Title: Quantum mechanics and the principle of maximal variety

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Abstract:

Quantum mechanics and the principle of maximal variety or: Modular hidden variables

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Concepts and paradoxes in a quantum universe

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hep-th/9203041 arXiv:1205.3707 arXiv:1104.2822, arXiv:1506.02938

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Thanks to the organizers.

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Three roads to Quantum Foundations:

• ***Discover new quantum phenomena that challenges our understanding.***

• ***Interpretational approaches:*** Assume the dynamics and kinematics of QM are correct and that the measurement problem and other foundational issues reflect a defect in our understanding of the theory. Seek to reformulate the theory, keeping its physical content unchallanged.

Copenhagen, Everett, information theoretic reconstructions

• ***Dynamical approaches:*** Assume the measurement problem and other issues arise because QM is an incomplete description of nature. Seek to find the correct completion, which will resolve the measurement problem and have an unproblematic interpretation.

deBroglie-Bohm, spontaneous collapse, hidden variables, Nelson, many classical interacting worlds (MIW), matrix models...

The challenge of non-locality

- Physics is a weird combination of local and non-local.
- Local propagation of energy and information
- Non-local entanglement.

- We learn from GR that space and time are relational. So locality must be relational too.
- Quantum gravity suggests space is emergent from a network of relations or interactions. But then locality must be emergent too.

- Valentini tells us that the price or reward of going out of quantum equilibrium is non-local signaling, ie locality is a feature only of quantum equilibrium. Is the world awash in non-local interactions that are hidden by equilibrium.

  - Locality is also relative to position and motion of the observer.

Are there defects in locality? Events that are far away in the emergent geometry of space that are causal neighbours? Is this the origin of entanglement?

A relational view of locality

The ***view of an event*** is the information available there from the past, containing information about the past causal neighbourhood.

Events nearby in spacetime have similar views.

But so do similarly prepared quantum systems in similar environments.

So we can reverse this and make locality a consequence of similarity of views.

Events that have similar views can interact, by virtue of that similarity.

Sometimes this results in their being nearby in the emergent spacetime.

But sometimes it is just because they have similar preparations, even if they are far away.

An heuristic Principle:

Leibnitz's identity of the indiscernible.

Principle of the identity of the indiscernible (PII): *any two events or objects with isomorphic relational properties are to be identified.*

- *Global symmetries cannot be fundamental. Indeed GR has none and all the global symmetries in the standard model are accidental or broken.*

- *Relative locality: Localization is a consequence of identity, ie something is uniquely localized if it is distinguished by having a unique causal neighborhood.*

- *Hypothesis: the fundamental geometry is built from distinctiveness based on causal neighborhoods. Distance is a consequence of having disimilar causal neighborhoods.*

- *There are defects in this causal geometry. Two systems with very similar causal neighborhoods are nearby causally, even if distant in the coarse grained macroscopic metric. Hence they interact.*

- *The interactions induced between two similar systems are repulsive in that they act to increase their distinctiveness. Thus the PII is protected dynamically.*

The PII forces local physics to be non-deterministic:

- *By the PII each event has a unique causal neighborhood (arXiv:1307.6167):*
  - *Suppose two events A and B have isomorphic causal pasts:*

$$P(A) = P(B)$$

Then to prevent a violation of the PII their causal futures must be different

$$\Rightarrow F(A) \neq F(B)$$

Thus the same causal past implies a different causal future. Hence local physics cannot be deterministic. It must be anti-deterministic.

The basic hypothesis: there is a non-local interaction between similar systems which acts to increase their differences. This is the origin of quantum physics. This interaction is driven by a potential energy which measures the distinctiveness of all the pairs of similar subsystems in nature.

QM from 4 principles

1) Quantum mechanics is necessarily a description of subsystems of the universe. It is an approximation to some other, different theory, which might be applied to the universe as a whole.

2) *The real ensemble hypothesis:* *A quantum state refers to an ensemble of similar systems present in the universe at a given time.* By similar systems we mean systems with the same constituents, whose dynamics are subject to (within errors that can be ignored) the same Hamiltonian, and which have very similar histories and hence, in operational terms, the same preparation. arXiv:1104.2822

3) ***Locality comes from similarity of views:*** Similar systems have a new kind of interaction with each other, just by virtue of their similarities. This interaction takes place amongst similar systems, regardless of how far apart they may be situated in space, and thus, this is how non-locality enters quantum phenomena. *These interactions prevent similar systems from becoming identical and hence protect the principle of the identity of the indiscernible.*

The variety of a network, G, representing a system of relations.

- N_l (k) is the l'th neighborhood of node k.
- This is the subgraph of G including those nodes l steps from k.

- For any pair of nodes,
- n_{kl} is the smallest n such that
- N_n (k) is not isomorphic to N_n (l).

- The distinctiveness of the pair is

$$D(k,l) = \frac{1}{n_{kl}}$$

- The variety of G is

$$\mathcal{V} = \frac{1}{N(N-1)} \sum_{k \neq l} D(k,l) = \frac{1}{N(N-1)} \sum_{k \neq l} \frac{1}{n_{kl}}$$

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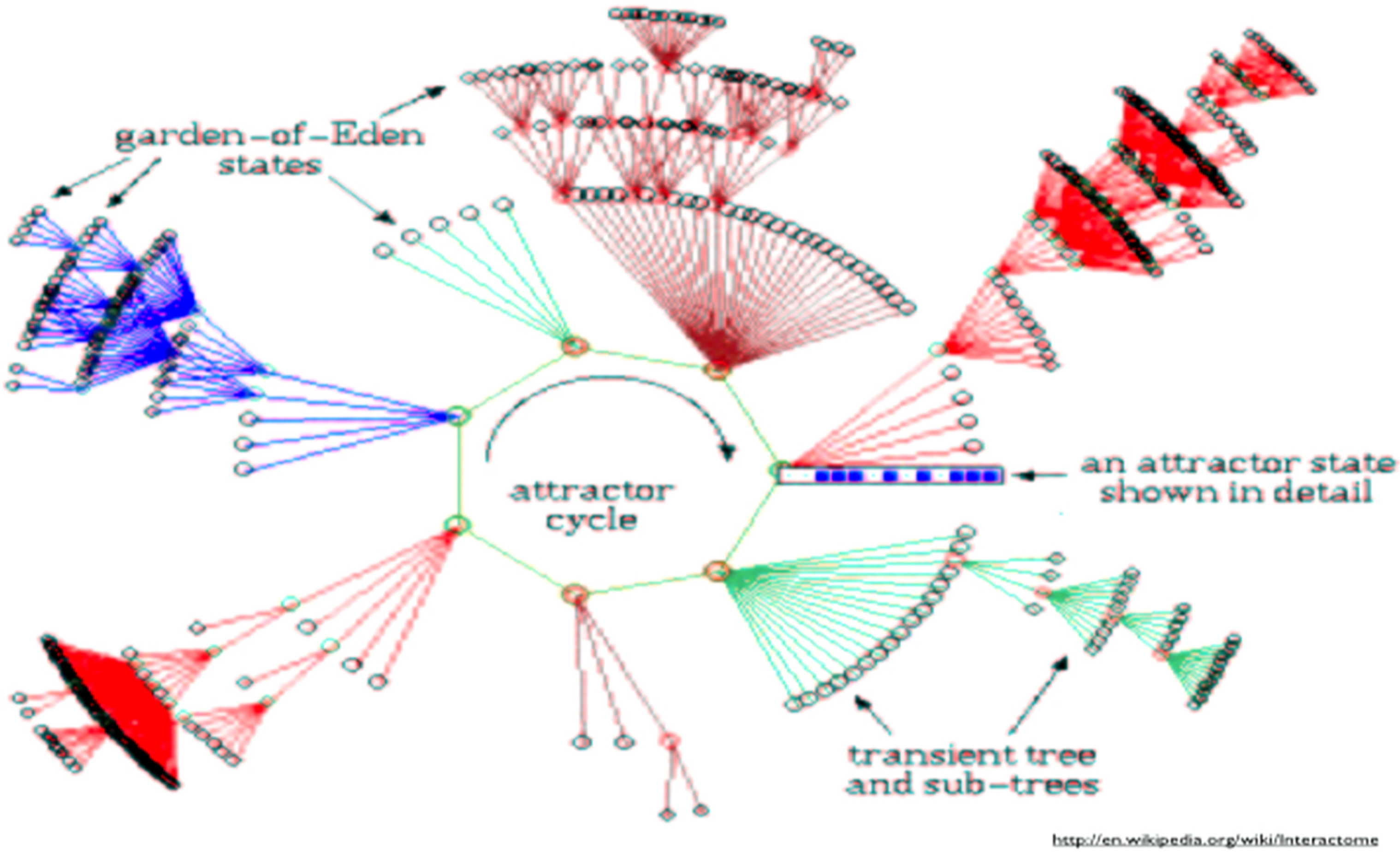
http://en.wikipedia.org/wiki/Interactome

High variety

healthandsociety.columbia.edu



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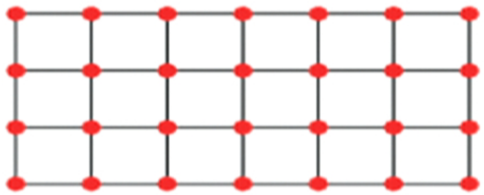
http://mathworld.wolfram.com/HexagonalGrid.html



Low variety

The dynamics of maximal variety

Configuration beables, views and variety.

Our system is an ensemble consisting of N similar subsystems.

Each has a d-dimensional configuration beable $x^a{}_k$. $k=1, \ldots N$, $a = 1,\ldots,d$.
We assume these live in a vector space with metric.

The momentum information is carried by a single modular phase:

$$w_k = e^{\frac{i}{\hbar} S_k}$$

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- The subsystems are similar systems with the same constituents, subject to the same forces, internally and externally so that, when expressed in centre of mass coordinates, they are, up to negligible errors, described by the same Hamiltonian. They also have the same preparations so that they fit the operational definition of having the same quantum state.

- Alternative interpretation: the subsystems are near copies of our universe which exist simultaneously and interact with each other through new interactions.

    (MIW hypothesis: Hall, Decker, Wiseman; Holland, Piorier)

Each subsystem, i, has a ***view*** of the rest of the system:

$$\boxed{V_i^{ka} = \frac{x_i^a - x_k^a}{D(i,k)^2} = \frac{x_i^a - x_k^a}{|x_i^a - x_k^a|^2}\Theta(R - |x_i^a - x_k^a|)}$$

Differences between views give the ***distinctiveness*** of a pair of subsystems:

$$\mathcal{I}_{ij} = \frac{1}{N}\sum_k \left(V_i^{ka} - V_j^{ka}\right)^2$$

Sum this over all pairs to define the ***variety:***

$$\boxed{\mathcal{V} = \frac{A}{N^2}\sum_{i\neq j}\mathcal{I}_{ij} = \frac{A}{N^3}\sum_{i\neq j}\sum_k \left(V_i^k - V_j^k\right)^2}$$

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From the variety define an interaction between subsystems in terms of a potential energy. This is the inter-ensemble interaction.

$$\mathcal{U}^{\mathcal{V}} = -\frac{\hbar^2}{8m}\mathcal{V} = -\frac{\hbar^2}{8m}\frac{A}{N^3}\sum_{i\neq j}\sum_{k}\left(V_i^k - V_j^k\right)^2$$

$$= -\frac{\hbar^2}{8m}\frac{A}{N^3}\sum_{k}\sum_{i\neq j}\left(\frac{x_i^a - x_k^a}{|x_i^a - x_k^a|^2} - \frac{x_j^a - x_k^a}{|x_j^a - x_k^a|^2}\right)^2$$

Energy is minimized by maximizing the variety!

Modular phase beable:

If this were classical mechanics we would define a momenta beable $p_a{}^k$ conjugate to the configuration beable $x^a{}_k$.

But in quantum mechanics the momentum density is the gradient of a phase, and this imposes restrictions on the $p_a{}^k$ (Wallstrom).

So we posit that each subsystem has a **modular phase beable**

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From these and the views we define a ***relational momenta:***

$$\boxed{p_{ka} = -\imath \frac{1}{N} \sum_{j \neq k} V_k^{aj} \ln \left(\frac{w_j}{w_k} \right)}$$

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These give us the ***kinetic energy:***

$$\boxed{K.E. = \mathcal{R}e \frac{Z\hbar^2}{2mN^2} \sum_{k \neq j} \frac{1}{(x_k - x_j)^2} \left[\ln \left(\frac{w_j}{w_k} \right) \right]^2}$$

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The fundamental action

$$S(w,x) = \int dt \sum_k \left\{ -Z_0 \sum_{j \neq k} x_k^a \imath V_k^{ja} \frac{d}{dt} \left[\ln \left(\frac{w_j}{w_k} \right) \right] - H[x,w] \right\}$$

The Hamiltonian

$$\boxed{H[x,w] = \mathcal{R}e \frac{\hbar^2}{2m} \left(Z \sum_{k \neq j} (V_k^{ja})^2 \left[\ln \left(\frac{w_j}{w_k} \right) \right]^2 - \frac{A}{4N} \sum_k \sum_{i \neq j} \left(V_i^{ka} - V_j^{ka} \right)^2 \right) + \sum_k U(x_k)}$$

U(x) is a standard potential, within each subsystem.

In the large N limit this reproduces quantum dynamics

Deriving quantum mechanics

The large N limit and the continuum approximation.

The average of a function is given by a probability density $\rho(x)$.

$$< \phi >= \frac{1}{N} \sum_k \phi(x_k) \to \int d^d z \rho(z) \phi(z)$$

For double sums we use:

$$\frac{1}{N} \sum_i \phi(x_{k+i}, x_k) \to \int_a^R d^d x \rho(z+x) \phi(z+x, z)$$

The short distance cutoff is a, reflecting the fact that for finite N nearest neighbors are not likely to come nearer to each other than

$$a(z) = \frac{1}{(N\rho(z))^{\frac{1}{d}}}$$

For the large scale cutoff, we rescale R by $N^{1/d}$:

$$R = \frac{r'}{N^{\frac{1}{d}}}$$

We hold r' fixed as we vary N.

for large R, r' is independent of $\rho(z)$

The results:

The Hamiltonian

$$H[x,w] = \mathcal{R}e\frac{\hbar^2}{2m}\left(Z\sum_{k\neq j}(V_k^{ja})^2\left[\ln\left(\frac{w_j}{w_k}\right)\right]^2 - \frac{A}{4N}\sum_k\sum_{i\neq j}\left(V_i^{ka}-V_j^{ka}\right)^2\right) + \sum_k U(x_k)$$

becomes to leading order:

$$H = \int d^d z\rho(z)\left[\frac{(\partial_a S)^2}{2m} + \frac{\hbar^2}{8m}(\frac{1}{\rho}\partial_a\rho)^2 + V + O(\frac{1}{r}) + O(\frac{1}{N})\right]$$

The symplectic structure:

$$S^0(w,x) = -Z_0 \int dt \sum_k \dot{p}_a^k x_k^a$$

where: $$\dot{p}_a^k = \frac{1}{\hbar} \sum_{j \neq k} \left[-\imath V_k^{ja} \left(\dot{S}_j - \dot{S}_k \right) \right]$$

In the continuum:

$$\boxed{\begin{aligned} S^0 &\rightarrow -Z_0 N \int dt \int d^d z \rho(z) z^a \dot{p}_a(z) \\ &= \int dt \int d^d z \rho(z) \dot{S}(z) \end{aligned}}$$

$$Z_0 = \frac{dN}{\Omega(r^d - 1)}$$

Putting everything together:

$$\boxed{S = \int dt \int d^d z \rho(z) \left[\dot{S} + \frac{(\partial_a S)^2}{2m} + \frac{\hbar^2}{8m}(\frac{1}{\rho}\partial_a \rho)^2 + U + O(\frac{1}{r}) + O(\frac{1}{N}) \right]}$$

Equations of motion:

quantum potential

$$\dot{\rho}(x^a) = \partial_a(\rho \frac{1}{m} g^{ab} \partial_b S(x^a))$$

$$-\dot{S} = \frac{1}{2m} g^{ab} (\frac{\partial S}{\partial x_{a\alpha}})(\frac{\partial S}{\partial x_{b\alpha}}) - \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + U$$

These are the real and imaginary parts of the Schroedinger equation:

$$\boxed{\imath\hbar \frac{d\Psi}{dt} = \left(-\frac{\hbar^2}{2m} \nabla^2 + U \right) \Psi}$$

$$\Psi(x,t) = \sqrt{\rho} e^{\frac{\imath}{\hbar} S}$$

Some comments

Probabilities and ergodisity:

$\rho(x)$ is the ensemble probability distribution.

$\rho_k(x)$ is the probability distribution in the k'th subsystem.

We must posit an ergodic hypothesis:

Over time, for all k, $\boldsymbol{\rho_k(x) \to \rho(x)}$

Quantum statistics: ρ(x)

Let us have a system of M identical particles with configurations
$x^a{}_{lk}$ $l=1,...M,$ $a=1,...d,$ $k=1,...n$ and phases w_k

PII: there is no effect of switching the coordinates of two identical particles, so we require

$$\rho(x^a_{k1}, x^a_{k2}, \ldots, t) = \rho(x^a_{k2}, x^a_{k1}, \ldots, t)$$
$$\dot{\rho}(x^a_{k1}, x^a_{k2}, \ldots, t) = \dot{\rho}(x^a_{k2}, x^a_{k1}, \ldots, t)$$

We plug these into the equations of motion to deduce

$$S(x^a_{k1}, x^a_{k2}, \ldots, t) = S(x^a_{k2}, x^a_{k1}, \ldots, t) + \phi$$

Doing this twice, and recalling that S is defined up to 2πn, we find

$$w(x_{k1}, x_{k2}) = e^{\frac{\imath}{\hbar}S(x_{k1}, x_{k2})} \rightarrow w(x_{k2}, x_{k1}) = \pm w(x_{k1}, x_{k2})$$

ie bosons and fermions.

The solution to the measurement problem:

Microscopic systems are quantum because they have large numbers of near copies in the universe, hence the variety interaction works on them and they find themselves members of large ensembles of similar systems.

Macroscopic systems are unique. They have no copies and are not parts of any ensembles. Hence they are not subject to quantum uncertainty. Their collective coordinates obey the fundamental deterministic dynamics.