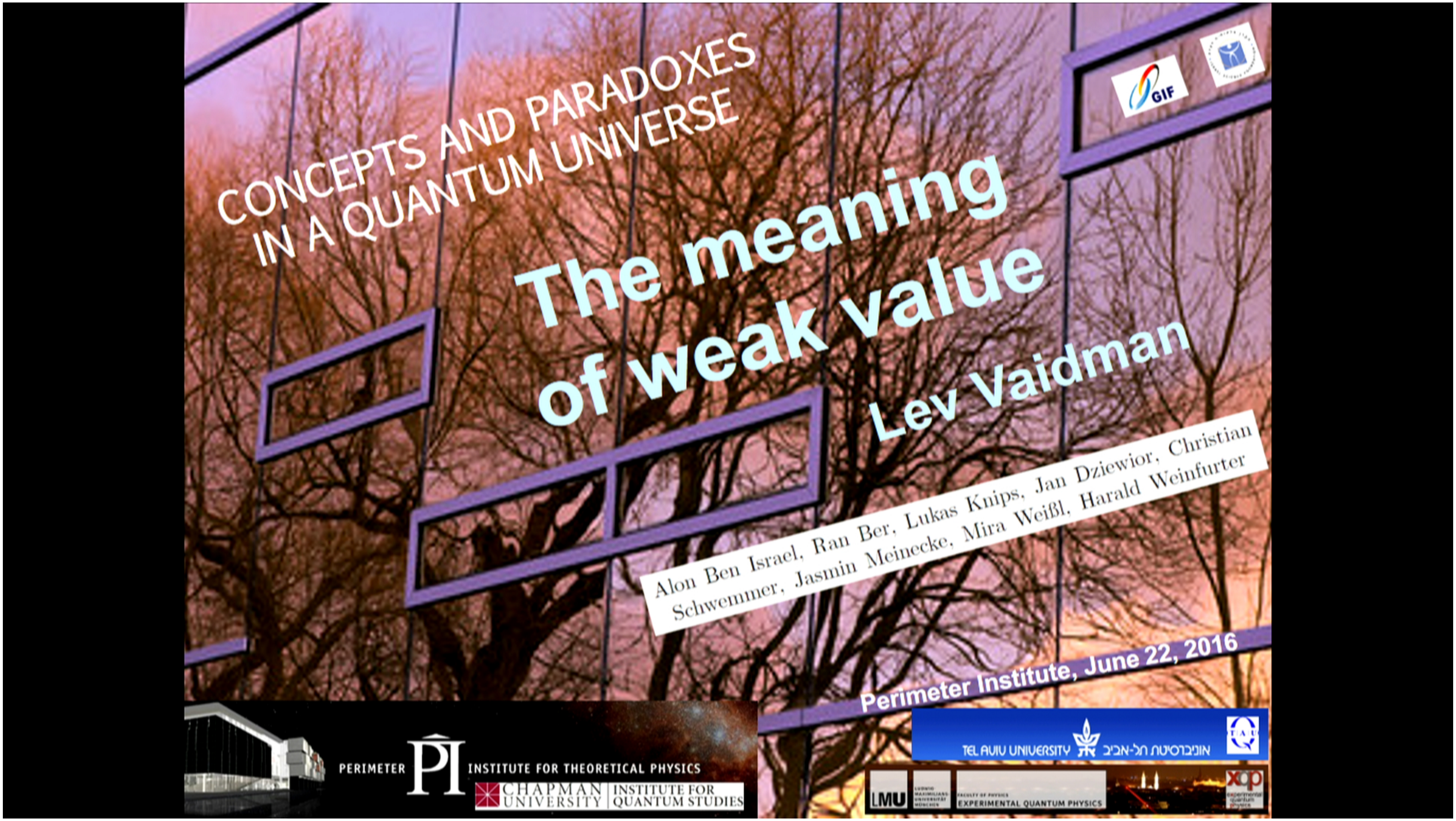
Title: The meaning of weak values

Date: Jun 22, 2016 11:45 AM

URL: http://pirsa.org/16060051

Abstract: The weak value, as an expectation value, requires an ensemble to be found. Nevertheless, we argue that the physical meaning of the weak value is much more close to the physical meaning of an eigenvalue than to the physical meaning of an expectation value. Theoretical analysis and experimental results performed in the MPQ laboratory of Harald Weinfurter are presented. Quantum systems described by numerically equal eigenvalue, weak value and expectation value cause identical average shift of an external system interacting with them during an infinitesimal time. However, there are differences between the final states of the external system. In the case of an eigenvalue, the shift is the only change in the wavefunction of the external system. In case of the expectation value, there is an additional change in the quantum state of the same order, while in the case of the weak value the additional distortion is negligible. The understanding of weak value as a property of a single system refutes recent claims that there exist classical statistical analogue to the weak value.


CONCEPTS AND PARADOXES
IN A QUANTUM UNIVERSE
The meaning
of weak value
Lev Vaidman
Alon Ben Israel, Ran Ber, Lukas Knips, Jan Dziewior, Christian
Schwemmer, Jasmin Meinecke, Mira Weißl, Harald Weinfurter
Perimeter Institute, June 22, 2016
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The meaning of weak value

What is “a value of a variable”?

Weak value controversy

Weak value as an outcome of weak measurements

Weak value as a property of a single system

Preliminary experimental results

A value of a physical observable C

C is classical $C = c$ Classical values always have definite values

---

C is quantum The system is pre-selected $|\Psi\rangle = |\Psi_k\rangle$

$C = c_k$ C has a definite value eigenvalue

C is quantum The system is pre-selected $|\Psi\rangle = \sum_k \alpha_k |\Psi_k\rangle$

$C = \langle C \rangle = \langle \Psi | C | \Psi \rangle$ $\langle C \rangle = \sum_k \mathrm{prob}(k)\, c_k = \sum_k |\alpha_k|^2 c_k$

C has no definite value $\langle C \rangle$ is a statistical expectation value

---

C is quantum The system is pre-selected $|\Psi\rangle$ and post-selected $|\Phi\rangle$

$C = C_w \equiv \dfrac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$ The system is described by the weak value

---

Is the weak value statistical as the expectation value or definite as the eigenvalue?

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

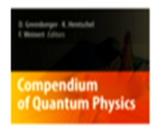
How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.



Compendium of Quantum Physics

Concepts, Experiments, History and Philosophy

Editors: **Greenberger**, Daniel, **Hentschel**, Klaus, **Weinert**, Friedel (Eds.)

Weak Value and Weak Measurements

Lev Vaidman

The weak value of a variable O is a description of an effective interaction with that variable in the limit of weak coupling. For a pre- and post-selected system described at time t by the two-state vector $\langle\Phi| \; |\Psi\rangle$ [1], the weak value is [2]:

$$O_w \equiv \frac{\langle\Phi|O|\Psi\rangle}{\langle\Phi|\Psi\rangle}. \quad (1)$$

Colloquium: Understanding quantum weak values: Basics and applications

Justin Dressel, Mehul Malik, Filippo M. Miatto, Andrew N. Jordan, and Robert W. Boyd
Rev. Mod. Phys. **86**, 307 – Published 28 March 2014

ABSTRACT

Since its introduction 25 years ago, the quantum weak value has gradually transitioned from a theoretical curiosity to a practical laboratory tool. While its utility is apparent in the recent explosion of weak value experiments, its interpretation has historically been a subject of confusion. Here a pragmatic introduction to the weak value in terms of measurable quantities is presented, along with an explanation for how it can be determined in the laboratory. Further, its application to three distinct experimental techniques is reviewed. First, as a large interaction parameter it can amplify small signals above technical background noise. Second, as a measurable complex value it enables novel techniques for direct quantum state and geometric phase determination. Third, as a conditioned average of generalized observable eigenvalues it provides a measurable window into nonclassical features of quantum mechanics. In this selective review, a single experimental configuration to discuss and clarify each of these applications is used.

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Quantum Physics

Weak values considered harmful

Christopher Ferrie, Joshua Combes

(Submitted on 15 Jul 2013 (this version), **latest version 22 Jan 2014** *(v3))*

For the task of parameter estimation, we show using statistically rigorous arguments that the process of postselection (a pre-requisite for so-called weak value amplification) can be no better on average than

PRL **112,** 040406 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Weak Value Amplification is Suboptimal for Estimation and Detection

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 25 July 2013; revised manuscript received 21 November 2013; published 31 January 2014)

We show by using statistically rigorous arguments that the technique of weak value amplification does not perform better than standard statistical techniques for the tasks of single parameter estimation and signal detection. Specifically, we prove that postselection, a necessary ingredient for weak value amplification, decreases estimation accuracy and, moreover, arranging for anomalously large weak values is a suboptimal strategy. In doing so, we explicitly provide the optimal estimator, which in turn allows us to identify the optimal experimental arrangement to be the one in which all outcomes have equal weak values (all as small as possible) and the initial state of the meter is the maximal eigenvalue of the square of the system observable. Finally, we give precise quantitative conditions for when weak measurement (measurements without postselection or anomalously large weak values) can mitigate the effect of uncharacterized technical noise in estimation.

arXiv:1402.0199 [pdf, ps, other]

Comment on "Weak value amplification is suboptimal for estimation and detection"
L.Vaidman

PRL **113,** 120404 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

How the Result of a Single Coin Toss Can Turn Out to be 100 Heads

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 16 March 2014; revised manuscript received 18 July 2014; published 18 September 2014)

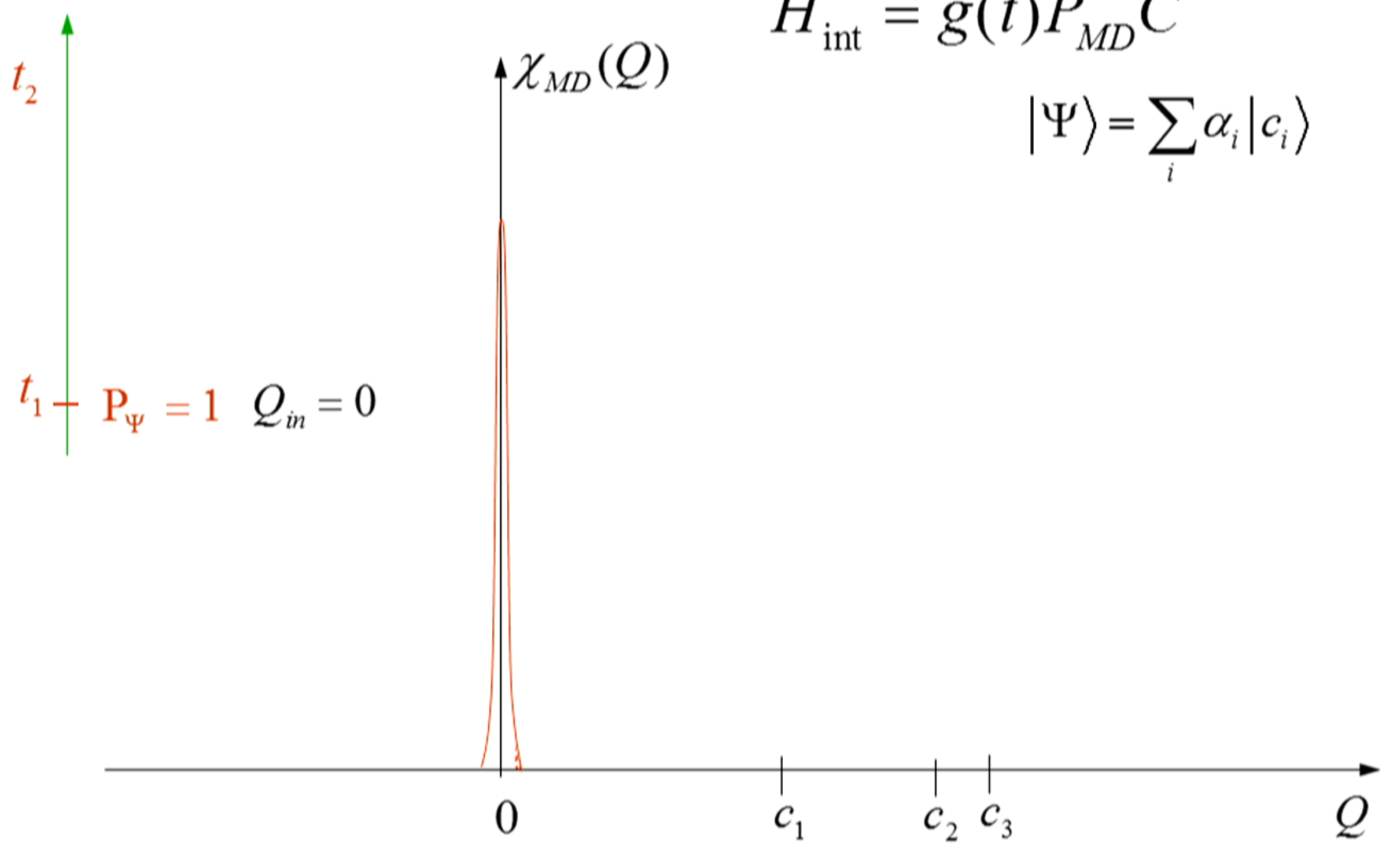
We show that the phenomenon of anomalous weak values is not limited to quantum theory. In particular, we show that the same features occur in a simple model of a coin subject to a form of classical backaction with pre- and postselection. This provides evidence that weak values are not inherently quantum but rather a purely statistical feature of pre- and postselection with disturbance.

Weak value as an outcome of a weak measurement

Quantum measurement of C

$$H_{\text{int}} = g(t) P_{MD} C$$

$$|\Psi\rangle = \sum_i \alpha_i |c_i\rangle$$



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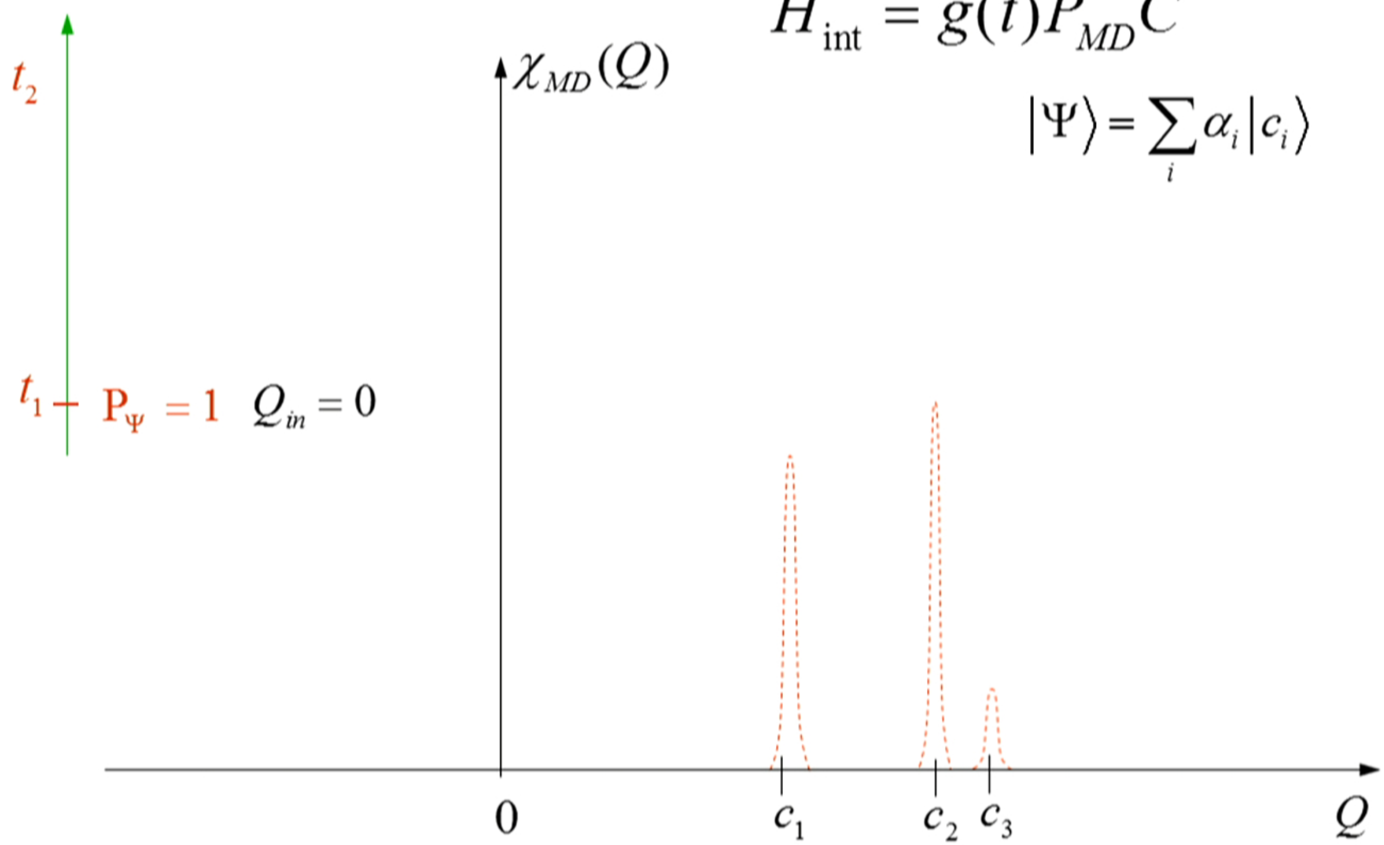

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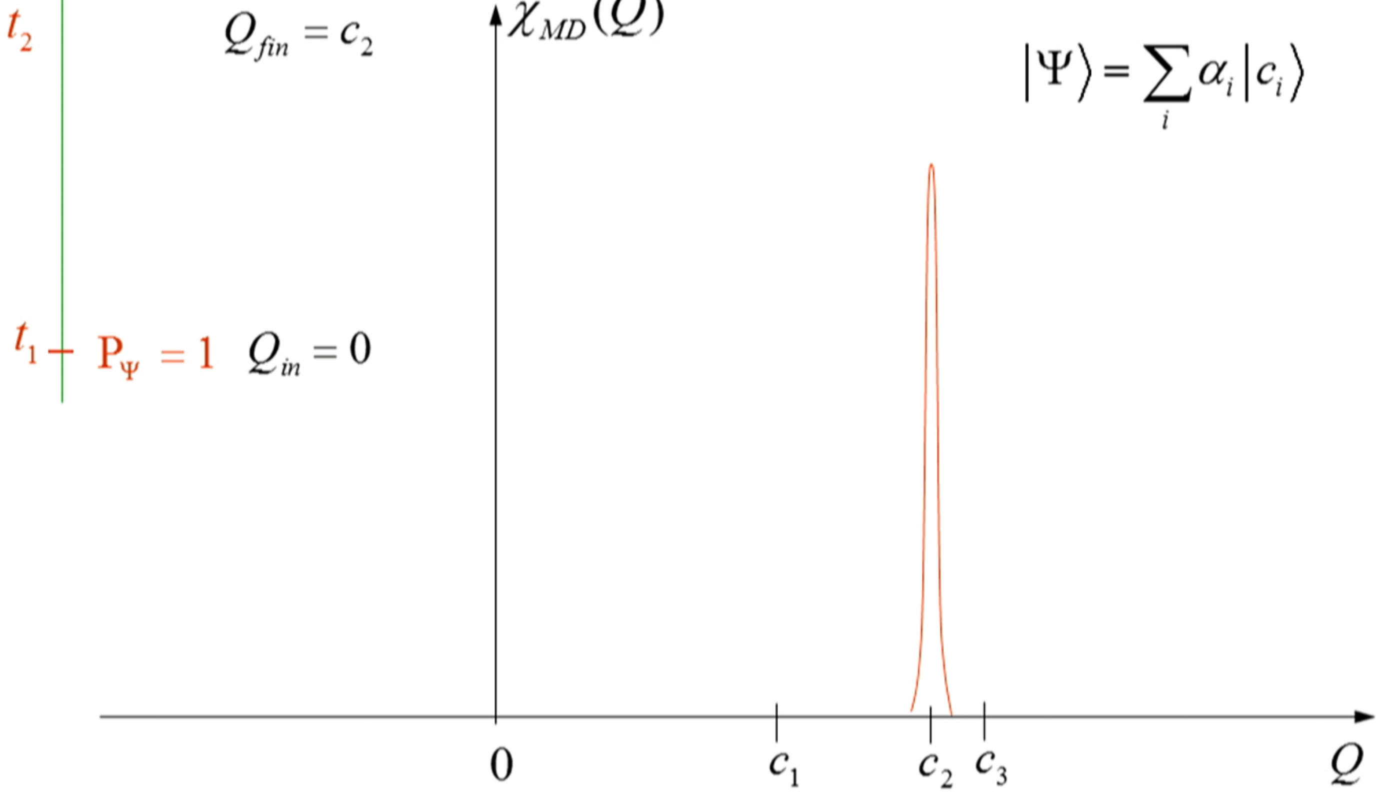

$$|\Psi\rangle |Q=0\rangle \rightarrow \sum_i \alpha_i |c_i\rangle |Q=c_i\rangle \xrightarrow{\text{Collapse!}} |c_2\rangle |Q=c_2\rangle$$

Quantum measurement of C

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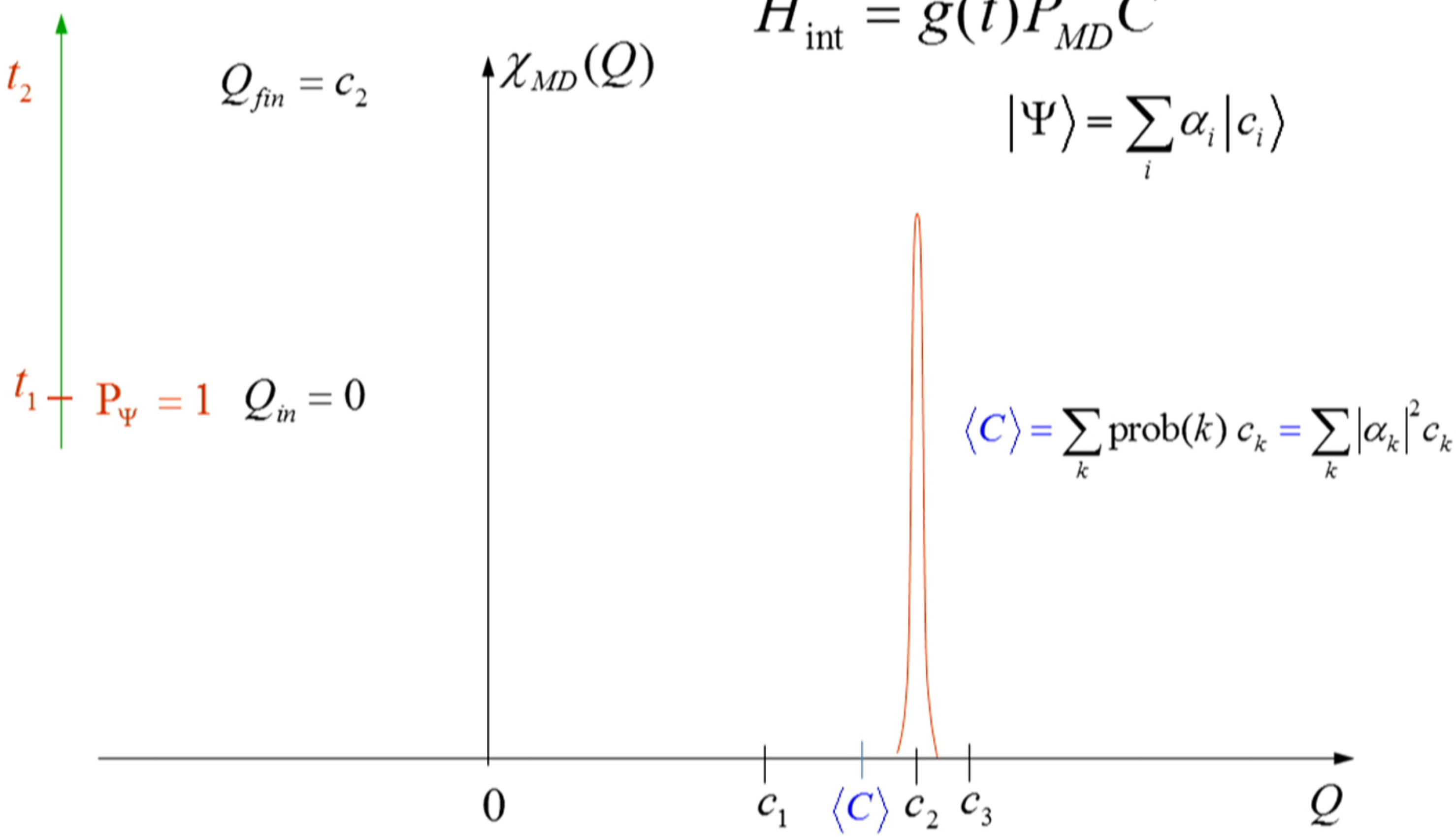
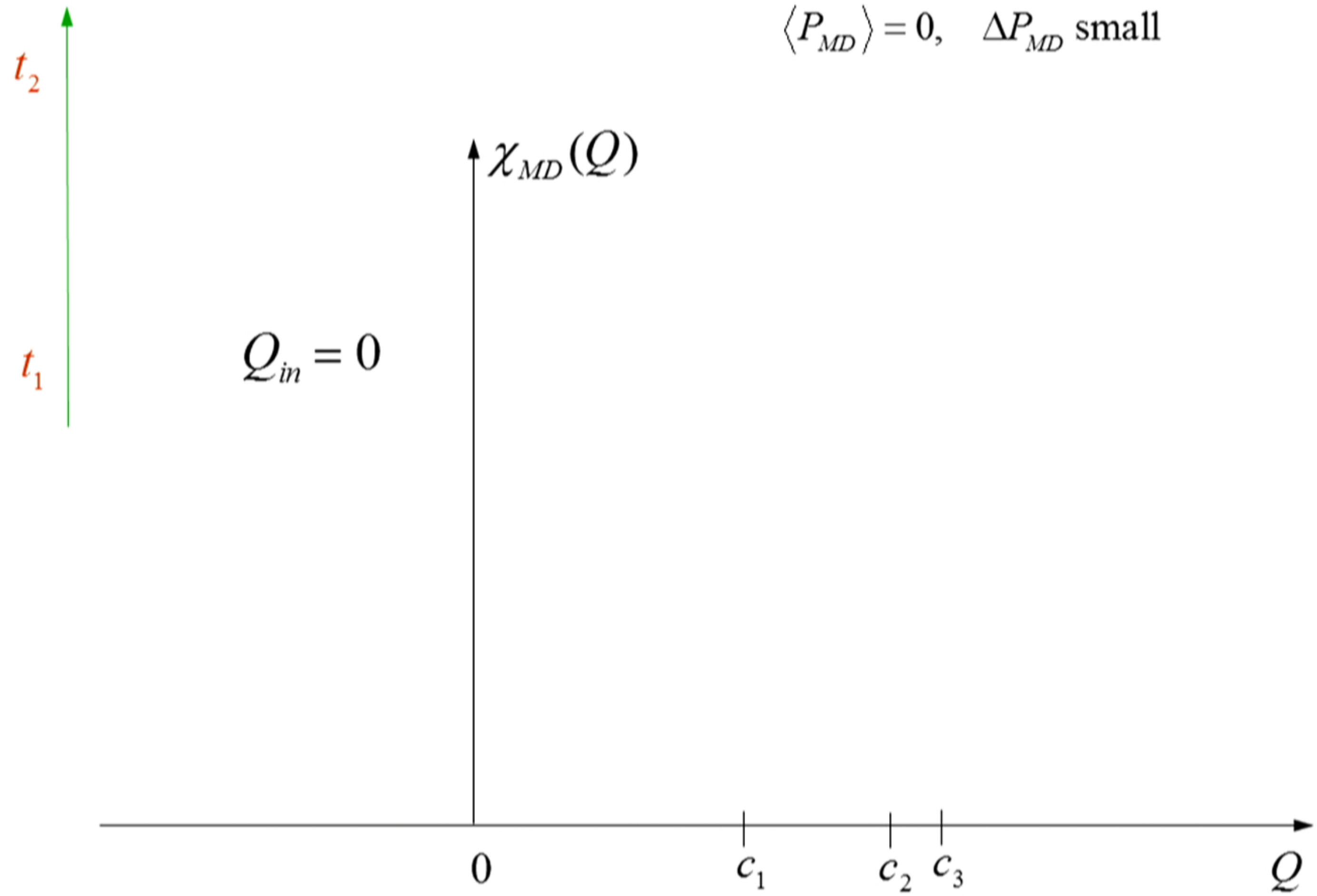


$$|\Psi\rangle |Q=0\rangle \rightarrow \sum_i \alpha_i |c_i\rangle |Q=c_i\rangle \xrightarrow{\text{Collapse!}} |c_2\rangle |Q=c_2\rangle \qquad \mathrm{prob}(2) = |\alpha_2|^2$$

Weak measurement of C with post-selection

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$



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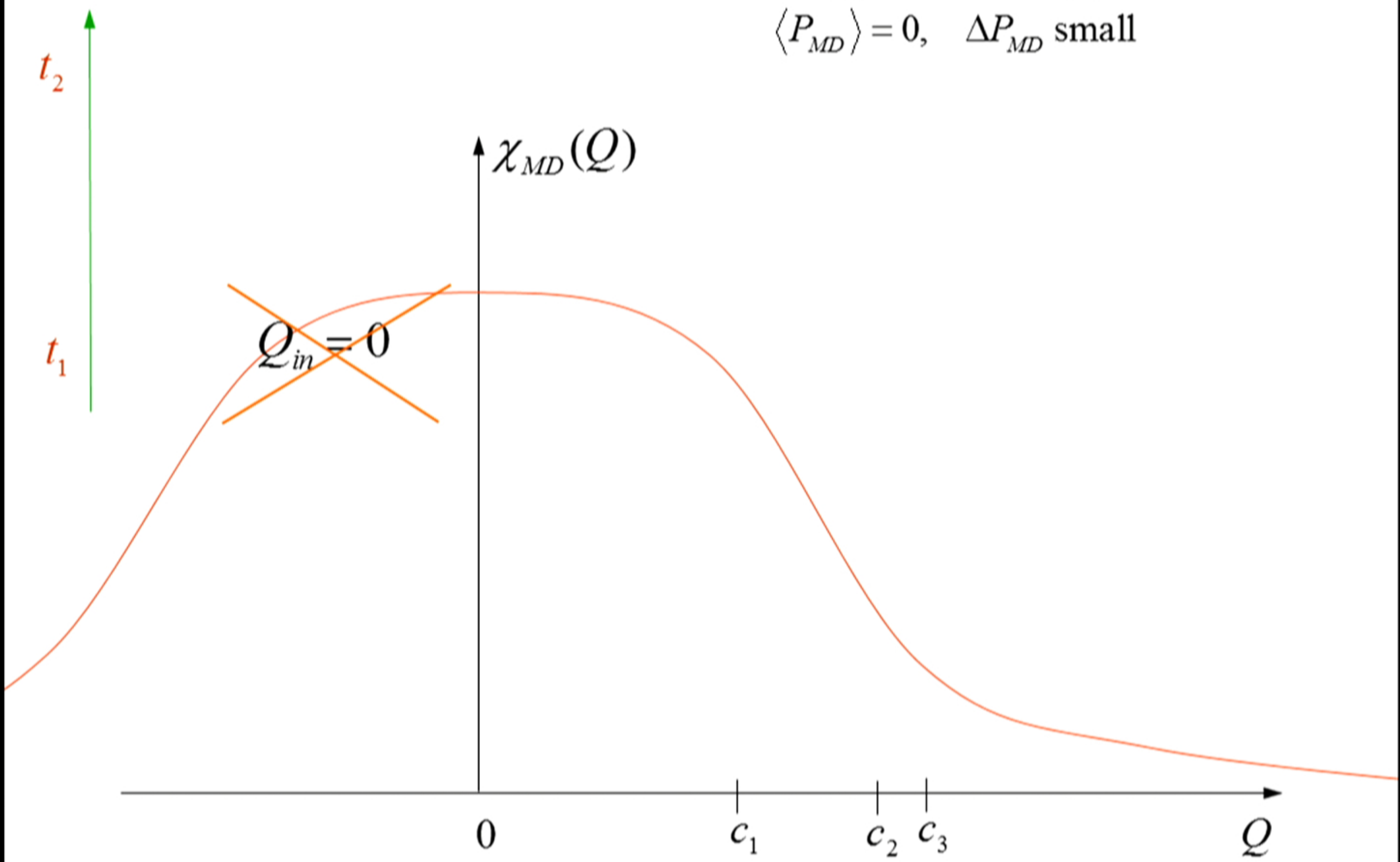

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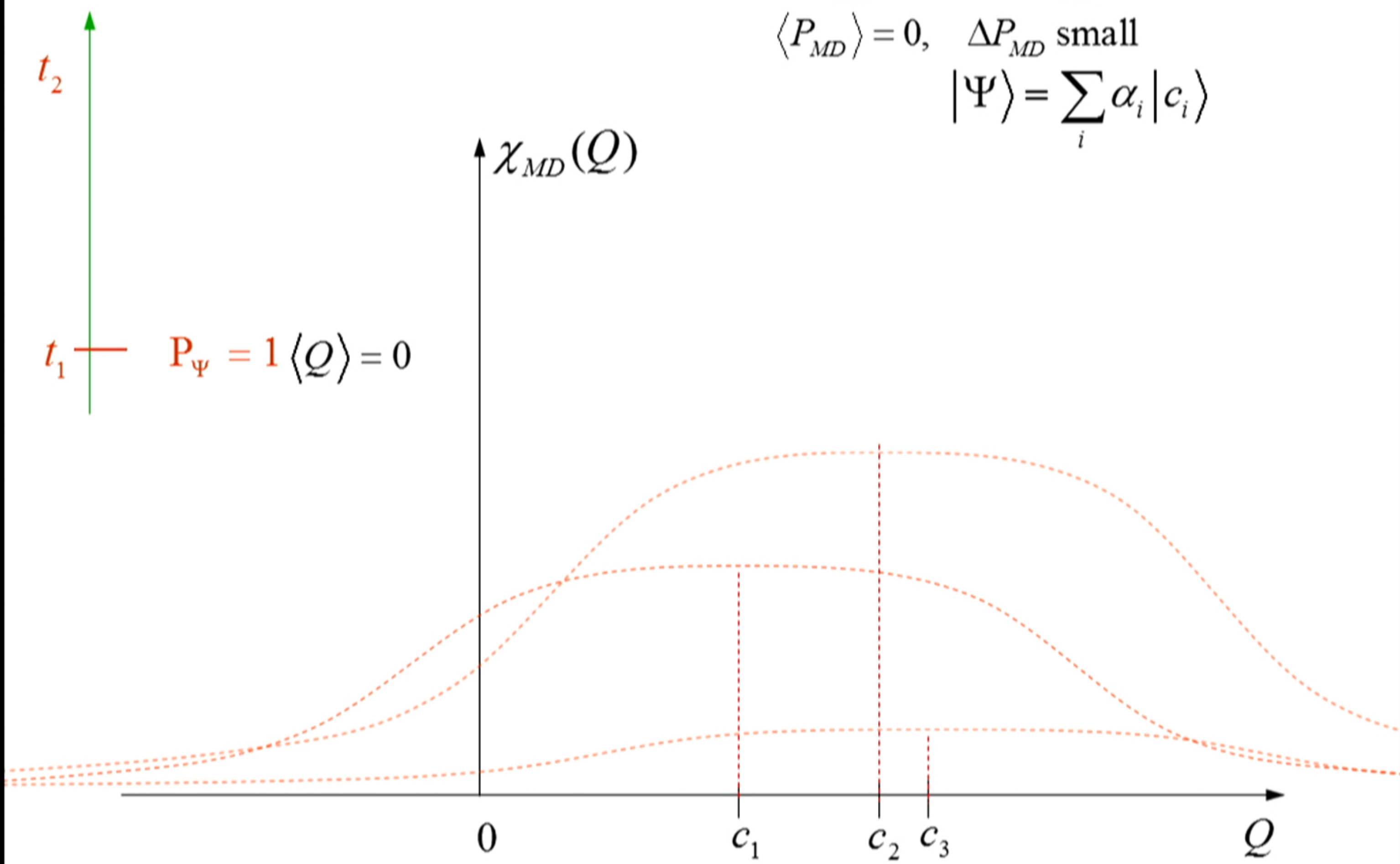

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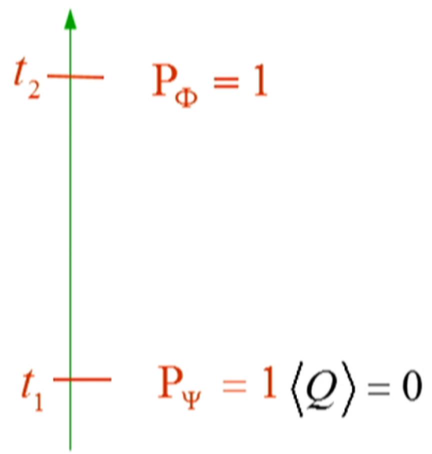

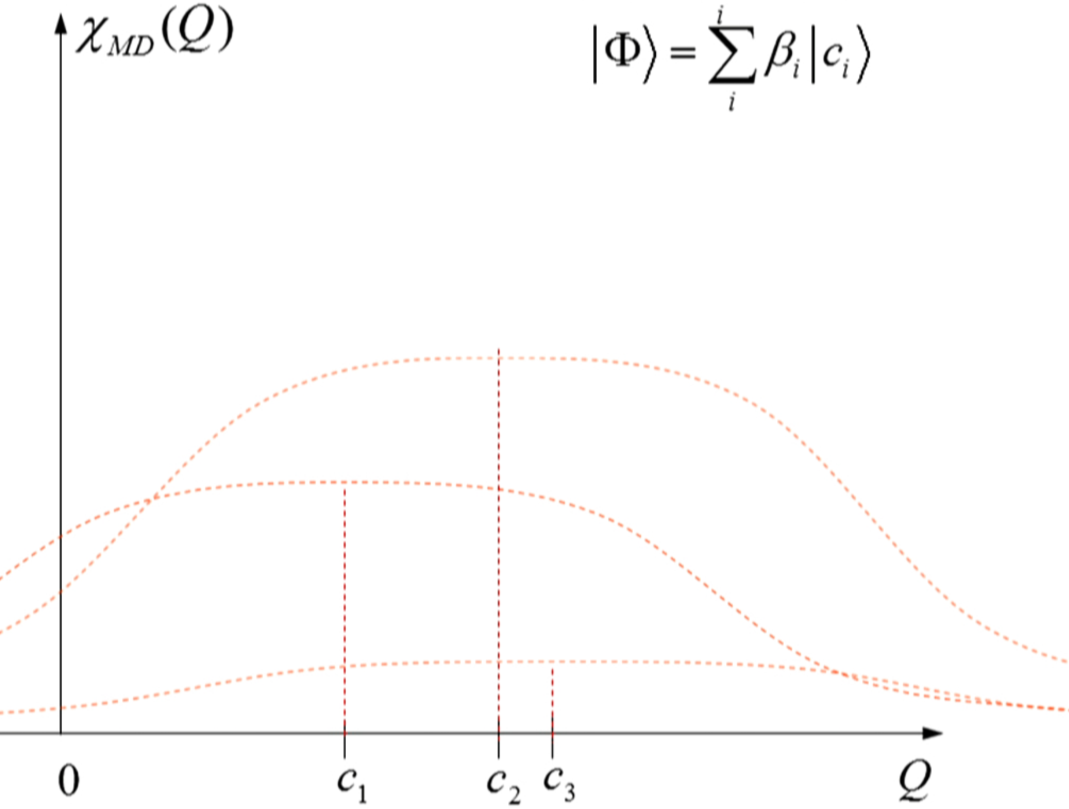

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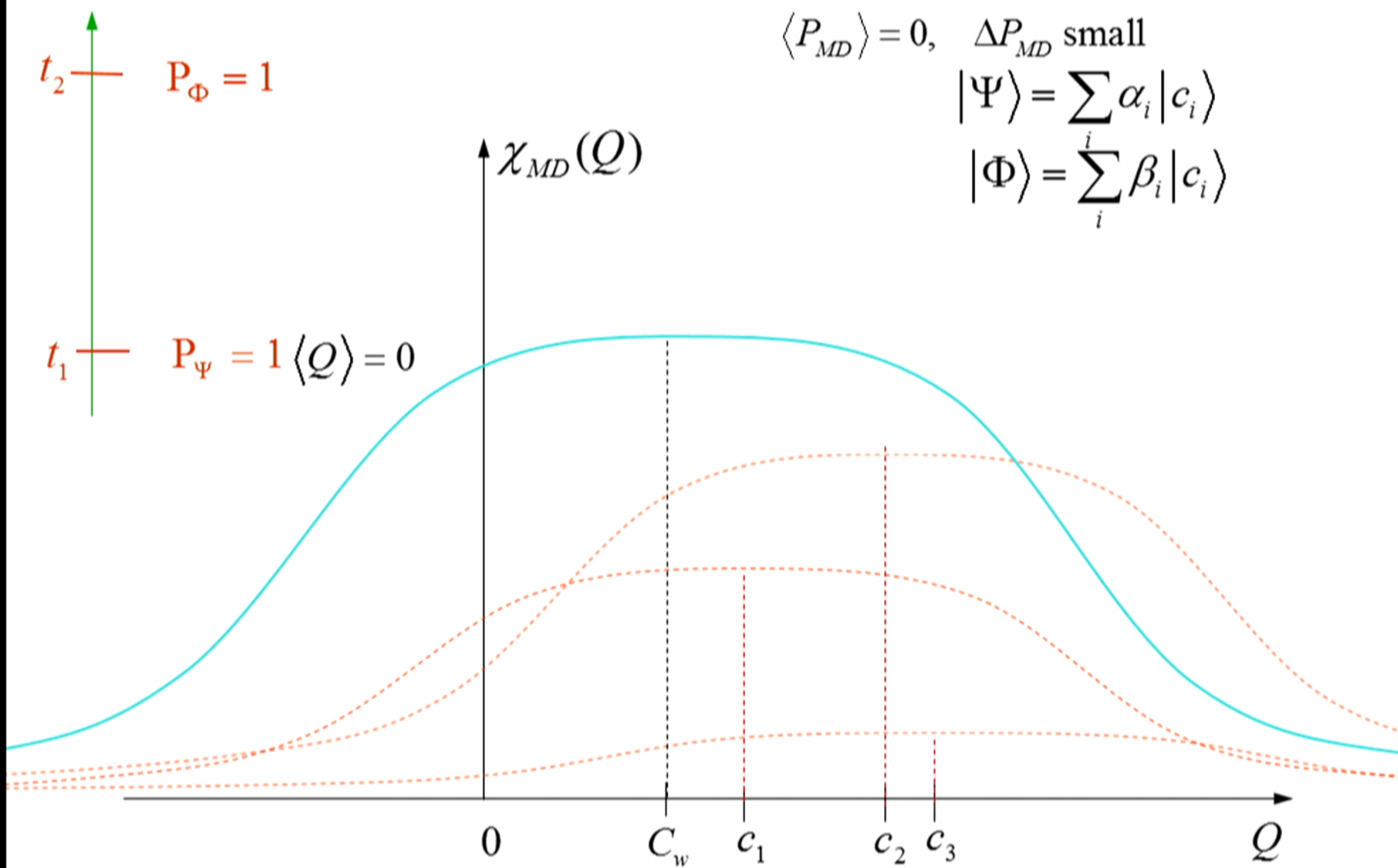

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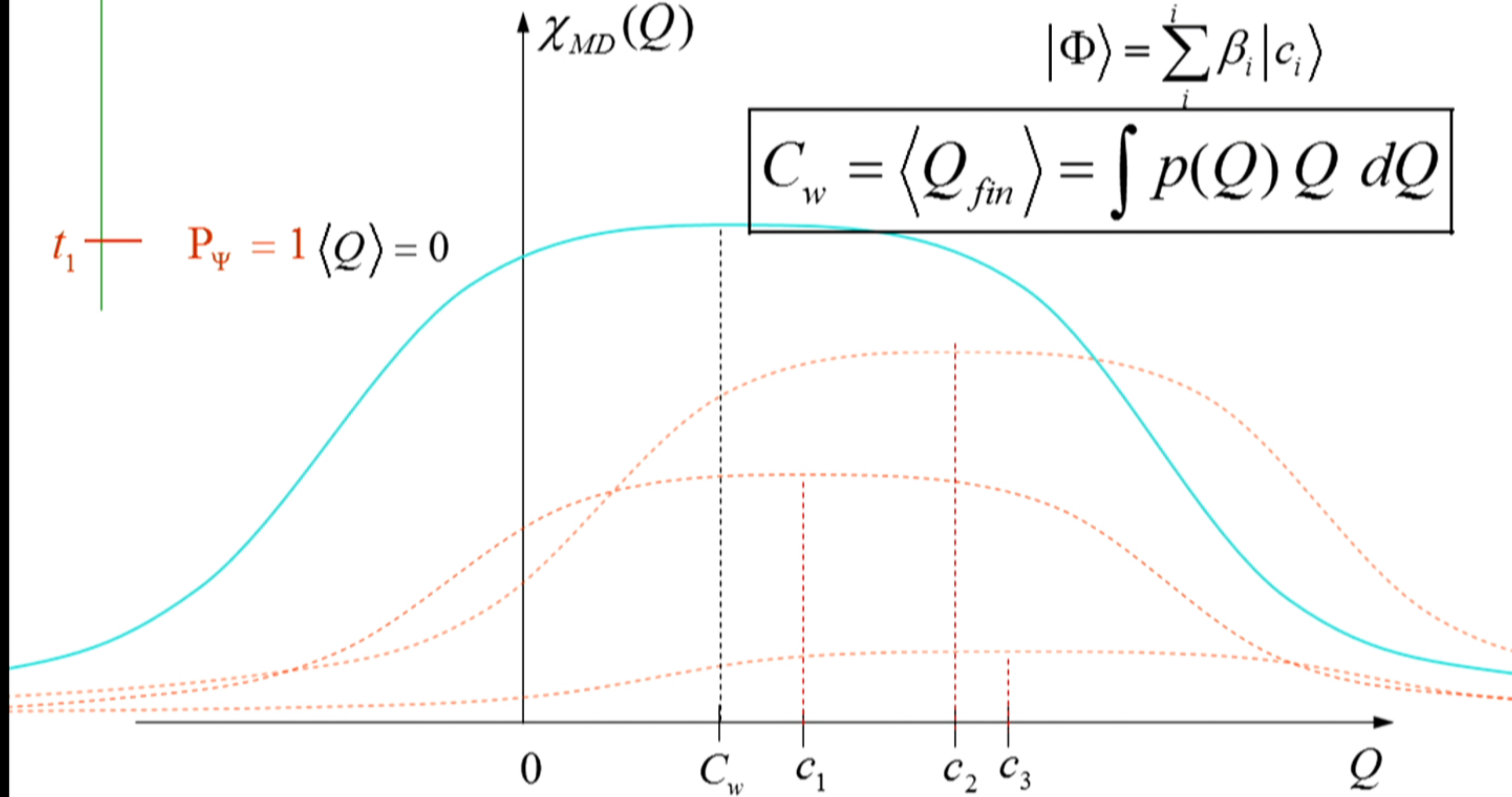
t_2 — $P_\Phi = 1 \ \langle Q_{fin} \rangle = C_w$

t_1 — $P_\Psi = 1 \ \langle Q \rangle = 0$

$$\boxed{C_w = \langle Q_{fin} \rangle = \int p(Q)\, Q \, dQ}$$

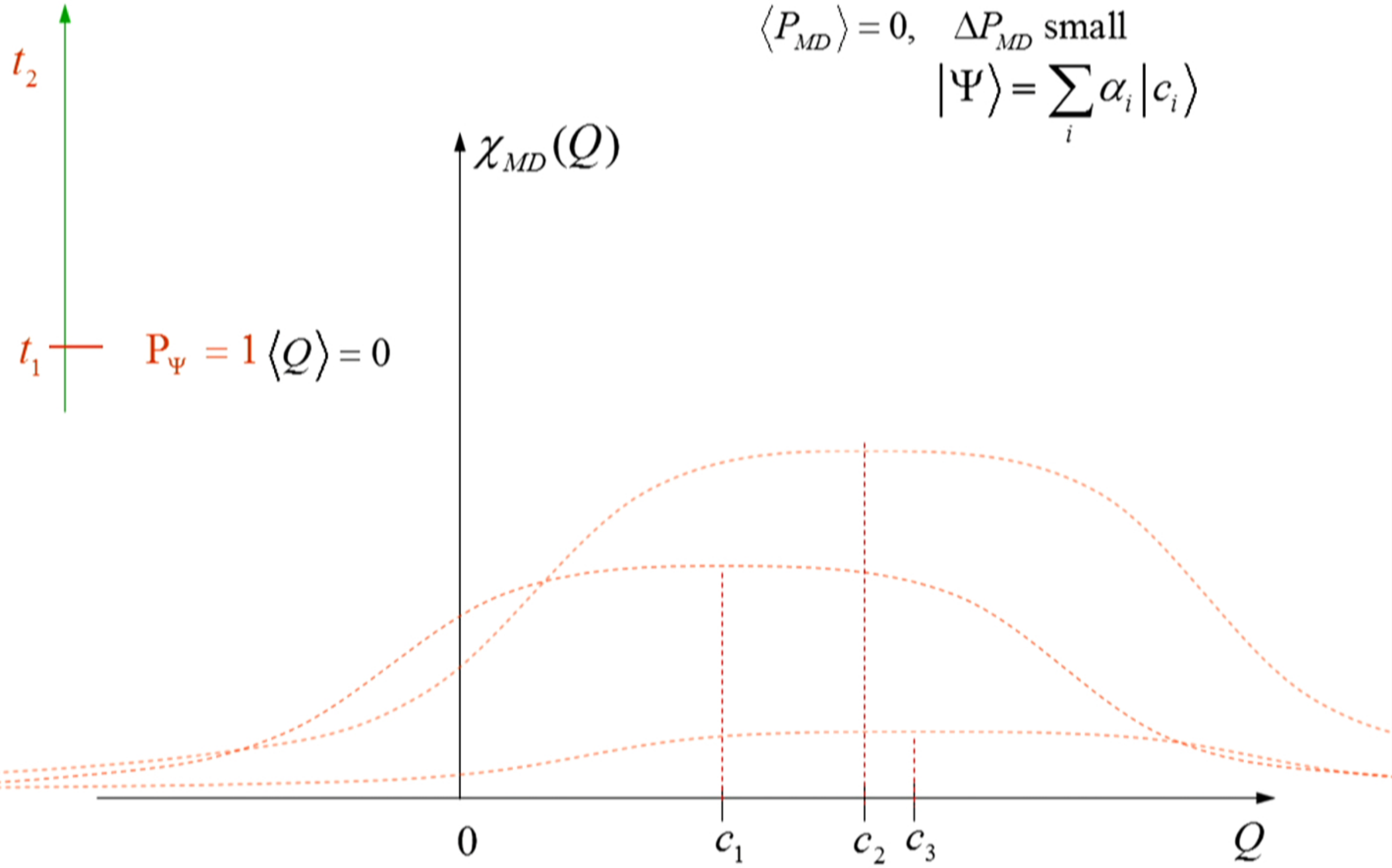


Weak measurement of C without post-selection

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$|\Psi\rangle = \sum_i \alpha_i |c_i\rangle$$



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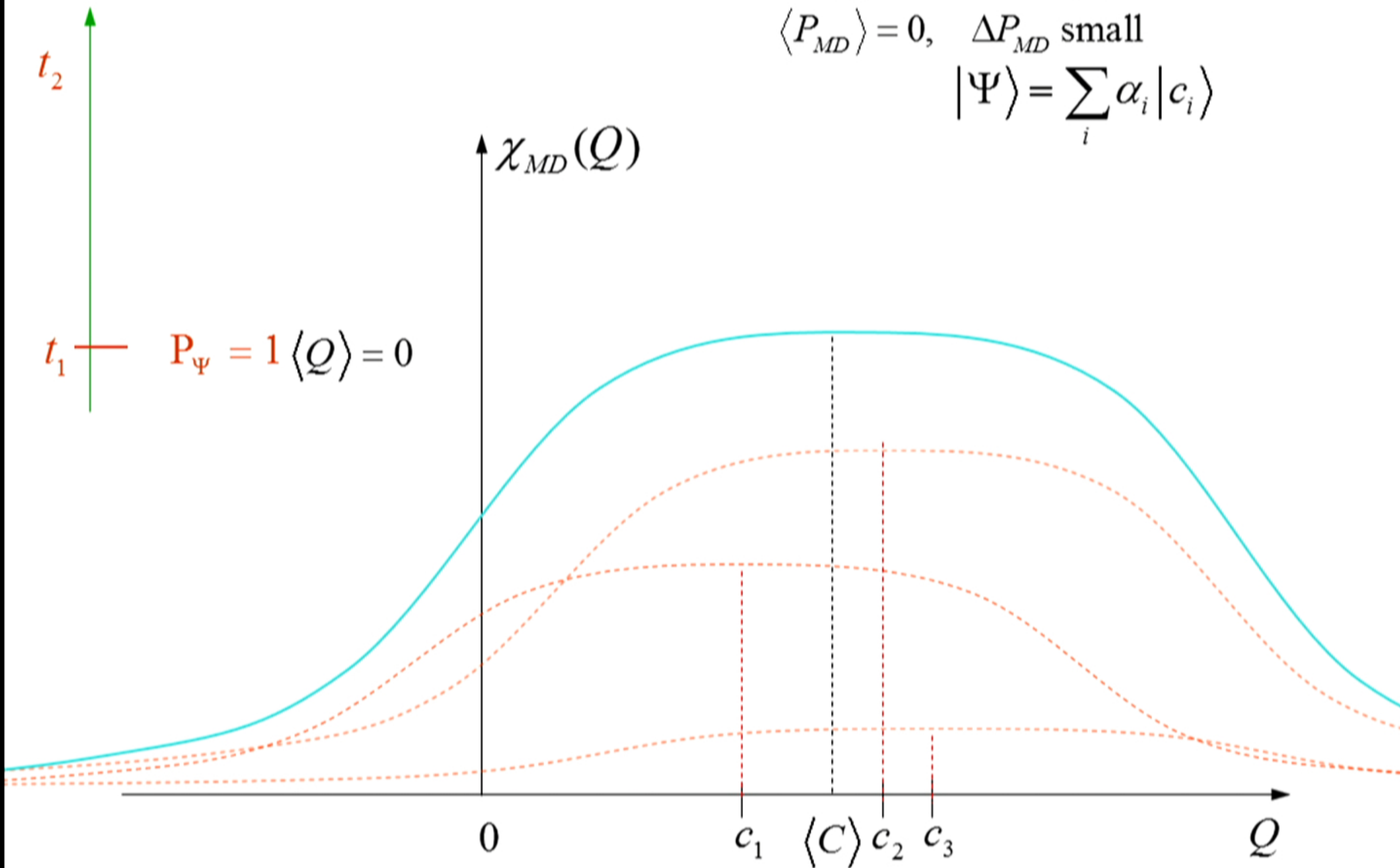

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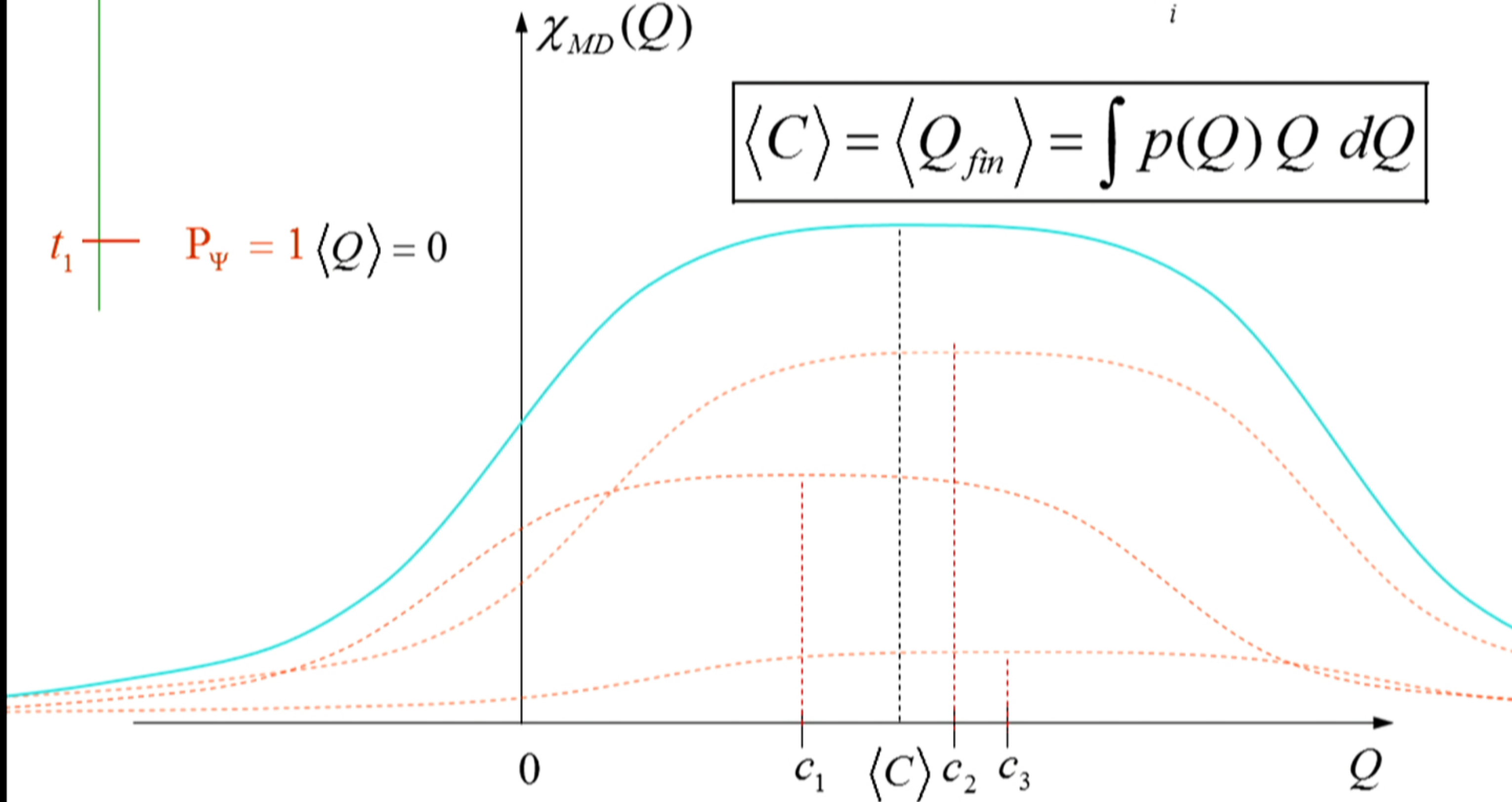

Weak value as an outcome of a weak measurement

$$\boxed{C_w = \left\langle Q_{fin} \right\rangle = \int p(Q)\, Q\, dQ}$$

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on condition of post-selection

Weak value is a conditional expectation value

Comment on "How the result of a single coin toss can turn out to be 100 heads"

In a recent Letter, Ferrie and Combes [1] claimed to show "that weak values are not inherently quantum, but rather a purely statistical feature of pre- and post-selection with disturbance." In this Comment I will show that this claim is not valid. It follows from Ferrie and Combes misunderstanding of the concept of weak value.

Weak value of a variable A is a property of a single quantum system pre-selected in a state $|\psi\rangle$ and post-selected in a state $|\phi\rangle$:

$$A_w \equiv \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}. \tag{1}$$

Weak value as an outcome of a weak measurement

$$\boxed{C_w = \langle Q_{fin} \rangle = \int p(Q)\, Q\, dQ}$$

$$\boxed{\langle C \rangle = \langle Q_{fin} \rangle = \int p(Q)\, Q\, dQ}$$

on condition of post-selection

Weak value is a conditional expectation value

Comment on "How the result of a single coin toss can turn out to be 100 heads"

In a recent Letter, Ferrie and Combes [1] claimed to

FC: Now we demonstrate that it is possible to find anomalous weak values for pre- and postselected states in the same basis provided there is classical disturbance. In particular, we take $A = Z$, $|\psi\rangle = |+1\rangle$, and later we will postselect on $|\phi\rangle = |-1\rangle$. By using the probabilities in Eq. (11), the

Weak value of a variable A is a property of a single quantum system pre-selected in a state $|\psi\rangle$ and post-selected in a state $|\phi\rangle$:

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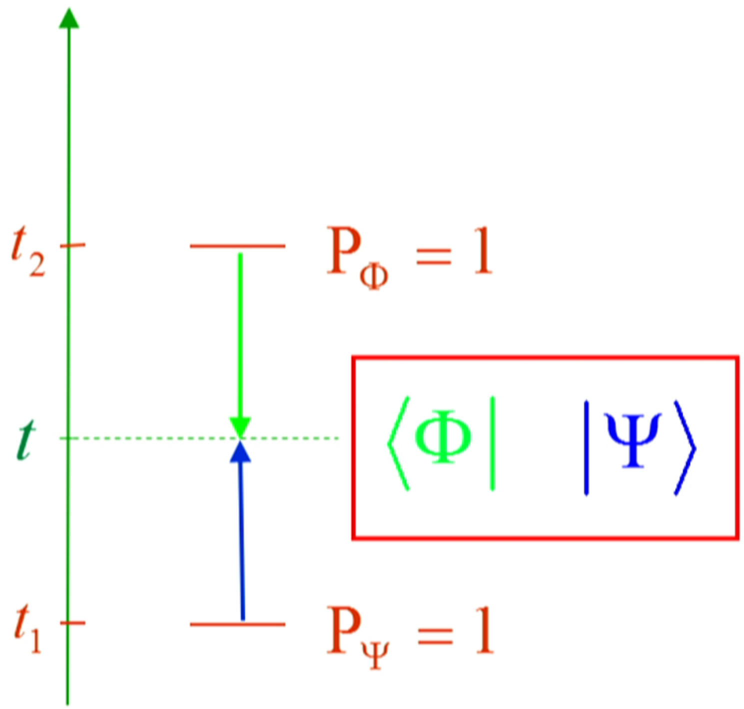
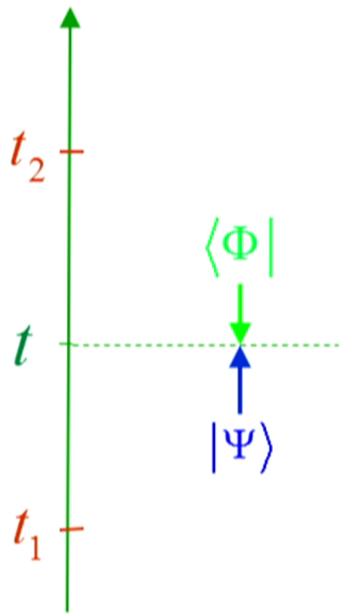
The two-state vector



The weak value as a property of a single system

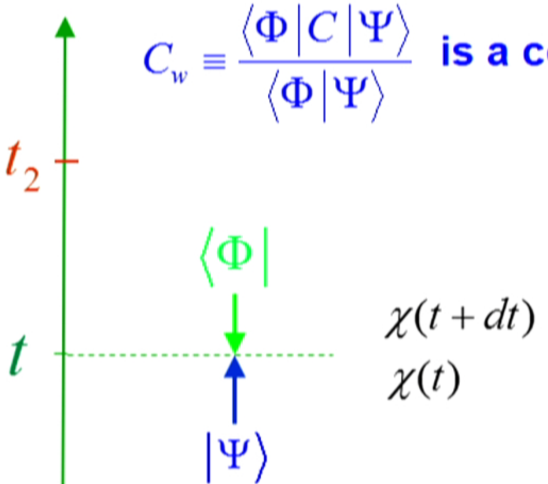
$\langle\Phi| \; |\Psi\rangle$ **is a complete description at a particular time *t***



The weak value as a property of a single system

$\langle\Phi| \quad |\Psi\rangle$ **is a complete description at a particular time *t***

$$C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}$$ **is a complete description of coupling to *C* at time *t***



$$H_{\text{int}} = \hat{C}\hat{B}$$

t_1

$$\chi(t+dt) = \langle\Phi| \; e^{-i\hat{C}\hat{B}dt} \; |\Psi\rangle\,\chi(t) = \langle\Phi|\left(1 - i\hat{C}\hat{B}dt\right)|\Psi\rangle\,\chi(t) = \langle\Phi|\Psi\rangle\left(1 - i\frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}Bdt\right)\chi(t)$$

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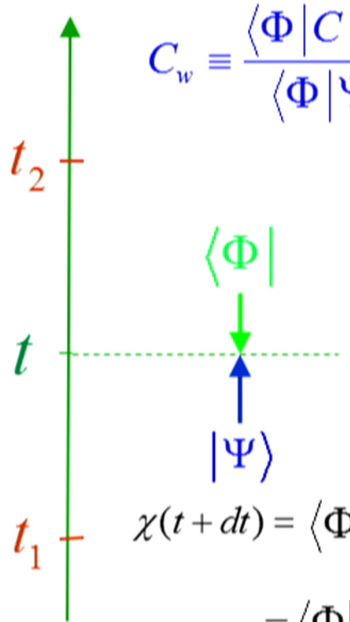

$\chi(t+dt)$

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$$= \langle\Phi|\Psi\rangle e^{-iC_w\hat{B}dt}\chi(t) \rightarrow e^{-iC_w\hat{B}dt}\chi(t)$$

$$\boxed{\chi_w(t+dt) \simeq e^{-iC_w\hat{B}dt}\chi(t)}$$

Comparing states of external system after dt

$$H_{\text{int}} = \hat{C}\hat{B} \qquad C_w = c_k = \langle C \rangle = c$$

weak value

$$C = C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$$

The system is pre-selected $|\Psi\rangle$ **and post-selected** $|\Phi\rangle$

$$\boxed{\chi_{\mathrm{w}}(t+dt) \simeq e^{-iC_w \hat{B} dt} \chi(t)}$$

eigenvalue

$$C = c_k$$

The system is pre-selected $|\Psi\rangle = |\Psi_k\rangle$

$$\boxed{\chi_{\mathrm{e}}(t+dt) = e^{-ic_k \hat{B} dt} \chi(t)}$$

expectation value

$$C = \langle C \rangle = \langle \Psi | C | \Psi \rangle$$

The system is pre-selected $|\Psi\rangle = \sum_k \alpha_k |\Psi_k\rangle$

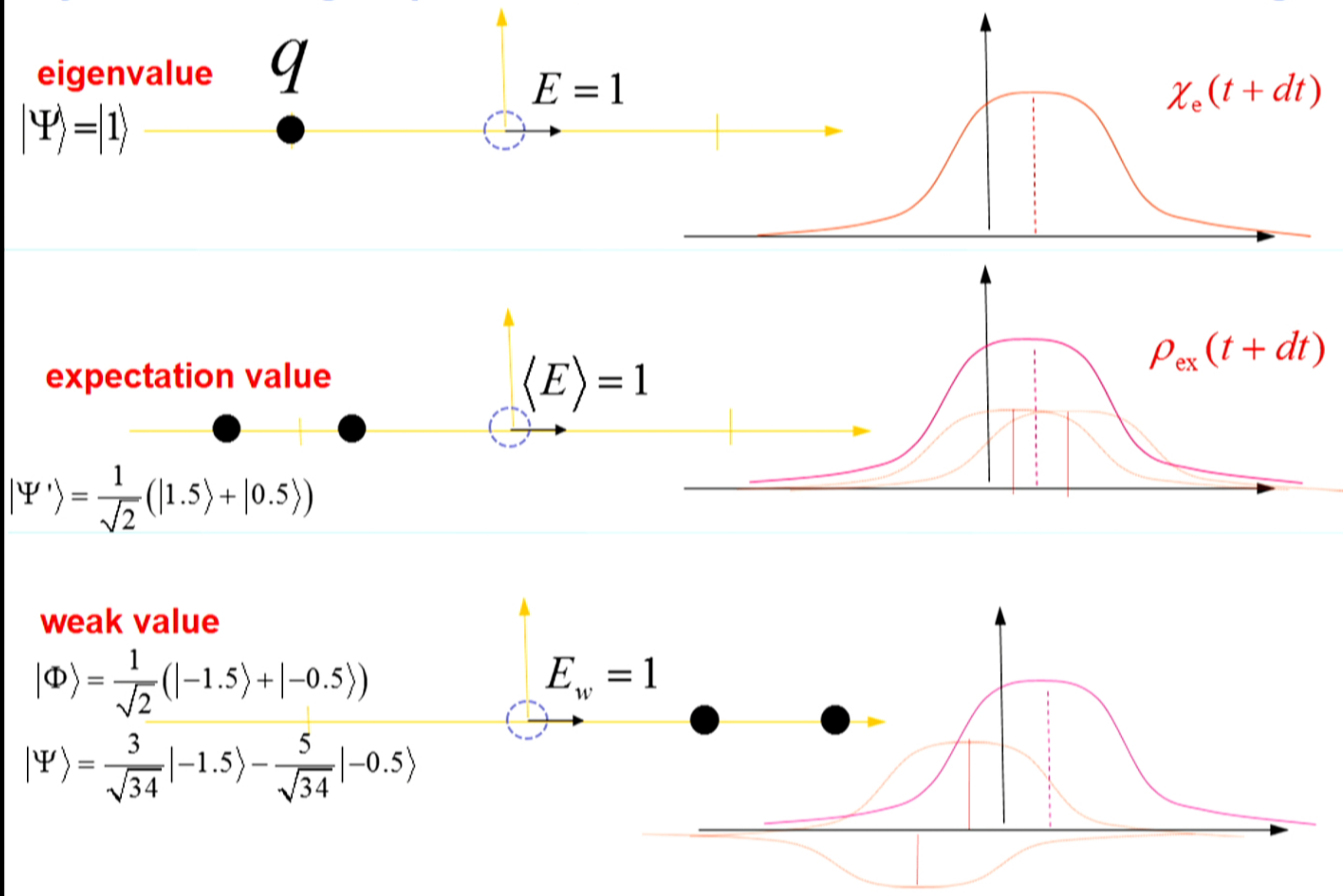
$$\boxed{\rho_{\mathrm{ex}}(t+dt) \overset{?}{\simeq} e^{-i\langle C \rangle \hat{B} dt} \chi(t)}$$

$$\chi_{\mathrm{w}}(t+dt) \simeq \rho_{\mathrm{ex}}(t+dt) \simeq \chi_{\mathrm{e}}(t+dt) \quad \simeq \chi(t)$$

$$\chi_{\mathrm{w}}(t+dt) \simeq \chi_{\mathrm{e}}(t+dt) \qquad\qquad \rho_{\mathrm{ex}}(t+dt) \simeq \chi_{\mathrm{e}}(t+dt)$$

Which equality is much closer than $\chi_{\mathrm{e}}(t+dt) \simeq \chi(t)$ **?**

System: charged particle, variable: electric field at the origin



Comparing states of external system after dt

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The system is pre-selected $|\Psi\rangle = \sum_k \alpha_k |\Psi_k\rangle$

$$\boxed{\rho_{\text{ex}}(t+dt) \simeq e^{-i\langle C \rangle \hat{B} dt} \chi(t)}$$

Bures angle distance

$$D(\chi, \xi) \equiv \arccos |\langle \chi | \xi \rangle|$$

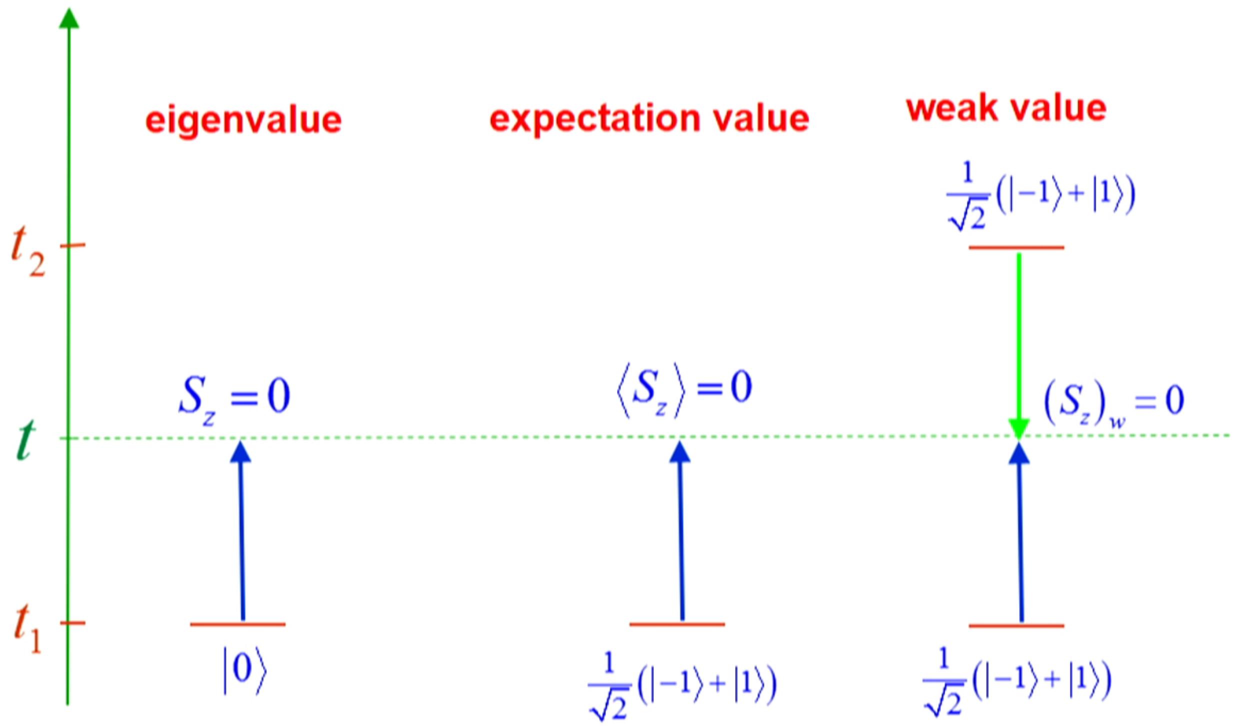
$$D(\chi, \rho) \equiv \arccos \sqrt{|\langle \chi | \rho | \chi \rangle|}$$

$$D\big(\chi(t), \chi_{\text{e}}(t+dt)\big) \simeq c\Delta B dt$$

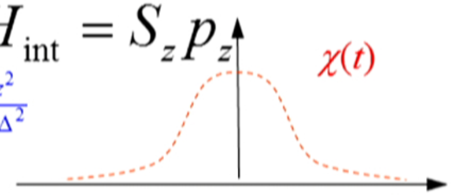
$$D\big(\chi_{\text{e}}(t+dt), \chi_{\text{w}}(t+dt)\big) \simeq \frac{1}{2}\left|\left(C^2\right)_w - c^2\right| \sqrt{\langle B^4 \rangle - \langle B^2 \rangle^2}\,(dt)^2$$

$$D\big(\chi_{\text{e}}(t+dt), \rho_{\text{ex}}(t+dt)\big) \simeq \Delta C\,\Delta B\,dt$$

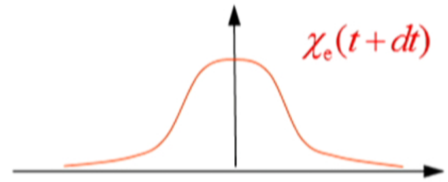
Example: spin 1 particle $C = S_z$



Example: spin 1 particle $C = S_z$ $\quad H_{\text{int}} = S_z p_z$

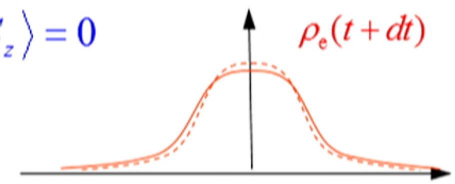


$$C_w = c_k = \langle C \rangle = 0 \qquad \chi(t) = e^{-\frac{z^2}{2\Delta^2}}$$



Eigenstate $|\Psi\rangle = |0\rangle$ **eigenvalue** $S_z = 0$

$$\chi_e(t+dt) = e^{-i0\hat{p}_z dt}\chi(t) = e^0 e^{-\frac{z^2}{2\Delta^2}} = e^{-\frac{z^2}{2\Delta^2}}$$



Pre-selection $|\Psi\rangle = \frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle)$ **expectation value** $\langle S_z \rangle = 0$

$$\left(|-1\rangle e^{i\hat{p}_z dt} + |1\rangle e^{-i\hat{p}_z dt}\right) e^{-\frac{z^2}{2\Delta^2}} = |-1\rangle e^{-\frac{(z+dt)^2}{2\Delta^2}} + |1\rangle e^{-\frac{(z-dt)^2}{2\Delta^2}}$$

Pre-selection $|\Psi\rangle = \frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle)$ **post-selection** $|\Psi\rangle = \frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle)$

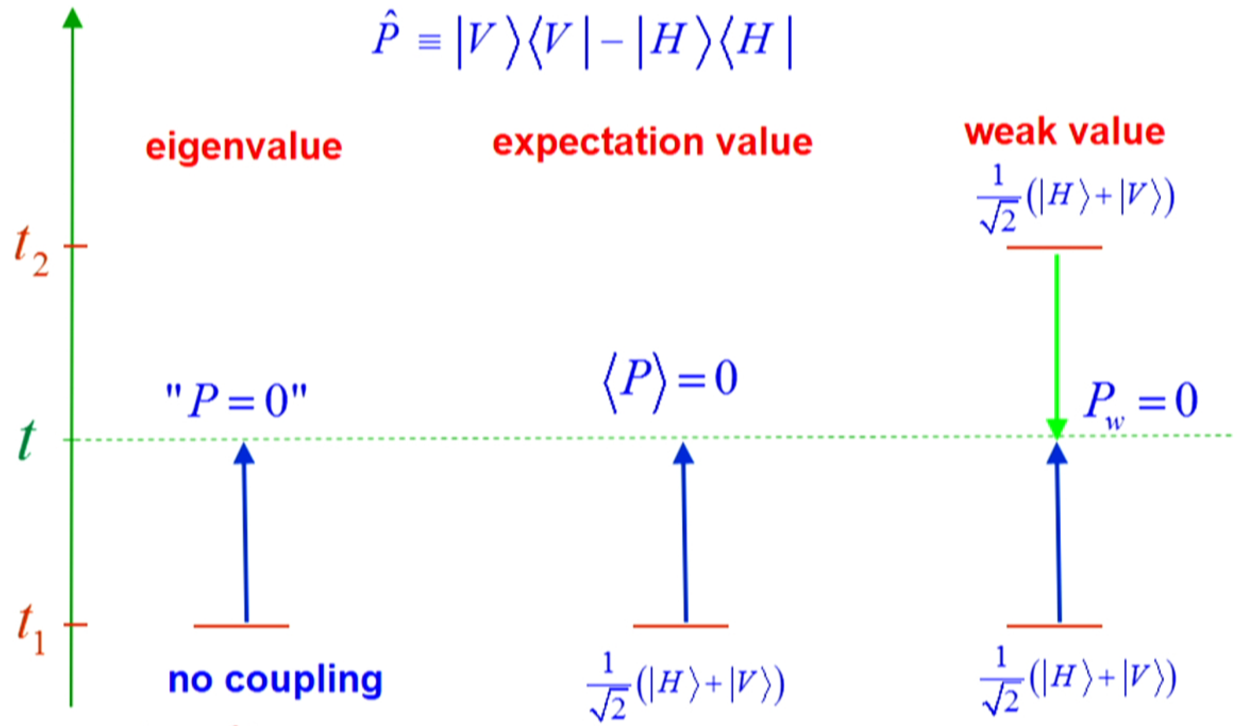
weak value $$(S_z)_w = \frac{\left(\langle -1| + \langle 1|\right) S_z \left(|-1\rangle + |1\rangle\right)}{\left(\langle -1| + \langle 1|\right)\left(|-1\rangle + |1\rangle\right)} = 0$$

$$\left(\langle -1| + \langle 1|\right)\left(|-1\rangle e^{-\frac{(z+dt)^2}{2\Delta^2}} + |1\rangle e^{-\frac{(z-dt)^2}{2\Delta^2}}\right) = e^{-\frac{(z+dt)^2}{2\Delta^2}} + e^{-\frac{(z-dt)^2}{2\Delta^2}}$$

Imitation of spin 1 particle by photon polarization

$|S_z=-1\rangle \Leftrightarrow |H\rangle \qquad |S_z=1\rangle \Leftrightarrow |V\rangle \qquad |S_z=0\rangle \Leftrightarrow$ **no coupling**

$$\hat{P} \equiv |V\rangle\langle V| - |H\rangle\langle H|$$



$$\chi_e(t+dt) = e^{-\frac{z^2}{2\Delta^2}}$$

$$\rho_{ex}(t+dt) = |-1\rangle e^{-\frac{(z+dt)^2}{2\Delta^2}} + |1\rangle e^{-\frac{(z-dt)^2}{2\Delta^2}}$$

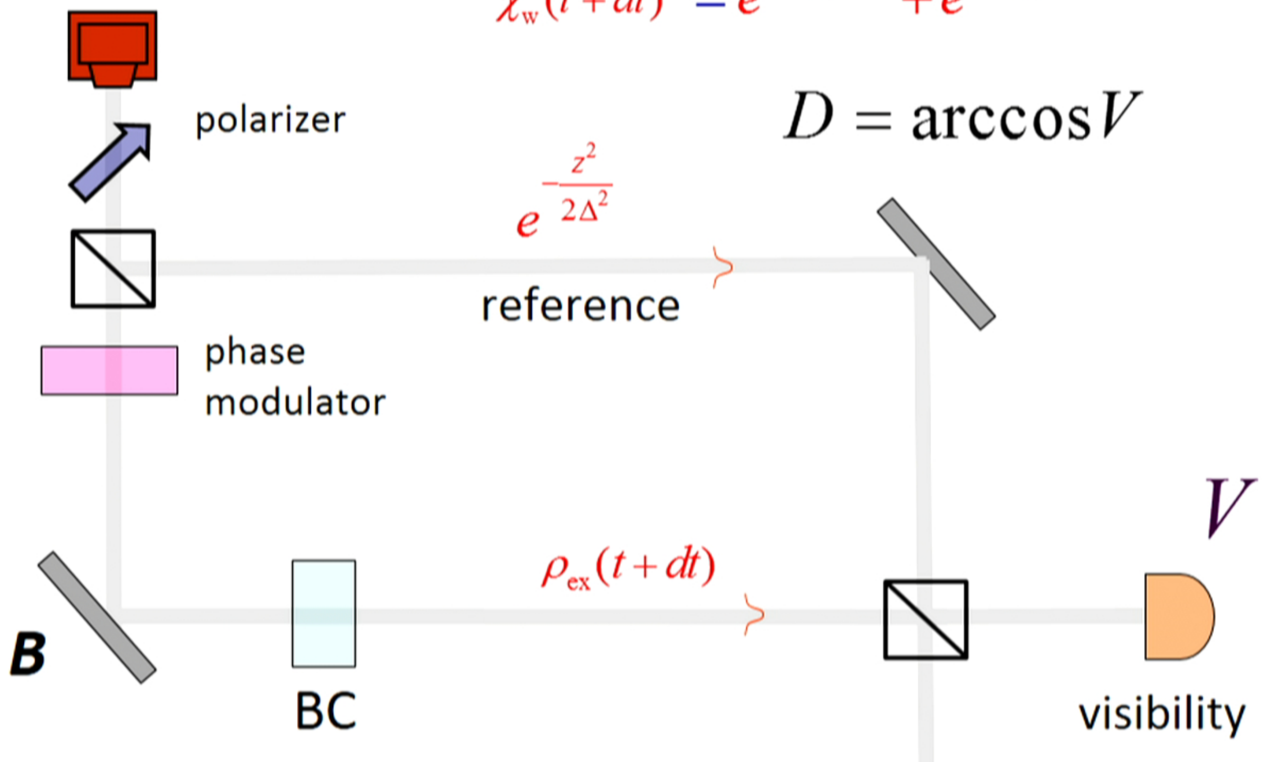
$$\chi_w(t+dt) = e^{-\frac{(z+dt)^2}{2\Delta^2}} + e^{-\frac{(z-dt)^2}{2\Delta^2}}$$

Experiment comparing $\chi_{\mathrm{e}}(t+dt) = e^{-\frac{z^2}{2\Delta^2}}$

with $\rho_{\mathrm{ex}}(t+dt) = |-1\rangle e^{-\frac{(z+dt)^2}{2\Delta^2}} + |1\rangle e^{-\frac{(z-dt)^2}{2\Delta^2}}$

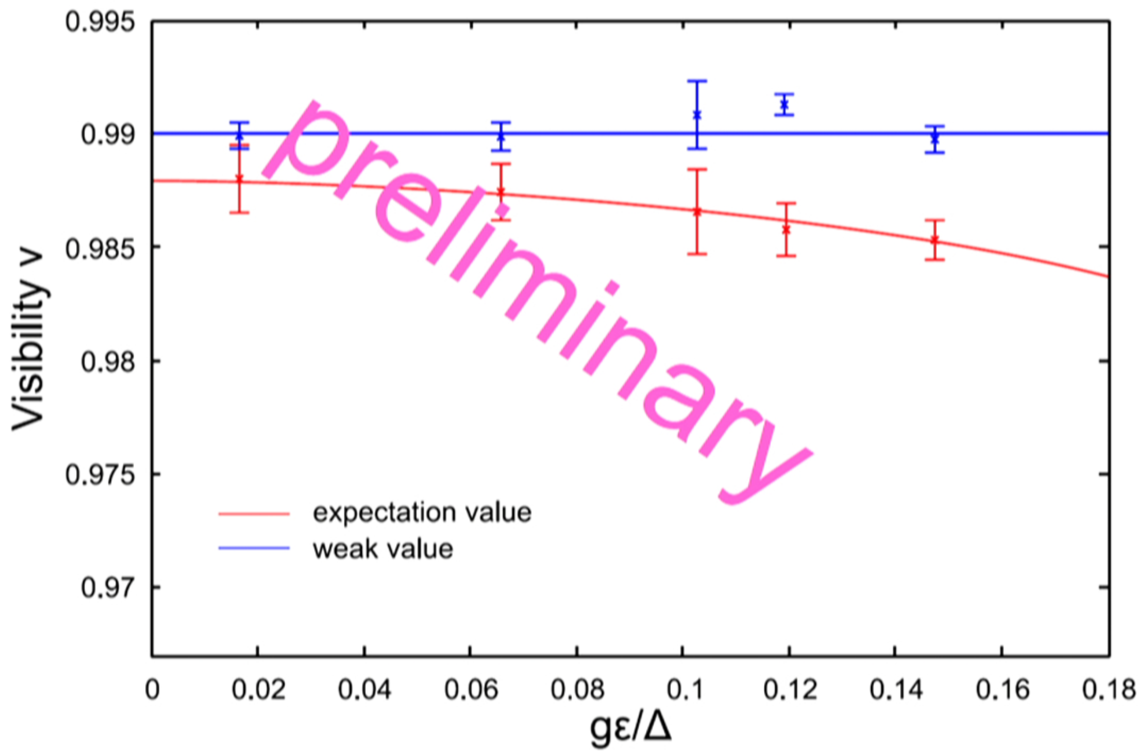
$$\chi_{\mathrm{w}}(t+dt) = e^{-\frac{(z+dt)^2}{2\Delta^2}} + e^{-\frac{(z-dt)^2}{2\Delta^2}}$$

$$D = \arccos V$$



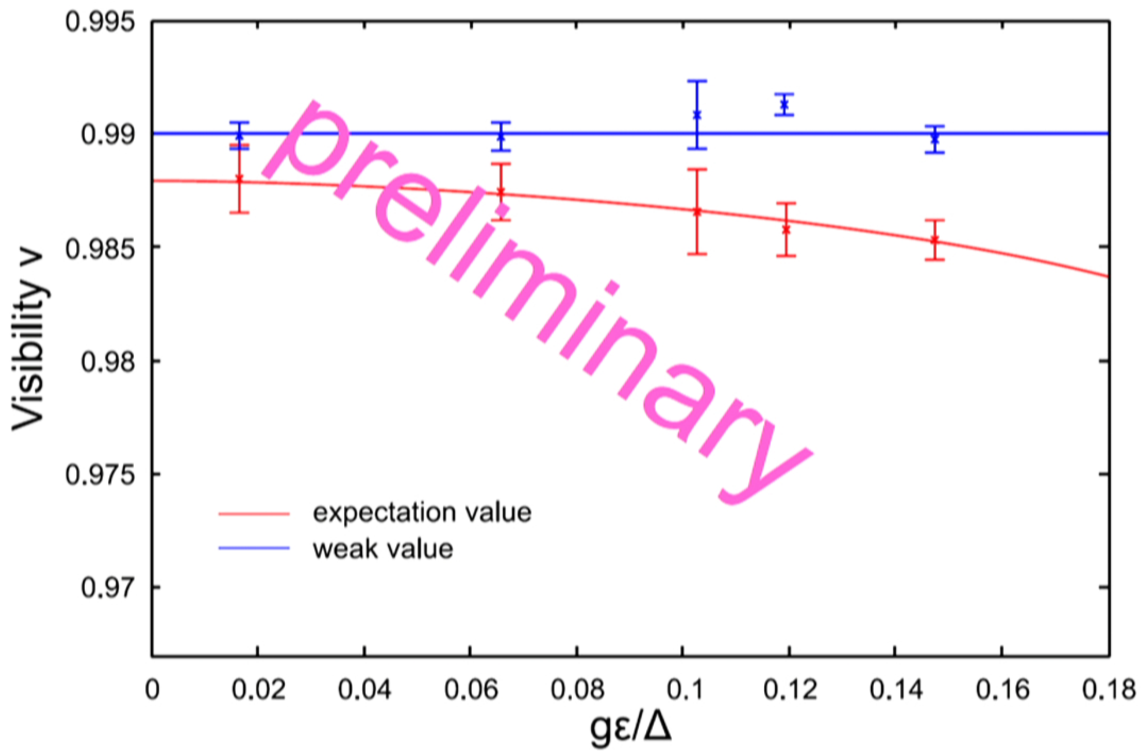
$$D\left(\chi_{\mathrm{e}}(t+dt), \rho_{\mathrm{ex}}(t+dt)\right) \simeq \Delta C\ \Delta B\ dt$$

$$D\left(\chi_{\mathrm{e}}(t+dt), \chi_{\mathrm{w}}(t+dt)\right) \simeq \frac{1}{2}\left|\left(C^2\right)_w - c^2\right| \sqrt{\left\langle B^4\right\rangle - \left\langle B^2\right\rangle^2}\ (dt)^2$$



$$D\left(\chi_{\mathrm{e}}(t+dt),\rho_{\mathrm{ex}}(t+dt)\right)\simeq \Delta C\;\Delta B\;dt\simeq \Delta S_z\;\Delta p_z\;dt=\frac{1}{2\Delta}dt$$

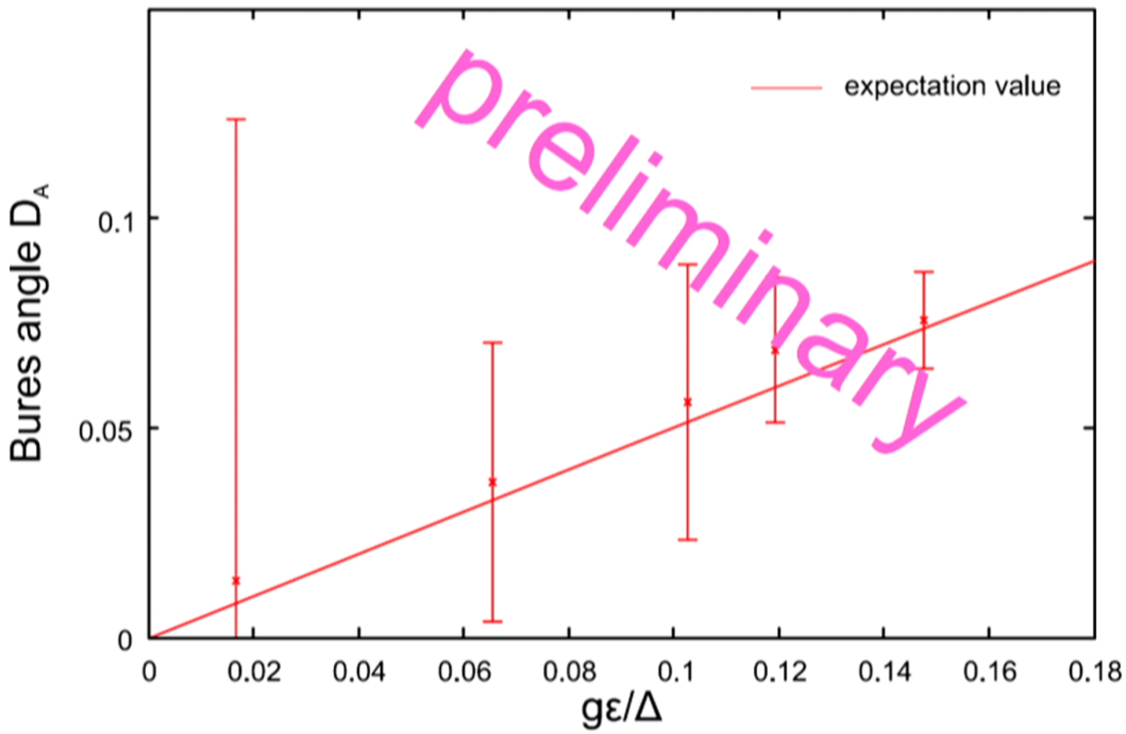
$$D\left(\chi_{\mathrm{e}}(t+dt),\chi_{\mathrm{w}}(t+dt)\right)\simeq \frac{1}{2}\left|\left(C^2\right)_w-c^2\right|\sqrt{\left\langle B^4\right\rangle-\left\langle B^2\right\rangle^2}\,(dt)^2\simeq \frac{1}{4\sqrt{2}\Delta^2}(dt)^2$$



$$D\left(\chi_{\mathrm{e}}(t+dt), \rho_{\mathrm{ex}}(t+dt)\right) \simeq \Delta C\ \Delta B\ dt \simeq \Delta S_z\ \Delta p_z\ dt = \frac{1}{2\Delta} dt$$

$$D\left(\chi_{\mathrm{e}}(t+dt), \chi_{\mathrm{w}}(t+dt)\right) \simeq \frac{1}{2}\left|\left(C^2\right)_w - c^2\right| \sqrt{\left\langle B^4\right\rangle - \left\langle B^2\right\rangle^2}\ (dt)^2 \simeq \frac{1}{4\sqrt{2}\Delta^2}(dt)^2$$

$$D = \arccos V$$



Summary

Weak value is a property of a single pre- and post-selected system

(It is not a property of a measuring procedure)

$$\boxed{C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}}$$

An ensemble is needed only to test that it is true

A pre and post-selected system interacts with external systems during infinitesimal time as if the weak value is an eigenvalue.

It is significantly different from coupling of pre-selected only system with expectation value equal to the weak value.

Weak value of an observable at time *t* represents an effective coupling to this observable during infinitesimal time.