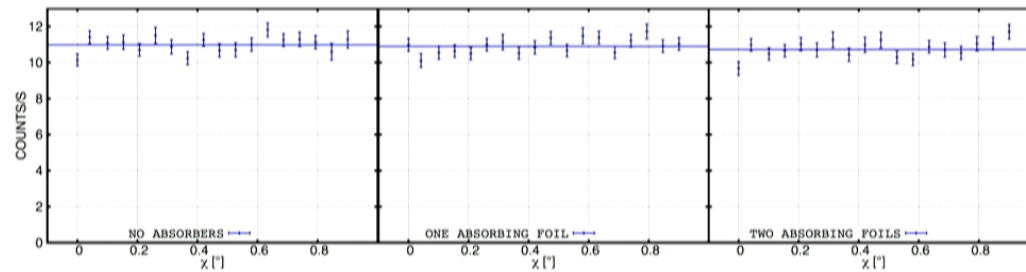
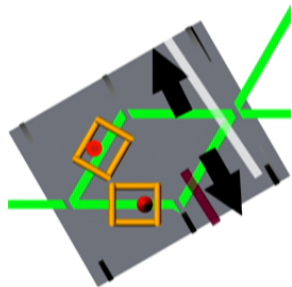


T=1

T=0.8

T=0.6

Quantum Cheshire-cat: neutron(cat) in upper path

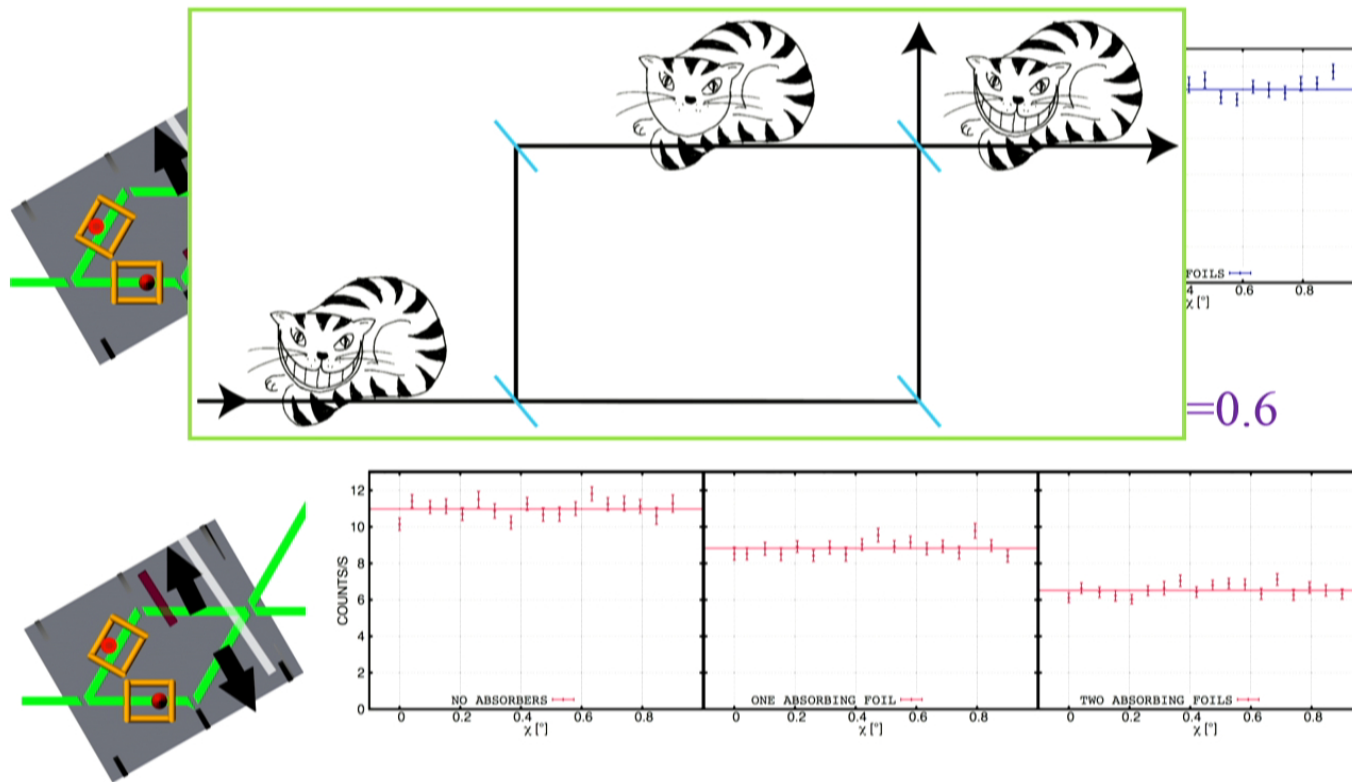


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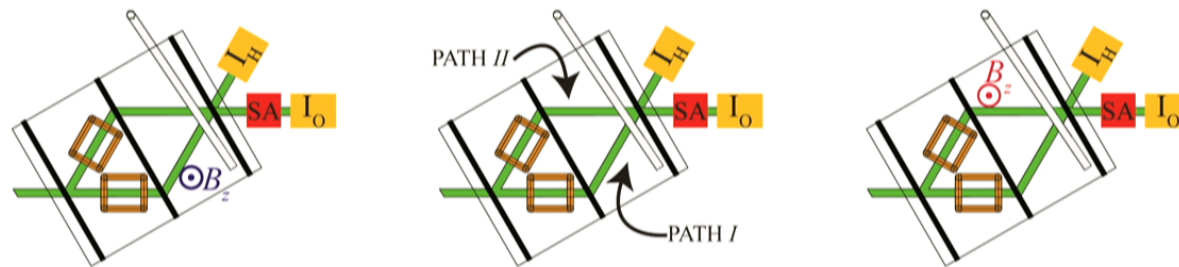
Quantum Cheshire-cat: neutron(cat) in upper path



=0.6

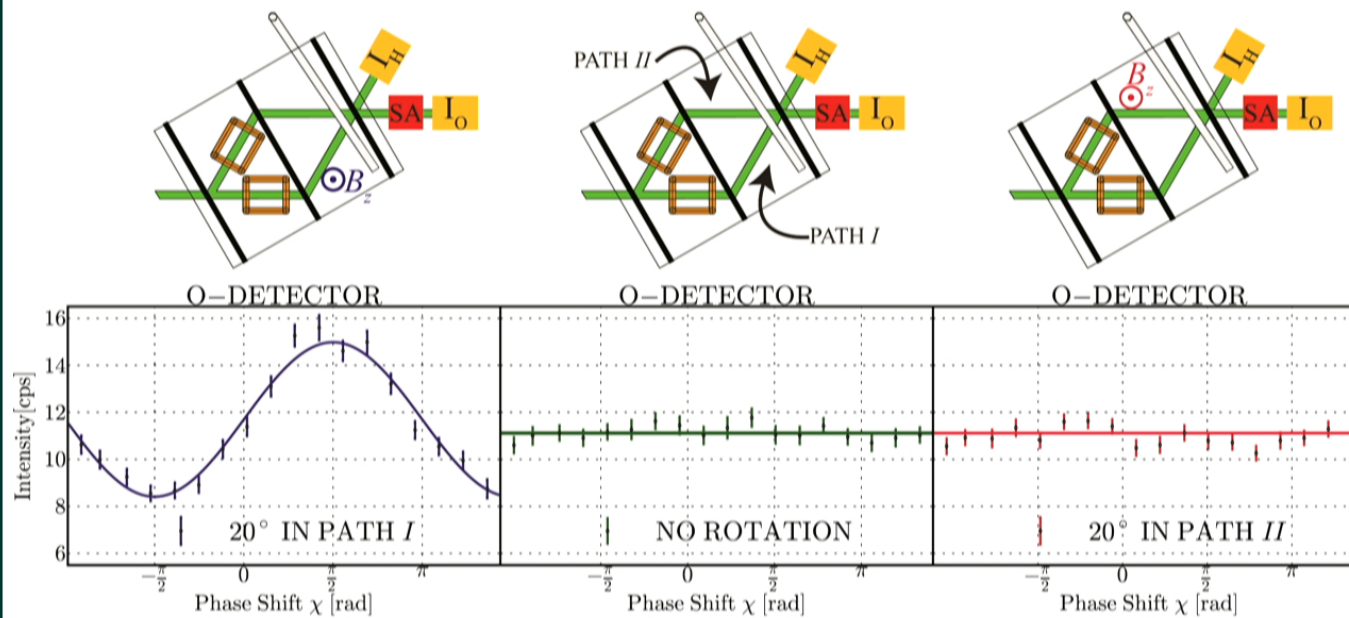
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Quantum Cheshire-cat: spin(smile) in lower path



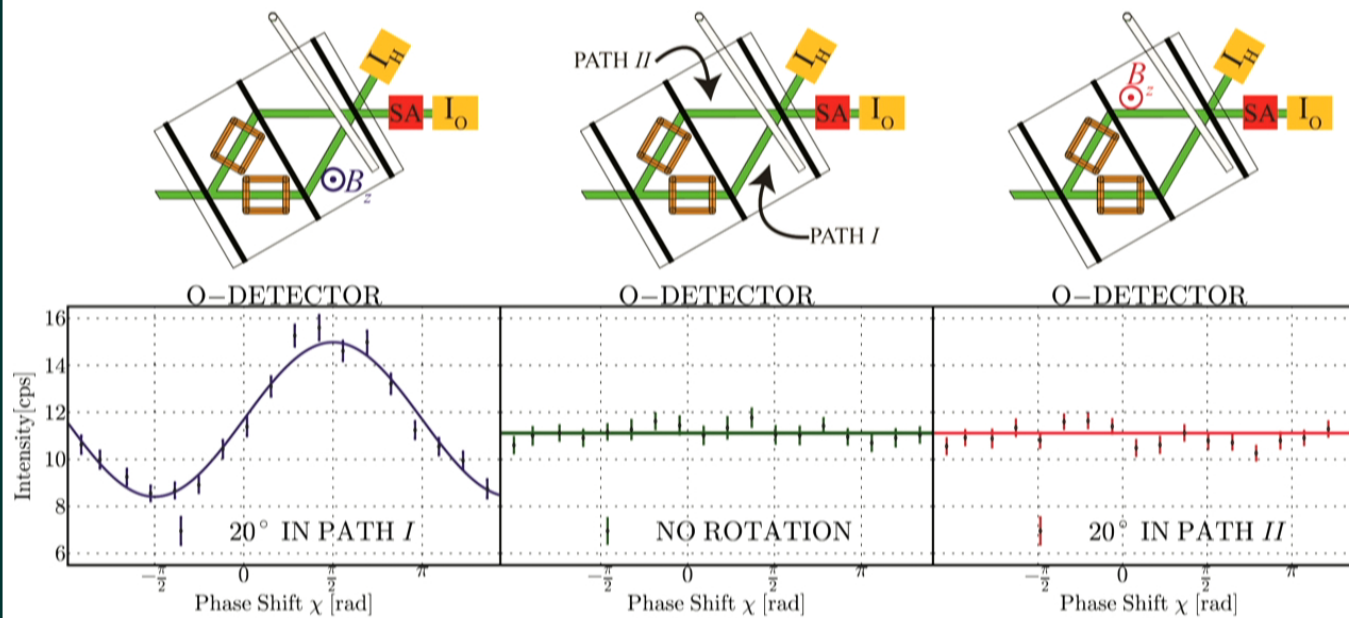
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Quantum Cheshire-cat: spin(smile) in lower path



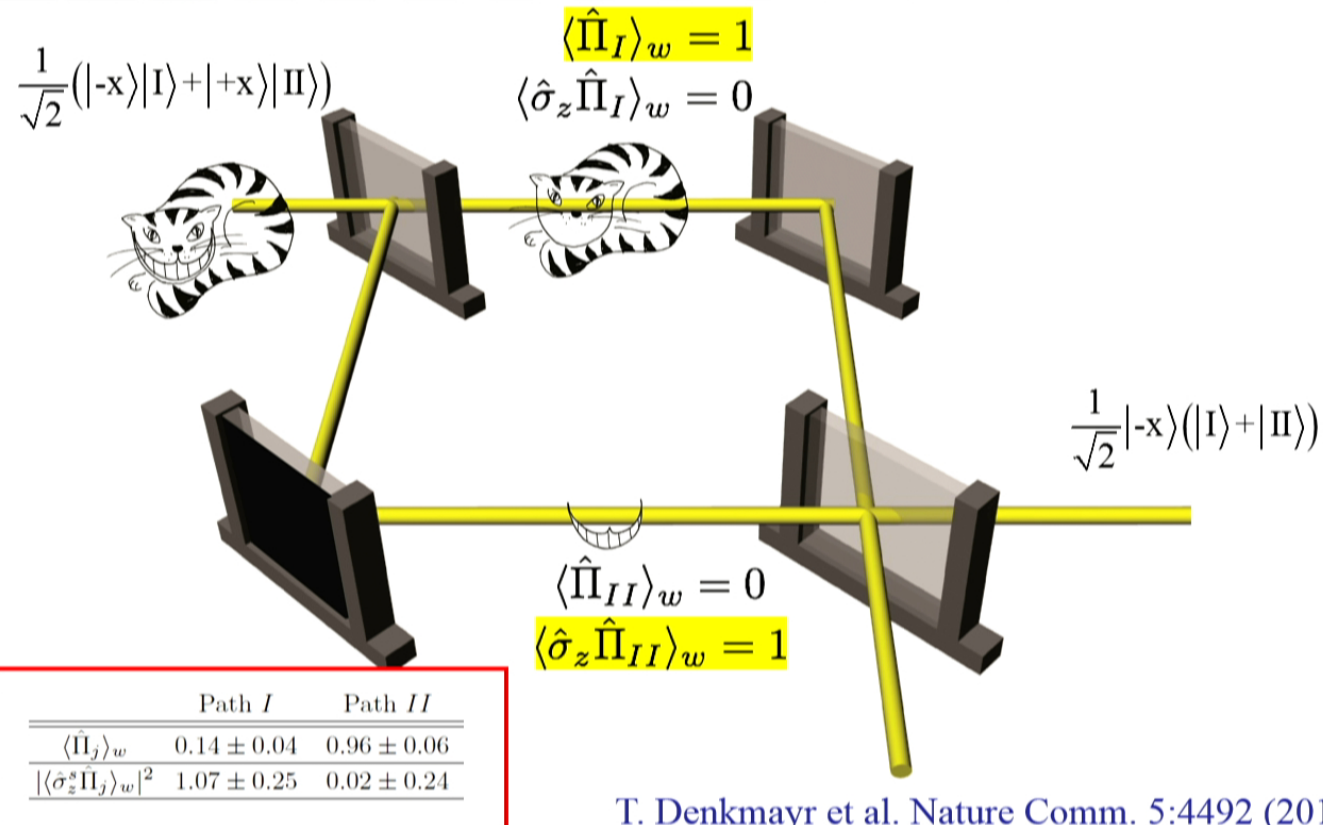
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Quantum Cheshire-cat: spin(smile) in lower path



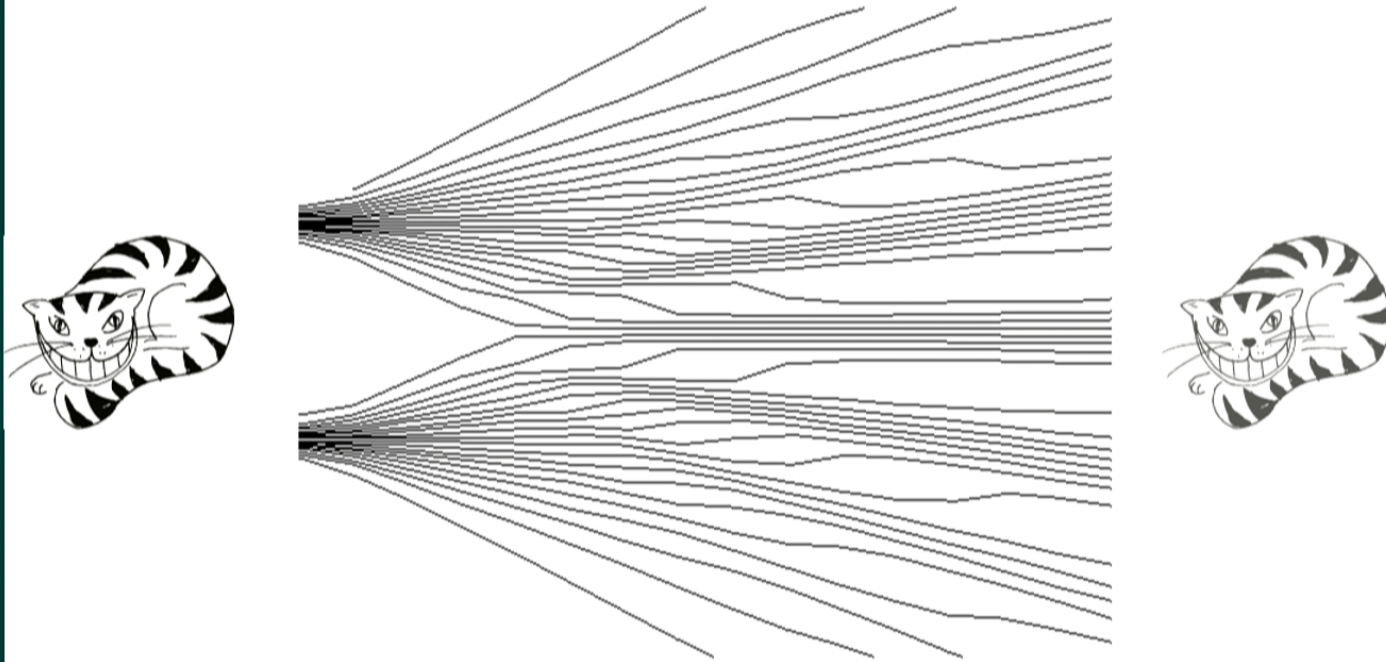
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Quantum Cheshire-cat: final results



T. Denkmayr et al. Nature Comm. 5:4492 (2014).

Quantum Cheshire-cat: invisible cat/spin ???



<http://Bohmian-mechanics.net/>

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FWF



Weak measurement, weak value

VOLUME 60, NUMBER 14

PHYSICAL REVIEW LETTERS

4 APRIL 1988

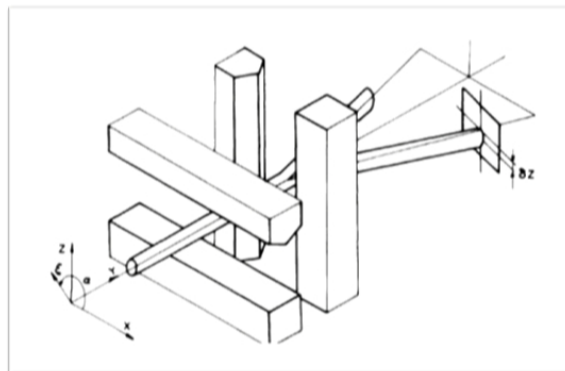
How the Result of a Measurement of a Component of the Spin of a Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

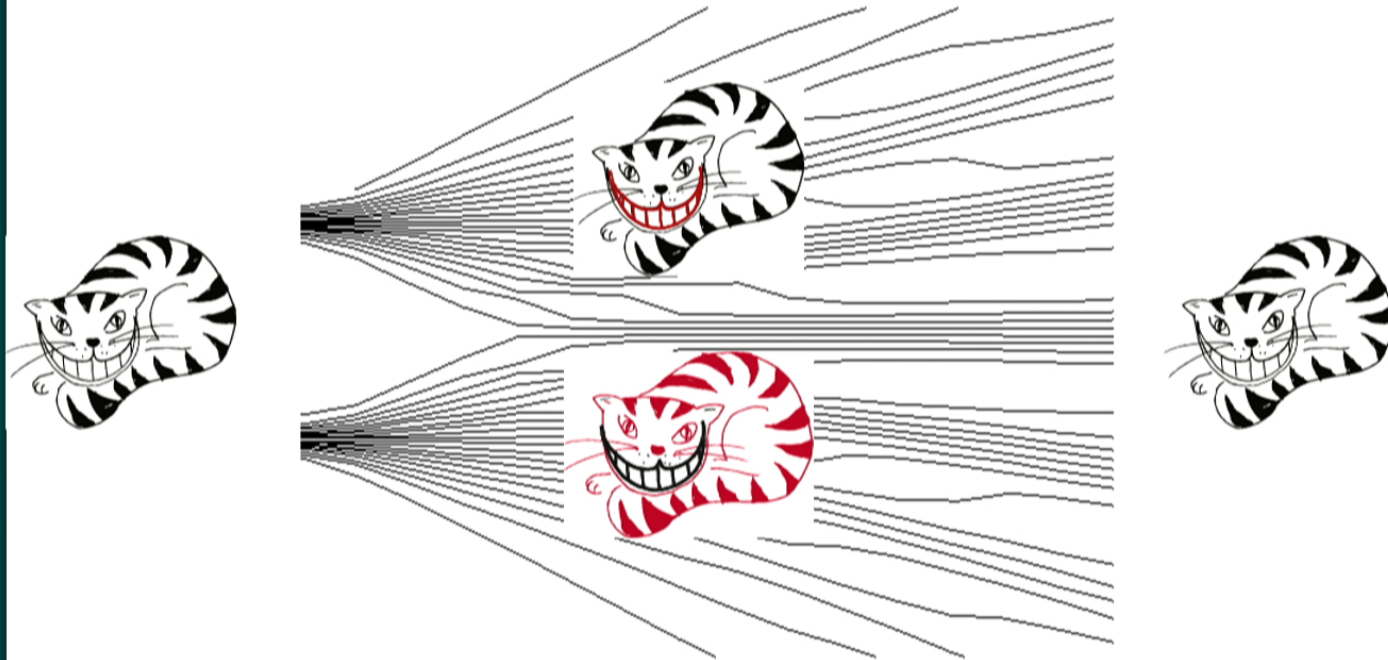
*Physics Department, University of South Carolina, Columbia, South Carolina 29208, and
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel*

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin- $\frac{1}{2}$ particles is presented.



Quantum Cheshire-cat: invisible cat/spin ???



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Weak measurement, weak value

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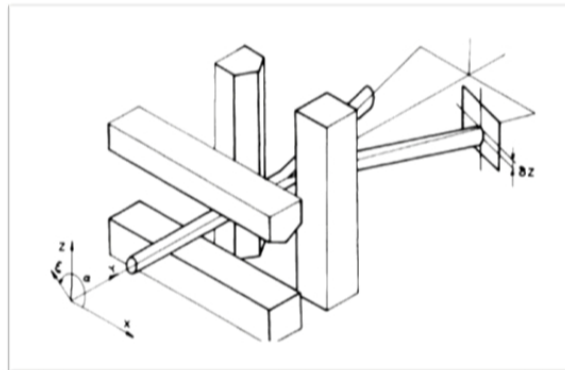
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Physics Department, University of South Carolina, Columbia, South Carolina 29208, and

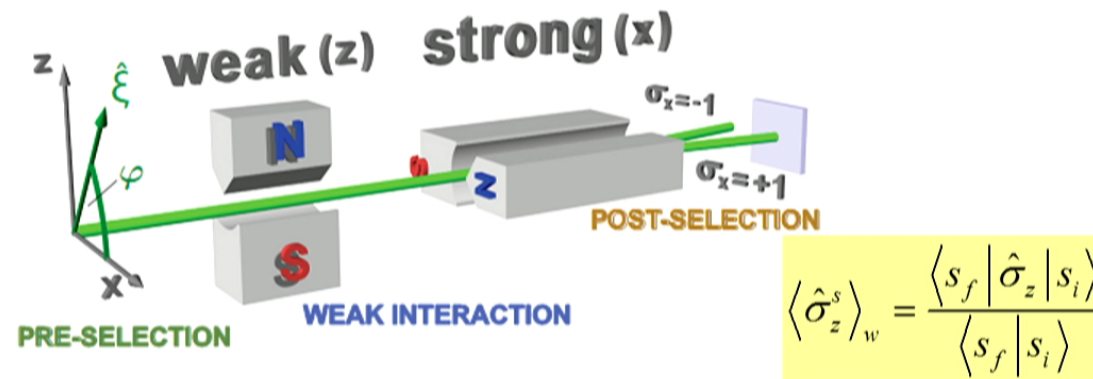
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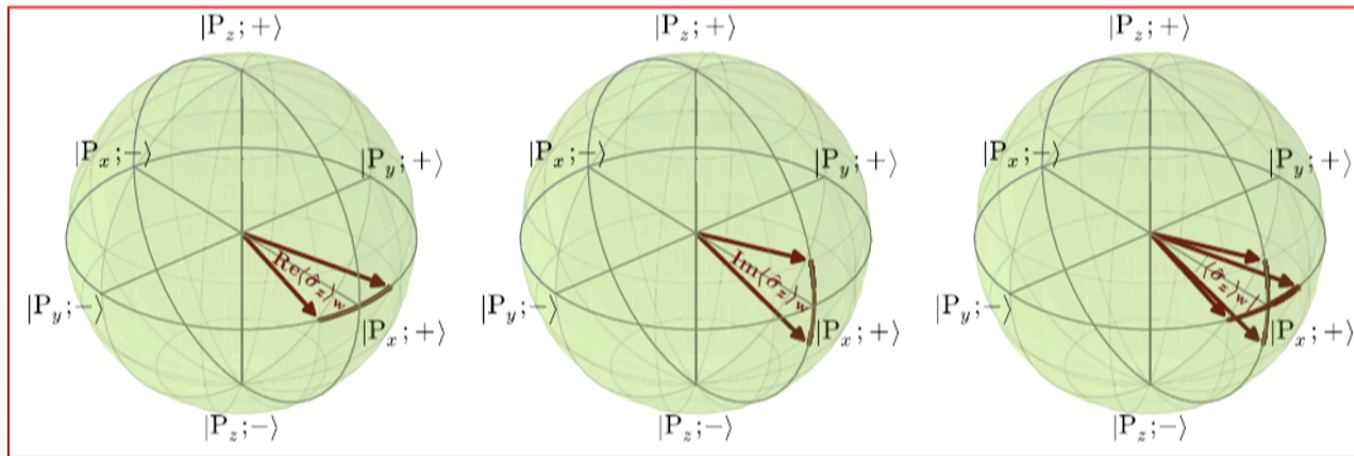


Weak measurement of neutron's $\frac{1}{2}$ -spin

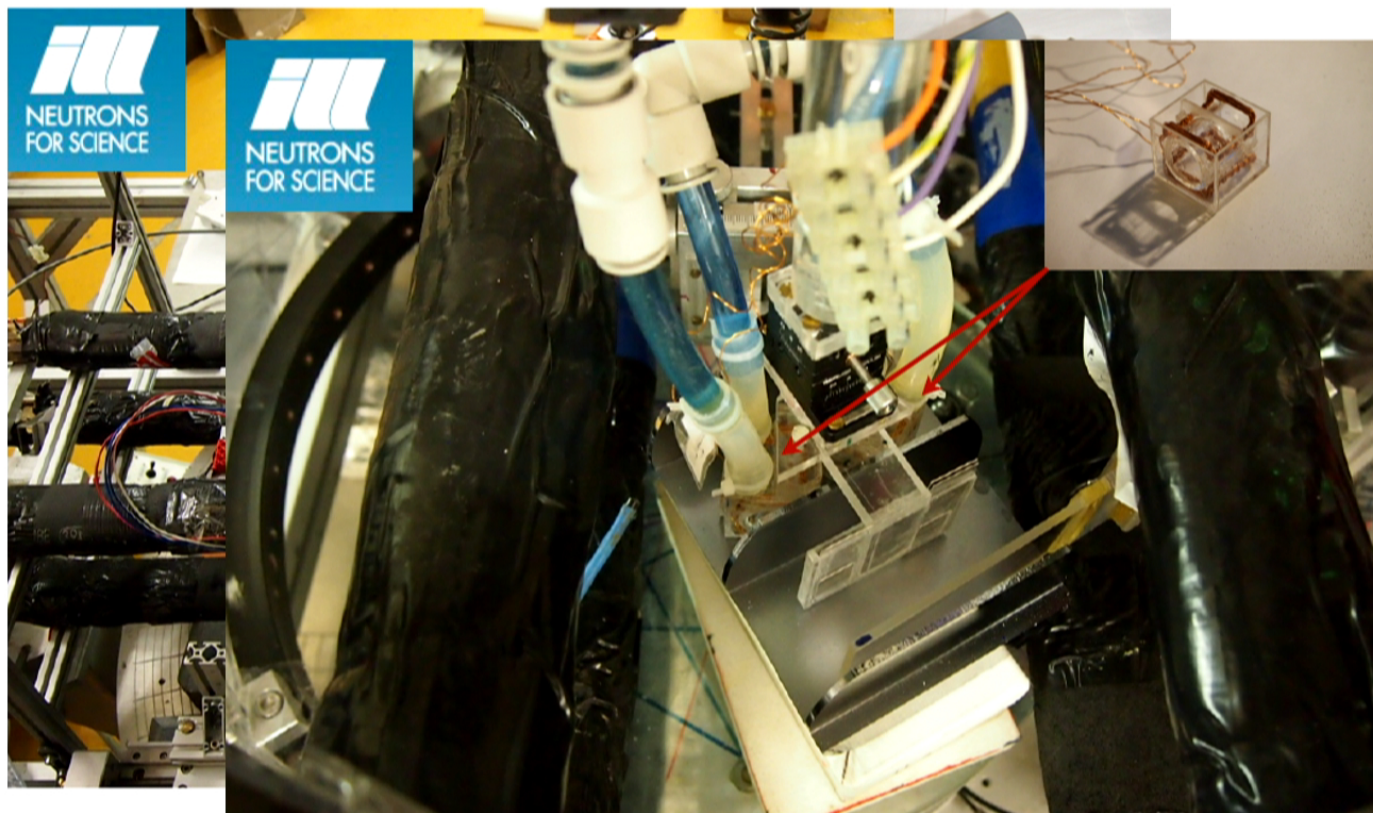


Weak measurement of $\frac{1}{2}$ -spin: real/imaginary/absolute

$$|\phi\rangle = \frac{\langle S_f | S_i \rangle}{\sqrt{2}} \left(e^{-i\alpha \langle \hat{\sigma}_z^s \rangle_w / 2} |I\rangle + e^{i\alpha \langle \hat{\sigma}_z^s \rangle_w / 2} |II\rangle \right)$$

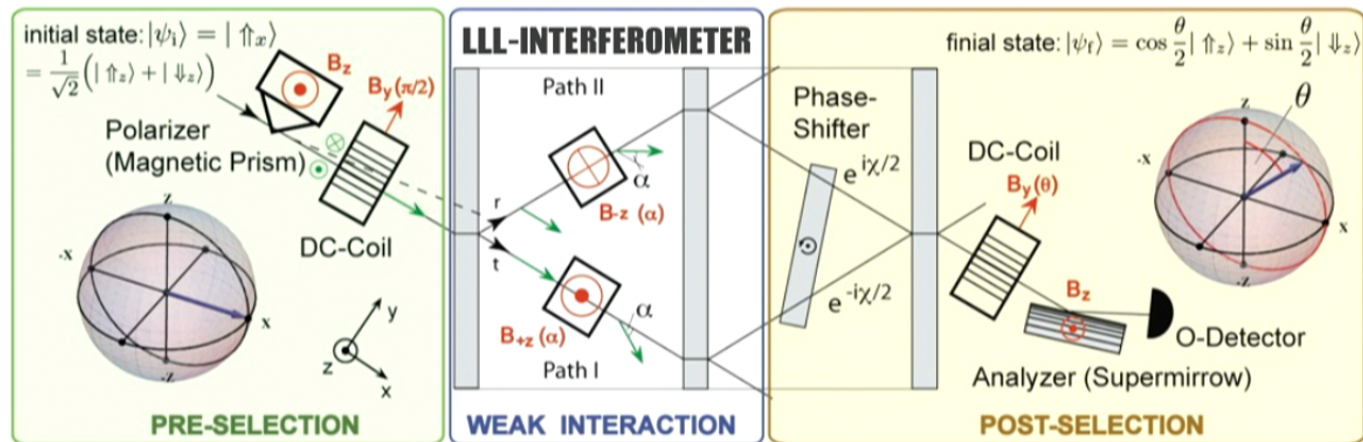


Pictures of experimental setup



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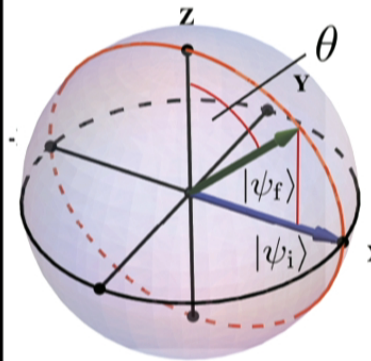
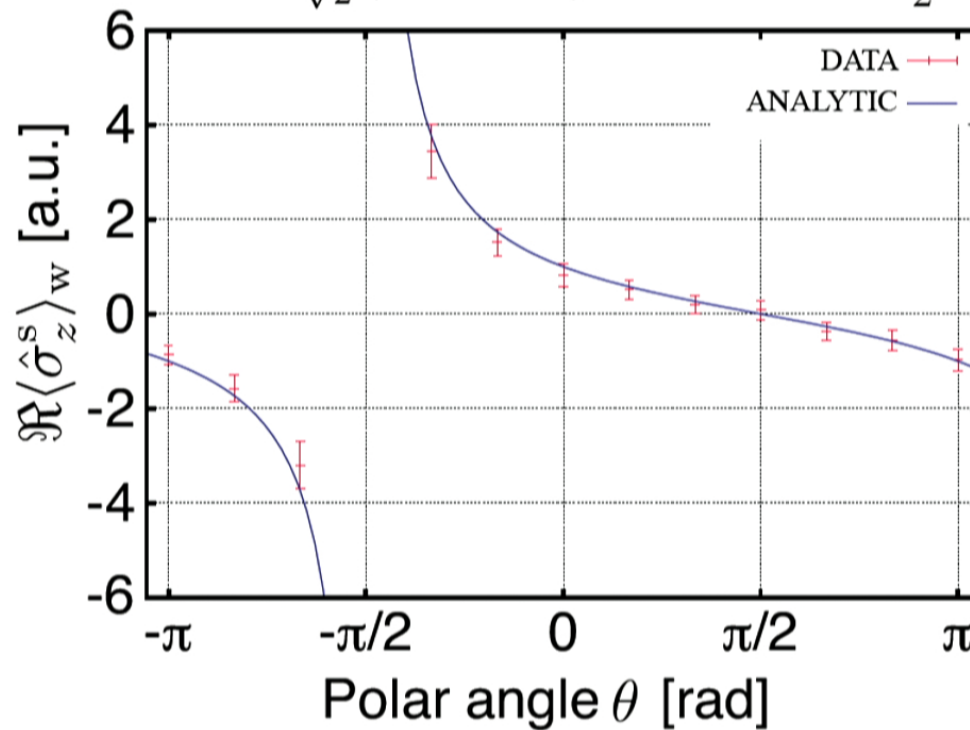
Experimental scheme



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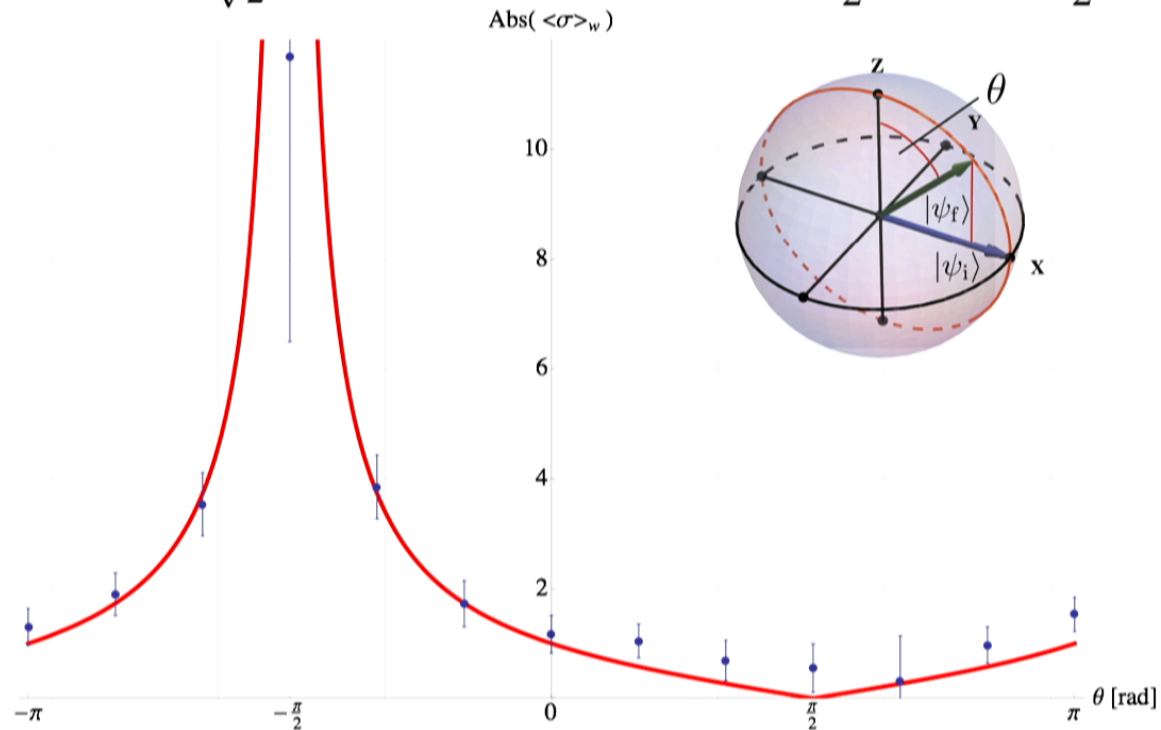
Weak measurement of $\frac{1}{2}$ -spin: real part

$\bullet |\psi_i\rangle = |\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$
 $\bullet |\psi_f\rangle = \cos \frac{\theta}{2} |\uparrow_z\rangle + \sin \frac{\theta}{2} |\downarrow_z\rangle$



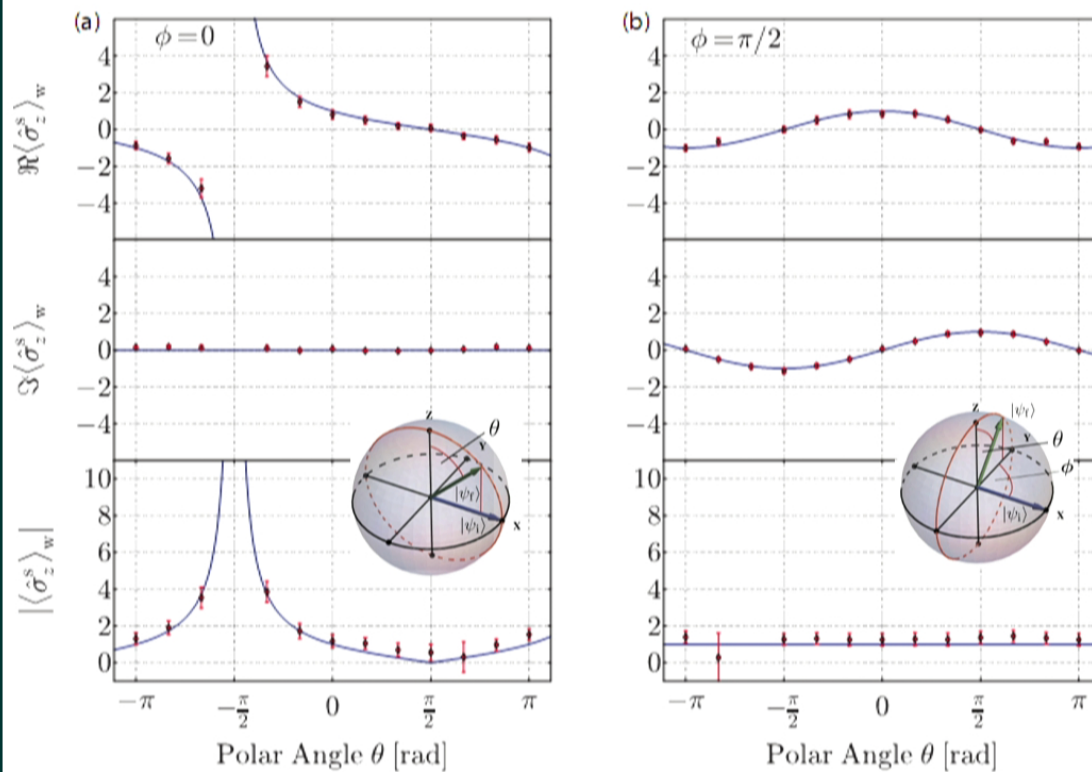
Weak measurement of $\frac{1}{2}$ -spin: absolute value

$$|\psi_i\rangle = |\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle) \quad |\psi_f\rangle = \cos\frac{\theta}{2}|\uparrow_z\rangle + \sin\frac{\theta}{2}|\downarrow_z\rangle$$



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Weak measurement of $\frac{1}{2}$ -spin: final results



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Quantum pigeonhole effect 1

PNAS

Quantum violation of the pigeonhole principle and the nature of quantum correlations

Yakir Aharonov^{a,b,c,1}, Fabrizio Colombo^d, Sandu Popescu^{c,e}, Irene Sabadini^d, Daniele C. Struppa^{b,c}, and Jeff Tollaksen^{b,c}

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Contributed by Yakir Aharonov, November 12, 2015 (sent for review April 3, 2015; reviewed by Charles H. Bennett and Lucien Hardy)

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Quantum pigeonhole effect 1

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Quantum violation of the pigeonhole principle and the nature of quantum correlations

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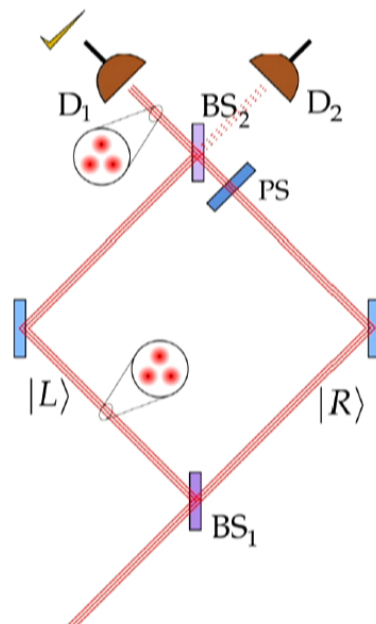
Contributed
532–535

The pigeonhole principle: “If you put three pigeons in two pigeonholes, at least two of the pigeons end up in the same hole,” is an obvious yet fundamental principle of nature as it captures the very essence of counting. Here however we show that in quantum mechanics this is not true! We find instances when three quantum particles are put in two boxes, yet no two particles are in the same box. Furthermore, we show that the above “quantum pigeonhole principle” is only one of a host of related quantum effects, and points to a very interesting structure of quantum mechanics that was hitherto unnoticed. Our results shed new light on the very notions of separability and correlations in quantum mechanics and on the nature of interactions. It also presents a new role for entanglement, complementary to the usual one. Finally, interferometric experiments that illustrate our effects are proposed.

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Quantum pigeonhole effect 2

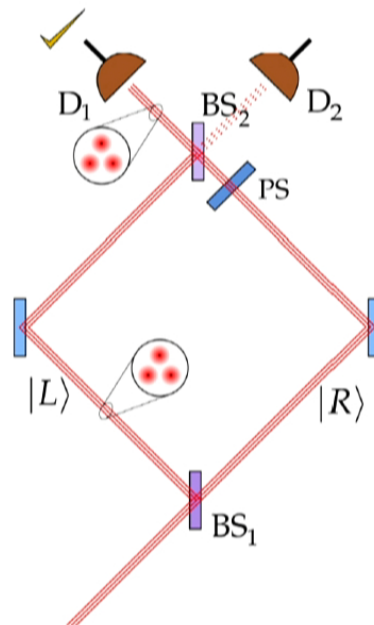


$$|\Psi\rangle = |+\rangle_1 |+\rangle_2 |+\rangle_3 \quad \text{where } |\pm\rangle \equiv \frac{1}{\sqrt{2}} \{|L\rangle \pm |R\rangle\}$$

Y. Aharonov et al. PNAS **113**, 532 (2016).

Quantum pigeonhole effect 2

$$|\Phi\rangle = |+\rangle_1 |+\rangle_2 |+\rangle_3 \quad \text{where } |\pm\rangle \equiv \frac{1}{\sqrt{2}} \{|L\rangle \pm |R\rangle\}$$

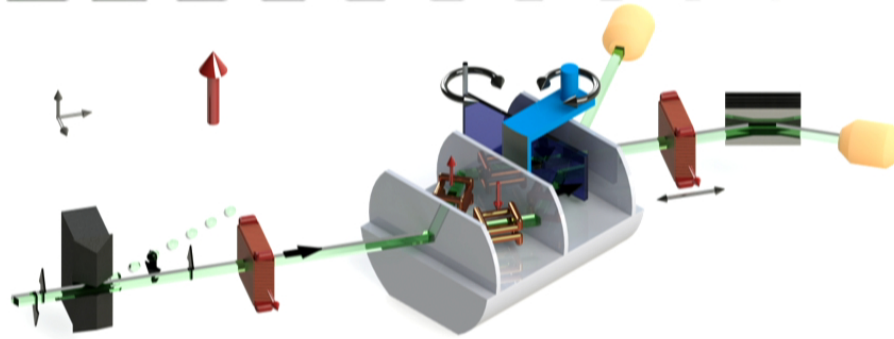


$$\begin{cases} \hat{\Pi}_{12}^{same} = \hat{\Pi}_{12}^{LL} + \hat{\Pi}_{12}^{RR} \\ \hat{\Pi}_{12}^{diff} = \hat{\Pi}_{12}^{LR} + \hat{\Pi}_{12}^{RL} \end{cases} \quad \text{where } \hat{\Pi}_{12}^{LL} \equiv |L\rangle\langle L| \text{ etc.}$$

$$|\Psi\rangle = |+\rangle_1 |+\rangle_2 |+\rangle_3 \quad \text{where } |\pm\rangle \equiv \frac{1}{\sqrt{2}} \{|L\rangle \pm |R\rangle\}$$

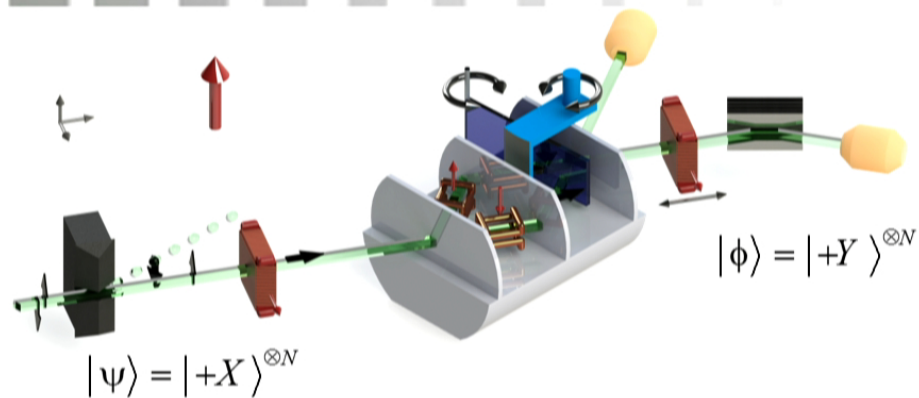
Y. Aharonov et al. PNAS **113**, 532 (2016).

Quantum pigeonhole effect in neutron interferometer



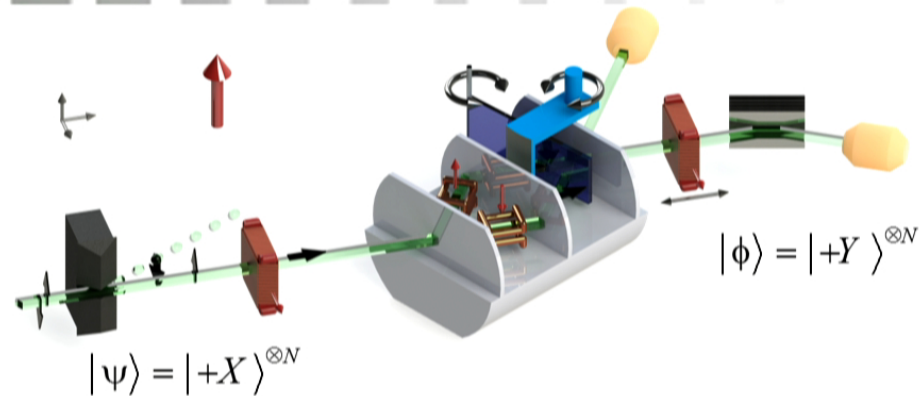
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Quantum pigeonhole effect in neutron interferometer



$$\begin{cases} \hat{\Pi}^{even} = \hat{\Pi}_+ \hat{\Pi}_+ + \hat{\Pi}_- \hat{\Pi}_- \\ \hat{\Pi}^{odd} = \hat{\Pi}_+ \hat{\Pi}_- + \hat{\Pi}_- \hat{\Pi}_+ \end{cases} \quad \text{where } \hat{\Pi}_{\pm} \equiv |\pm Z\rangle\langle\pm Z|$$

Quantum pigeonhole effect in neutron interferometer



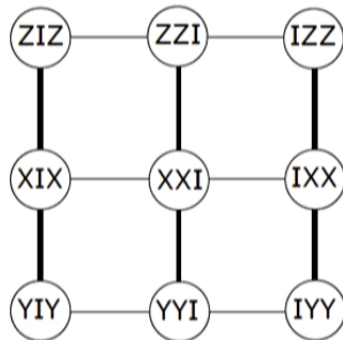
$$\begin{cases} \hat{\Pi}^{even} = \hat{\Pi}_+ \hat{\Pi}_+ + \hat{\Pi}_- \hat{\Pi}_- \\ \hat{\Pi}^{odd} = \hat{\Pi}_+ \hat{\Pi}_- + \hat{\Pi}_- \hat{\Pi}_+ \end{cases} \quad \text{where } \hat{\Pi}_{\pm} \equiv |\pm Z\rangle \langle \pm Z|$$

$$\hat{\Pi}_w^{even} = 0 \ \& \ \hat{\Pi}_w^{odd} = 1, \text{ then } \left(\hat{\sigma}_z^{path} \bullet \hat{\sigma}_z^{path} \right)_w = -1$$

No two pigeons ever seem to be in the same box!!!

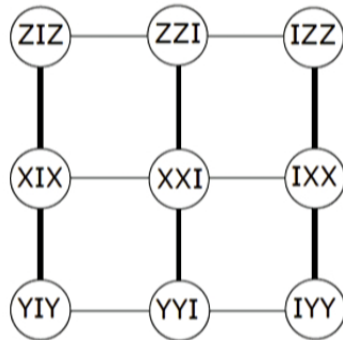
Quantum contextuality in neutron interferometer

(a) Magic square

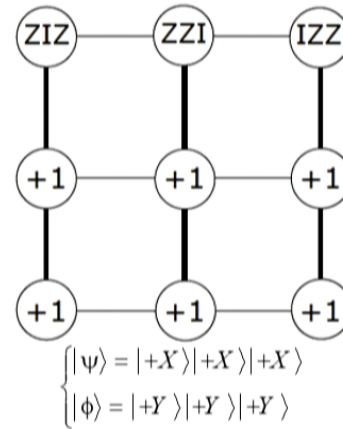


Quantum contextuality in neutron interferometer

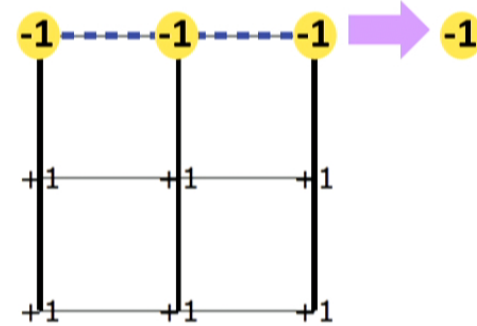
(a) Magic square



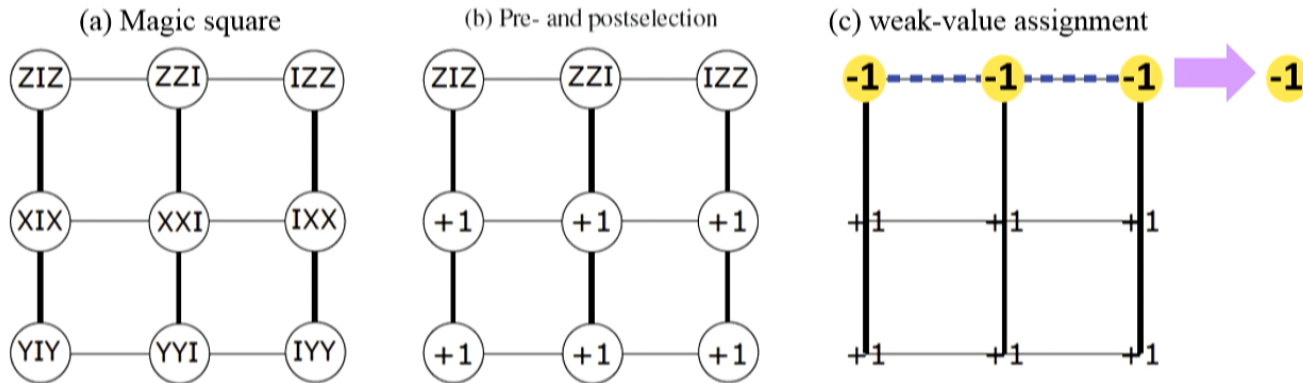
(b) Pre- and postselection



(c) weak-value assignment



Quantum contextuality in neutron interferometer



$$\begin{cases} |\psi\rangle = |+X\rangle|+X\rangle|+X\rangle \\ |\phi\rangle = |+Y\rangle|+Y\rangle|+Y\rangle \end{cases}$$

$$\Pi_1^{(3)} = | +Z, +Z, +Z \rangle \langle +Z, +Z, +Z | + | -Z, -Z, -Z \rangle \langle -Z, -Z, -Z |,$$

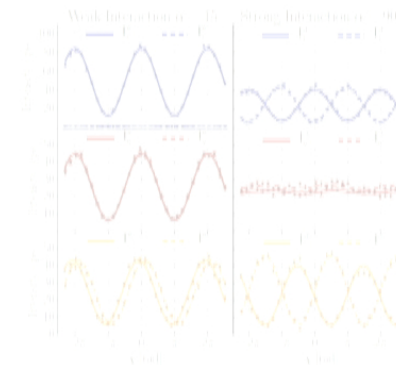
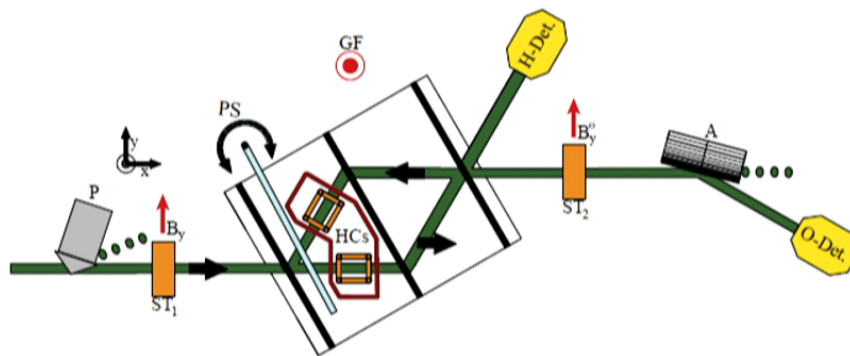
$$\Pi_2^{(3)} = | +Z, -Z, +Z \rangle \langle +Z, -Z, +Z | + | -Z, +Z, -Z \rangle \langle -Z, +Z, -Z |,$$

$$\Pi_3^{(3)} = | -Z, +Z, +Z \rangle \langle -Z, +Z, +Z | + | +Z, -Z, -Z \rangle \langle +Z, -Z, -Z |,$$

$$\Pi_4^{(3)} = | -Z, -Z, +Z \rangle \langle -Z, -Z, +Z | + | +Z, +Z, -Z \rangle \langle +Z, +Z, -Z |,$$

$$(\Pi_1^{(3)})_w = \prod_{n=1}^3 \frac{1 + Z_w}{2} + \prod_{n=1}^3 \frac{1 - Z_w}{2} = -\frac{1}{2} \notin [0, 1]$$

WV via weak/strong measurements: experiment



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Concluding remarks

Neutron interferometer is effective tool for studies of quantum paradoxes

- **Quantum Cheshire-Cat was observed in neutron interferometer.**
- **Weak-values of $\frac{1}{2}$ -spin allow to demonstrate quantum pigeonhole effect and contextuality**
→ further development & applications

