

Title: Integrable systems and vacua of $N=2^*$ theories

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URL: <http://pirsa.org/16060031>

Abstract:

GAUGE THEORY w/ 8 SUPERCHARGES

ON $\mathbb{R}^2 \times T^{d-2}$

$d=2,3,4$

MASS DEFORM

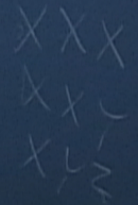
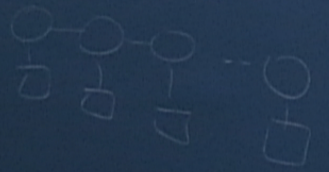
($N=2^*$ BREAKS
& OF SUPERCHARGES)

LOOK AT VACUA / TWISTED CHIRAL RING RELATION /

EQUILIBRIUM QUANTUM COHOMOLOGY RING
(\mathbb{R}^- / \mathbb{C}^-)



BETHE EQUATIONS FOR SOME INTEGRABLE SYSTEM



SPIN CHAIN

Gauge theory with 8 supercharges

on $\mathbb{R}^2 \times T^{d-2}$

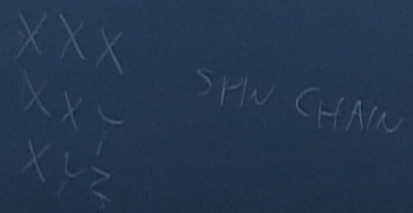
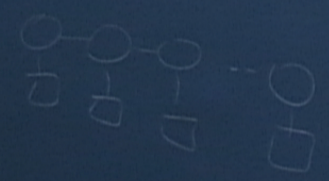
$d=2,3,4$
Mass deform
($N=2^*$ breaks
4 of supercharges)

Look at vacua / Twisted chiral ring relation /

Equilibrium quantum cohomology ring
(K - E rr-)

Examples of integrable system

Bethe equations for some integrable system



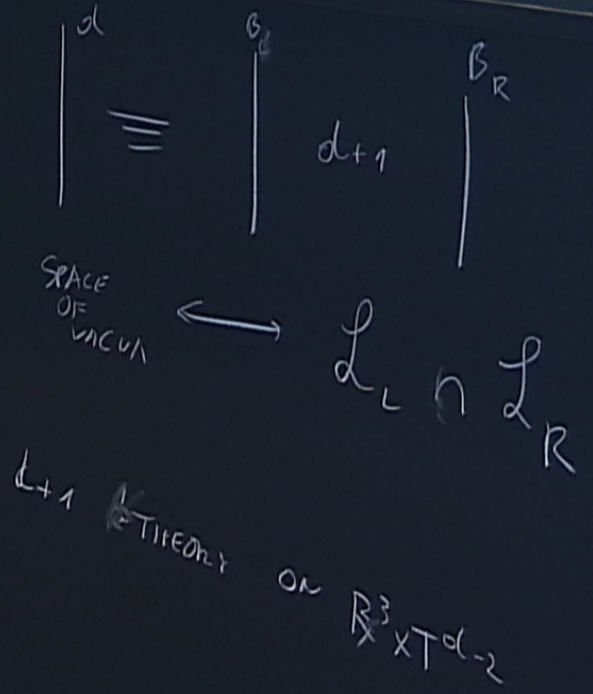
Spin chain

d

$d+1$ $SU(N)$ SYM (16 SUPERCHARGES)

VACUA OF d -DIMENSIONAL THEORY \longleftrightarrow

$\mathcal{L} \subset$ COULOMB BRANCH OF $d=3$ ERS



$\mathcal{L}_e \cap \mathcal{L}_e^v$ IN $M(SU(L))$

2d CP^1 σ -MODEL \leftrightarrow $U(1)$ GAUGE THEORY
+ 2 CHIRAL S

$$W(s, m) = (s+m) \ln(s+m) + (s-m) \ln(s-m) - t s$$

$$e^{\frac{\partial W}{\partial s}} = 1 \Rightarrow (s+m)(s-m) = e^t$$

$$s^2 = m^2 + e^t$$

$$W(s_+, m, t) = W^*(s_-, m, t)$$

$$\mathcal{L} \subset \mathbb{C} \times \mathbb{C}^*$$

$$p_m = e^{\frac{\partial W}{\partial m}} = \frac{s+m}{s-m}$$

$$\frac{p_m^{\frac{1}{2}} - p_m^{-\frac{1}{2}}}{2m} = e^{-\frac{t}{2}}$$

$$M_{1p} = e^{-\frac{t}{2}}$$

LAGRANGIAN IN C.B. OF PURE $SU(2)$

	$U(1)_{\text{GAUGE}}$	$U(1) = SU(2)$
X_1	1	1
X_2	1	-1

$$\frac{p_m^{\frac{1}{2}} - p_m^{-\frac{1}{2}}}{2m} = e^{-\frac{t}{2}}$$

$$p_m = e^{\frac{\partial \mathcal{L}}{\partial m}} = \frac{s+m}{s-m}$$

$$M_{10} = e^{-\frac{t}{2}}$$

$$p_t = \frac{\partial \mathcal{L}}{\partial t} = s$$

LAGRANGIAN IN C.B. OF PURE SU(2)
GIVENTAL, TELEMAN

DIFFERENCE EQN. FOR J-FUNCT

$T^*CP^1 \longleftrightarrow$

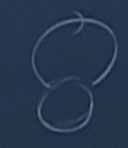
$$e^{\frac{\partial \mathcal{L}}{\partial s}} = 1 \Rightarrow [(s+\epsilon)^2 - m^2] = [(s-\epsilon)^2 - m^2] e^t$$

$$p_m = \frac{(s^2 - (\epsilon+m)^2)}{s^2 - (m-\epsilon)^2}$$

	$U(1)_y$	$U(1)_m \in SU(2)$
X_1	1	1
X_2	1	1
Y_1	-1	-1
Y_2	-1	-1
Z	-1	1
Φ	0	0

$U(1)_\epsilon$
1
1
1
1
1
-2

$$W^{\text{NOT TWISTED}} = \phi X - Y$$



$$\frac{(m-\epsilon) p_m^{\frac{1}{2}} + (m+\epsilon) p_m^{-\frac{1}{2}}}{2m} = e^{\frac{t}{2}} e^{-\frac{t}{2}}$$

LAGRANGIAN IN C.B. OF 3d SU(2) + AdJ
OF MASS \in

$$2m = e^{-\frac{t}{2}}$$

$$m_{10} = e^{-\frac{t}{2}}$$

$$p_t = \frac{\partial U}{\partial t} = 5$$

LAGRANGIAN IN C.B. OF PURE SU(2)
GIVENTAL, TELEMAN

3d
on $\mathbb{R}^2 \times S^1$

$$x \text{ has } \rightarrow$$

$$\partial_x(xh_2) = h_2x$$

$$l(x)$$

$$\downarrow$$

$$\partial_x l = h_2 \text{sinh } x$$

T^*CP^1

$$(\sigma \eta - \mu)(\sigma \eta - \mu^{-1}) = \tau (\eta \mu - \sigma)(\eta \mu^{-1} - \sigma)$$

- $\sigma : e^s$
- $\eta : e^{\sigma}$
- $\mu : e^{\eta}$
- $\tau : e^t$

$$p_\mu = e \frac{\partial U}{\partial \mu}$$

$$p_\tau = e \frac{\partial U}{\partial \tau}$$

$$\frac{\eta \tau - \eta^{-1}}{\tau - 1} p_\tau^{\frac{1}{2}} + \frac{\eta - \tau \eta^{-1}}{1 - \tau} p_\tau^{-\frac{1}{2}} = \mu + \mu^{-1}$$

$$\frac{\eta^{-1} \mu - \eta}{\mu - 1} p_\mu^{\frac{1}{2}} + \frac{\eta^{-1} - \mu \eta}{1 - \mu} p_\mu^{-\frac{1}{2}} = \tau + \tau^{-1}$$

$$2m = e^{-\frac{t}{2}}$$

$$M_{10} = e^{-\frac{t}{2}}$$

$$P_t = \frac{\partial L}{\partial t} = S$$

LAGRANGIAN IN CB. OF PURE SU(2)
GIVENTAL, TELEMAN

3d
on $\mathbb{R}^2 \times S^1$

$$x \in \mathbb{R}^2 \rightarrow l(x)$$

$$\partial_x(x \cdot h_x) = h_x$$

$$\partial_x l = h_x \text{ on } x$$

T^*CP^1

$$(\sigma \eta - \mu)(\sigma \eta - \mu^{-1}) = \tau (\eta \mu - \sigma)(\eta \mu^{-1} - \sigma)$$

- $\sigma : e^s$
- $\eta : e^{\mu s}$
- $\mu : e^{\mu s}$
- $\tau : e^t$

$$P_\mu = e^{\frac{\partial U}{\partial \mu}} = \sigma = \frac{\eta \mu - \sigma}{\eta \sigma - \mu^{-1}}$$

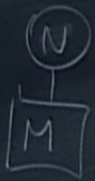
$$P_\tau = e^{\frac{\partial U}{\partial t}} = \sigma$$

$$T_2 U = T_2 U'$$

$$\frac{\eta \tau - \eta^{-1}}{\tau - 1} P_\tau^{\frac{1}{2}} + \frac{\eta - \tau \eta^{-1}}{1 - \tau} P_\tau^{-\frac{1}{2}} = \mu + \mu^{-1}$$

$$\frac{\eta^{-1} \mu - \eta}{\mu - 1} P_\mu^{\frac{1}{2}} + \frac{\eta^{-1} - \mu \eta}{1 - \mu} P_\mu^{-\frac{1}{2}} = \tau + \tau^{-1}$$

$SU(2)_- \quad | \quad SU(2)_+$
 $UVU^{-1}V^{-1} = E$
 $LOC_{SU(2)}(\odot)$



Φ - ADJOINT OF $U(N)$

$X = C^M \times C^M$

Y

$$\prod_{l=1}^M \frac{s_i - s_j - 2\epsilon}{s_j - s_i - 2\epsilon} = \prod_{a=1}^M \frac{s_i - m_a + \epsilon}{m_a - s_i + \epsilon} = e^t$$



BETHE EQNS

WITH

$$C_{m_1}^2 \times C_{m_2}^2 \times \dots \times C_{m_M}^2$$

s_i - RAPIDITIES

m_a - DEGREE

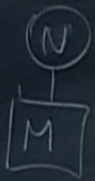
PARAMETERS

SECTOR WITH N

SPINS UP

$M-N$ DOWN

$SU(2)$ XXX SPIN CHAIN



Φ - ADJOINT OF $U(N)$

$X \in \mathbb{C}^M \times \mathbb{C}^M$

Y

$l=1 \dots N$

$$\prod_{j=1}^M \frac{s_i - s_j - 2\epsilon}{s_j - s_i + 2\epsilon} = \prod_{a=1}^M \frac{s_i - m_a + \epsilon}{m_a - s_i + \epsilon} = e^t$$

$$Q(z) = \prod (z - s_i)$$

$$T(z)Q(z) = Q(z - 2\epsilon) \prod (z - m_a + \epsilon) - e^t Q(z + 2\epsilon) \prod (z - m_a - \epsilon)$$



BETHE EQNS

WITH

$$\mathbb{C}_{m_1}^2 \times \mathbb{C}_{m_2}^2 \times \dots \times \mathbb{C}_{m_M}^2$$

s_i - RAPIDITIES

m_a - DEFECTS

PARAMETERS

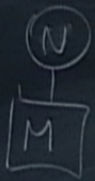
SECTOR

WITH N

SPINS UP

$M-N$ DOWN

$SU(2)$ XXX SPIN CHAIN



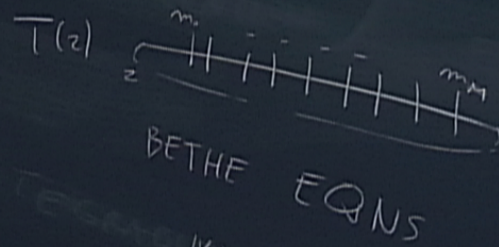
Φ - ADJOINT OF $U(N)$

$X = C^M \times C^M$

Y

$l=1 \dots N$

$$\prod_{j=1}^M \frac{s_i - s_j - 2\epsilon}{s_j - s_i + 2\epsilon} \prod_{a=1}^M \frac{s_i - m_a + \epsilon}{m_a - s_i + \epsilon} = e^t$$



BETHE EQNS

$Q(z) = \prod_{i=1}^N (z - s_i)$

$T(z)Q(z) = Q(z-2\epsilon) \prod_{i=1}^M \pi(z - m_i + \epsilon) - e^t Q(z+2\epsilon) \prod_{i=1}^M \pi(z - m_i - \epsilon)$

$\tilde{Q}(z) = \prod_{i=1}^{M-N} (z - \tilde{s}_i)$

$Q(z - \frac{\epsilon}{2}) \tilde{Q}(z + \epsilon) - Q(z + \frac{\epsilon}{2}) \tilde{Q}(z - \epsilon) = \prod (z - m_n)$

$T(z) = Q(z + \epsilon) \tilde{Q}(z + \epsilon) - Q(z - \epsilon) \tilde{Q}(z - \epsilon)$

SECTOR

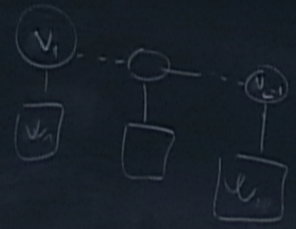
S_i - RAPIDITIES
 m_n - DEFECTS

PARAMETERS
 N SPINS UP

$SU(2)$ XXX SPIN CHAIN

$M-N$ POWERS

$T(z) = Q(z+\epsilon)\tilde{Q}(z+\epsilon) - Q(z-\epsilon)\tilde{Q}(z-\epsilon) = \prod (z-m_n)$
 SECTOR WITH N SPINS UP M-N POWERS



$$\prod_{j=1}^{v_a} \frac{s_i^{(a)} - s_j^{(a)} - 2\epsilon}{\dots}$$

$$\prod_{j=1}^{w_a} \frac{s_i^{(a)} - m_j^{(a)} + \epsilon}{\dots}$$

$$\prod_{j=1}^{v_{a+1}} \frac{s_i^{(a)} - s_j^{(a+1)} + \epsilon}{\dots}$$

$$\prod_{j=1}^{v_{a+1}} \frac{s_i^{(a)} - s_j^{(a+1)} + \epsilon}{\dots} = e^{t_n}$$

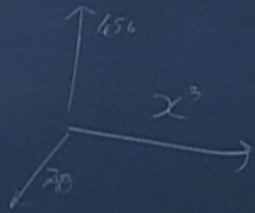
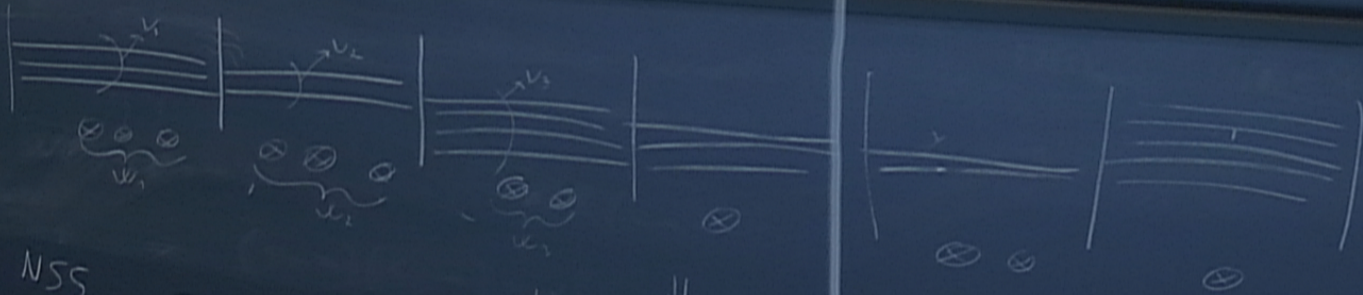
BETHE FOR SL(L)

- $\psi_1 \in \mathbb{C}^L$
- $\psi_2 \in \mathbb{C}^{L^2}$
- $\psi_3 \in \mathbb{C}^{L^3}$
- \vdots
- $\psi_{L-1} \in \mathbb{C}^{L^{L-1}}$

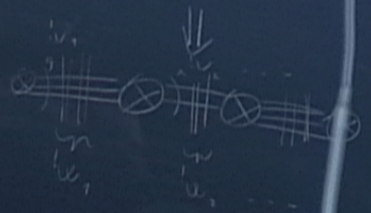
XXX SPIN CHAIN

$$\frac{(m-c)p_m^{\frac{1}{2}} + (m+c)p_m^{-\frac{1}{2}}}{2m} = e^{\frac{t}{2m}} e^{-\frac{t}{2m}}$$

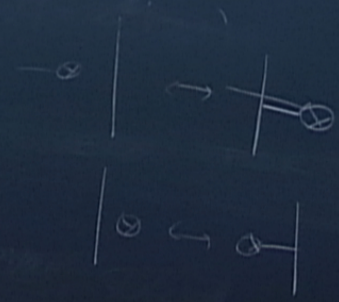
LAGRANGIAN IN C.B. OF 3L SU(2) + Adj
OF MASS ϵ



NSS
 ⊗ D5 012 456
 — D3 012 3 789



HAMMUR WITTEN



$$(v, u) \leftrightarrow (l, l')$$

$$(v, w) \leftrightarrow (e)$$