Title: Confined contextuality: How specific counterfactual paradoxes in pre- and post-selected Kochen-Specker sets give rise to experimentally observable consequences.

Date: Jun 20, 2016 03:30 PM

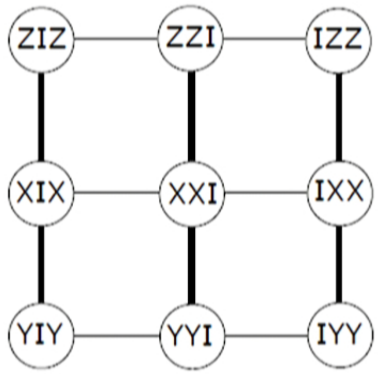
URL: http://pirsa.org/16060039

Abstract: The Kochen-Specker (KS) theorem can gives rise to logical paradoxes under pre- and post-selection in which the contextual behavior is confined to specific observables of a system. Weak measurements allow direct experimental observation of the nonclassical behavior of these specific observables. This presents an experimental advantage over other tests of KS inequalities which rule out a particular class of counterfactual noncontextual hidden variable models, but can never specify where the contradiction occurs, nor make any direct observation of its consequences. The confined contextuality can always be interpreted as a logical pre- and post-selection paradox, such as the 3-box paradox, the Quantum Cheshire Cat, or the Quantum Pigeonhole Effect. This confined contextuality was recently observed using neutron inferometry for KS sets of up to 17 qubits. Details of the theory and experimental results will be presented.

# Overview

- The Bell-Kochen-Specker (BKS) theorem shows that quantum mechanics forbids a noncontextual hidden variable theory (NCHVT) with outcome determinism
- This is because the predictions of any such NCHVT disagree with quantum predictions by violating the 'sum rule' or 'product rule' in at least one measurable basis, and such violations are never observed in an experiment.
- The only way that the NCHVT can remain plausible is if the violation is counterfactual, in the sense that is is never located in the basis that is measured.
- By considering sub-ensembles that are both pre- and post-selected, it is possible to confine this violation to a specific basis, where it can be observed by performing weak measurements of the projectors in that basis, and finding anomalous weak values.

# The 3-qubit Square




- Each row and column of the Square is a set of three mutually commuting observables that define a joint measurement basis.
- Any NCHVT with outcome determinism must assign a predicted eigenvalue $\pm 1$ to each of the nine observables of the square. These assignments are noncontextual because the prediction is the same if either the row or column is measured.

# The Weak Value

We call an ensemble of experiments in which a given state $|\psi\rangle$ is prepared in each run, and the outcome $|\phi\rangle$ is obtained by a final measurement, a pre- and post-selected ensemble. For such an ensemble, the weak value of every observable $\hat{A}$ of a system, with spectral decomposition $\hat{A} = \sum_i \lambda_i \Pi^i$, is defined as,

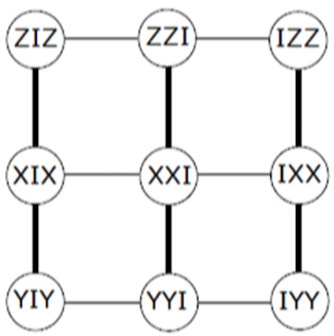
$$\hat{A}_w \equiv \frac{\langle\phi|\hat{A}|\psi\rangle}{\langle\phi|\psi\rangle} = \sum_i \lambda_i \frac{\langle\phi|\Pi^i|\psi\rangle}{\langle\phi|\psi\rangle} = \sum_i \lambda_i \Pi^i_w. \tag{1}$$

If a pointer is weakly coupled to the system during the time between pre- and post-selection, such that the interaction introduces almost no disturbance, the ensemble average shift of the pointer is given by the weak value.

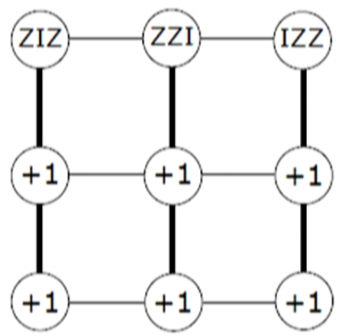
A recent result from Pusey [Phys. Rev. Lett. 113, 200401 (2014)] shows that any projector with a weak value whose real part is above 1 or below 0 is a proof of contextuality, using Spekkens generalized definitions of noncontextuality [Physical Review A 71.5 (2005): 052108].

# Confined Contextuality

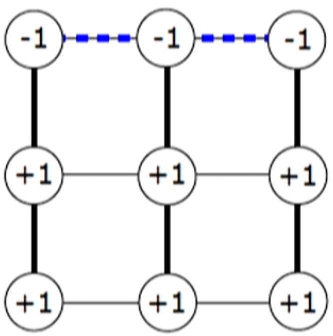
- Choosing a pre-selection $|\psi\rangle = |+X\rangle|+X\rangle|+X\rangle$ and post-selection $|\phi\rangle = |+Y\rangle|+Y\rangle|+Y\rangle$ fixes some of the predictions of the 3-qubit Square.
- From the Aharonov-Bergmann-Lebowitz (ABL) probability rule, it also follows that certain other eigenvalue assignments are forced, which results in a violation of the product rule confined to a specific basis.
- We directly observed this confined contextuality in a neutron interferometer setup by measuring anomalous weak values of projectors in this basis.



(a) No Predictions



(b) Pre- and Post-Selection



(c) ABL Rule

## Projectors in the Classical Basis

The rank-2 projectors in the $Z$ basis (or classical basis) of the 3-qubit Square can be written as,

$$B_C^3 = \left\{ \begin{array}{l} |+Z,+Z,+Z\rangle\langle +Z,+Z,+Z| + |-Z,-Z,-Z\rangle\langle -Z,-Z,-Z|, \\ |+Z,-Z,+Z\rangle\langle +Z,-Z,+Z| + |-Z,+Z,-Z\rangle\langle -Z,+Z,-Z|, \\ |-Z,+Z,+Z\rangle\langle -Z,+Z,+Z| + |+Z,-Z,-Z\rangle\langle +Z,-Z,-Z|, \\ |-Z,-Z,+Z\rangle\langle -Z,-Z,+Z| + |+Z,+Z,-Z\rangle\langle +Z,+Z,-Z| \end{array} \right\}.$$

The weak value of the first projector in $B_C^3$ is,

$$\Pi_w^1 = \frac{\langle\phi|\Pi^1|\psi\rangle}{\langle\phi|\psi\rangle} = \prod_{j=1}^{\otimes 3}\left(\frac{\langle +Y|+Z\rangle\langle +Z|+X\rangle}{\langle +Y|+X\rangle}\right)^j + \prod_{k=1}^{\otimes 3}\left(\frac{\langle +Y|-Z\rangle\langle -Z|+X\rangle}{\langle +Y|+X\rangle}\right)^k$$

$$= \prod_{j=1}^{\otimes 3}(|+Z\rangle\langle +Z|)_w^j + \prod_{k=1}^{\otimes 3}(|-Z\rangle\langle -Z|)_w^k = -\frac{1}{2}$$

# Single-Qubit Weak Measurements

This factorization shows that when weakly measuring these projectors, each qubit can be measured independently. We measured the weak values of each $Z^j$ (the $j$th qubit) separately in our experiment, and computed the weak values of the single-qubit projectors as,

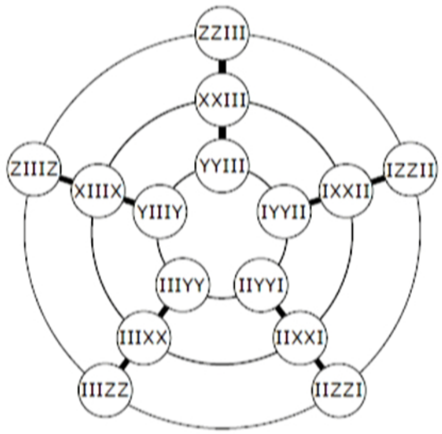
$$(|\pm Z\rangle\langle\pm Z|)^j_w = (1 \pm Z^j_w)/2.$$

This method enabled us to compute the weak value of any 3-qubit projector in $B^3_C$ from single-qubit measurement data.

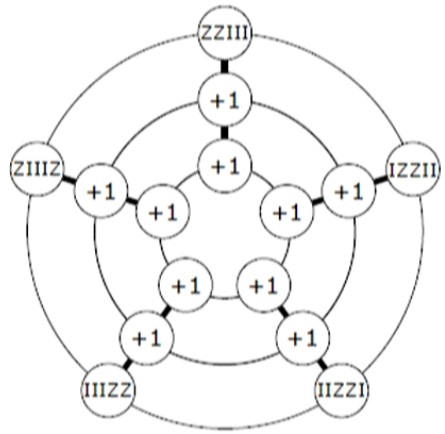
The 3-qubit Square can be generalized into the $N$-qubit Wheel for all odd $N \geq 3$, and all of the preceding logic for the 3-qubit cases follows exactly. We measured $Z^j_w \approx i$ for 17 different qubits, which were actually 17 different ensembles of many runs through our apparatus, and thus we consider all $N$-qubit Wheels for odd $3 \leq N \leq 17$.

# 5-qubit Wheel

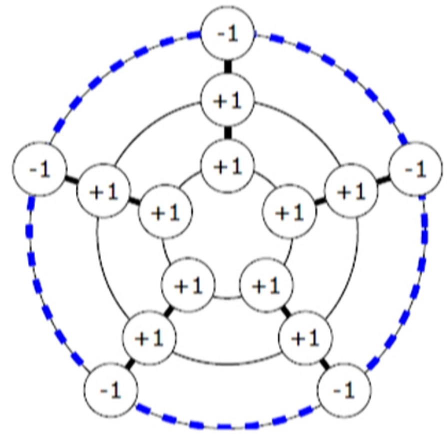
For $N$ qubits the pre-selection is $|\psi\rangle = |+X\rangle^{\otimes N}$, and the post-selection is $|\phi\rangle = |+Y\rangle^{\otimes N}$.




(d) No Predictions




(e) Pre- and Post-Selection




(f) ABL Rule

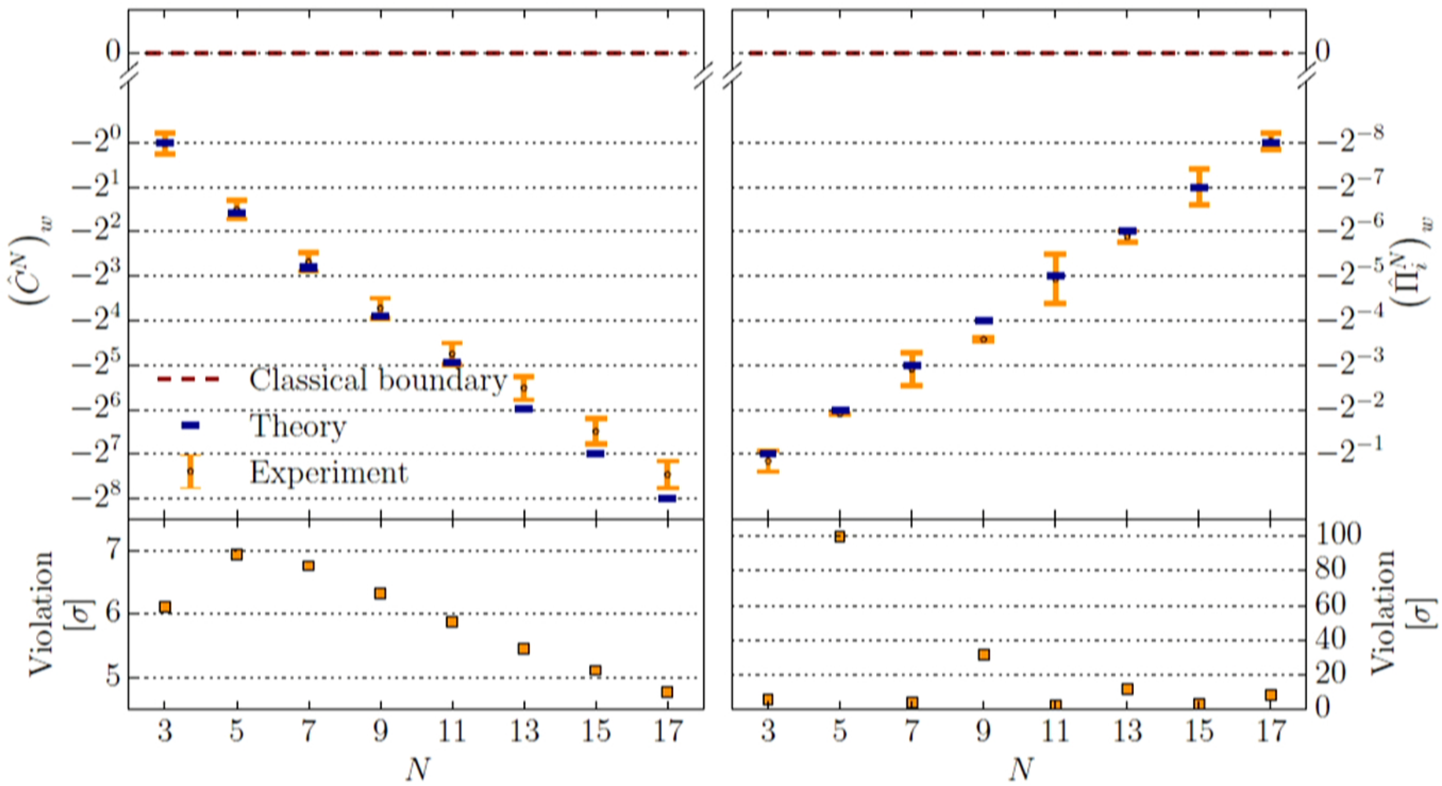
## Contextuality Witness

For each of the $N$-qubit Wheels we obtain a classical basis $B_C^N = \{\Pi^i\}$ containing $2^{N-1}$ rank-2 projectors. We construct the general contextuality observable $\hat{\mathcal{C}}^N$ for any specific choice of basis $\{\Pi^i\}$ and pre- and post-selection as,

$$\hat{\mathcal{C}}^N = I - \sum_{i=1}^{2^{N-1}} s_i \Pi^i, \tag{4}$$

with $s_i = \text{sign}[\text{Re}(\Pi_w^i)]$, using the theoretically predicted value of $\Pi_w^i$. Regardless of the signs $s_i$, if all $0 \leq \text{Re}(\Pi_w^i) \leq 1$ (noncontextual), then $\text{Re}(\hat{\mathcal{C}}_w^N) \geq 0$. Therefore, any $\text{Re}(\hat{\mathcal{C}}_w^N) < 0$ is a proof of contextuality.

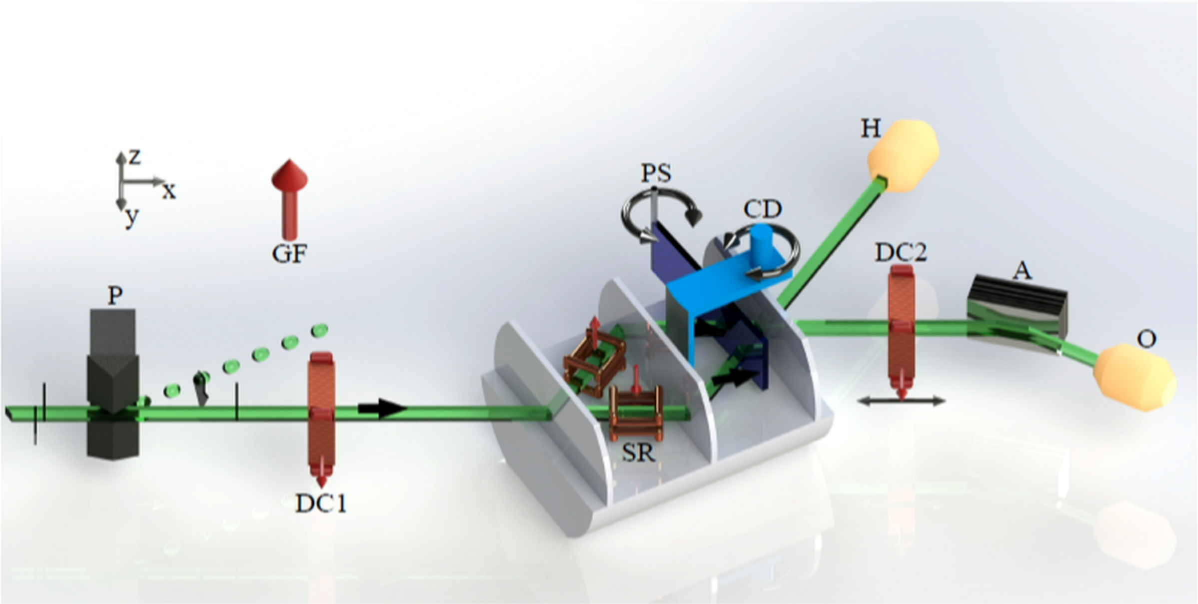
# Results



Classical boundary
Theory
Experiment
$(\hat{C}^N)_w$
$(\hat{\Pi}_i^N)_w$
Violation $[\sigma]$
$N$

# Experimental Setup

The experiment was conducted at the Institut Laue-Langevin in Grenoble France using neutron interferometry techniques. The quantum system in the experiment was the neutron spin degree of freedom (DOF), and the weak measurements were performed by coupling this to the path DOF in the interferometer.



## Generalized Contextuality Witness

The confined contextuality proof we used in our experiment can be generalized to a test that uses all pre- and post-selections that occur in the experimental setup, such that the complete ensemble is considered in every experiment. Let the pre-selection $|\psi_j\rangle$ be any of the $d = 2^N$ possible outcomes of an initial measurement of $\prod_j^{\otimes N} X_j$ on $N$ qubits, and also let the post-selection $|\phi_k\rangle$ be any of the $d$ possible outcomes of a final measurement of $\prod_k^{\otimes N} Y_k$. For each case, the Contextuality Witness can be written as,

$$\hat{C}_{jk}^N = I - \sum_{i=1}^{2^{N-1}} s_{ijk}\Pi^{ijk}. \tag{5}$$

Combining the data from all runs of the experiment, we arrive at the quantity,

$$W = \sum_j^d \sum_k^d \frac{\langle\phi_k|\hat{C}_{jk}^N|\psi_j\rangle}{\langle\phi_k|\psi_j\rangle} = d^2 I - \sum_{i,j,k} s_{ijk}\Pi_w^{ijk}, \tag{6}$$

and any $W < 0$ is a proof of contextuality.

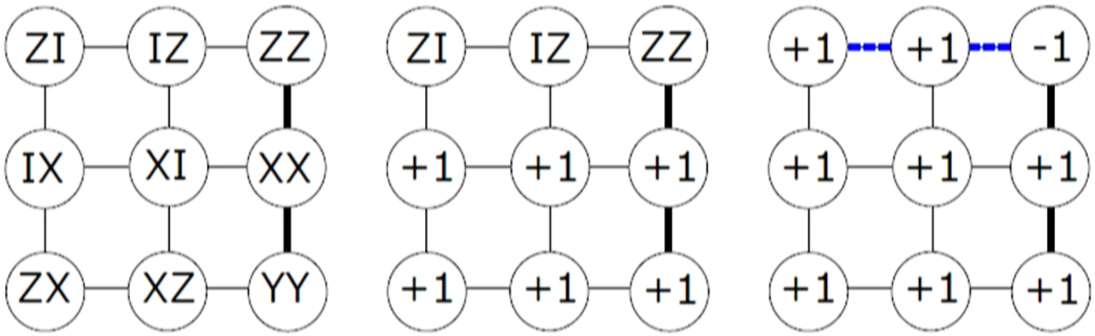
# The Quantum Pigeonhole Effect

We now consider the ontological interpretation of these eigenvalue assignments to the observables of the $N$-qubit Wheel, including the logical inconsistency of the product rule being violated in the classical basis.
The pigeonhole principle states that if one places all of $M > N$ pigeons into $N$ boxes, then at least one box must contain more than one pigeon.

Let each qubit be a pigeon, and consider the two $Z$ eigenstates as $|+Z\rangle \equiv |L\rangle$ and $|-Z\rangle \equiv |R\rangle$ as corresponding to two boxes, left and right. The correlation $ZZ = -1$ between every pair of qubits in this model, obtained using the ABL rule, then implies that regardless of how many pigeons we place in the two boxes, no two pigeons are ever in the same box, in direct violation of the pigeonhole principle. We call this violation the quantum pigeonhole effect.

# The Quantum Cheshire Cat

We can also consider confined contextuality in the Peres-Mermin Square, a different proof of the BKS theorem for 2 qubits, by pre- and post-selecting the states that assign eigenvalues +1 to six observables of the two bottom rows.




(g) No Predictions (h) Pre- and Post-Selection (i) ABL Rule

# The Quantum Cheshire Cat (continued)

The ABL rule has forced us to conclude that $ZI = +1$, $IZ = +1$, and $ZZ = -1$, which is an obvious violation of the product rule. We again consider an ontological interpretation of these contradictory eigenvalue assignments.

Let the first qubit represent the path ($|L\rangle$ or $|R\rangle$) a neutron takes through an interferometer, and the second qubit represent its spin ($|\uparrow\rangle$ or $|\downarrow\rangle$). From the first two forced values, we would conclude that neutron takes the left path and has spin up, however from the last term we would conclude that spin up can only be on the right path (or spin down on the left). This logical inconsistency can be interpreted as indicating that the spin becomes disembodied from the neutron, allowing the spinless neutron to take the left path, while the neutronless spin takes the right path. We refer to this phenomenon as the quantum Cheshire Cat.

# Pre- and Post-Selection Paradoxes

There is also a general class of logical pre- and post-selection paradoxes were developed using the time symmetric formalism of quantum mechanics, including the 3-box paradox. These paradoxes are 'logical' in the sense that the relevant projectors are assigned only values 0 and 1 by the pre- and post-selection and the ABL rule.

It has recently been shown by Leifer and Pusey [arXiv:1506.07850] that every logical pre-and post-selection paradox can be generalized to a state-dependent proof that rules out measurement-noncontextual models with outcome determinism. Every such proof depends crucially on projectors that have anomalous weak values given the pre-and post-selection, and so these are simply further examples of confined contextuality.

## The 3-box Paradox

Consider a pre-selection $|\psi\rangle = (|1\rangle + |2\rangle + |3\rangle)/\sqrt{3}$ and post-selection $|\psi\rangle = (|1\rangle + |2\rangle - |3\rangle)/\sqrt{3}$, such that a particle begins and ends with equal probability 1/3 to be found in each of three boxes. The weak values of the projectors onto the three boxes are, $|1\rangle\langle 1|_w = 1$, $|2\rangle\langle 2|_w = 1$, and $|3\rangle\langle 3|_w = -1$.

To arrive at the paradox, we use the ABL rule for two different coarse-grained bases with cardinality 2: The basis $B_1 = (|1\rangle\langle 1| + |3\rangle\langle 3|, |2\rangle\langle 2|)$, and the basis $B_2 = (|2\rangle\langle 2| + |3\rangle\langle 3|, |1\rangle\langle 1|)$. The ABL formula gives probabilities,

$$P_{ABL}(|2\rangle\langle 2| = 1|B_1) = P_{ABL}(|1\rangle\langle 1| = 1|B_2) = \frac{|1|^2}{|1-1|^2 + |1|^2} = 1, \tag{7}$$

but these two statements are contradictory - the particle cannot always be found in both boxes. Note that without the negative weak value of projector $|3\rangle\langle 3|$ it would be impossible to construct this paradox or any other like it.

## Interpreting the 3-box Paradox

Instead of assigning ontological realism only to ABL-forced eigenvalues of projectors, we may consider treating the weak value as ontological assignments. For the 3-box paradox, the interpretation is that boxes 1 and 2 literally do always contain the a particle, while box 3 contains a 'phantom' particle with negative properties for all of its observables that give rise to the weak value $|3\rangle\langle 3|_w = -1$. The sum of the weak values of the three projectors in this basis must be still 1, and so we can conclude that there was a pair creation from the vacuum of an extra particle and its negative counterpart, and that this pair annihilated prior to the post-selection.

If we consider the confined contextuality in the 3-qubit Square and the Peres-Mermin Square, the weak values in the classical bases are $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$, which give rise to similar 4-box paradoxes, but we obtain an ontological interpretation in terms of phantom particles which is quite different from the pigeonhole effect or the Cheshire cat. Determining which of these ontologies is more physical remains an interesting question.

# Conclusions

- We have shown that pre- and post-selection can be used with certain proofs of the BKS theorem to confined the conflict between quantum mechanics and noncontextual realism to specific observables of the system.
- We have shown that this confined contextuality can be observed by performing weak measurements of those observables in a neutron interferometry setup and obtaining anomalous weak values.
- We obtained the conditional correlations $ZZ \approx -1$ for all pairs of neutron spins, even though these neutrons never interacted, nor in fact were they even in the interferometer at the same time.
- We have examined ontological interpretations of confined contextuality, including the quantum pigeonhole effect, the Cheshire cat, and the 3-box paradox with phantom particles.