Title: Ubiquity of Weak Values

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Abstract: In this brief talk we will show how weak values appear in a wide range of physical contexts beyond the usual context of weak measurements. Among others, we will discuss how weak values appear in: the physics of classical parameters in a quantum evolution; the statistics of strong measurements; formulas for probability amplitudes in quantum mechanics; and finally, in the classical correspondence of quantum mechanics.

Ubiquity of Weak Values

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Universidad de los Andes

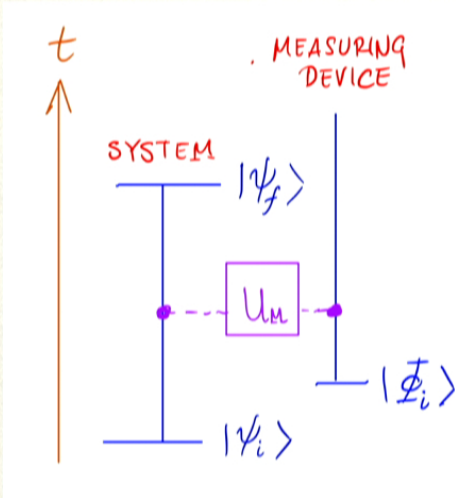
Concepts and Paradoxes in a Quantum Universe.

Perimeter Institute, June 21,2016

Main idea

- Weak values are more ubiquitous than we think
- Hint: Pay more attention to variable coupling to system in Hamiltonian (i.e., conjugate to "pointer")

Von Neumann measurement conditional on pre- and post-selection



Measuring device

Measured system

$$H_M = -\delta(t)\, g x \hat{A}$$

$$[x, p] = i$$

“Reaction variable"

"Pointer variable"

Amplitude determines conditional state of the mirror

Initial State

$$|\Phi_0\rangle$$

Conditional Final State

$$|\Phi_c\rangle = \frac{K(x)}{\sqrt{P_{fi}(M)}}|\Phi_0\rangle$$

$$K(x) = \langle \psi_f \left| e^{igxA} \right| \psi_i \rangle$$

$$P_{fi}(M) = \langle K^\dagger K \rangle_{\Phi_0}$$

Polar decomposition of amplitude tells a story

$$K(x) = |K(x)|e^{iS(x)}$$

Likelihood factor for updating probabilities

Mechanical effect on pointer in terms of weak values

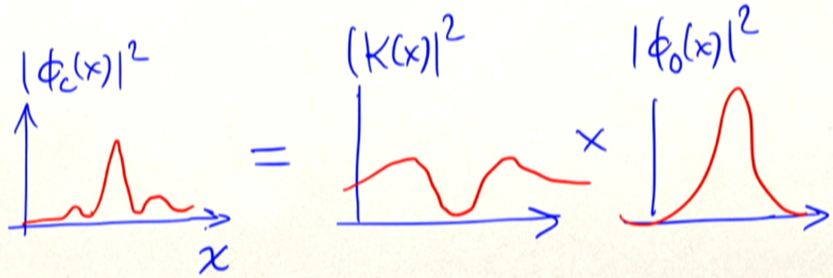
The Modulus

$$|\Phi_c(x)|^2 = \frac{|K(x)|^2}{P_{fi}(M)}|\Phi_0(x)|^2$$

Bayes' Theorem

$$P(a|bc) = \frac{P(c|ab)}{P(c|b)}P(a|b)$$

Posterior = Likelihood x Prior

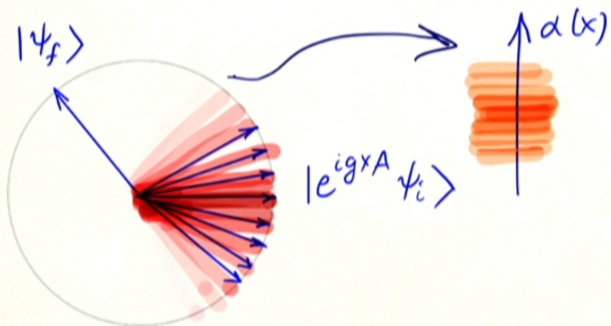


The Phase

$$K(x) = |K(x)|e^{iS(x)}$$

$$\frac{dS}{dx} = \mathrm{Re}\frac{\langle\psi_f|A|e^{igxA}\psi_i\rangle}{\langle\psi_f|e^{igxA}\psi_i\rangle} \equiv \alpha(x)$$

x- dependent weak value

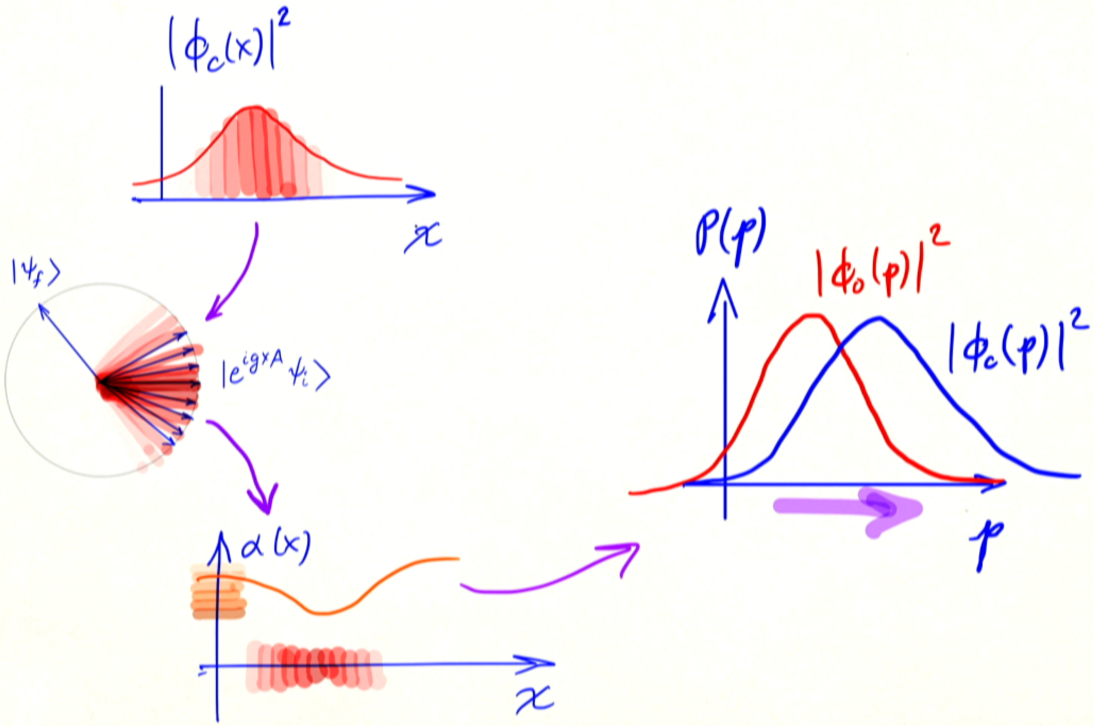


Pointer shift from phase

Phase determines a unitary
(and hence reversible) operation

$$e^{-iS(x)} p e^{iS(x)} = p + \alpha(x)$$

A rough picture



Aharonov and Botero, Phys. Rev. A 72, 052111 (2005)

More precisely

Conditional initial state

$$|\tilde{\Phi}_0\rangle \propto |K(x)||\Phi_0\rangle$$

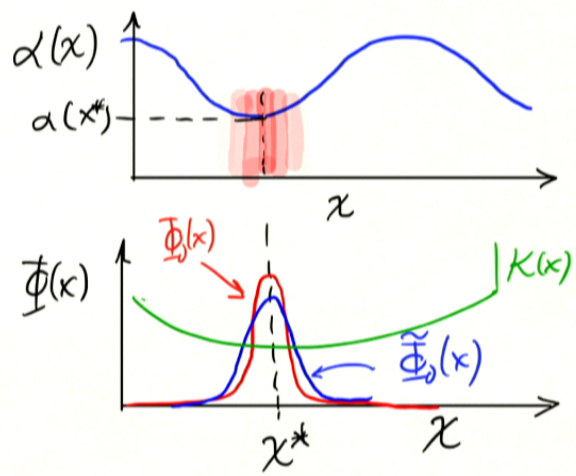
Pointer distribution

$$P(p) = \langle\tilde{\phi}_0|\delta(p - \hat{p} - \alpha(\hat{x}))|\tilde{\Phi}_0\rangle$$

Average pointer shift

$$\langle\delta p\rangle = \langle\tilde{\phi}_0|\alpha(\hat{x})|\tilde{\Phi}_0\rangle$$

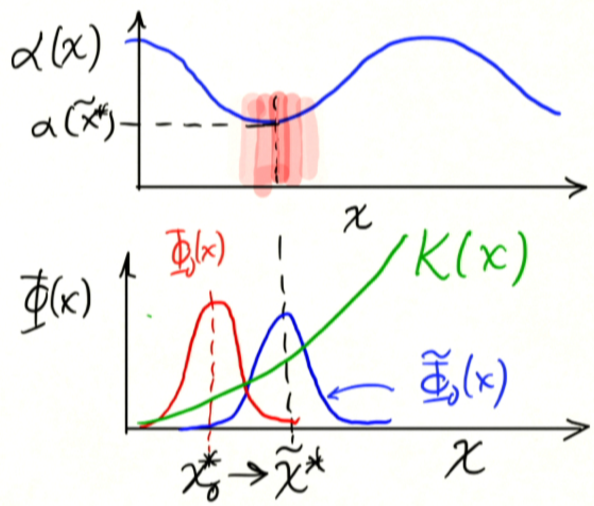
Weak Measurement (standard def.)



$$\Phi_c(p) \simeq \Phi_0(p - \alpha(x^*))$$

(up to phase)

Weak Measurement (more generally)



$$\Phi_c(p) \simeq \tilde{\Phi}_0(p - \alpha(\tilde{x}^*))$$

(up to phase)

General Evolution parameters

x: "Classical parameter"
(sharp continuous quantum variable)

Suppose $H(t, x)$

$$U(t_f, t_i, \vec{r}) = T\left[\exp\left(-i\int_{t_i}^{t_f} dt'\, H(t', x)\right)\right]$$

$$\Rightarrow \quad K(x) = \langle \psi_f | U(t_f, t_i, x) | \psi_i \rangle$$

$$= |K(x)| e^{iS(x)}$$

General Evolution parameters

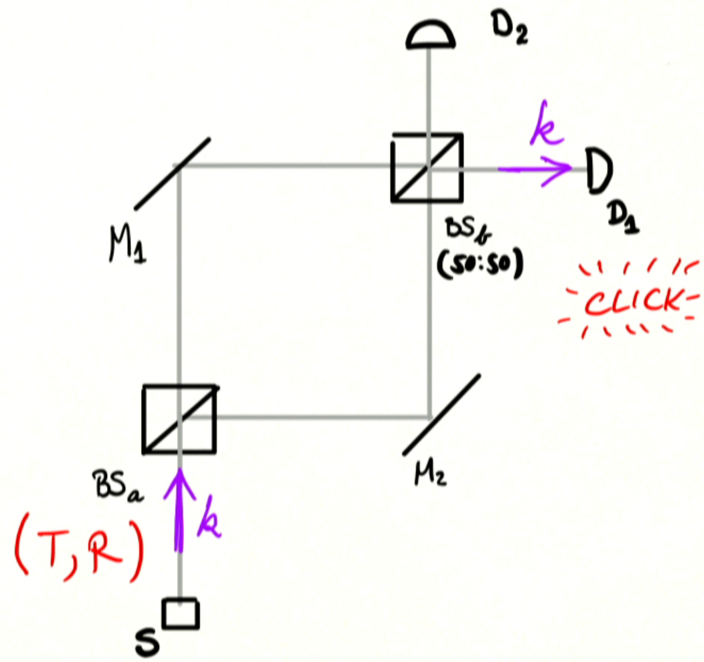
Back-action on p conjugate to x:

$$\frac{\partial S}{\partial x} = -i \int_{t_i}^{t_f} dt' \, \mathrm{Re} \frac{\langle \psi_f \left| U(t_f, t', x) \frac{\partial H}{\partial x} U(t', t_i, x) \right| \psi_i \rangle}{\langle \psi_f \left| U(t_f, t_i, x) \right| \psi_i \rangle}$$

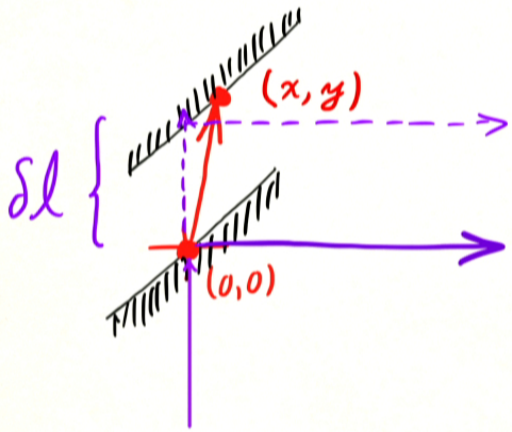
$$= -i \int_{t_i}^{t_f} dt' \, \mathrm{Re} \frac{\langle \psi_2(t', x) \left| \frac{\partial H}{\partial x} \right| \psi_1(t', x) \rangle}{\langle \psi_2(t', x) | \psi_1(t', x) \rangle}$$

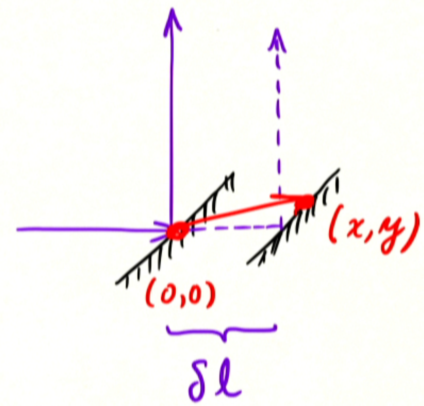
A weak value !

How much momentum is transferred to the optical elements ?



Optical length shifts



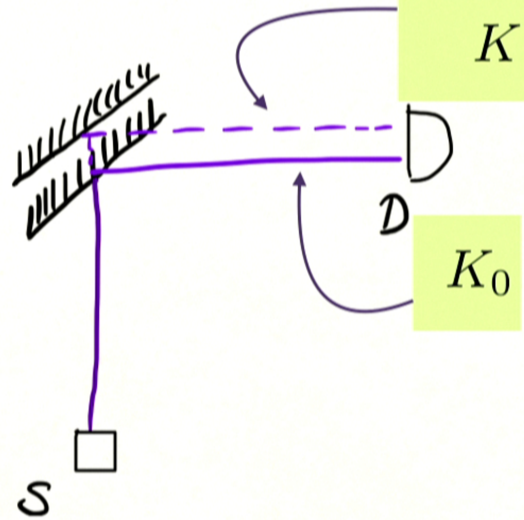


$$\delta l = y - x$$

$$\delta l = x - y$$

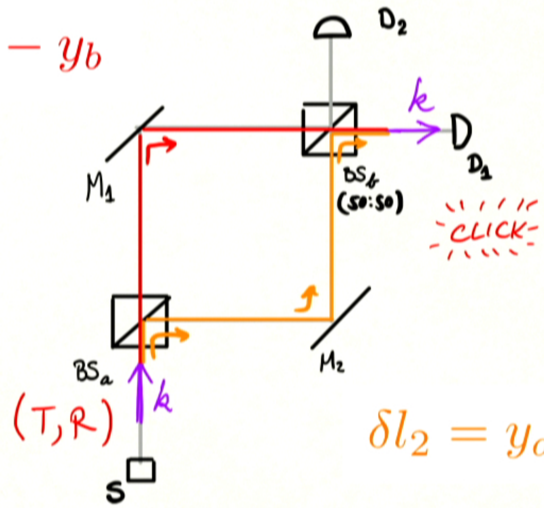
Photon amplitude is sensitive to mirror displacements

$$K(x,y) = K_0 e^{ik(y-x)}$$



The Amplitude

$$\delta l_1 = y_1 - x_1 + x_b - y_b$$



$$\delta l_2 = y_a - x_a + x_2 - y_2$$

$$K = \sqrt{T} e^{ik\delta l_1} - \sqrt{R} e^{ik\delta l_2}$$

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Momentum shifts for $\Delta x_i \ll \lambda$ (and hence $\Delta p_i \gg k$):

$$\delta p_{x_1} = \left(\frac{\partial S}{\partial x_1}\right)_{\vec{R}_0=0}$$

$$= \mathrm{Im}\left[\frac{\partial K}{\partial x_1}\frac{1}{K}\right]\Bigg|_{\vec{R}_0=0}$$

$$= \mathrm{Im}\left[\frac{-ik\sqrt{T}e^{ik\delta\ell_1}}{\sqrt{T}e^{ik\delta\ell_1}-\sqrt{R}e^{ik\delta\ell_2}}\right]\Bigg|_{\vec{R}=0}$$

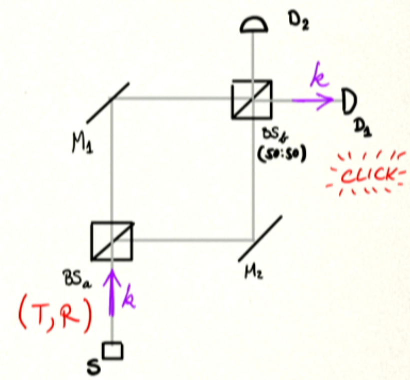
$$= -k\frac{\sqrt{T}}{\sqrt{T}-\sqrt{R}}$$

etc...

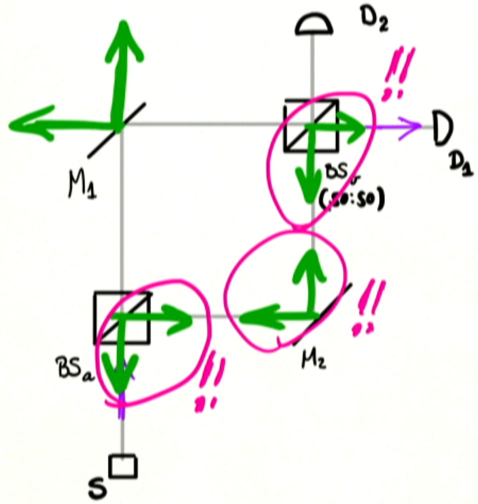
Computing all momenta this way

$\delta p_{x_1} = -k\frac{\sqrt{T}}{\sqrt{T}-\sqrt{R}}$	$\delta p_{y_1} = k\frac{\sqrt{T}}{\sqrt{T}-\sqrt{R}}$
$\delta p_{x_2} = -k\frac{\sqrt{R}}{\sqrt{T}-\sqrt{R}}$	$\delta p_{y_2} = k\frac{\sqrt{R}}{\sqrt{T}-\sqrt{R}}$
$\delta p_{x_a} = k\frac{\sqrt{R}}{\sqrt{T}-\sqrt{R}}$	$\delta p_{y_a} = -k\frac{\sqrt{R}}{\sqrt{T}-\sqrt{R}}$
$\delta p_{x_b} = k\frac{\sqrt{R}}{\sqrt{T}-\sqrt{R}}$	$\delta p_{y_b} = -k\frac{\sqrt{R}}{\sqrt{T}-\sqrt{R}}$

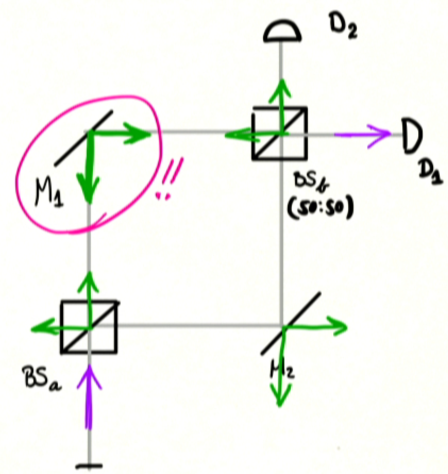
$\sum_i \delta p_{x_i} = -k$	$\sum_i \delta p_{y_i} = k$



Eccentric effects

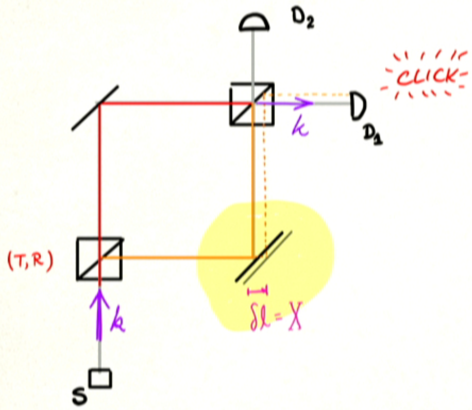


$T > R$

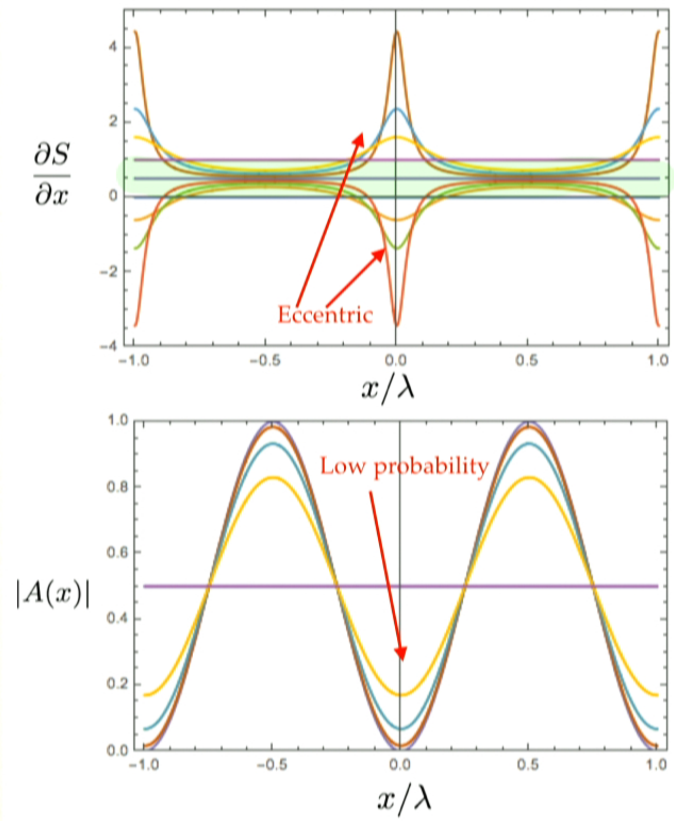


$R > T$

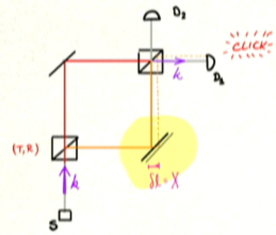
Eccentricity vs. likelihood



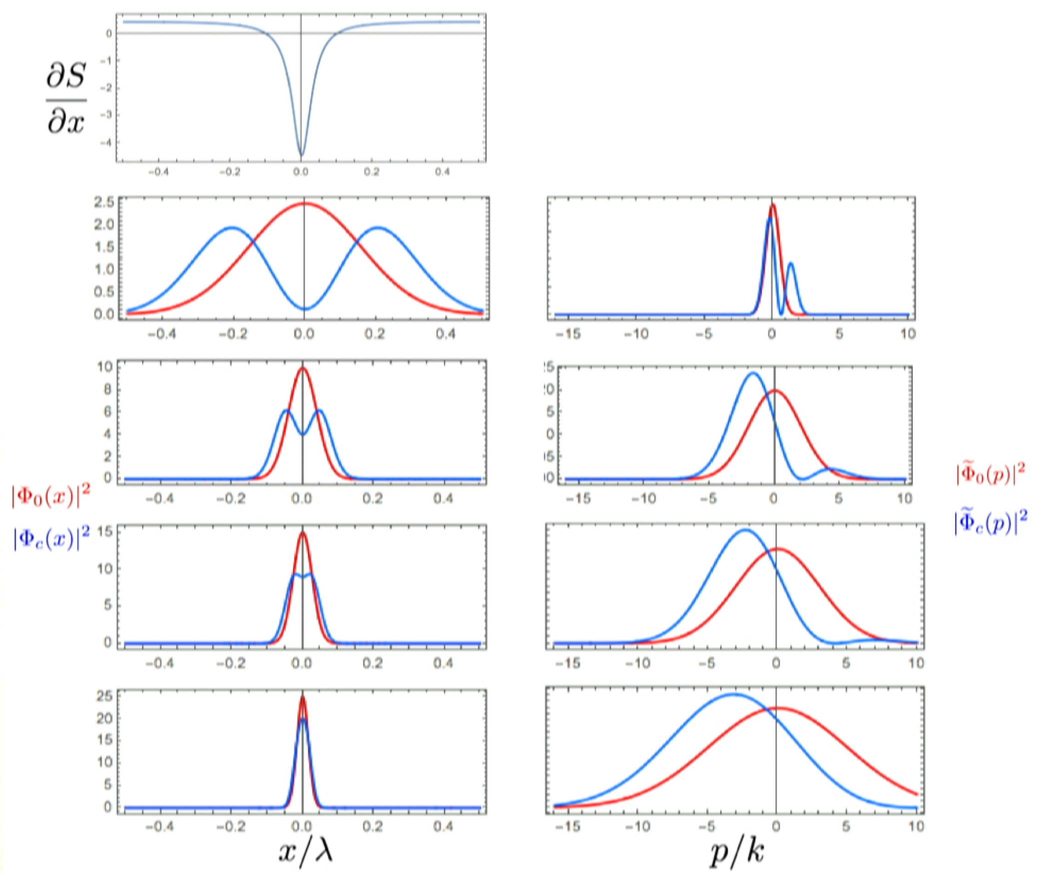
$$A(x) = \sqrt{T} - \sqrt{R}e^{ikx}$$



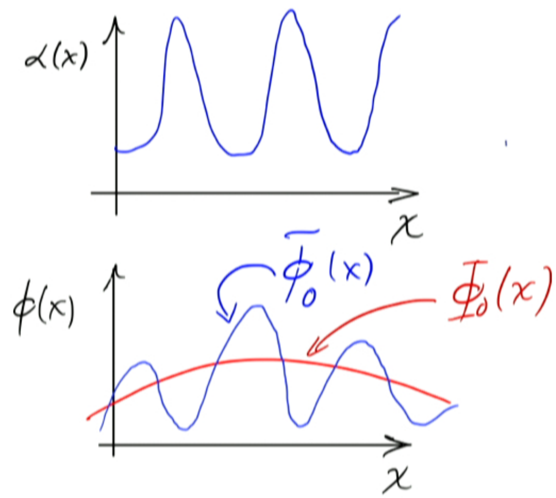
Eccentricity vs. likelihood



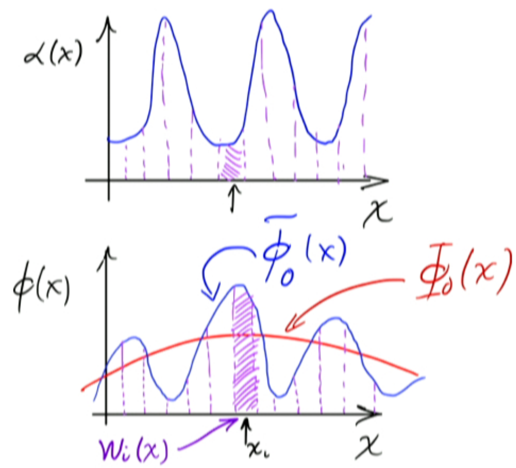
$$A(x) = \sqrt{0.6} - \sqrt{0.4}e^{ikx}$$



What about strong measurement?

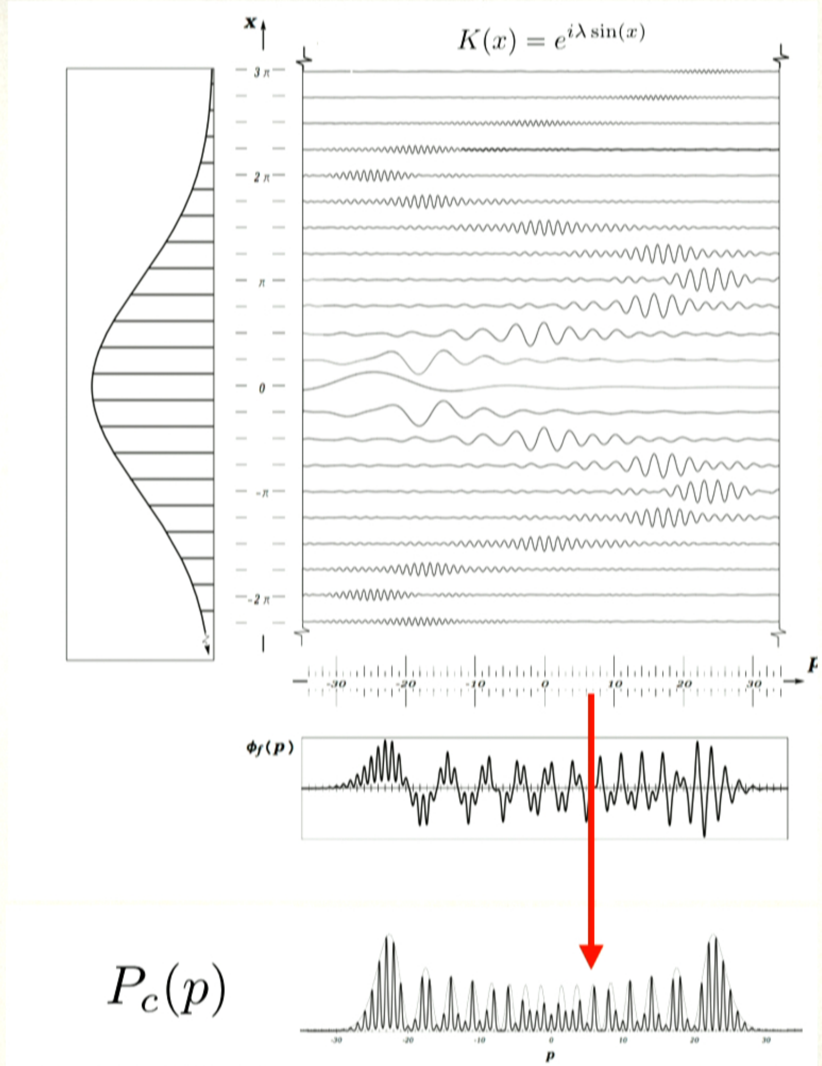


What about strong measurement?



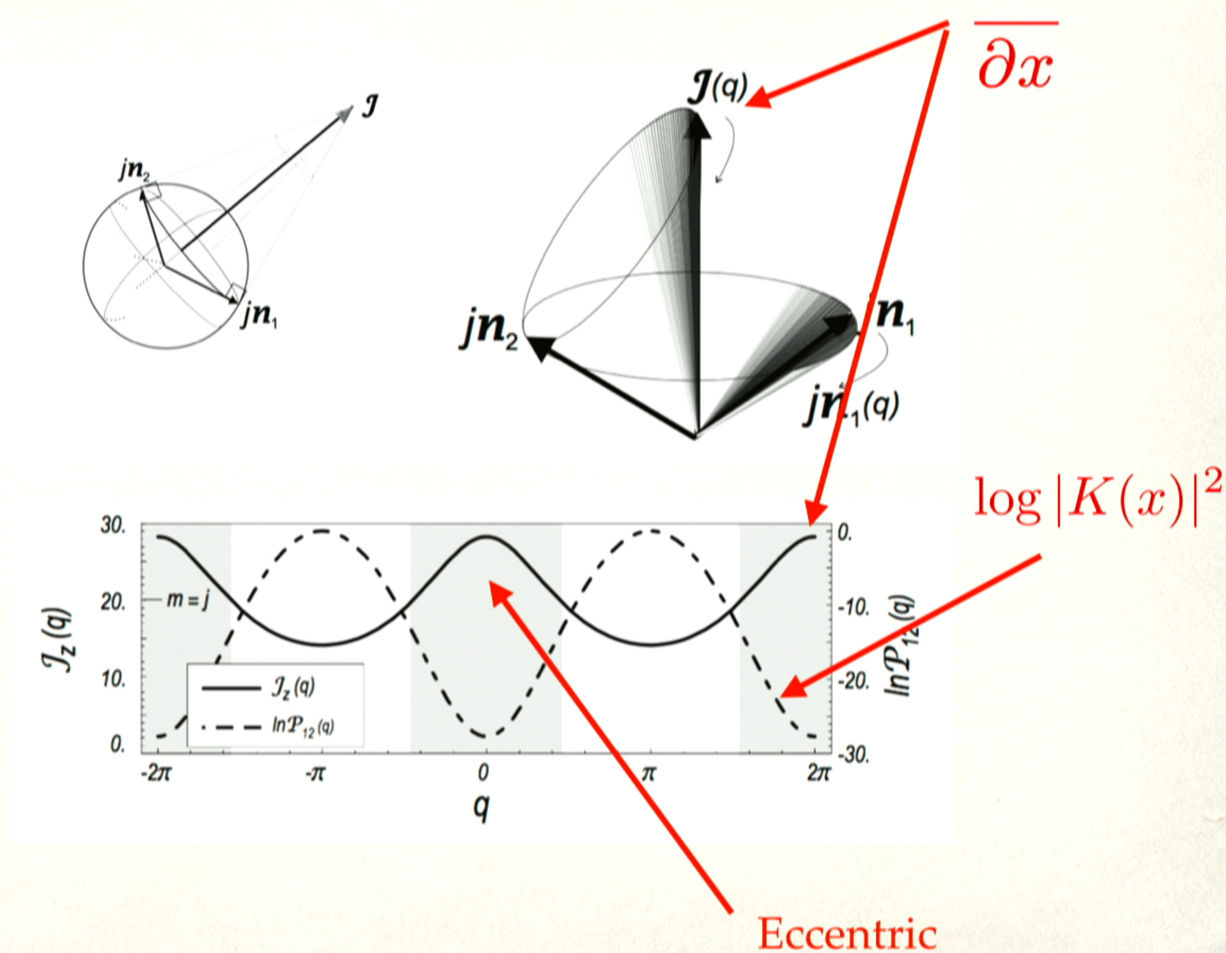
$$\Phi_c(p) \simeq \sum_i W_i(p - \alpha(x_i))e^{i\eta_i}$$

Strong measurement as a superposition of weak measurements

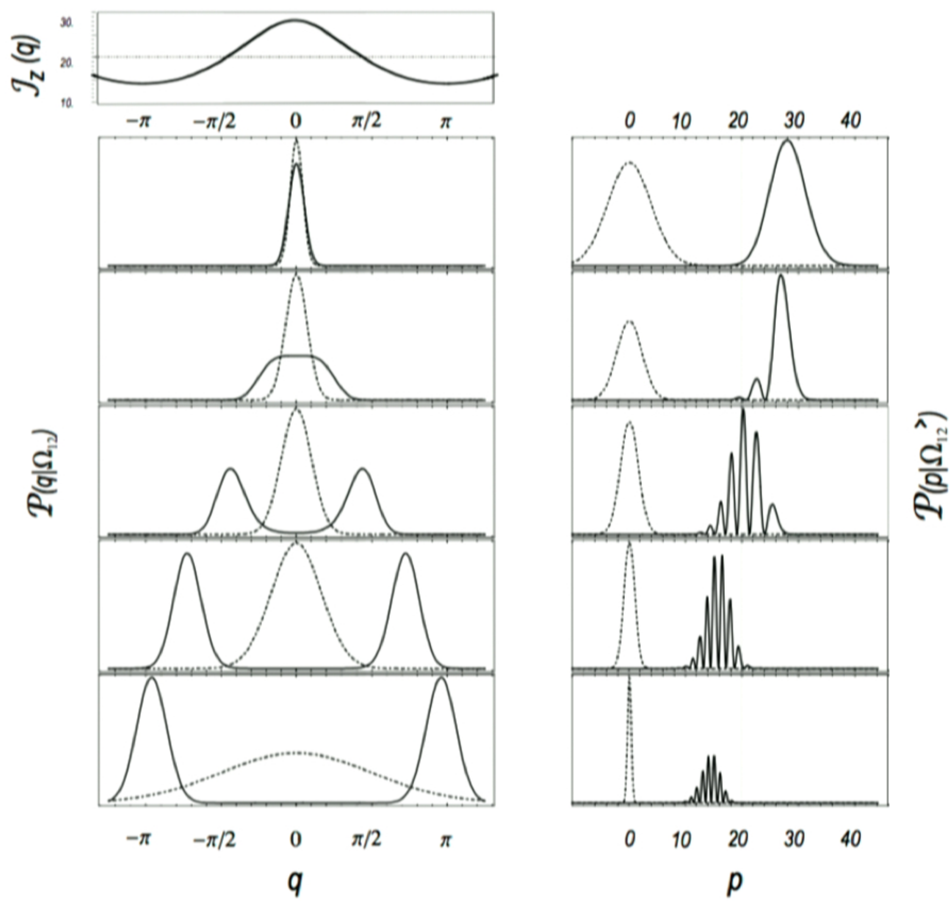


$K(x) = e^{i\lambda \sin(x)}$
x
3π
2π
π
0
$-\pi$
-2π
p
-30
-20
-10
0
10
20
30
$\phi_f(p)$
$P_c(p)$

Measuring Angular Momentum



Weak to strong transition



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Amplitudes in terms of weak values

For $|\psi\rangle \in \mathbb{C}^n$ and $U(\mathbf{x}) \in SU(n)$:

$$\langle\psi_f|U(\mathbf{x})|\psi_i\rangle = \frac{e^{iS(\mathbf{x})}}{\sqrt{|\nabla S|^2 - (n-2)/2}}$$

Sum of squares of weak values of *SU(n)* generators

A. Botero, J. Math. Phys. 44 , pp. 5279-5295 (2003) + ArXiv:math-ph/031006

Summary

- Weak values underlie the statistics of arbitrary-strength quantum measurements (not only "weak measurements")
- Back-action on a system represented classically in a quantum evolution is a weak value
- Probability amplitudes in quantum mechanics are functions of weak values