Title: A Perfect Quantum Cosmological Bounce

Date: Jun 23, 2016 11:00 AM

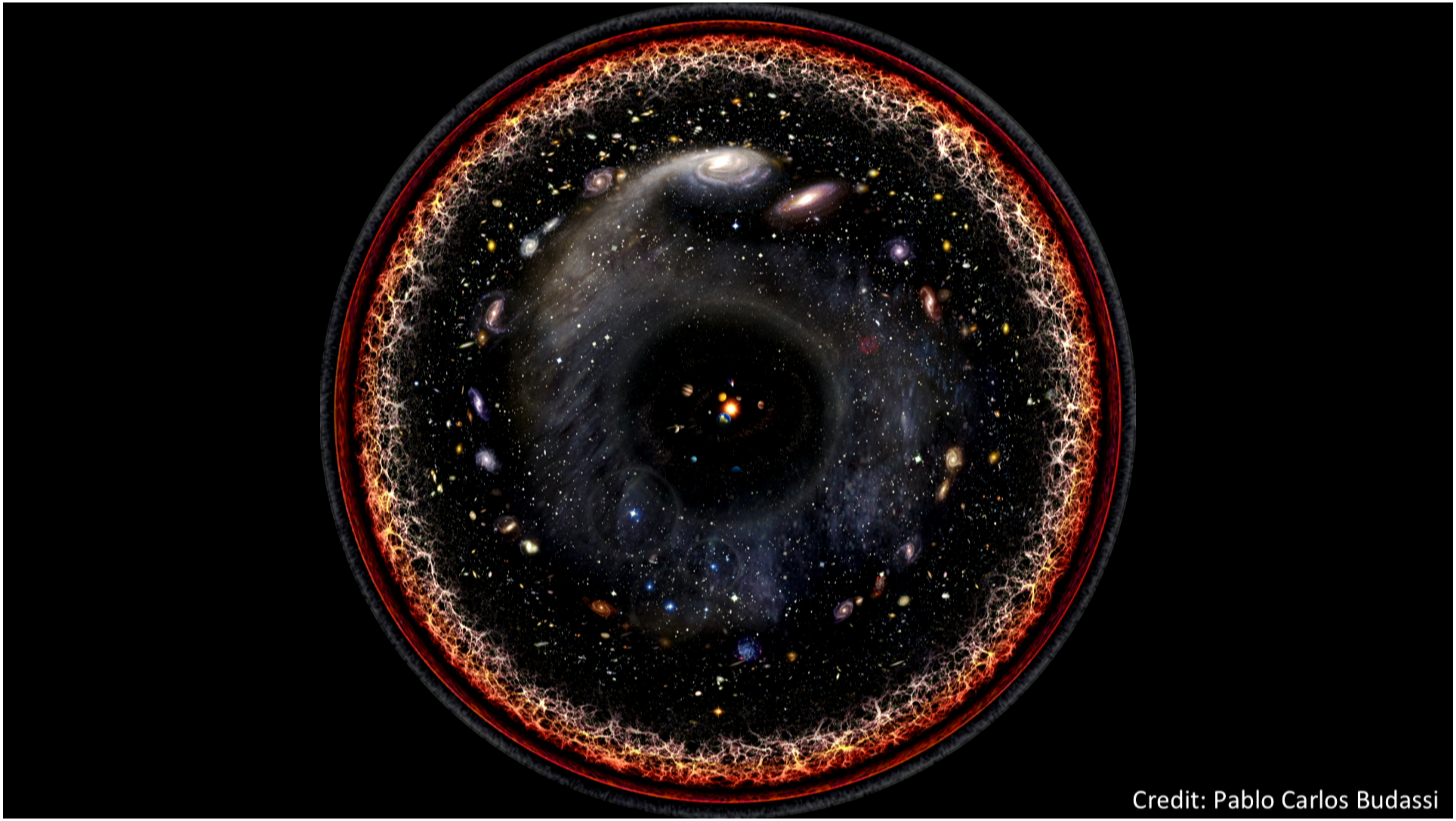
URL: http://pirsa.org/16060063

Abstract:

A perfect quantum cosmological bounce

w/ Steffen Gielen (Imperial College)
1510.00699, PRL to appear

w/ J. Feldbrugge (Perimeter), to appear


Credit: Pablo Carlos Budassi

Just six numbers

			Observational Uncertainty
Matter	Radiation density	9.1±0.1 x 10^{-5} x critical	1%
	Baryon density:	0.048±0.0005 x critical	1%
	Dark matter density:	0.26±0.005 x critical	2%
	Dark energy density:	0.69±0.006 x critical	1%
Geometry	Scalar amplitude :	4.6±0.006 x 10^{-5}	1%
	Scalar spectral index : (scale invariant = 0)	-.033±0.004	12%

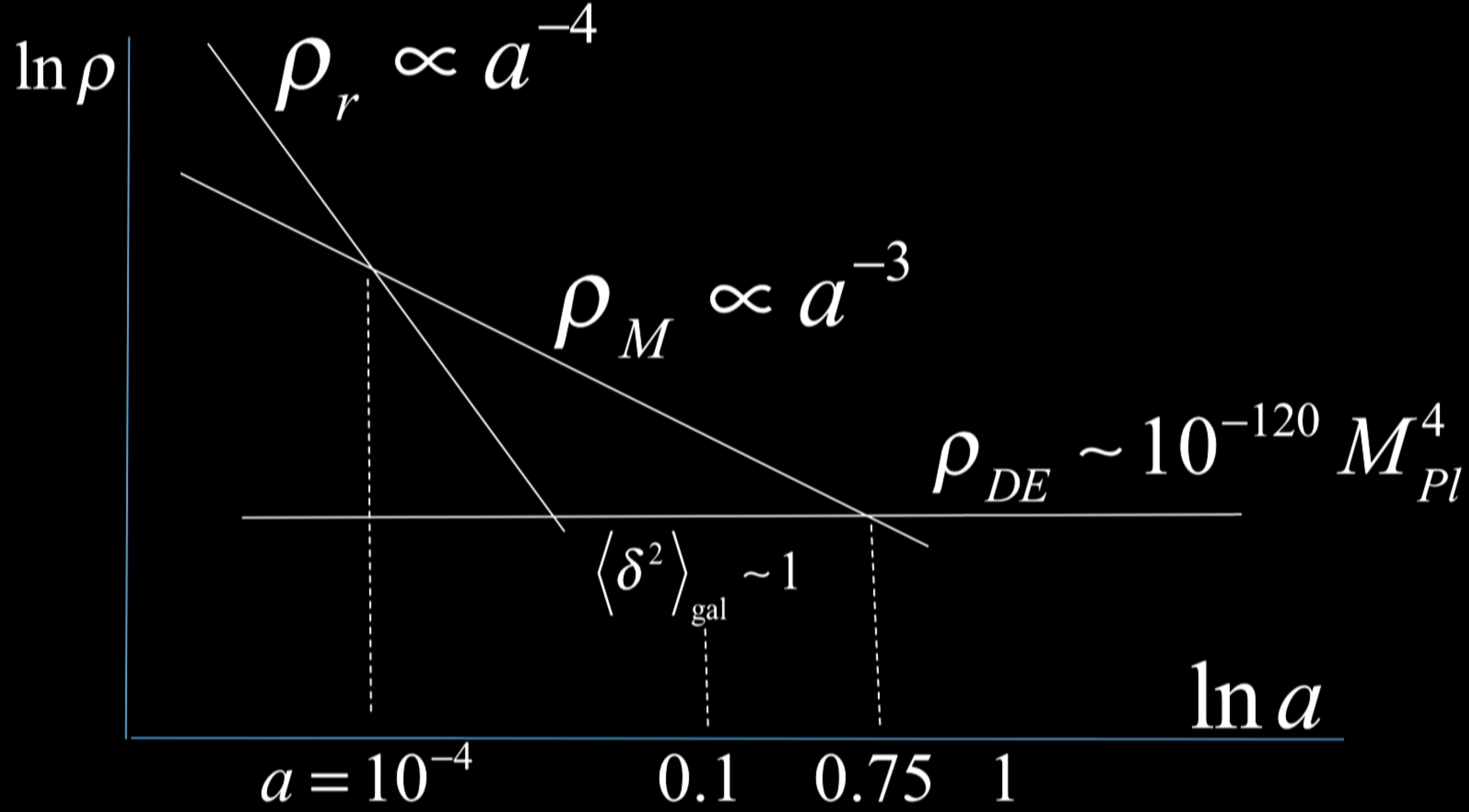
A_s

n_s

and a lot of zeros! (so far)

$$\Omega_k;\ \left(\frac{P+\rho}{\rho}\right)_{DE};\ r=\frac{A_{gw}}{A_S};\ \frac{dn_s}{d\ln k};\ \frac{\langle\delta^3\rangle}{\left(\langle\delta^2\rangle\right)^{\frac{3}{2}}};\ etc+m_\nu\text{'s}$$

Strange coincidences!



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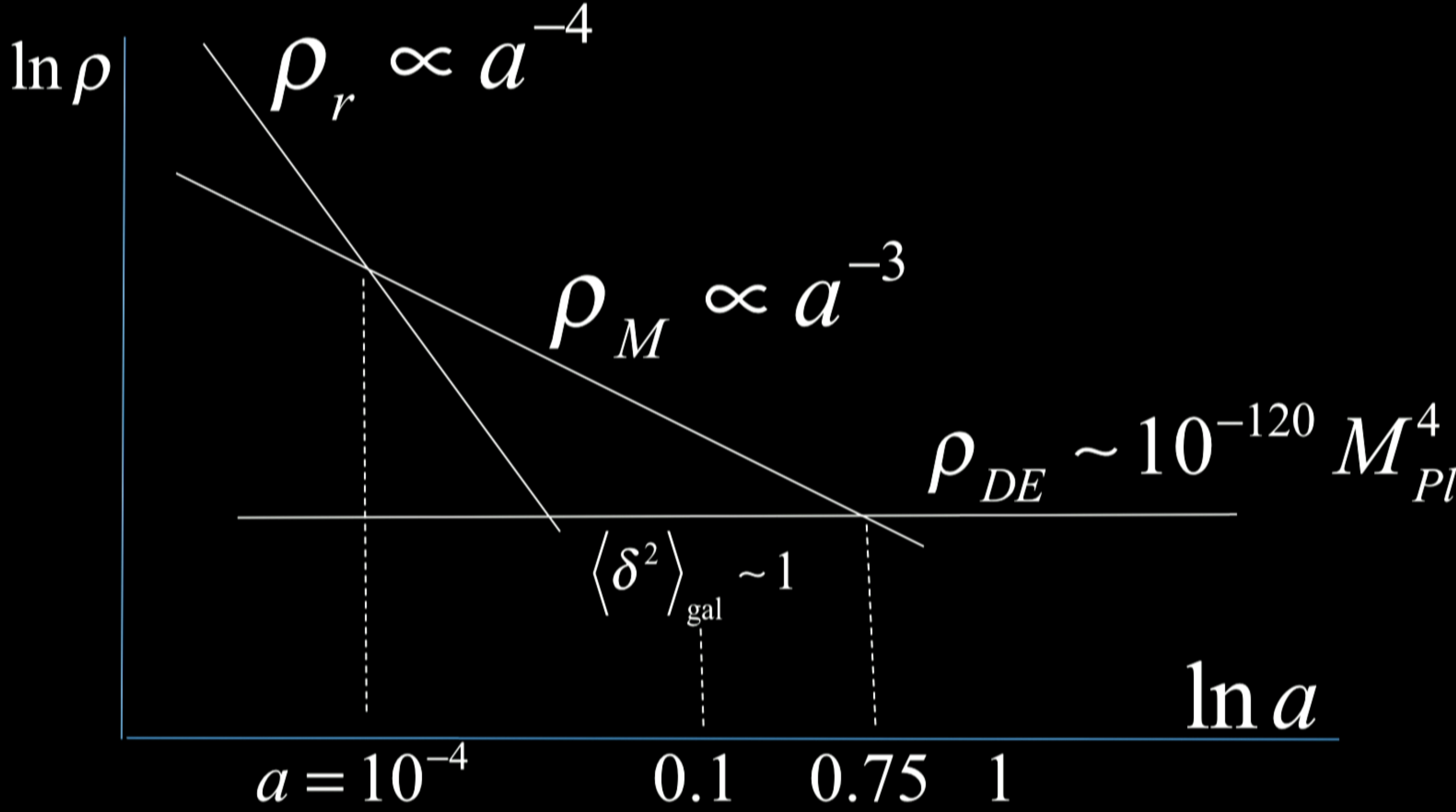

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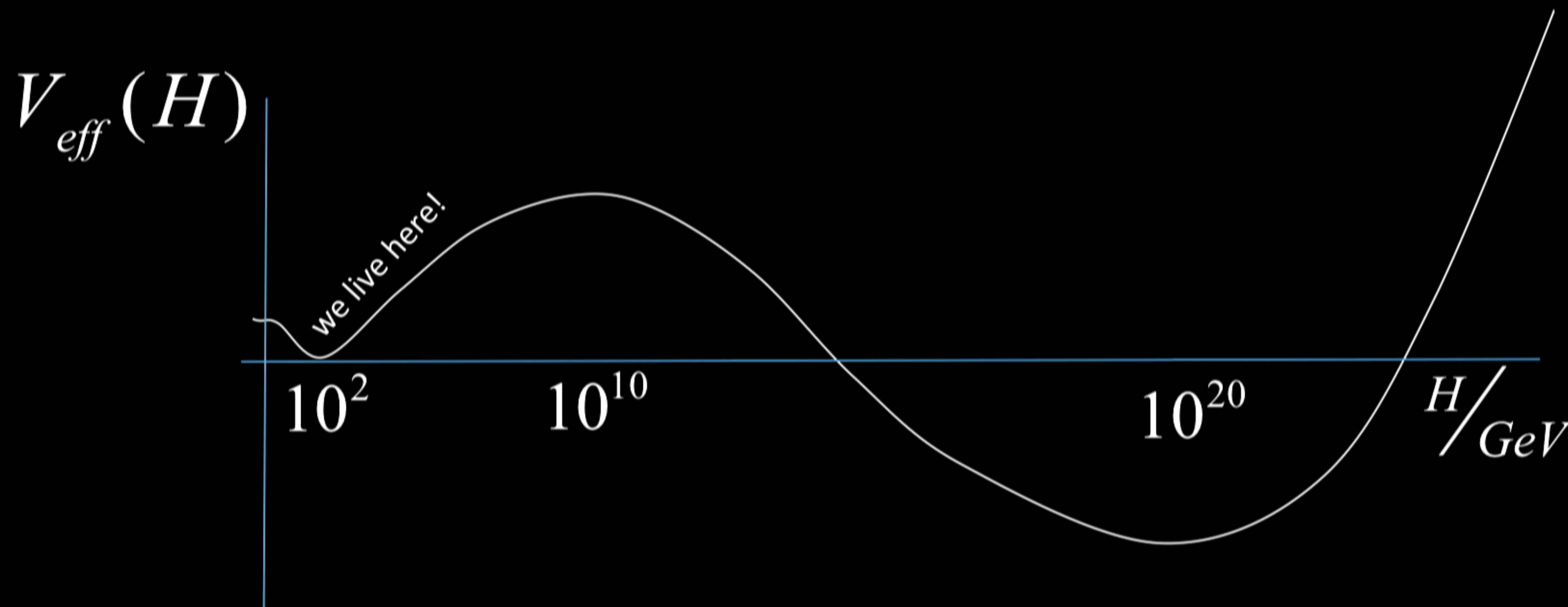
all known physics

$$\Psi = \int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi} i \not{D} \psi - \lambda H \overline{\psi}\psi + |DH|^2 - V(H) \right)}$$

Schrödinger Feynman Einstein Maxwell-Yang-Mills Kobayashi-Maskawa Lagrange Euler Planck Newton Dirac Yukawa Higgs

(+ neutrino masses and dark matter!)

Extrapolating the minimal standard model from LHC measurements to Planck scale, Higgs vacuum is metastable:



Buttazzo et al arXiv 1307.3536

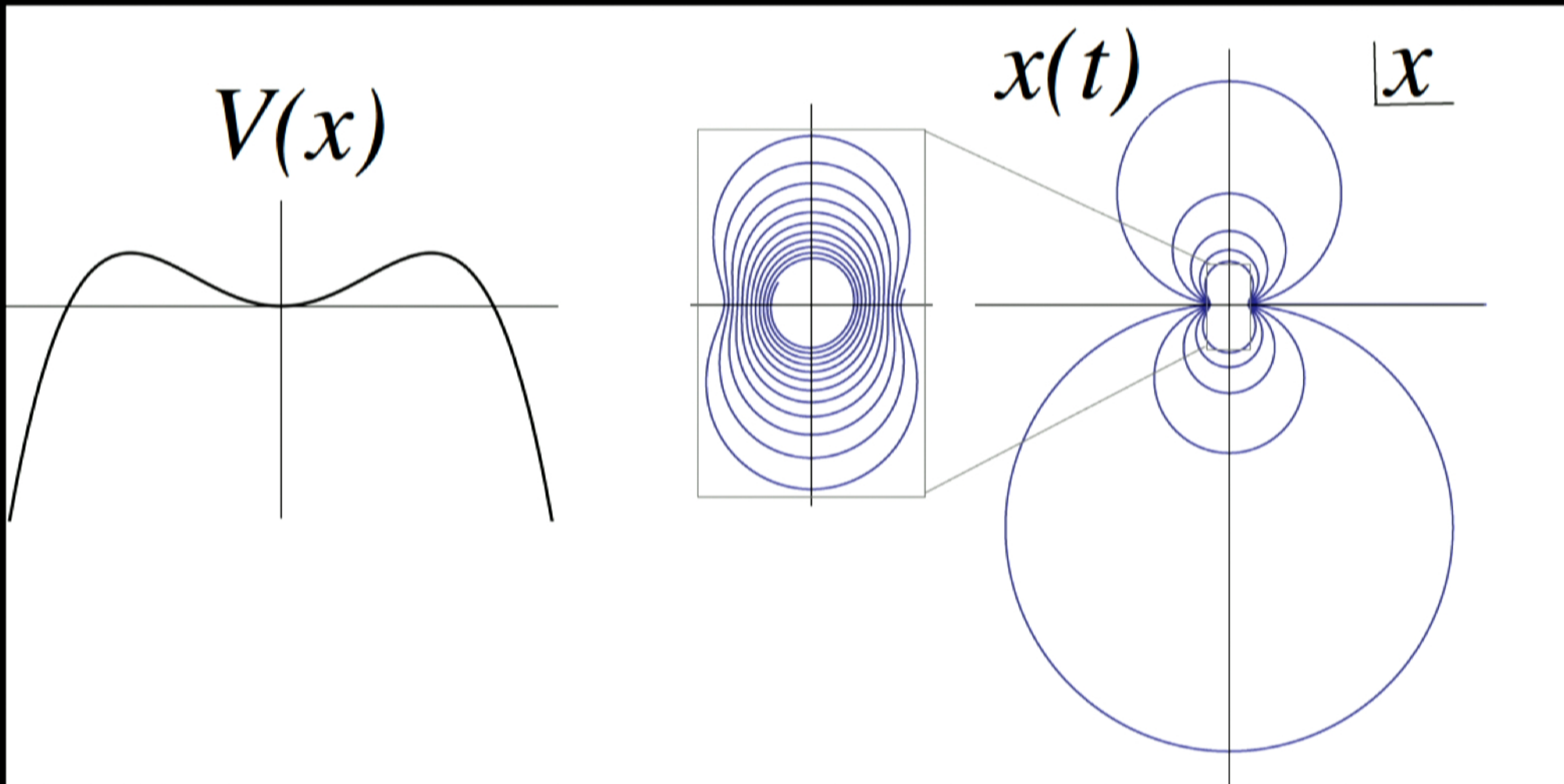
The Feynman path integral

$$\langle f|i\rangle = \iint dx_f\, dx_i \Psi^*_f(x_f)\Psi_i(x_i)\int DxDpe^{\frac{i}{\hbar}S_i^f}$$

Naturally incorporates pre- and post-selection

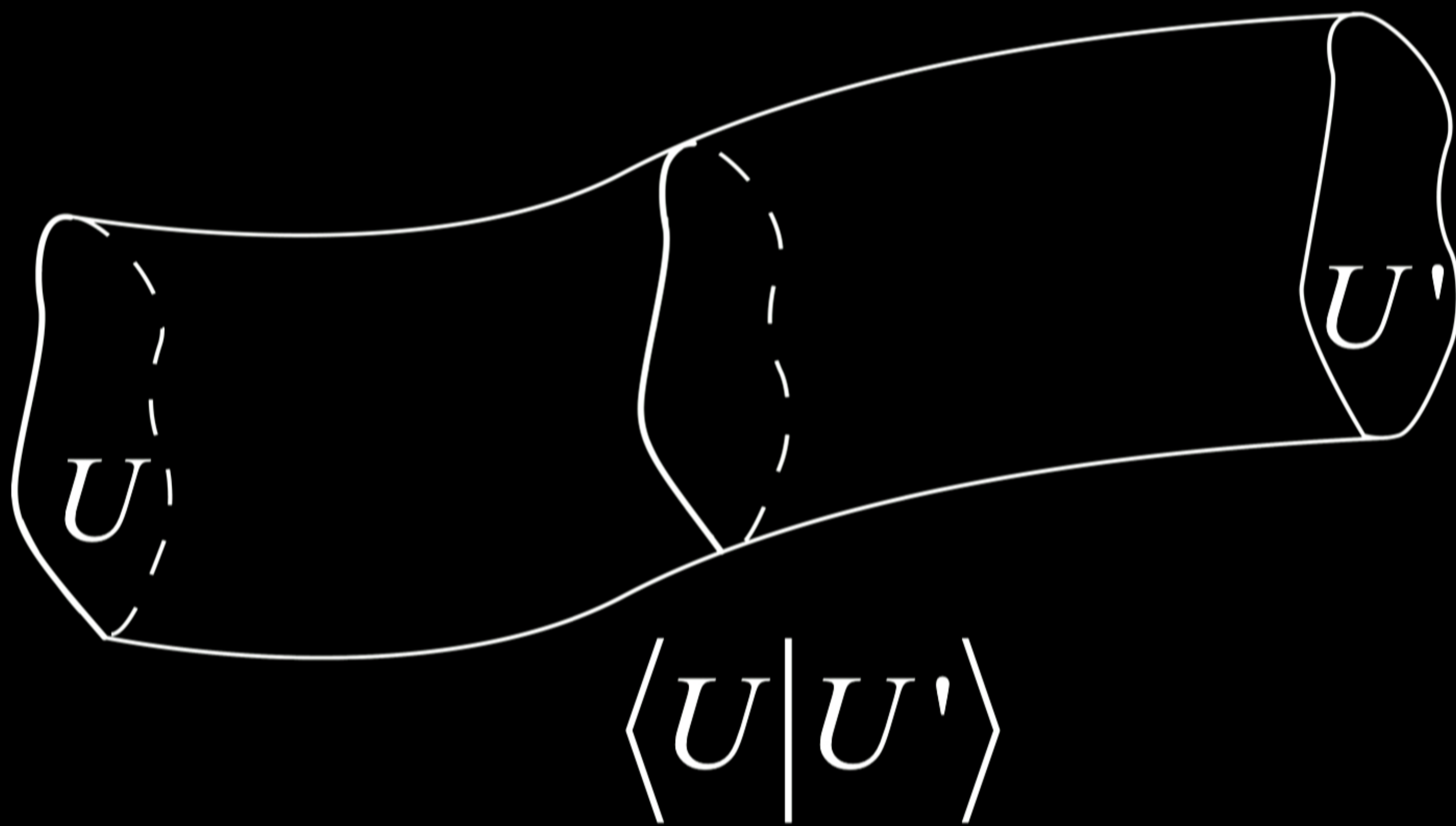
In the semi-classical approximation, classical saddle point trajectories are generically *complex*

quantum tunneling in real time



NT, arXiv 1312.1772, New J. Physics **16** (2014) 063006

quantum gravity amplitudes



Wheeler-DeWitt equation (for pure gravity)

$$\left(G_{ijkl}\frac{\delta}{\delta\gamma_{ij}}\frac{\delta}{\delta\gamma_{kl}}+\gamma^{1/2}\,{}^{(3)}R\right)\Psi[{}^{(3)}\mathcal{G}]=0\,,$$

$$G_{ijkl}\equiv\tfrac{1}{2}\gamma^{-1/2}(\gamma_{ik}\gamma_{jl}+\gamma_{il}\gamma_{jk}-\gamma_{ij}\gamma_{kl})\,.$$

Hamiltonian is zero on physical states

(Klein-Gordon-like equation)

DeWitt conserved inner product

$$(\Psi_b,\Psi_a)=Z\int_\Sigma \Psi_b{}^*[{}^{(3)}\mathcal{G}]\prod_{\mathbf{x}}\left(d\Sigma^{ij}G_{ijkl}\frac{\vec{\delta}}{i\delta\gamma_{kl}}-\frac{\overleftarrow{\delta}}{i\delta\gamma_{kl}}G_{ijkl}d\Sigma^{ij}\right)\Psi_a[{}^{(3)}\mathcal{G}]\,.$$

(but $\left(\Psi,\Psi\right)$ not generally positive)

Perhaps the most impressive fact which emerges from a study of the quantum theory of gravity is that it is an extraordinarily economical theory. It gives one just exactly what is needed in order to analyze a particular physical situation, but not a bit more. Thus it will say nothing about time unless a clock to measure time is provided, and it will say nothing about geometry unless a device (either a material object, gravitational waves, or some other form of radiation) is introduced to tell when and where the geometry is to be measured.[50] In view of the strongly operational foundations of both the quantum theory and general relativity this is to be expected. When the two theories are united the result is an operational theory *par excellence*.[51]

B.S. DeWitt, Phys Rev 160, 1967 (p 1140)

Conformal matter in quantum cosmology

QFTs which are conformal invariant in the UV, like QCD, can be defined independently of any UV cutoff

They also have a special quantum cosmology, related to their Weyl-invariance (matter "does not see" expansion!)

Truncating to cosmological symmetry (homogeneity and isotropy), with conformal matter (pure radiation and M free, conformally coupled scalar fields), the quantum behavior of the background FRW universe can be solved *exactly*

Propagator describes a "perfect cosmological bounce"

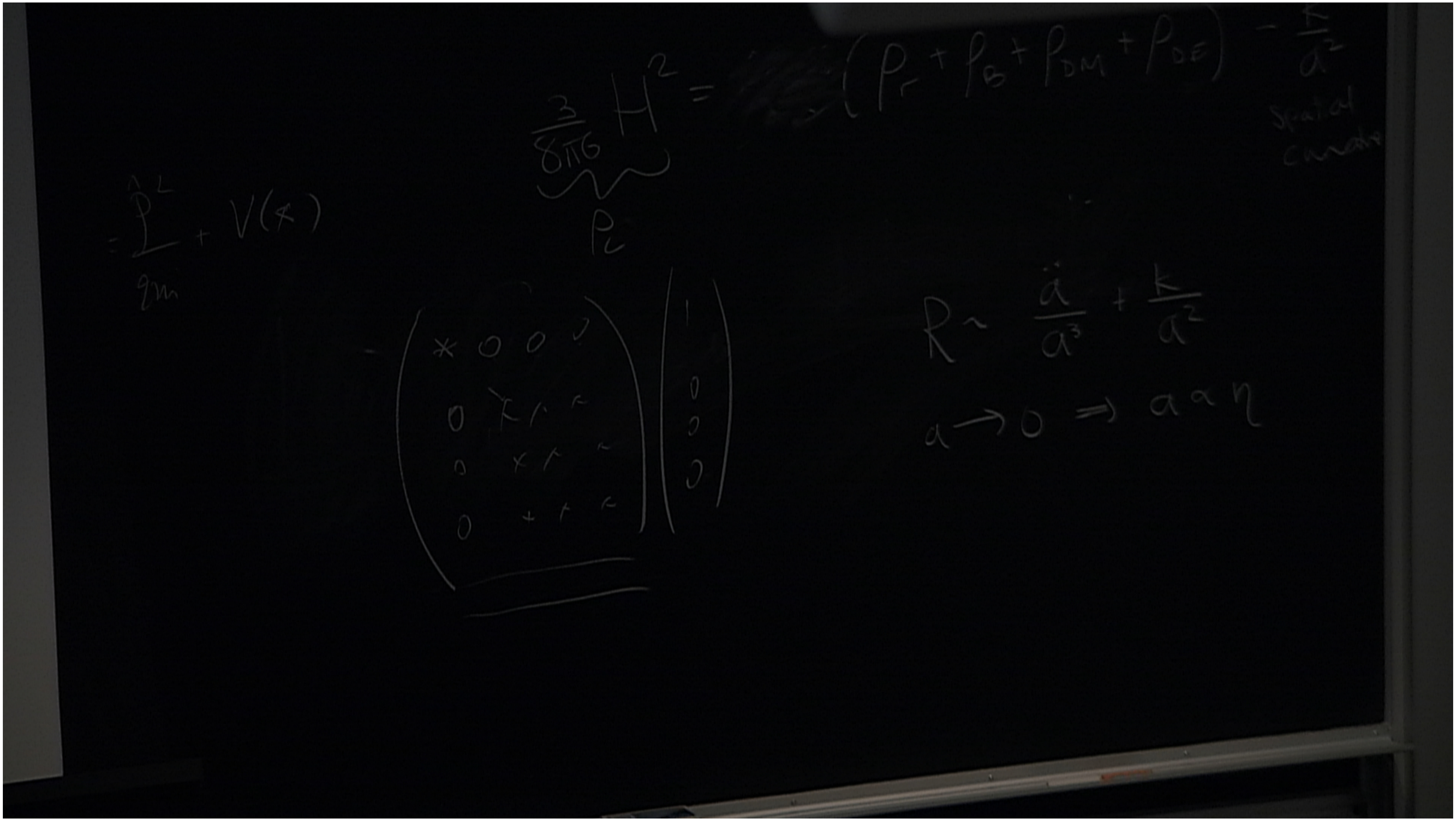
Simplest case: pure radiation

FRW $ds^2 \sim a(\eta)^2(-d\eta^2 + \gamma_{ij}dx^i dx^j); \quad R^{(3)} = 6\kappa \quad \underset{\kappa<0,=0,>0}{H^3, E^3, S^3}$

Trace of Einstein equations $R \propto T_\lambda^\lambda = 0 \Rightarrow \; a \propto \eta \text{ as } a \to 0$

"perfect bounce" $ds^2 \sim \eta^2(-d\eta^2 + \gamma_{ij}dx^i dx^j)$

unique analytic extension from negative to positive a


3/8πG H² = (ρ_r + ρ_B + ρ_DM + ρ_DE) − k/a²
spatial curvature
R ~ ä/a³ + k/a²
a → 0 ⇒ a ∝ η

add M conformally coupled scalars

"Weyl lift"

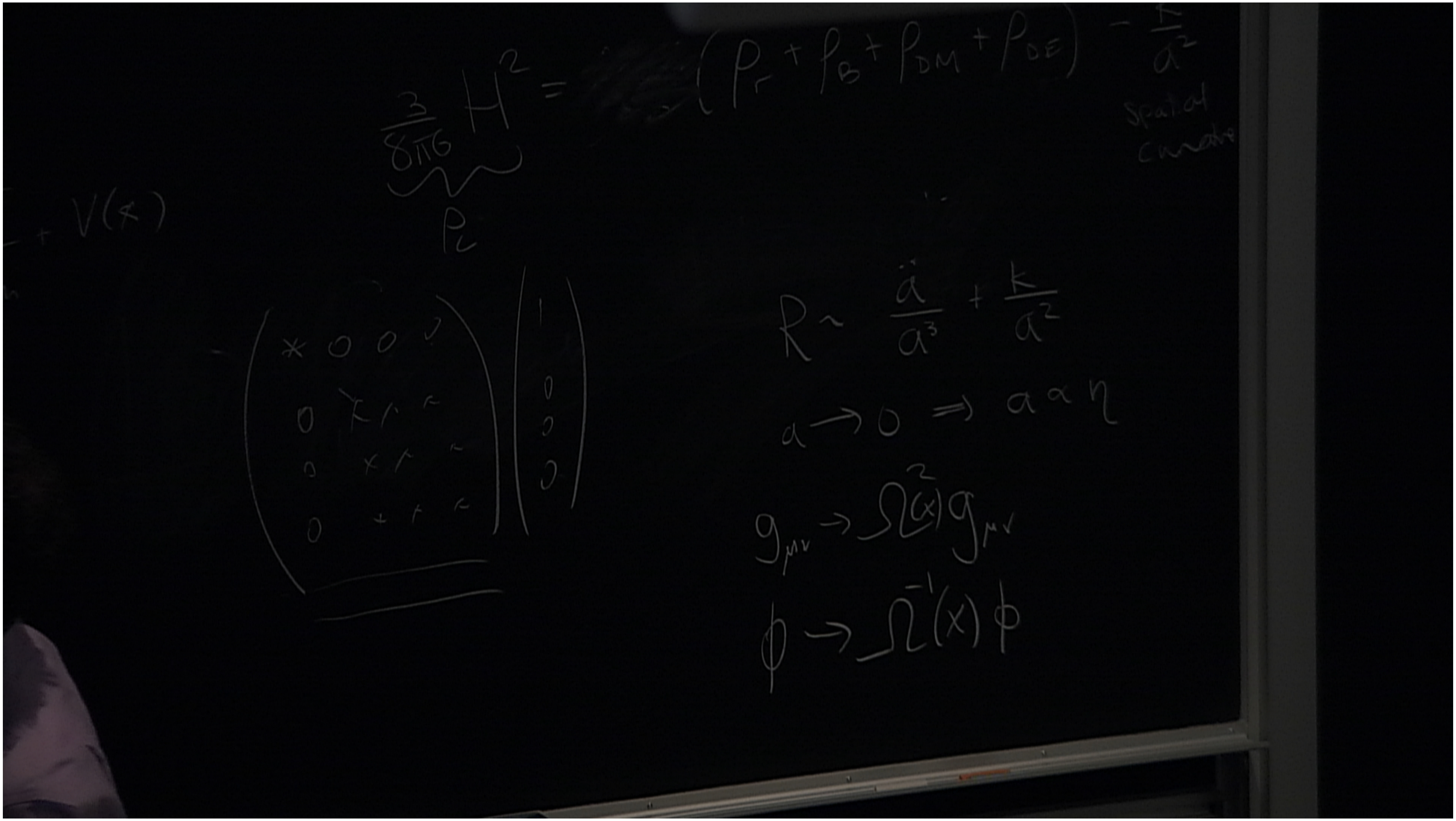
$$S = \int \left[\tfrac{1}{2}\left((\partial\phi)^2 - (\partial\vec{\chi})^2 \right) + \tfrac{1}{12}\left(\phi^2 - \vec{\chi}^2 \right) R - \rho(n) - nU^{\mu}\,\partial_{\mu}\varphi \right]$$

Pick Weyl gauge in which 3-metric is static

$$ds^2 = -N(t)^2 dt^2 + \gamma_{ij} dx^i dx^j;\ \gamma_{ij} \text{ metric on } S^3, E^3 \text{or } H^3;\ \rho_r = r = const$$

define $x^{\alpha} = \frac{1}{\sqrt{2r}}(\phi, \vec{\chi});\ \eta_{\alpha\beta} = \mathrm{diag}(-1,1,\ldots,1)$

action $S = \frac{m}{2}\int dt \left[N^{-1}\dot{x}^{\alpha}\dot{x}_{\alpha} - N(\kappa x^{\alpha}x_{\alpha} + 1) \right]$


spatial curvature

Hamiltonian $H = \frac{P_\alpha P^\alpha}{2m} + \frac{m}{2}(1+\kappa x^\alpha x_\alpha), \quad m = 2V_0 r$

comoving volume

radiation density parameter

Gauge choice $N = \tau$ proper time $0 < \tau < \infty$

Feynman propagator

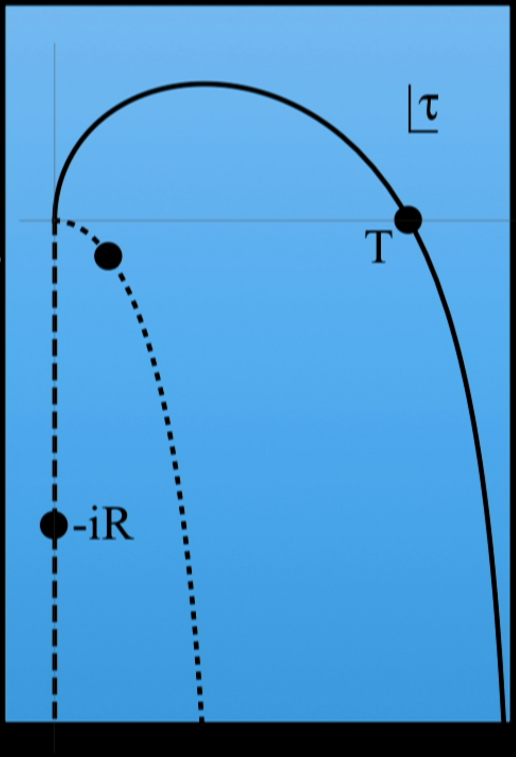
$$\langle x, m | x', m' \rangle = \delta(m - m') \int_0^\infty d\tau \int DxDPe^{i\int_{-\frac{1}{2}}^{+\frac{1}{2}} dt\left(\dot{x}^\alpha P_\alpha - \tau H(x,P)\right)}$$

- path integrals are Gaussian so semi-classical approximation is exact
- if m is large, FRW background is "heavy": $\frac{3}{8\pi G} H_E V_E >> 1$ quantum spreading and back-reaction are small

Flat radiation-dominated FRW corresponds to free, massive relativistic particle

$$G(x,x') = \int_0^{"\infty"} d\tau \, Dx e^{\frac{im}{2\hbar}\int dt\left(\frac{\dot{x}^2}{\tau}-\tau\right)} = i\int_0^{"\infty"} d\tau \left(\frac{m}{2\pi i\tau}\right)^{\frac{M+1}{2}} e^{-i\frac{m}{2}\left(\frac{\sigma}{\tau}+\tau\right)},$$

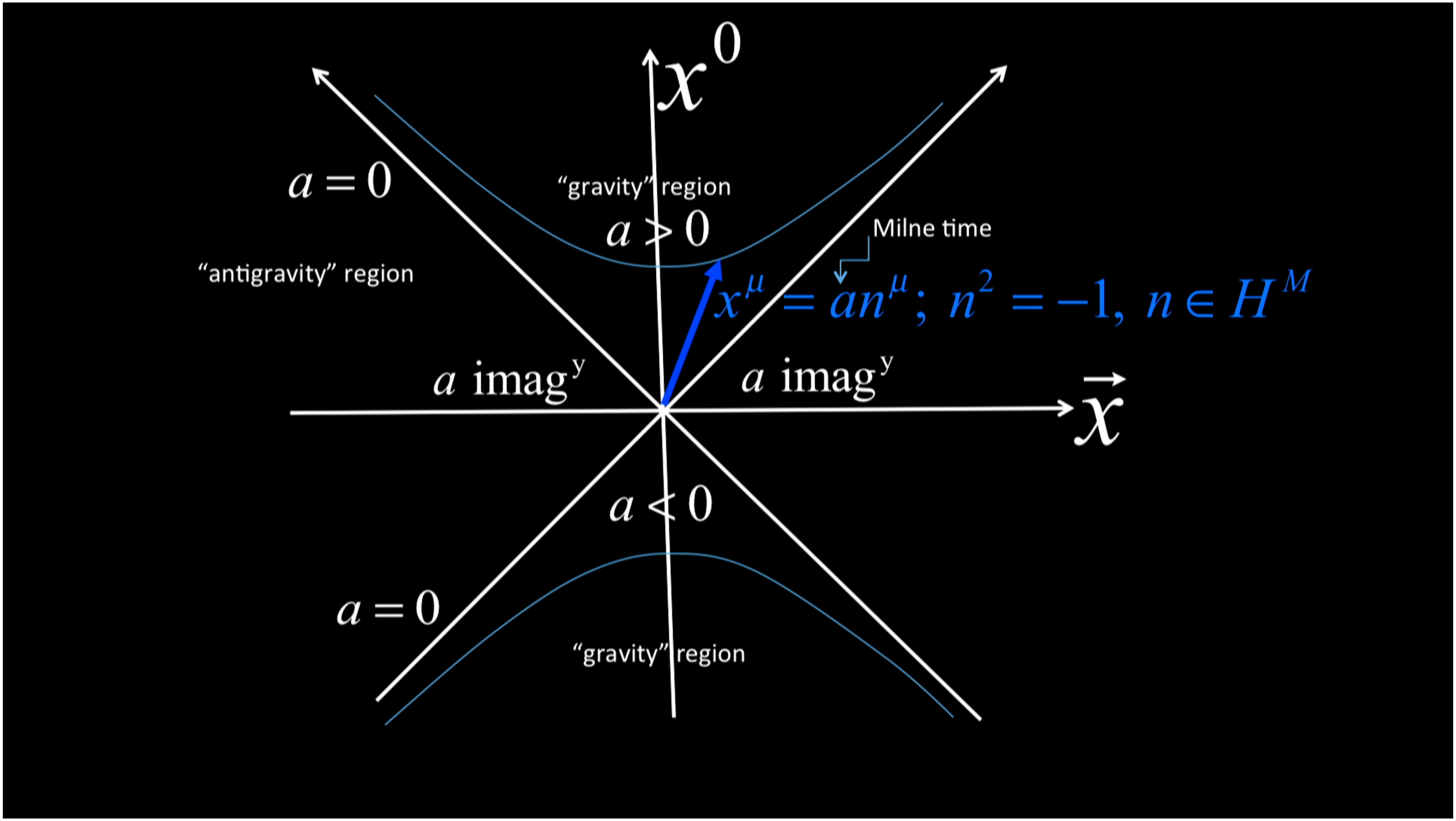
$$\text{where } \sigma \equiv -(x-x')^2 = \begin{cases} T^2 & \text{timelike separations} \\ -R^2 & \text{spacelike separations} \end{cases}$$



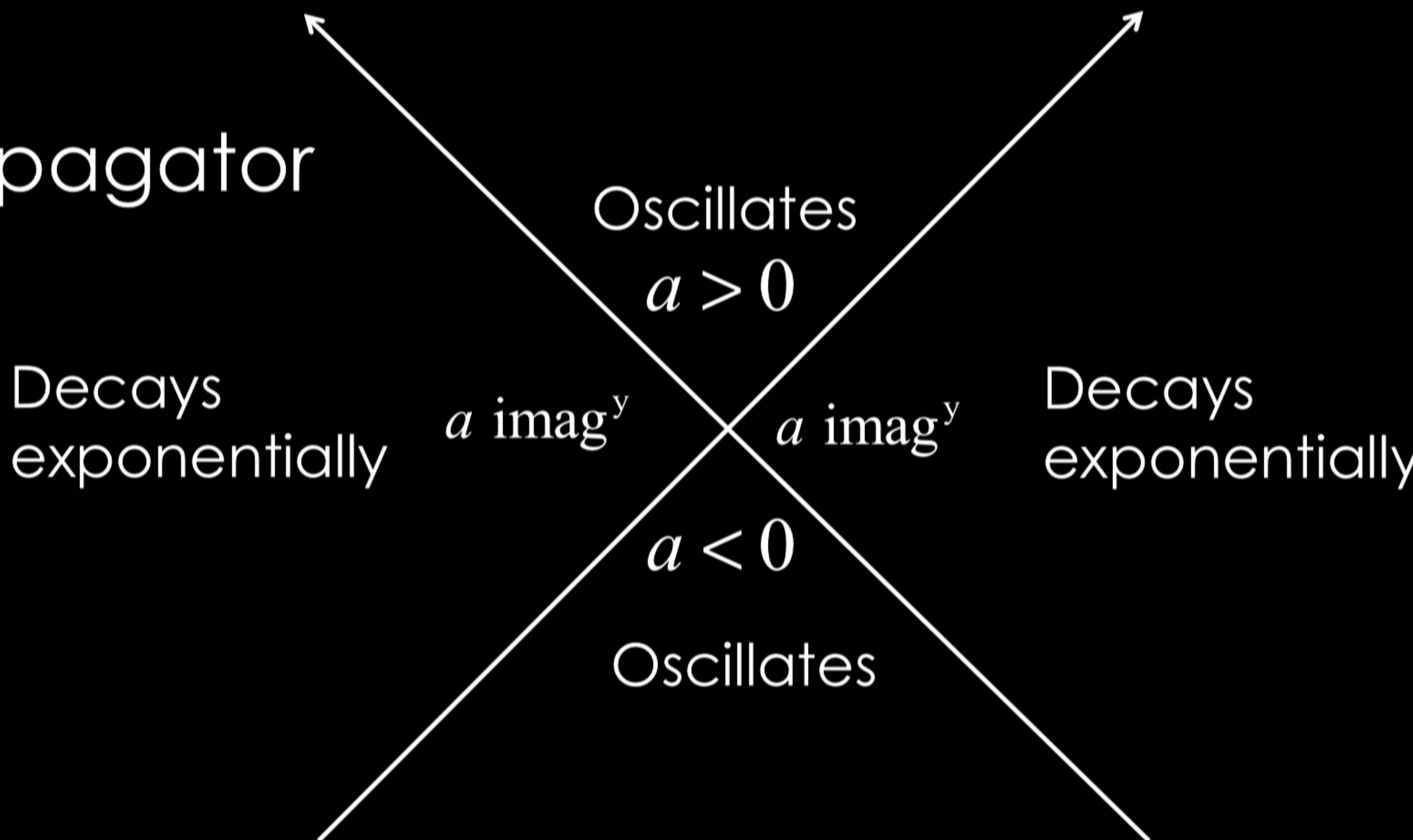
Uniquely defined by saddle point and associated steepest descent contour. No need for $i\varepsilon$.

Formally, $G(x,x') = \langle x | \int_0^{\infty} d\tau e^{-iH\tau} | x' \rangle = -i \langle x | H^{-1} | x' \rangle$

So $H_x G(x,x') = -i\delta^{M+1}(x-x')$

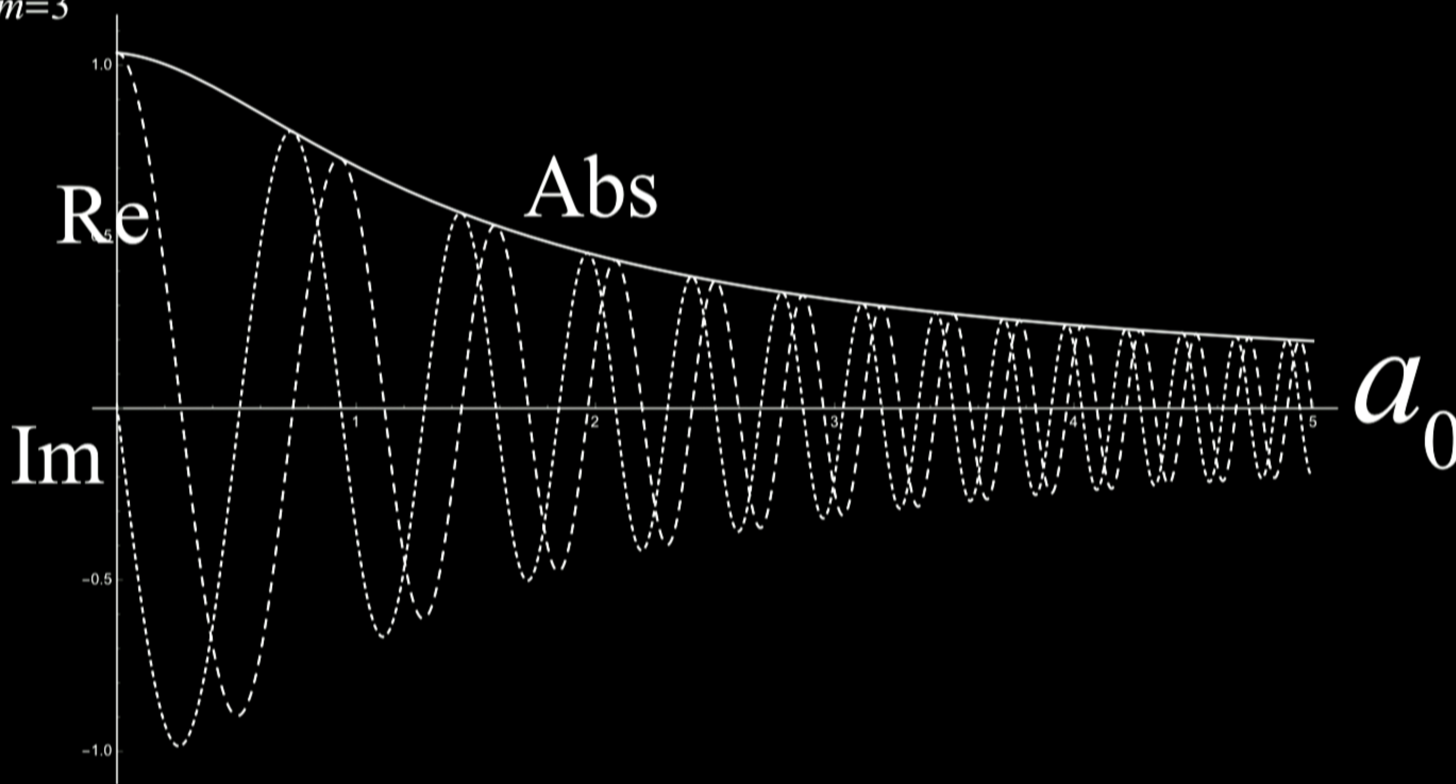

x^0
$\vec{x}$
$a = 0$
"gravity" region
$a > 0$
Milne time
"antigravity" region
$x^\mu = an^\mu;\ n^2 = -1,\ n \in H^M$
a imag^y
a imag^y
$a < 0$
$a = 0$
"gravity" region

Propagator


Oscillates
$a > 0$
Decays exponentially
a imag^y
a imag^y
Decays exponentially
$a < 0$
Oscillates

e.g. open universe $(\kappa = -1)$ propagator

$$\langle -a_0 | a_0 \rangle_{m=3}$$



(Expressible in parabolic cylinder functions)

Now include anisotropies:

Consider deviations around flat ($\kappa = 0$) FRW

Anisotropies: $ds^2 \sim a(\eta)^2(-d\eta^2 + \sum_{i=1}^{3} e^{c\lambda_i} dx_i^2); \quad \sum \lambda_i = 0$

Line element on space of "moduli":

$$ds^2_{\text{mod}} = -da^2 + a^2(dH_M^2 + dE_2^2)$$

scalars aniso-moduli

(Note: standard model Higgs sufficient to remove BKL chaos)

Propagator uniquely determined in terms of positive and negative frequency modes

$$G(a,m|a',m') = \sqrt{2m}\,\delta(m-m')\exp\left(-\frac{m\pi}{4\sqrt{-\kappa}}\right)(\psi_3(a')\psi_4(a)\theta(a-a') + \psi_3(a)\psi_4(a')\theta(a'-a))\,.$$

where

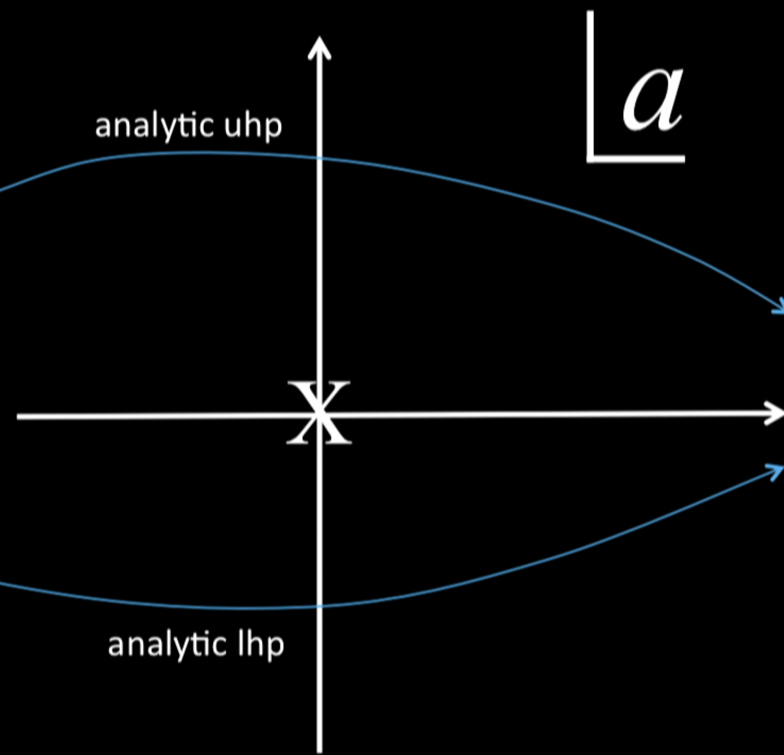
$$\psi_3(a) = D_{\mathrm{i}\frac{m}{2\sqrt{-\kappa}}-\frac{1}{2}}((1-\mathrm{i})\sqrt{m}\,(-\kappa)^{1/4}a)\,,$$
$$\psi_4(a) = D_{-\mathrm{i}\frac{m}{2\sqrt{-\kappa}}-\frac{1}{2}}((1+\mathrm{i})\sqrt{m}\,(-\kappa)^{1/4}a)$$

Analytic continuation of propagator

$$a > a'$$

$$G_F(a,a') \sim \Sigma \Psi^{(+)}(a)\Psi^{(-)}(a')$$

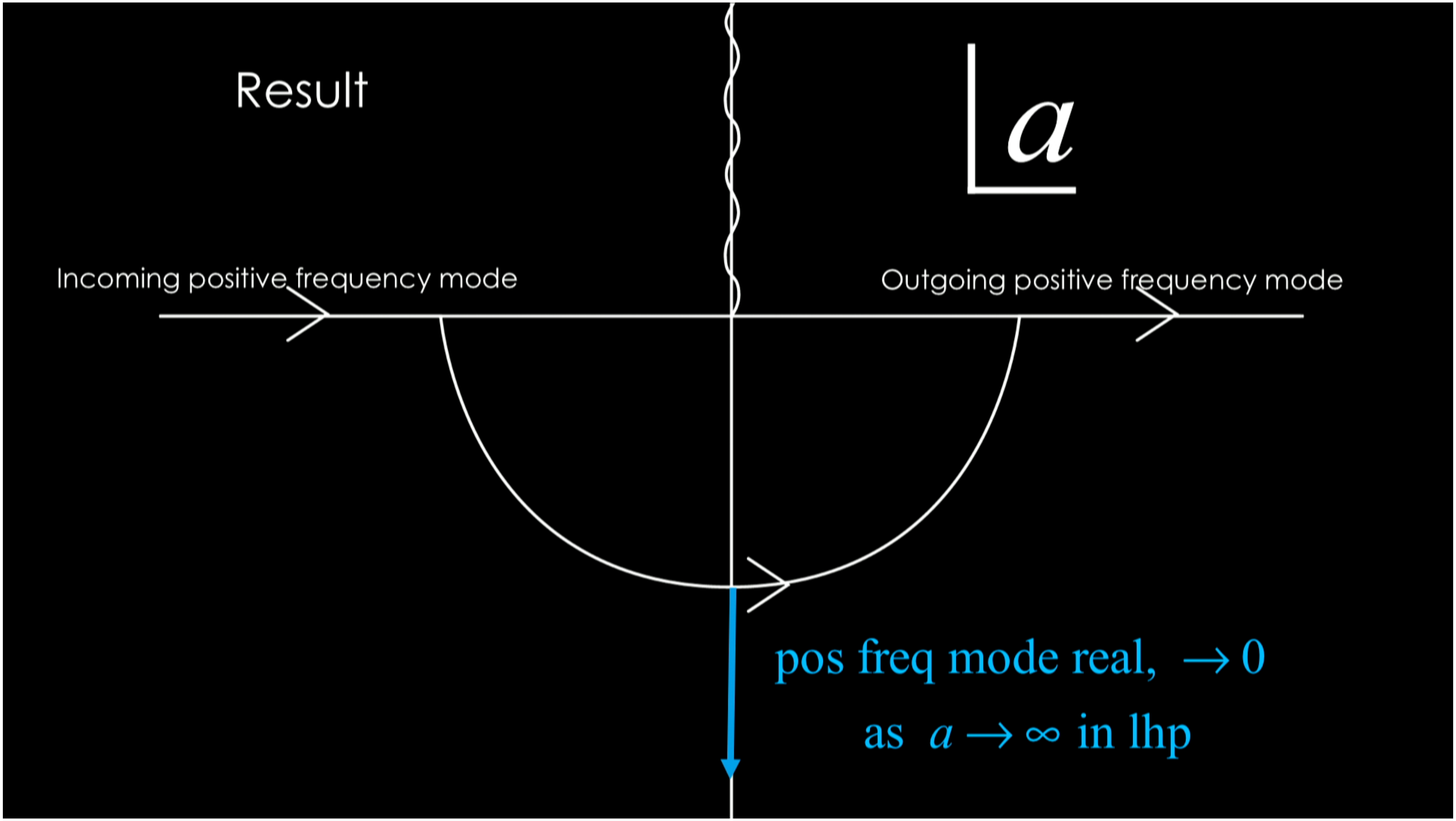
[Positive frequency: $\Psi^{(+)}$; Negative frequency: $\Psi^{(-)}$]

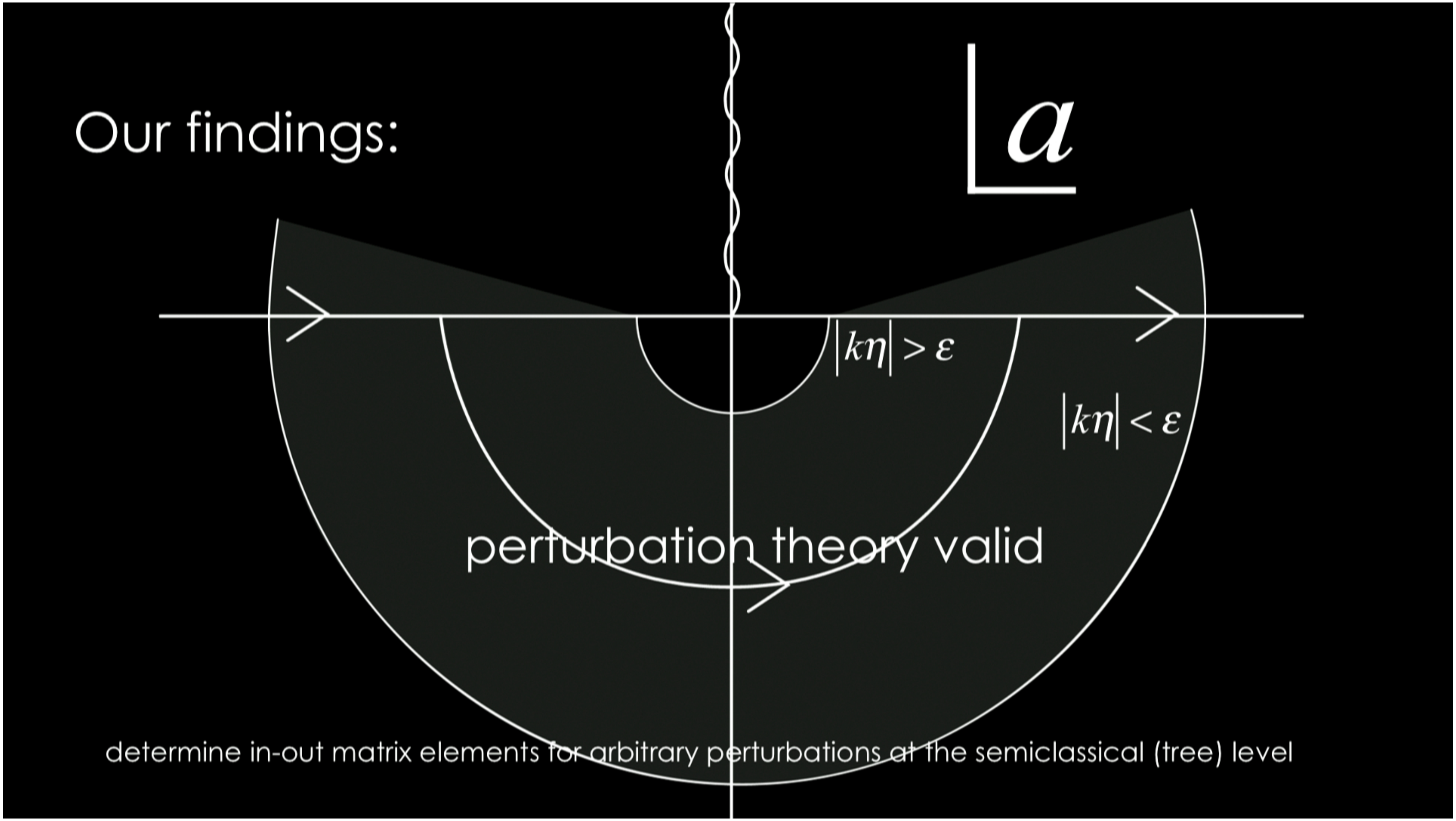


Generic inhomogeneous metric, in perturbation theory

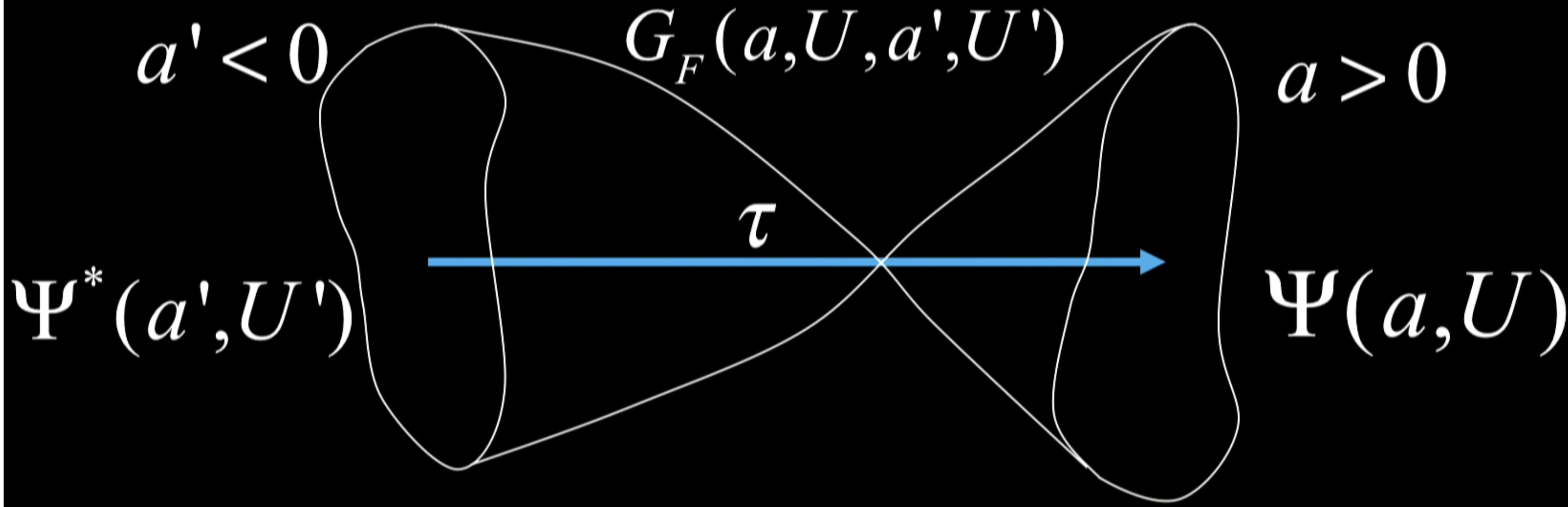
$$ds^2 = a^2(\eta)\begin{pmatrix} -1+2\epsilon\phi(\eta,x) & & & \\ & 1+2\epsilon(\psi(\eta,x)+\gamma(\eta,x)) & & \\ & & 1+\epsilon(2\psi(\eta,x)+\frac{1}{2}h^T(\eta,x)) & \frac{\epsilon}{2}h^\times(\eta,x) \\ & & \frac{\epsilon}{2}h^\times(\eta,x) & 1+\epsilon(2\psi(\eta,x)-\frac{1}{2}h^T(\eta,x)) \end{pmatrix}$$

$$\begin{aligned}
\frac{3}{\eta^2}\delta_r^{(n)} - \frac{6}{\eta^2}\phi^{(n)} + 2(\psi^{(n)})'' - \frac{2}{\eta}\dot\gamma^{(n)} - \frac{6}{\eta}\dot\psi^{(n)} &= J_1^{(n)} \\
\frac{1}{\eta}(\phi^{(n)})' + (\dot\psi^{(n)})' &= J_2^{(n)} \\
\frac{1}{\eta^2}\delta_r^{(n)} - \frac{2}{\eta^2}\phi^{(n)} + \frac{2}{\eta}\dot\phi^{(n)} + \frac{4}{\eta}\dot\psi^{(n)} + 2\ddot\psi^{(n)} &= J_3^{(n)} \\
\ddot h^{T(n)} + \frac{2}{\eta}\dot h^{T(n)} - (h^{T(n)})'' &= J_4^{(n)} \\
\ddot h^{\times(n)} + \frac{2}{\eta}\dot h^{\times(n)} - (h^{\times(n)})'' &= J_5^{(n)} \\
-\frac{1}{\eta^2}\delta_r^{(n)} + \frac{2}{\eta^2}\phi^{(n)} + (\psi^{(n)})'' - (\phi^{(n)})'' - \frac{2}{\eta}\dot\gamma^{(n)} - \frac{2}{\eta}\dot\phi^{(n)} - \frac{4}{\eta}\dot\psi^{(n)} - \ddot\gamma^{(n)} - 2\ddot\psi^{(n)} &= J_6^{(n)}
\end{aligned}$$


Result
a
Incoming positive frequency mode
Outgoing positive frequency mode
pos freq mode real, → 0
as a → ∞ in lhp


Our findings:
$\lfloor a$
$|k\eta| > \varepsilon$
$|k\eta| < \varepsilon$
perturbation theory valid
determine in-out matrix elements for arbitrary perturbations at the semiclassical (tree) level

Probability of a quantum state

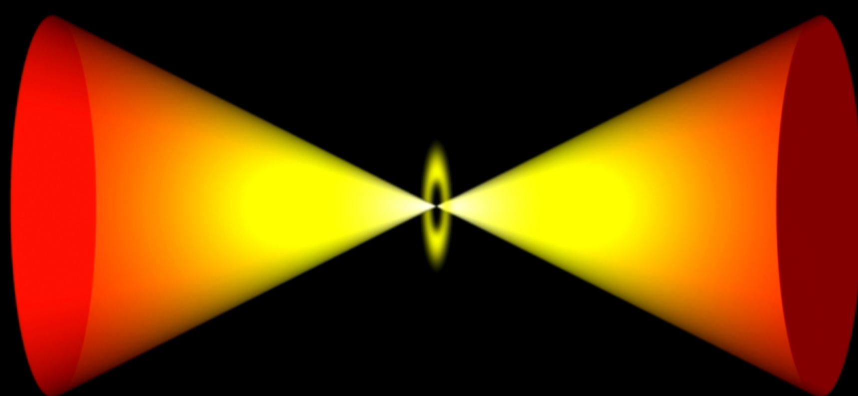


$$\Pr_\Psi = \Psi^*(a) \circ G(a,a') \circ \Psi(a'); \text{ where } \circ = \text{DeWitt inner product}$$

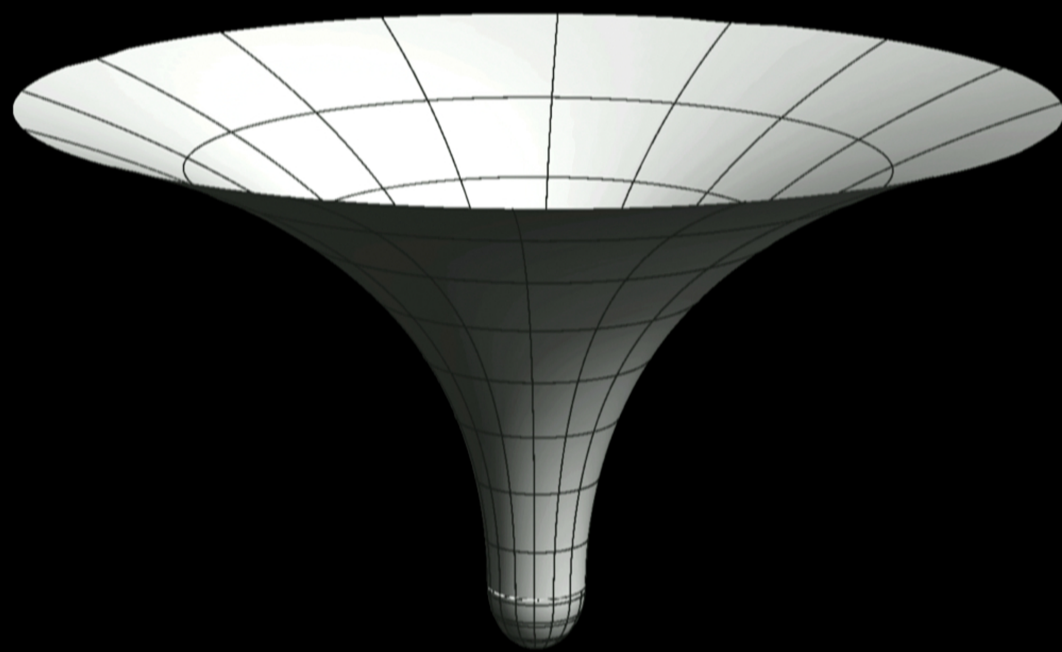
This norm is conserved *and* positive
It incorporates the big bang

Positive frequency condition is *global*, and strongly constrains the outgoing state. Nevertheless there are interesting opportunities for post-selection

This suggests a new proposal for the quantum state of the Universe, i.e., for understanding how the laws of physics might determine their own initial conditions



Compare with Hartle and Hawking's no boundary proposal



thank you

summary

We have shown how a quantum universe filled with conformal matter propagates through a bounce

Can treat generic inhomogenous metrics in cosmological perturbation theory, at linear and nonlinear order

This suggests a promising proposal for the "quantum state of the universe"