Title: How to count one photon and get a(n average) result of 1000...

Date: Jun 20, 2016 10:00 AM

URL: http://pirsa.org/16060034

Abstract: I will present our recent experimental work using electromagnetically induced transparency in laser-cooled atoms to measure the nonlinear phase shift created by a single post-selected photon, and its enhancement through "weak-value amplification." Put simply, due to the striking effects of "post-selective" quantum measurements, a (very uncertain) measurement of photon number can yield an average value much larger than one, even when it is carried out on a single photon. I will say a few words about possible practical applications of this "weak value amplification" scheme, and their limitations.
Time permitting, I will also describe other future and past work using "weak measurement," such as our studies quantifying the disturbance due to a measurement and what happens when it destroys interference; and our project to measure "where a particle has been" as it tunnels through a classically forbidden region â€“ our prediction being that it will make it from one side of the barrier to the other without spending any significant time in the middle.

Outline: weak measurement & weak value amplification

How to count a single photon and get a result of 1000

SOME INTRO / REMINDERS

(0) Conditional measurements

Why should we be interested?

Connections to other quantities of interest

(1) Do they tell us something more about reality?

Welcher Weg question: momentum transfer

(2) Addressing new questions

Precision-disturbance relations, etc.

WHEN IS 1 PHOTON WORTH MORE THAN 1 PHOTON?

(3) Amplifying the effect of a single photon

Operational significance made clear.

Fully quantum regime.

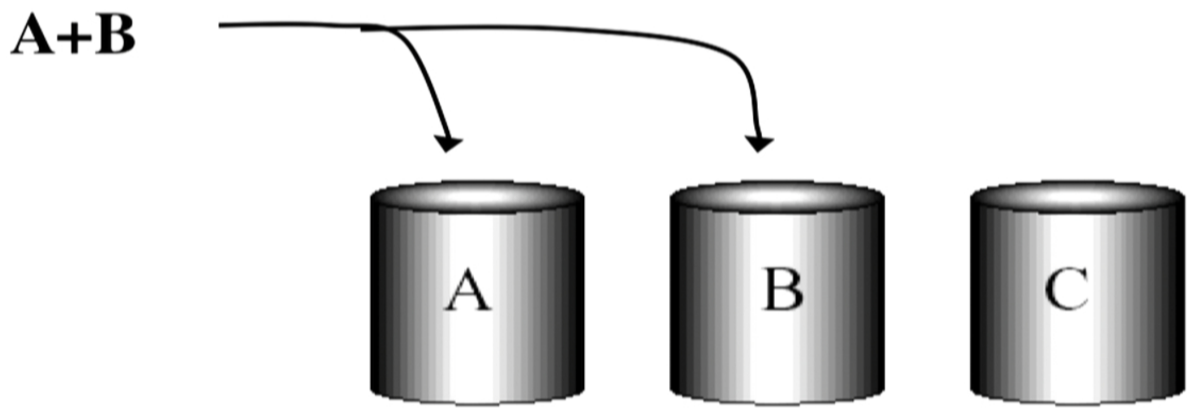
[Practical uses of WVA?]

FUTURE: (4) *New experiment still ahead: tunneling times*

Can we ask what a photon was doing before we observed it?

(How should one describe post-selected states?)

Predicting the past...



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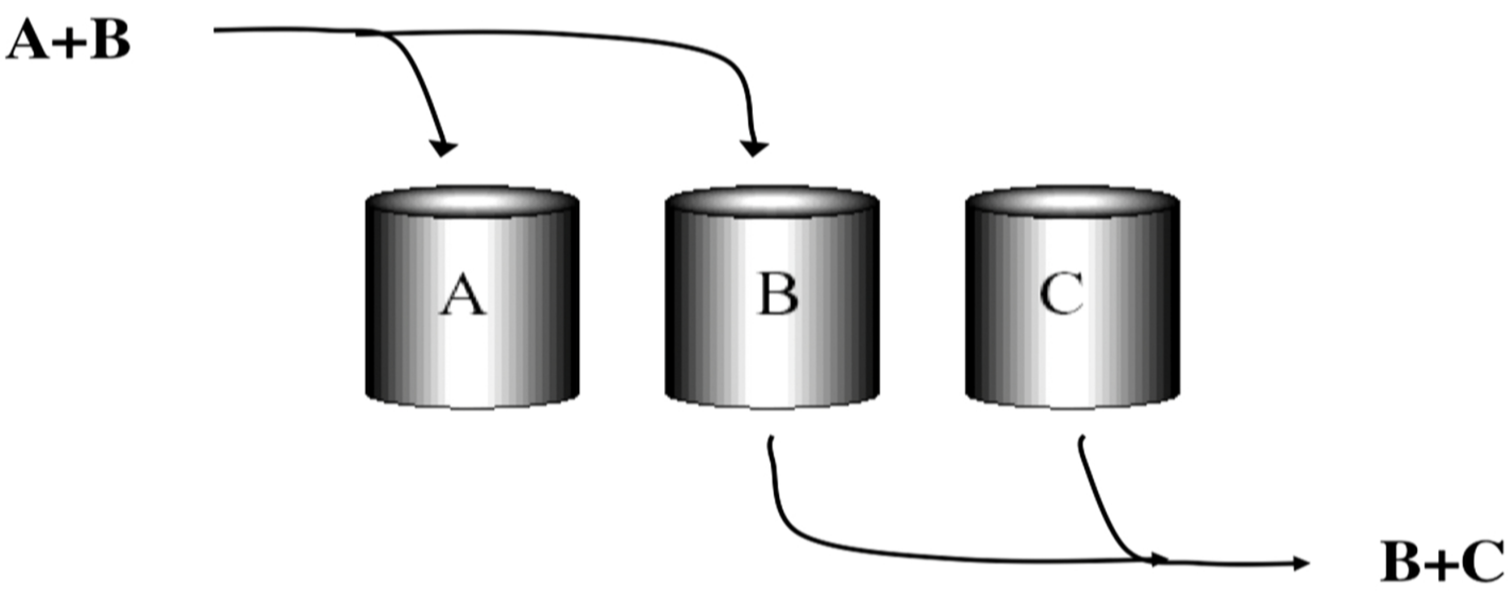

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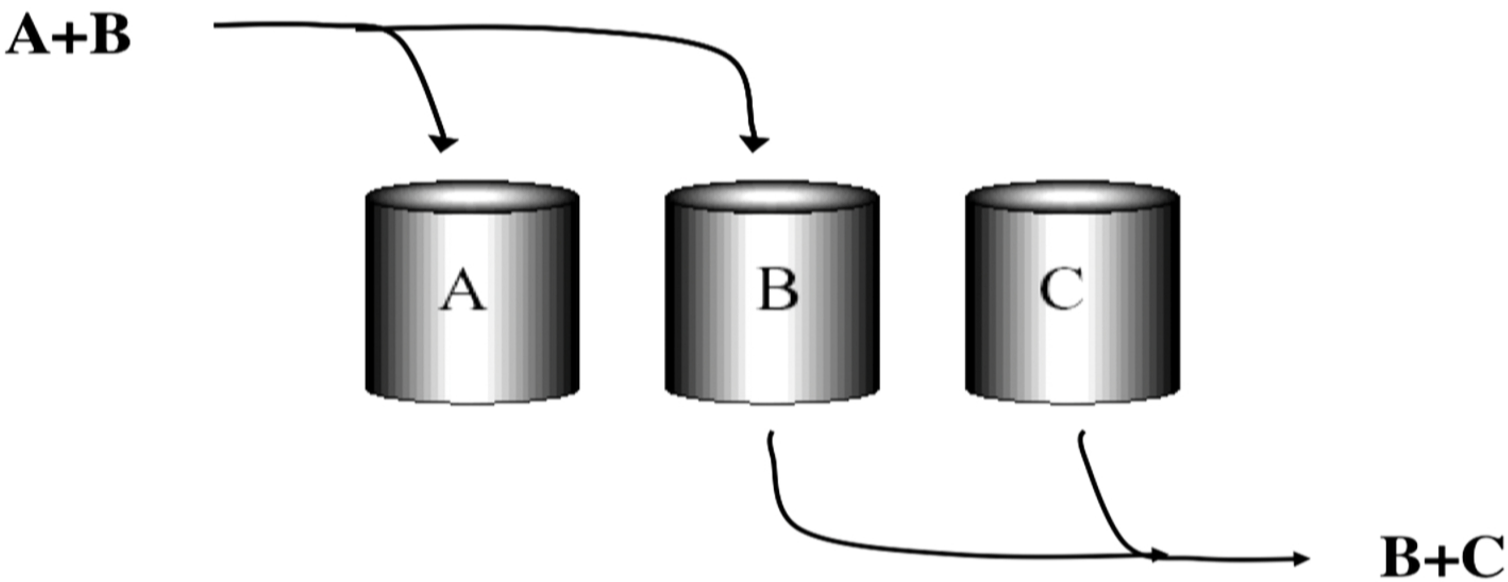

What are the odds that the particle was in a given box (e.g., box B)?

It had to be in B, with 100% certainty.

Bayesian Approach to Conditional Expectation Values

$$P(A|B) \equiv \frac{P(A\&B)}{P(B)} \,. \qquad \langle A\rangle_{\rm wk} = \sum_j a_j P(j|i,\, f),$$

$$\begin{aligned} P(A) &= \langle Proj(A)\rangle = \langle |A\rangle\langle A|\rangle \\ &= \langle\psi|A\rangle\langle A|\psi\rangle = |\langle A|\psi\rangle|^2 \,. \end{aligned}$$

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$$P(A\&B) = \langle Proj(B)Proj(A) \rangle = \langle \psi|B\rangle\langle B|A\rangle\langle A|\psi\rangle$$

$$P(A_i|f) = \frac{\langle \mathrm{Proj}(f)\mathrm{Proj}(A_i) \rangle}{\langle \mathrm{Proj}(f) \rangle} .$$

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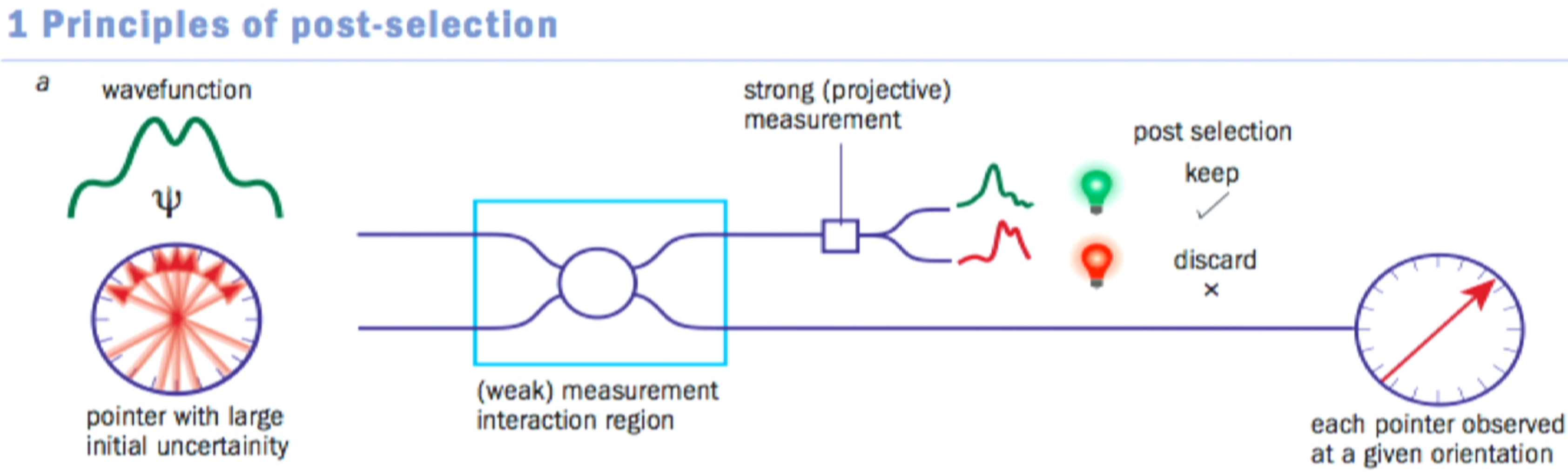
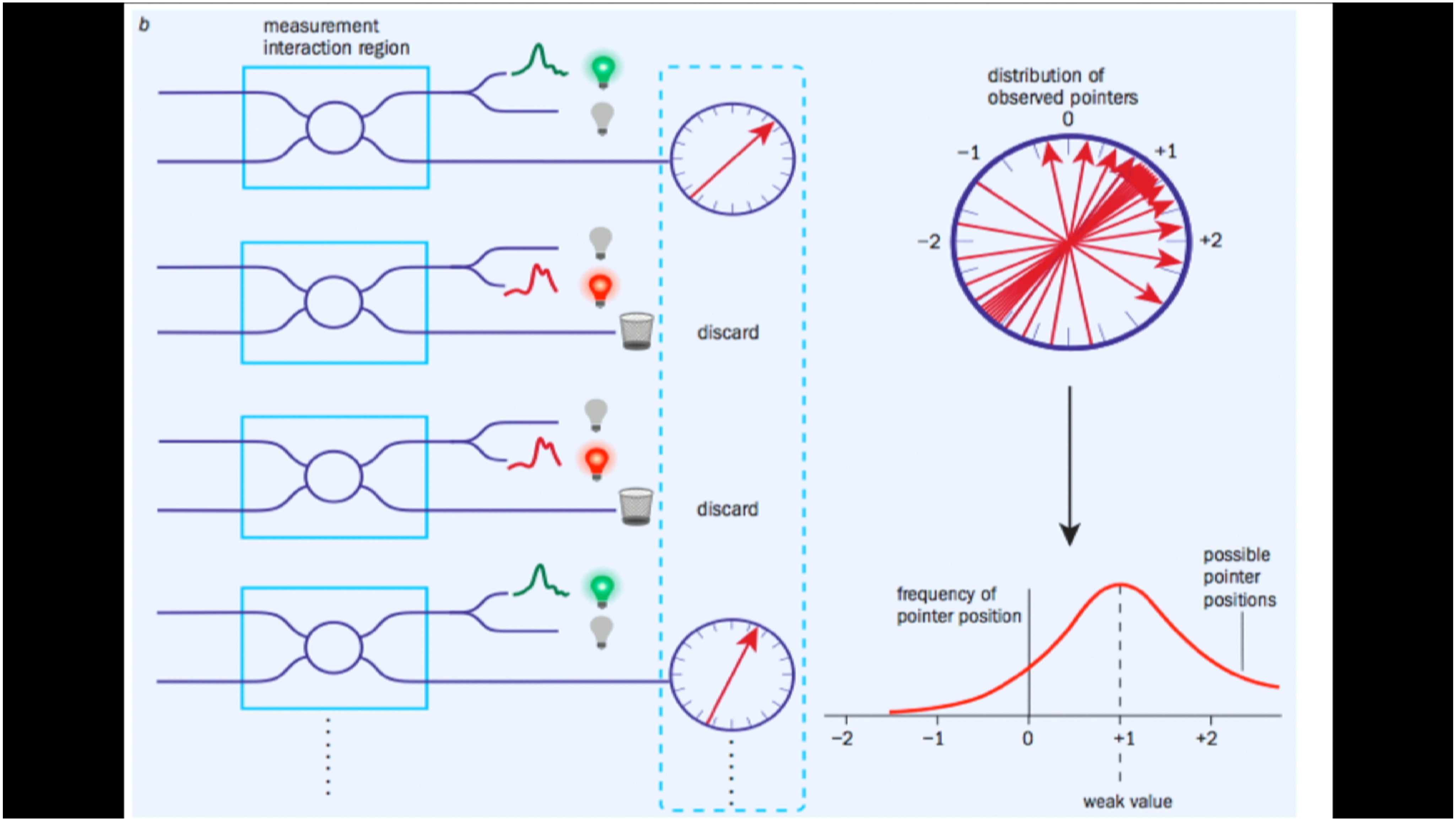
$$\langle A \rangle_f = \sum_i a_i \frac{\langle \mathrm{Proj}(f)\mathrm{Proj}(A_i) \rangle}{\langle \mathrm{Proj}(f) \rangle} = \frac{\langle |f\rangle\langle f|A \rangle}{\langle |f\rangle\langle f| \rangle} \, .$$

$$\langle A \rangle_{fi} = \frac{\langle i| \, |f\rangle\langle f|A \, |i\rangle}{\langle i| \, |f\rangle\langle f| \, |i\rangle} = \frac{\langle f|A|i\rangle}{\langle f|i\rangle} \, .$$

Operational effects of post-selecting on a particular final state



And now, even though each pointer position seems to be pretty random, if you make millions of measurements and build up statistics, you can figure out the average shift --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measurement
interaction region
discard
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0
−1
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frequency of
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possible
pointer
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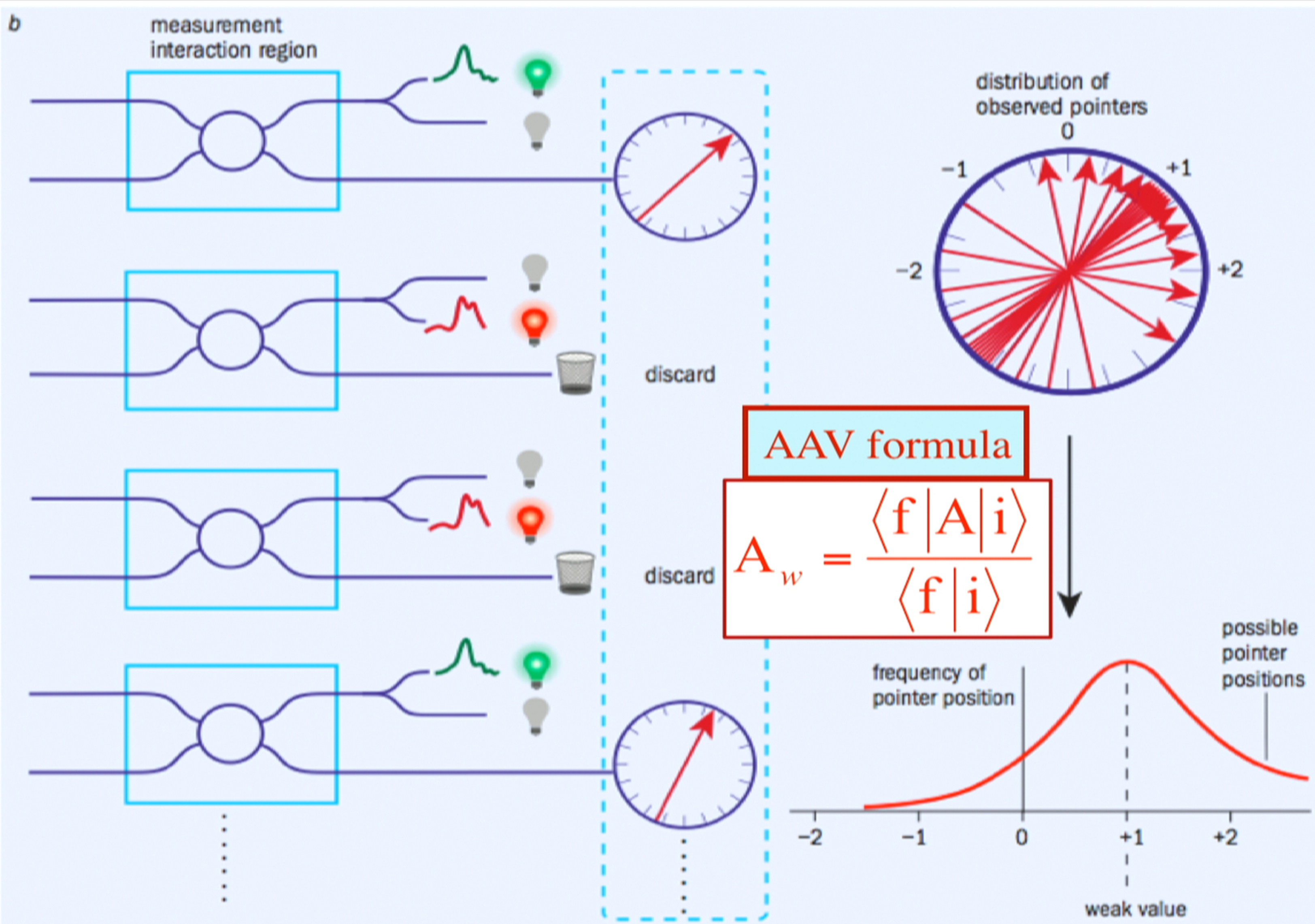
weak value


b
measurement interaction region
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AAV formula
$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}$
frequency of pointer position
possible pointer positions
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−1
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+1
+2
weak value

Grounding Bohmian mechanics in weak values and bayesianism

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New Journal of Physics **9** (2007) 165

Consider a naive experimentalist, with no knowledge of QM

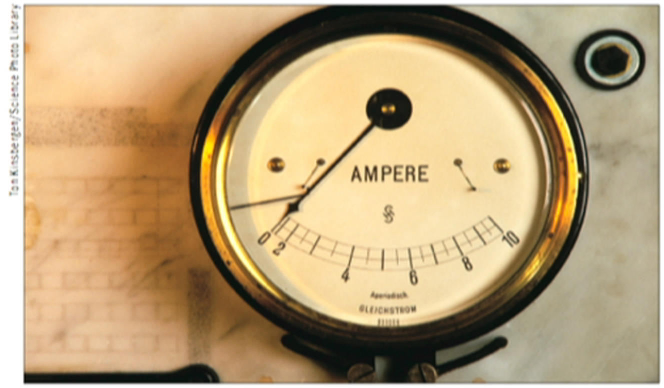
For such an experimentalist, equation (5) would be the only sensible way to measure the velocity of a particle at position $\mathbf{x}$.

$$\mathbf{v}(\mathbf{x};t) \equiv \lim_{\tau\to 0} \tau^{-1} \mathrm{E}[\mathbf{x}_{\mathrm{strong}}(t+\tau) - \mathbf{x}_{\mathrm{weak}}(t) | \mathbf{x}_{\mathrm{strong}}(t+\tau) = \mathbf{x}].$$

$$\mathbf{v}(\mathbf{x};t) = \mathrm{Re}\frac{\langle\psi(t)|\mathbf{x}\rangle\langle\mathbf{x}|\mathrm{i}[\hat{H},\hat{\mathbf{x}}]|\psi(t)\rangle}{\hbar\langle\psi(t)|\mathbf{x}\rangle\langle\mathbf{x}|\psi(t)\rangle}.$$

For $\hat{H} = \hat{\mathbf{p}}^2/2m + V(\hat{\mathbf{x}})$, we have $\mathrm{i}[\hat{H},\hat{\mathbf{x}}]/\hbar = \hat{\mathbf{p}}/m$, so equation (7) immediately reduces to Bohmian velocity (3) for the standard probability current (1).

Some would argue that whatever this Byzantine strategy yields, it is not really a "measurement" of anything (it's not on page 36 of the QM textbooks yet)...



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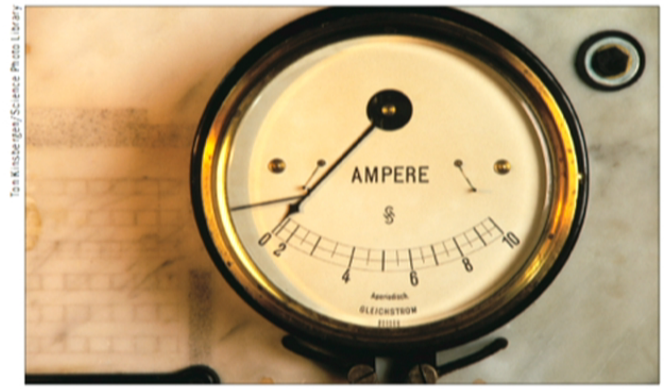
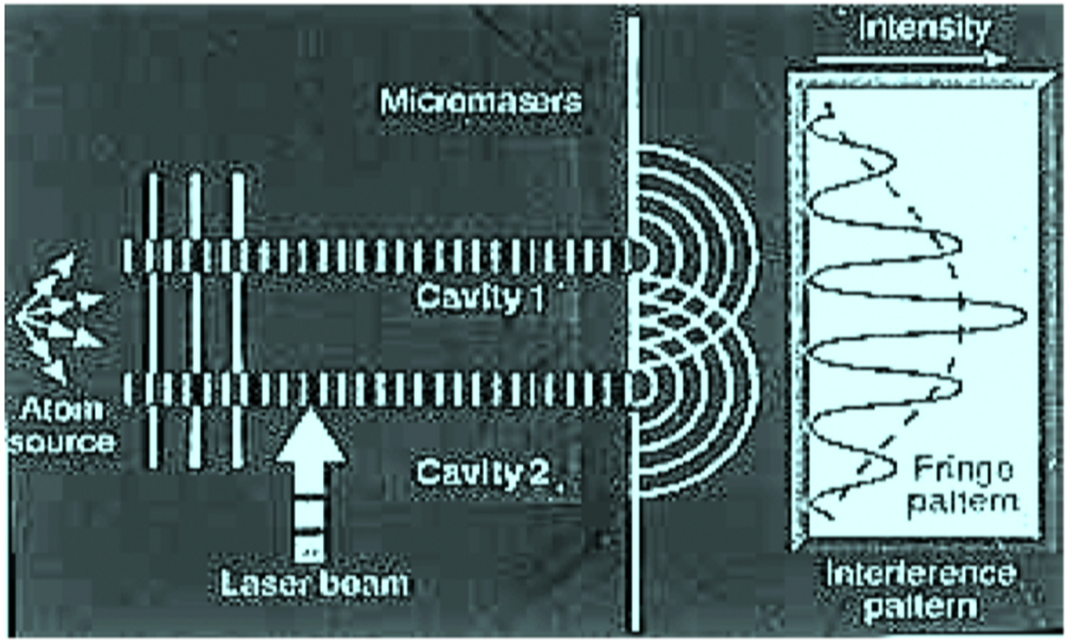


Some of us instead maintain that the QM definition of measurement has only ever aimed to model what happens when we really interact with measuring devices, and if interacting with them strongly changes the results, it's only natural to investigate what interacting with them weakly does.

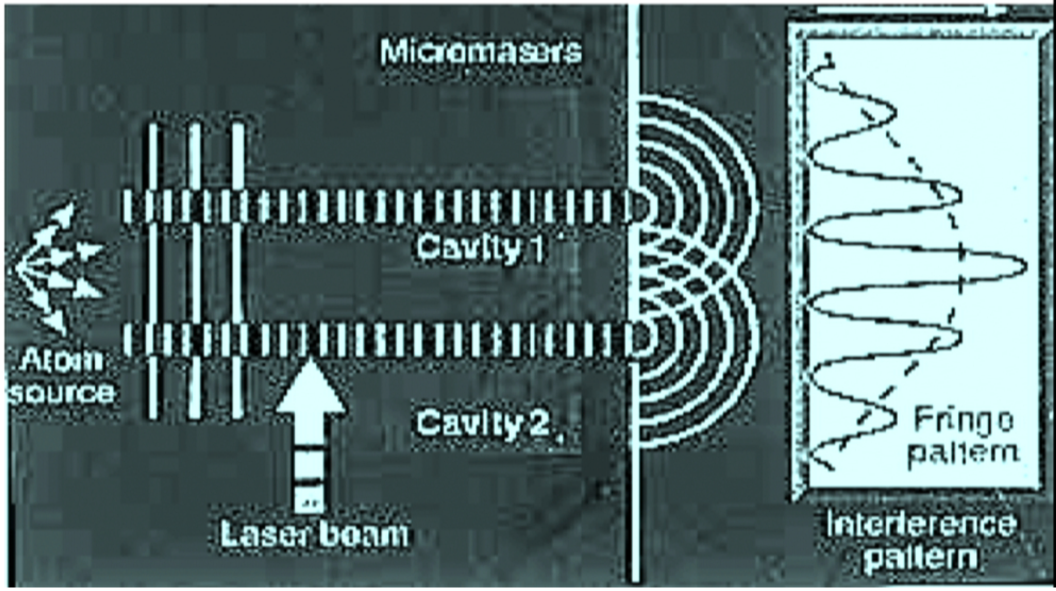
Which-path controversy
(Scully, Englert, Walther 1991)



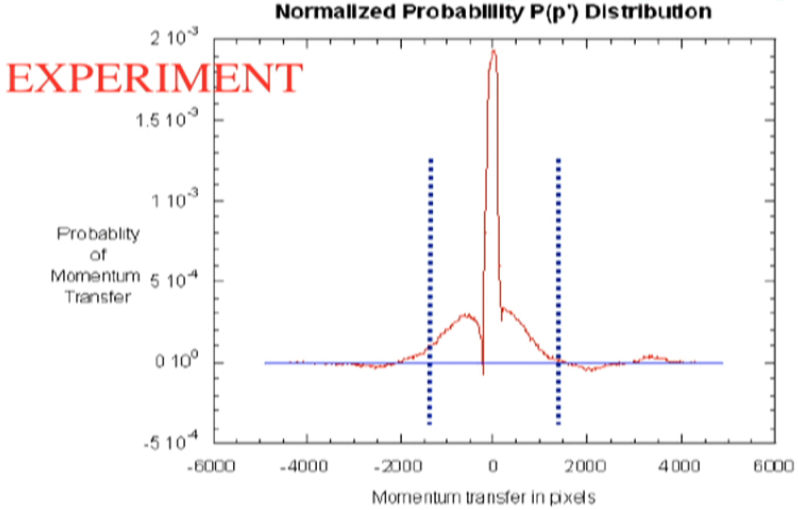
Which-path controversy (Scully, Englert, Walther 1991)

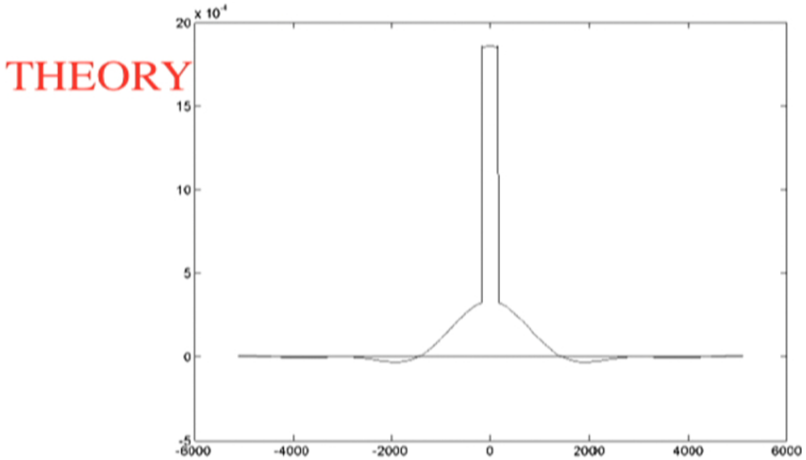
[Reza Mir *et al.*, New. J. Phys. 9, 287 (2007)]

Which-path measurements destroy interference.
This is usually explained via measurement backaction & HUP.
Suppose we use a *microscopic* pointer.
Is this really irreversible, as Bohr would have all measurements?
Need it disturb momentum?
Which is «more fundamental» – uncertainty or complementarity?



The distribution of the integrated momentum-*transfer*





Note: the distribution extends well beyond h/d.

On the other hand, all its moments are (at least in theory, so far) 0.

The former fact agrees with Walls *et al*; the latter with Scully *et al*.

For weak distributions, they may be reconciled *because the distributions may take negative values* in weak measurement.

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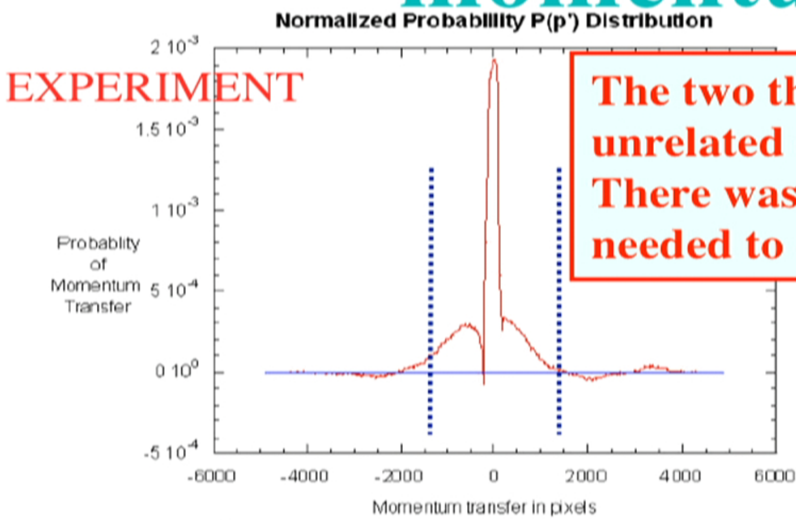

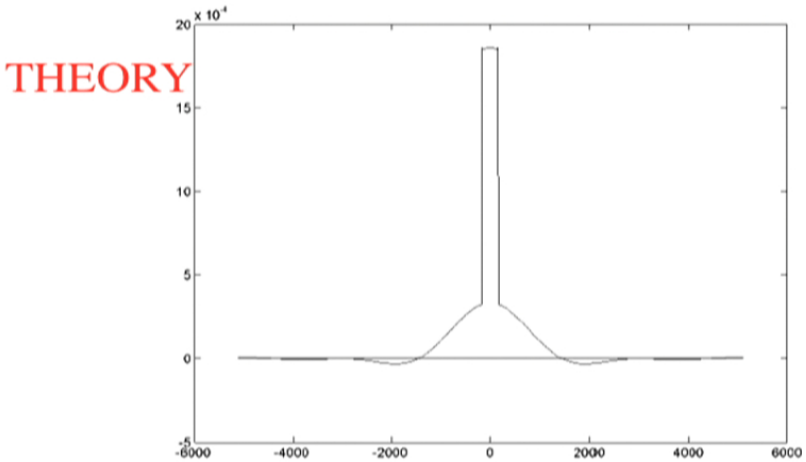

The two theoretical calculations addressed entirely unrelated definitions of momentum transfer. There was no reason a 3rd definition (weak values) needed to obey those theorems...

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Uncertainty about uncertainty

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A similar relation for measurement precision $\varepsilon(A)$ of the probe vs. disturbance to the system $\eta(B)$ is, however, false:

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Ozawa, PRA 67, 042105 (2003):

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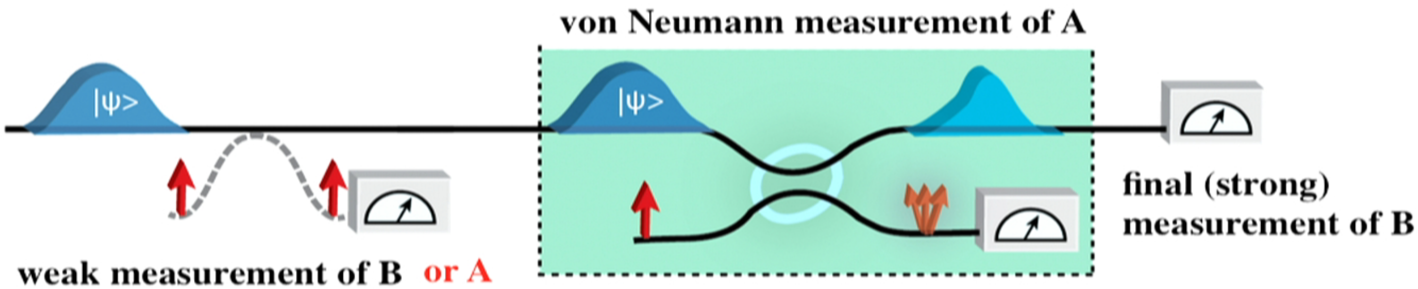
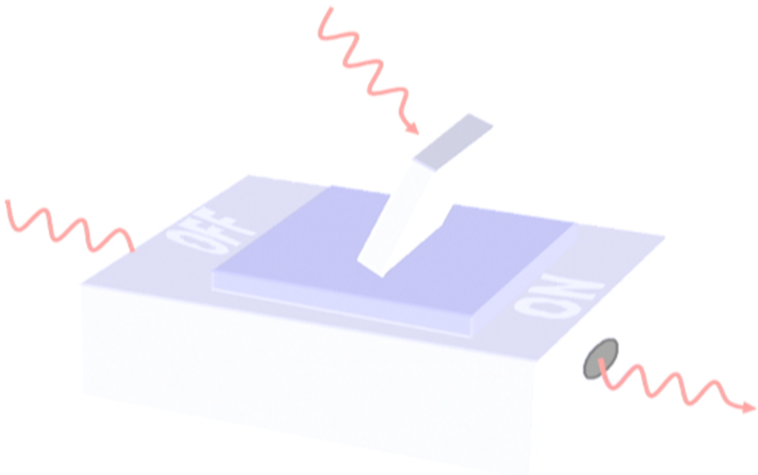
Proposal Using Weak Measurements



- **Define disturbance to B as the RMS difference between the value of B before and after the measurement**
- **Define precision of A as the RMS difference between the value of A of the system before the measurement and the value of A on the probe**

Proposal: Lund & Wiseman, NJP 12, 093011 (2010)
Our experiment: Rozema et al., PRL 109, 100404 (2012)
Hasegawa group: Erhart et al., Nature Physics 8, 185 (2012)
(and much more work since then....)

Observing the nonlinear effect of a single photon



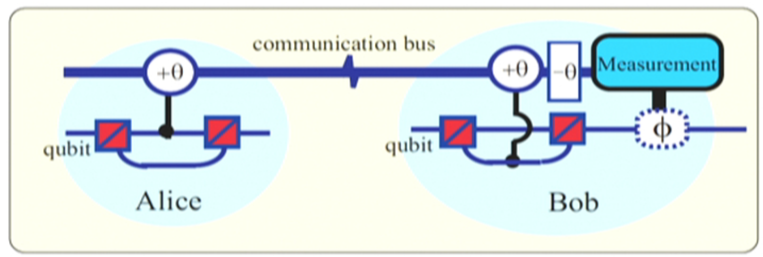
Motivation: quantum NLO (e.g., weak "giant nonlinearities")

"Giant" optical nonlinearities...
(a route to optical quantum computation;
and in general, to a new field of *quantum nonlinear optics*)
– cf. Ray Chiao, Ivan Deutsch, John Garrison)


New Journal of Physics
The open-access journal for physics

Weak nonlinearities: a new route to optical quantum computation

W J Munro[1,2,3], K Nemoto[1] and T P Spiller[2]



Munro, Nemoto, Spiller, NJP 7, 137 (05)

(Also of course, cf. "giant giant nonlinearities,"
e.g., Lukin & Vuletic and Rempe with Rydberg atoms;
Jeff Kimble *et al*. on nanophotonic approaches; Gaeta Rb in hollow-core fibres; et cetera)

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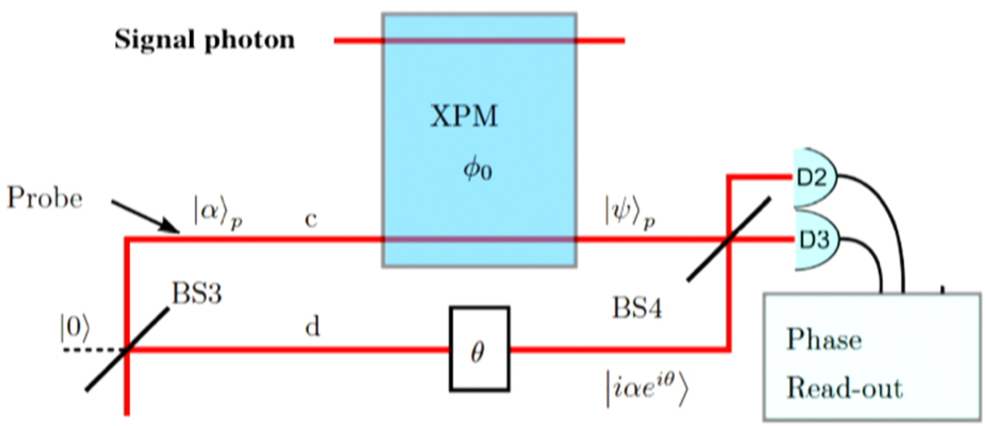

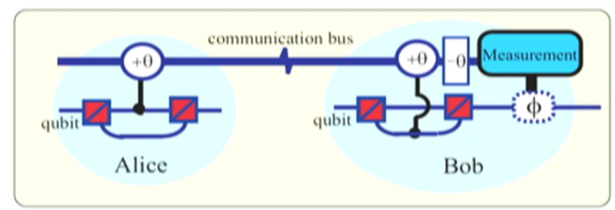

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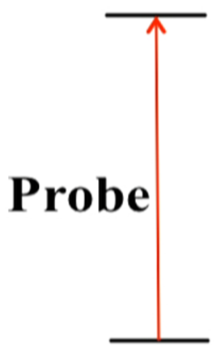
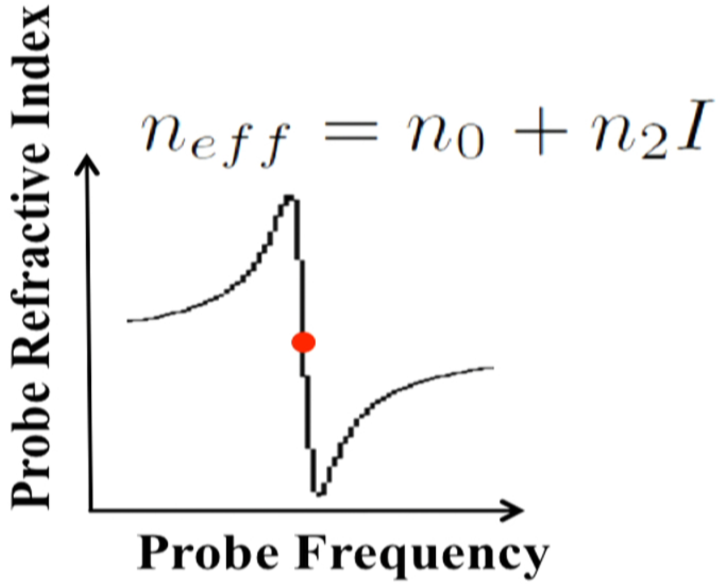
Cross-phase modulation (XPM)





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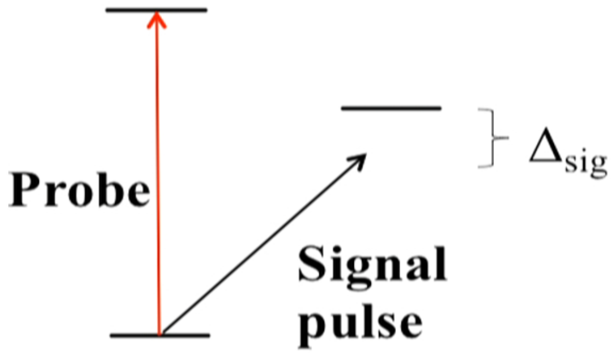

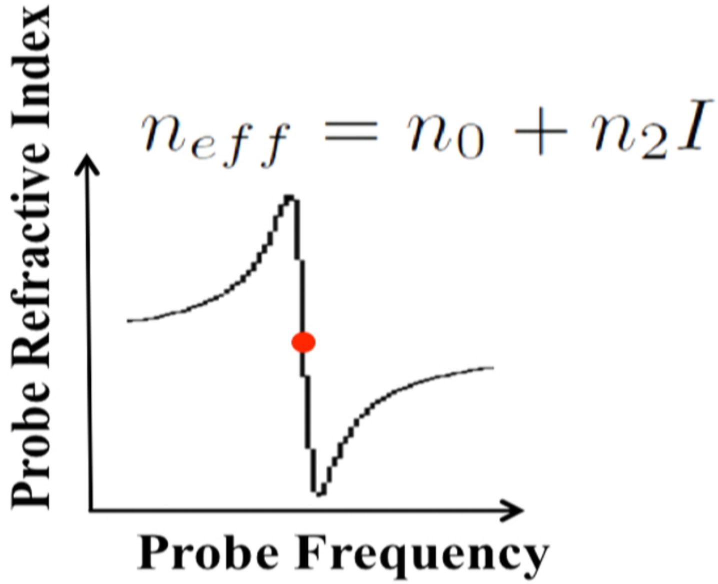

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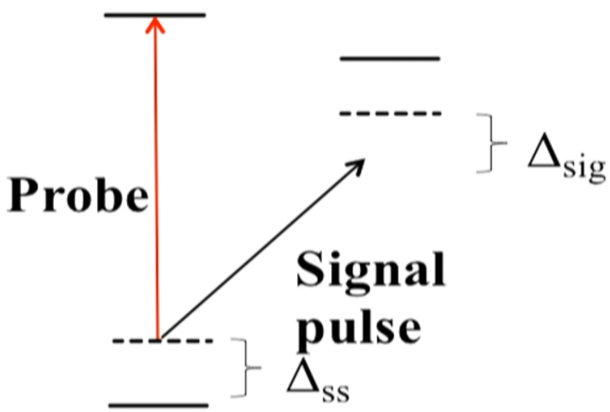

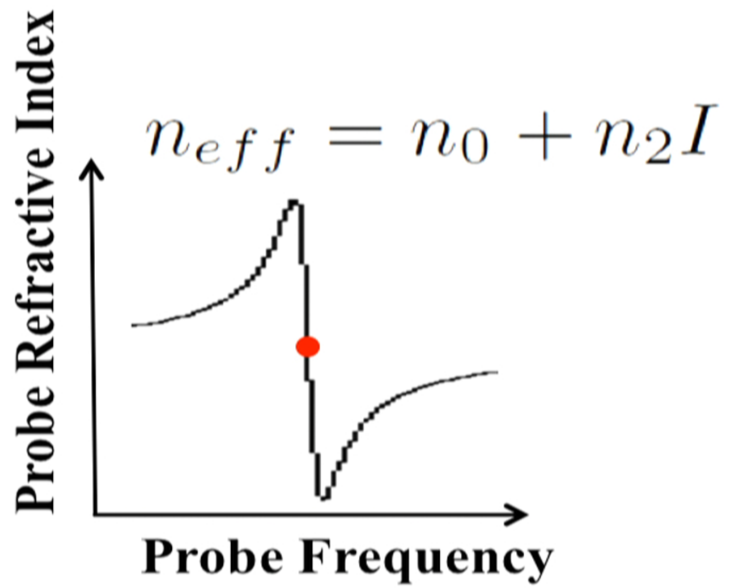

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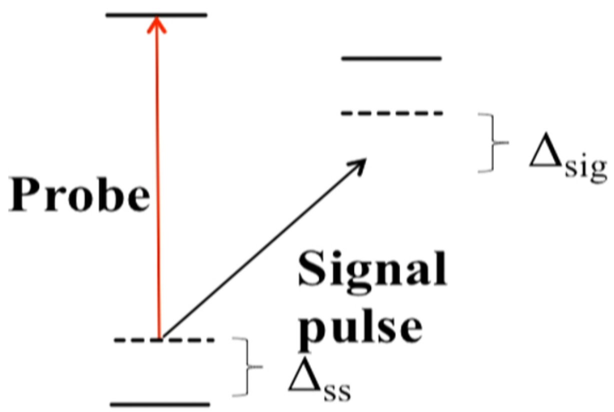

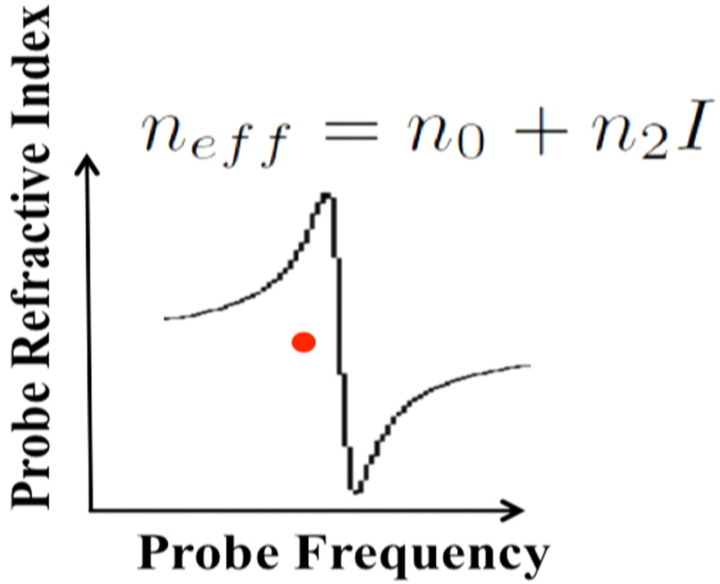

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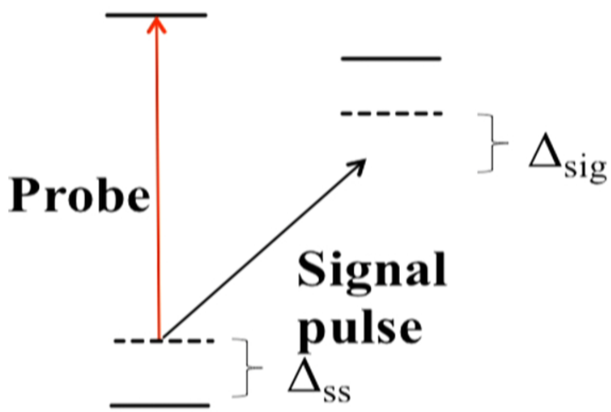

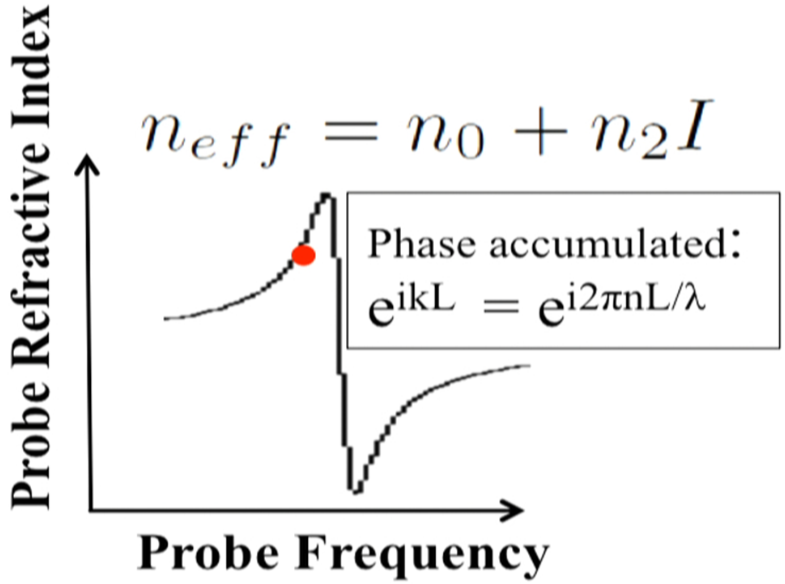

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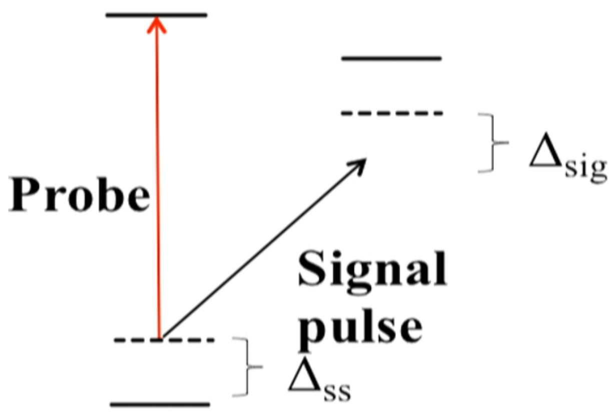
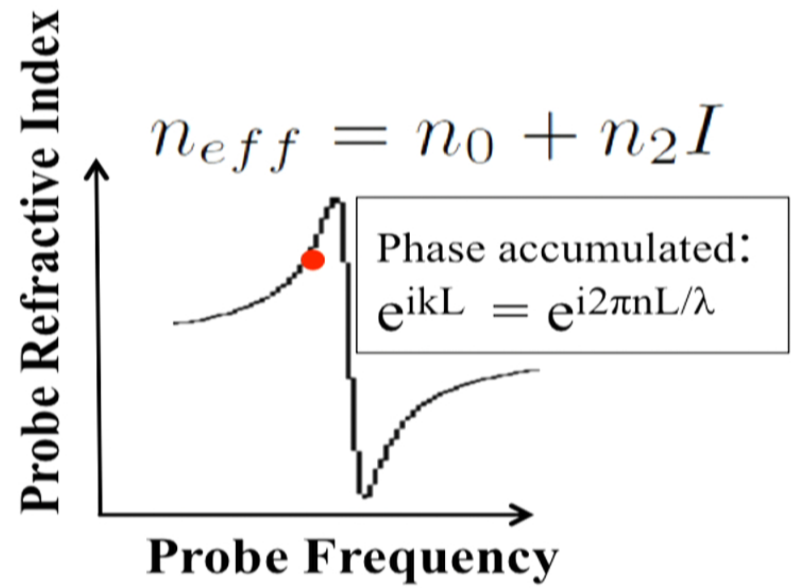
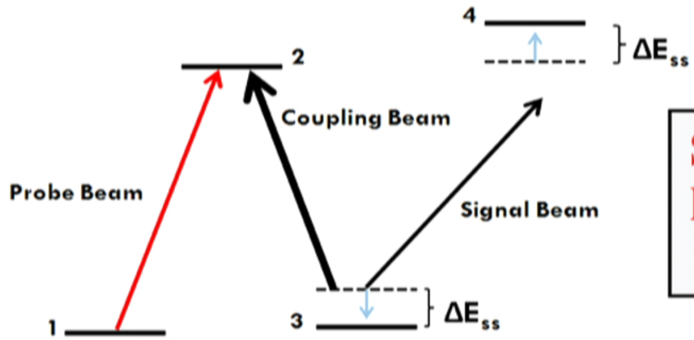




AC Stark shift changes effective detuning, changing index of refraction experienced by probe

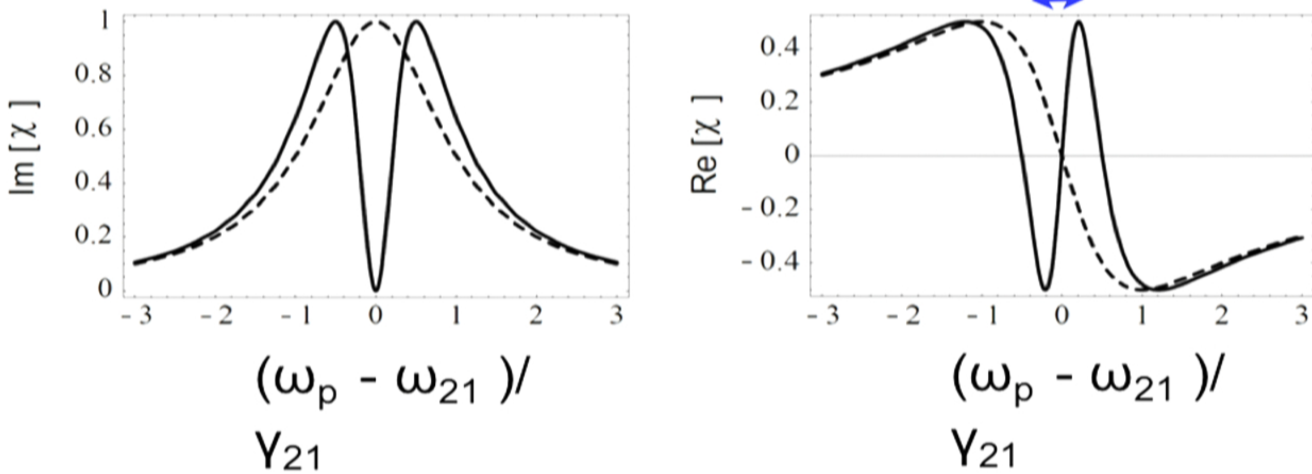
EIT-enhanced XPM



e.g., Schmidt & Imamoglu, Opt. Lett. 21, 1936 (96)

Steep slope of dispersion curve -> higher sensitivity to AC Stark shift (& transparency too)

EIT width -> 0 as I_{coup} -> 0



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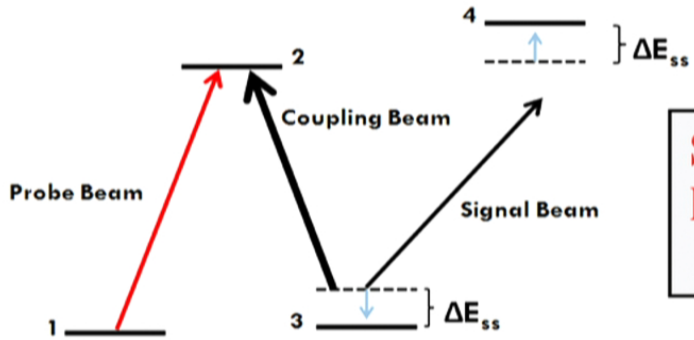

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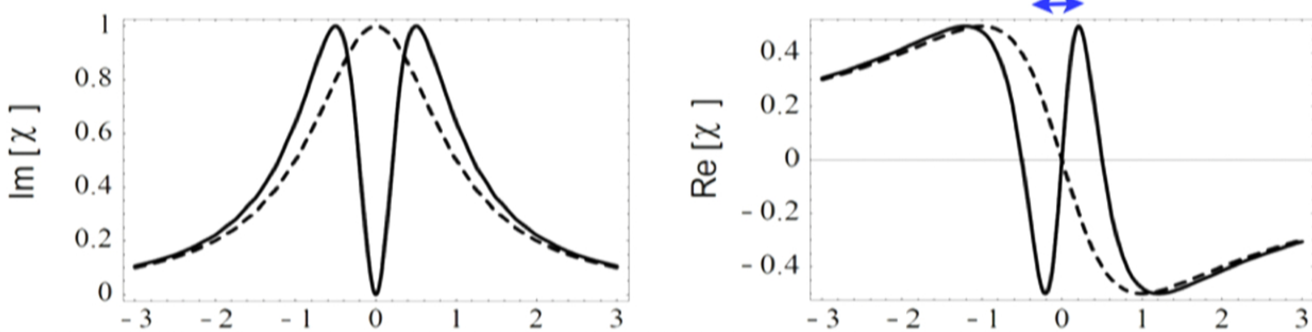
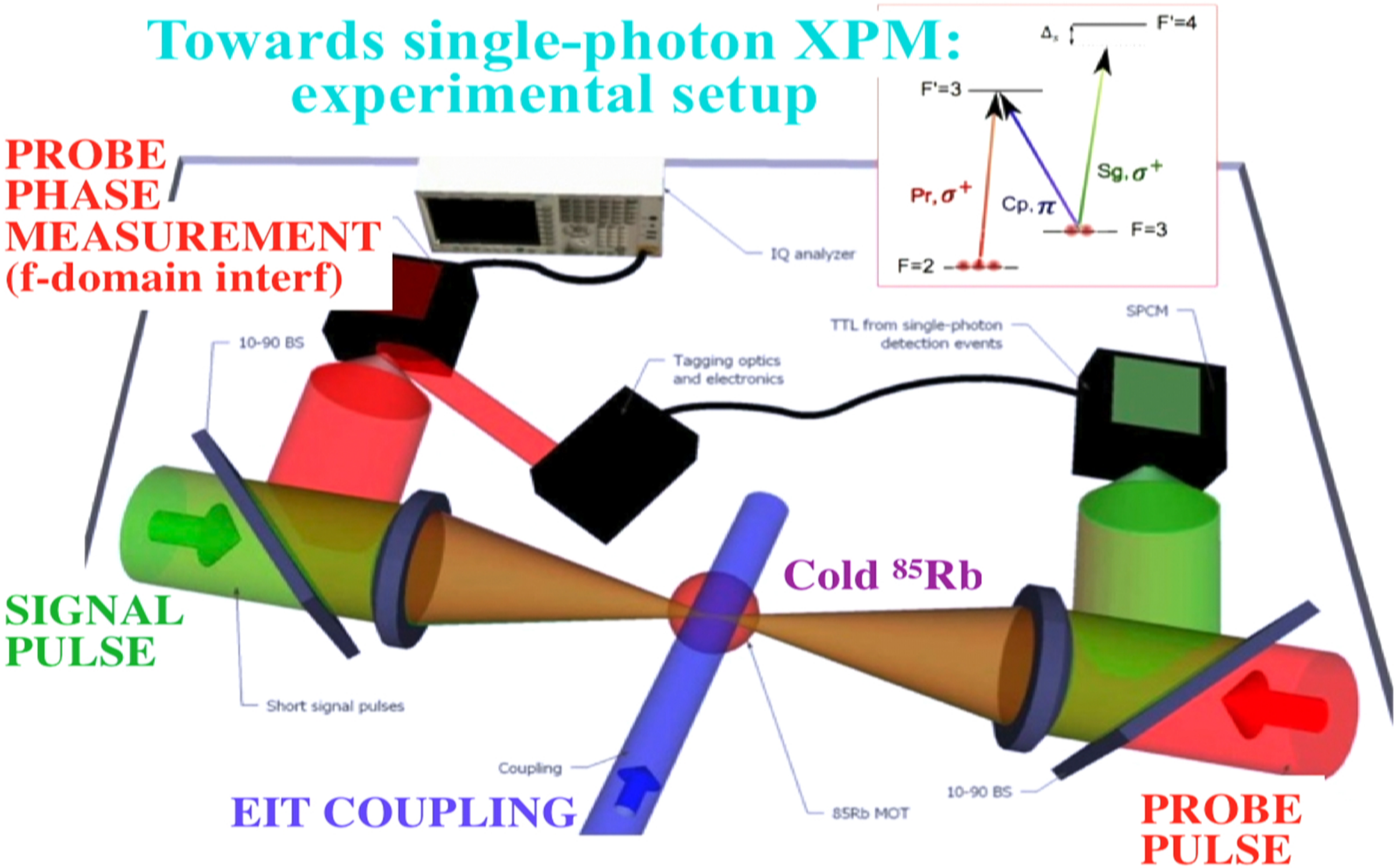


Narrower transparency windows yield larger cross-phase shifts

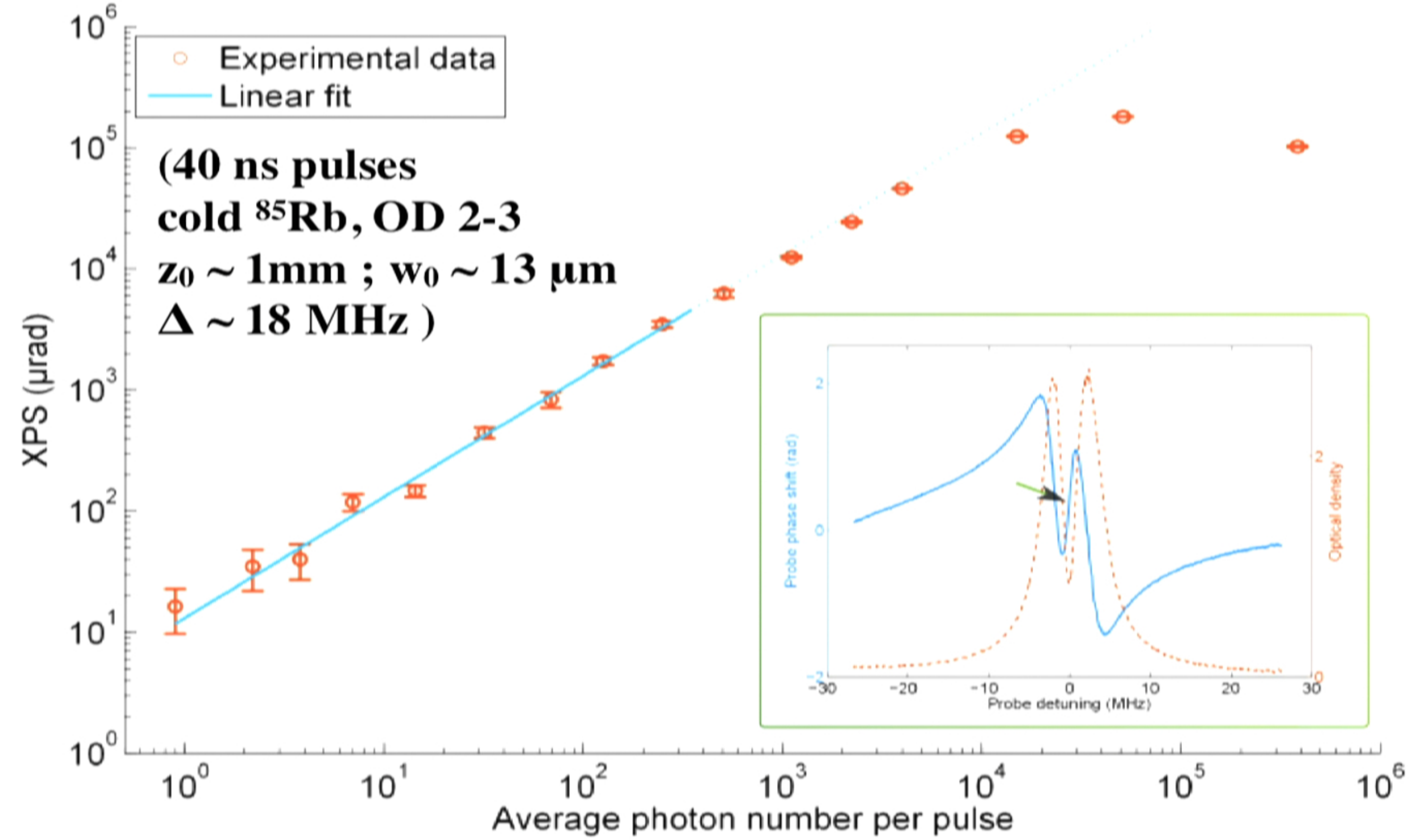
AC Stark shift is intensity-dependent – i.e. shift acts as a measure of photon number

Towards single-photon XPM: experimental setup

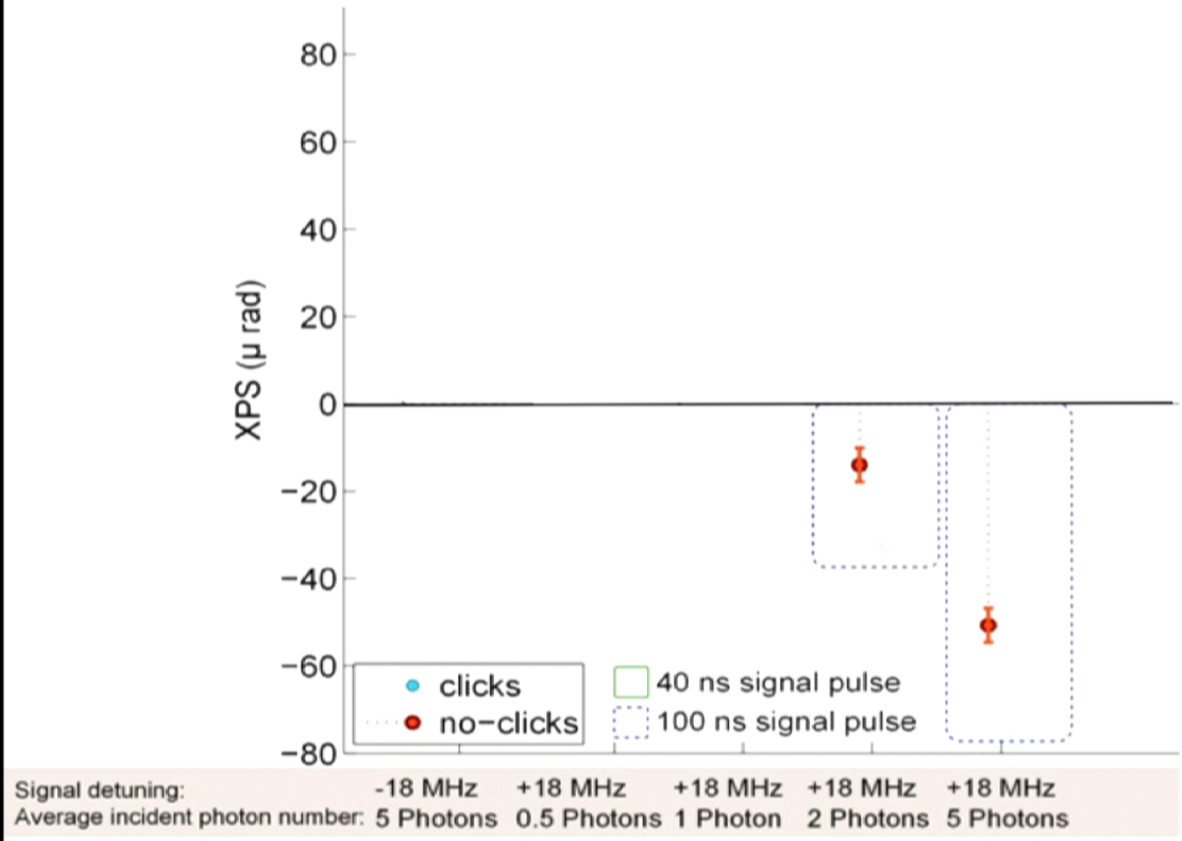


A. Feizpour et al., Nature Physics, DOI: 10.1038/nphys3433 (2015)

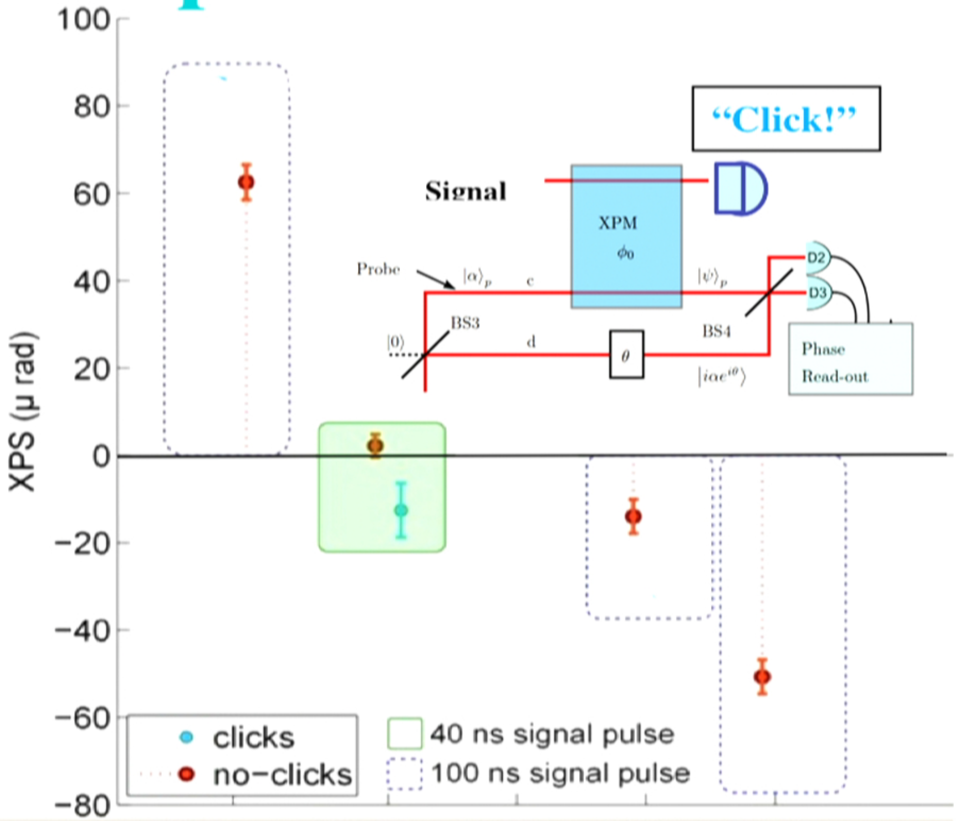
Measurement of cross phase shift, down to signal pulses with <n> = 1



Non-linear phase shift due to single photons

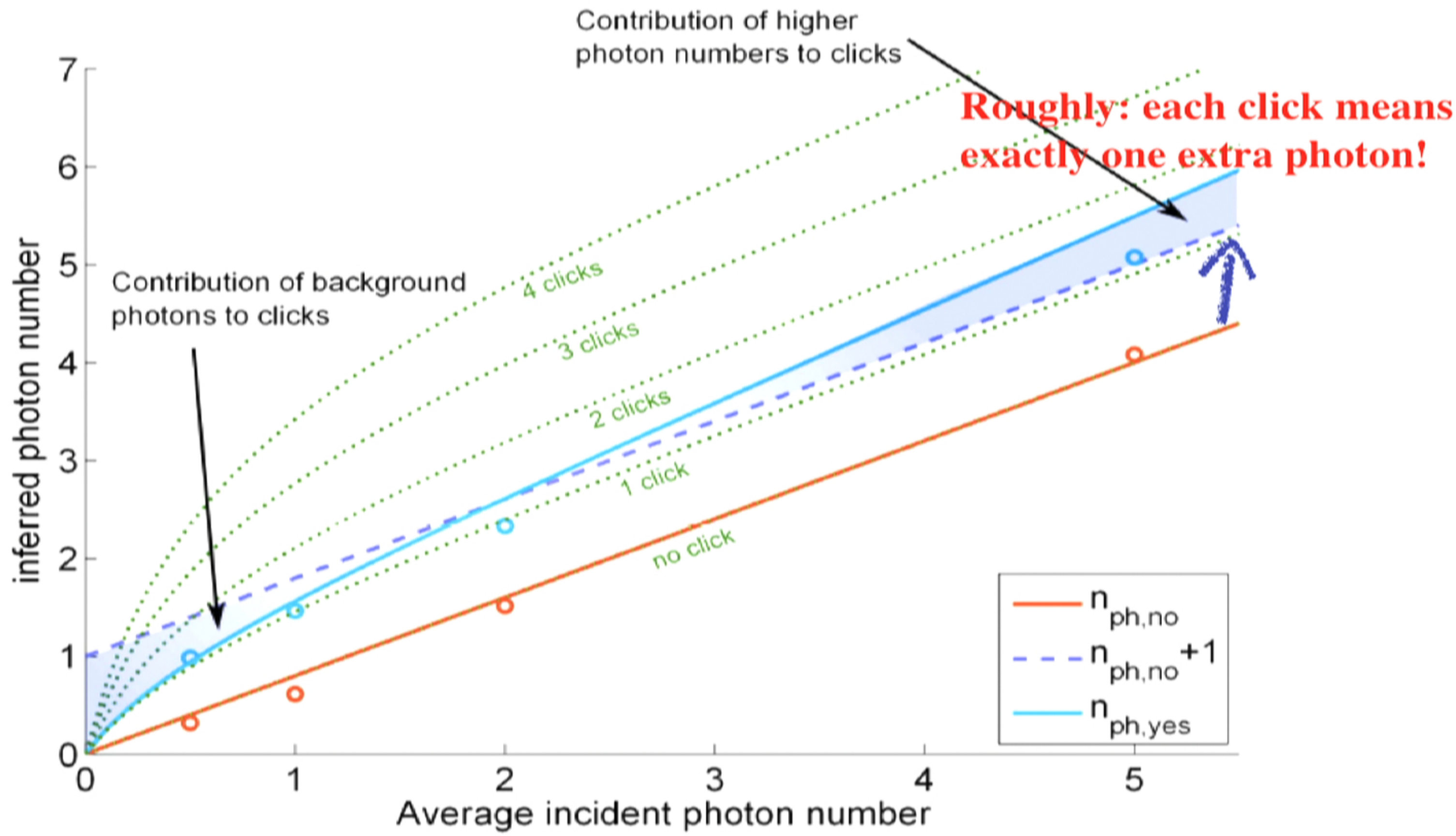


Non-linear phase shift due to a single post-selected photon



Signal detuning:	-18 MHz	+18 MHz	+18 MHz	+18 MHz	+18 MHz
Average incident photon number:	5 Photons	0.5 Photons	1 Photon	2 Photons	5 Photons

Post-selected single photons



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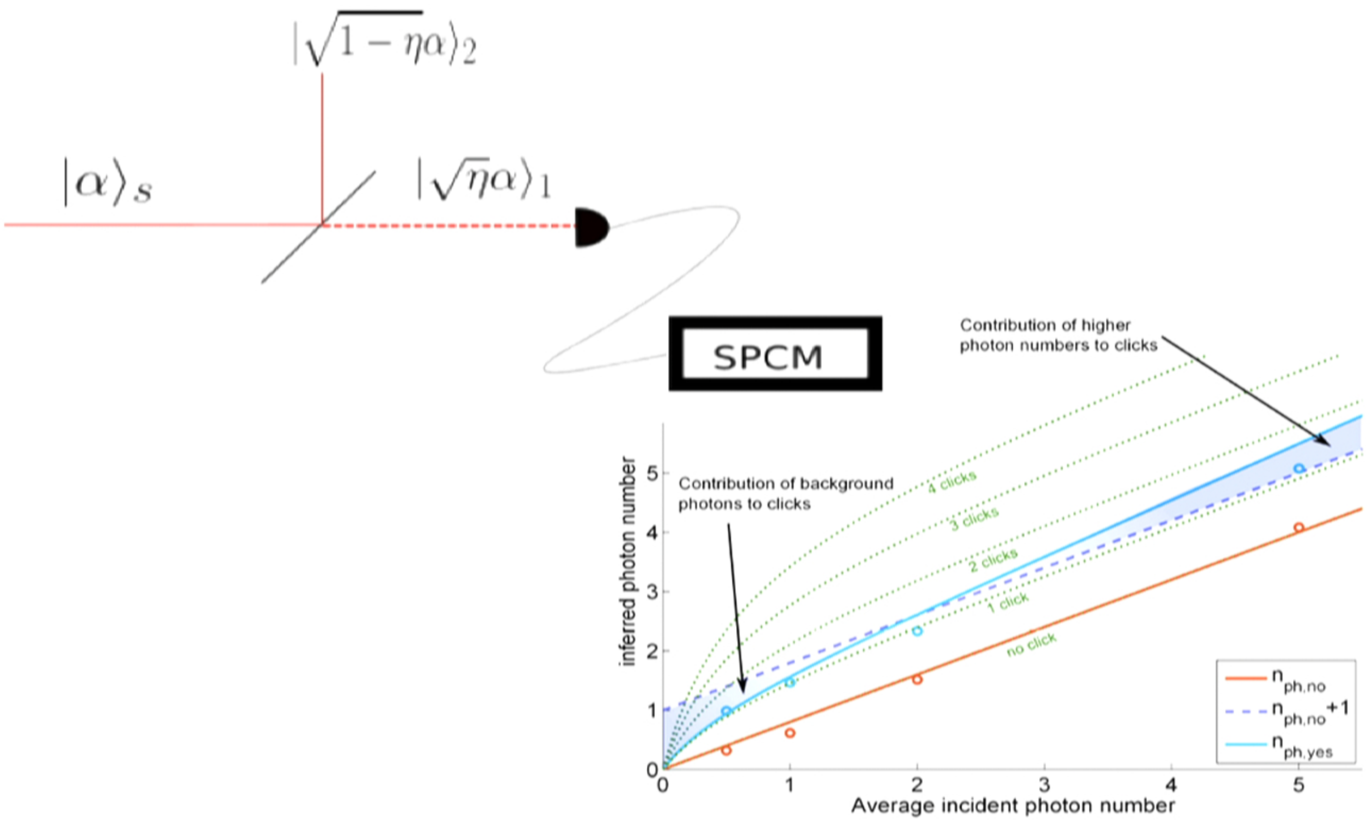

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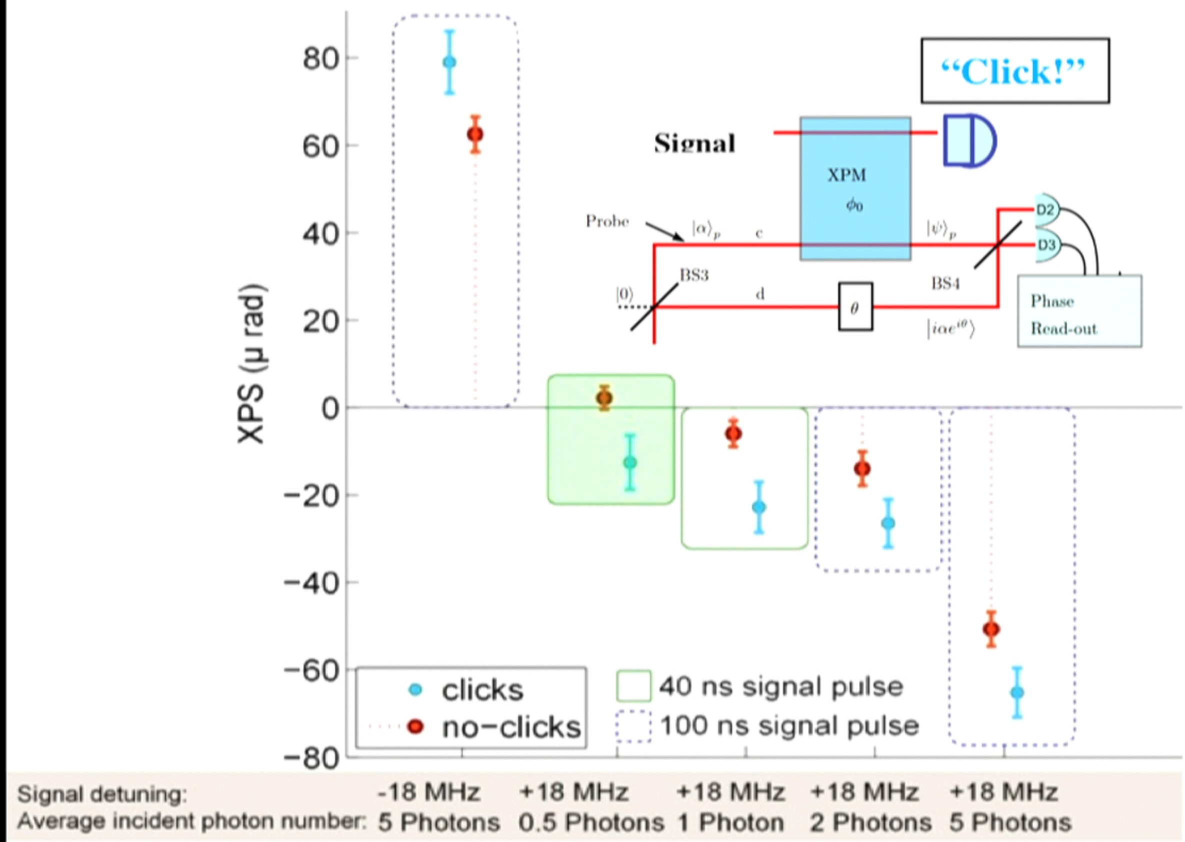

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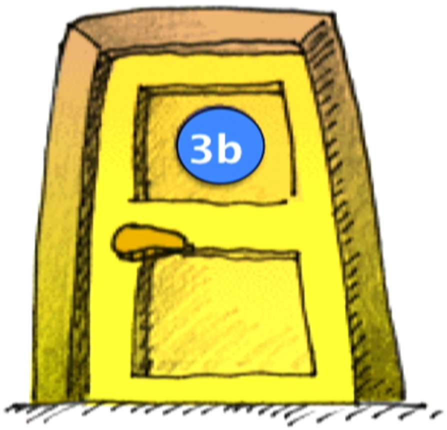

A photon in the hand
is worth 1000* in the vacuum chamber

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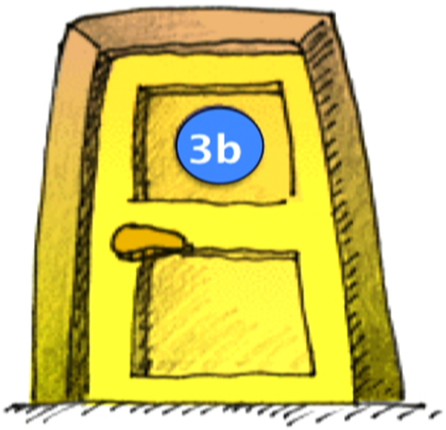

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Aharonov, Albert, & Vaidman, PRL 60, 1351 (1988)

IBER 14 PHYSICAL REVIEW LETTERS

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

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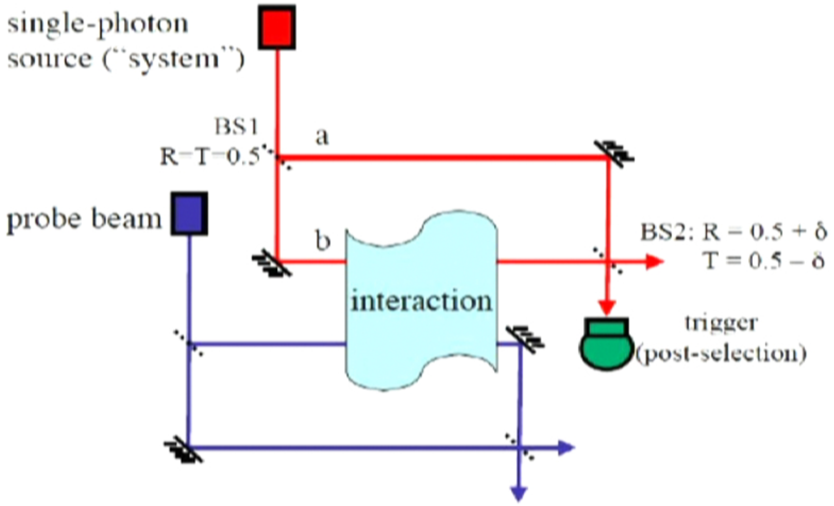
"Weak value amplification" – pioneering applications, e.g.,
Hosten & Kwiat, Science 319, 5864 (08);
Ben Dixon, Starling, Jordan, & Howell, PRL 102, 173601 (09); etc

How the result of the measurement of the number of 1 photon can be 100



<u>**Weak Measurement Amplification of Single-Photon Nonlinearity**</u>,
Amir Feizpour, Xingxing Xing, and Aephraim M. Steinberg
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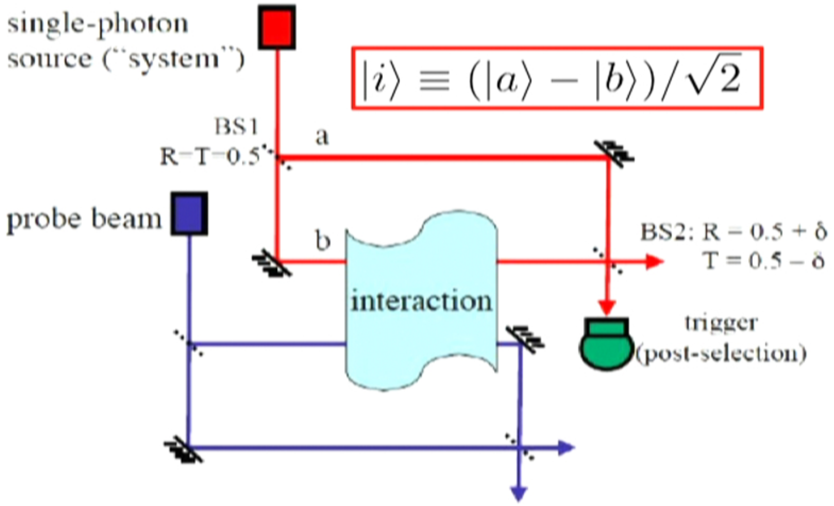

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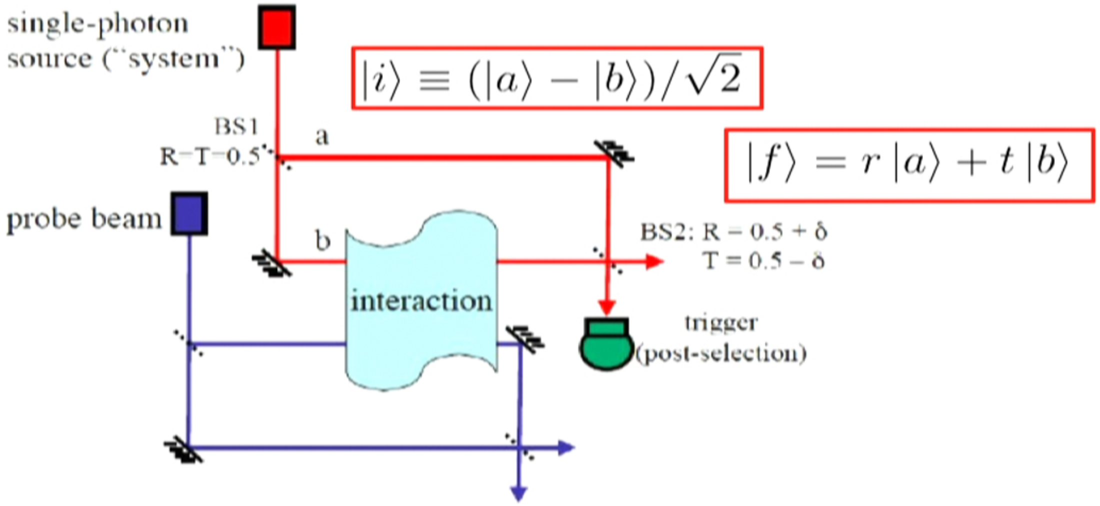

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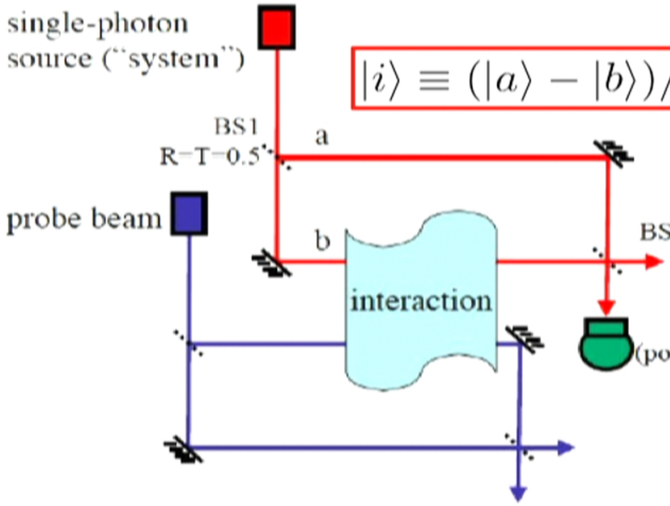

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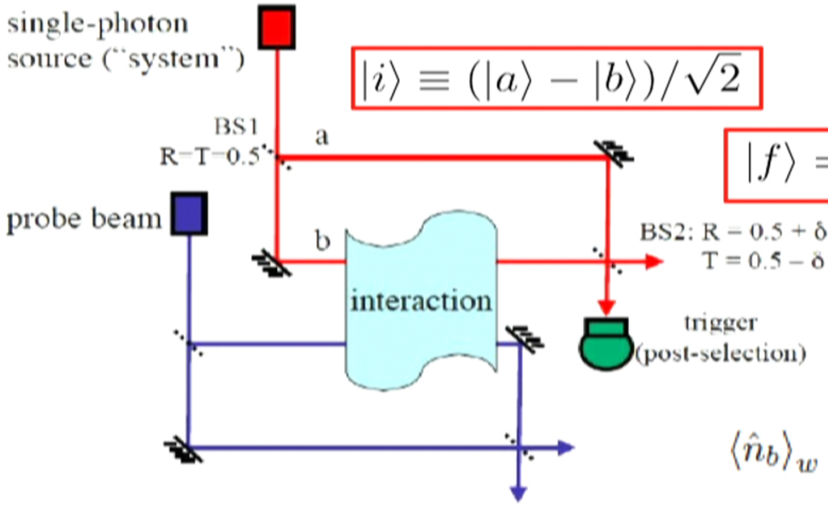

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When the post-selection succeeds, the phase shift on the probe may be much larger than the phase shift due to a single photon -- ***even though there only ever*** **is** ***at most one signal photon*****!**

$\langle n \rangle_w$ may be >> 1.

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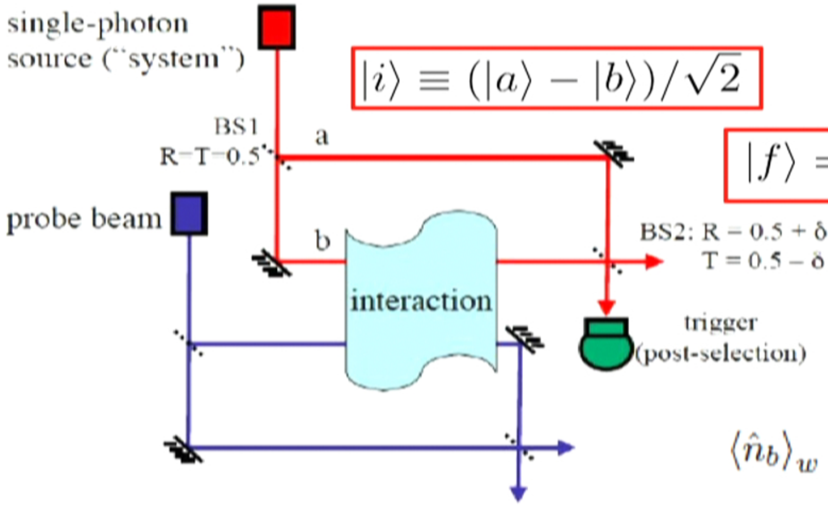

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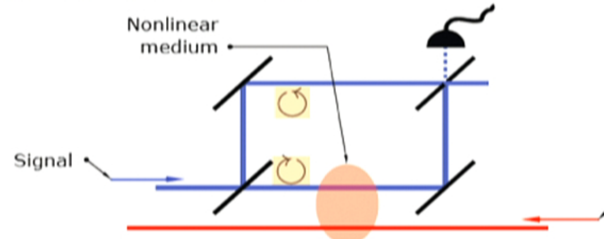
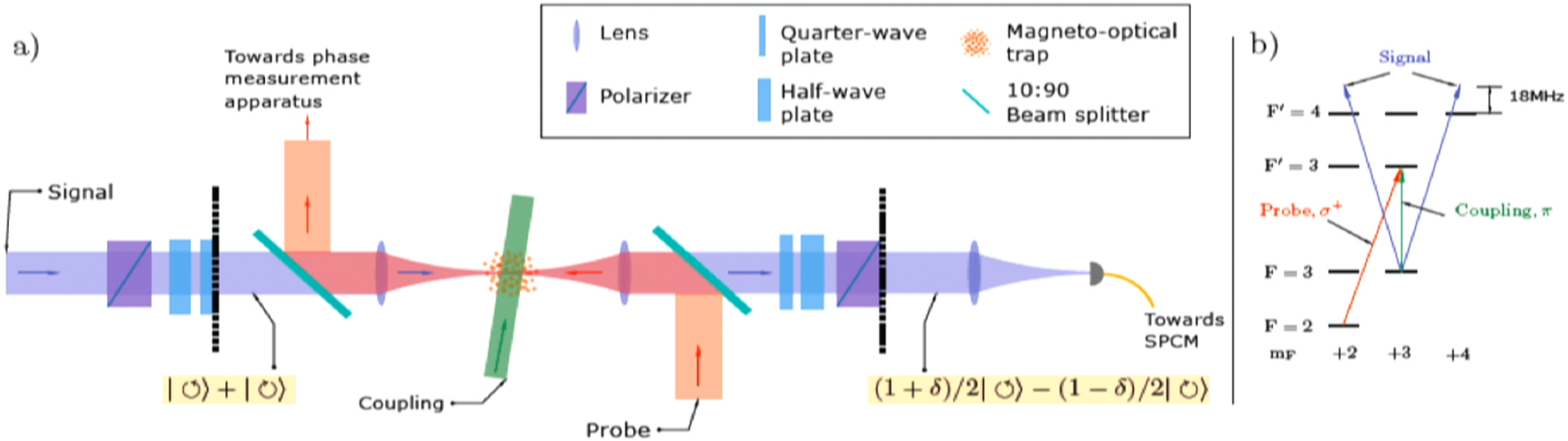
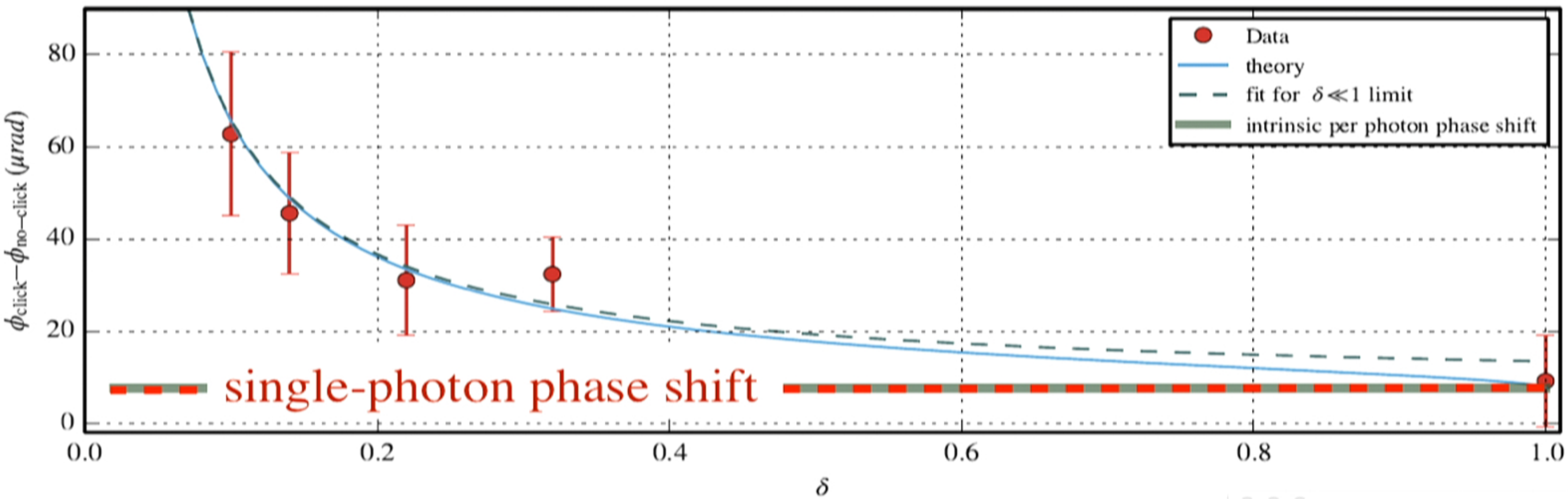
Weak Measurement Amplification of Single-Photon Nonlinearity, Amir Feizpour, Xingxing Xing, and Aephraim M. Steinberg Phys Rev Lett 107, 133603 (2011)

Polarisation interferometer





The phase shift due to an appropriately post-selected photon



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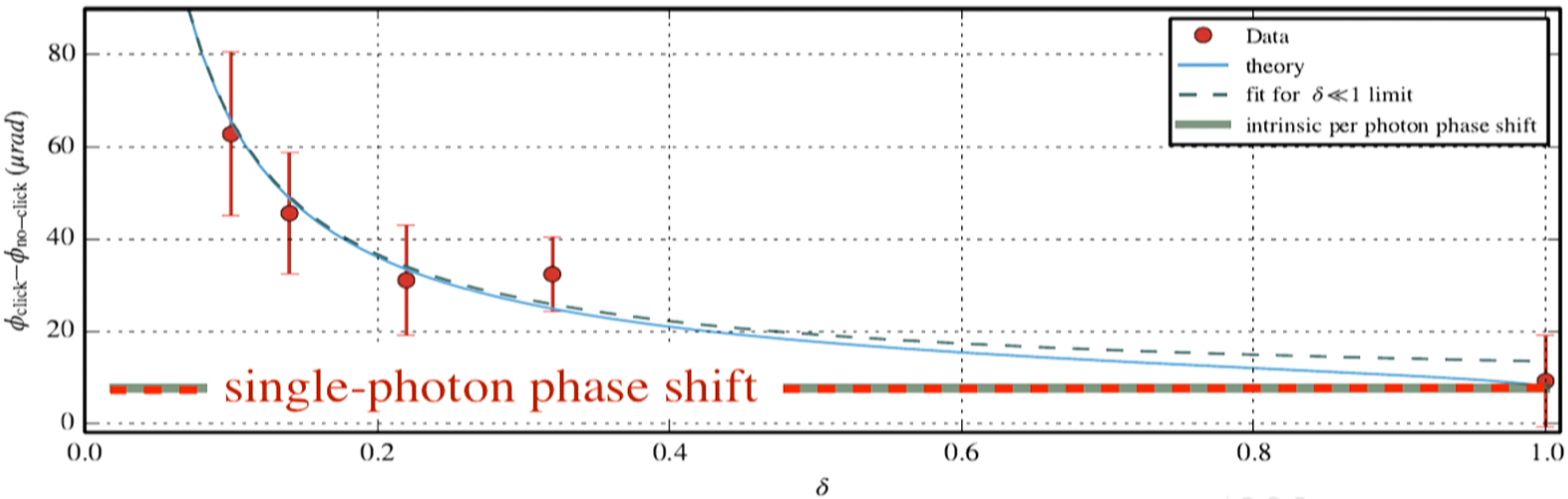

NOTE: this amplification shows up in the *difference* between the "click" and "no-click" cases; it is "the post-selected photon" which gets amplified, while all the other contributions cancel out.

Is weak measurement good for anything *practical*?

"Weak value amplification" has been proposed as a way to enhance the signals of small effects (like our nonlinearity...?):

Hosten & Kwiat, Science 319, 5864 (08); and, more quantitatively --

Selected for a Viewpoint in *Physics*
RL **102,** 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA
(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam's transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to 400 ± 200 frad and the linear travel of a piezo actuator down to 14 ± 7 fm.

DOI: 10.1103/PhysRevLett.102.173601 PACS numbers: 42.50.Xa, 03.65.Ta, 06.30.Bp, 07.60.Ly

PRL 112, 040406 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Weak Value Amplification is Suboptimal for Estimation and Detection

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 25 July 2013; revised manuscript received 21 November 2013; published 31 January 2014)

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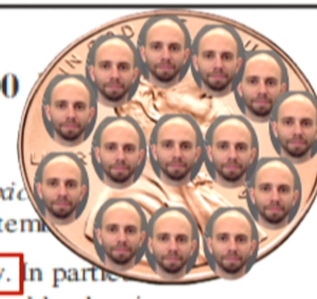
PRL 113, 120404 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

How the Result of a Single Coin Toss Can Turn Out to be 100



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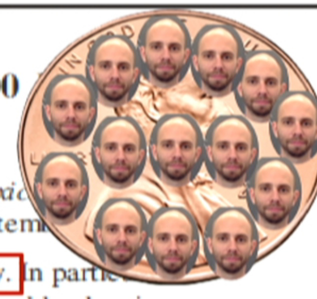

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Anomalous weak values are proofs of contextuality

Matth

Perimeter Institute for Theoretical Physics, 31

(Dated: S

The average result of a weak measurement
measured quantum system, exceed the largest
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Comment on "How the result of a single coin toss can turn out to be 100 heads"

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Experimentally quantifying the advantages of weak-value-based metrology

Gerardo I. Viza, Julián Martínez-Rincón, Gabriel B. Alves, Andrew N. Jordan, and John C. Howell
Phys. Rev. A **92**, 032127 – Published 22 September 2015

Center for Quantum Information and Control, University of New Mexic
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SNR controversy: the short version

Weak value ~ 1 / <f|i>
Success probability ~ |<f|i>|^2

Pointer shift gets 10 times bigger,
as data rate gets 100 times smaller; noise 10 times bigger too.

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TRUE IF --- **the noise is "statistical," as opposed to "technical."**

Early conjectures: something like pixel size in a detector array is insurmountable. Use WVA to make shift > pixel size ("technical")
Truth: you can still fit the center of a distribution to better than the pixel size, and $1/N^{1/2}$ still applies in principle.

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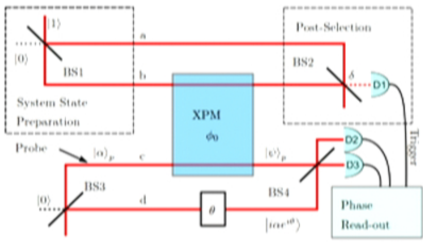
BUT: noise only drops as $1/N^{1/2}$ because of the random walk, i.e., the fact that the noise on different data points is uncorrelated. Adding more data points within a noise correlation time *does not* let you keep averaging the noise away; better to post-select, and get a bigger signal.

One (of many) perspective(s) on the signal-to-noise issues... "technical noise"

NOTE: some language issues?
To most theorists, "postselection" means "throwing something out"; to some experimentalists, it means "doing a measurement on the system at all" (and perhaps choice of basis)

A. Feizpour et al., Phys. Rev. Lett. 107, 133603 (2011) + experiment & theory to appear...



WE CONTEND WVA IS USEFUL IN THE FOLLOWING SITUATIONS:

(1) limited by detector saturation
(2) most bins "empty" anyway
(3) noise correlation time > time between photons

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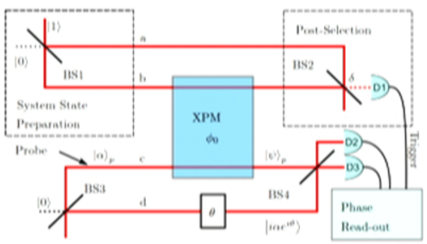

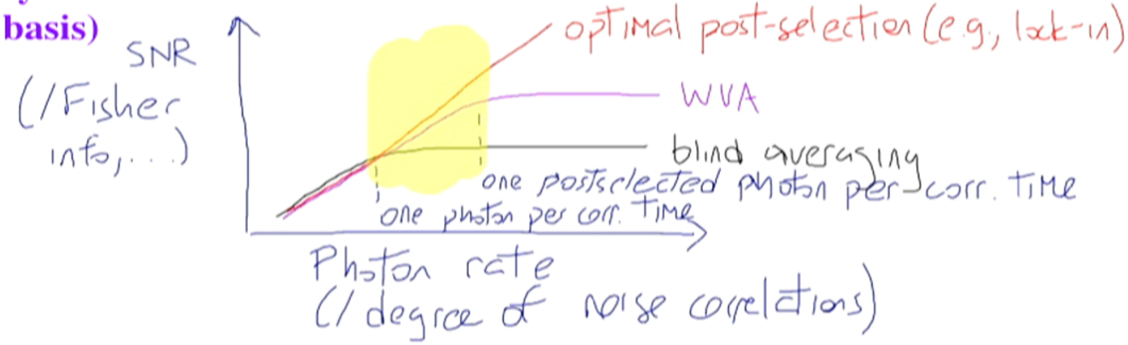

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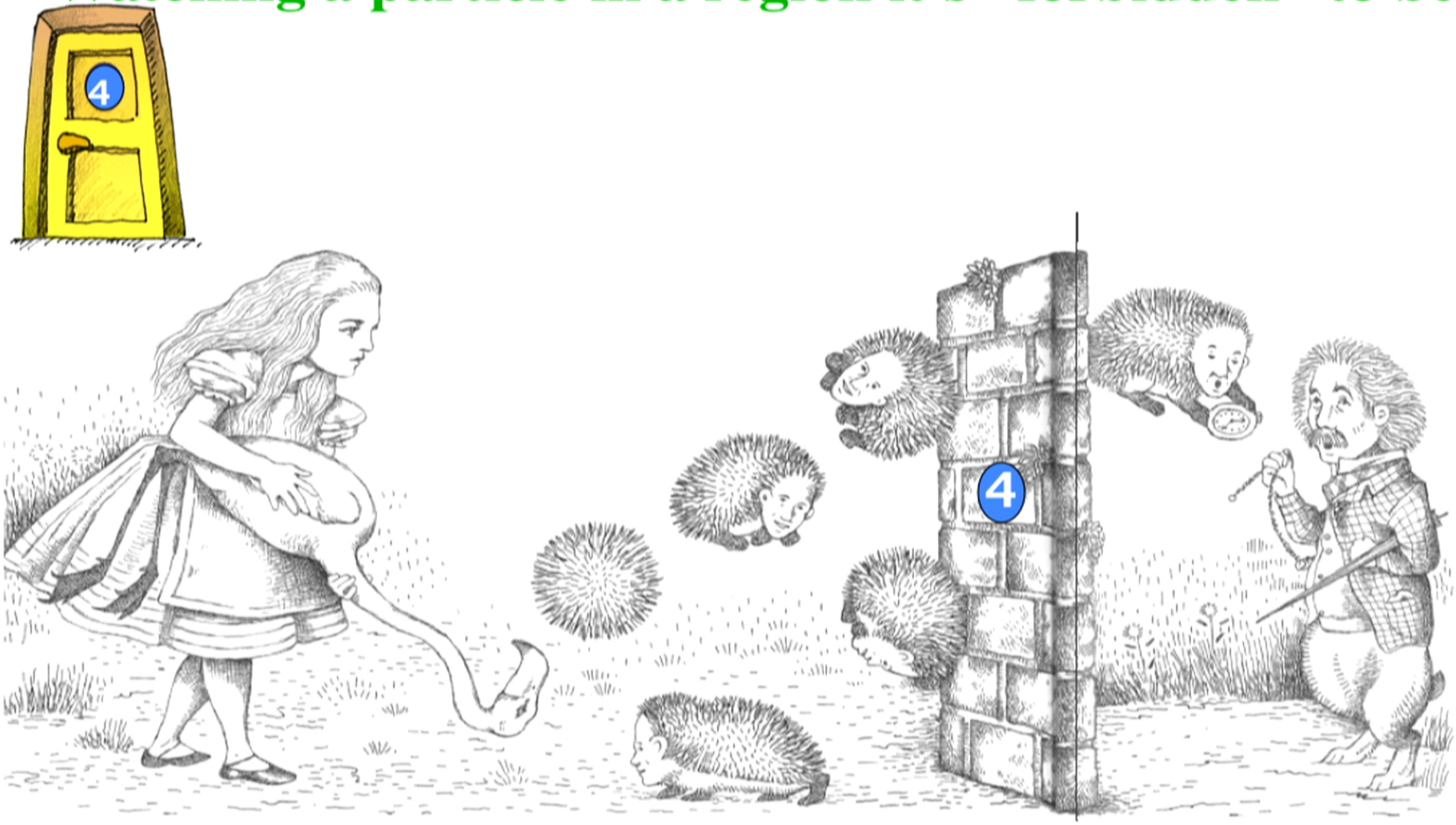
One unexpected advantage

Given the extensive discussion in recent years over the possible merits of WVA for making sensitive measurements of small parameters, it is interesting to contrast the present experiment with an earlier one, in which we measured the nonlinear phase shift due to post-selected single-photons, but without any weak-value amplification (*31*). In our previous experiment, a total of approximately 1 billion trials (300 million events with post-selected photons, and 700 million without) were used to measure the XPS due to σ^+-polarized photons. By looking at the difference between the XPS measured for "click" and "no-click" events, we measured peak XPS ϕ_+ of $18 \pm 4\mu$rad. In this experiment, where we use the WVA technique, we used a total of around 830 million trials (200 million successful post-selections) to extract an average XPS ϕ_+ of $10.0\pm0.6\mu$rad (for more information regarding the reported average XPS see the Probe phase measurement section in the supplementary material). Note that this number it agrees well with our classical calibration of the peak XPS of $13.0\pm1.5\mu$rad (*31*). It is evident that the WVA

Watching a particle in a region it's "forbidden" to be in



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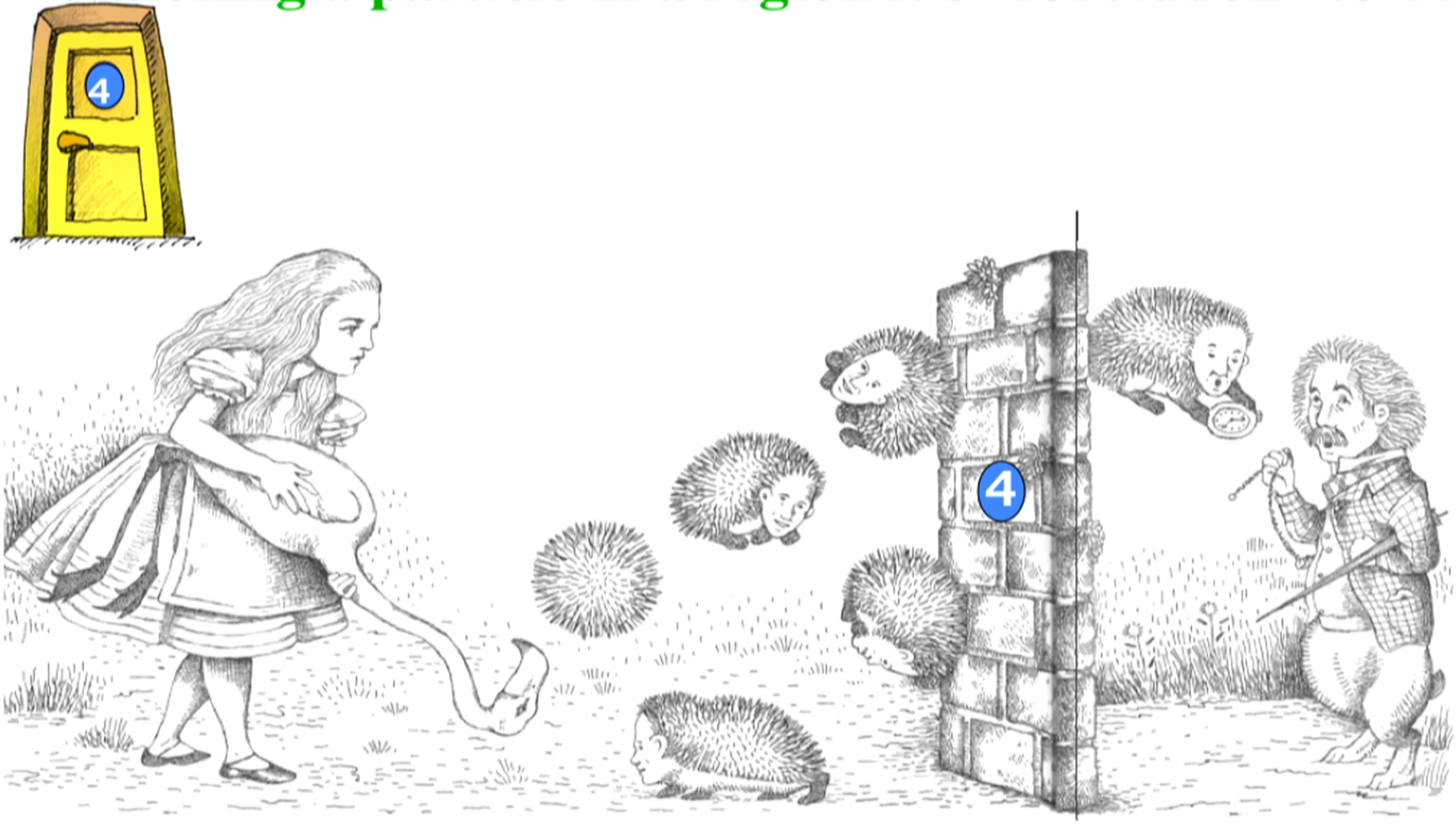
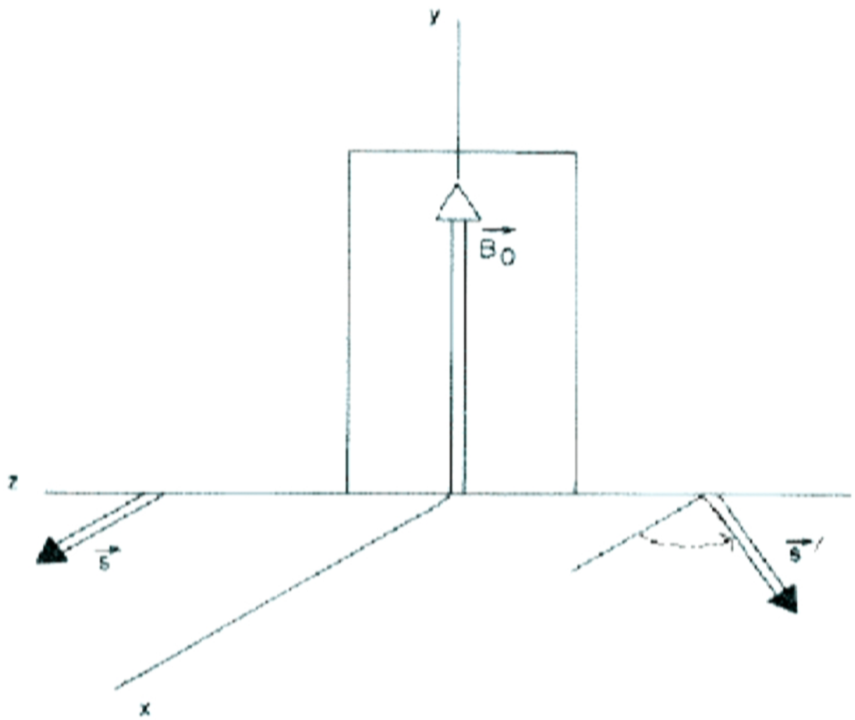


How long has the transmitted particle spent in the region?
Need a clock...

"Larmor Clock"
(Baz'; Rybachenko; Büttiker 1983)



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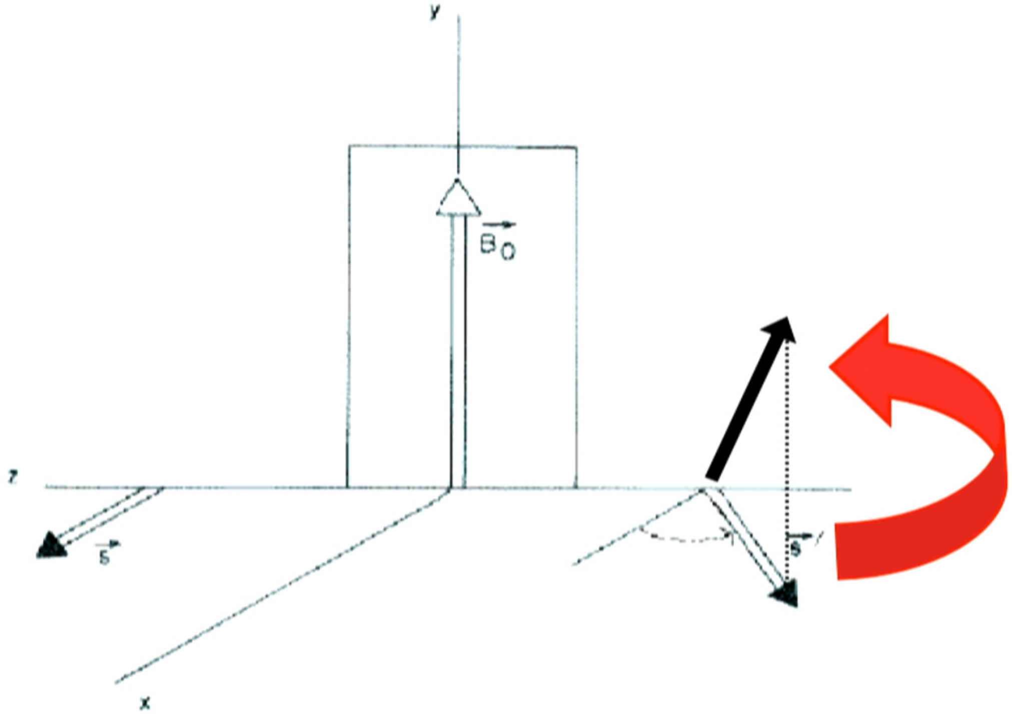
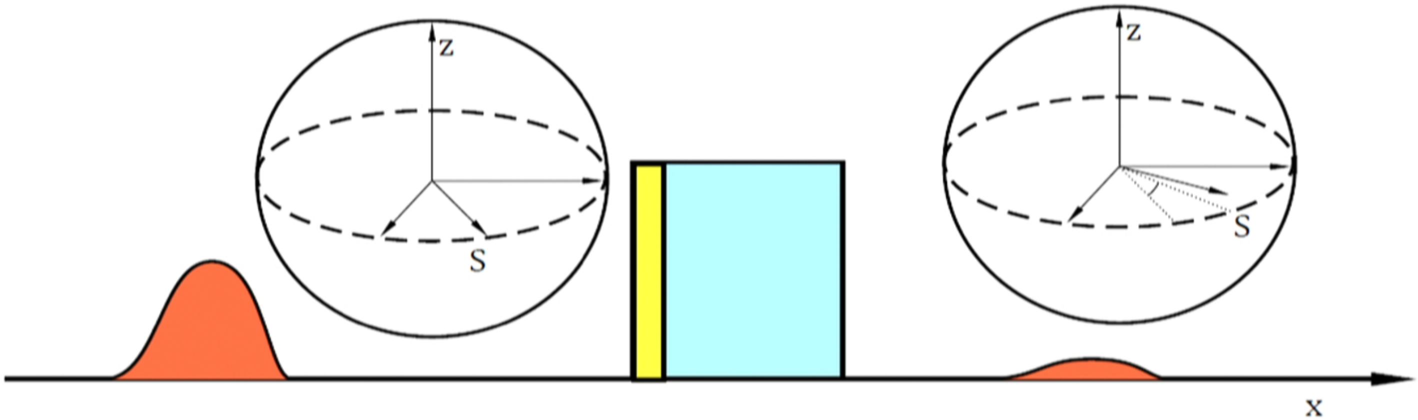


Local "Larmor Clock" – how much time spent in any given region?



- $\tau = \theta_{\text{rot}}/\omega_l$
  - In plane rotation measures the tunneling time
  - Spin aligns along z axis; back-action of the measurement.

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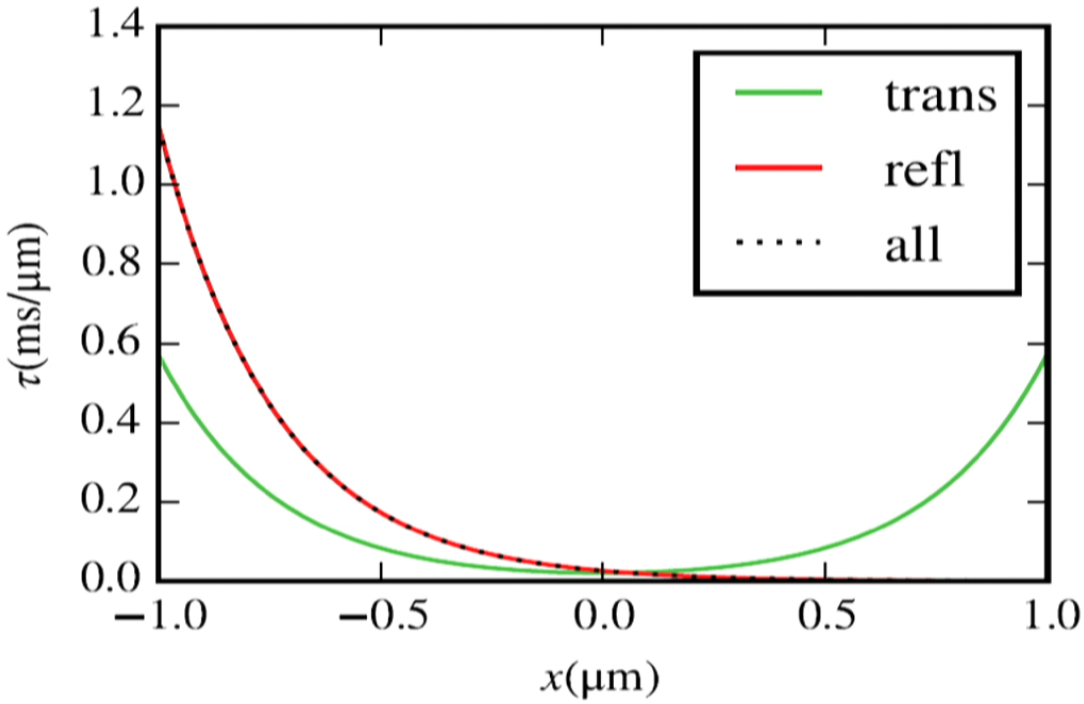
Where does a particle spend time inside the barrier?



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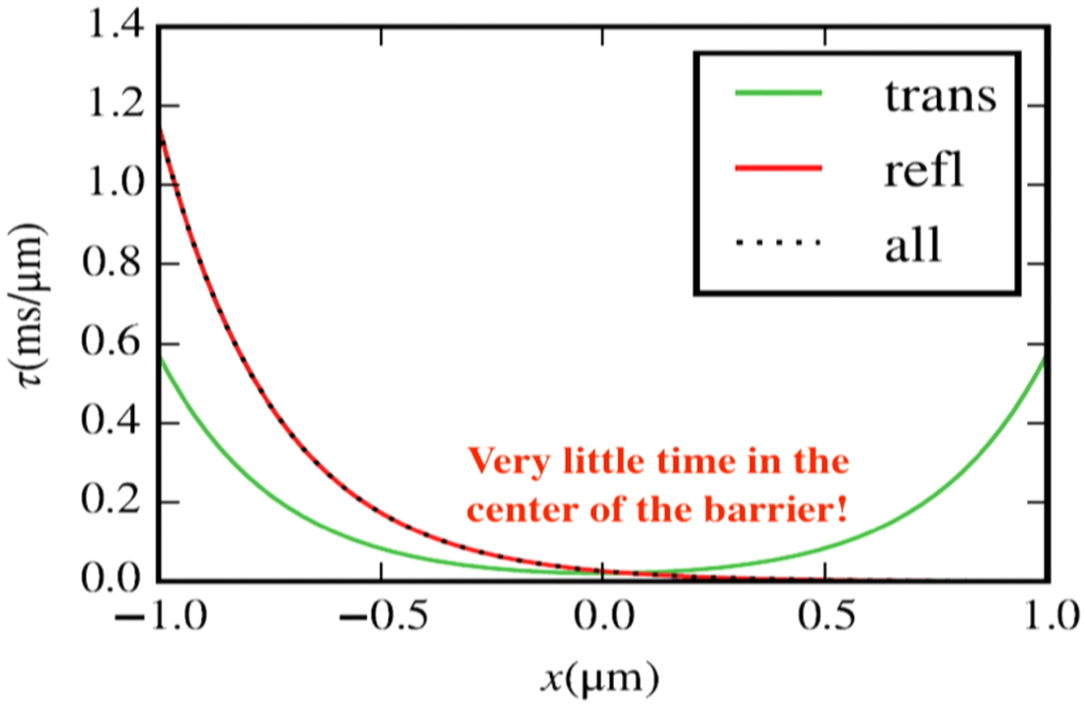

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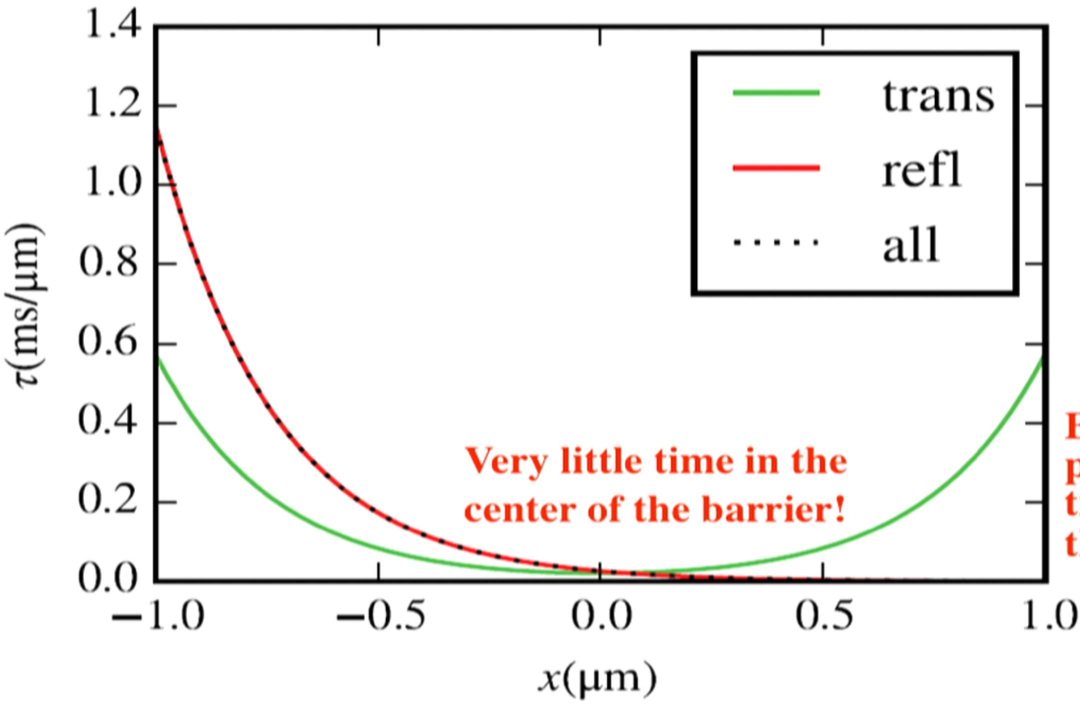
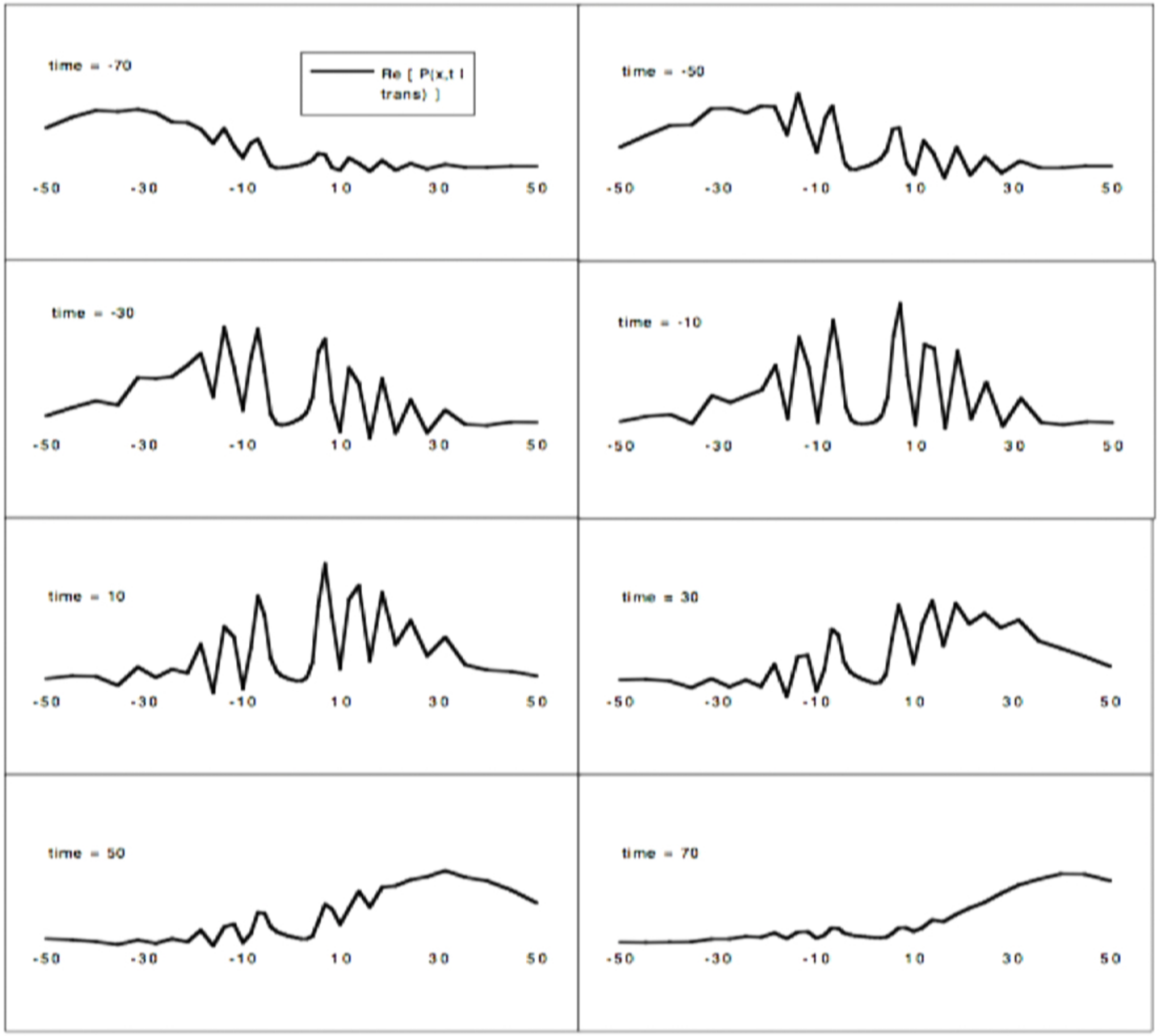


Very little time in the center of the barrier!

But – unlike the reflected particles – the transmitted ones "see" the region near the exit!

AMS, *Phys. Rev. Lett.*, *74*(13), 2405–2409, *Phys. Rev. A*, *52*(1), 32–42.

Conditional-probability "movie" of tunneling



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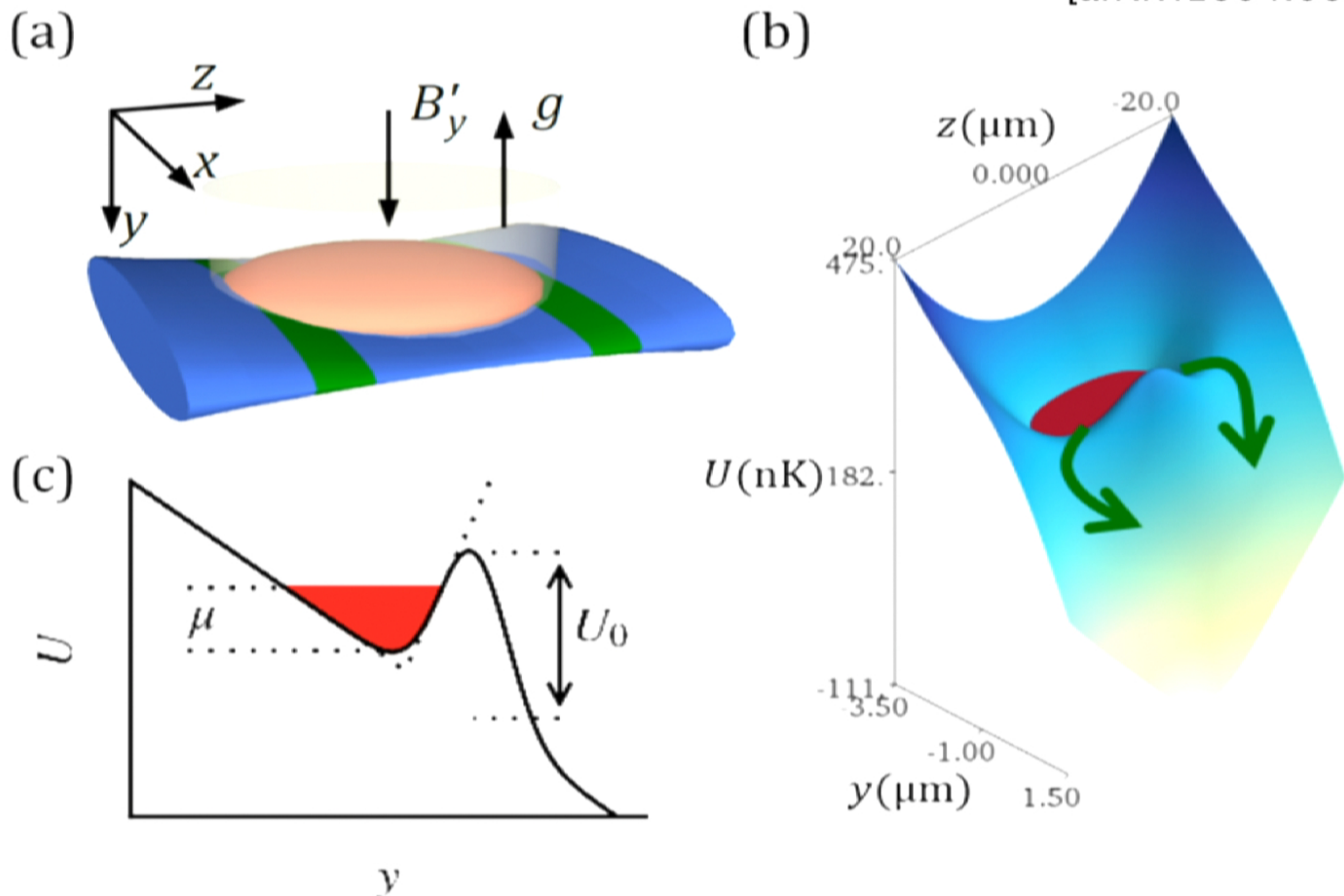
Our latest tunneling geometry

[arXiv:1604.06388]



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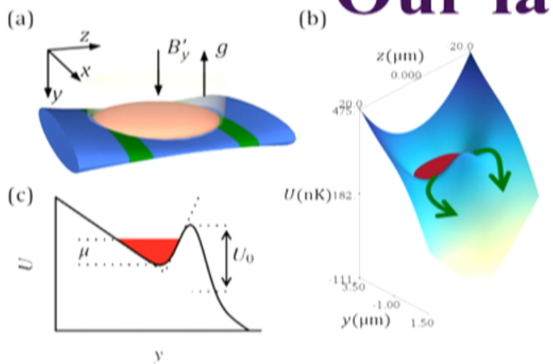

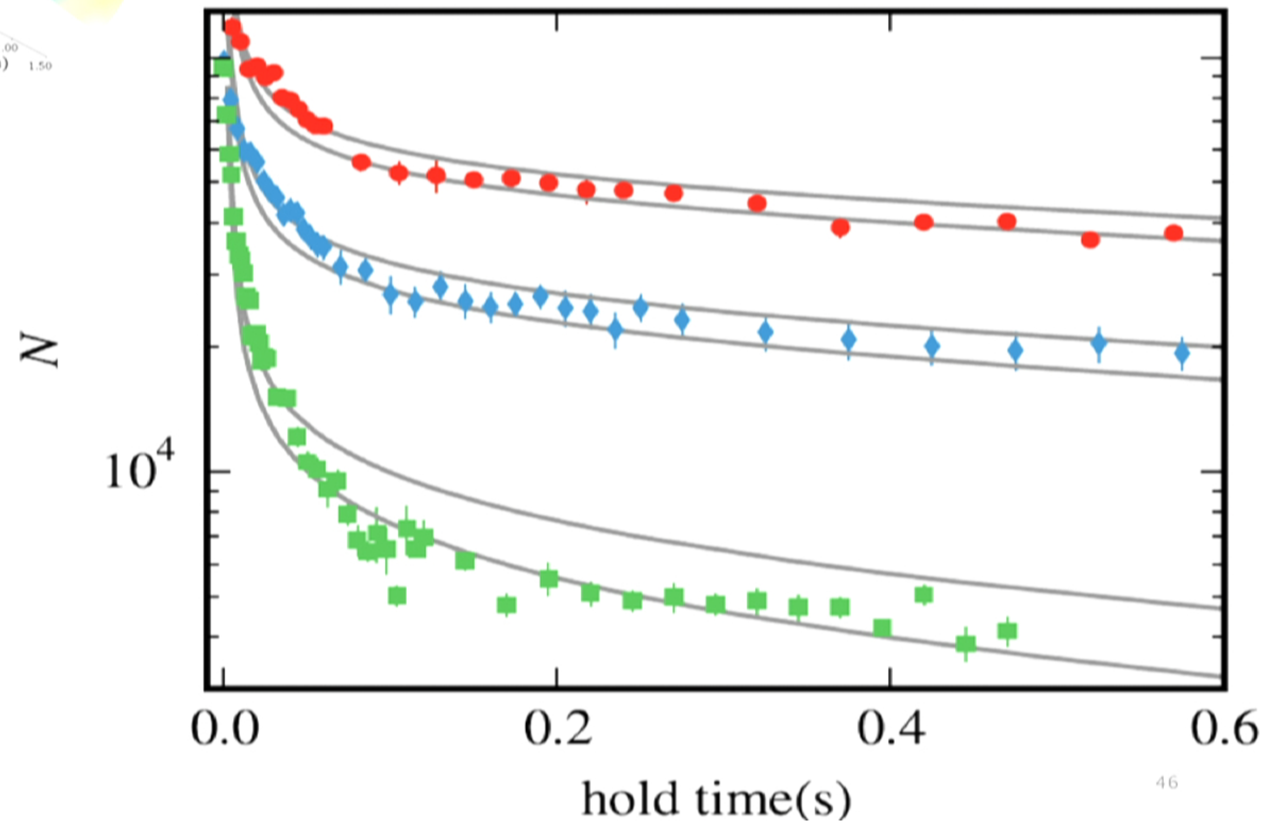

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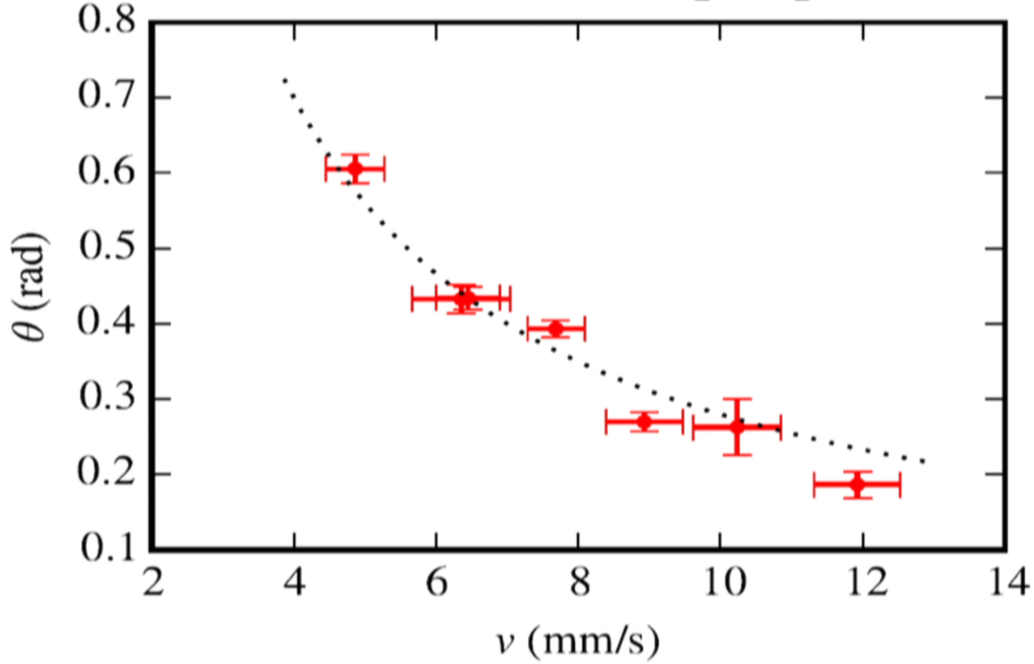
Calibration of Larmor clock for free propagation

$$\tau[\text{ms}] = 1.9(2) \times \theta[\text{rad}]$$



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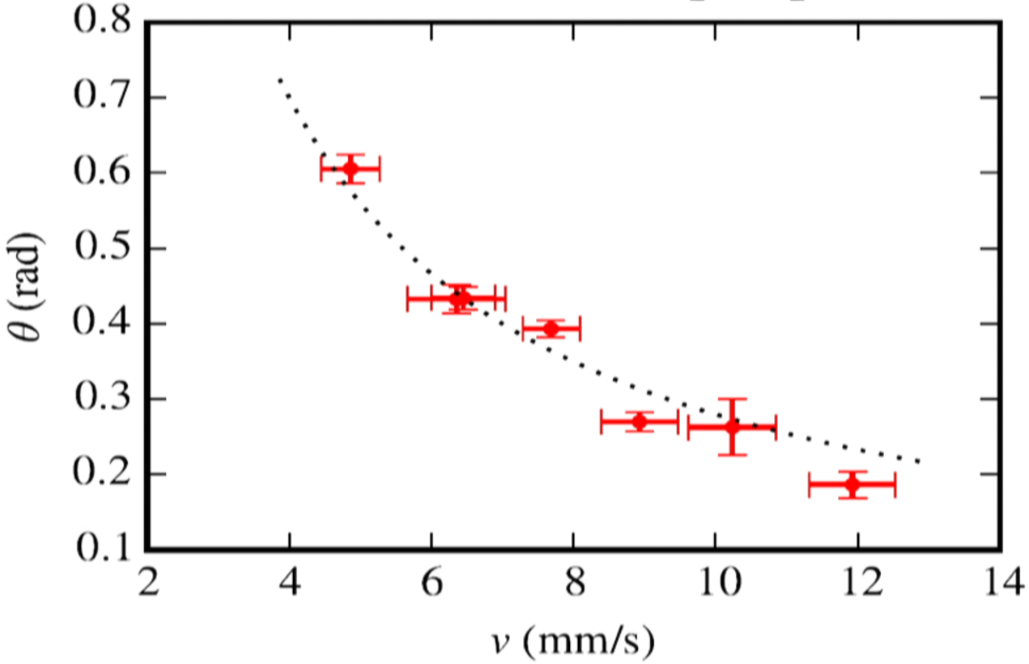

(A [very low-precision] confirmation that : t = L / v !)

Summary

If there is some deeper reality underlying QM,
what is revealed (to us naïve experimentalists)
by weak measurements should presumably tell us something about it.

- We were able to generate a "big" (10^{-5} rad) per-photon nonlinear phase shift, and measure it – and confirm that properly post-selected photons may have an amplified effect on the probe, as per the weak value.

- After talking about it for 20 years, we are getting close to being able to probe atoms while they tunnel through an optical barrier, using weak measurement to ask "where they were" before being transmitted