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Abstract:

PHYSICAL REVIEW VOLUME 155, NUMBER 5 25 MARCH 1967

Charge Superselection Rule*

YAKIR AHARONOV AND LEONARD SUSSKIND
Belfer Graduate School of Science, Yeshiva University, New York, New York
(Received 25 August 1966)

Questions concerning superselection rules are considered. Two experiments are discussed. In the first, coherent superpositions of different angular-momentum states are constructed. In the second, coherent superpositions of states with different charge are constructed in complete analogy with the angular-momentum case. We suggest that, contrary to a widespread belief, interference may be possible between states with different charges.

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Quantum frames of reference

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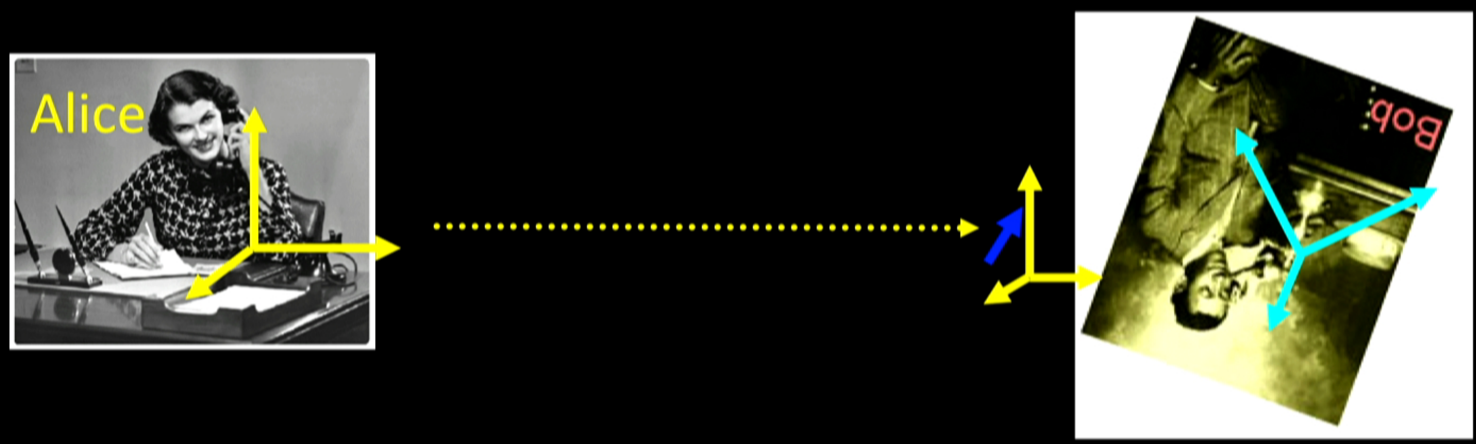
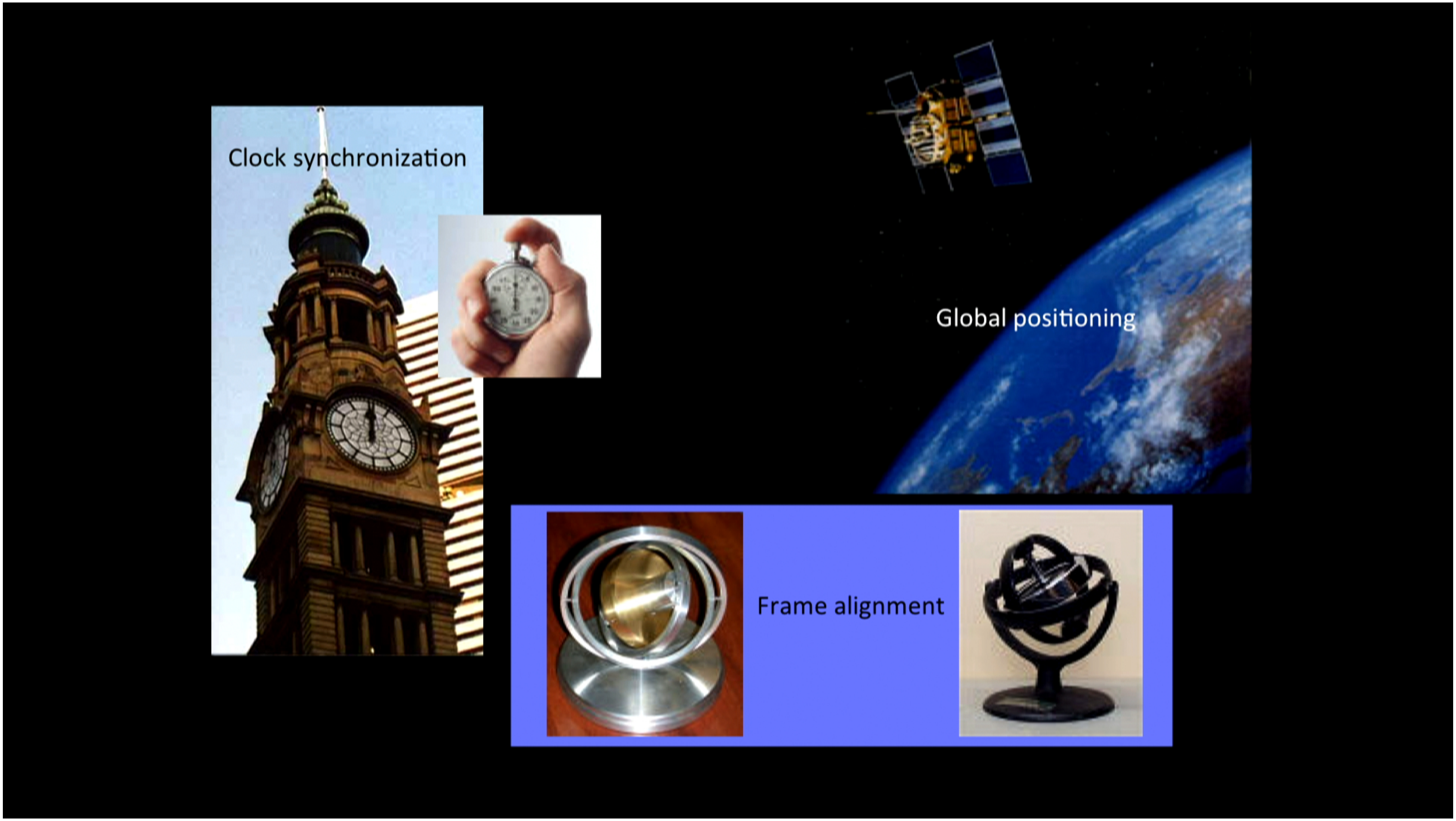
Frames of reference attached to quantum-mechanical objects of finite mass are considered. A consistent description of such frames is obtained which resolves a variety of apparent paradoxes associated with such a description. The main result of the present work is a formalism wherein the principle of equivalence is extended to reference frames described by quantum states.


Alice
Bob


Clock synchronization
Global positioning
Frame alignment

REVIEWS OF MODERN PHYSICS, VOLUME 79, APRIL–JUNE 2007

Reference frames, superselection rules, and quantum information

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(Published 5 April 2007)

Recently, there has been much interest in a new kind of "unspeakable" quantum information that stands to regular quantum information in the same way that a direction in space or a moment in time stands to a classical bit string: the former can only be encoded using particular degrees of freedom while the latter are indifferent to the physical nature of the information carriers. The problem of correlating distant reference frames, of which aligning Cartesian axes and synchronizing clocks are important instances, is an example of a task that requires the exchange of unspeakable information and for which it is interesting to determine the fundamental quantum limit of efficiency. There have also been many investigations into the information theory that is appropriate for parties that lack reference frames or that lack correlation between their reference frames, restrictions that result in global and local superselection rules. In the presence of these, quantum unspeakable information becomes a new kind of resource that can be manipulated, depleted, quantified, etc. Methods have also been developed to contend with these restrictions using relational encodings, particularly in the context of computation, cryptography, communication, and the manipulation of entanglement. This paper reviews the role of reference frames and superselection rules in the theory of

Quantum resource theories

	Entanglement	Asymmetry
Restriction	Local Operations and classical communication (No quantum channel)	Symmetric operations (No reference frame)
Resource	Entangled states	Asymmetric states

The resource theory of quantum reference frames: manipulations and monotones

Gilad Gour[1] **and Robert W Spekkens**[2]

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Abstract. Every restriction on quantum operations defines a resource theory, determining how quantum states that cannot be prepared under the restriction may be manipulated and used to circumvent the restriction. A superselection rule (SSR) is a restriction that arises through the lack of a classical reference frame and the states that circumvent it (the resource) are quantum reference frames. We consider the resource theories that arise from three types of SSRs, associated respectively with lacking: (i) a phase reference, (ii) a frame for chirality, and (iii) a frame for spatial orientation. Focusing on pure unipartite quantum states

The resource theory of quantum reference frames: manipulations and monotones

Gilad Gour[1] and Robert W Spekkens[2]
[1] Institute for Quantum Information Science and Department of Mathematics

The theory of manipulations of pure state asymmetry: I. Basic tools, equivalence classes and single copy transformations

Iman Marvian[1,2,3] and Robert W Spekkens[1]
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University

nature COMMUNICATIONS

ARTICLE
Received 9 Dec 2013 | Accepted 7 Apr 2014 | Published 13 May 2014
DOI: 10.1038/ncomms4821

Extending Noether's theorem by quantifying the asymmetry of quantum states

Iman Marvian[1,2,3] & Robert W. Spekkens[1]

he final state of
ıs broken by the
than that of the
consequences of

Pierre Curie
(1859 –1906)

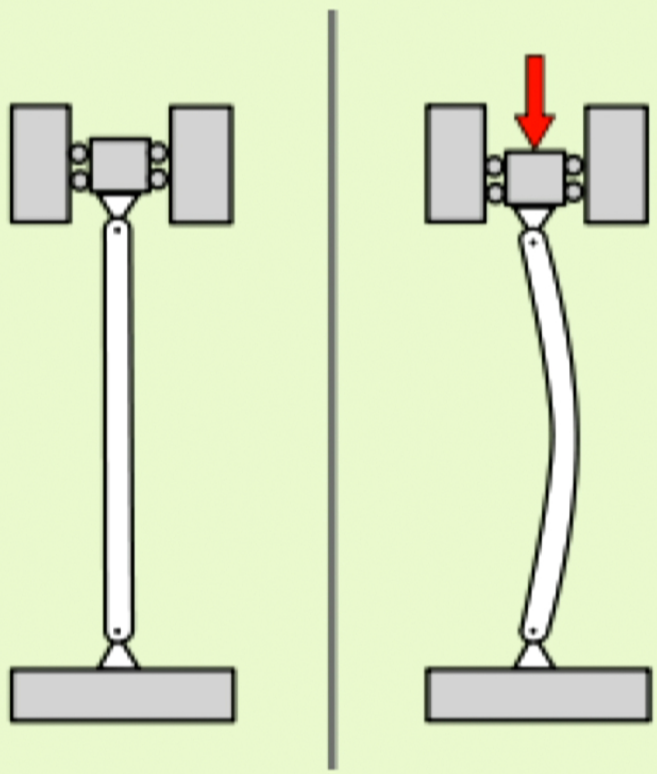
Curie's principle

Any asymmetry in a physical effect must be found in its causes

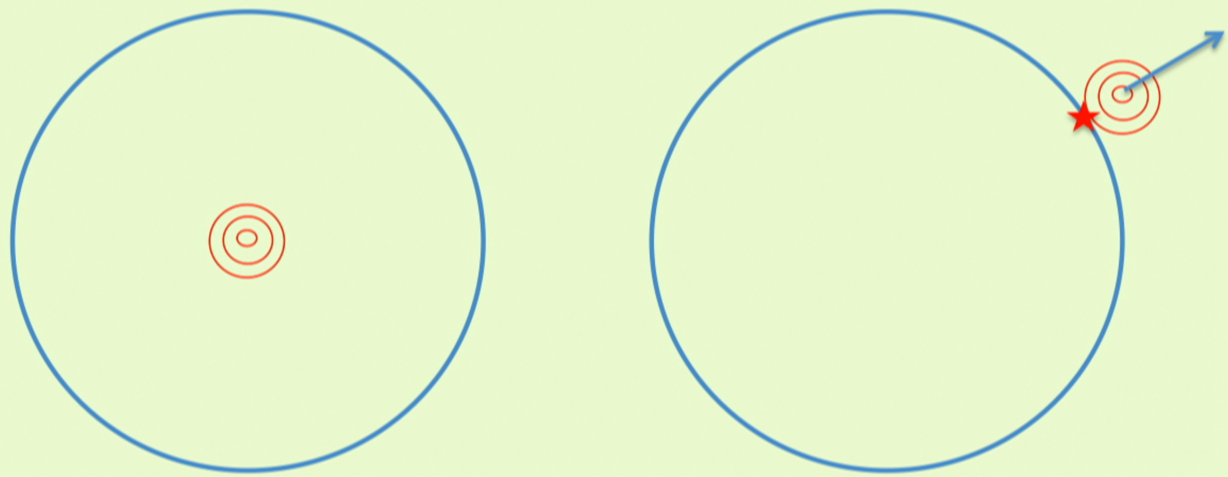
A quantitative version of Curie's principle

The measure of asymmetry in a physical effect cannot be greater than the measure of asymmetry in its cause

Violation of Curie's principle?



Violation of Curie's principle?



Quantum resource theories

	Entanglement	Asymmetry
Restriction	Local Operations and classical communication (No quantum channel)	Symmetric operations (No reference frame)
Resource	Entangled states	Asymmetric states

Symmetric operations

A symmetry is defined by a group G
and a unitary representation $g \in G \to U(g)$

A symmetric operation is any completely-positive trace-preserving map $\mathcal{E}$ that commutes with the action of the group

$$\forall g \in G: \quad \mathcal{E}[U(g)\rho U^\dagger(g)] = U(g)\mathcal{E}[\rho]U^\dagger(g)$$

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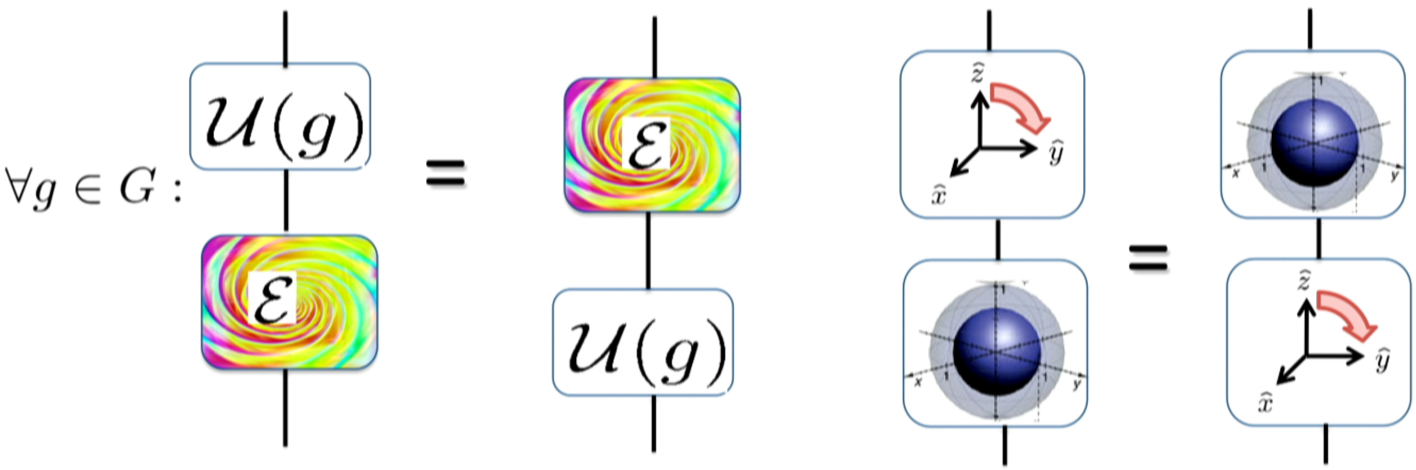
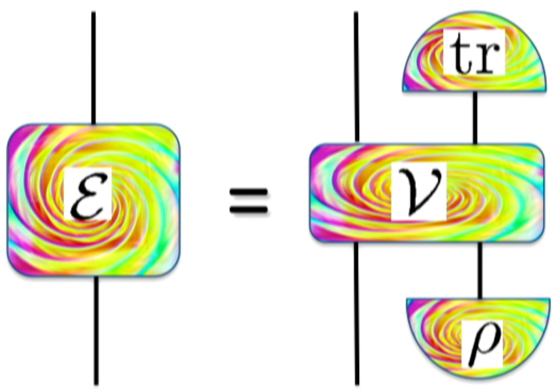


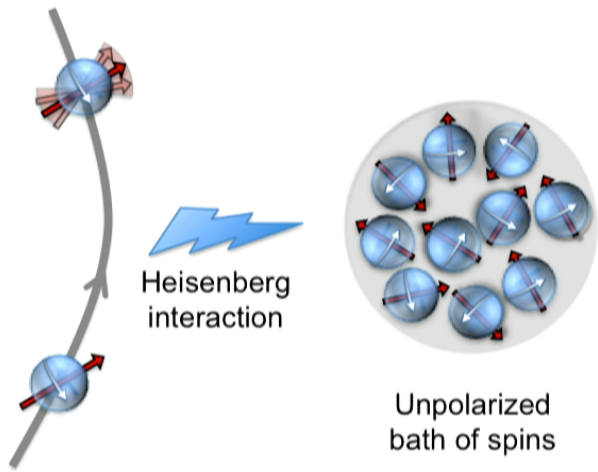
Every such operation can be achieved as follows:

$$\mathcal{E}(\rho_S) = \mathrm{tr}_{A'}\left(V_{SA}(\rho_S \otimes \rho_A)V_{SA}^{\dagger}\right)$$

ρ_A a symmetric state

V_{SA} a symmetric unitary





Noether's theorem

Suppose the symmetry transformation has *L* as a generator $U(g) = e^{i\theta L}$

$$[V, U(g)] = 0 \; \forall g \in G \Rightarrow [V, L] = 0$$

$$\rho \overset{\text{sym}}{\longleftrightarrow} \sigma \quad \Rightarrow \quad tr(\rho L^k) = tr(\sigma L^k) \;\; \forall k \in \mathbb{N}$$

Symmetry	Conservation law
Rotational	Angular momentum
Spatial Translation	Linear momentum
Time translation	Energy
Phase shift	Number

Two deficiencies:

- It is silent about open-system dynamics
- Even for closed-system dynamics, it does not capture all of the consequences of symmetry

Measures of asymmetry

Def'n: A function *A* from states to the reals is a measure of asymmetry if $\rho \xrightarrow{sym} \sigma \quad \Rightarrow \quad A(\rho) \geq A(\sigma)$

Open-system dynamics (irreversible): every measure yields a **monotonicity constraint**

Closed-system dynamics (reversible) : every measure yields a **conservation law**

Are the Noether conserved quantities $tr(\rho L^k)$ nontrivial measures of asymmetry? NO!

$$U(\theta) = e^{i\theta L} \qquad \rho = \sum_l |c_l|^2 |l\rangle\langle l| \qquad \sigma = \sum_{l,l'} c_l c_{l'}^* |l\rangle\langle l'|$$

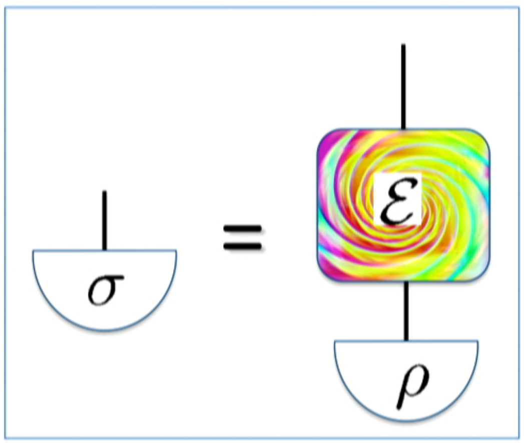
The difference is not seen by any fn' of the Noether quantities

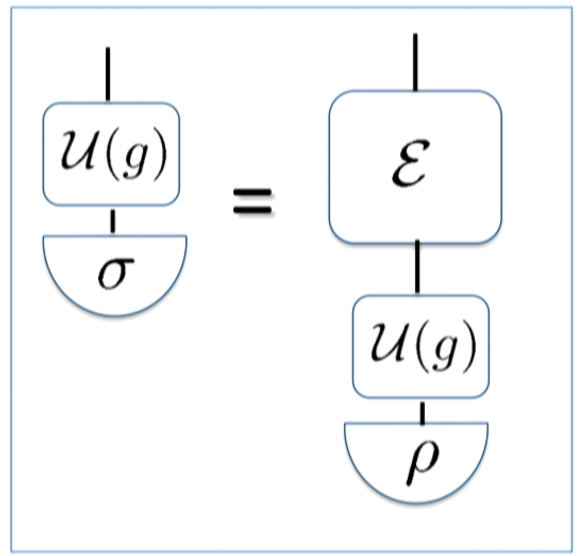
$$\forall k \in \mathbb{N}: \quad tr(\rho L^k) = tr(\sigma L^k)$$

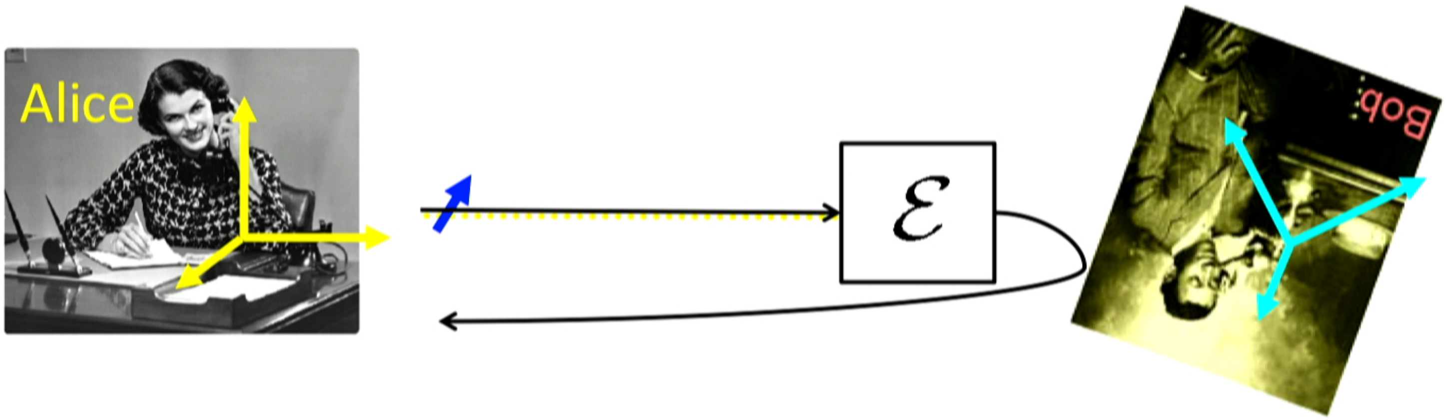
How can we find measures of asymmetry?

Bridge lemma:

$$\rho \xrightarrow{sym} \sigma \quad \overset{\Rightarrow}{\Leftarrow} \quad \forall g \in G : \; U(g)\rho U^\dagger(g) \rightarrow U(g)\sigma U^\dagger(g)$$

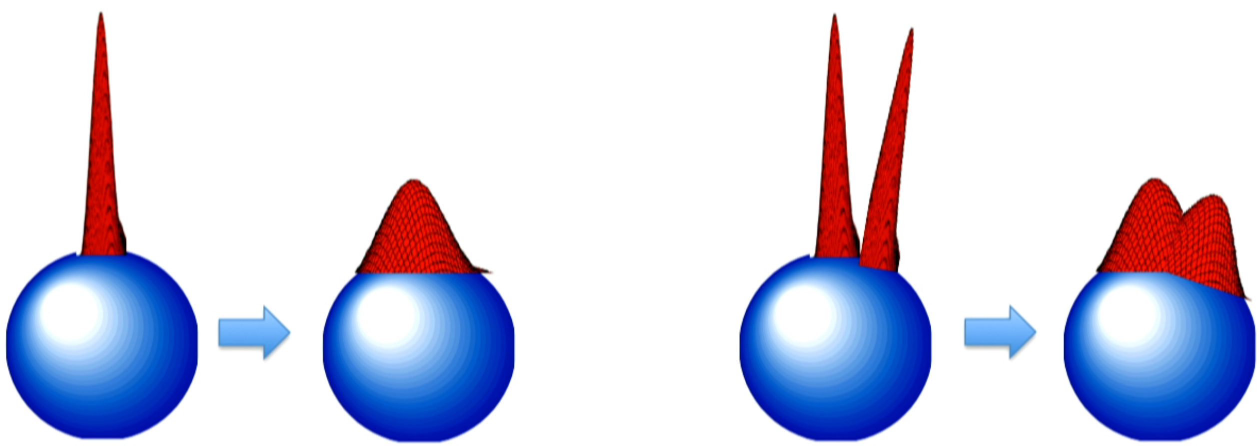





Alice
$\mathcal{E}$
Bob

$$\rho \xrightarrow{sym} \sigma$$

$$\forall g \in G :$$
$$U(g)\rho U^\dagger(g) \to U(g)\sigma U^\dagger(g)$$



Holevo asymmetry

$$A_p^{\mathrm{Hol}}(\rho) \equiv S(\mathcal{G}_p(\rho)) - S(\rho)$$

$$\mathcal{G}_p(\rho) \equiv \int dg\ p(g)\ U(g)\rho U^\dagger(g)$$

$$S(\rho) \equiv -\mathrm{tr}(\rho \log \rho)$$

l_1-norm-based asymmetry

$$A_L^{l_1}(\rho) \equiv \|[\rho, L]\|_1$$

$$\|A\|_1 \equiv \mathrm{tr}(\sqrt{A^\dagger A})$$

Wigner-Yanase-Dyson skew information

$$A_{L,s}^{\mathrm{skew}}(\rho) \equiv \mathrm{tr}(\rho L^2) - \mathrm{tr}(\rho^s L \rho^{1-s} L)$$

$$s \in (0,1) \cup (1,\infty)$$

Marvian and RWS, Nature Commun. 5, 3821 (2014)

Symmetric operations

$$\rho_1 \to \rho_2,\quad \rho_1 \to \rho_3$$

$$\rho_2 \to \rho_4,\quad \rho_3 \to \rho_4$$

$$\rho_4 \to \rho_5$$

Data processing

$$\{U(g)\rho_1 U^\dagger(g)\} \to \{U(g)\rho_2 U^\dagger(g)\},\quad \{U(g)\rho_1 U^\dagger(g)\} \to \{U(g)\rho_3 U^\dagger(g)\}$$

$$\{U(g)\rho_2 U^\dagger(g)\} \to \{U(g)\rho_4 U^\dagger(g)\},\quad \{U(g)\rho_3 U^\dagger(g)\} \to \{U(g)\rho_4 U^\dagger(g)\}$$

$$\{U(g)\rho_4 U^\dagger(g)\} \to \{U(g)\rho_5 U^\dagger(g)\}$$

Rehabilitated version of Noether's theorem

$$\rho \xrightarrow{sym} \sigma \quad \substack{\Rightarrow \\ \Leftarrow} \quad$$ Finite set of conditions on ρ and σ

So far, only solved for the case of pure states

$$|\psi\rangle \xrightarrow{sym} |\phi\rangle \quad \substack{\Rightarrow \\ \Leftarrow} \quad \forall g \in \mathrm{G} : \langle\psi|U(g)|\psi\rangle = e^{i\theta(g)}\langle\phi|U(g)|\phi\rangle$$

I. Marvian and RWS, New J. Phys. 15, 033001 (2013)

Symmetric operations

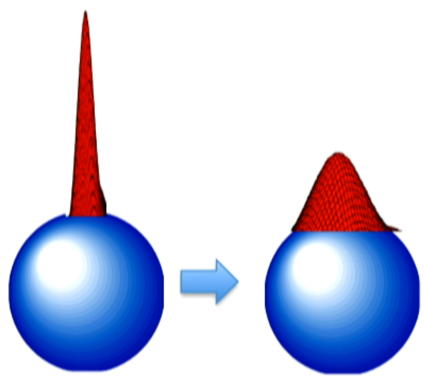
$$\rho_1 \to \rho_2,\ \rho_3 \to \rho_4 \to \rho_5$$

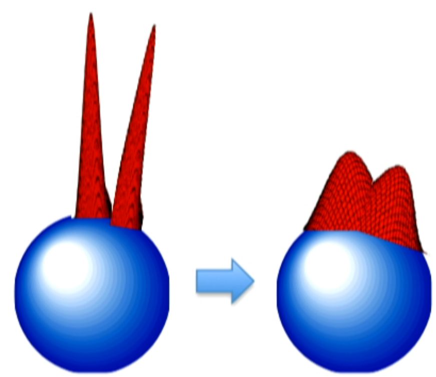
Data processing

$$\{U(g)\rho_1 U^\dagger(g)\} \to \{U(g)\rho_2 U^\dagger(g)\},\ \{U(g)\rho_3 U^\dagger(g)\} \to \{U(g)\rho_4 U^\dagger(g)\} \to \{U(g)\rho_5 U^\dagger(g)\}$$

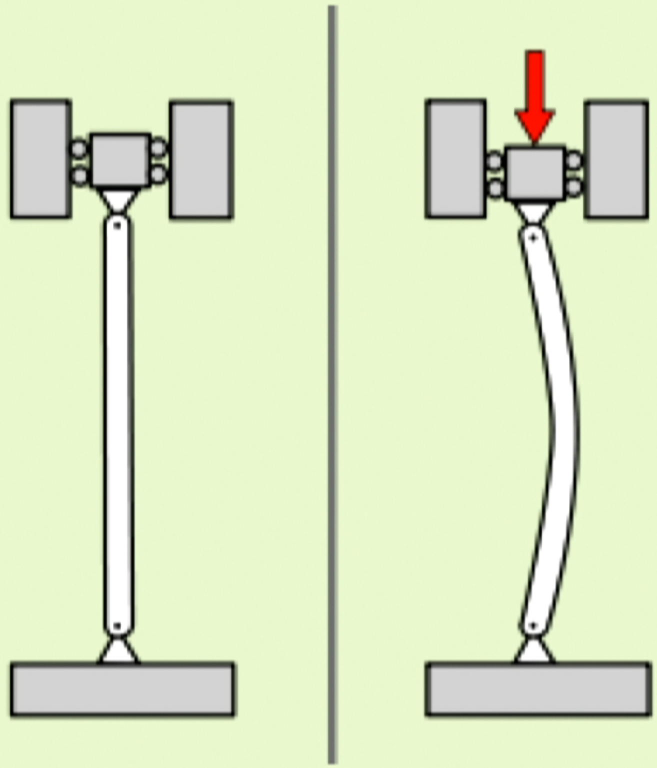
No increase of asymmetry under symmetric processing

No increase of information under data processing

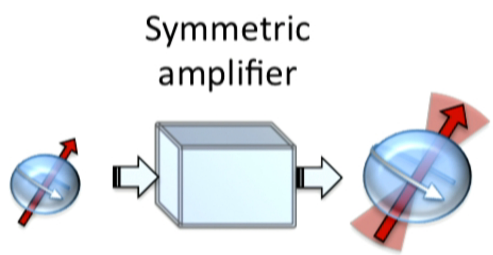




Violation of Curie's principle?



Fundamental noise limits in quantum amplifiers

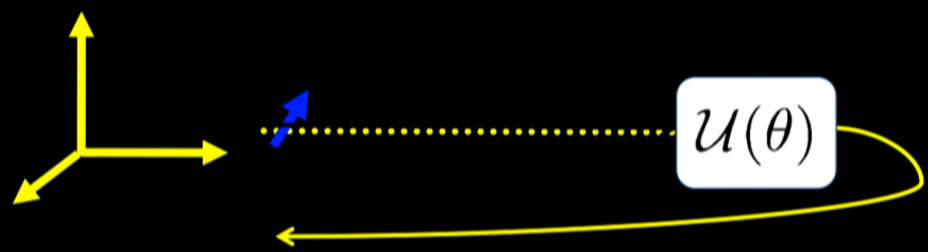


$$\rho \xrightarrow{sym} \sigma \quad \Rightarrow \quad S(\mathcal{G}_p[\rho]) - S(\rho) \;\geq\; S(\mathcal{G}_p[\sigma]) - S(\sigma)$$

$$S(\sigma) - S(\rho) \;\geq\; S(\mathcal{G}_p[\sigma]) - S(\mathcal{G}_p[\rho])$$

Marvian and RWS, Nature Commun. 5, 3821 (2014)

Quantum Metrology



Variance in unbiased estimator of a phase θ $\leq \dfrac{1}{4\left(\mathrm{tr}(\rho L^2)-\mathrm{tr}(\rho^{1/2}L\rho^{1/2}L)\right)}$

Marvian and RWS, Nature Commun. 5, 3821 (2014)

Quantum speed limits

L. I. Mandelstam and I. E. Tamm, J. Phys. (USSR) 9, 249 (1945)
N. Margolus and L. B. Levitin, Physica D 120, 188 (1998)
J. Anandan and Y. Aharonov, Phys. Rev. Lett. 65, 1697 (1990)

Mandelstam-Tamm bound

$$\tau_\perp(\rho) \geq \frac{\pi}{2\Delta E(\rho)}$$

$$\Delta E(\rho) = \mathrm{tr}(\rho H^2) - \mathrm{tr}^2(\rho H)$$

Margolus-Levitin bound

$$\tau_\perp(\rho) \geq \frac{\pi}{2[E(\rho) - E_{\min}(\rho)]}$$

$$E(\rho) = \mathrm{tr}(\rho H)$$
$$E_{\min}(\rho) = \lambda_{\min}(H) \;\; \text{on } \mathrm{supp}(\rho)$$

Define the speed of evolution as a measure of time-translation asymmetry:

Minimum time t such that ρ(t) is ϵ-distinguishable from ρ(0) relative to measure of distinguishability D

$$\tau_\epsilon^D(\rho) \equiv \begin{cases} \infty \ , & \text{if} \ \ \forall t \in \mathbb{R}^+ : \quad D\left(\rho, \rho(t)\right) < \epsilon \\ \min\{t : t \in \mathbb{R}^+ \ , \ D\left(\rho, \rho(t)\right) \geq \epsilon\} \ , & \text{otherwise.} \end{cases}$$

Quantum speed limits from measures of time-translation asymmetry

$$\tau_{\epsilon}^{l_1}(\rho) \geq \frac{\epsilon}{\|[H,\rho]\|_1}$$

$$\tau_{\epsilon}^{\mathrm{Ren}}(\rho) \geq \frac{\sqrt{1-e^{-\frac{\epsilon}{2}}}}{\sqrt{\mathrm{tr}(\rho H^2)-\mathrm{tr}(\sqrt{\rho}H\sqrt{\rho}H)}}$$

Marvian, RWS, and Zanardi, Phys. Rev. A 93, 052331 (2016)

Quantum speed limits
from measures of time-translation asymmetry

$$\tau_\epsilon^{l_1}(\rho) \geq \frac{\epsilon}{\|[H,\rho]\|_1}$$

$$\|[H,\rho]\|_1 = \left[\frac{d}{dt}\|\rho - \rho(t)\|_1\right]_{t=0}$$

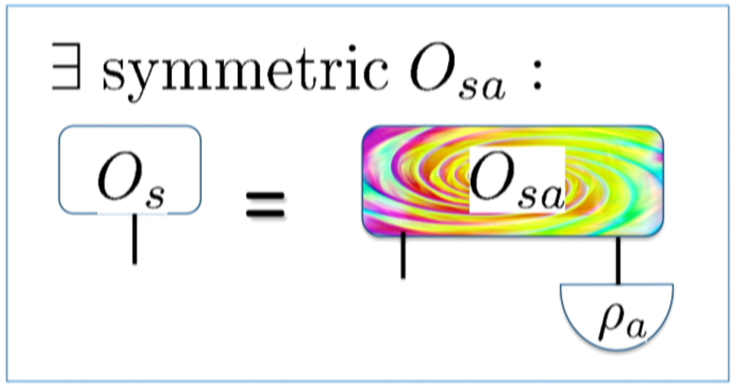
$$\tau_\epsilon^{\mathrm{Ren}}(\rho) \geq \frac{\sqrt{1-e^{-\frac{\epsilon}{2}}}}{\sqrt{\mathrm{tr}(\rho H^2)-\mathrm{tr}(\sqrt{\rho}H\sqrt{\rho}H)}}$$

$$\mathrm{tr}(\rho H^2) - \mathrm{tr}(\sqrt{\rho}H\sqrt{\rho}H) = \frac{1}{4}\left[\frac{d^2}{dt^2}D_{\mathrm{Ren}}(\rho,\rho(t))\right]_{t=0}$$

Marvian, RWS, and Zanardi, Phys. Rev. A 93, 052331 (2016)

Limits to simulating asymmetric measurements

$$\rho_a \xrightarrow{\text{sym}} O_s$$

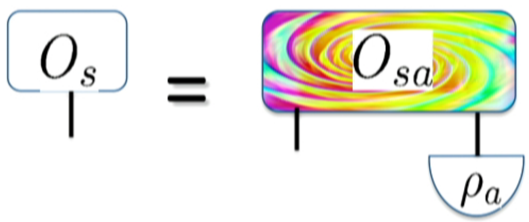


$$\forall g \in G :$$

$$U_a(g)\rho_a U_a^\dagger(g) \rightarrow U_s(g) O_s U_s^\dagger(g)$$

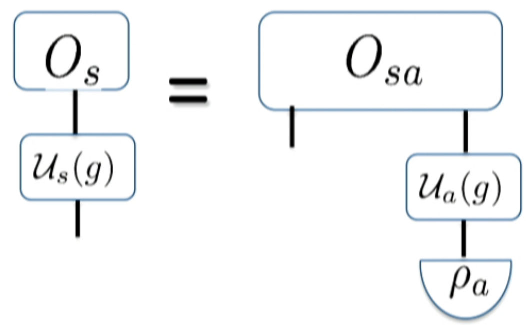
$$\rho_a \xrightarrow{\text{sym}} O_s$$

$\exists$ symmetric O_{sa} :



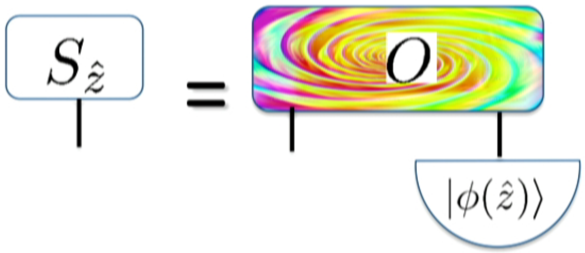
$\Rightarrow$
$\Leftarrow$

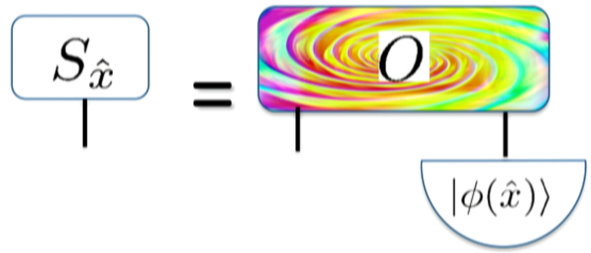
$\exists O_{sa}$: $\forall g \in G$:

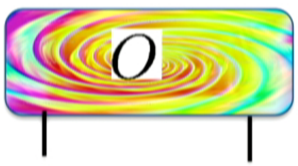


The no-programming theorem for measurements

M. Dusek and V. Buzek, Phys. Rev. A 66, 022112 (2002)
G. M. DAriano and P. Perinotti, Phys. Rev. Lett. 94, 90401 (2005)







must be able to perfectly discriminate

$$\{|+\hat{z}\rangle|\phi(\hat{z})\rangle, |+\hat{x}\rangle|\phi(\hat{x})\rangle\}$$

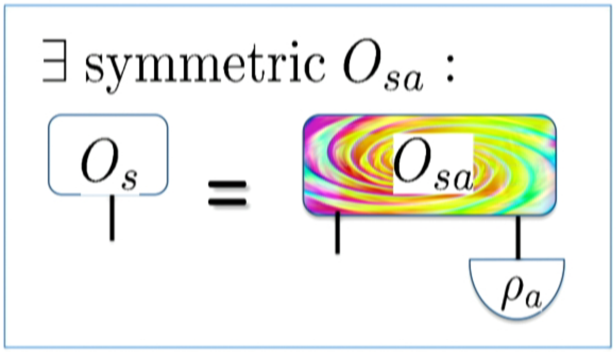
from

$$\{|-\hat{z}\rangle|\phi(\hat{z})\rangle, |-\hat{x}\rangle|\phi(\hat{x})\rangle\}$$

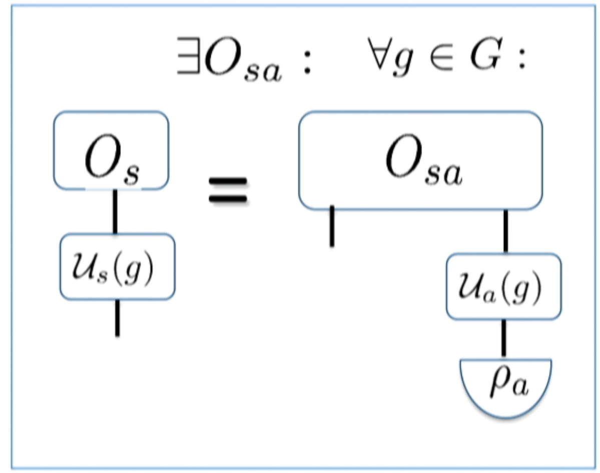
and this requires $\langle\phi(\hat{z})|\phi(\hat{x})\rangle = 0$

$$\rho_a \xrightarrow{\text{sym}} O_s$$

$$\forall g \in G : U_a(g)\rho_a U_a^\dagger(g) \to U_s(g) O_s U_s^\dagger(g)$$



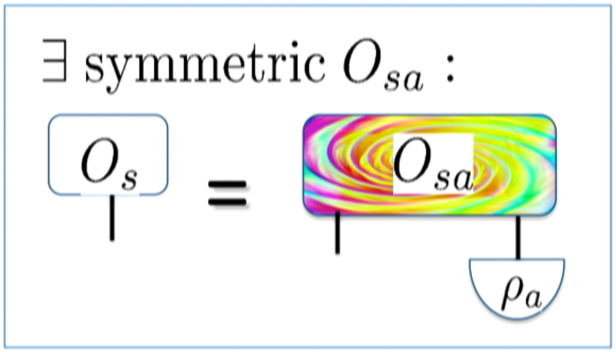
$$\overset{\Rightarrow}{\Leftarrow}$$



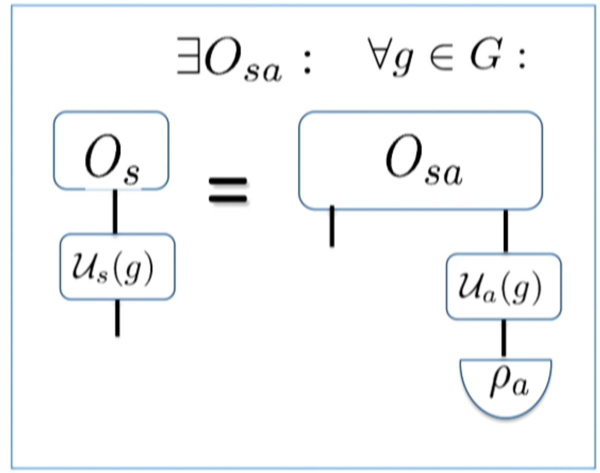
Requires classical reference frame ← Requires classical encoding of g

$$\rho_a \xrightarrow{\text{sym}} O_s$$

$$\forall g \in G : U_a(g)\rho_a U_a^\dagger(g) \to U_s(g) O_s U_s^\dagger(g)$$



$\Rightarrow$
$\Leftarrow$



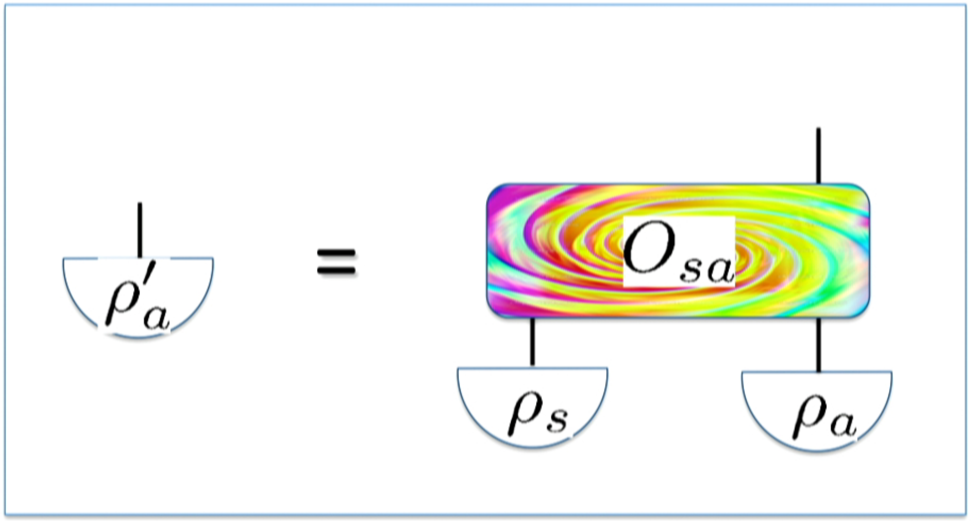
Requires classical reference frame

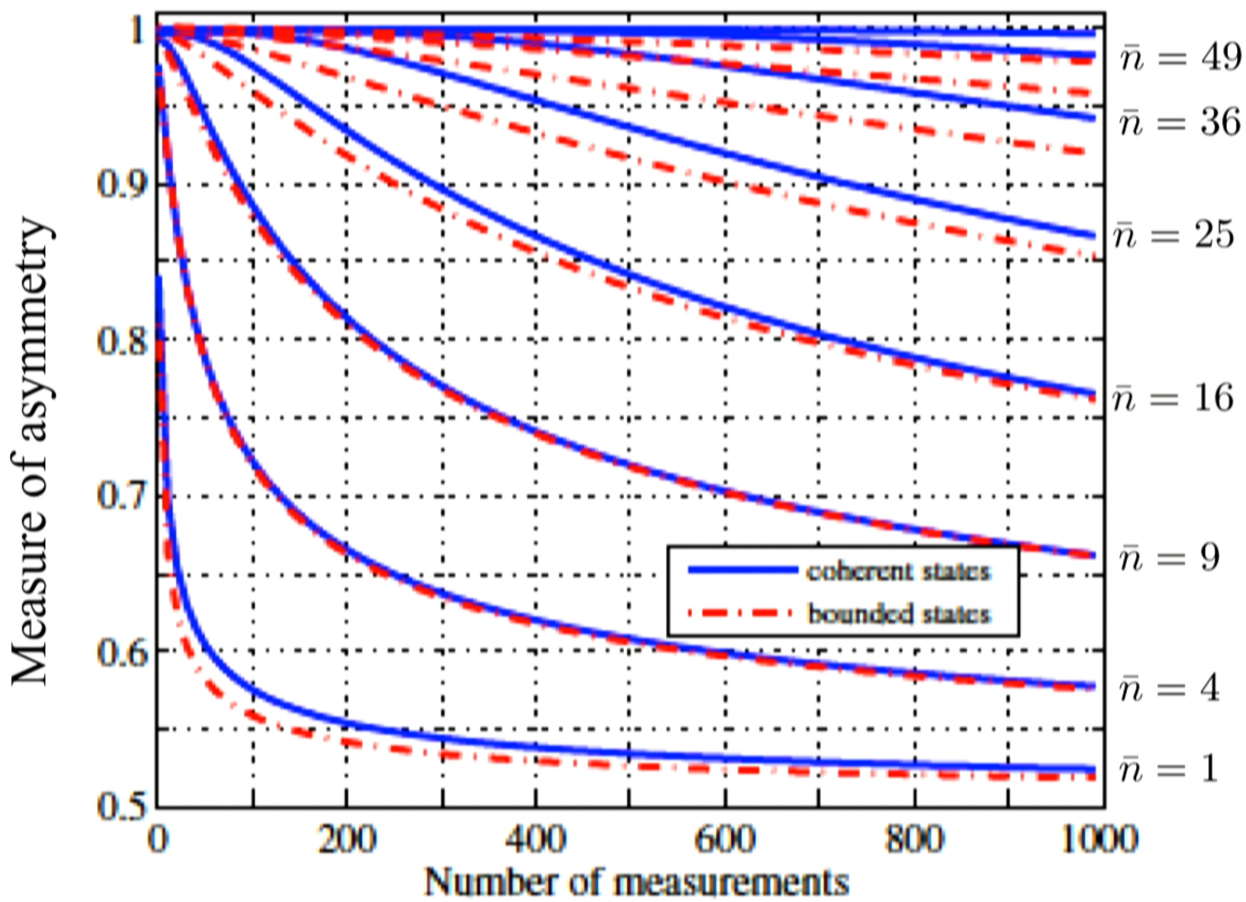
Requires classical encoding of g

This is a generalization of the Wigner-Araki-Yanase theorem

E. P. Wigner, Z. Phys. 133, 101 (1952)
H. Araki and M. M. Yanase, Phys. Rev. 120, 622 (1960)
Marvian and RWS, arXiv:1212.3378 (quant-ph)

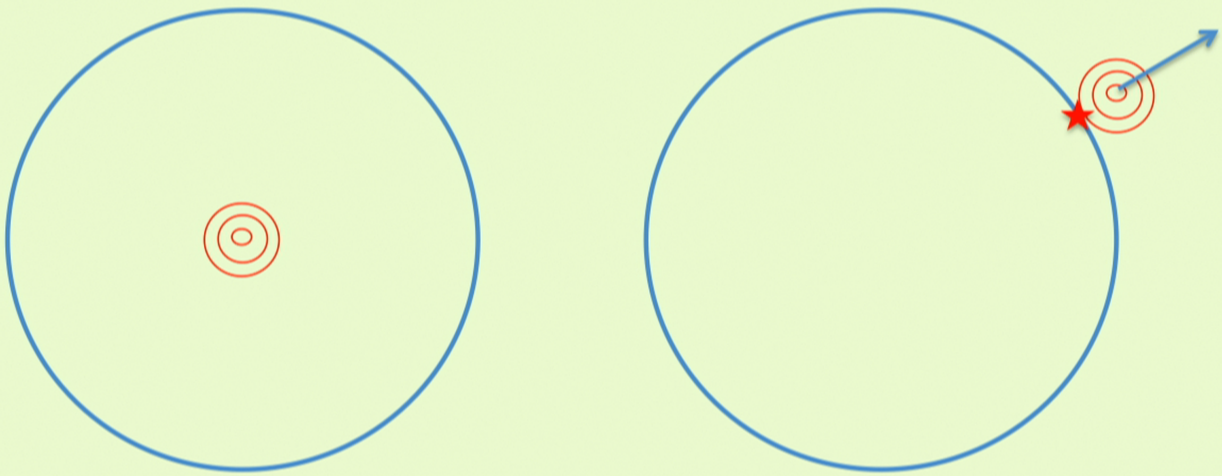
Degradation of quantum reference frames





Bartlett, Rudolph, RWS, Turner, New J. Phys. 8, 58 (2006)

Violation of Curie's principle?



Conclusions

The quality of a quantum reference frame is captured by its asymmetry properties.

The asymmetry of a state can be understood as its capacity to encode information about the symmetry group.

The quantitative Curie's principle ultimately follows from the principle that information cannot be increased by data processing.

The resource theory of asymmetry constitutes a useful tool for many problems