Title: Gauge invariant nonlocal nonlocal quantum dynamics of the Aharonov-Bohm effect and how it may be tested

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Abstract: The gauge invariant nonlocal quantum dynamics that is responsible for the Aharonov-Bohm effect is described. It is shown that it may be verified experimentally.

# Gauge invariant nonlocal quantum dynamics of the Aharonov-Bohm effect and how it may be tested

**Tirzah Kaufherr**

T. Kaufherr 2016, Int. j. of quantum information, in press

T. Kaufherr 2014, Quantum Studies: Mathematics and Foundations **1**, 3, 187

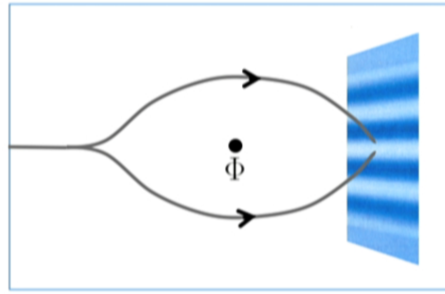
Y. Aharonov, T. Kaufherr 2004, Phys. Rev. Lett. 92, 070404

**The AB effect is the shift of the interference pattern of a charged particle moving in the field free region surrounding an infinitely long flux line.**

The shift $\alpha = -\frac{e\Phi}{\hbar c} = -\frac{e}{\hbar c}\oint \vec{A}\cdot d\vec{l},$

$\Phi$ - magnetic flux in the z direction

$e$ - charge $\vec{A}$ - vector potential




**When and how exactly does the AB effect occur?**

since the electron's wave packets have to encircle the flux line before the shift of the interference pattern may be observed

In every simply connected region the evolution of a wave packet is given by

$$\psi(\vec{r},t) = \psi_0 e^{-iS(\vec{r})} = \psi_0 e^{-i\int_{\vec{r}_0}^{\vec{r}} \vec{A}\cdot d\vec{l}}$$

$\psi_0$ - "free" solution $\frac{e}{c}\vec{A} =$ grad $S$

**In every gauge the relative phase evolves differently**

- **Modulo momentum**
- **The change of modulo velocity in the AB effect**
- **Measuring the velocity distribution**
- **A Gedanken experiment**

# Modulo momentum

Consider a particle in a superposition of two non-overlapping wave packets

$$\Psi_\alpha = \psi(x) + e^{i\alpha}\psi(x-L)$$


**Modulo momentum, p modulo $\frac{h}{L}$, is represented by the displacement operator** whose average depends on the relative phase $\alpha$:

$$\left\langle \Psi_\alpha \mid e^{\frac{i}{\hbar}pL} \mid \Psi_\alpha \right\rangle = \frac{e^{i\alpha}}{2}$$

since $[x,p] = i\hbar, \quad e^{ipL}\psi(x) = \psi(x+L).$

$< x^n >, < p^n >$ do not depend on $\alpha$.

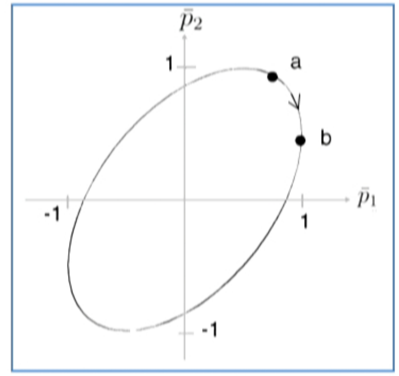
**Modulo momentum has a non-local equation of motion**

$$H = \frac{p^2}{2m} + V(x), \quad \frac{d}{dt}e^{ipL} = i\left[V(x) - V(x+L)\right]e^{ipL}$$

## Conservation law

In a collision between two particles, modulo momentum is “exchanged on an ellipse”

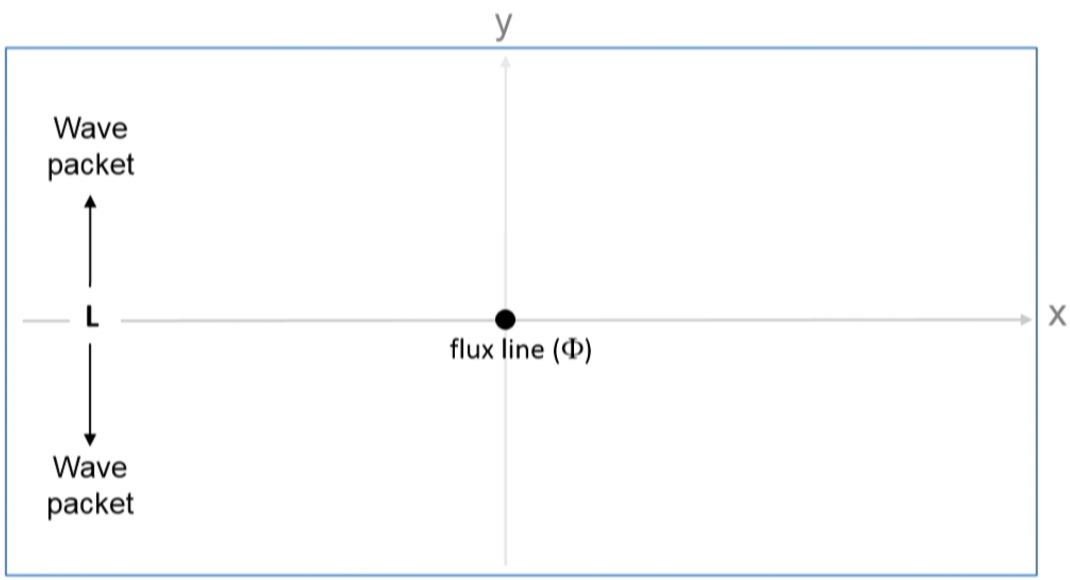
The system shifts from a to b



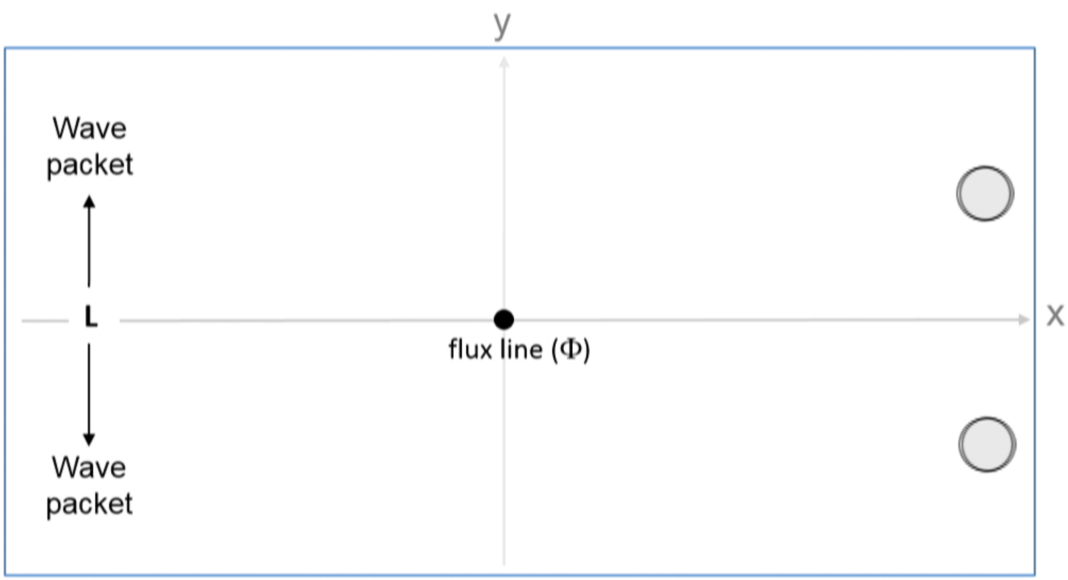

$$\bar{p}_1 \equiv \cos p_1 L$$
$$\bar{p}_2 \equiv \cos p_2 L$$

**Modulo momentum is the dynamical variable that describes interference in the Heisenberg picture**

# The change of modulo velocity in the AB effect



# The change of modulo velocity in the AB effect



- An abrupt change of the modulo velocities of the charged particle occurs, when the imaginary line connecting the wave packets crosses the flux line
- The velocity distribution changes as well
- The change of modulo velocity is responsible for the shift of the interference pattern of the AB Effect

**An abrupt change of the modulo velocities of the charged particle occurs, when the imaginary line connecting the wave packets crosses the flux line**

$$\delta\langle e^{\pm imv_yL}\rangle = \frac{1}{2}(e^{\mp i\frac{e}{c}\oint \vec{A}\cdot d\vec{l}} - 1) \equiv \frac{1}{2}(e^{\pm i\alpha} - 1)$$

$\alpha = -\frac{e\Phi}{\hbar c}$, $\Phi$ - magnetic flux in the $z$ direction, $e$ - electric charge

Having used $e^{imv_yL} = e^{i(p_y - \frac{e}{c}A_y)L} = e^{-i\frac{e}{c}\int_y^{y+L} A_y dy} e^{ip_yL}$

and Stokes's theorem

**The change occurs via non-local interaction**

**The average modulo velocity is the Fourier transform of the velocity distribution**

$$\langle e^{imv_yL}\rangle = \int P(v_y) e^{imv_yL} dmv_y$$

The corresponding velocity distributions can then be calculated

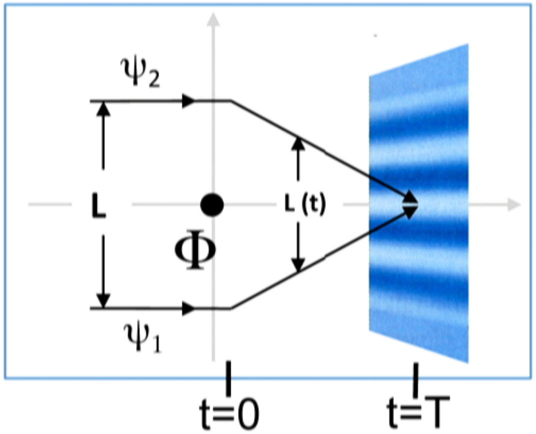
$$P_{before}(v_y) = 2P_o(v_y)\cos^2\frac{mv_yL}{2\hbar}$$

$$P_{after}(v_y) = 2P_o(v_y)\cos^2\frac{(mv_y - \alpha\frac{h}{L})L}{2\hbar}$$

$P_o(v_y)$ - velocity distribution of a single wave packet.

**The velocity distribution can be measured: the shift is observable**

## The change of modulo velocity is responsible for the shift of the interference pattern of the AB Effect



## Measuring the velocity distribution

$N$ - Total number of particles measured
$Ns$ - Number of particles for which the result $v_{y,s}$ was obtained

$$P(v_{y,s}) = \lim_{N\to\infty} \frac{N_s}{N}$$

- **To relate the change of the velocity distribution to the event, the measurement of the velocity should be performed within a very short time, and shortly after the event**
- **Measuring the velocity using two position measurements**

$$v_y(0) = \lim_{T\to 0} \frac{y(T) - y(0)}{T}$$

## Measuring the velocity distribution after the event

System:

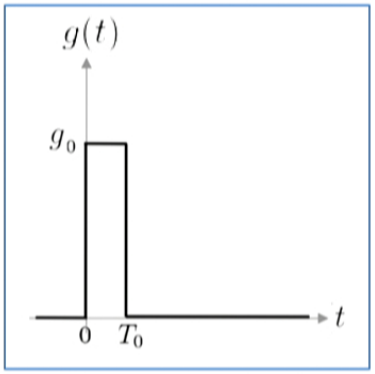
**Heavy non-recoiling measuring device** $(q, \pi_q)$

**Measured particle** m $(y, p_y)$

$$H = \frac{p_y^2}{2m} + g(t)qy - g(t-T)qy - \delta(t-T)\frac{aq^2}{2}$$

$\delta(t-T)$ - Dirac's $\delta$ function $\quad T \gg T_0$

Gate function




$$G = \int_0^{T_0} g(t)dt = g_0 T_0$$

$$G \geq 1 \quad (T_0 \to 0)$$

**After the first position measurement the particle's velocity is very uncertain:**

$$v_y \begin{pmatrix}\text{during}\\ \text{measurement}\end{pmatrix} = \frac{p_y}{m} + \frac{Gq}{m}$$

Perturbation term

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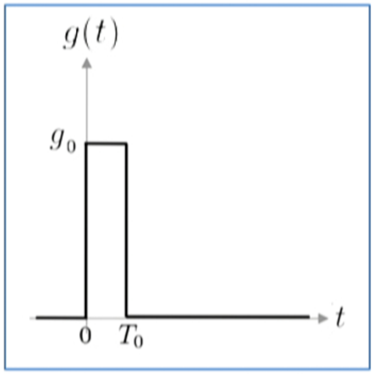


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↑ Perturbation term

The evolution of the system's state

$$|\Psi_s(y,\pi_q,t)\rangle = \mathcal{T} e^{-i\int_0^t H(t')dt'} |\Psi_s(y,\pi_q,0)\rangle$$

$|\Psi_s(y,\pi_q,0)\rangle = |\phi(\pi_q)\psi'(y,0)\rangle$ - The initial state

$\phi(\pi_q)$ - The device's wave function

$\psi'(y,0)$ - The particle's wave function

$\mathcal{T}$ - Time ordering

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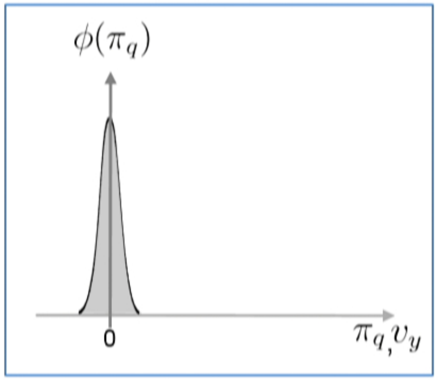
After the measurement

$$|\Psi_s(y,\pi_q,T)\rangle = e^{i\frac{aq^2}{2}} e^{iGqy} e^{-i\frac{p_y^2}{2m}T} e^{-iGqy} |\Psi_s(y,\pi_q,0)\rangle$$
$$= e^{-i\frac{p_y^2}{2m}T} e^{iG(\frac{p_y}{m}T)q} \left|\phi(\pi_q)\psi'(y,0)\right\rangle$$
$$= \int \tilde{\psi}'(p_y,0) \mid \phi(\pi_q - GT\frac{p_y}{m}) > e^{ip_y y} e^{-i\frac{p_y^2}{2m}t} \mid p_y > dp_y$$

provided $a = \frac{G^2 T}{m}$. $GT \geq 1$

The measurement




$\tilde{\psi}'(p_y,0)$ - The Fourier transform of $\psi'(y,0)$

$$\psi'(y,0) = \int \tilde{\psi}'(p_y,0) e^{ip_y y} dp_y$$

**The displacement of the device's wave packet $\phi(\pi_q)$ is proportional to the particle's velocity**

**$\pi_q$ is the device's pointer**

After the measurement

$$|\Psi_s(y,\pi_q,T)\rangle = e^{i\frac{aq^2}{2}} e^{iGqy} e^{-i\frac{p_y^2}{2m}T} e^{-iGqy} |\Psi_s(y,\pi_q,0)\rangle$$

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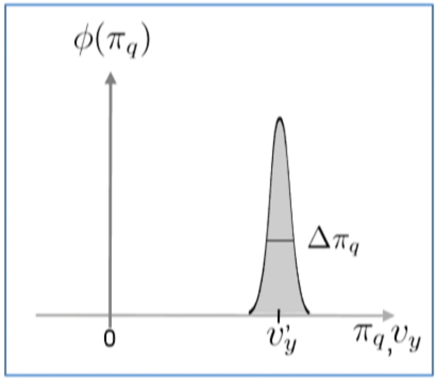


$\tilde{\psi}'(p_y,0)$ - The Fourier transform of $\psi'(y,0)$

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**The displacement of the device's wave packet $\phi(\pi_q)$ is proportional to the particle's velocity**

$\pi_q$ **is the device's pointer**

- **The pointer's displacement is proportional to the particle's velocity before the measurement**

- **The probability to measure a given value of the velocity is**

$$P(\frac{p_y}{m}) = \left|\tilde{\psi}'(p_y, 0)\right|^2$$

**This is the particle's velocity distribution before the measurement**

**Compare**

**with the distribution measured shortly before the particle passed by the flux line**

**Is it shifted as our calculation predicts?**

## Relativistic limitation on the duration of the measurement

The particle's very uncertain velocity during the measurement must not exceed the speed of light

$$T > \frac{L}{c} \sim 10^{-14} sec$$

For an electron, with $L \sim 6\mu m$ as in Tonomura's set-up

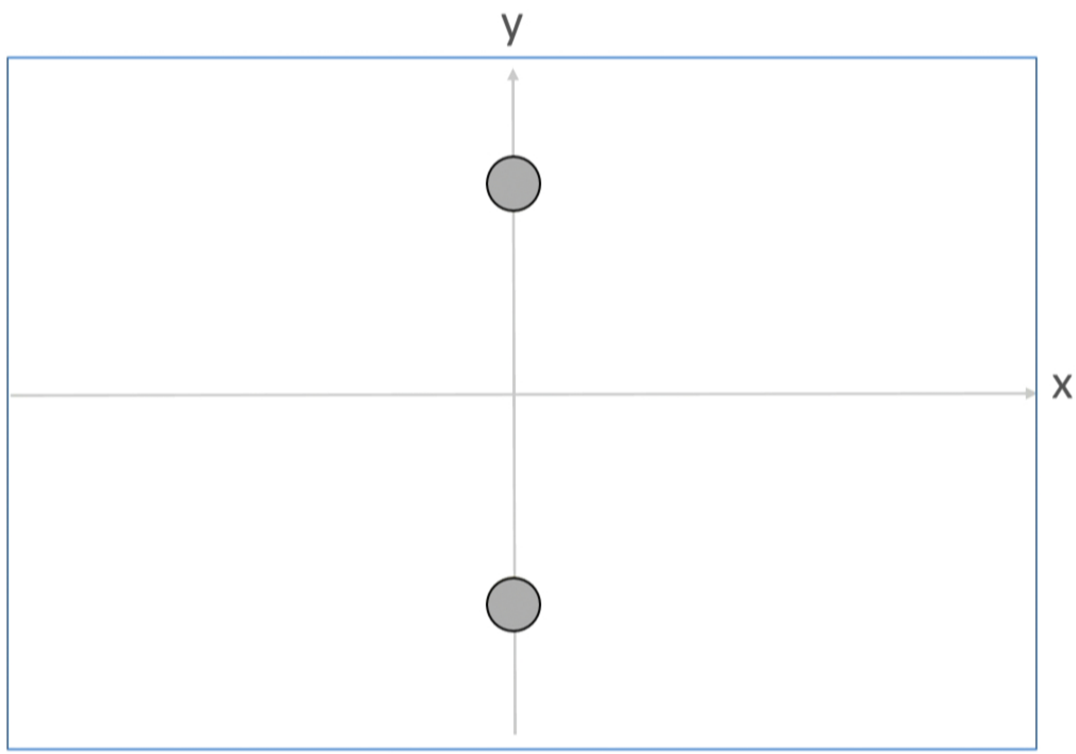
## Accuracy needed to measure in the "modulo region"

The pointer's uncertainty must satisfy

$$\Delta\pi_q \ll GT\frac{h}{mL} \approx GT \cdot 10^4 \frac{cm}{sec}$$

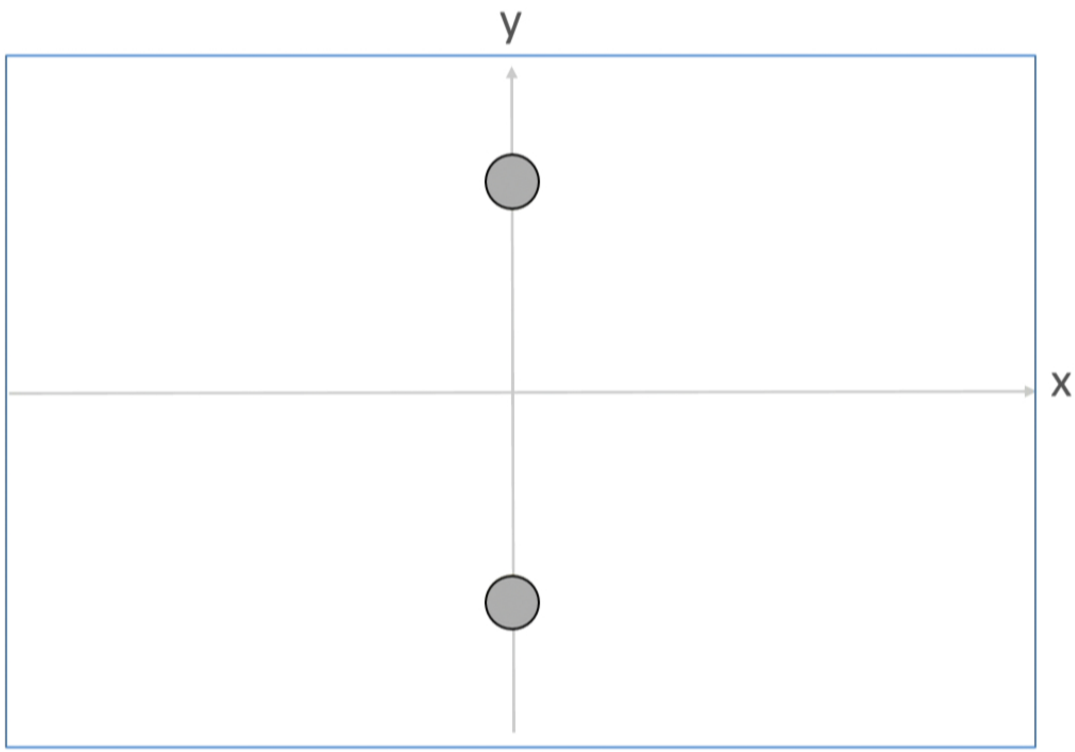
For same set-up

# A Gedanken experiment




Measuring the velocity $v_y$ in the particle's rest frame

## A Gedanken experiment




Measuring the velocity $v_y$ in the particle's rest frame

## To conclude

- **The shift of the interference pattern of the AB effect is due to an abrupt, nonlocal dynamical change of the gauge invariant modulo velocity**

- **The change may be observed by measuring the particle's velocity distribution shortly after it passed by the flux line**

**On a basic level this work relates to the role of the electromagnetic potentials in quantum theory**

Thank You